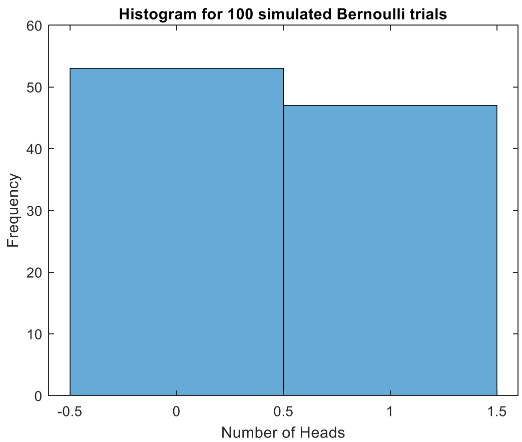
EE 511 Simulation Methods for Stochastic System
Project #1

[A Few Coins]

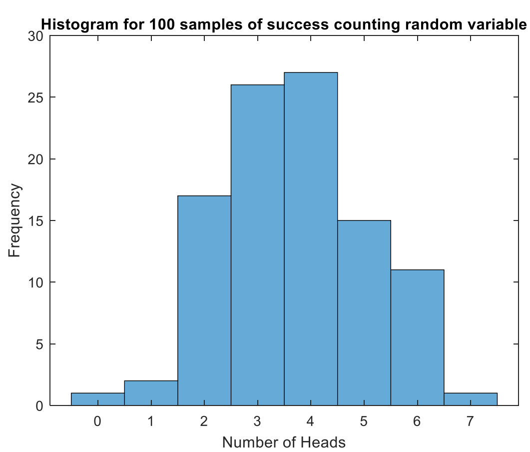
1.1 Routine to simulate a fair Bernoulli trial

- Generate a random number from uniform distribution U [0,1]
- If generated random number is less than 0.5, assign random variable X=0 indicating tail, else X=1 indicating head in a Bernoulli trial
- Histogram for 100 simulated bernoulli trial is plotted.
- Histogram has Bernoulli distribution



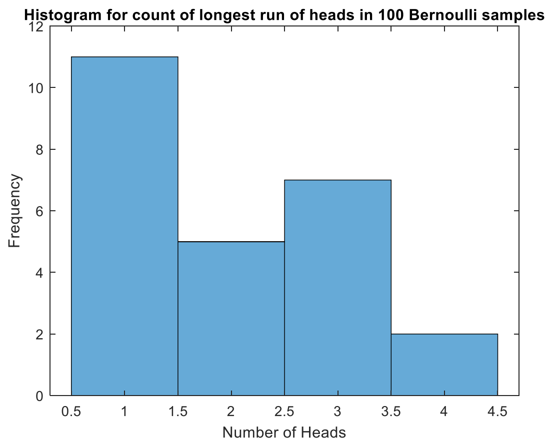
1.2 Routine to count the number of successes (*success-counting* random variable) in 7 fair Bernoulli trials

- Generate a random number from uniform distribution U [0,1]
- If generated random number is less than 0.5, assign random variable X=0 indicating tail, else X=1 indicating head in a Bernoulli trial
- The above stated procedure is repeated for 7 times and the total number of heads (X=1) in the 7 trials are calculated.
- Histogram for 100 samples is plotted
- Histogram has Binomial distribution



