ESc201, Lecture 8: Sinusoidal steady state analysis

Phasor algebra :

$v(t) = 3\cos(\omega t + 45) \Longrightarrow 3\angle 45 \Longrightarrow 3\cos(45) + j3\sin(45)$

$5\angle -60 \Longrightarrow v(t) = 5\cos(\omega t - 60)$

Let $v_1(t)=5\cos(\omega t + \pi/6)$ and $v_2(t)=10\cos(\omega t - \pi/4)$

Since both signals have the same frequency and both are in *cosine representation* *Phasor algebra* can be performed on them i.e.

$\mathbb{V}_1(j\omega) = 5\angle 30^0 = 5\cos 30^0 + j5\sin 30^0 = 4.33 + j2.5$

$\mathbb{V}_2(j\omega) = 10\angle -45^0 = 10\cos 45^0 - j10\sin 45^0 = 7.07 - j7.07$

$v_1(t) + v_2(t) = \mathbb{V}_1(j\omega) + \mathbb{V}_2(j\omega) = 4.33 + j2.5 + 7.07 - j7.07$

$= 11.41 - j4.57 = (11.41^2 + 4.57^2)^{1/2}\angle \tan^{-1}(-4.57/11.41) = 12.29\angle 22.83^0.$

$(a + jb)/(x + jy) = (a + jb)(x - jy) / (x^2 + y^2) = [(ax + by) + j(bx - ay)] / (x^2 + y^2)$

OR $[\sqrt{(a^2 + b^2)}.\ \angle \tan^{-1}(b/a)] / [\sqrt{(x^2 + y^2)}.\ \angle -\tan^{-1}(y/x)]$

$= [\sqrt{\{(a^2 + b^2)/(x^2 + y^2)\}}\ ]\angle\{[\tan^{-1}(b/a)] - [-\tan^{-1}(y/x)]\}$

$= K.\cos\{\tan^{-1}(b/a) + \tan^{-1}(y/x)\} + jK.\sin\{[\tan^{-1}(b/a)] - [-\tan^{-1}(y/x)]\}$

Remember Impedances are written as : $\dfrac{\mathbb{V}_R}{\mathbb{I}_R} = R \qquad \dfrac{\mathbb{V}_C}{\mathbb{I}_C} = \dfrac{1}{j\omega C} = -j\left(\dfrac{1}{\omega C}\right) = -jX_C \qquad \dfrac{\mathbb{V}_L}{\mathbb{I}_L} = j\omega L = jX_L$

Admitances are written as : $(1/R) = G$, $j\omega C = j(1/X_c)$, and $-j(1/\omega L) = -j(1/X_L)$

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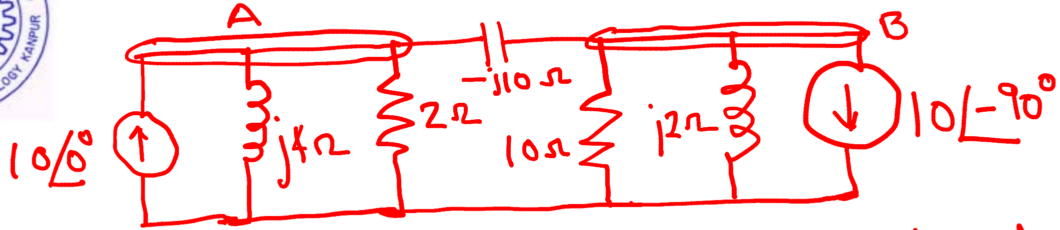

