Math Document Template

C ANISH

***Abstract*—This is a document explaining a question about the concept of cyclic quadilateral.**

Download all python codes from

```
svn co https://github.com/chakki1234/summer
    -2020/trunk/Circle/codes
```

and latex-tikz codes from

```
svn co https://github.com/chakki1234/summer
    -2020/trunk/Circle/figs
```

1 Problem

If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

2 Construction

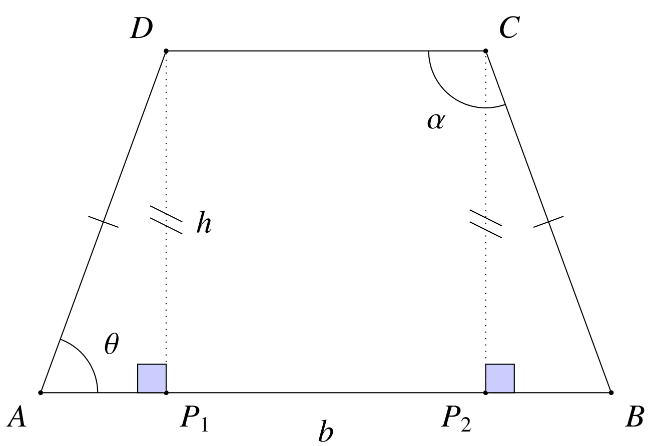

Fig. 2.0: Trapezium by Latex-Tikz

2.1. The figure obtained looks like Fig. 2.0. $AD = BC$, $AB \parallel DC$.

2.2. The design parameters used for construction See Table. 2.2.

2.3. Find the coordinates of the various points in Fig

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.3.1}$$

$$\mathbf{B} = \mathbf{A} + \begin{pmatrix} b \\ 0 \end{pmatrix}, \tag{2.3.2}$$

Design Parameters	
Parameters	Value
b	5
h	3
$\angle A$	70

TABLE 2.2: Trapezium $ABCD$

$$\mathbf{C} = \mathbf{B} + \begin{pmatrix} -h\cot\theta \\ h \end{pmatrix} \tag{2.3.3}$$

$$\implies \mathbf{C} = \begin{pmatrix} b - h\cot\theta \\ h \end{pmatrix} \tag{2.3.4}$$

$$\mathbf{D} = \mathbf{A} + \begin{pmatrix} h\cot\theta \\ h \end{pmatrix} \tag{2.3.5}$$

$$\implies \mathbf{D} = \begin{pmatrix} h\cot\theta \\ h \end{pmatrix} \tag{2.3.6}$$

2.4. **Solution:** From the given information, The values are listed in 2.4

Ouput values	
Parameter	Value
$\mathbf{C}$	$\begin{pmatrix} 3.9 \\ 3 \end{pmatrix}$
$\mathbf{D}$	$\begin{pmatrix} 1.09 \\ 3 \end{pmatrix}$

TABLE 2.4: Values of $\mathbf{C}$ *and* $\mathbf{D}$

2.5. Draw Fig. 2.0.

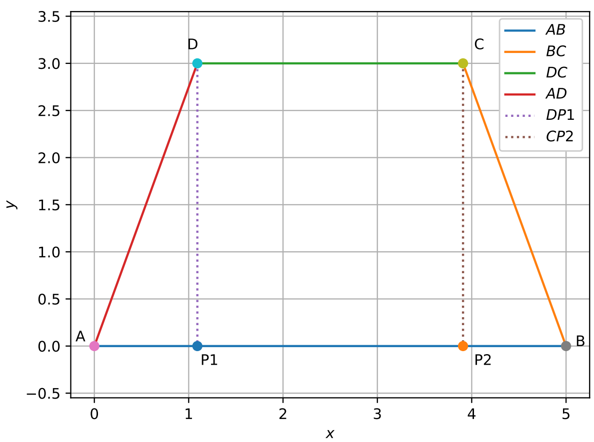

Fig. 2.5: Trapezium generated using python

Solution: The following Python code generates Fig. 2.5

```
codes/quad.py
```

and the equivalent latex-tikz code generating Fig. 2.5 is

```
figs/trapezium_altitude_fig.tex
```

The above latex code can be compiled as a standalone document as

```
figs/trapezium_final_altitude.tex
```

3 Solution

Theorem 3.1. *In a cyclic quadrilateral, the sum of each pair of opposite angles is* $180°$.

Solution: From theorem 3.1 to prove $ABCD$ is a cyclic quadilateral, it is sufficient to prove that sum of opposite angles is $180°$.

From 2.3.1, 2.3.2, 2.3.4, 2.3.6

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} b \\ 0 \end{pmatrix} \tag{5.1}$$

$$\mathbf{D}-\mathbf{C} = \begin{pmatrix} 2h\cot\theta - b \\ 0 \end{pmatrix} \tag{5.2}$$

$$\mathbf{D}-\mathbf{A} = \begin{pmatrix} h\cot\theta \\ h \end{pmatrix} \tag{5.3}$$

$$\mathbf{B}-\mathbf{C} = \begin{pmatrix} h\cot\theta \\ -h \end{pmatrix} \tag{5.4}$$

Finding the scalar products:

$$(\mathbf{D}-\mathbf{A})^T(\mathbf{B}-\mathbf{A}) = \|\mathbf{D}-\mathbf{A}\|\,\|\mathbf{B}-\mathbf{A}\|\cos\theta \tag{5.5}$$

$$(\mathbf{D}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = \|\mathbf{D}-\mathbf{C}\|\,\|\mathbf{B}-\mathbf{C}\|\cos\alpha \tag{5.6}$$

Dividing 5.6 with 5.5:

$$\frac{(\mathbf{D}-\mathbf{A})^T(\mathbf{B}-\mathbf{A})}{(\mathbf{D}-\mathbf{C})^T(\mathbf{B}-\mathbf{C})} = \frac{\|\mathbf{D}-\mathbf{A}\|\,\|\mathbf{B}-\mathbf{A}\|\cos\theta}{\|\mathbf{D}-\mathbf{C}\|\,\|\mathbf{B}-\mathbf{C}\|\cos\alpha} \tag{5.7}$$

Since $\|\mathbf{D}-\mathbf{A}\| = \|\mathbf{B}-\mathbf{C}\|$, 5.7 can be simplified to the form:

$$\frac{(\mathbf{D}-\mathbf{A})^T(\mathbf{B}-\mathbf{A})}{(\mathbf{D}-\mathbf{C})^T(\mathbf{B}-\mathbf{C})} = \frac{\|\mathbf{B}-\mathbf{A}\|\cos\theta}{\|\mathbf{D}-\mathbf{C}\|\cos\alpha} \tag{5.8}$$

Substituting values from 5.1, 5.2, 5.3 and 5.4:

$$\frac{bh\cot\theta}{(2h\cot\theta - b)\,h\cot\theta} = \frac{b\cos\theta}{b-2h\cot\theta} \tag{5.9}$$

$$\implies \cos\alpha = -\cos\theta \tag{5.10}$$

$$\implies \alpha+\theta = 180° \tag{5.11}$$

$\therefore$ $ABCD$ is a cyclic quadilateral.