

Math Document Template

C ANISH

Abstract—This is a document explaining a question about the concept of sum of angles in a triangle.

Download all python codes from

svn co <https://github.com/chakki1234/summer-2020/trunk/linearalg/codes>

and latex-tikz codes from

svn co <https://github.com/chakki1234/summer-2020/trunk/linearalg/figs>

1 PROBLEM

In a $ABCD$ is a cyclic quadrilateral with

$$\angle A = 4y + 20 \quad (1.0.1)$$

$$\angle B = 3y - 5 \quad (1.0.2)$$

$$\angle C = -4x \quad (1.0.3)$$

$$\angle D = -7x + 5 \quad (1.0.4)$$

Find its angles.

2 CONSTRUCTION

2.1. The figure obtained looks like Fig. 2.0.

2.2. The design parameters used for construction
See Table. 2.2.

Design Parameters	
Parameters	Value
a	10
b	8

TABLE 2.2: Quadilateral $ABCD$

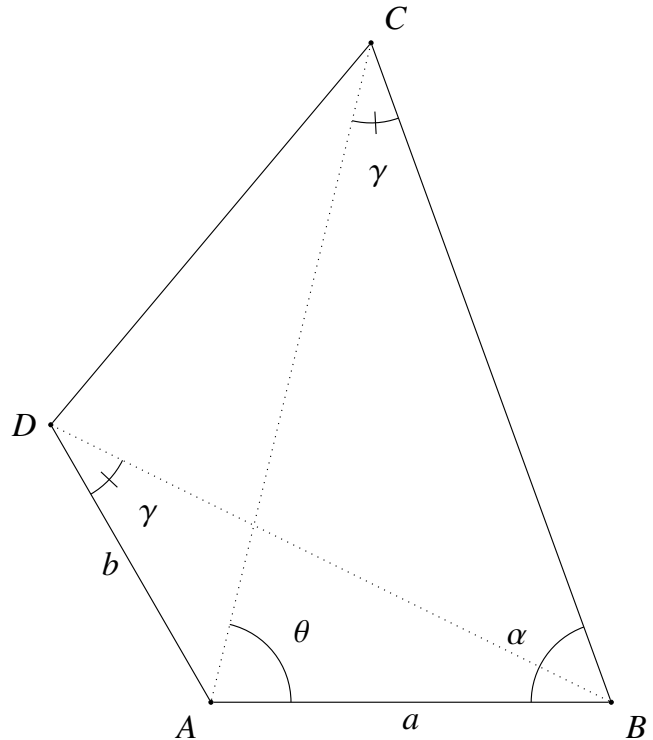


Fig. 2.0: Cyclic quadrilateral by Latex-Tikz

2.3. Coordinates of cyclic quadrilateral Fig2.0.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.3.1)$$

$$\mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (2.3.2)$$

$$\mathbf{C} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (2.3.3)$$

$$\mathbf{D} = \begin{pmatrix} b \cos \theta \\ b \sin \theta \end{pmatrix} \quad (2.3.4)$$

2.4. To find the coordinates of \mathbf{C} .

Theorem 2.1. Angles formed in the same segment of a circle are always equal in measure.

2.6. Draw Fig. 2.6.

$$\cos \gamma = \frac{(A - D)^T (B - D)}{\|A - D\| \|B - D\|} \quad (2.4.1)$$

$$\theta = 180^\circ - \gamma - \angle B \quad (2.4.2)$$

In $\triangle ACB$. Finding the Scalar Products:

$$\begin{aligned} (\mathbf{B} - \mathbf{A})^T (\mathbf{C} - \mathbf{A}) &= \\ \|\mathbf{B} - \mathbf{A}\| \|\mathbf{C} - \mathbf{A}\| \cos \theta \end{aligned} \quad (2.4.3)$$

$$\begin{aligned} (\mathbf{C} - \mathbf{B})^T (\mathbf{A} - \mathbf{B}) &= \\ \|\mathbf{C} - \mathbf{B}\| \|\mathbf{A} - \mathbf{B}\| \cos \alpha \end{aligned} \quad (2.4.4)$$

On simplifying equation 2.4.3 and 2.4.4:

$$x^2 \tan^2 \theta = y^2 \quad (2.4.5)$$

$$(x - a)^2 = ((x - a)^2 + y^2) \cos^2 \alpha \quad (2.4.6)$$

Substituting 2.4.5 in 2.4.6:

$$\begin{aligned} x^2 (1 - \cos^2 \alpha - \tan^2 \theta \cos^2 \alpha) \\ + x(2a \cos^2 \alpha - 2a) + a^2 \sin^2 \alpha \end{aligned} \quad (2.4.7)$$

If θ and α are acute angles:

$$x = \frac{(-b - \sqrt{b^2 - 4ac})}{2a} \quad (2.4.8)$$

else:

$$x = \frac{(-b + \sqrt{b^2 - 4ac})}{2a} \quad (2.4.9)$$

The value of x can then be substituted in 2.4.5 to find the coordinates of \mathbf{C}

2.5. From the given information, The values are listed in 2.5

Output values	
Parameter	Value
C	$\begin{pmatrix} 4 \\ 16.47 \end{pmatrix}$
D	$\begin{pmatrix} -4 \\ 6.93 \end{pmatrix}$

TABLE 2.5: Values of \mathbf{C} and \mathbf{D}

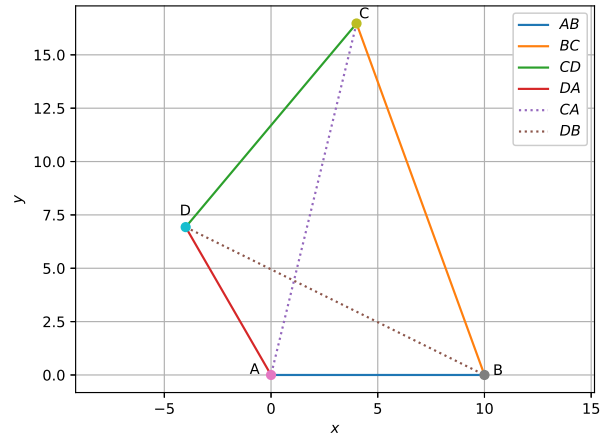


Fig. 2.6: Triangle generated using python

Solution: The following Python code generates Fig. 2.6

```
codes/cyclic_quad.py
```

and the equivalent latex-tikz code generating Fig. 2.6 is

```
figs/cyclic_quad_fig.tex
```

The above latex code can be compiled as a standalone document as

```
figs/cyclic_quad_final.tex
```

3 SOLUTION

Theorem 3.1. Sum of opposite angles in a cyclic quadrilateral equals 180° .

Solution: From theorem 3.1

$$\angle A + \angle C = 180^\circ \quad (6.1)$$

$$\angle B + \angle D = 180^\circ \quad (6.2)$$

From the given information:

$$4y + 20 - 4x = 180^\circ \quad (6.3)$$

$$3y - 5 - 7x + 5 = 180^\circ \quad (6.4)$$

Solving equations 6.3 and 6.4:

$$x = -15 \quad (6.5)$$

$$y = 25 \quad (6.6)$$

$$\implies \angle A = 120^\circ \quad (6.7)$$

$$\implies \angle B = 70^\circ \quad (6.8)$$

$$\implies \angle C = 60^\circ \quad (6.9)$$

$$\implies \angle D = 110^\circ \quad (6.10)$$