

Math Document Template

C ANISH

Abstract—This is a document explaining questions about the concept of Linear algebra.

Download all python codes from

svn co <https://github.com/chakki1234/summer-2020/trunk/linearalg/codes>

and latex-tikz codes from

svn co <https://github.com/chakki1234/summer-2020/trunk/linearalg/figs>

1 TRIANGLE

1.1 Problem

In a $\triangle ABC$, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

1.2 Solution

Theorem 1.1. Sum of all angles in a triangle equals 180° . 1.2.2.

1.2.1. **Solution:** From theorem 1.1

$$\angle A + \angle B + \angle C = 180^\circ \quad (1.2.1.1)$$

From the given information:

$$\angle A = \angle C \quad (1.2.1.2)$$

$$\angle B = \angle C \quad (1.2.1.3)$$

$$\angle A + \angle B + \angle C = 180^\circ \quad (1.2.1.4)$$

In vector form:

$$\begin{pmatrix} 6 & 0 & -1 \\ 0 & 3 & -1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 180 \end{pmatrix} \quad (1.2.1.5)$$

To find the angles:

$$\begin{pmatrix} 6 & 0 & -1 & 0 \\ 0 & 3 & -1 & 0 \\ 1 & 1 & 1 & 180 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{R_1}{6}} \begin{pmatrix} 1 & 0 & -\frac{1}{6} & 0 \\ 0 & 3 & -1 & 0 \\ 1 & 1 & 1 & 180 \end{pmatrix} \quad (1.2.1.6)$$

$$\xrightarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 1 & 0 & -\frac{1}{6} & 0 \\ 0 & 3 & -1 & 0 \\ 0 & 1 & \frac{7}{6} & 180 \end{pmatrix} \quad (1.2.1.7)$$

$$\xrightarrow{R_2 \leftarrow \frac{R_2}{3}} \begin{pmatrix} 1 & 0 & -\frac{1}{6} & 0 \\ 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 1 & \frac{7}{6} & 180 \end{pmatrix} \quad (1.2.1.8)$$

$$\xrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 0 & -\frac{1}{6} & 0 \\ 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{3}{2} & 180 \end{pmatrix} \quad (1.2.1.9)$$

$$\xrightarrow{R_3 \leftarrow \frac{2R_3}{3}} \begin{pmatrix} 1 & 0 & -\frac{1}{6} & 0 \\ 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 120 \end{pmatrix} \quad (1.2.1.10)$$

$$\xrightarrow{\begin{matrix} R_1 \leftarrow R_1 + \frac{R_3}{6} \\ R_2 \leftarrow R_2 + \frac{R_3}{3} \end{matrix}} \begin{pmatrix} 1 & 0 & 1 & 20 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & 120 \end{pmatrix} \quad (1.2.1.11)$$

$$\therefore \angle C = 120^\circ \quad \angle A = 20^\circ \quad \angle B = 40^\circ$$

1.2.3. The following Python code generates Fig. 1.2.3

codes/triangle_ex/triangle_linearalg.py

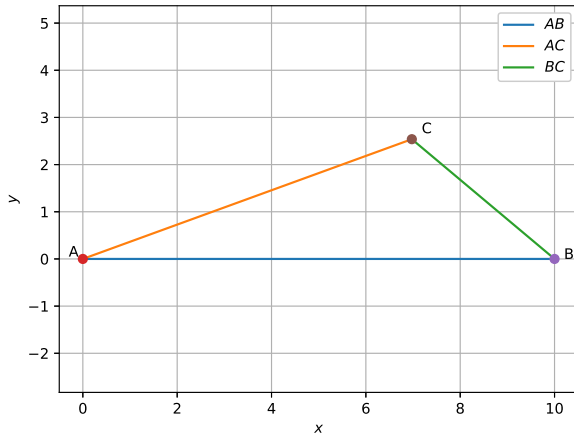


Fig. 1.2.3: Triangle generated using python

2 QUADILATERAL

2.1 Problem

In a $ABCD$ is a cyclic quadrilateral with

$$\angle A = 4y + 20 \quad (3.1)$$

$$\angle B = 3y - 5 \quad (3.2)$$

$$\angle C = -4x \quad (3.3)$$

$$\angle D = -7x + 5 \quad (3.4)$$

Find its angles.

