

Reference

- Chapters 9, 10, 11 of Sutton and Barto
 - Provide general understanding of function approximation
- Practical advances over past decade
 - Research articles best source
- Chapters 6 and 7, Lapan
 - Focusses on problems needing high-computational resources (1-2 days to converge)
 - Review it for computational efficiency
 - Use of wrappers to simplify code writing
 - Use of PTAN libraries for simplifying functionality
 - For this class: I will instead use simple example (computationally doable)

Recollect Q-learning (also called off-policy temporal difference (TD) control)

- Set step size: $\alpha \in (0,1]$, small $\epsilon > 0$
- Set episode length; If there is absorbing state, episode ends if reaches terminal state or reaches episode length
- Initialize Q(s,a), $\forall (s,a)$ pairs; except $Q(terminal\ state,.) = 0$
- Loop for each episode
 - Initialize state, say s
 - Loop for each step of episode (until episode length or terminal state)
 - Choose action α given state s, using action selection method
 - Take action a and observe r, s'
 - Update $Q(s, a) = Q(s, a) + \alpha \left[r(s, a, s') + \lambda \max_{a'} Q(s', a') Q(s, a) \right]$
 - Set $s \leftarrow s'$
 - Update α

Q-Learning uses Q-table

	a=0	a=1	a=2
s=0	13774.6	13760.6	14085
s=1	14187.7	14183.6	14478.7
s=2	14558.7	14522.4	14824.2
s=3	14950.3	14886.4	15095.4
s=4	15307.8	15218.8	15229.7

Overview TabularRL v DeepRL

function approximation

Tabular methods Q-Learning

	a=0	a=1	a=2
s=0	13774.6	13760.6	14085
s=1	14187.7	14183.6	14478.7
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Q-Learning with function approximation

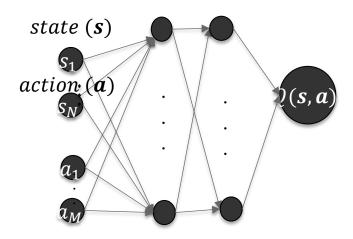
If analytical form is known, solve for coefficients (**w**) using Widrow-Hoff

$$\widetilde{Q}(s, A = a; \mathbf{w})$$

= $w_1 s^2 + w_2 s^3$

DeepRL

If analytical form is not known, use deep neural network (DNN) for function approximation of either state-value function (v), action value function (Q) or policy function (π)



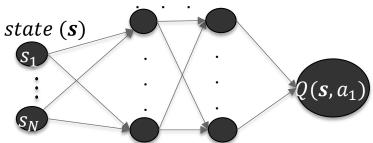
Q-learning with function approximation using W-H

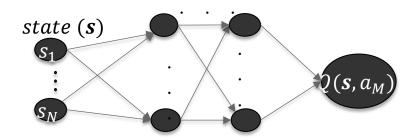
- Suppose we know functional form
 - $\widetilde{Q}(s, a = 1; \mathbf{w}) = w_1 s^2 + w_2 s^3$
 - Note: addition of w in Q-value to represent coefficient of the function approximation
 - Apply incremental Widrow-Hoff
 - Recollect W-H uses steepest descent (SD)
 - Objective (loss function E)
 - $\mathbf{E} = \mathbf{Min}_{\mathbf{w}} [\widetilde{\mathbf{Q}}(s, a = 1; \mathbf{w}) Q(s, a = 1)]^2$ for every $a \in A$
 - $-\widetilde{Q}(s, a = 1; w)$ estimated Q
 - -Q(s, a=1) actual Q
 - Bellman equation: $Q(s, a) = (1 \alpha)Q(s, a) + \alpha \left[r + \lambda \max_{a'} Q(s', a')\right]$
 - Main transformation: $\mathbf{w} \leftarrow \mathbf{w} \mu \frac{\partial E}{\partial \mathbf{w}}$

