Hypothesis Testing

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Overview

1. Analysis of variance
   - One-way with 2 levels
   - One-way with 4 levels
   - Between-subjects
   - Two-way
2. Assumption check
   - Normality check
   - Homogeneity of variances
3. Non-parametric tests

Sources

- Mackenzie, Chapter 6, **Hypothesis Testing**, Human Computer Interaction: An Empirical Research Perspective, 1st ed. (2013)
- Yatani, Advanced Topics in Human-Computer Interaction, `http://yatani.jp/teaching/doku.php?id=2016hci:start`

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Analysis of Variance

- **ANOVA**, or **F-test**, is the main statistical test for factorial experiment
- **T test** is similar but only two levels
- The main motivation to use statistical test is - to check that the difference in mean **occur by chance** or is **significant**?
- Some definition: **Null hypothesis** is an assumption of no difference in mean. **One**-way ANOVA refers to **one** factor; **two**-way ANOVA to **two** factors, etc.

Why test?

Example: One-way with 2 levels

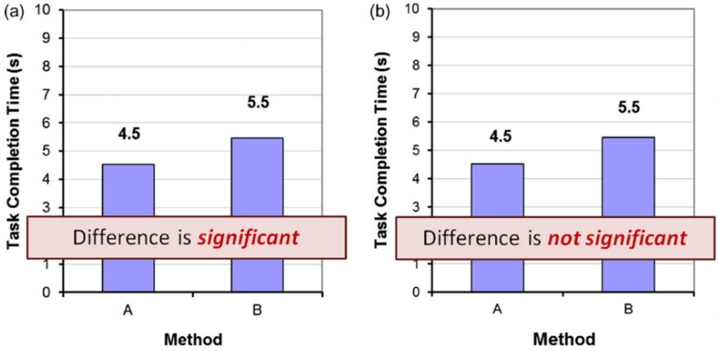


FIGURE 6.2

Difference in task completion time (in seconds) across two test conditions, Method A and Method B. Two hypothetical outcomes are shown: (a) The difference is statistically significant. (b) The difference is not statistically significant.

Figure: Source: Fg. 6.2 (Mackenzie)

Example: One-way with 2 levels

(a)

Participant	Method A	Method B
1	5.3	5.7
2	3.6	4.8
3	5.2	5.1
4	3.6	4.5
5	4.6	6.0
6	4.1	6.8
7	4.0	6.0
8	4.8	4.6
9	5.2	5.5
10	5.1	5.6
Mean	4.5	5.5
SD	0.68	0.72

(b)

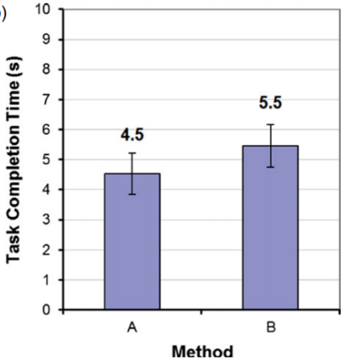


FIGURE 6.3

(a) Data for simulation in Figure 6.2a. (b) Bar chart with error bars showing ±1 standard deviation.

Figure: Source: Fg. 6.3 (Mackenzie)

Example: One-way with 2 levels with sig

ANOVA Table for Task Completion Time (s)

	DF	Sum of Squares	Mean Square	F-Value	P-Value	Lambda	Power
Subject	9	5.080	.564				
Method	1	4.232	4.232	9.796	.0121	9.796	.804
Method * Subject	9	3.888	.432				

FIGURE 6.4

Analysis of variance table for data in Figure 6.3a.

Figure: Source: Fg. 6.4 (Mackenzie): P-value of 0.0121 means that there is less than 2% that the difference occurs by chance. By convention requires less than 0.05 to reject null hypothesis

> The mean task completion time for Method A was 4.5 s. This was 20.1% less than the mean of 5.5 s observed for Method B. The difference was statistically significant ($F_{1,9} = 9.80$, $p < .05$).

