Math Document Template

C ANISH

***Abstract*—This is a document explaining questions about the concept of Probability and Statistics.**

Download all python codes from

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svn co https://github.com/chakki1234/summer
    −2020/trunk/Probability_and_statistics/
    probability/codes
```

and latex-tikz codes from

```
svn co https://github.com/chakki1234/summer
    −2020/trunk/Probability_and_statistics/
    probability/figs
```

1 Example - Problem 30

1.1 Problem

A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?

1.2 Solution

1) Let:

$$T_1 : Event\ the\ doctor\ came\ by\ train$$
$$T_2 : Event\ the\ doctor\ came\ by\ Bus$$
$$T_3 : Event\ the\ doctor\ came\ by\ Scooter$$
$$T_4 : Event\ the\ doctor\ came\ by\ other\ means\ of\ transport$$
$$L : Event\ the\ doctor\ arrives\ late$$

2)

$$P(T_1) = \frac{3}{10}$$
$$P(T_2) = \frac{1}{5}$$
$$P(T_3) = \frac{1}{10}$$
$$P(T_4) = \frac{2}{5}$$
$$P(L|T_1) = \frac{1}{4}$$
$$P(L|T_2) = \frac{1}{3}$$
$$P(L|T_3) = \frac{1}{12}$$
$$P(L|T_4) = 0$$

3) To find the probability the doctor comes by train and he is late = $P(T_1|L)$ using Baye's Theorem:

$$P(T_1|L) = \frac{P(T_1) \times P(L|T_1)}{P(T_1) \times P(L|T_1) + P(T_2) \times P(L|T_2) + P(T_3) \times P(L|T_3) + P(T_4) \times P(L|T_4)}$$

$$P(T_1|L) = \frac{\frac{3}{10} \times \frac{1}{4}}{\frac{3}{10} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{3} + \frac{1}{10} \times \frac{1}{12} + \frac{2}{5} \times 0}$$

$$= \frac{\frac{3}{40}}{\frac{18}{40}}$$

$$= \frac{3}{40} \times \frac{120}{18}$$

$$= \frac{360}{720}$$

$$= \frac{1}{2}$$

The python implementation:

codes/example_prob/prob30.py

2 Example - Problem 31

2.1 Problem

A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

2.2 Solution

1)

$$S_1 : man\ speaks\ the\ truth$$
$$S_2 : man\ lies$$
$$E : six\ on\ the\ die$$

2)

$$P(S_1) = \frac{3}{4}$$
$$P(S_2) = 1 - \frac{3}{4} = \frac{1}{4}$$
$$P(E|S_1) = \frac{1}{6}$$
$$P(E|S_2) = 1 - \frac{1}{6} = \frac{5}{6}$$

3) Applying Baye's theorem, we get the required probability as

$$P(S_1|E) = \frac{P(S_1).P(E|S_1)}{P(S_1).P(E|S_1) + P(S_2).P(E|S_2)}$$

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}}$$
$$= \frac{3}{8}$$

The python implementation:

codes/example_prob/prob31.py

3 Example - Problem 32

3.1 Problem

A person plays a game of tossing a coin thrice. For each head, he is given Rs 2 by the organiser of the game and for each tail, he has to give Rs 1.50 to the organiser. Let X denote the amount gained or lost by the person. Show that X is a random variable and exhibit it as a function on the sample space of the experiment.

3.2 Solution

1) The sample space is:

$$\begin{bmatrix} (HHH) & (HHT) & (HTH) & (THH) \\ (TTH) & (THT) & (HTT) & (TTT) \end{bmatrix} \tag{1.1}$$

2) Let X be the amount gain or lost by a person. To calculate values of X for different outcomes.

a) For HHH:
b)

$$2 + 2 + 2 = 6 \tag{2.1}$$

a) For HHT:
b)

$$2 + 2 - 1.5 = 2.5 \tag{2.2}$$

a) For HTH:
b)

$$2 - 1.5 + 2 = 2.5 \tag{2.3}$$

a) For THH:
b)

$$-1.5 + 2 + 2 = 2.5 \tag{2.4}$$

a) For TTH:
b)

