

Math Document Template

C ANISH

Abstract—This is a document explaining questions about the concept of Probability and Statistics.

Download all python codes from

svn co https://github.com/chakki1234/summer-2020/trunk/Probability_and_statistics/codes

and latex-tikz codes from

svn co https://github.com/chakki1234/summer-2020/trunk/Probability_and_statistics/figs

1 EXERCISE - PROBLEM 10

1.1 Problem

The following table shows the ages of the patients admitted in a hospital during a year. Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Age(in years)	Number of patients
5-15	6
15-25	11
25-35	21
35-45	23
45-55	14
55-65	5

1.2 Solution

1) $Mean = \frac{\sum f_i x_i}{\sum f_i}$

2) To calculate class mark - x_i and $f_i \times x_i$

a) For the class 5 – 15:

$$class\ mark = \frac{5 + 15}{2} = 10$$

$$f_i \times x_i = 6 \times 10 = 60$$

b) For the class 15 – 25:

$$class\ mark = \frac{15 + 25}{2} = 20$$

$$f_i \times x_i = 11 \times 20 = 220$$

c) For the class 25 – 35:

$$class\ mark = \frac{25 + 35}{2} = 30$$

$$f_i \times x_i = 21 \times 30 = 630$$

d) For the class 35 – 45:

$$class\ mark = \frac{35 + 45}{2} = 40$$

$$f_i \times x_i = 23 \times 40 = 920$$

e) For the class 45 – 55:

$$class\ mark = \frac{45 + 55}{2} = 50$$

$$f_i \times x_i = 14 \times 50 = 700$$

f) For the class 55 – 65:

$$class\ mark = \frac{55 + 65}{2} = 60$$

$$f_i \times x_i = 5 \times 60 = 300$$

3) Mean:

$$\sum f_i x_i = 2830$$

$$\sum f_i = 80$$

$$Mean = \bar{x} = \frac{2830}{80} = 35.375$$

4) To find the Mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$:

5) Where:

Modal class = interval with highest frequency

l = lower limit of modal class

h = class interval

f₁ = frequency with modal class

f₀ = frequency of class

before modal class

f₂ = frequency of class

after modal class

6)

$$\text{Modal class} = 35 - 45 \quad (6.1)$$

$$l = 35 \quad (6.2)$$

$$h = 10 \quad (6.3)$$

$$f_1 = 23 \quad (6.4)$$

$$f_0 = 21 \quad (6.5)$$

$$f_2 = 14 \quad (6.6)$$

$$\text{Mode} = 36.8 \quad (6.7)$$

The python implementation:

```
codes/prob10.py
```

2 EXERCISE - PROBLEM 11

2.1 Problem

The following data gives the information on the observed lifetimes (in hours) of 225 electrical components. Determine the modal lifetimes of the components.

Lifetime(in hours)	Frequency
0-20	10
20-40	35
40-60	52
60-80	61
80-100	38
100-120	29

2.2 Solution

1) To find the Mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$:

2) Where:

Modal class = interval with highest frequency

l = lower limit of modal class

h = class interval

f₁ = frequency with modal class

f₀ = frequency of class

before modal class

f₂ = frequency of class

after modal class

3)

$$\text{Modal class} = 60 - 80 \quad (3.1)$$

$$l = 60 \quad (3.2)$$

$$h = 20 - 0 = 20 \quad (3.3)$$

$$f_1 = 61 \quad (3.4)$$

$$f_0 = 52 \quad (3.5)$$

$$f_2 = 38 \quad (3.6)$$

$$\text{Mode} = 60 + \frac{61 - 52}{2(61) - 52 - 38} \times 20 \quad (3.7)$$

$$= 65.625 \quad (3.8)$$

The python implementation:

```
codes/prob11.py
```

3 EXERCISE - PROBLEM 12

3.1 Problem

The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure:

Expenditure(in Rs.)	Number of families
1000-1500	24
1500-2000	40
2000-2500	33
2500-3000	28
3000-3500	30
3500-4000	22
4000-4500	16
4500-5000	7

3.2 Solution

1) To find the Mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$:

2) Where:

Modal class = interval with highest frequency

l = lower limit of modal class

h = class interval

f₁ = frequency with modal class

f₀ = frequency of class

before modal class

f₂ = frequency of class

after modal class

3)

$$\text{Modal class} = 1500 - 2000 \quad (3.1)$$

$$l = 1500 \quad (3.2)$$

$$h = 1500 - 1000 = 500 \quad (3.3)$$

$$f_1 = 40 \quad (3.4)$$

$$f_0 = 24 \quad (3.5)$$

$$f_2 = 33 \quad (3.6)$$

$$\text{Mode} = 1500 + \frac{40 - 24}{2(40) - 24 - 33} \times 500 \quad (3.7)$$

$$= 1847.83 \quad (3.8)$$

$$4) \text{ Mean} = a + h \times \frac{\sum f_i u_i}{\sum f_i}$$

5) Where:

$$h = \text{class interval} = 1500 - 1000 = 500$$

$$a = \text{assumed mean}$$

$$= \text{mean of } 3000 - 3500 = 3250$$

$$u_i = \frac{x_i - a}{h}$$

6) To calculate class mark - x_i , $x_i - a$, u_i and $f_i \times u_i$

a) For the class 1000 – 1500:

$$\text{class mark} = \frac{1000 + 1500}{2} = 1250$$

$$x_i - a = 1250 - 3250 = -2000$$

$$u_i = \frac{-2000}{500} = -4$$

$$f_i \times u_i = 24 \times -4 = -96$$

b) For the class 1500 – 2000:

$$\text{class mark} = \frac{1500 + 2000}{2} = 1750$$

$$x_i - a = 1750 - 3250 = -1500$$

$$u_i = \frac{-1500}{500} = -3$$

$$f_i \times u_i = 40 \times -3 = -120$$

c) For the class 2000 – 2500:

$$\text{class mark} = \frac{2000 + 2500}{2} = 2250$$

$$x_i - a = 2250 - 3250 = -1000$$

$$u_i = \frac{-1000}{500} = -2$$

$$f_i \times u_i = 33 \times -2 = -66$$

d) For the class 2500 – 3000:

$$\text{class mark} = \frac{2500 + 3000}{2} = 2750$$

$$x_i - a = 2750 - 3250 = -500$$

$$u_i = \frac{-500}{500} = -1$$

$$f_i \times u_i = 28 \times -1 = -28$$

e) For the class 3000 – 3500:

$$\text{class mark} = \frac{3000 + 3500}{2} = 3250$$

$$x_i - a = 3250 - 3250 = 0$$

$$u_i = \frac{0}{500} = 0$$

$$f_i \times u_i = 30 \times 0 = 0$$

f) For the class 3500 – 4000:

$$\text{class mark} = \frac{3500 + 4000}{2} = 3750$$

$$x_i - a = 3750 - 3250 = 500$$

$$u_i = \frac{500}{500} = 1$$

$$f_i \times u_i = 22 \times 1 = 22$$

g) For the class 4000 – 4500:

$$\text{class mark} = \frac{4000 + 4500}{2} = 4250$$

$$x_i - a = 4250 - 3250 = 1000$$

$$u_i = \frac{1000}{500} = 2$$

$$f_i \times u_i = 16 \times 2 = 32$$

h) For the class 4500 – 5000:

$$\text{class mark} = \frac{4500 + 5000}{2} = 4750$$

$$x_i - a = 4750 - 3250 = 1500$$

$$u_i = \frac{1500}{500} = 3$$

$$f_i \times u_i = 7 \times 3 = 21$$

7)

$$\sum f_i = 200$$

$$\sum f_i \times u_i = -235$$

$$\begin{aligned}\bar{x} &= 3250 + 500 \times \frac{-235}{200} \\ &= 2662.5\end{aligned}$$

8) The modal monthly expenditure of families is Rs1847.83 and the mean monthly expenditure is Rs2662.5

The python implementation:

codes/prob12.py

4 EXERCISE - PROBLEM 13

4.1 Problem

The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

No of students/teacher	No of States/U.T
15-20	3
20-25	8
25-30	9
30-35	10
35-40	3
40-45	0
45-50	0
50-55	2

4.2 Solution

1) To find the Mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$:

2) Where:

