RECAP

- Markov property:

$$P(X_n = j | X_1 = x_1, .., X_{n-1} = x_{n-1}) = P(X_n = j | X_{n-1} = x_{n-1})$$

- You need $\bar{\mu} = (\mu_1, \ldots, \mu_M)$ and t.p.m P do write down the finite dimensional distributions

$$P(X_0 = x_0, X_1 = x_1, \ldots X_k = x_k) = p_{x_{k-1}, x_k} \times \ldots \times p_{x_0, x_1} \mu_{x_0}$$

- Chapman Kolmogorov Equations tell us that the n-step t.p.m is just P^n.

Classification of states

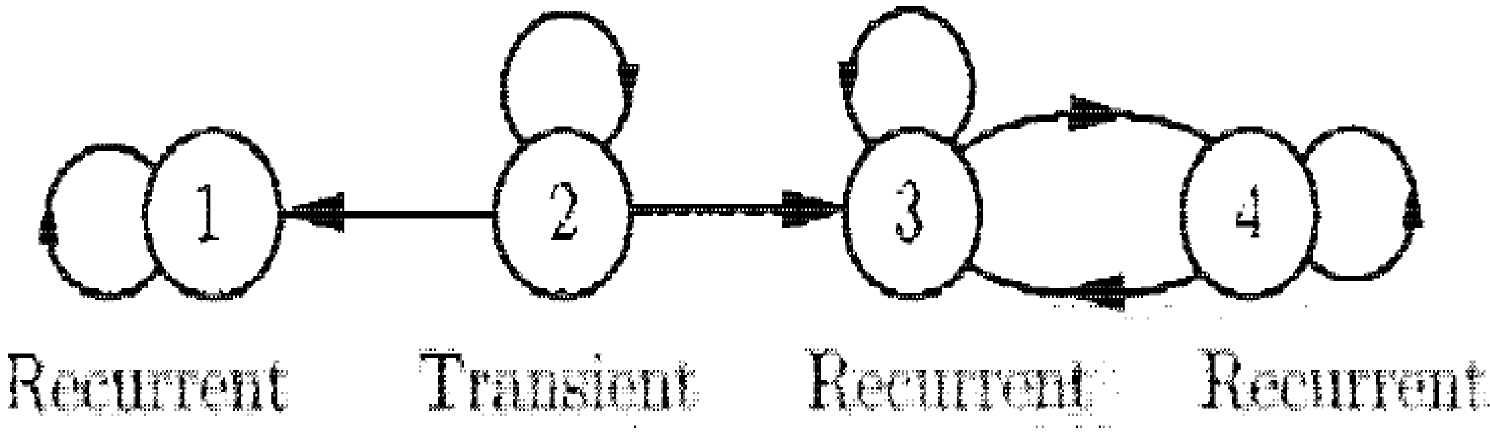
- Consider a Markov process with state space $\mathcal{S}$
- We say that j is accessible from i if $p_{ij}^n > 0$ for some n.
- This is denoted by $i \to j$.
- if $i \to j$ and $j \to i$ then we say that i and j communicate. This is denoted by $i \leftrightarrow j$.

  A chain is said to be irreducible if $i \leftrightarrow j$ for all $i, j \in \mathcal{S}$.

- Are the examples of Markovian coin and dice we have considered till now irreducible? check!

Recurrent and Transient states

- We say that a state i is recurrent if $F_{ii} = P($ ever returning to i having started in i $) = 1$.
- F_{ii} is not easy to calculate. (Not part of this course)
- If a state is not recurrent, it is transient.
- For a transient state i, $F_{ii} < 1$.
- If $i \leftrightarrow j$ and i is recurrent, then j is recurrent.



Limiting probabilities

- $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.6 & 0.4 \\ 0.6 & 0.4 & 0 \end{bmatrix}$ $P^5 = \begin{bmatrix} .06 & .3 & .64 \\ .18 & .38 & .44 \\ .38 & .44 & .18 \end{bmatrix}$ $P^{30} = \begin{bmatrix} .23 & .385 & .385 \\ .23 & .385 & .385 \\ .23 & .385 & .385 \end{bmatrix}$

- $P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$ $\lim_{n\to\infty} P^n = \begin{bmatrix} \frac{b}{a+b} & \frac{a}{a+b} \\ \frac{b}{a+b} & \frac{a}{a+b} \end{bmatrix}$

- What is the interpretation of $\lim_{n\to\infty} p_{ij}^{(n)} = [\lim_{n\to\infty} P^n]_{ij}$?

- $\alpha_j = \lim_{n\to\infty} p_{ij}^{(n)}$ denotes the probability of being in state j after a large time from starting in state i.

- For an M state DTMC, $\alpha = (\alpha_1, \ldots, \alpha_M)$ denotes the limiting distribution.

- How do we obtain the limiting distribution α? Does it always exist?

Stationary distribution

The **stationary distribution** of a Markov chain is defined as a solution to the equation $\pi = \pi P$.

- πP is essentially the p.m.f of X_1 having picked X_0 according to π.
- $\pi = \pi P$ says that, if the initial distribution is π, then the distribution of X_1 is also π.
- Continuing this argument, the p.m.f of X_n for any n is π and there is no dependence on the starting state.
- MCMC algorithms use this idea (at stationarity successive states of the Markov chain have p.m.f π) to sample from target distribution π.

Limiting vs Stationary distribution

- Obtain stationary distribution for the Markov Chain with transition probability $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.6 & 0.4 \\ 0.6 & 0.4 & 0 \end{bmatrix}$
- The limiting distribution α need not exist for some Markov chains, but the stationary distribution π exists. For example for $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.
- The limiting distribution if it exists, is same as the stationary distribution, i.e. $\alpha_i = \pi_i$ for all i.