Math Document Template

C ANISH

Abstract—**This is a document explaining questions about the concept of Linear algebra.**

Download all python codes from

```
svn co https://github.com/chakki1234/summer
    -2020/trunk/linearalg/codes
```

and latex-tikz codes from

```
svn co https://github.com/chakki1234/summer
    -2020/trunk/linearalg/figs
```

1 Triangle

1.1 Problem

In a $\triangle ABC$, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

1.2 Solution

Theorem 1.1. *Sum of all angles in a triangle equals* $180°$.

1.2.1. **Solution:** From theorem 1.1

$$\angle A + \angle B + \angle C = 180° \tag{1.2.1.1}$$

1.2.2. From the given information:

$$\frac{\angle C}{6} + \frac{\angle C}{3} + \angle C = 180° \tag{1.2.2.1}$$

$$\therefore \angle C = 120° \ \angle A = 20° \ \angle B = 40°$$

1.2.3. The following Python code generates Fig. 1.2.3

```
codes/triangle_ex/triangle_linearalg.py
```

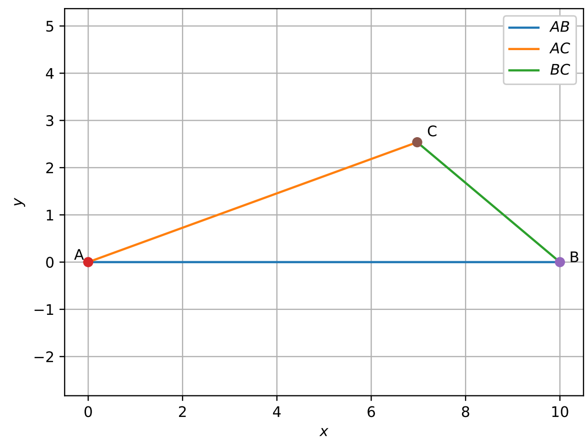

Fig. 1.2.3: Triangle generated using python

2 Quadilateral

2.1 Problem

In a $ABCD$ is a cyclic quadilateral with

$$\angle A = 4y + 20 \tag{3.1}$$
$$\angle B = 3y - 5 \tag{3.2}$$
$$\angle C = -4x \tag{3.3}$$
$$\angle D = -7x + 5 \tag{3.4}$$

Find its angles.

2.2 Solution

Theorem 2.1. *Sum of opposite angles in a cyclic quadilateral equals* $180°$.

2.2.1. **Solution:** From theorem 2.1

$$\angle A + \angle C = 180° \tag{2.2.1.1}$$
$$\angle B + \angle D = 180° \tag{2.2.1.2}$$

2.2.2. From the given information:

$$4y + 20 - 4x = 180° \tag{2.2.2.1}$$
$$3y - 5 - 7x + 5 = 180° \tag{2.2.2.2}$$

2.2.3. Solving equations 2.2.2.1 and 2.2.2.2:

$$x = -15 \tag{2.2.3.1}$$
$$y = 25 \tag{2.2.3.2}$$
$$\implies \angle A = 120^\circ \tag{2.2.3.3}$$
$$\implies \angle B = 70^\circ \tag{2.2.3.4}$$
$$\implies \angle C = 60^\circ \tag{2.2.3.5}$$
$$\implies \angle D = 110^\circ \tag{2.2.3.6}$$

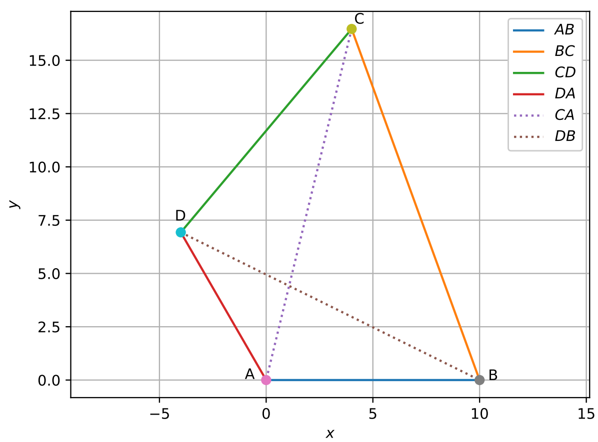

Fig. 2.2.4: Quadilateral generated using python

2.2.4. The following Python code generates Fig. 2.2.4

```
codes/quadilateral_ex/cyclic_quad.py
```

3 Line

3.1 Points and vectors

3.1.1 Problem:

Find the distance between the points $\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}36\\15\end{pmatrix}$.

