Math Document Template

C ANISH

Abstract—This is a document explaining for a question about the concept of congruence.

Download all python codes from

svn co https://github.com/chakki1234/summer -2020/trunk/traingle/codes

and latex-tikz codes from

svn co https://github.com/chakki1234/summer –2020/trunk/traingle/figs

1 Problem

In quadilateral ACBD, AC = AD and AB bisects $\angle A$. Show that $\triangle ABC \cong \triangle ABD$. What can you say about BC and BD?

2 Construction

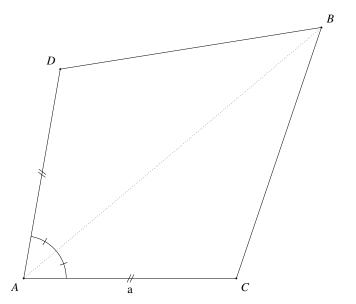


Fig. 2.0: Quadilateral by Latex-Tikz

- 2.1. The figure obtained looks like Fig. 2.0. AC = AD = a, $\angle DAB = \angle BAC$ and $\angle A = \theta$.
- 2.2. The design parameters used for construction **Solution:** See Table. 2.2.

Design Parameters	
Parameters	Value
AC	5
∠A	80
λ	6

TABLE 2.2: Quadilateral ACBD

2.3. Find the coordinates of the various points in Fig

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.3.1}$$

$$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \tag{2.3.2}$$

$$\mathbf{D} = a \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \tag{2.3.3}$$

2.4. **Proof**: To find the angle bisector if three points are know.

Solution: Consider $\triangle DAC$ from the quadrilateral, let O be the incenter and P_1 be the point where the perpendicular from O intersects AC. Similarly let P_2 be the point where the perpendicular from O intersects AC. Let A be at the orign.

$$\mathbf{P_1} = \frac{\mathbf{C}}{\parallel \mathbf{C} \parallel} \tag{2.4.1}$$

$$\mathbf{P_2} = \frac{\mathbf{D}}{\parallel \mathbf{D} \parallel} \tag{2.4.2}$$

$$\mathbf{P_1P_2} \perp \mathbf{AB} \tag{2.4.3}$$

B
$$\epsilon$$
 (**P**₁ - **P**₂) (2.4.4)

$$\|\mathbf{P_1}\| = \|\mathbf{P_2}\| \tag{2.4.5}$$

$$||\mathbf{P_1}||^2 = ||\mathbf{P_2}||^2 \tag{2.4.6}$$

$$(\mathbf{P_1} - \mathbf{P_2})^T (\mathbf{P_1} + \mathbf{P_2}) = 0$$
 (2.4.7)

$$P_1 + P_2 \perp P_1 - P_2$$
 (2.4.8)

From 2.4.4 and 2.4.8, The equation of AB is:

$$\mathbf{AB} = \lambda \left(\mathbf{P_1} + \mathbf{P_2} \right) \tag{2.4.9}$$

$$\implies \mathbf{B} = \lambda \left(\frac{\mathbf{C}}{\parallel \mathbf{C} \parallel} + \frac{\mathbf{D}}{\parallel \mathbf{D} \parallel} \right) \qquad (2.4.10)$$

If A is not at the origin:

$$\mathbf{B} = \mathbf{A} + \lambda \left(\frac{\mathbf{C}}{\parallel \mathbf{C} \parallel} + \frac{\mathbf{D}}{\parallel \mathbf{D} \parallel} \right) \tag{2.4.11}$$

 λ is the length of the angle bisector

Solution: From the given information, The values are listed in 2.4

Ouput values	
Parameter	value
В	$\begin{pmatrix} 7 \\ 5.9 \end{pmatrix}$

TABLE 2.4: B

2.5. Draw Fig. 2.5.

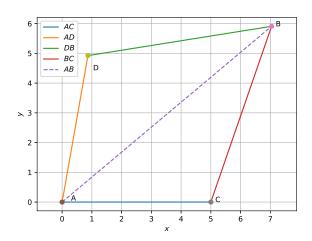


Fig. 2.5: quadilateral generated using python

Solution: The following Python code generates Fig. 2.5

codes/quad.py

and the equivalent latex-tikz code generating Fig. 2.5 is

figs/quad fig.tex

The above latex code can be compiled as a standalone document as

figs/quad final.tex

3 Solution

 $\triangle ABC \cong \triangle ABD$ by SAS congruence:

$$AD = AC$$

$$\angle DAB = \angle BAC$$

DB is common

$$\implies BC = BD$$