Find V_A & V_B by nodal Analysis

KCL at node A

$$10\angle 0^\circ = \frac{V_A}{j4} + \frac{V_A}{2} + \frac{V_A - V_B}{-j10}$$

$$\text{or } 10 = (0.5 - j0.25 + j0.1)V_A - j0.1V_B$$

$$= (0.5 - j0.15)V_A - j0.1V_B$$

$$10 = (0.5 - j0.15)V_A - \frac{(j-4)(j0.1)}{0.17} - \frac{j0.1V_A(-0.01 - j0.04)}{0.17}$$

$$\therefore 1.7 = (0.85 - j0.0255)V_A + 0.1 + j0.4 - (0.004 - 0.001j)V_A$$

$$\text{or } 1.6 - j0.4 = (0.846 - j0.0245)V_A$$

$$1.65\angle -14 = 0.846\angle -1.66\, V_A$$

$$\therefore V_A = 1.95\angle -12.34^\circ = 1.9 - j0.417\text{ V}$$

$$V_B = \frac{9.12\angle 3^\circ}{0.1\angle -90^\circ} = 91.2\angle 93^\circ = -4.77 + j91\text{ V}$$

KCL at node B

$$\frac{V_B - V_A}{-j10} + \frac{V_B}{10} + \frac{V_B}{j2} + 10(0 - j) = 0$$

$$\text{or } -j0.1V_A + (j0.1 + 0.1 - j0.5)V_B - j10 = 0$$

$$\text{or } V_B = \frac{(j10 + j0.1V_A)(0.1 + j0.4)}{(0.1 - j0.4)(0.1 + j0.4)}$$

$$= \frac{j(1 + j0.01V_A + j4 - 0.04V_A)}{0.17}$$

$$V_B = \frac{\frac{(-4 + j)}{0.17} - (0.01 + 0.04)V_A}{0.17}$$

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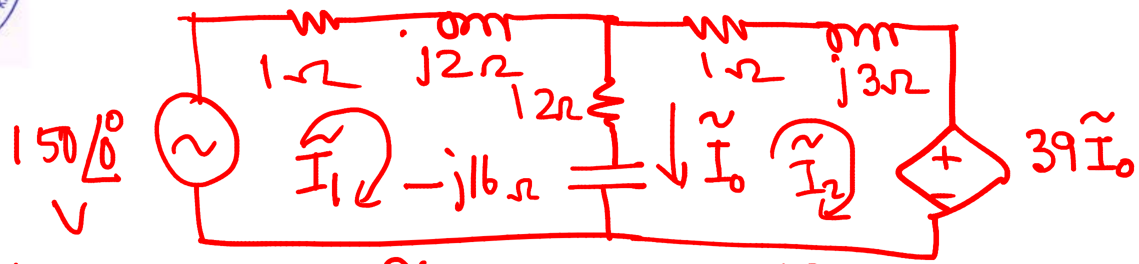

Loop ① $150 = (1+j2)\tilde{I}_1 + (12-j16)(\tilde{I}_1 - \tilde{I}_2) \rightarrow 150 = (13-j14)\tilde{I}_1 - (12-j16)\tilde{I}_2$

Loop ② $0 = (12-j16)(\tilde{I}_2 - \tilde{I}_1) + (1+j3)\tilde{I}_2 + 39\tilde{I}_o \qquad \tilde{I}_o = \tilde{I}_1 - \tilde{I}_2$

$0 = (27+j16)\tilde{I}_1 - (26+j13)\tilde{I}_2 \quad \text{--- I}$

So $(27+j16)\tilde{I}_1 = (26+j13)\tilde{I}_2$ or $\tilde{I}_2 = \left(\dfrac{27+j16}{26+j13}\right)\tilde{I}_1 \quad \text{--- II}$

$150 = (13-j14)\tilde{I}_1 - \dfrac{(12-j16)(27+j16)}{(26+j13)}\tilde{I}_1$

$- \dfrac{20\angle -53.1^\circ \times 31.4\angle 30.7^\circ}{29\angle 26.6^\circ}\tilde{I}_1$

$\tilde{I}_o = -1.8 + j7 \text{ A}$

$- 21.66\angle -49^\circ$

$= (13-j14)\tilde{I}_1 + (-14.2 + j16.35)\tilde{I}_1$

$150 = [-1.2 + 2.35j]\tilde{I}_1 \Rightarrow \tilde{I}_1 = 56.82\angle -117^\circ$ *