2.2 Solution

Theorem 2.1. *Sum of opposite angles in a cyclic quadrilateral equals 180° .*

2.2.1. **Solution:** From theorem 2.1

$$\angle A + \angle C = 180^\circ \quad (2.2.1.1)$$

$$\angle B + \angle D = 180^\circ \quad (2.2.1.2)$$

2.2.2. From the given information:

$$\begin{pmatrix} -4 & 4 \\ -7 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 160 \\ 180 \end{pmatrix} \quad (2.2.2.1)$$

To find the angles:

$$\begin{pmatrix} -4 & 4 & 160 \\ -7 & 3 & 180 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{-R_1}{4}} \begin{pmatrix} 1 & -1 & -40 \\ -7 & 3 & 180 \end{pmatrix} \quad (2.2.2.2)$$

$$\xrightarrow{R_2 \leftarrow R_2 + 7R_1} \begin{pmatrix} 1 & -1 & -40 \\ 0 & -4 & -100 \end{pmatrix} \quad (2.2.2.3)$$

$$\xrightarrow{R_2 \leftarrow \frac{-R_2}{4}} \begin{pmatrix} 1 & -1 & -40 \\ 0 & 1 & 25 \end{pmatrix} \quad (2.2.2.4)$$

$$\xrightarrow{R_1 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 0 & -15 \\ 0 & 1 & 25 \end{pmatrix} \quad (2.2.2.5)$$

2.2.3.

$$x = -15 \quad (2.2.3.1)$$

$$y = 25 \quad (2.2.3.2)$$

$$\Rightarrow \angle A = 120^\circ \quad (2.2.3.3)$$

$$\Rightarrow \angle B = 70^\circ \quad (2.2.3.4)$$

$$\Rightarrow \angle C = 60^\circ \quad (2.2.3.5)$$

$$\Rightarrow \angle D = 110^\circ \quad (2.2.3.6)$$

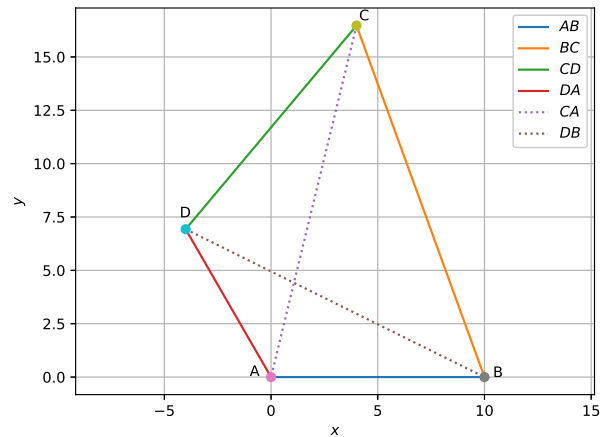


Fig. 2.2.4: Quadrilateral generated using python

2.2.4. The following Python code generates Fig. 2.2.4

```
codes/quadrilateral_ex/cyclic_quad.py
```

3 LINE

3.1 Complex Numbers

3.1.1 Problem:

Find $\begin{pmatrix} -\sqrt{3} \\ \sqrt{2} \end{pmatrix} \begin{pmatrix} 2\sqrt{3} \\ -1 \end{pmatrix}$

3.1.2 Solution: A complex number $\begin{pmatrix} a \\ b \end{pmatrix}$ can be represented as 2 x 2 matrix:

$$\begin{pmatrix} a & -b \\ b & a \end{pmatrix} \quad (4.1)$$

- 1) Multiplying the given matrices after converting them to a 2 x 2 matrix:

$$\begin{pmatrix} -\sqrt{3} & -\sqrt{2} \\ \sqrt{2} & -\sqrt{3} \end{pmatrix} \begin{pmatrix} 2\sqrt{3} & 1 \\ -1 & 2\sqrt{3} \end{pmatrix} \quad (1.1)$$

$$\Rightarrow \begin{pmatrix} \sqrt{2} - 6 & -\sqrt{3} - 2\sqrt{6} \\ \sqrt{3} + 2\sqrt{6} & \sqrt{2} - 6 \end{pmatrix} \quad (1.2)$$

- 2) Matrix 1.2 can be represented as a vector:

$$\therefore \begin{pmatrix} \sqrt{2} - 6 \\ \sqrt{3} + 2\sqrt{6} \end{pmatrix} \quad (2.1)$$

- 3) Python code to multiply two complex numbers:

```
codes/line_ex/complex_ex/complex_ex.py
```

3.2 Points and vectors

3.2.1 Problem:

Find the distance between the points $\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 36 \\ 15 \end{pmatrix}$.