FIGURE 6.5

Example of how to report the results of an analysis of variance in a research paper.

Figure: Source: Fg. 6.5 (Mackenzie): F-value is calculated = between-group variances / within-group variances = 4.232 / .432

Example: One-way with 2 levels with sig

Reporting format (APA):

- If **significant**, use threshold set .05, .01, .005, .001, .0005, .0001. p is cited as $p < .05$ instead of $p = .0121$.
- If **not significant though**, say "n.s." instead
- If **very close to significant**, report exact value.
- Plot with **standard error bars**
- Report **mean** and **std** (same unit)
- Common nowadays to report **effect size**
  - **Effect size** measures how "strong" is the significance. SPSS reports **Partial Eta Squared** (η_p^2) - .02 means that the factor X by itself accounted for only 2% of the overall (effect + error) variance. Usually around > 0.09 is considered moderate, while > 0.25 is large.

Wide format

Since we are doing a **within-subject** design, this is also sometimes called **Repeated Measures ANOVA.** In RP ANOVA, it uses **wide format** data structure.

A	B
5.3	5.7
3.6	4.8
5.2	5.1
3.6	4.5
4.6	6
4.1	6.8
4	6
4.8	4.6
5.2	5.5
5.1	5.6

Figure: Wide format structure: Cols depicting possible combinations

Long format

Between-subject ANOVA (or ANOVA) uses **long format**.

A	5.3
A	3.6
A	5.2
A	3.6
A	4.6
A	4.1
A	4
A	4.8
A	5.2
A	5.1
B	5.7
B	4.8
B	5.1
B	4.5
B	6
B	6.8
B	6
B	4.6
B	5.5
B	5.6

Figure: Long format structure: one col for each factor

Example: One-way with 2 levels with no sig

(a)

Participant	Method A	Method B
1	2.4	6.9
2	2.7	7.2
3	3.4	2.6
4	6.1	1.8
5	6.4	7.8
6	5.4	9.2
7	7.9	4.4
8	1.2	6.6
9	3.0	4.8
10	6.6	3.1
Mean	4.5	5.5
SD	2.23	2.45

(b)

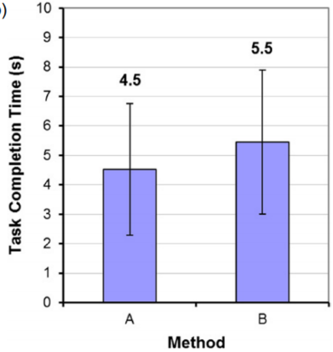


FIGURE 6.6

(a) Data for simulation in Figure 6.2b. (b) Bar chart with error bars showing ±1 standard deviation.

Figure: Source: Fg. 6.6 (Mackenzie)

Example: One-way with 2 levels with no sig

ANOVA Table for Task Completion Time (s)

	DF	Sum of Squares	Mean Square	F-Value	P-Value	Lambda	Power
Subject	9	37.372	4.152				
Method	1	4.324	4.324	.626	.4491	.626	.107
Method * Subject	9	62.140	6.904				

FIGURE 6.7

Analysis of variance for data in Figure 6.3b.

Figure: Source: Fg. 6.7 (Mackenzie). $F = 4.324/6.904 = .626$. Given p-value of .4491, there is around 45% that the difference occurs by chance.

The mean task completion times were 4.5 s for Method A and 5.5 s for Method B. As there was substantial variation in the observations across participants, the difference was not statistically significant as revealed in an analysis of variances ($F_{1,9}$ = 0.626, ns).

FIGURE 6.8

Reporting a non-significant ANOVA result.

Figure: Source: Fg. 6.8 (Mackenzie). It means that we have not enough evidence to reject null hypothesis, but it **does not mean that null hypothesis is true either**.

