

# Math Document Template

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**Abstract**—This is a document explaining a question about the concept of sum of angles in a triangle.

Download all python codes from

svn co <https://github.com/chakki1234/summer-2020/trunk/linearalg/codes>

and latex-tikz codes from

svn co <https://github.com/chakki1234/summer-2020/trunk/linearalg/figs>

## 1 PROBLEM

In a  $ABCD$  is a cyclic quadrilateral with

$$\angle A = 4y + 20 \quad (1.0.1)$$

$$\angle B = 3y - 5 \quad (1.0.2)$$

$$\angle C = -4x \quad (1.0.3)$$

$$\angle D = -7x + 5 \quad (1.0.4)$$

Find its angles.

## 2 CONSTRUCTION

2.1. The figure obtained looks like Fig. 2.0.

2.2. The design parameters used for construction  
See Table. 2.2.

| Design Parameters |       |
|-------------------|-------|
| Parameters        | Value |
| a                 | 10    |
| b                 | 8     |

TABLE 2.2: Quadilateral  $ABCD$

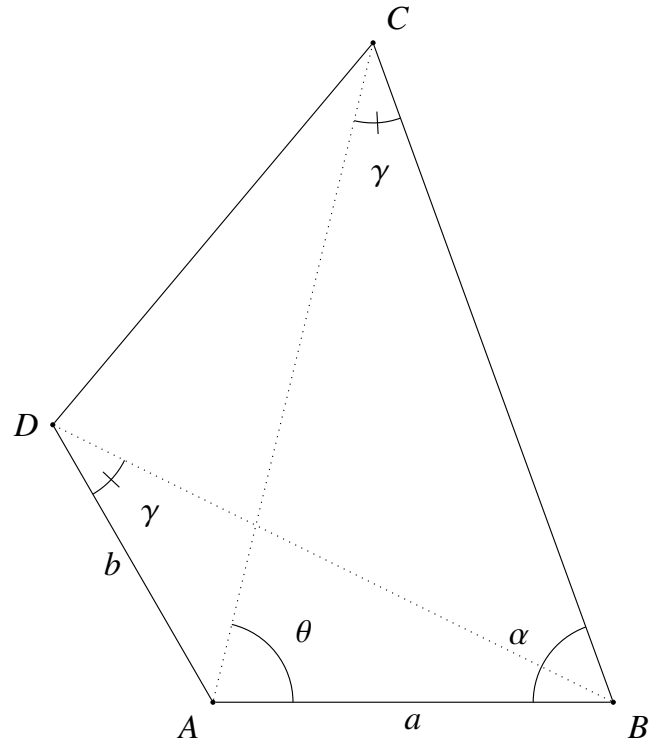


Fig. 2.0: Cyclic quadilateral by Latex-Tikz

2.3. Coordinates of cyclic quadilateral Fig2.0.

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.3.1)$$

$$\mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (2.3.2)$$

$$\mathbf{C} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (2.3.3)$$

$$\mathbf{D} = \begin{pmatrix} b \cos \theta \\ b \sin \theta \end{pmatrix} \quad (2.3.4)$$

2.4. To find the coordinates of  $\mathbf{C}$ .

**Property:**  $\angle ADB = \angle BCA$  since  $ABCD$  is a cyclic quadilateral.

$$\cos \gamma = \frac{(\mathbf{A} - \mathbf{D})^T (\mathbf{B} - \mathbf{D})}{\|\mathbf{A} - \mathbf{D}\| \|\mathbf{B} - \mathbf{D}\|} \quad (2.4.1)$$

$$\theta = 180^\circ - \gamma - \angle B \quad (2.4.2)$$

In  $\triangle ACB$ . Finding the Scalar Products:

$$\begin{aligned} (\mathbf{B} - \mathbf{A})^T (\mathbf{C} - \mathbf{A}) &= \\ \|\mathbf{B} - \mathbf{A}\| \|\mathbf{C} - \mathbf{A}\| \cos \theta \end{aligned} \quad (2.4.3)$$

$$\begin{aligned} (\mathbf{C} - \mathbf{B})^T (\mathbf{A} - \mathbf{B}) &= \\ \|\mathbf{C} - \mathbf{B}\| \|\mathbf{A} - \mathbf{B}\| \cos \alpha \end{aligned} \quad (2.4.4)$$