3.1.2 Solution:

1)

$$\mathbf{A} = \begin{pmatrix}0\\0\end{pmatrix} \tag{1.1}$$
$$\mathbf{B} = \begin{pmatrix}36\\15\end{pmatrix} \tag{1.2}$$

2) Distance between $\mathbf{A}$ and $\mathbf{B}$ is:

$$\|\mathbf{A} - \mathbf{B}\| \tag{2.1}$$

3) From the given information:

$$\|\begin{pmatrix}0\\0\end{pmatrix} - \begin{pmatrix}36\\15\end{pmatrix}\| = 39 \tag{3.1}$$

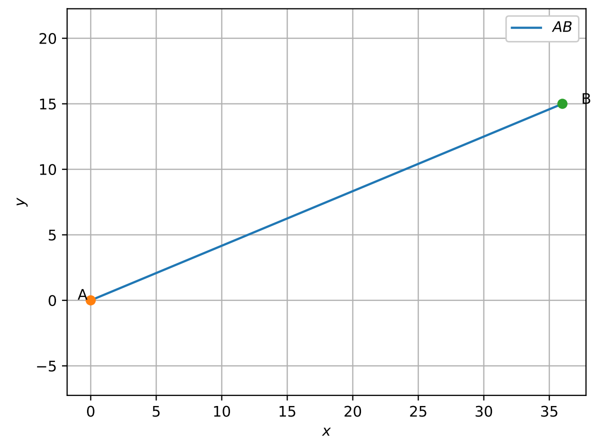

Fig. 4: Line AB generated using python

4) The following Python code generates Fig. 4

```
codes/line_ex/pts_and_vectors/
    dist_btw_pts.py
```

3.2 Points on a line

3.2.1 Problem:

Find the coordinates of the points of trisection of the line segment joining $\begin{pmatrix}4\\-1\end{pmatrix}$ and $\begin{pmatrix}-2\\-3\end{pmatrix}$.

3.2.2 Solution:

1) Let $\mathbf{E}$ be a point which divides line segment AB in the ratio $k : 1$:
2)

$$\mathbf{E} = \frac{k\mathbf{A} + \mathbf{B}}{k+1} \tag{2.1}$$

3) $\mathbf{C}$ divides the line in the ratio $\frac{1}{2} : 1$ and $\mathbf{D}$ divides the line in the raio $\frac{2}{1} : 1$
4)

$$\mathbf{C} = \frac{0.5\mathbf{A} + \mathbf{B}}{0.5+1} \tag{4.1}$$
$$\mathbf{D} = \frac{2\mathbf{A} + \mathbf{B}}{2+1} \tag{4.2}$$
$$\therefore \mathbf{C} = \begin{pmatrix}0\\-2.33\end{pmatrix} \tag{4.3}$$
$$\therefore \mathbf{D} = \begin{pmatrix}2\\-1.66\end{pmatrix} \tag{4.4}$$

5) The following Python code generates Fig. 5

```
codes/line_ex/pts_on_a_line/trisection.py
```

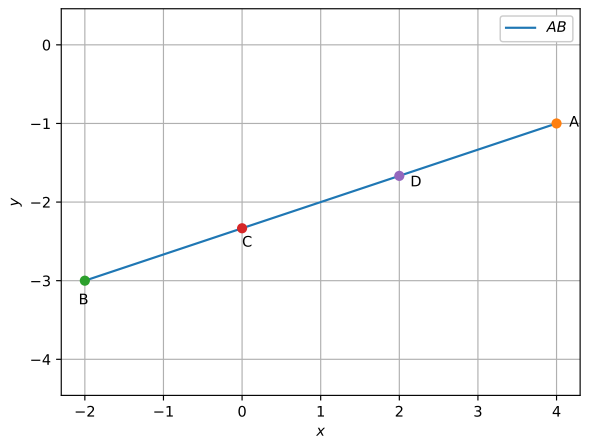

Fig. 5: Line AB trisected - generated using python

3.3 Lines and planes

3.3.1 Problem:

Find the zero of the polynomial in each of the following cases:

$$p(x) = x + 5$$
$$p(x) = x - 5$$
$$p(x) = 2x + 5$$
$$p(x) = 3x - 2$$
$$p(x) = 3x$$