Example: One-way with 4 levels

Participant	Test Condition			
	A	B	C	D
1	11	11	21	16
2	18	11	22	15
3	17	10	18	13
4	19	15	21	20
5	13	17	23	10
6	10	15	15	20
7	14	14	15	13
8	13	14	19	18
9	19	18	16	12
10	10	17	21	18
11	10	19	22	13
12	16	14	18	20
13	10	20	17	19
14	10	13	21	18
15	20	17	14	18
16	18	17	17	14
Mean	14.25	15.13	18.75	16.06
SD	3.84	2.94	2.89	3.23

Figure: Source: Fg. 6.9a (Mackenzie)

Example: One-way with 4 levels

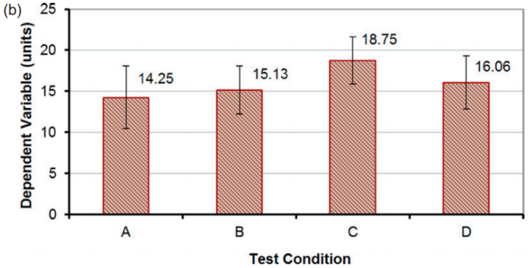


Figure: Source: Fg. 6.9b (Mackenzie)

ANOVA Table for Dependent Variable (units)

	DF	Sum of Squares	Mean Square	F-Value	P-Value	Lambda	Power
Subject	15	81.109	5.407				
Test Condition	3	182.172	60.724	4.954	.0047	14.862	.896
Test Condition * Subject	45	551.578	12.257				

Figure: Source: Fg. 6.9c (Mackenzie)

Example: One-way with 4 levels

After ANOVA, to determine exactly which condition is different with which condition, a **posthoc analysis** is required - either **Tukey's test** or **pairwise comparison with the Bonferroni correction**

Scheffe for Dependent Variable (units)
Effect: Test Condition
Significance Level: 5 %

	Mean Diff.	Crit. Diff.	P-Value	
A, B	-.875	3.302	.9003	
A, C	-4.500	3.302	.0032	S
A, D	-1.813	3.302	.4822	
B, C	-3.625	3.302	.0256	S
B, D	-.938	3.302	.8806	
C, D	2.688	3.302	.1520	

Figure: Source: Fg. 6.11 (Mackenzie)

Example: Between-subjects designs

To check whether handedness has a effect on task completion time.

(a)

Participant	Task Completion Time (s)	Handedness
1	23	L
2	19	L
3	22	L
4	21	L
5	23	L
6	20	L
7	25	L
8	23	L
9	17	R
10	19	R
11	16	R
12	21	R
13	23	R
14	20	R
15	22	R
16	21	R
Mean	20.9	
SD	2.38	

(b)

Handedness	Task Completion Time (s)	
	Mean	*SD*
Left	22.0	1.93
Right	19.9	2.42

(c)

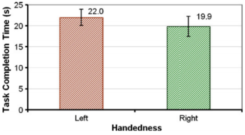


Figure: Source: Fg. 6.12 (Mackenzie)

ANOVA Table for Task Completion Time (s)

	DF	Sum of Squares	Mean Square	F-Value	P-Value	Lambda	Power
Handedness	1	18.063	18.063	3.781	.0722	3.781	.429
Residual	14	66.875	4.777				

Figure: Source: Fg. 6.13 (Mackenzie)

Two-way ANOVA

- Experiments with two IVs (factors) is called a **two-way design**
- Analysis of variance of two-way design will give us **main effects** of each factor and **interaction effect**
- Interaction effect indicates a **relational effect** between the IV on the DV

Interaction effects

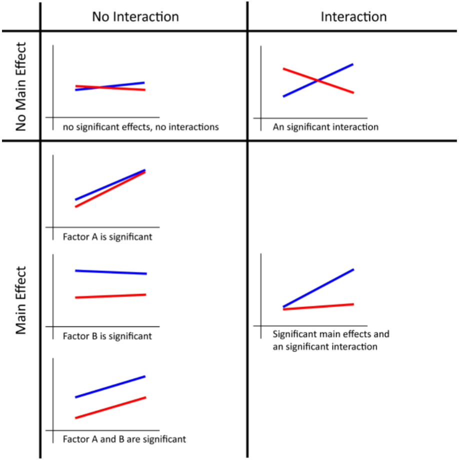


Figure: Source: Yatani's post-hoc tests

Example: 3 x 2 within-subjects design

Let's take both factors as within-subjects, the first factor is device with 3 levels - mouse, trackball, and stylus, and second factor is task with 2 levels - point-select and drag-select. We called this a 3 x 2 within-subjects design.