On simplifying equation 2.4.3 and 2.4.4:

$$x^2 \tan^2 \theta = y^2 \quad (2.4.5)$$

$$(x - a)^2 = ((x - a)^2 + y^2) \cos^2 \alpha \quad (2.4.6)$$

Substituting 2.4.5 in 2.4.6:

$$\begin{aligned} x^2 (1 - \cos^2 \alpha - \tan^2 \theta \cos^2 \alpha) \\ + x(2a \cos^2 \alpha - 2a) + a^2 \sin^2 \alpha \end{aligned} \quad (2.4.7)$$

If  $\theta$  and  $\alpha$  are acute angles:

$$x = \frac{(-b - \sqrt{b^2 - 4ac})}{2a} \quad (2.4.8)$$

else:

$$x = \frac{(-b + \sqrt{b^2 - 4ac})}{2a} \quad (2.4.9)$$

The value of  $x$  can then be substituted in 2.4.5 to find the coordinates of  $\mathbf{C}$

2.5. From the given information, The values are listed in 2.5

| Output values |  |
|---------------|--|
| Parameter     | Value                                      |
| C             | $\begin{pmatrix} 4 \\ 16.47 \end{pmatrix}$ |
| D             | $\begin{pmatrix} -4 \\ 6.93 \end{pmatrix}$ |

TABLE 2.5: Values of  $\mathbf{C}$  and  $\mathbf{D}$

2.6. Draw Fig. 2.6.

**Solution:** The following Python code generates Fig. 2.6

```
codes/cyclic_quad.py
```

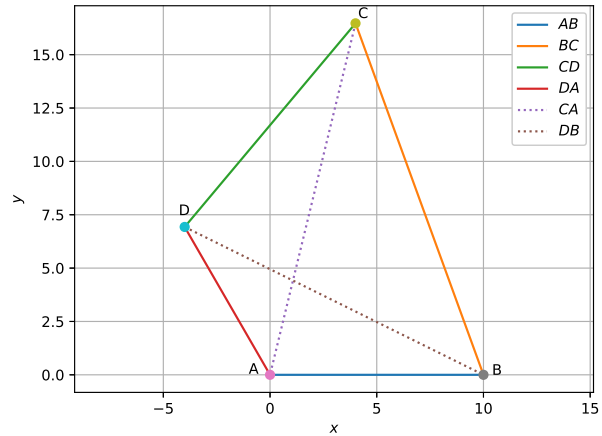


Fig. 2.6: Triangle generated using python

and the equivalent latex-tikz code generating Fig. 2.6 is

```
figs/cyclic_quad_fig.tex
```

The above latex code can be compiled as a standalone document as

```
figs/cyclic_quad_final.tex
```

### 3 SOLUTION

**Theorem 3.1.** *Sum of opposite angles in a cyclic quadrilateral equals  $180^\circ$ .*

**Solution:** From theorem 3.1

$$\angle A + \angle C = 180^\circ \quad (6.1)$$

$$\angle B + \angle D = 180^\circ \quad (6.2)$$

From the given information:

$$4y + 20 - 4x = 180^\circ \quad (6.3)$$

$$3y - 5 - 7x + 5 = 180^\circ \quad (6.4)$$

Solving equations 6.3 and 6.4:

$$x = -15 \quad (6.5)$$

$$y = 25 \quad (6.6)$$

$$\Rightarrow \angle A = 120^\circ \quad (6.7)$$

$$\Rightarrow \angle B = 70^\circ \quad (6.8)$$

$$\Rightarrow \angle C = 60^\circ \quad (6.9)$$

$$\Rightarrow \angle D = 110^\circ \quad (6.10)$$