3.2.2 Solution:

- 1)

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (1.1)$$

$$\mathbf{B} = \begin{pmatrix} 36 \\ 15 \end{pmatrix} \quad (1.2)$$

- 2) Distance between \mathbf{A} and \mathbf{B} is:

$$\|\mathbf{A} - \mathbf{B}\| \quad (2.1)$$

- 3) From the given information:

$$\left\| \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 36 \\ 15 \end{pmatrix} \right\| = 39 \quad (3.1)$$

- 4) The following Python code generates Fig. 4

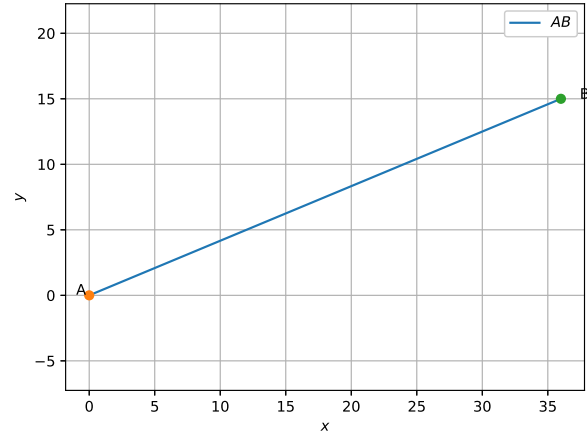


Fig. 4: Line AB generated using python

```
codes/line_ex/pts_and_vectors/
dist_bt看_pts.py
```

3.3 Points on a line

3.3.1 Problem:

Find the coordinates of the points of trisection of the line segment joining $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$.

3.3.2 Solution:

- 1) Let \mathbf{E} be a point which divides line segment AB in the ratio $k : 1$:

- 2)

$$\mathbf{E} = \frac{k\mathbf{A} + \mathbf{B}}{k + 1} \quad (2.1)$$

- 3) \mathbf{C} divides the line in the ratio $\frac{1}{2} : 1$ and \mathbf{D} divides the line in the ratio $\frac{2}{1} : 1$

- 4)

$$\mathbf{C} = \frac{0.5\mathbf{A} + \mathbf{B}}{0.5 + 1} \quad (4.1)$$

$$\mathbf{D} = \frac{2\mathbf{A} + \mathbf{B}}{2 + 1} \quad (4.2)$$

$$\therefore \mathbf{C} = \begin{pmatrix} 0 \\ -2.33 \end{pmatrix} \quad (4.3)$$

$$\therefore \mathbf{D} = \begin{pmatrix} 2 \\ -1.66 \end{pmatrix} \quad (4.4)$$

- 5) The following Python code generates Fig. 5

```
codes/line_ex/pts_on_a_line/trisection.py
```

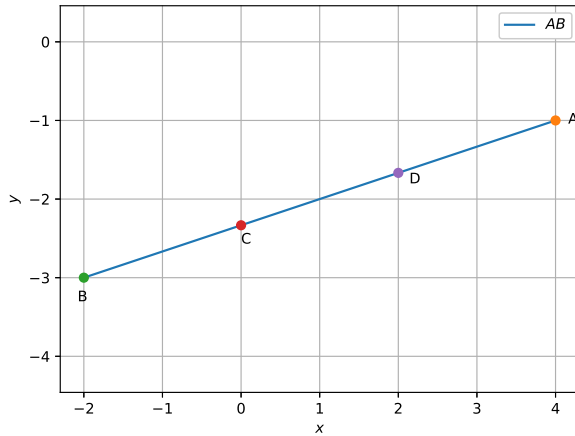


Fig. 5: Line AB trisected - generated using python

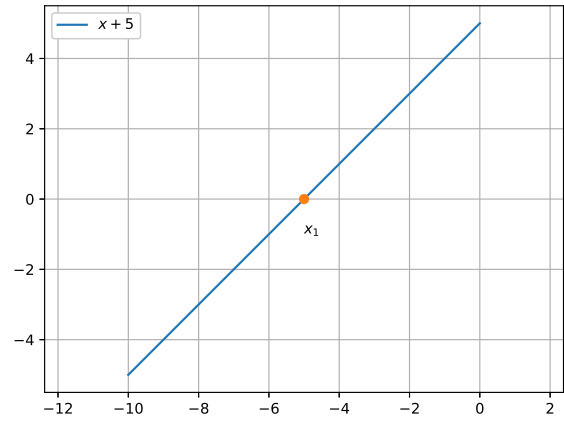


Fig. 1: $x + 5$ generated using python

3.4 Lines and planes

3.4.1 Problem:

Find the zero of the polynomial in each of the following cases:

$$p(x) = x + 5$$

$$p(x) = x - 5$$

$$p(x) = 2x + 5$$

$$p(x) = 3x - 2$$

$$p(x) = 3x$$

3.4.2 Solution:

1) **Solution:** For $p(x) = x + 5$

The given equation can be represented as follows in the vector form:

$$(5 \quad -1)\mathbf{x} + 5 = 0 \quad (1.1)$$

To find the roots $y = 0$:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix} \quad (1.2)$$

$$x_1 + 5 = 0 \quad (1.3)$$

$$x_1 = -5 \quad (1.4)$$

2) **Solution:** For $p(x) = x - 5$

The given equation can be represented as follows in the vector form:

$$(5 \quad -1)\mathbf{x} - 5 = 0 \quad (2.1)$$

To find the roots $y = 0$:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix} \quad (2.2)$$

$$x_1 - 5 = 0 \quad (2.3)$$

$$x_1 = 5 \quad (2.4)$$

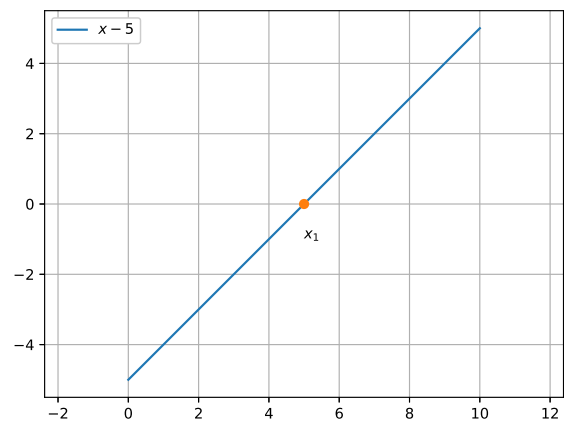


Fig. 2: $x - 5$ generated using python

3) **Solution:** For $p(x) = 2x + 5$

The given equation can be represented as follows in the vector form:

$$(2 \ -1)\mathbf{x} + 5 = 0 \quad (3.1)$$

To find the roots $y = 0$:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix} \quad (3.2)$$

$$2x_1 + 5 = 0 \quad (3.3)$$

$$x_1 = \frac{-5}{2} \quad (3.4)$$

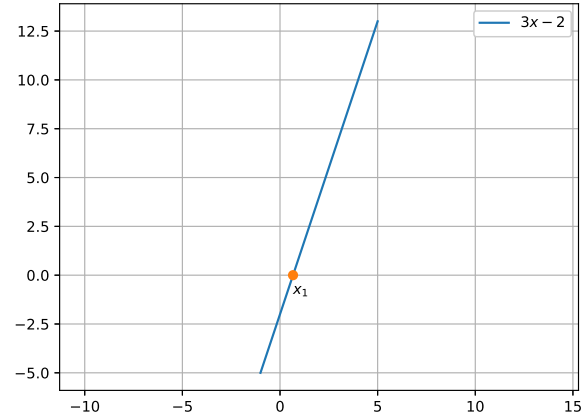


Fig. 4: $3x - 2$ generated using python

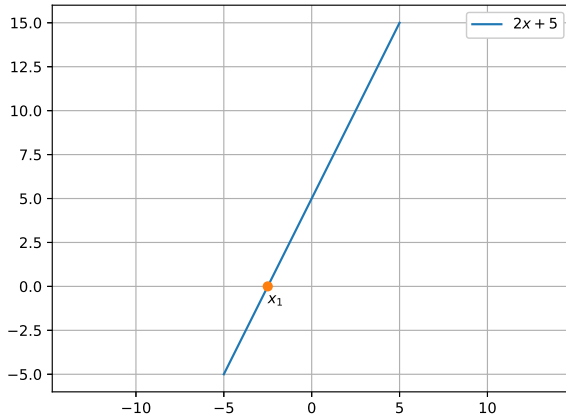


Fig. 3: $2x + 5$ generated using python

5) **Solution:** For $p(x) = 3x$

The given equation can be represented as follows in the vector form:

$$(3 \ -1)\mathbf{x} = 0 \quad (5.1)$$

To find the roots $y = 0$:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix} \quad (5.2)$$

$$3x_1 = 0 \quad (5.3)$$

$$x_1 = 0 \quad (5.4)$$

4) **Solution:** For $p(x) = 3x - 2$

The given equation can be represented as follows in the vector form:

$$(3 \ -1)\mathbf{x} - 2 = 0 \quad (4.1)$$

To find the roots $y = 0$:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ 0 \end{pmatrix} \quad (4.2)$$

