Task 1: Mathematical Truth

Prove by strong induction

We want to prove If a binary tree has n nodes, and each node has either 0 or 2 children, then the number of leaves is $\frac{n+1}{2}$.

Base case: let n = 1

1+1 /2 = 0.

The only tree with one node is just the root, which is a leaf (0 children)

1 leaf.

Inductive Hypothesis:

Assume for all trees with $n' < n$ nodes, the number of leaves is $\frac{n\prime+1}{2}$.

Inductive step:

Consider a tree with n nodes where each node has either 0 or 2 children. Let he left and right subtree have n_L and n_R nodes respectively, then.

$$n = 1 + \ n_L + \ n_R$$

By inductive hypothesis, the left and right subtrees have $\frac{n_L+1}{2}$ and $\frac{n_R+1}{2}$ leaves respectively. So total leaves:

$$\frac{n_L + 1}{2} + \ \frac{n_R + 1}{2} = \ \frac{n_L + \ n_R + 2}{2} = \ \frac{(n-1) + 2}{2} = \frac{n + 1}{2}$$

Hence, the proposition holds for all n.