CS 170: Algorithms

CS 170: Algorithms

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CS 170: Algorithms

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CS 170: Algorithms

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CS 170: Algorithms

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CS 170: Algorithms

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CS 170: Algorithms

Hello and ...

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CS 170: Algorithms

Hello and ...

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More divide and conquer: mergesort.

Sort items in n elt array: $A = [a_1, \ldots, a_n]$,

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Choose lowest from two lists, cross out, repeat.

Sorted SubArray 1: $3\,, 7\,, 8\,, 10, 11, \ldots$
Sorted Subarray 2: $4\,, 5\,, 9, 19, 20, \ldots$

$, \;\; , \;\; , \;\; ,$

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Masters: $d == \log_b a \implies O(n^d \log_b n) \implies T(n) = O(n \log n)$.

Check it out...

Iterative Mergesort: Bottom up, use of queues.

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Make each element into list and put lists in queue.

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Merge first two lists, put in queue (at end).

$\boxed{5}$ $\boxed{9}$ $\cdots\cdots$ $\boxed{3,8}$

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Rinse.

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Rinse. Repeat.

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And next pass through queue...

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Rinse. Repeat.

$\cdots\cdots\ \boxed{3,8}\ \boxed{5,9}$

And next pass through queue...

$\cdots\cdots\ \boxed{3,5,8,9}$

Each pass through queue:

Check it out...

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Rinse. Repeat.

$\cdots\cdots\ \boxed{3,8}\ \boxed{5,9}$

And next pass through queue...

$\cdots\cdots\ \boxed{3,5,8,9}$

Each pass through queue: each element touched once.

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$\cdots\cdots$ $\boxed{3,8}$ $\boxed{5,9}$

And next pass through queue...

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Each pass through queue: each element touched once. $O(n)$ time.

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Each pass through queue: each element touched once. $O(n)$ time.

Each pass halves number of lists.

Check it out...

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Each pass through queue: each element touched once. $O(n)$ time.

Each pass halves number of lists. $\implies$ $O(\log n)$ passes

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Each pass through queue: each element touched once. $O(n)$ time.
Each pass halves number of lists. $\implies$ $O(\log n)$ passes $\implies$ $O(n \log n)$ time

Sorting lower bound.

Can we do better?

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Comparison sorting algorithm only compares numbers.

Sorting lower bound.

Can we do better?

Comparison sorting algorithm only compares numbers.
Does not look at bits.

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Merge:
Compare two first elts and then output first.

Sorting lower bound.

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Comparison sort?

Sorting lower bound.

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Compare two first elts and then output first.

Comparison sort? Yes.

Sorting lower bound.

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Comparison sorting algorithm only compares numbers.
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Merge:
Compare two first elts and then output first.

Comparison sort? Yes.

“Radix” Sort.

Sorting lower bound.

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Merge:
Compare two first elts and then output first.

Comparison sort? Yes.

“Radix” Sort.
Bucket according to whether begins with “A”, “B”....

Sorting lower bound.

Can we do better?

Comparison sorting algorithm only compares numbers.
Does not look at bits.

Merge:
Compare two first elts and then output first.

Comparison sort? Yes.

“Radix” Sort.
Bucket according to whether begins with “A”, “B”....
Repeat in each bucket with next characters.

Sorting lower bound.

Can we do better?

Comparison sorting algorithm only compares numbers.
Does not look at bits.

Merge:
Compare two first elts and then output first.

Comparison sort? Yes.

"Radix" Sort.
Bucket according to whether begins with "A", "B"....
Repeat in each bucket with next characters.
Looks at characters...

Sorting lower bound.

Can we do better?

Comparison sorting algorithm only compares numbers.
Does not look at bits.

Merge:
Compare two first elts and then output first.

Comparison sort? Yes.

“Radix” Sort.
Bucket according to whether begins with “A”, “B”....
Repeat in each bucket with next characters.
Looks at characters... or looks at “bits”.

Sorting lower bound.

Can we do better?

Comparison sorting algorithm only compares numbers.
Does not look at bits.

Merge:
Compare two first elts and then output first.

Comparison sort? Yes.

“Radix” Sort.
Bucket according to whether begins with “A”, “B”....
Repeat in each bucket with next characters.
Looks at characters... or looks at “bits”.
Not a comparision sort.!

Sorting lower bound.

Thm: Comparison sort requires $\Omega(n \log n)$ comparisons.

Sorting lower bound.