$= -25.8 - 51j \text{ A}$

$\tilde{I}$

$\tilde{I}_2 = -24 - j58 \text{ A}$

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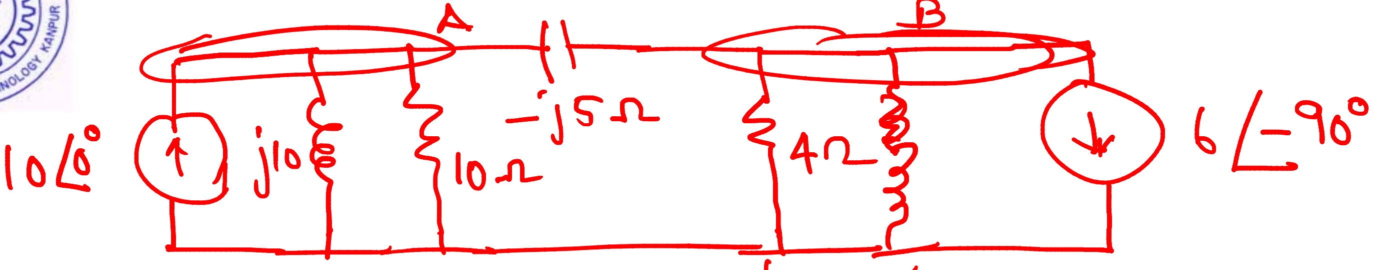


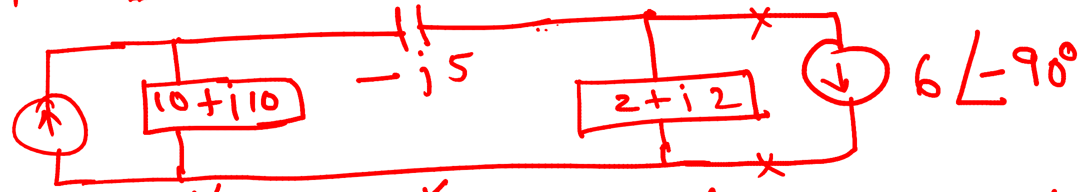
V_A & V_B = ?

Z_{eq_1} Z_{eq_2}

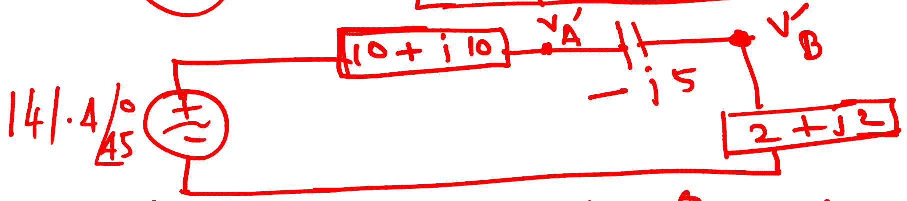
$$Z_{eq_1} = \frac{j10\times 10}{10+j10} = 14.14\angle 45 = 10+j10$$

$$Z_{eq_2} = \frac{j4\times 4}{4+j4} = \frac{4j}{1+j} = \frac{4\angle 90^\circ}{\sqrt{2}\angle 45^\circ} = 2.83\angle 45^\circ = 2+j2$$