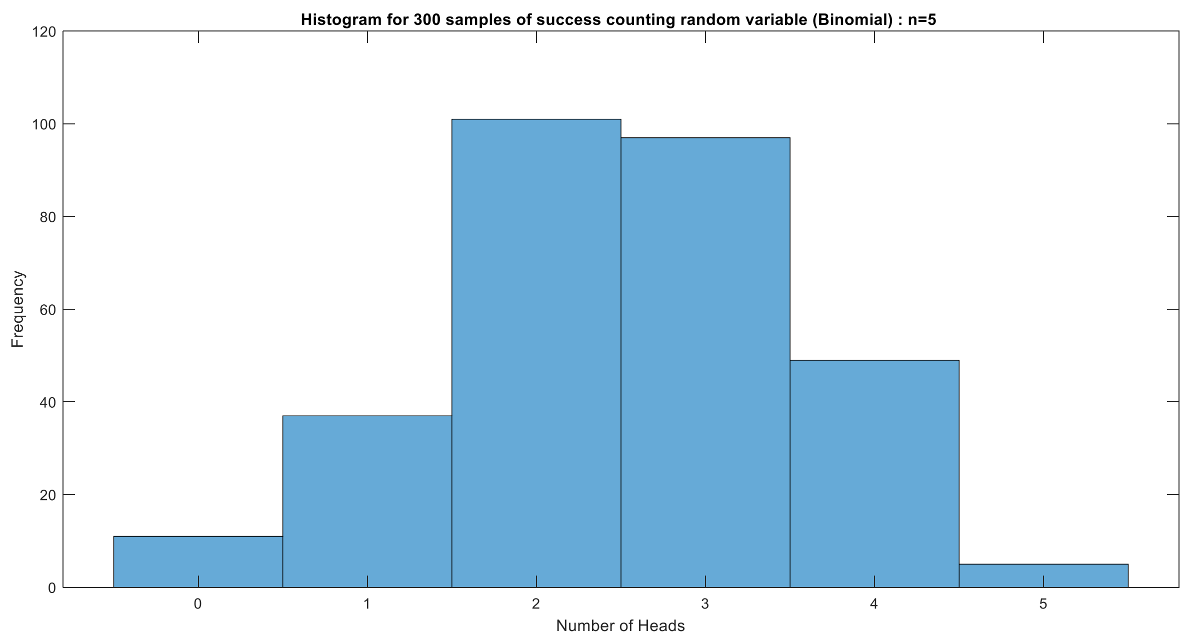
1.3 Routine to count the longest run of heads in 100 Bernoulli samples

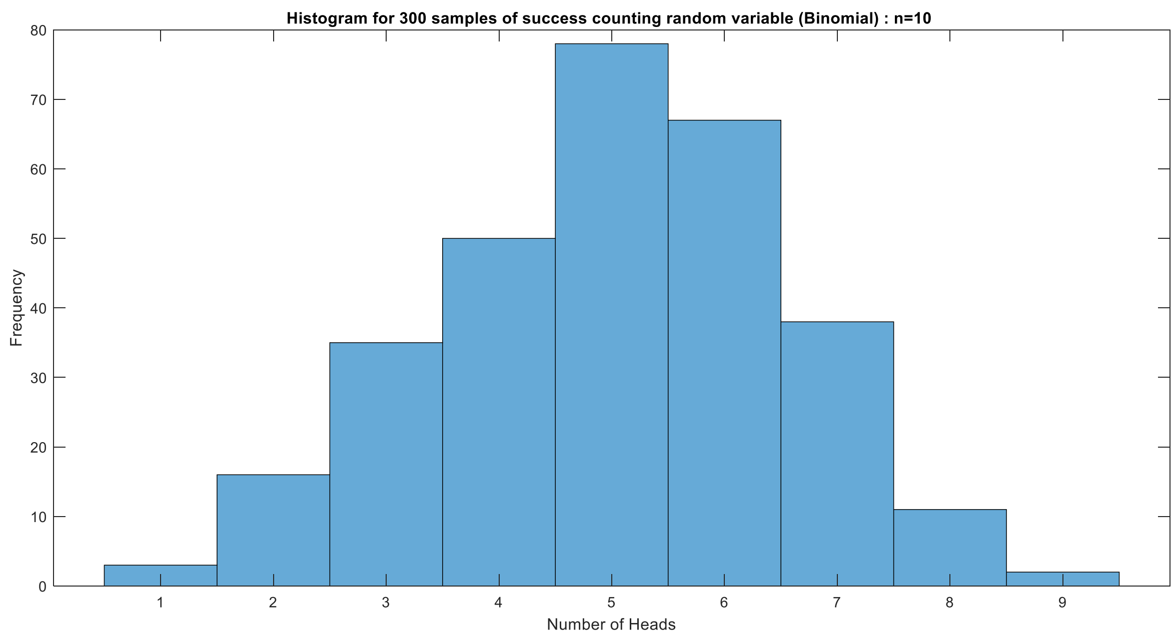
- Generate a random number from uniform distribution U [0,1]
- If generated random number is less than 0.5, assign random variable X=0 indicating tail, else X=1 indicating head in a Bernoulli trial
- Initialize a counter equal to 0. Increment the counter only if X=1 for each trial and set it to 0 if tail (X=0) occurs
- Histogram has Poisson distribution used modelling counts or events that occur randomly over a fixed period of time

