Math Document Template

C ANISH

***Abstract*—This is a document explaining for a question about the concept of congruence.**

Download all python codes from

```
svn co https://github.com/chakki1234/summer
    −2020/trunk/traingle/codes
```

and latex-tikz codes from

```
svn co https://github.com/chakki1234/summer
    −2020/trunk/traingle/figs
```

1 Problem

In quadilateral $ACBD$, $AC = AD$ and AB bisects $\angle A$.Show that $\triangle ABC \cong \triangle ABD$.What can you say about BC and BD?

2 Construction

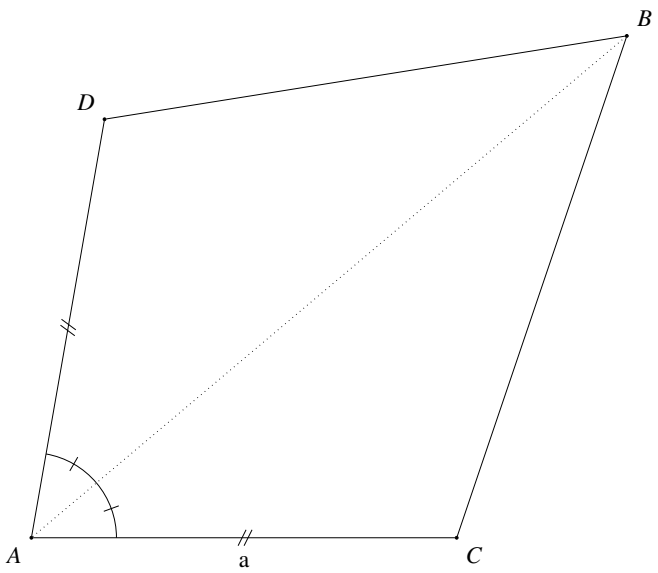

Fig. 2.0: Quadilateral by Latex-Tikz

2.1. The figure obtained looks like Fig. 2.0.
$AC = AD = a$, $\angle DAB = \angle BAC$ and $\angle A = \theta$.

2.2. The design parameters used for construction
Solution: See Table. 2.2.

Design Parameters	
Parameters	Value
AC	5
$\angle A$	80
AB	6

TABLE 2.2: Quadilateral $ACBD$

2.3. Find the coordinates of the various points in Fig

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.3.1}$$

$$\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \tag{2.3.2}$$

$$\mathbf{D} = a\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{2.3.3}$$

2.4. **Proof**: To find the angle bisector if three points are know.
Solution: Consider $\triangle DAC$ from the quadrilateral, let O be the incenter and P_1 be the point where the perpendicular from O intersects AC.Similarly let P_2 be the point where the perpendicular from O intersects AC.Let A be at the orign.

$$\mathbf{P_1} = \frac{\mathbf{C}}{\| \mathbf{C} \|} \tag{2.4.1}$$

$$\mathbf{P_2} = \frac{\mathbf{D}}{\| \mathbf{D} \|} \tag{2.4.2}$$

$$\mathbf{P_1P_2} \perp \mathbf{AB} \tag{2.4.3}$$

$$\mathbf{B} \in (\mathbf{P_1} - \mathbf{P_2}) \tag{2.4.4}$$

$$\|\mathbf{P_1}\| = \|\mathbf{P_2}\| \tag{2.4.5}$$

$$\|\mathbf{P_1}\|^2 = \|\mathbf{P_2}\|^2 \tag{2.4.6}$$

$$(\mathbf{P_1} - \mathbf{P_2})^T(\mathbf{P_1} + \mathbf{P_2}) = 0 \tag{2.4.7}$$

$$\mathbf{P_1} + \mathbf{P_2} \perp \mathbf{P_1} - \mathbf{P_2} \tag{2.4.8}$$

From **2.4.4** and **2.4.8**, The equation of AB is:

$$\mathbf{AB} = \lambda(\mathbf{P_1} + \mathbf{P_2}) \tag{2.4.9}$$

$$\implies \mathbf{B} = \lambda\left(\frac{\mathbf{C}}{\| \mathbf{C} \|} + \frac{\mathbf{D}}{\| \mathbf{D} \|}\right) \tag{2.4.10}$$

If A is not at the origin :

$$\mathbf{B} = \mathbf{A} + \lambda\left(\frac{\mathbf{C}}{\| \mathbf{C} \|} + \frac{\mathbf{D}}{\| \mathbf{D} \|}\right) \tag{2.4.11}$$

λ is the length of the angle bisector

Solution: From the given information, The values are listed in 2.4

Ouput values	
Parameter	value
B	$\begin{pmatrix} 7 \\ 5.9 \end{pmatrix}$

TABLE 2.4: **B**

2.5. Draw Fig. 2.5.

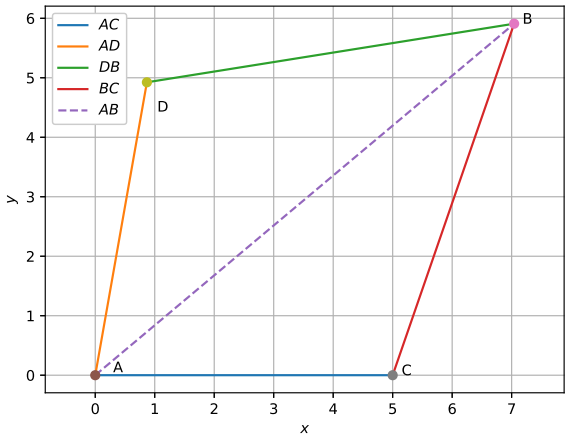

Fig. 2.5: quadilateral generated using python

Solution: The following Python code generates Fig. 2.5

codes/quad.py

and the equivalent latex-tikz code generating Fig. 2.5 is

figs/quad_fig.tex

The above latex code can be compiled as a standalone document as

figs/quad_final.tex

3 Solution

$\triangle ABC \cong \triangle ABD$ by SAS congruence:

$$AD = AC$$

$$\angle DAB = \angle BAC$$

$$DB \text{ is common}$$

$$\implies BC = BD$$