(a)

Participant	Device 1		Device 2		Device 3	
	Task 1	Task 2	Task 1	Task 2	Task 1	Task 2
1	11	18	15	13	20	14
2	10	14	17	15	11	13
3	10	23	13	20	20	16
4	18	18	11	12	11	10
5	20	21	19	14	19	8
6	14	21	20	11	17	13
7	14	16	15	20	16	12
8	20	21	18	20	14	12
9	14	15	13	17	16	14
10	20	15	18	10	11	16
11	14	20	15	16	10	9
12	20	20	16	16	20	9
Mean	15.4	18.5	15.8	15.3	15.4	12.2
SD	4.01	2.94	2.69	3.50	3.92	2.69

(b)

	Task 1	Task 2	*Mean*
Device 1	15.4	18.5	17.0
Device 2	15.8	15.3	15.6
Device 3	15.4	12.2	13.8
Mean	15.6	15.3	15.4

(c)

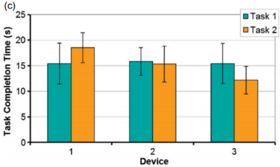


Figure: Source: Fg. 6.14 (Mackenzie)

Example: 3 x 2 within-subjects design

Three effects were observed - the main effect of device and task, and the interaction effect between device and task.

ANOVA Table for Task Completion Time (s)

	DF	Sum of Squares	Mean Square	F-Value	P-Value	Lambda	Power
Subject	11	134.778	12.253				
Device	2	121.028	60.514	5.865	.0091	11.731	.831
Device * Subject	22	226.972	10.317				
Task	1	.889	.889	.076	.7875	.076	.057
Task * Subject	11	128.111	11.646				
Device * Task	2	121.028	60.514	5.435	.0121	10.869	.798
Device * Task * Subject	22	244.972	11.135				

Figure: Source: Fg. 6.15 (Mackenzie)

Example: 3 x 2 within-subjects design

Reporting:

The grand mean for task completion time was 15.4 seconds. Device 3 was the fastest at 13.8 seconds, while device 1 was the slowest at 17.0 seconds. The main effect of device on task completion time was statistically significant ($F_{2,22}$ = 5.865, p <.01). The task effect was modest, however. Task completion time was 15.6 seconds for task 1. Task 2 was slightly faster at 15.3 seconds; however, the difference was not statistically significant ($F_{1,11}$ = 0.076, ns). The results by device and task are shown in Figure x. There was a significant Device × Task interaction effect ($F_{2,22}$ = 5.435, p < .05), which was due solely to the difference between device 1 task 2 and device 3 task 2, as determined by a Scheffé post hoc analysis.

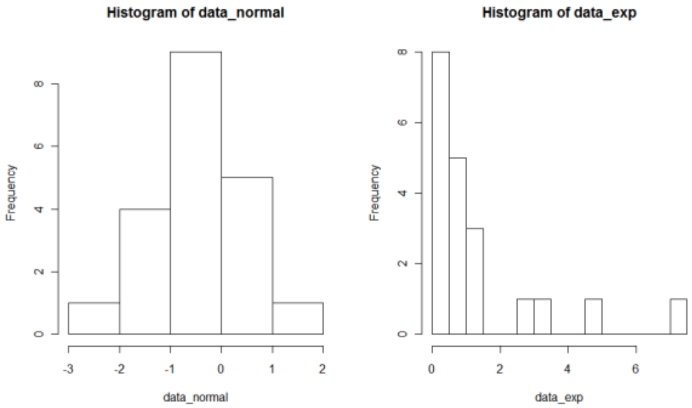
Figure: Source: Fg. 6.16 (Mackenzie)

Assumption check

- To decide whether we can use ANOVA (also called parametric tests), we check the assumption of **normality** and **homogenity of variances**.