$$-1.5 - 1.5 + 2 = -1 \tag{2.5}$$

a) For THT:
b)

$$-1.5 + 2 - 1.5 = -1 \tag{2.6}$$

a) For HTT:
b)

$$2 - 1.5 - 1.5 = 6 \tag{2.7}$$

a) For TTT:
b)

$$-1.5 - 1.5 - 1.5 = -4.5 \tag{2.8}$$

3) The values of X can be 6, 2.5, -1, -4.5. Hence X = $(6, 2.5, -1, -4.5)$

4) Thus X is a real valued function whose domain is sample space. Hence X, is a random variable. The python implementation:

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codes/example_prob/prob32.py
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4 Example - Problem 33

4.1 Problem

A bag contains 2 white and 1 red balls. One ball is drawn at random and then put back in the box after noting its colour. The process is repeated again. If X denotes the number of red balls recorded in the two draws, describe X.

4.2 Solution

1) Let two white balls be denoted by W_1, W_2 and red ball by r. X is the number of red balls in two draws.
2) If we draw the two balls one after the other the sample space is:

$$\begin{bmatrix} (W_1, W_1) & (W_2, W_2) & (W_1, W_2) & (W_2, W_1) \\ (r, W_1) & (W_1, r) & (r, W_2) & (W_2, r) \\ (r, r) & & & \end{bmatrix} \tag{2.1}$$

3) The values of X can be 0 red ball, 1 red ball and 2 red balls.
4) $X = (0, 1, 2)$

5 Example - Problem 34

5.1 Problem

Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the probability distribution of the number of aces.

5.2 Solution

1) Let the number of aces be a random variable. Let it it be denoted by X. Let T_1 be a event of not getting an ace and T_2 be a event of getting an ace.
2)

$$\begin{aligned} P(X = 0) &= P(T_1) \times P(T_1) \\ &= \frac{48}{52} \times \frac{48}{52} = \frac{144}{169} \\ P(X = 1) &= P(T_2) \times P(T_1) + P(T_1) \times P(T_2) \\ &= \frac{24}{169} \\ P(X = 2) &= P(T_2) \times P(T_2) \\ &= \frac{4}{52} \times \frac{4}{52} \\ &= \frac{1}{169} \end{aligned}$$

3) The probability distribution of X

X	**0**	**1**	**2**
P(X)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$

TABLE 3: The probability distribution of X

The python implementation:

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codes/example_prob/prob34.py
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6 Example - Problem 35

6.1 Problem

Find the probability distribution of number of doublets in three throws of a pair of dice?

6.2 Solution

1) Let X denote the number of doublets. Possible doublets are $(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)$. As the dice is thrown thrice, therefore X can take the value $0, 1, 2$ *or* 3.
2)

$$Probability\ of\ getting\ a\ doublet = \frac{1}{6} \tag{2.1}$$

$$Probability\ of\ not\ getting\ a\ doublet = \frac{5}{6} \tag{2.2}$$

$$P(X=0) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$$
$$P(X=1) = \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{1}{6} \times \frac{5}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{75}{216}$$
$$P(X=2) = \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} + \frac{1}{6} \times \frac{5}{6} \times \frac{1}{6} + \frac{5}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{15}{216}$$
$$P(X=3) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$$