3.3.2 Solution:

1) **Solution:** For $p(x) = x + 5$

   The given equation can be represented as follows in the vector form:

$$\begin{pmatrix} 5 & -1 \end{pmatrix} x + 5 = 0 \tag{1.1}$$

   To find the roots $y = 0$:

$$x + 5 = 0 \tag{1.2}$$
$$x = -5 \tag{1.3}$$

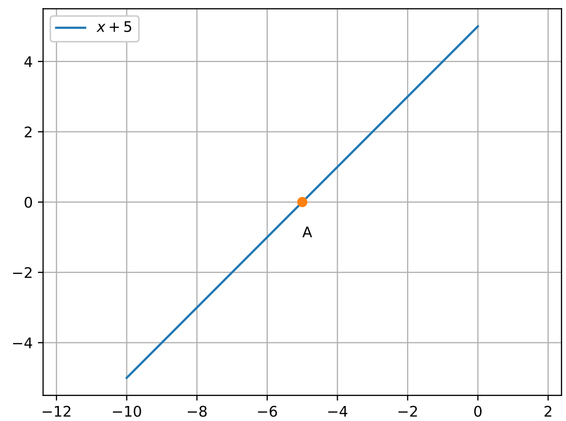

Fig. 1: $x + 5$ generated using python

2) **Solution:** For $p(x) = x - 5$
   The given equation can be represented as follows in the vector form:

$$\begin{pmatrix} 5 & -1 \end{pmatrix} x - 5 = 0 \tag{2.1}$$

   To find the roots $y = 0$:

$$x - 5 = 0 \tag{2.2}$$
$$x = 5 \tag{2.3}$$

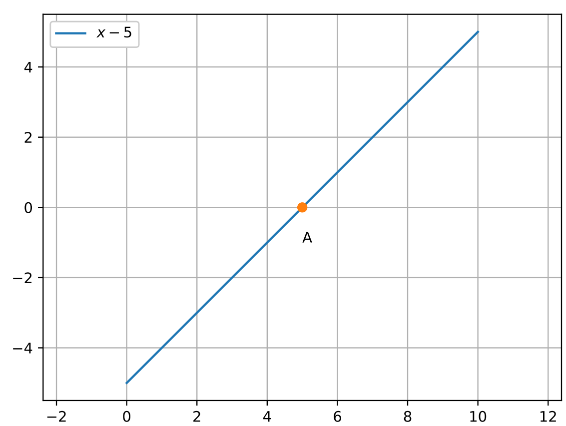

Fig. 2: $x - 5$ generated using python

3) **Solution:** For $p(x) = 2x + 5$
   The given equation can be represented as follows in the vector form:

$$\begin{pmatrix} 2 & -1 \end{pmatrix} x + 5 = 0 \tag{3.1}$$

To find the roots $y = 0$:

$$2x + 5 = 0 \tag{3.2}$$

$$x = \frac{-5}{2} \tag{3.3}$$

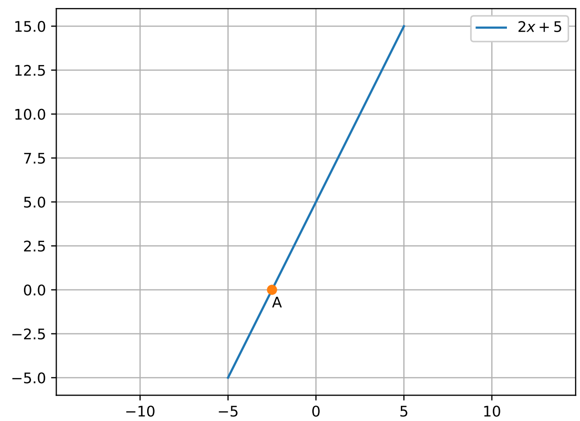

Fig. 3: $2x + 5$ generated using python

4) **Solution:** For $p(x) = 3x - 2$

   The given equation can be represented as follows in the vector form:

$$\begin{pmatrix} 3 & -1 \end{pmatrix} x - 2 = 0 \tag{4.1}$$

   To find the roots $y = 0$:

$$3x - 2 = 0 \tag{4.2}$$

$$x = \frac{2}{3} \tag{4.3}$$

5) **Solution:** For $p(x) = 3x$

   The given equation can be represented as follows in the vector form:

$$\begin{pmatrix} 3 & -1 \end{pmatrix} x = 0 \tag{5.1}$$

   To find the roots $y = 0$:

$$3x = 0 \tag{5.2}$$

$$x = 0 \tag{5.3}$$

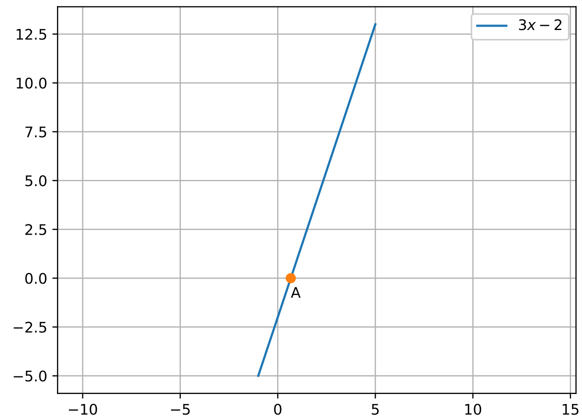

Fig. 4: $3x - 2$ generated using python

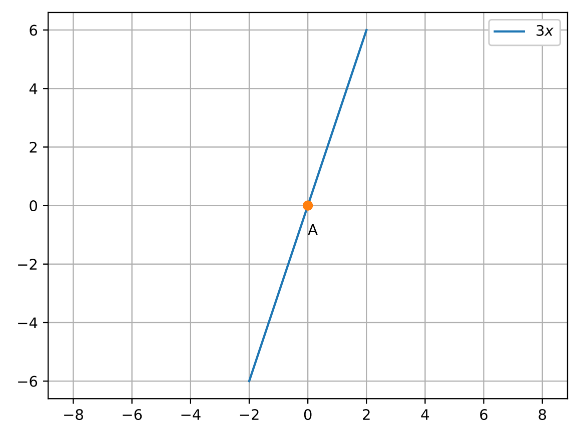

Fig. 5: $3x$ generated using python

3.4 Motion in a plane

3.4.1 Problem:

A motorboat is racing towards north at 25 km/h and the water current in that region is 10 km/h in the direction of 60° east of south. Find the resultant velocity of the boat.

3.4.2 Solution:

1) **A** denotes the velocity of the boat and **B** denotes the water current and **C** represents the resultant velocity.

2)

$$\mathbf{A} = \begin{pmatrix} 0 \\ 25 \end{pmatrix} \tag{2.1}$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ -8.67 \end{pmatrix} \tag{2.2}$$

$$\mathbf{C} = \mathbf{A} + \mathbf{B} \tag{2.3}$$

$$\mathbf{C} = \begin{pmatrix} 5 \\ 16.34 \end{pmatrix} \tag{2.4}$$

3) Magnitude of resulant velocity:

$$\|\mathbf{C}\| = 17.08 \tag{3.1}$$

4) Direction of resultant velocity:

$$\cos\theta = \frac{(\mathbf{A})^T (\mathbf{C})}{\|\mathbf{A}\|\ \|\mathbf{C}\|} \tag{4.1}$$

$$\theta = 17.01° \tag{4.2}$$

5) $\therefore$ The resulant velocity is 17.08 km/h at an angle of 17.01° east of north.

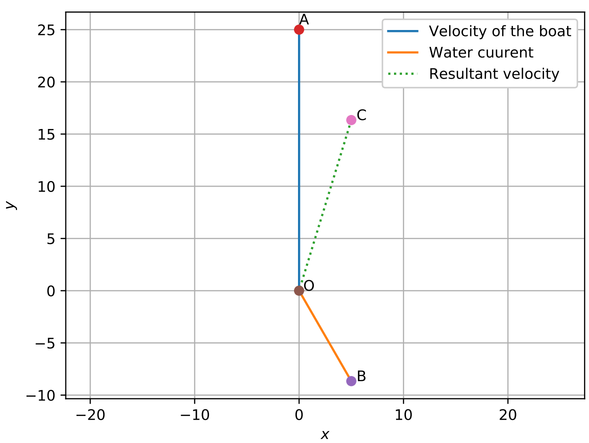

Fig. 6: Vectorial representation of velocities generated using python

6) The following Python code generates Fig. 6

```
codes/line_ex/motion_in_a_plane/
    motion_plane.py
```

3.5 Matrix

3.5.1 Problem:

If a matrix has 24 elements, what are the possible orders it can have ? What, if it has 13 elements.

3.5.2 Solution:

The total number of elements in a matrix is $m\ x\ n$.