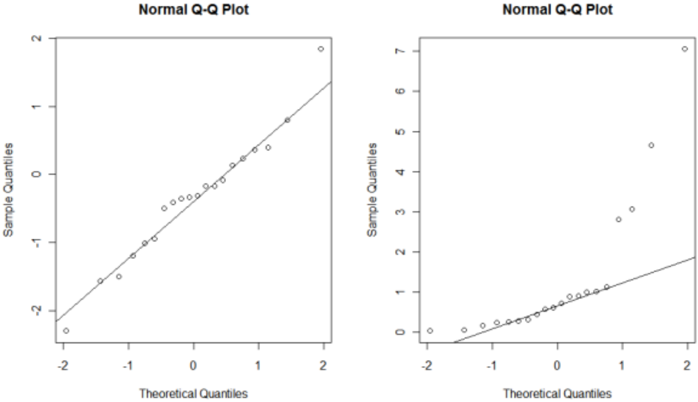
Normality check

- First easy way is to use **histogram** to check skewness



Normality check

- Another way is to use **Q-Q plot**.



Normality check

- Two common tests for normality is **Shapiro Wilk** and **Kolmogorov-Smirnov** test
- Shapiro-Wilk is more appropriate for small sample sizes (< 50)
- For example, the null hypothesis of Shapiro-Wilk is that samples are taken from a normal distribution. Here, **the p-value is larger than .05, thus is safe to say it's normal.** The null hypothesis is same for Kolmogorov-Smirnov

Tests of Normality

Course		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Time	Beginner	.177	10	.200*	.964	10	.827
	Intermediate	.166	10	.200*	.969	10	.882
	Advanced	.151	10	.200*	.965	10	.837

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

Homogeneity of variances

- t-test and ANOVA can handle differences in variances up to 4 times between smallest and largest (Howell, 2007)
- In a **between-subject** experiment, tests that can be use is **Levene's test** and **Bartlett's test** (p-value over 0.05 means that the variances are equal)
- In a **repeated measures** experiment, **Sphericity test** is used instead (p-value over .05 means that sphericity has not been violated). Note that in sphericity test, factors must have **more than 2 levels**.

Non-parametric tests for ordinal data

- **Non-parametric tests** make no assumptions for probability distribution
- Downsides of non-parametric tests are **loss of information**
- For example, 49, 81, 82 are transformed to 1, 2, 3
- In HCI, non-parametric tests are often used for **questionnaires data** (e.g., using Likert scale) since they are **ordinal** data.

Non-parametric tests for ordinal data

Four most common non-parametric procedures that work based on the number of conditions and design

Design	Conditions	
	2	3 or more
Between-subjects (independent samples)	Mann-Whitney U	Kruskal-Wallis
Within-subjects (correlated samples)	Wilcoxon Signed-Rank	Friedman

Figure: Source: Fg. 6.29 (Mackenzie)

Example: Mann-Whitney U

10 Mac users and 10 PC users are interviewed about their political views on a 10-point linear scale (1 = very left, 2 = very right). Turns out PC users are a little more "right-leaning"!

Mac Users	PC Users
2	4
3	6
2	5
4	4
9	8
2	3
5	4
3	2
4	4
3	5

Figure: Source: Fg. 6.30 (Mackenzie)

Example:Mann-Whitney U

- Given 2 levels and between subject designs, **Mann-Whitney U** is suitable
- Here we found that $p = .1418$, thus we conclude that no differences were found.

(a)

Mann-Whitney U for Response
Grouping Variable: Category for Response

U	31.000
U Prime	69.000
Z-Value	-1.436
P-Value	.1509
Tied Z-Value	-1.469
Tied P-Value	.1418
# Ties	4

Figure: Source: Fg. 6.31 (Mackenzie)

Example: Wilcoxon Signed-Rank

10 users rated the design of two media players on a 10-point linear scale (1 = not cool, 10 = really cool). Which test should we use?