3) The probability distribution of X

X	**0**	**1**	**2**	**3**
P(X)	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

TABLE 3: The probability distribution of X

The python implementation:

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codes/example_prob/prob35.py
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7 Example - Problem 36

7.1 Problem

Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form, where k is some unknown constant.

$$\begin{pmatrix} 0.1, if\ \ x = 0 \\ kx, if\ \ x = 1\ or\ 2 \\ k(5-x), if\ \ x = 3\ or\ 4 \\ 0, otherwise \end{pmatrix} \tag{3.1}$$

1) Find the value of k.
2) What is the probability that you study at least two hours ? Exactly two hours? At most two hours?

7.2 Solution

1) Since X is a random variable, its Sum of probabilities is equal to 1. $\sum_0^4 P(x) = 1$.
2) If we draw the two balls one after the other the sample space is:

$$P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) = 1 \tag{2.1}$$
$$0.1 + k + 2k + 2k + k = 1 \tag{2.2}$$
$$6k = 1 - 0.1 \tag{2.3}$$
$$6k = 0.9 \tag{2.4}$$
$$k = \frac{0.9}{6} \tag{2.5}$$
$$k = 0.15 \tag{2.6}$$

3) $P(X \geq 2)$:

$$= P(X=2) + P(X=3) + P(X=4) \tag{3.1}$$
$$= 2k + 2k + k \tag{3.2}$$
$$= 0.75 \tag{3.3}$$

4) $P(X = 2)$:

$$= 2k \tag{4.1}$$
$$= 0.3 \tag{4.2}$$

5) $P(X \leq 2)$:

$$= P(X=0) + P(X=1) + P(X=2) \tag{5.1}$$
$$= 0.1 + 2k + k \tag{5.2}$$
$$= 0.55 \tag{5.3}$$

The python implementation:

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codes/example_prob/prob36.py
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8 Example - Problem 37

8.1 Problem

Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the mean or expectation of X.

8.2 Solution

1) Let X be equal to sum of numbers on two dices.
2) $P(X = 2)$:

$$Outcomes = (1, 1) \tag{2}$$
$$Probability = \frac{1}{36} \tag{2}$$

3) $P(X = 3)$:

$$Outcomes = (1,2), (2,1) \tag{3}$$

$$Probability = \frac{2}{36} \tag{3}$$

4) $P(X = 4)$:

$$Outcomes = (1,3), (2,2), (3,1) \tag{4}$$

$$Probability = \frac{3}{36} \tag{4}$$

5) $P(X = 5)$:

$$Outcomes = (1,4), (2,3), (3,2), (4,1) \tag{5}$$

$$Probability = \frac{4}{36} \tag{5}$$

6) $P(X = 6)$:

$$Outcomes = (1,5), (2,4), (3,3), (4,2), (5,1) \tag{6}$$

$$Probability = \frac{5}{36} \tag{6}$$

7) $P(X = 7)$:

$$\begin{aligned} Outcomes = (1,6), (2,5), (3,4) \\ (4,3), (5,2), (6,1) \end{aligned} \tag{7}$$

$$Probability = \frac{6}{36} \tag{7}$$

8) $P(X = 8)$:

$$Outcomes = (2,6), (3,5), (4,4), (5,3), (6,2) \tag{8}$$

$$Probability = \frac{5}{36} \tag{8}$$

9) $P(X = 9)$:

$$Outcomes = (3,6), (4,5), (5,4), (6,3) \tag{9}$$

$$Probability = \frac{4}{36} \tag{9}$$

10) $P(X = 10)$:

$$Outcomes = (4,6), (5,5), (6,4) \tag{10}$$

$$Probability = \frac{3}{36} \tag{10}$$

11) $P(X = 11)$:

$$Outcomes = (5,6), (6,5) \tag{11}$$

$$Probability = \frac{2}{36} \tag{11}$$

12) $P(X = 12)$:

$$Outcomes = (6,6) \tag{12}$$

$$Probability = \frac{1}{36} \tag{12}$$

13) Mean $= \sum_{i=2}^{n} X_i P_i$.

$$= \frac{252}{36} \tag{13}$$

$$= 7 \tag{13}$$

The python implementation:

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codes/example_prob/prob37.py
```