$$3x_1 - 2 = 0 \quad (4.3)$$

$$x_1 = \frac{2}{3} \quad (4.4)$$

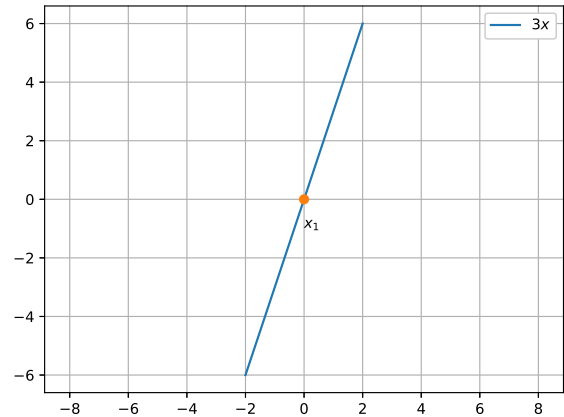


Fig. 5: $3x$ generated using python

The following Python code generates the figures

```
codes/line_ex/lines_and_planes/
linear_eq_roots.py
```

3.5 Motion in a plane

3.5.1 Problem:

A motorboat is racing towards north at 25 km/h and the water current in that region is 10 km/h in the direction of 60° east of south. Find the resultant velocity of the boat.

3.5.2 Solution:

- 1) **A** denotes the velocity of the boat and **B** denotes the water current and **C** represents the resultant velocity.
- 2)

$$\mathbf{A} = \begin{pmatrix} 0 \\ 25 \end{pmatrix} \quad (2.1)$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ -8.67 \end{pmatrix} \quad (2.2)$$

$$\mathbf{C} = \mathbf{A} + \mathbf{B} \quad (2.3)$$

$$\mathbf{C} = \begin{pmatrix} 5 \\ 16.34 \end{pmatrix} \quad (2.4)$$

- 3) Magnitude of resultant velocity:

$$\|\mathbf{C}\| = 17.08 \quad (3.1)$$

- 4) Direction of resultant velocity:

$$\cos \theta = \frac{(\mathbf{A})^T (\mathbf{C})}{\|\mathbf{A}\| \|\mathbf{C}\|} \quad (4.1)$$

$$\theta = 17.01^\circ \quad (4.2)$$

- 5) \therefore The resultant velocity is 17.08 km/h at an angle of 17.01° east of north.

- 6) The following Python code generates Fig. 6

```
codes/line_ex/motion_in_a_plane/
motion_plane.py
```

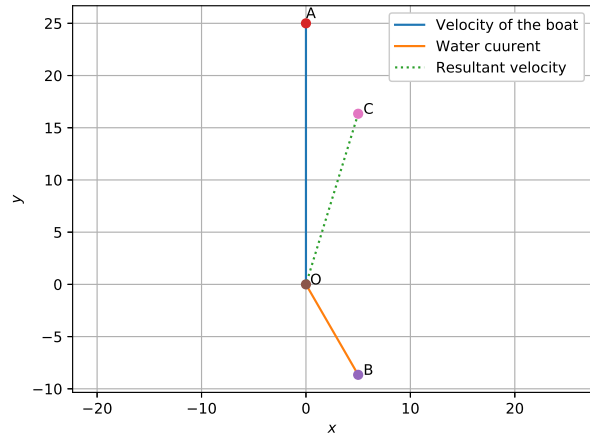


Fig. 6: Vectorial representation of velocities generated using python

3.6 Matrix

3.6.1 Problem:

If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements.

3.6.2 Solution: The total number of elements in a matrix is $m \times n$.

- 1) If the total number of elements is 24. The possible orders are:

$$1 \times 24 = 24 \quad (1.1)$$

$$24 \times 1 = 24 \quad (1.2)$$

$$2 \times 12 = 24 \quad (1.3)$$

$$12 \times 2 = 24 \quad (1.4)$$

$$3 \times 8 = 24 \quad (1.5)$$

$$8 \times 3 = 24 \quad (1.6)$$

$$4 \times 6 = 24 \quad (1.7)$$

$$6 \times 4 = 24 \quad (1.8)$$

- 2) If the total number of elements is 13. The possible orders are:

$$1 \times 13 = 13 \quad (2.1)$$

$$13 \times 1 = 13 \quad (2.2)$$

3.7 Determinants

3.7.1 Problem:

Find the determinant of

$$(i) \begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} \quad (ii) \begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$$

3.7.2 Solution:

1) Determinant of a 2×2 matrix is obtained as follows

2)

$$A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\det A = a_{11}a_{22} - a_{12}a_{21} \quad (2.1)$$

3) From 2.1:

$$(i) \det = \cos \theta^2 + \sin \theta^2 = 1 \quad (3.1)$$

$$(ii) \det = x^3 - x^2 + 2 \quad (3.2)$$

Python code to calculate the determinant of a matrix:

```
codes/line_ex/determinants/det.py
```