I_1



I_2



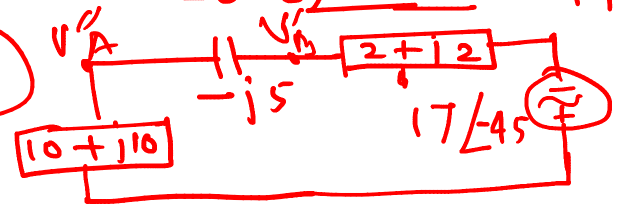
$$V_A' = \frac{141.4\angle 45}{10+j10-j5+2+j2}\times(2+j2-j5)$$

$$= 36.6\angle -41.55^\circ = 27.4 - j24.3$$

$$V_B' = \frac{141.4\angle 45^\circ}{10+j10-j5+2+j2}\times(2+j2)$$

$$= 28.8\angle 59.75^\circ = 14.5 + j27$$

I_2



$$V_A'' = -17.3\angle 30.25^\circ = -14.94 + j8.715$$

$$V_B'' = -13.67\angle 48.65^\circ = -12.5 - j15.6$$

$$V_A = V_A' + V_A'' \qquad V_B = V_B' + V_B''$$

$$V_A = 12.5 - j15.6, \quad V_B = 5.5 - j37.26$$

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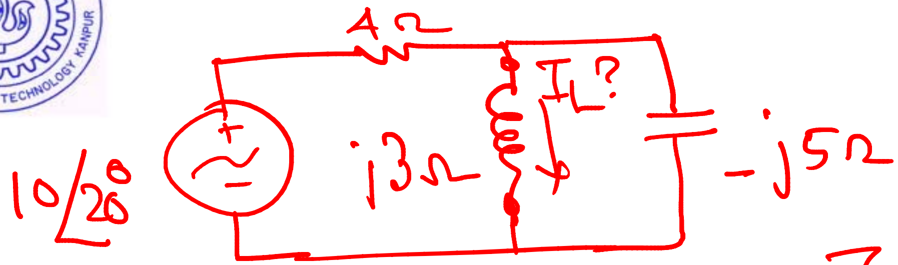
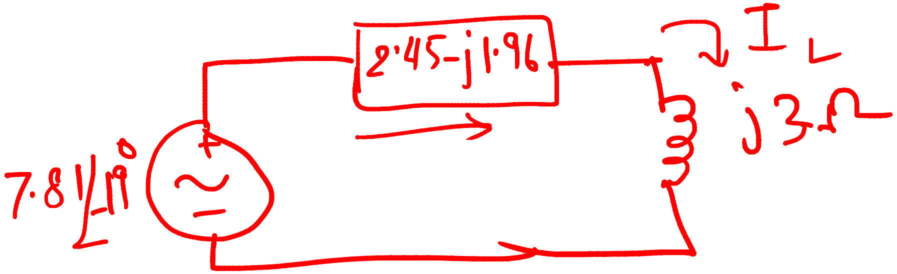
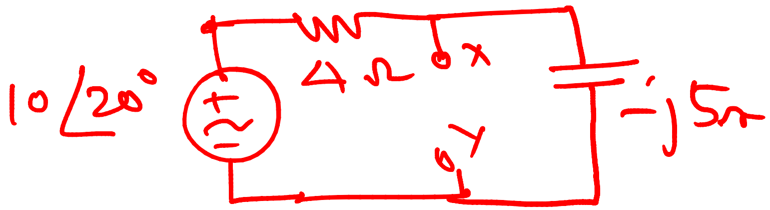


$$V_{Th} = \frac{10\angle 20^\circ}{4+(-j5)}(-j5)$$

$$= \frac{10\angle 20^\circ \times 5\angle -90^\circ}{6.4\angle -51.34^\circ}$$

$$= 7.81\angle -19^\circ$$





$$Z_{Th} = 4\Omega \,\|\, -j5\Omega$$

$$= \frac{4\times(-j5)}{4+(-j5)} = \frac{-20j}{4-5j} = \frac{20\angle -90^\circ}{6.4\angle -51.34^\circ}$$

$$= 3.14\angle -38.66^\circ = 2.45 - j1.96\ \Omega$$

$$I_L = \frac{7.81\angle -19^\circ}{2.45 - j1.96 + j3} \quad \cancel{(-j3)}$$

$$= \frac{7.81\angle -19}{2.66\angle 23^\circ}$$

$$= 2.94\angle -42$$

$$= 2.18 - j1.97 \text{ A}$$