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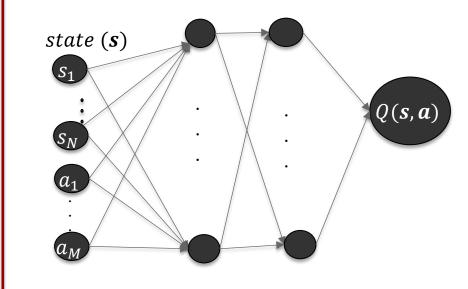
Deep RL- earlier architectures

One network for each action





Include action in input layer



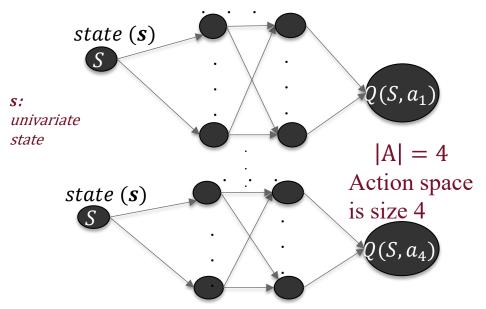
s: state vector

a: action vector

$$\mathbf{s} = [s_1, ... s_N]$$
 $\mathbf{a} = [a_1, ... a_M]$

$$\boldsymbol{a} = [a_1, \dots a_M]$$

Example of uni-variate state and action



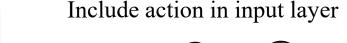
Suppose,

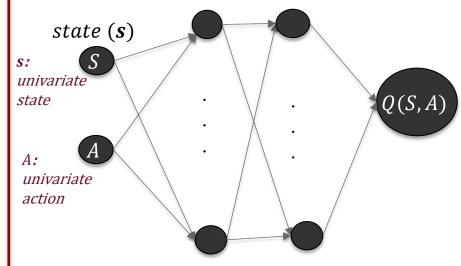
 X_t =proportion of people with active infection

 D_t =how often to test

State space: $S \in \mathbb{R}^1$; $S \in [0,1]$

Action space:A = {test once a week, twice a week, three times a week, daily}

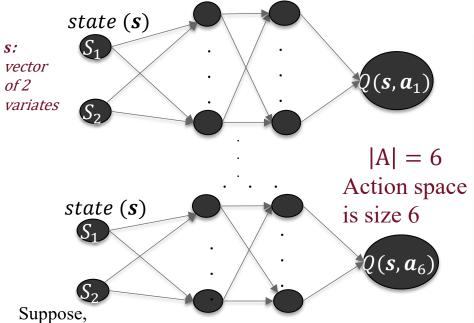




Action space: $A \in \mathbb{R}^1$; $A \in [1,30]$

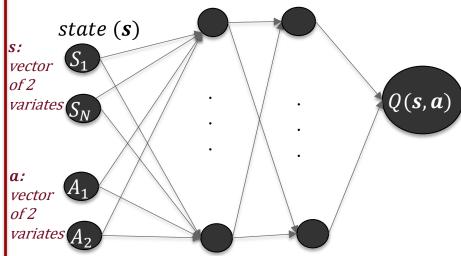
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Example of multi-variate state and action



 $X_t = [proportion with active infection, proportion recovered]$ $D_t = [\text{how often to test in a week, what \%lockdown}]$ State space: $S \in \mathbb{R}^2$; (vector of 2 real random variables) Action space: $A = \{[once, 25\%], [twice, 25\%], [thrice, 25\%], [thrice, 25\%], [twice, 25\%], [twice, 25\%], [thrice, 25\%], [twice, 25\%], [tw$ [once, 50%], [twice, 50%], [thrice, 50%] }

Include action in input layer



Action space: $a \in \mathbb{R}^2$; (vector of 2 real random variables)

s: state vector

a: action vector

$$\mathbf{s} = [S_1, \dots S_N]$$

$$s = [S_1, ... S_N]$$
 $a = [A_1, ... A_M]$

Notations

- Random variables in Capital
- Vector in bold small

$$s: state \ vector$$
 $a: action \ vector$
 $s = [S_1, ... S_N]$ $a = [A_1, ... A_M]$

Challenges with these earlier architectures of Deep RL

- Disadvantages of earlier architectures?
- If action space is large, computationally burdensome
 - Train a separate network for each action
 - Need to do a forward pass for each action

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