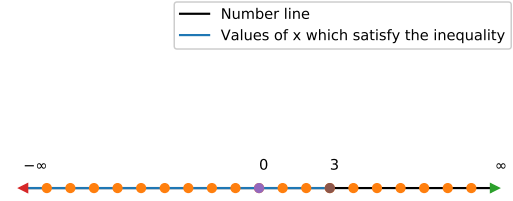


Fig. 2: Values of x satisfying the inequality in the number line generated using python

3.9 Miscellaneous

3.9.1 Problem:

Solve the following pair of equations:

$$\begin{aligned} (a-b \quad a+b) \mathbf{x} &= a^2 - 2ab - b^2 \\ (a+b \quad a+b) \mathbf{x} &= a^2 + b^2 \end{aligned} \quad (2.1)$$

3.9.2 Solution:

3.9.1. Vector form of the given equations:

$$\begin{pmatrix} a-b & a+b \\ a+b & a+b \end{pmatrix} \mathbf{x} = \begin{pmatrix} a^2 - 2ab - b^2 \\ a^2 + b^2 \end{pmatrix} \quad (3.9.1.1)$$

3.9.2. To find \mathbf{x} :

$$\begin{pmatrix} a-b & a+b & a^2 - 2ab - b^2 \\ a+b & a+b & a^2 + b^2 \end{pmatrix} \xrightarrow[R_2 \leftarrow \frac{R_2}{a+b}]{R_1 \leftarrow \frac{R_1}{a-b}} \begin{pmatrix} 1 & \frac{a+b}{a-b} & \frac{a^2 - 2ab - b^2}{a-b} \\ 1 & 1 & \frac{a^2 + b^2}{a+b} \end{pmatrix} \quad (3.9.2.1)$$

$$\xrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & \frac{a+b}{a-b} & \frac{a^2 - 2ab - b^2}{a-b} \\ 0 & \frac{-2b}{a-b} & \frac{4ab^2}{a^2 - b^2} \end{pmatrix} \quad (3.9.2.2)$$

$$\xrightarrow{R_2 \leftarrow \frac{-(a-b)R_2}{2b}} \begin{pmatrix} 1 & \frac{a+b}{a-b} & \frac{a^2 - 2ab - b^2}{a-b} \\ 0 & 1 & \frac{a^2 - 2ab - b^2}{a+b} \end{pmatrix} \quad (3.9.2.3)$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & a+b \\ 0 & 1 & \frac{a^2 - 2ab - b^2}{a+b} \end{pmatrix} \quad (3.9.2.4)$$

$$\therefore \mathbf{x} = \begin{pmatrix} a+b \\ \frac{a^2 - 2ab - b^2}{a+b} \end{pmatrix} \quad (3.9.2.5)$$

3.8 Linear inequalities

3.8.1 Problem:

Solve $7x + 3 < 5x + 9$. Show the graph of the solutions on number line.

3.8.2 Solution:

1)

$$7x + 3 < 5x + 9 \quad (1.1)$$

$$2x - 6 < 0 \quad (1.2)$$

$$x < 3 \quad (1.3)$$

$$\therefore x \in \{3, -\infty\} \quad (1.4)$$

2) The following Python code to generate the figures:

```
codes/line_ex/lin_ineq/dist_btw_pts.py
```

4 CIRCLE

4.1 Problem

Find the center of a circle passing through the points $\begin{pmatrix} 6 \\ -6 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -7 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$.

4.2 Solution

4.2.1.

$$\mathbf{P}_1 = \begin{pmatrix} 6 \\ -6 \end{pmatrix} \quad (4.2.1.1)$$

$$\mathbf{P}_2 = \begin{pmatrix} 3 \\ -7 \end{pmatrix} \quad (4.2.1.2)$$

$$\mathbf{P}_3 = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad (4.2.1.3)$$

4.2.2. The general of a circle equation is $x^2 + y^2 + Dx + Ey + F$, the equation can be represented as follow in the vector form:

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} D & E \end{pmatrix} \mathbf{x} + F = 0 \quad (4.2.2.1)$$

Substituting the P_1, P_2, P_3 in equation 4.2.2.1

$$\begin{pmatrix} 6 & -6 & 1 \\ 3 & -7 & 1 \\ 3 & 3 & 1 \end{pmatrix} \begin{pmatrix} D \\ E \\ F \end{pmatrix} = \begin{pmatrix} -72 \\ -58 \\ -18 \end{pmatrix} \quad (4.2.2.2)$$