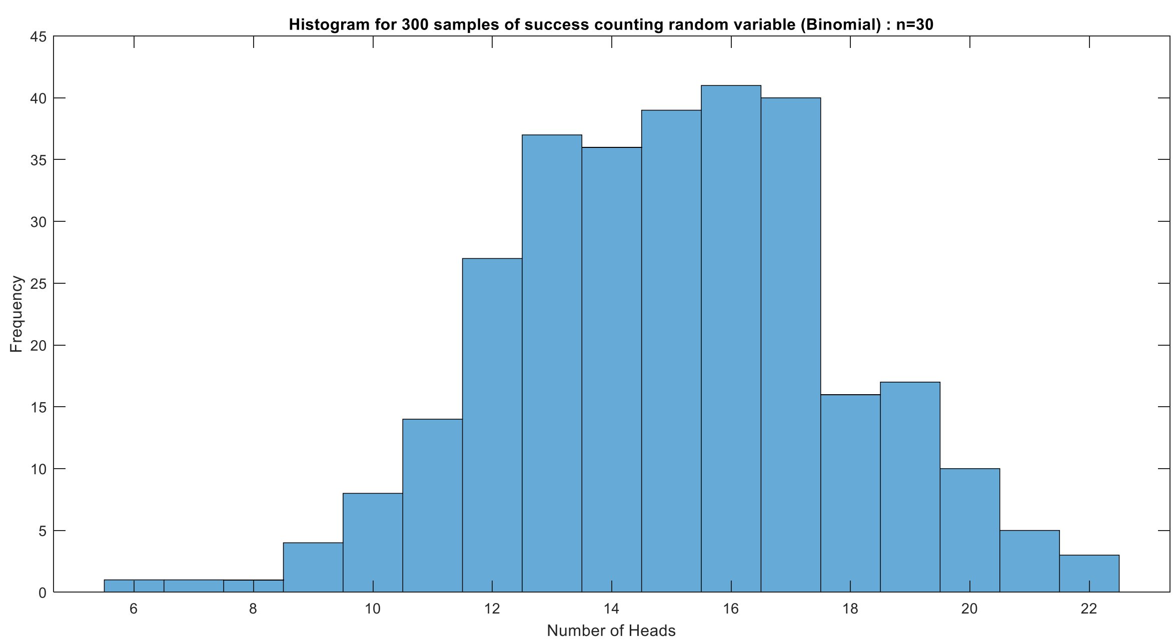


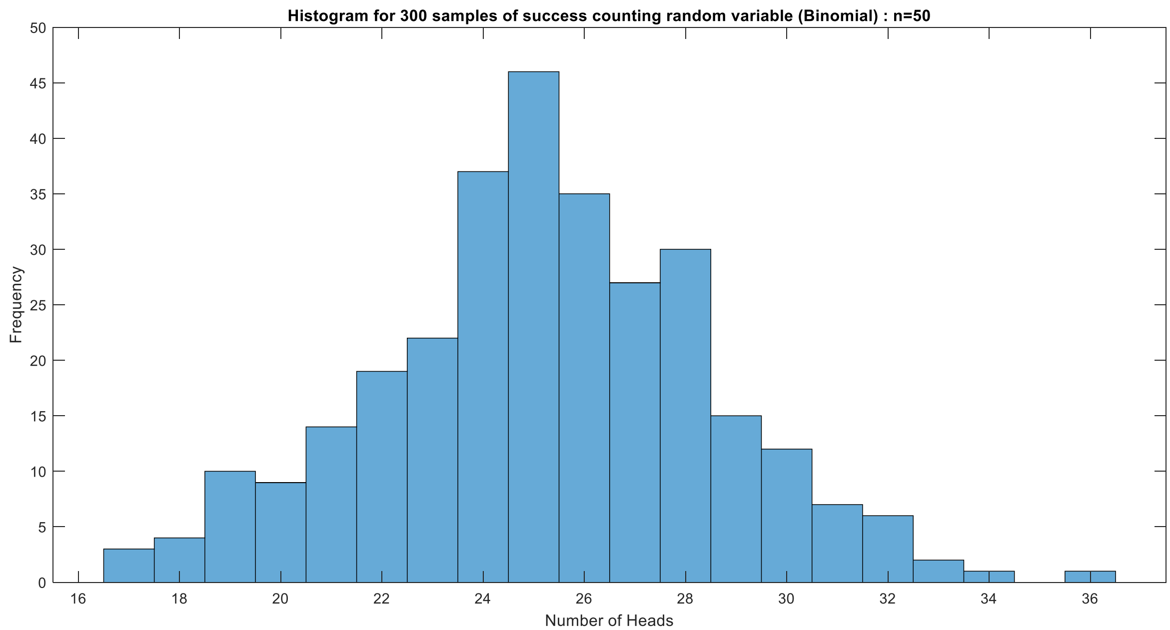
[Counting Successes]

2.




Histogram for 300 samples of success counting random variable (Binomial) : n=10
Frequency
Number of Heads
0
10
20
30
40
50
60
70
80
1
2
3
4
5
6
7
8