Modal class = interval with highest frequency

l = lower limit of modal class

h = class interval

f₁ = frequency with modal class

f₀ = frequency of class

before modal class

f₂ = frequency of class

after modal class

3)

$$\text{Modal class} = 30 - 35 \quad (3.1)$$

$$l = 30 \quad (3.2)$$

$$h = 20 - 15 = 5 \quad (3.3)$$

$$f_1 = 10 \quad (3.4)$$

$$f_0 = 9 \quad (3.5)$$

$$f_2 = 3 \quad (3.6)$$

$$\text{Mode} = 30 + \frac{10 - 9}{2(10) - 9 - 3} \times 5 \quad (3.7)$$

$$= 30.625 \quad (3.8)$$

$$4) \text{ Mean} = a + h \times \frac{\sum f_i u_i}{\sum f_i}$$

5) Where:

$$h = \text{class interval} = 20 - 15 = 5$$

a = assumed mean

= mean of class 35 – 40

$$= 37.5$$

$$u_i = \frac{x_i - a}{h}$$

6) To calculate class mark - x_i , $x_i - a$, u_i and $f_i \times u_i$

a) For the class 15 – 20:

$$\text{class mark} = \frac{15 + 20}{2} = 17.5$$

$$x_i - a = 17.5 - 37.5 = -20$$

$$u_i = \frac{-20}{5} = -4$$

$$f_i \times u_i = 3 \times -4 = -12$$

b) For the class 20 – 25:

$$\begin{aligned} \text{class mark} &= \frac{20 + 25}{2} = 22.5 \\ x_i - a &= 22.5 - 37.5 = -15 \\ u_i &= \frac{-15}{5} = -3 \\ f_i \times u_i &= 8 \times -3 = -24 \end{aligned}$$

c) For the class 25 – 30:

$$\begin{aligned} \text{class mark} &= \frac{25 + 30}{2} = 27.5 \\ x_i - a &= 27.5 - 37.5 = -10 \\ u_i &= \frac{-10}{5} = -2 \\ f_i \times u_i &= 9 \times -2 = -18 \end{aligned}$$

d) For the class 30 – 35:

$$\begin{aligned} \text{class mark} &= \frac{30 + 35}{2} = 32.5 \\ x_i - a &= 32.5 - 37.5 = -5 \\ u_i &= \frac{-5}{5} = -1 \\ f_i \times u_i &= 10 \times -1 = -10 \end{aligned}$$

e) For the class 35 – 40:

$$\begin{aligned} \text{class mark} &= \frac{35 + 40}{2} = 37.5 \\ x_i - a &= 37.5 - 37.5 = 0 \\ u_i &= \frac{0}{5} = 0 \\ f_i \times u_i &= 3 \times 0 = 0 \end{aligned}$$

f) For the class 40 – 45:

$$\begin{aligned} \text{class mark} &= \frac{40 + 45}{2} = 42.5 \\ x_i - a &= 42.5 - 37.5 = 5 \\ u_i &= \frac{5}{5} = 1 \\ f_i \times u_i &= 0 \times 1 = 0 \end{aligned}$$

g) For the class 45 – 50:

$$\begin{aligned} \text{class mark} &= \frac{45 + 50}{2} = 47.5 \\ x_i - a &= 47.5 - 37.5 = 10 \\ u_i &= \frac{10}{5} = 2 \\ f_i \times u_i &= 0 \times 2 = 0 \end{aligned}$$

h) For the class 50 – 55:

$$\begin{aligned} \text{class mark} &= \frac{50 + 55}{2} = 52.5 \\ x_i - a &= 52.5 - 37.5 = 15 \\ u_i &= \frac{15}{5} = 3 \\ f_i \times u_i &= 2 \times 3 = 6 \end{aligned}$$

7)

$$\begin{aligned} \sum f_i &= 35 \\ \sum f_i \times u_i &= -58 \\ \bar{x} &= 37.5 + 5 \times \frac{-58}{35} \\ &= 29.214 \end{aligned}$$

8) Therefore most states have a student-teacher ratio 30.625 and average student-teacher ratio is 29.214.

The python implementation:

codes/13.py

5 EXERCISE - PROBLEM 14

5.1 Problem

The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches. Find the mode of the data.