To find D, E, F:

$$\begin{pmatrix} 6 & -6 & 1 & -72 \\ 3 & -7 & 1 & -58 \\ 3 & 3 & 1 & -18 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{R_1}{6}} \begin{pmatrix} 1 & -1 & \frac{1}{6} & -12 \\ 3 & -7 & 1 & -58 \\ 3 & 3 & 1 & -18 \end{pmatrix} \quad (4.2.2.3)$$

$$\begin{pmatrix} 1 & -1 & \frac{1}{6} & -12 \\ 0 & -4 & \frac{1}{2} & -22 \\ 0 & 6 & \frac{1}{2} & 18 \end{pmatrix} \xrightarrow{\begin{matrix} R_2 \leftarrow R_2 - 3R_1 \\ R_3 \leftarrow R_3 - 3R_1 \end{matrix}} \begin{pmatrix} 1 & -1 & \frac{1}{6} & -12 \\ 0 & -4 & \frac{1}{2} & -22 \\ 0 & 6 & \frac{1}{2} & 18 \end{pmatrix} \quad (4.2.2.4)$$

$$\begin{pmatrix} 1 & -3 & 0 & -18 \\ 0 & -10 & 0 & -40 \\ 0 & 6 & \frac{1}{2} & 18 \end{pmatrix} \xrightarrow{\begin{matrix} R_1 \leftarrow R_1 - \frac{R_3}{3} \\ R_2 \leftarrow R_2 - R_3 \end{matrix}} \begin{pmatrix} 1 & -3 & 0 & -18 \\ 0 & -10 & 0 & -40 \\ 0 & 6 & \frac{1}{2} & 18 \end{pmatrix} \quad (4.2.2.5)$$

$$\begin{pmatrix} 1 & -3 & 0 & -18 \\ 0 & 1 & 0 & 4 \\ 0 & 12 & 1 & 36 \end{pmatrix} \xrightarrow{\begin{matrix} R_3 \leftarrow 2R_3 \\ R_2 \leftarrow \frac{-R_2}{10} \end{matrix}} \begin{pmatrix} 1 & -3 & 0 & -18 \\ 0 & 1 & 0 & 4 \\ 0 & 12 & 1 & 36 \end{pmatrix} \quad (4.2.2.6)$$

$$\begin{pmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -12 \end{pmatrix} \xrightarrow{\begin{matrix} R_2 \leftarrow R_3 - 12R_2 \\ R_1 \leftarrow R_1 + 3R_2 \end{matrix}} \begin{pmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -12 \end{pmatrix} \quad (4.2.2.7)$$

$$\Rightarrow D = -6 \quad (4.2.2.8)$$

$$\Rightarrow E = 4 \quad (4.2.2.9)$$

$$\Rightarrow F = -12 \quad (4.2.2.10)$$

$$\mathbf{O} = \frac{-1}{2A} \begin{pmatrix} D \\ E \end{pmatrix} \quad (4.2.2.11)$$

Substituting values of D and E in equation 4.2.2.11

$$\therefore \mathbf{O} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad (4.2.2.12)$$

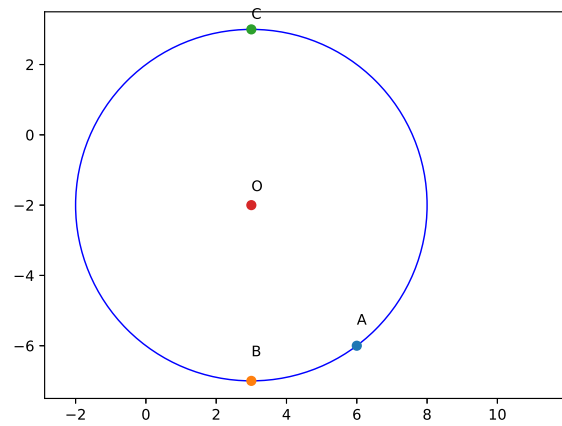


Fig. 4.2.3: Circumcircle generated using python

4.2.3. The following Python code generates Fig. 4.2.3

codes/circle_ex/circumcircle.py

5 CIRCLE-EXAMPLE

5.1 Problem

Find the center and radius of the circle

$$\mathbf{x}^T \mathbf{x} + \begin{pmatrix} 8 \\ 10 \end{pmatrix} \mathbf{x} - 8 = 0 \quad (3.1)$$

5.2 Solution

6 CONICS

5.2.1. The general of a circle equation is $Ax^2 + Ay^2 + Dx + Ey + F$, the equation can be represented as follow in the vector form:

$$\mathbf{x}^T \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix} \mathbf{x} + (D \ E) \mathbf{x} + F = 0 \quad (5.2.1.1)$$