9


Histogram for 300 samples of success counting random variable (Binomial) : n=30
Frequency
Number of Heads
0
5
10
15
20
25
30
35
40
45
6
8
10
12
14
16
18
20
22



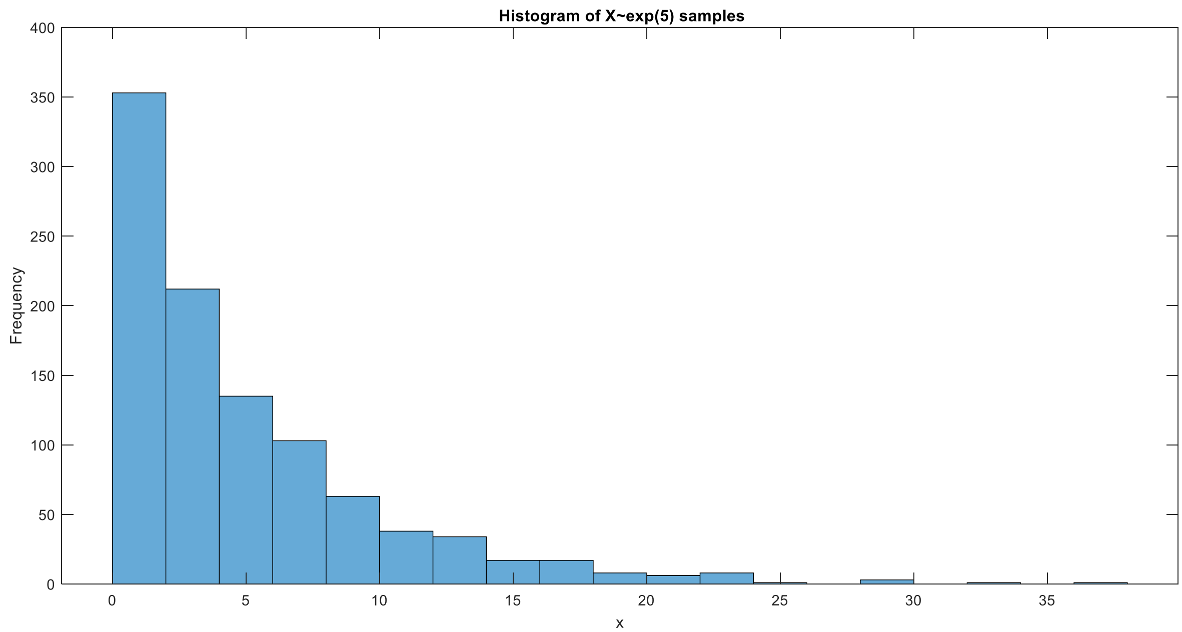
Comment on the histograms for the different values of n:

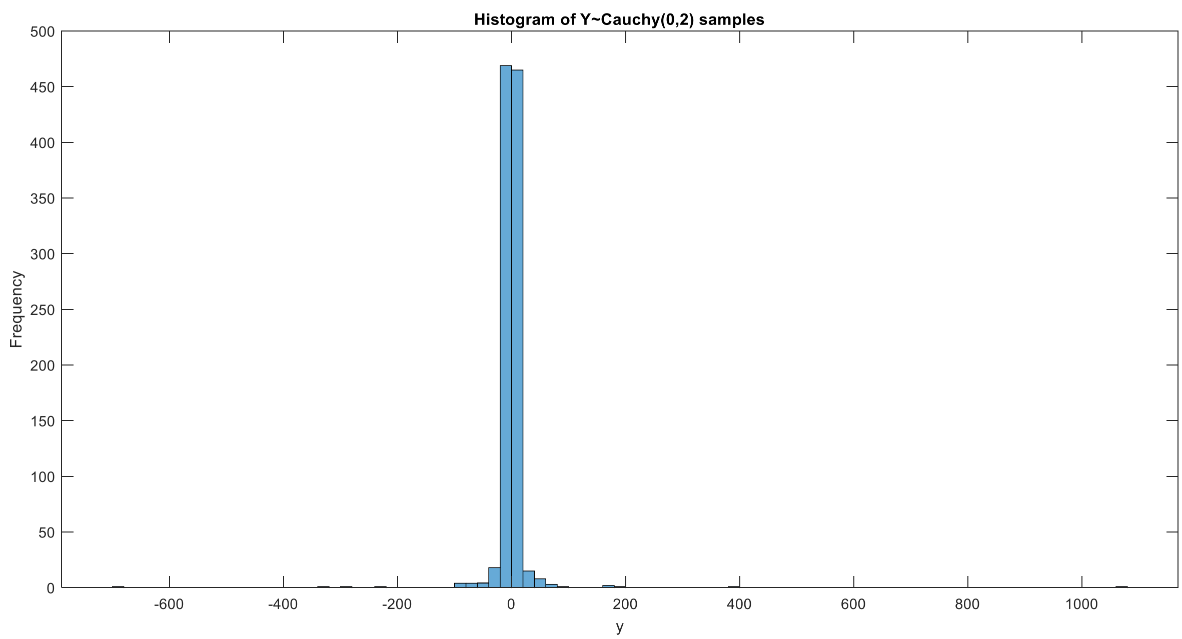
- n=5 trials has mean μ ~ 2.5 = 5*0.5
- n=10 trials has mean μ ~ 5 = 10*0.5
- n=30 trials has mean μ ~ 15 = 30*0.5
- n=50 trials has mean μ ~ 25 = 50*0.5
- As n increases, the distribution approximates more closer to Gaussian distribution with mean μ=n*p, where n is the number of Bernoulli trials and p is probability of getting heads in a Bernoulli trial.
- The range or the standard deviation of the distribution increases as n increases.