Runs scored	Number of batsmen
3000-4000	4
4000-5000	18
5000-6000	9
6000-7000	7
7000-8000	6
8000-9000	3
9000-10000	1
10000-11000	1

5.2 Solution

1) To find the Mode = $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$:

2) Where:

Modal class = interval with highest frequency

l = lower limit of modal class

h = class interval

f₁ = frequency with modal class

f₀ = frequency of class

before modal class

f₂ = frequency of class

after modal class

3)

$$\text{Modal class} = 4000 - 5000 \quad (3.1)$$

$$l = 4000 \quad (3.2)$$

$$h = 4000 - 3000 = 1000 \quad (3.3)$$

$$f_1 = 18 \quad (3.4)$$

$$f_0 = 4 \quad (3.5)$$

$$f_2 = 9 \quad (3.6)$$

$$\text{Mode} = 4000 + \frac{18 - 4}{2(18) - 4 - 9} \times 1000 \quad (3.7)$$

$$= 4608.69 \quad (3.8)$$

The python implementation:

codes/14.py

6 EXERCISE - PROBLEM 15

6.1 Problem

A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data:

Number of cars	Frequency
0-10	7
10-20	14
20-30	13
30-40	12
40-50	20
50-60	11
60-70	15

6.2 Solution

1) To find the Mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$:

2) Where:

Modal class = interval with highest frequency

l = lower limit of modal class

h = class interval

f₁ = frequency with modal class

f₀ = frequency of class

before modal class

f₂ = frequency of class

after modal class

3)

$$\text{Modal class} = 40 - 50 \quad (3.1)$$

$$l = 40 \quad (3.2)$$

$$h = 10 - 0 = 10 \quad (3.3)$$

$$f_1 = 20 \quad (3.4)$$

$$f_0 = 12 \quad (3.5)$$

$$f_2 = 11 \quad (3.6)$$

$$\text{Mode} = 40 + \frac{20 - 12}{2(20) - 12 - 11} \times 10 \quad (3.7)$$

$$= 44.7 \quad (3.8)$$

The python implementation:

codes/15.py

7 EXERCISE - PROBLEM 16

7.1 Problem

The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption	Number of consumers
65-85	4
85-105	5
105-125	13
125-145	20
145-165	14
165-185	8
185-205	4

7.2 Solution

- 1) To find the Mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$:

$$\text{Modal class} = 125 - 145 \quad (1.1)$$

$$l = 125 \quad (1.2)$$

$$h = 85 - 65 = 20 \quad (1.3)$$

$$f_1 = 20 \quad (1.4)$$

$$f_0 = 13 \quad (1.5)$$

$$f_2 = 14 \quad (1.6)$$

$$\text{Mode} = 125 + \frac{34 - 22}{2(20) - 13 - 14} \times 20 \quad (1.7)$$

$$= 143.46 \quad (1.8)$$

- 2) Where:

Modal class = interval with highest frequency

l = lower limit of modal class

h = class interval

f₁ = frequency with modal class

f₀ = frequency of class

before modal class

f₂ = frequency of class

after modal class

3) $\text{Median} = l + h \times \frac{\frac{n}{2} - cf}{f}$

- 4) Where:

$$n = \sum f_i = 68$$

$$\frac{n}{2} = \frac{68}{2} = 34$$

$\therefore 125 - 145$ is the median class

l = lower limit of median class = 125

h = class interval = 85 - 65 = 20

cf = cumulative frequency of the class before median class

f = frequency of median class = 20

- 5) To calculate class mark - cf

- a) For the class 65 - 85:

$$\text{Cumulative Frequency} = 4$$

- b) For the class 85 - 105:

$$\text{Cumulative Frequency} = 4 + 5 = 9$$

- c) For the class 105 - 125:

$$\text{Cumulative Frequency} = 9 + 13 = 22$$

- d) For the class 125 - 145:

$$\text{Cumulative Frequency} = 22 + 20 = 42$$

- e) For the class 145 - 165:

$$\text{Cumulative Frequency} = 42 + 14 = 56$$

- f) For the class 165 - 185:

$$\text{Cumulative Frequency} = 56 + 8 = 64$$

- g) For the class 185 - 205:

$$\text{Cumulative Frequency} = 4 + 64 = 68$$

- h) For the class 65 - 85:

$$\text{Cumulative Frequency} = 4$$

- i) $\therefore cf = 22$

- 6)

$$\begin{aligned} \text{Median} &= 125 + \frac{34 - 22}{20} \times 20 \\ &= 137 \end{aligned}$$