1) If the total number of elements is 24. The possible orders are:

$$1x24 = 24 \tag{1.1}$$
$$24x1 = 24 \tag{1.2}$$
$$2x12 = 24 \tag{1.3}$$
$$12x2 = 24 \tag{1.4}$$
$$3x8 = 24 \tag{1.5}$$
$$8x3 = 24 \tag{1.6}$$
$$4x6 = 24 \tag{1.7}$$
$$6x4 = 24 \tag{1.8}$$

2) If the total number of elements is 13. The possible orders are:

$$1x13 = 13 \tag{2.1}$$
$$13x1 = 13 \tag{2.2}$$

3.6 Determinants

3.6.1 Problem:

Find the determinant of

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ (ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

3.6.2 Solution:

1) Determinant of a $2x2$ matrix is obtained as follows
2)

$$A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$detA = a_{11}a_{22} - a_{12}a_{21} \tag{2.1}$$

3) From 2.1:

$$(i)\ det = \cos\theta^2 + \sin\theta^2 = 1 \tag{3.1}$$

$$(ii)\ det = x^3 - x^2 + 2 \tag{3.2}$$

3.7 Linear inequalities

3.7.1 Problem:

Solve $7x + 3\ <\ 5x + 9$.Show the graph of the solutions on number line.

3.7.2 Solution:

1)

$$
\begin{aligned}
7x+3 &< 5x+9 && (1.1)\\
2x-6 &< 0 && (1.2)\\
x &< 3 && (1.3)\\
\therefore\ x &\in \{3,-\infty\} && (1.4)
\end{aligned}
$$

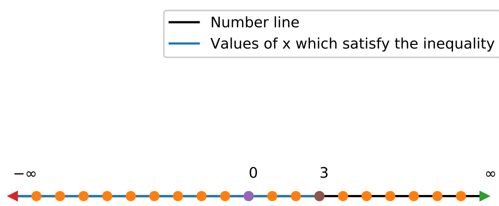

Fig. 2: Values of x satisfying the inequality in the number line generated using python

2) The following Python code generates Fig. 2

```
codes/line_ex/lin_ineq/dist_btw_pts.py
```

4 Circle

4.1 Problem

Find the center of a circle passing through the points $\begin{pmatrix}6\\-6\end{pmatrix}, \begin{pmatrix}3\\-7\end{pmatrix}$ and $\begin{pmatrix}3\\3\end{pmatrix}$.

4.2 Solution

4.2.1. **Solution:** A circle passing through three non-collinear points is the circumcircle and the center is the circumcenter.

$$\mathbf{O} = \frac{A\sin\angle 2A + B\sin\angle 2B + C\sin\angle 2C}{\sin\angle 2A + \sin\angle 2B + \sin\angle 2C}$$

4.2.2. To find the angles:

$$\cos\angle A = \frac{(\mathbf{C}-\mathbf{A})^T(\mathbf{B}-\mathbf{A})}{\|\mathbf{C}-\mathbf{A}\|\ \|\mathbf{B}-\mathbf{A}\|} \quad (4.2.2.1)$$

$$\cos\angle B = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{C}-\mathbf{B})}{\|\mathbf{A}-\mathbf{B}\|\ \|\mathbf{C}-\mathbf{B}\|} \quad (4.2.2.2)$$

$$\cos\angle C = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C})}{\|\mathbf{A}-\mathbf{C}\|\ \|\mathbf{B}-\mathbf{C}\|} \quad (4.2.2.3)$$

4.2.3. Substituting the give values:

$$\therefore \mathbf{O} = \begin{pmatrix}3\\-2\end{pmatrix} \quad (4.2.3.1)$$

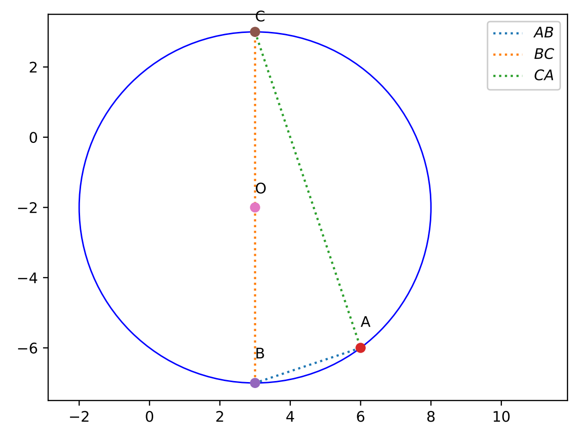

Fig. 4.2.4: Circumcircle generated using python

4.2.4. The following Python code generates Fig. 4.2.4

```
codes/circle_ex/circumcircle.py
```

5 Circle-example

5.1 Problem

Find the center and radius of the circle

$$x^T x + \begin{pmatrix}8\\10\end{pmatrix} x - 8 = 0 \quad (4.1)$$

5.2 Solution

5.2.1. **Solution:** The general of a circle equation is $Ax^2+Bxy+Ay^2+Dx+Ey+F$, the equation can be represented as follow in the vector form:

$$x^T\begin{pmatrix}A & \frac{B}{2}\\ \frac{B}{2} & A\end{pmatrix}x + \begin{pmatrix}D & E\end{pmatrix}x + F = 0 \quad (5.2.1.1)$$