9 Example - Problem 38

9.1 Problem

Find the variance of the number obtained on a throw of an unbiased die.

9.2 Solution

1) Let X be a number obtained on the throw of a dice.X can take the values 1, 2, 3, 4, 5 and 6.
2) The mean expectation value is given by $E(X) = \sum_{i=1}^{n} X_i P_i$:

$$1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} \tag{2.1}$$

$$= \frac{21}{6} \tag{2.2}$$

3) Variance $Var(X) = E\left(X^2\right) - E(X)^2$:

$$E\left(X^2\right) = \sum_{i=1}^{n} X_i^2 P_i \tag{3.1}$$

$$= \frac{1 + 4 + 9 + 16 + 25 + 36}{6} \tag{3.2}$$

$$= \frac{91}{6} \tag{3.3}$$

4) To find $Var(X)$:

$$= E\left(X^2\right) - E(X)^2 \tag{4.1}$$

$$= \frac{91}{6} - \left(\frac{21}{6}\right)^2 \tag{4.2}$$

$$= \frac{35}{12} \tag{4.3}$$

The python implementation:

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codes/example_prob/prob38.py
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10 Example - Problem 39

10.1 Problem

Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.

10.2 Solution

1) Let two white balls be denoted by W_1, W_2 and red ball by r. X is the number of red balls in two draws.
2) Let X be the number of kings obtained. X can be 0, 1, 2.
3) The total number of ways to draw 2 cards is ${}^{5}C_2 = 1326$.
4) $P(X = 0)$:

$$= {}^{48}C_2 \tag{4.1}$$
$$= 1128 \tag{4.2}$$
$$P(X = 0) = \frac{Number\ of\ ways\ to\ get\ 0\ kings}{Total\ nummber\ of\ ways} \tag{4.3}$$
$$= \frac{1128}{1326} \tag{4.4}$$

5) $P(X = 1)$:

$$= {}^{48}C_1 \times {}^{4}C_1 \tag{5.1}$$
$$= 4 \times 48 \tag{5.2}$$
$$P(X = 1) = \frac{Number\ of\ ways\ to\ get\ 1\ kings}{Total\ nummber\ of\ ways} \tag{5.3}$$
$$= \frac{192}{1326} \tag{5.4}$$

6) $P(X = 2)$:

$$= {}^{4}C_2 \tag{6.1}$$
$$= 6 \tag{6.2}$$
$$P(X = 2) = \frac{Number\ of\ ways\ to\ get\ 2\ kings}{Total\ nummber\ of\ ways} \tag{6.3}$$
$$= \frac{6}{1326} \tag{6.4}$$

7) Expectation value $E(X)$ is given by:

$$E(X) = \sum_{i=1}^{n} X_i P_i \tag{7.1}$$
$$= \frac{204}{1326} \tag{7.2}$$
$$= \frac{34}{221} \tag{7.3}$$

8) To find $E\left(X^2\right)$

$$E(X) = \sum_{i=1}^{n} {X_i}^2 P_i \tag{8.1}$$
$$= \frac{216}{1326} \tag{8.2}$$
$$= \frac{36}{221} \tag{8.3}$$

9) Variance is given by: $Var(X) = E\left(X^2\right) - E(X)^2$

$$= \frac{36}{221} - \left(\frac{34}{221}\right)^2 \tag{9.1}$$
$$= \frac{6800}{221^2} \tag{9.2}$$

The python implementation:

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codes/example_prob/prob39.py
```

11 Excercise - Problem 30

11.1 Problem

Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

11.2 Solution

1) The number of possible outcomes is ${}^{52}C_2 = 1326$:
2) The number of favorable outcomes are ${}^{26}C_2 = 325$:
3)

$$Probability = \frac{325}{1326} \tag{3.1}$$
$$= 0.245 \tag{3.2}$$

The python implementation:

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codes/excercise_prob/prob30.py
```

12 Excercise - Problem 31

12.1 Problem

A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box contaning 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

12.2 Solution

1) The number of elements in the Sample Space is ${}^{15}C_3 = 455$:
2) Favorable outcome is - Number of ways of selecting three good oranges = ${}^{12}C_3 = 220$
3)

$$Probability = \frac{220}{455} \tag{3.1}$$
$$= 0.483 \tag{3.2}$$

The python implementation:

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codes/excercise_prob/prob31.py
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13 Excercise - Problem 32

13.1 Problem

A fair coin and an unbaised die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not.