7) $\text{Mean} = a + h \times \frac{\sum f_i u_i}{\sum f_i}$

- 8) Where:

$$h = \text{class interval} = 85 - 65 = 20$$

a = assumed mean

= mean of class 125 - 145

$$= 135$$

$$u_i = \frac{x_i - a}{h}$$

- 9) To calculate class mark - x_i , $x_i - a$, u_i and $f_i \times u_i$

- a) For the class 65 - 85:

$$\text{class mark} = \frac{65 + 85}{2} = 75$$

$$x_i - a = 75 - 135 = -60$$

$$u_i = \frac{-60}{20} = -3$$

$$f_i \times u_i = 4 \times -3 = -12$$

b) For the class 85 – 105:

$$\begin{aligned} \text{class mark} &= \frac{85 + 105}{2} = 95 \\ x_i - a &= 95 - 135 = -40 \\ u_i &= \frac{-40}{20} = -2 \\ f_i \times u_i &= 5 \times -2 = -10 \end{aligned}$$

c) For the class 105 – 125:

$$\begin{aligned} \text{class mark} &= \frac{105 + 125}{2} = 115 \\ x_i - a &= 115 - 135 = -20 \\ u_i &= \frac{-20}{20} = -1 \\ f_i \times u_i &= 13 \times -1 = -13 \end{aligned}$$

d) For the class 125 – 145:

$$\begin{aligned} \text{class mark} &= \frac{125 + 145}{2} = 135 \\ x_i - a &= 135 - 135 = 0 \\ u_i &= \frac{0}{20} = 0 \\ f_i \times u_i &= 20 \times 0 = 0 \end{aligned}$$

e) For the class 145 – 165:

$$\begin{aligned} \text{class mark} &= \frac{145 + 165}{2} = 155 \\ x_i - a &= 155 - 135 = 20 \\ u_i &= \frac{20}{20} = 1 \\ f_i \times u_i &= 14 \times 1 = 14 \end{aligned}$$

f) For the class 165 – 185:

$$\begin{aligned} \text{class mark} &= \frac{165 + 185}{2} = 175 \\ x_i - a &= 175 - 135 = 40 \\ u_i &= \frac{40}{20} = 2 \\ f_i \times u_i &= 8 \times 2 = 16 \end{aligned}$$

g) For the class 185 – 205:

$$\begin{aligned} \text{class mark} &= \frac{185 + 205}{2} = 195 \\ x_i - a &= 195 - 135 = 60 \\ u_i &= \frac{60}{20} = 3 \\ f_i \times u_i &= 4 \times 3 = 12 \end{aligned}$$

10)

$$\begin{aligned} \sum f_i &= 68 \\ \sum f_i \times u_i &= 7 \\ \bar{x} &= 135 + 20 \times \frac{7}{68} \\ &= 137.05 \end{aligned}$$

11) Mean = 137.05 , Median = 137 , Mode = 135.77.

The python implementation:

codes/16.py

8 EXERCISE - PROBLEM 17

8.1 Problem

If the median of the distribution given below is 28.5, find the values of x and y.

Class interval	Frequency
0-10	5
10-20	x
20-30	20
30-40	15
40-50	y
50-60	5

8.2 Solution

$$1) \text{ Median} = l + h \times \frac{\frac{n}{2} - cf}{f} = 28.5$$

2) Where:

20 – 30 is the median class

$$n = \sum f_i = 60$$

$$\frac{n}{2} = \frac{60}{2} = 30$$

l = lower limit of median class = 20

h = class interval = 10 – 0 = 10

cf = cumulative frequency of the class before median class

f = frequency of median class = 20

3) To calculate Cumulative Frequency

a) For the class 0 – 10:

Cumulative Frequency = 5

b) For the class 10 – 20:

$$\text{Cumulative Frequency} = 5 + x$$

c) For the class 20 – 30:

$$\text{Cumulative Frequency} = 5 + x + 20 = 25 + x$$

d) For the class 30 – 40:

$$\text{Cumulative Frequency} = 25 + x + 15 = 40 + x$$

e) For the class 40 – 50:

$$\text{Cumulative Frequency} = 40 + x + y$$

f) For the class 50 – 60:

$$\begin{aligned}\text{Cumulative Frequency} &= 40 + x + y + 5 \\ &= 45 + x + y\end{aligned}$$

g) $\therefore cf = 5 + x$

4) To find the median:

$$\begin{aligned}28.5 &= 20 + \frac{30 - (5 + x)}{20} \times 10 \\ x &= 8\end{aligned}$$

5) To find y:

$$\sum f_i = 5 + x + 20 + 15 + y + 5 \quad (5.1)$$

$$y = 7 \quad (5.2)$$

The python implementation:

```
codes/17.py
```

9 EXERCISE - PROBLEM 18

9.1 Problem

A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years on wards but less than 60 year.