5.2.2. To find the center - **O** and radius - r of a circle:

$$\mathbf{O} = \frac{-1}{2A}\begin{pmatrix} D & E \end{pmatrix} \tag{5.2.2.1}$$

$$r = \frac{1}{A}\sqrt{\frac{1}{4}\|\begin{pmatrix} D \\ E \end{pmatrix}\|^2 - F^2} \tag{5.2.2.2}$$

5.2.3. The values given:

$$A = 1 \tag{5.2.3.1}$$
$$D = 8 \tag{5.2.3.2}$$
$$E = 10 \tag{5.2.3.3}$$
$$F = -8 \tag{5.2.3.4}$$

5.2.4. Substituting the values in equation 5.2.2.1 and 5.2.2.2:

$$\mathbf{O} = \begin{pmatrix} -4 \\ -5 \end{pmatrix} \tag{5.2.4.1}$$
$$r = 7 \tag{5.2.4.2}$$

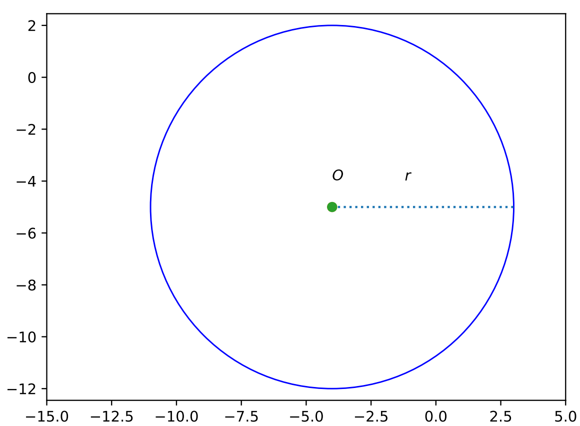

Fig. 5.2.5: Circle generated using python

5.2.5. The following Python code generates Fig. 5.2.5

```
codes/circle_exam.py
```

6 Conics

6.1 Problem

Verify whether 2 and 0 are zeroes of the polynomial $x^2 - 2x$.

6.2 Solution

6.2.1. **Solution:** $p(x, y) = Ax^2+Bxy+Cy^2+Dx+Ey+F$ can be represented as follow in the vector form:

$$x^T \begin{pmatrix} A & \frac{B}{2} \\ \frac{B}{2} & C \end{pmatrix} x + \begin{pmatrix} D & E \end{pmatrix} x + F = 0 \tag{6.2.1.1}$$

6.2.2. The given equation can be represented as follows in the vector form:

$$x^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} -2 & 0 \end{pmatrix} x + 0 = 0 \tag{6.2.2.1}$$

6.2.3. To find the roots $y = 0$:

$$x^2 - 2x = 0 \tag{6.2.3.1}$$
$$x(x-2) = 0 \tag{6.2.3.2}$$
$$x = 0, 2 \tag{6.2.3.3}$$

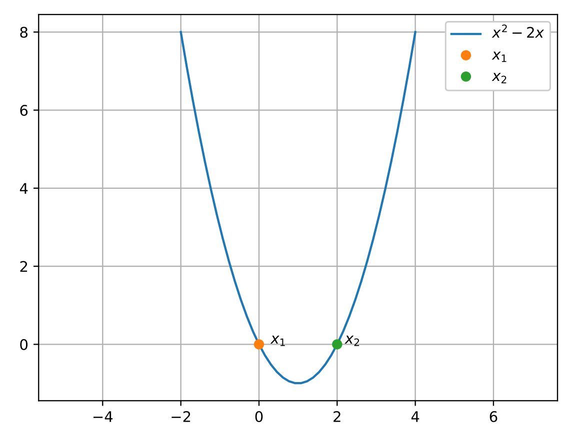

Fig. 6.2.4: $x^2 - 2x$ generated using python

6.2.4. The following Python code generates Fig. 6.2.4

```
codes/conics_example/conics.py
```