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Proof idea: Input: $a_1, a_2, \ldots, a_n$

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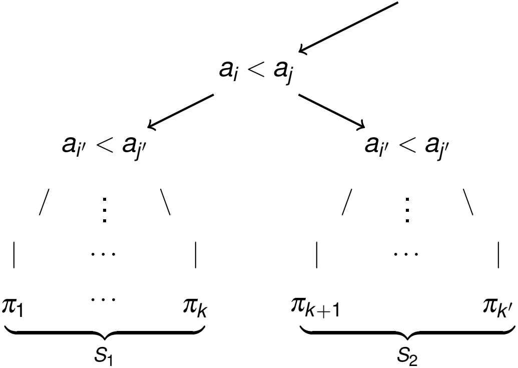
A figure proof.



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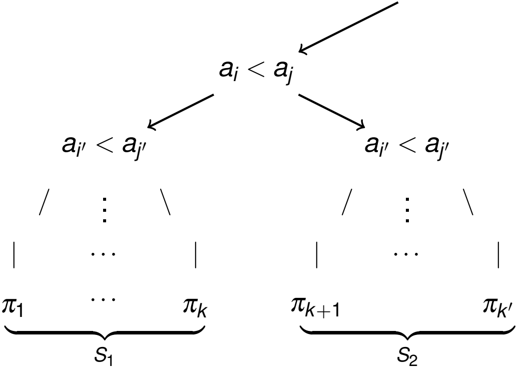

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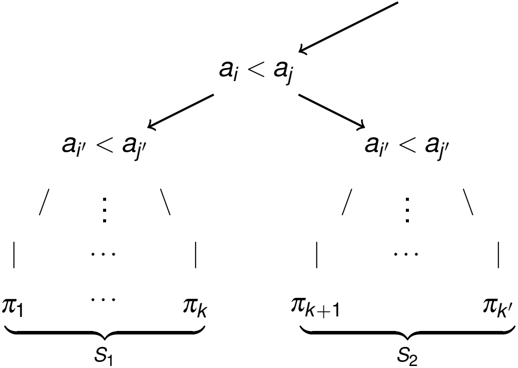

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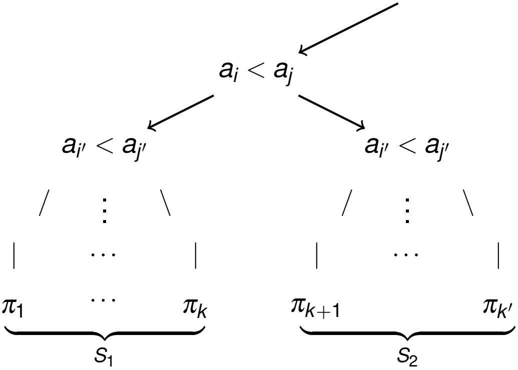

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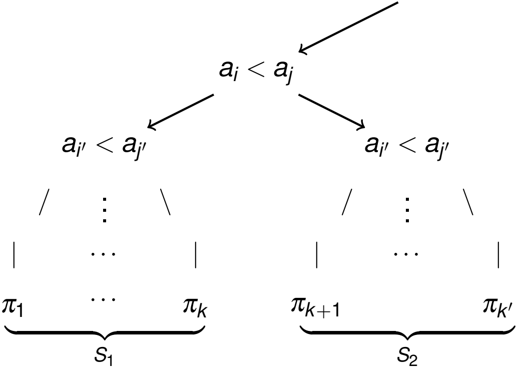

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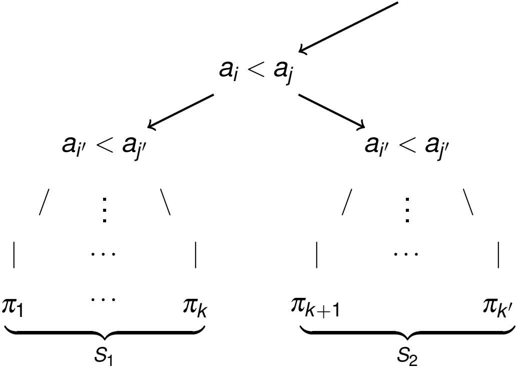

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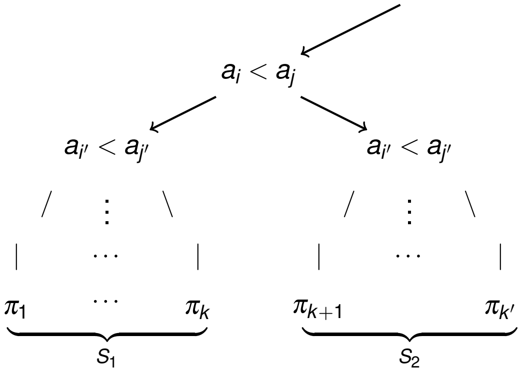