Mac Users	PC Users
2	4
3	6
2	5
4	4
9	8
2	3
5	4
3	2
4	4
3	5

Figure: Source: Fg. 6.32 (Mackenzie)

Example: Wilcoxon Signed-Rank

The Wilcoxon Signed-Rank test found that $p = .0242$, thus we conclude that no differences were found.

(a)

Wilcoxon Signed Rank Test for MPA, MPB

# 0 Differences	2
# Ties	2
Z-Value	-2.240
P-Value	.0251
Tied Z-Value	-2.254
Tied P-Value	.0242

Figure: Source: Fg. 6.33 (Mackenzie)

Example: Kruskal-Wallis

Is it significant?

A20-29	A30-39	A40-49
9	7	4
9	3	5
4	5	5
9	3	2
6	2	2
3	1	1
8	4	2
9	7	2

Figure: Source: Fg. 6-34 (Mackenzie).

(a)

Kruskal-Wallis Test for Acceptability
Grouping Variable: Category for Preference

DF	2
# Groups	3
# Ties	7
H	9.421
P-Value	.0090
H corrected for ties	9.605
Tied P-Value	.0082

Figure: Source: Fg. 6-35 (Mackenzie).

Example: Kruskal-Wallis

Since there are three conditions, we can further run post-hoc tests to find out the differences in pair. Here, we found the difference between group 1 and 3.

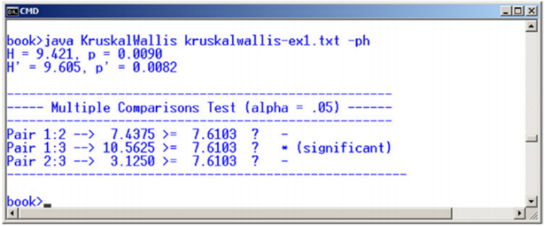


Figure: Source: Fg. 6.36 (Mackenzie)

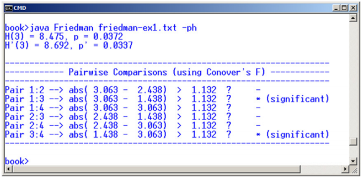
Example: Friedman Test

So, what's the conclusion?

Participant	A	B	C	D
1	66	80	67	73
2	79	64	61	66
3	67	58	61	67
4	71	73	54	75
5	72	66	59	78
6	68	67	57	69
7	71	68	59	64
8	74	69	69	66

Friedman Test for 4 Variables

DF	3
# Groups	4
# Ties	2
Chi Square	8.475
P-Value	.0372
Chi Square corrected for ties	8.692
Tied P-Value	.0337



```
CMD
book>java Friedman friedman-ex1.txt -ph
H(3) = 8.475, p = 0.0372
H'(3) = 8.692, p' = 0.0337

------------------------------------------------------------------
----------- Pairwise Comparisons (using Conover's F) ------------
------------------------------------------------------------------
Pair 1:2 --> abs( 3.063 -  2.438) >  1.132  ?    -
Pair 1:3 --> abs( 3.063 -  1.438) >  1.132  ?    * (significant)
Pair 1:4 --> abs( 3.063 -  3.063) >  1.132  ?    -
Pair 2:3 --> abs( 2.438 -  1.438) >  1.132  ?    -
Pair 2:4 --> abs( 2.438 -  3.063) >  1.132  ?    -
Pair 3:4 --> abs( 1.438 -  3.063) >  1.132  ?    * (significant)
------------------------------------------------------------------

book>
```

Figure: Source: Fg. 6-(37-39) (Mackenzie).

What's next

- Couple of workshops for ANOVA. Please take a look at the **Tutorials** folder before coming to the class. Make sure you have **JASP** installed.
- After we finish ANOVA, we gonna work on interaction and modeling, download **GoFitts.jar** from the **Download** folder and make sure you can run it (you need Java).

Questions