13.2 Solution

Theorem 13.1. *Two events A and B are said to be independent if $P(A \cap B) = P(A).P(B)$.*

1) The Sample Space is:

$$\begin{bmatrix} (H,1) & (H,2) & (H,3) & (H,4) & (H,5) & (H,6) \\ (T,1) & (T,2) & (T,3) & (T,4) & (T,5) & (T,6) \end{bmatrix} \tag{1.1}$$

2) Probability of event A:
Favourable outcomes are:

$$\begin{bmatrix} (H,1) & (H,2) & (H,3) & (H,4) & (H,5) & (H,6) \end{bmatrix} \tag{2.1}$$

$$P(A) = \frac{6}{12} = \frac{1}{2} \tag{2.2}$$

3) Probability of event B:
Favourable outcomes are:

$$\begin{bmatrix} (H,3) & (T,3) \end{bmatrix} \tag{3.1}$$
$$P(B) = \frac{2}{12} = \frac{1}{6} \tag{3.2}$$

4) $P(A \cap B)$:

$$A \cap B = (H,3) \tag{4.1}$$
$$P(A \cap B) = \frac{1}{12} \tag{4.2}$$

5) $P(A).P(B)$:

$$= \frac{1}{12} \times \frac{1}{6} \tag{5.1}$$
$$= \frac{1}{12} \tag{5.2}$$

6) Since $P(A \cap B) = P(A).P(B)$. Therefore A and B are independent events.
The python implementation:

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codes/excercise_prob/prob32.py
```

14 Excercise - Problem 33

14.1 Problem

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even', and B be the event, 'the number is red'. Are A and B independent?

14.2 Solution

Theorem 14.1. *Two events A and B are said to be independent if $P(A \cap B) = P(A).P(B)$.*

1) The Sample Space is:

$$\begin{bmatrix} (1,R) & (2,R) & (3,R) & (4,G) & (5,G) & (6,G) \end{bmatrix} \tag{1.1}$$

2) Probability of event A:
Favourable outcomes are:

$$\begin{bmatrix} (2,R) & (4,G) & (6,G) \end{bmatrix} \tag{2.1}$$
$$P(A) = \frac{3}{6} = \frac{1}{2} \tag{2.2}$$

3) Probability of event B:
Favourable outcomes are:

$$\begin{bmatrix} (1,R) & (2,R) & (3,R) \end{bmatrix} \tag{3.1}$$
$$P(B) = \frac{3}{6} = \frac{1}{2} \tag{3.2}$$

4) $P(A \cap B)$:

$$A \cap B = (2, R) \tag{4.1}$$
$$P(A \cap B) = \frac{1}{6} \tag{4.2}$$

5) $P(A).P(B)$:

$$= \frac{1}{2} \times \frac{1}{2} \tag{5.1}$$
$$= \frac{1}{4} \tag{5.2}$$

6) Since $P(A \cap B)$ is not equal to $P(A).P(B)$. Therefore A and B are not independent events. The python implementation:

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codes/excercise_prob/prob33.py
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15 Excercise - Problem 34

15.1 Problem

Let E and F be events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent?

15.2 Solution

Theorem 15.1. *Two events A and B are said to be independent if $P(A \cap B) = P(A).P(B)$.*

1) $P(E \cap F)$:

$$= \frac{1}{5} \tag{1.1}$$

2) $P(E).P(F)$:

$$= \frac{3}{5} \times \frac{3}{10} \tag{2.1}$$
$$= \frac{9}{50} \tag{2.2}$$

3) Since $P(E \cap F)$ is not equal to $P(E).P(F)$. Therefore A and B are not independent events. The python implementation:

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codes/excercise_prob/prob34.py
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16 Excercise - Problem 35

16.1 Problem

Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$.Find p if they are

1) mutually exclusive
2) independent

16.2 Solution

1) Condition for events to be mutually exclusive:

$$P(A \cup B) = P(A) + P(B) \tag{1.1}$$

2)

$$\frac{3}{5} = \frac{1}{2} + p \tag{2.1}$$
$$\implies p = \frac{1}{10} \tag{2.2}$$

3) Condition for events to be independent:

$$P(A \cap B) = P(A).P(B) \tag{3.1}$$

4)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{4.1}$$
$$\frac{3}{5} = \frac{1}{2} + p - \frac{p}{2} \tag{4.2}$$
$$\implies p = \frac{1}{5} \tag{4.3}$$

The python implementation:

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codes/excercise_prob/prob35.py
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17 Excercise - Problem 36

17.1 Problem

Given that the events A and B are independent events with $P(A) = 0.3$, $P(B) = 0.4$. Find

1) $P(A \cap B)$
2) $P(A \cup B)$
3) $P(A|B)$
4) $P(A|B)$

17.2 Solution

1) Since events A and B are independent:

$$P(A \cap B) = P(A).P(B) \tag{1.1}$$
$$= 0.3 \times 0.4 \tag{1.2}$$
$$= 0.12 \tag{1.3}$$

2)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{2.1}$$
$$= 0.3 + 0.4 - 0.12 \tag{2.2}$$
$$= 0.58 \tag{2.3}$$

3)

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = P(A) \quad (3.1)$$
$$= 0.3 \quad (3.2)$$

4)

$$P(B|A) = \frac{P(A \cap B)}{P(B)} = P(B) \quad (4.1)$$
$$= 0.4 \quad (4.2)$$

The python implementation:

codes/excercise_prob/prob36.py

18 Excercise - Problem 37

18.1 Problem

If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find P(not A and not B).

18.2 Solution

1) P(not A and not B) is:

$$= 1 - P(A \cup B)$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 1 - \left(\frac{1}{4} + \frac{1}{2} - \frac{1}{8}\right)$$
$$= \frac{3}{8}$$

The python implementation:

codes/excercise_prob/prob37.py

19 Excercise - Problem 38

19.1 Problem

Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and P(not A or not B) = $\frac{1}{4}$. State whether A and B are independent ?

19.2 Solution

Theorem 19.1. *Two events A and B are said to be independent if $P(A \cap B) = P(A).P(B)$.*

1) P(not A or not B) = $P(A \cap B)'$:

$$P(A \cap B) = 1 - \frac{1}{4}$$
$$P(A \cap B) = \frac{3}{4}$$

2) $P(A).P(B)$:

$$= \frac{1}{2} \times \frac{7}{12}$$
$$= \frac{7}{24}$$

3) Since $P(A \cap B)$ is not equal to $P(A).P(B)$. Therefore A and B are not independent events. The python implementation:

codes/excercise_prob/prob38.py

20 Excercise - Problem 39

20.1 Problem

Given two independent events A and B such that P(A) = 0.3, P(B) = 0.6. Find

1) P(A and B)
2) P(A and not B)
3) P(A or B)
4) P(neither A nor B)

20.2 Solution

Theorem 20.1. *Two events A and B are said to be independent if $P(A \cap B) = P(A).P(B)$.*

1) P(A and B) is:

$$= P(A \cap B) \quad (1.1)$$
$$= P(A).P(B) \quad (1.2)$$
$$= 0.18 \quad (1.3)$$

2) P(A and not B) is:

$$= P(A) - P(A \cap B) \quad (2.1)$$
$$= 0.12 \quad (2.2)$$

3) P(A or B) = $P(A \cup B)$:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (3.1)$$
$$P(A \cup B) = 0.72 \quad (3.2)$$

4) P(neither A nor B) = $P(A \cup B)'$:

$$= 1 - 0.72 \tag{4.1}$$
$$= 0.28 \tag{4.2}$$

The python implementation:

```
codes/excercise_prob/prob39.py
```