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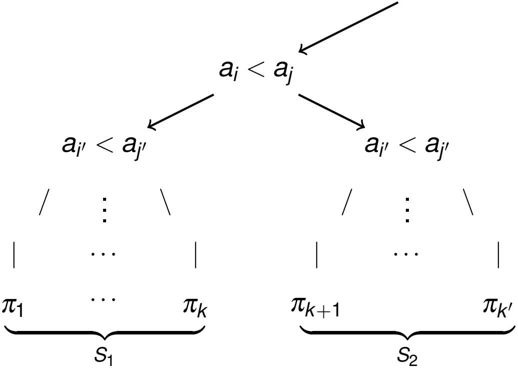

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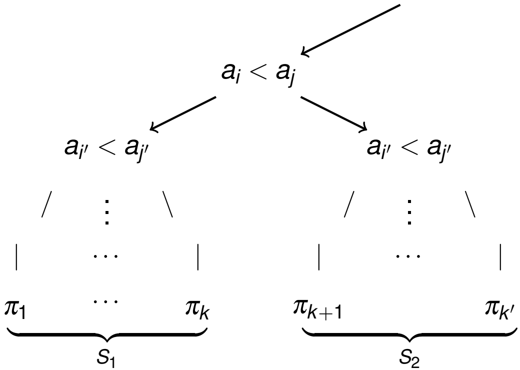

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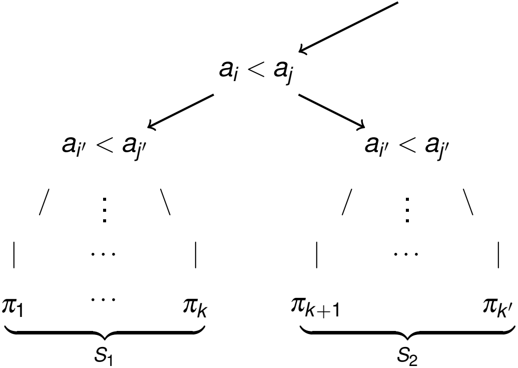

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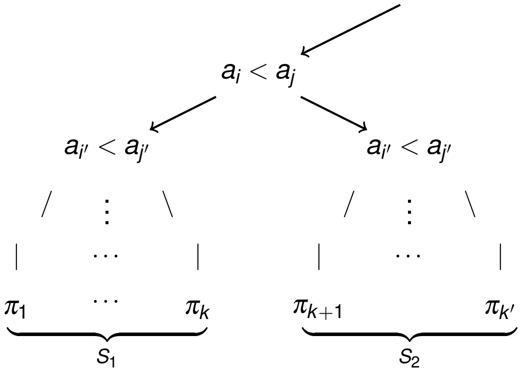

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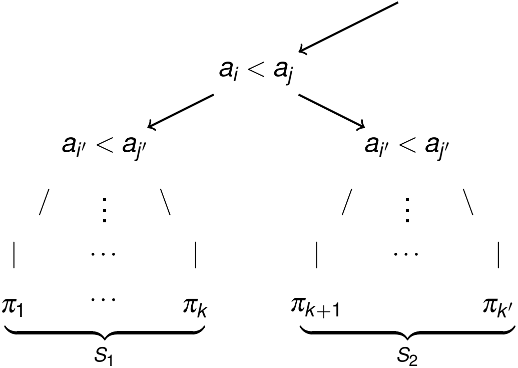

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Find the median element of a set of elements: $a_1, \ldots, a_n$.

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Better algorithm?

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On average?

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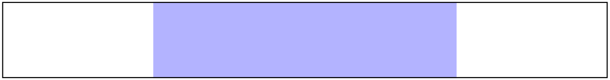

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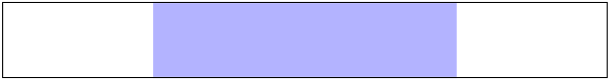

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Masters or just thinking $(n + (3/4)n + (3/4)^2 n + \cdots = O(n))$

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If pick in middle half subproblem size is $\leq \frac{3}{4}n$.

Expected time recurrence:

$$T(n) \leq T(\frac{3}{4}n) + O(n).$$

Masters or just thinking $(n + (3/4)n + (3/4)^2 n + \cdots = O(n))$
$\implies T(n) = O(n)$.

See you ..

...on Wednesday..