[Continuous Distributions]

3. Inverse CDF method to generate 1000 samples of the X~exp(5) and Y~Cauchy(0,2)

- For any continuous distribution function F, the random variable X is evaluated using U uniform random variable X = F^{-1}(U)
- To generate samples from X~exp(5) such that x = F^{-1}(U) ,
  x= -5 * log (1 – u)
- To generate samples from Y~Cauchy(0,2) such that y = F^{-1}(U) ,
  y= 2*tan(pi*(u-0.5))





Pearson's chi-squared test, Kolmogorov–Smirnov test, Deviance statistic G^2 (the likelihood-ratio test statistic) are few methods for testing goodness-of-fit.

Code:

```
%Question 1

for i=1:100
   p=rand;
   if(p<0.5)
       X1(1,i)=0;
   else
       X1(1,i)=1;
   end
end
figure(1)
h=histogram(X1);
title(' Histogram for 100 simulated Bernoulli trials ');
ylabel('Frequency');
xlabel('Number of Heads');

for i=1:100
    count=0;
    for j=1:7
      p=rand;
      if(p<0.5)
       X2(1,j)=0;
      else
       X2(1,j)=1;
       count=count+1;
      end
    end
    success_count1(1,i)=count;
end
figure(2)
h=histogram(success_count1);
title(' Histogram for 100 samples of success counting random variable ');
ylabel('Frequency');
xlabel('Number of Heads');

count=0;
m=1;
for i=1:100
   p=rand;
   if(p<0.5)
       X3(1,i)=0;
       if(count~=0)
       m=m+1;
       end
       count=0;
   else
       X3(1,i)=1;
       count=count+1;
       longest_run_count(1,m)=count;
   end
```

```matlab
end
figure(3)
h=histogram(longest_run_count);
title(' Histogram for count of longest run of heads in 100 Bernoulli samples ');
ylabel('Count of Longest run of heads ');
xlabel('Number of Heads');

%Question 2

for i=1:300
    count=0;
    for j=1:5
      p=rand;
      if(p<0.5)
       X4(1,j)=0;
      else
       X4(1,j)=1;
       count=count+1;
      end
    end
    success_count2(1,i)=count;
end
figure(4)
h=histogram(success_count2);
title('Histogram for 300 samples of success counting random variable (Binomial) :
n=5 ');
ylabel('Frequency');
xlabel('Number of Heads');

for i=1:300
    count=0;
    for j=1:10
      p=rand;
      if(p<0.5)
       X5(1,j)=0;
      else
       X5(1,j)=1;
       count=count+1;
      end
    end
    success_count3(1,i)=count;
end
figure(5)
h=histogram(success_count3);
title('Histogram for 300 samples of success counting random variable (Binomial) :
n=10 ');
ylabel('Frequency');
xlabel('Number of Heads');

for i=1:300
    count=0;
    for j=1:30
      p=rand;
      if(p<0.5)
       X6(1,j)=0;
      else
       X6(1,j)=1;
```

```
       count=count+1;
      end
    end
    success_count4(1,i)=count;
end
figure(6)
h=histogram(success_count4);
title('Histogram for 300 samples of success counting random variable (Binomial) : n=30');
ylabel('Frequency');
xlabel('Number of Heads');

for i=1:300
    count=0;
    for j=1:50
      p=rand;
      if(p<0.5)
       X7(1,j)=0;
      else
       X7(1,j)=1;
       count=count+1;
      end
    end
    success_count5(1,i)=count;
end
figure(7)
h=histogram(success_count5);
title('Histogram for 300 samples of success counting random variable (Binomial) : n=50');
ylabel('Frequency');
xlabel('Number of Heads');


%Question 3

X=zeros(1,1000);
Y=zeros(1,1000);
for i=1:1000
    u=rand;
    X(i)=-5*log(1-u);
    Y(i)=2*tan(pi*(u-0.5));
end
figure(8);
h=histogram(X);
title(' Histogram of X~exp(5) samples');
ylabel('Frequency');
xlabel('x');

figure(9);
h=histogram(Y);
title(' Histogram of Y~Cauchy(0,2) samples ');
ylabel('Frequency');
xlabel('y');
```