9.2 Solution

1) The given data has been reorganized and tabulated.

2) $\text{Median} = l + h \times \frac{\frac{n}{2} - cf}{f}$

Age(in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

Age(in years)	Number of policy holders
0-20	2
20-25	4
25-30	18
30-35	21
35-40	33
40-45	11
45-50	3
50-55	6
55-60	2

3) Where:

$$n = \sum f_i = 100$$

$$\frac{n}{2} = \frac{100}{2} = 50$$

$\therefore 35 - 40$ is the median class

l = lower limit of median class = 35

h = class interval = $25 - 20 = 5$

cf = cumulative frequency of the class before median class

f = frequency of median class = 33

4) To calculate Cumulative Frequency

a) For the class 0 – 20:

$$\text{Cumulative Frequency} = 2$$

b) For the class 20 – 25:

$$\text{Cumulative Frequency} = 2 + 4 = 6$$

c) For the class 25 – 30:

$$\text{Cumulative Frequency} = 6 + 18 = 24$$

d) For the class 30 – 35:

$$\text{Cumulative Frequency} = 24 + 21 = 45$$

e) For the class 35 – 40:

$$\text{Cumulative Frequency} = 45 + 33 = 78$$

f) For the class 40 – 45:

$$\text{Cumulative Frequency} = 78 + 11 = 89$$

g) For the class 45 – 50:

$$\text{Cumulative Frequency} = 89 + 3 = 92$$

h) For the class 50 – 55:

$$\text{Cumulative Frequency} = 92 + 6 = 98$$

i) For the class 55 – 60:

$$\text{Cumulative Frequency} = 98 + 2 = 100$$

j) $\therefore cf = 45$

5) To find the median:

$$\begin{aligned} \text{Median} &= 35 + \frac{50 - 45}{33} \times 5 \\ &= 35.76 \end{aligned}$$

The python implementation:

```
codes/prob18.py
```

10 EXERCISE - PROBLEM 19

10.1 Problem

The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table. Find the median length of the leaves.

Length (in mm)	Number or leaves
118-126	3
127-135	5
136-144	9
145-153	12
154-162	5
163-171	4
172-180	2

Length (in mm)	Number or leaves
117.5-126.5	3
126.5-135.5	5
135.5-144.5	9
144.5-153.5	12
153.5-162.5	5
162.5-171.5	4
171.5-180.5	2

10.2 Solution

1) The given data has been reorganized to a continous form and tabulated by the following procedure:

$$\text{gap} = 127 - 126$$

$$\frac{\text{gap}}{2} = \frac{1}{2} = 0.5$$

\therefore We need to add 0.5 to each upper class limit and subtrack 0.5 from each lower class limit.

2) $\text{Median} = l + h \times \frac{\frac{n}{2} - cf}{f}$

3) Where:

$$n = \sum f_i = 40$$

$$\frac{n}{2} = \frac{40}{2} = 20$$

\therefore 144.5 – 153.5 is the median class

l = lower limit of median class = 144.5

h = class interval = 126.5 – 117.5 = 9

cf = cumulative frequency of the class before median class

f = frequency of median class = 12

4) To calculate Cumulative Frequency

a) For the class 117.5 – 126.5:

$$\text{Cumulative Frequency} = 3$$

b) For the class 126.5 – 135.5:

$$\text{Cumulative Frequency} = 3 + 5 = 8$$

c) For the class 135.5 – 144.5:

$$\text{Cumulative Frequency} = 8 + 9 = 17$$

d) For the class 144.5 – 153.5:

$$\text{Cumulative Frequency} = 17 + 12 = 29$$

e) For the class 153.5 – 162.5:

$$\text{Cumulative Frequency} = 29 + 5 = 34$$

f) For the class 162.5 – 171.5:

$$\text{Cumulative Frequency} = 34 + 4 = 38$$

g) For the class 171.5 – 180.5:

$$\text{Cumulative Frequency} = 38 + 2 = 40$$

h) For the class 50 – 55:

$$\text{Cumulative Frequency} = 92 + 6 = 98$$

i) For the class 55 – 60:

$$\text{Cumulative Frequency} = 98 + 2 = 100$$

j) $\therefore cf = 17$

5) To find the median:

$$\begin{aligned} \text{Median} &= 144.5 + \frac{20 - 17}{12} \times 9 \\ &= 146.75 \end{aligned}$$

The python implementation:

codes/prob19.py