5.2.2. To find the center - \mathbf{O} and radius - r of a circle:

$$\mathbf{O} = \frac{-1}{2A} \begin{pmatrix} D \\ E \end{pmatrix} \quad (5.2.2.1)$$

$$r = \frac{1}{A} \sqrt{\frac{1}{4} \left\| \begin{pmatrix} D \\ E \end{pmatrix} \right\|^2 - F^2} \quad (5.2.2.2)$$

5.2.3. The values given:

$$A = 1 \quad (5.2.3.1)$$

$$D = 8 \quad (5.2.3.2)$$

$$E = 10 \quad (5.2.3.3)$$

$$F = -8 \quad (5.2.3.4)$$

5.2.4. Substituting the values in equation 5.2.2.1 and 5.2.2.2:

$$\mathbf{O} = \begin{pmatrix} -4 \\ -5 \end{pmatrix} \quad (5.2.4.1)$$

$$r = 7 \quad (5.2.4.2)$$

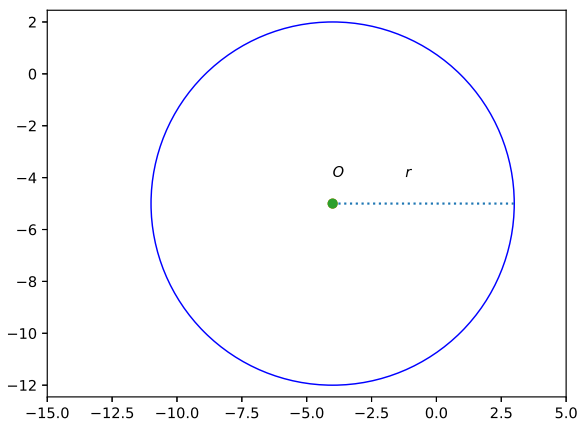


Fig. 5.2.5: Circle generated using python

5.2.5. The following Python code generates Fig. 5.2.5

codes/circle_exam.py

6.1 Problem

Verify whether 2 and 0 are zeroes of the polynomial $x^2 - 2x$.

6.2 Solution

6.2.1. **Solution:** $p(x, y) = Ax^2 + Bxy + Cy^2 + Dx + Ey + F$ can be represented as follow in the vector form:

$$\mathbf{x}^T \begin{pmatrix} A & \frac{B}{2} \\ \frac{B}{2} & C \end{pmatrix} \mathbf{x} + (D \ E) \mathbf{x} + F = 0 \quad (6.2.1.1)$$

6.2.2. The given equation can be represented as follows in the vector form:

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + (-2 \ 0) \mathbf{x} + 0 = 0 \quad (6.2.2.1)$$

6.2.3. To find the roots $y = 0$:

$$x^2 - 2x = 0 \quad (6.2.3.1)$$

$$x(x - 2) = 0 \quad (6.2.3.2)$$

$$x = 0, 2 \quad (6.2.3.3)$$

6.2.4. To verify:

a) Substitute $\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ in 6.2.2.1

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + (-2 \ 0) \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 0 \quad (6.2.4.1)$$

$$\Rightarrow 0 \quad (6.2.4.2)$$

b) Substitute $\mathbf{x} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ in 6.2.2.1

$$\begin{pmatrix} 2 \\ 0 \end{pmatrix}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} + (-2 \ 0) \begin{pmatrix} 2 \\ 0 \end{pmatrix} + 0 \quad (6.2.4.3)$$

$$\Rightarrow 0 \quad (6.2.4.4)$$

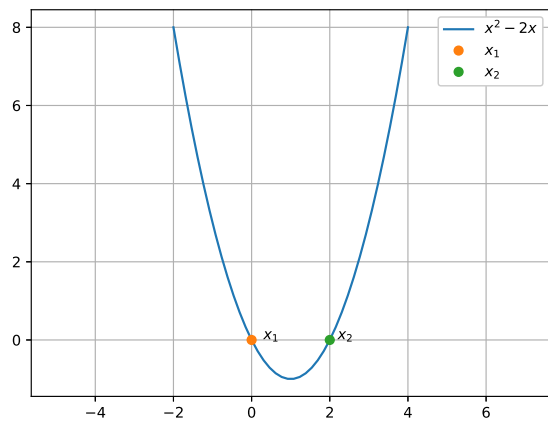


Fig. 6.2.5: $x^2 - 2x$ generated using python

6.2.5. The following Python code generates Fig. 6.2.5

```
codes/conics_example/conics.py
```