CS170 Fall 2013 Solutions to Homework 12

Zackery Field, section Di, 103, `cs170-fe`

December 1, 2013

1 [10 pts.] Making DAGs is hard

Consider the search problem Max-Acyclic-Induced-Subgraph:

INPUT: A *directed* graph $G = (V, E)$, and a positive integer k.

OUTPUT: A subset $S \subseteq V$ of size k such that the graph G_S obtained from G by keeping exactly those edges whose endpoints are in S is a DAG.

Show that Max-Acyclic-Induced-Subgraph is NP-complete.

- Show that the Max-Acyclic-Induced-Subgraph (MAIS) problem is in NP.
- Reduce to Independent Set / Vertex Cover / Clique

Continued on Page 4

2 [10 pts.] Reductions redux

Show that you can not hope to do much better than insertion or deletion worst case complexity $Omega(\log n)$, where n is the number of elements in queue. Prove that in the "comparison model" , there cannot exist a priority queue implementation in which both Insert and Delete-Min operations have worst case complexity $O(1)$. Hint: Do an appropriate reduction from sorting in the comparison mode.

Continued on Page 5

3 [10 pts.] Finding Zero(s)

Consider the problem Integer-Zeros.

INPUT: A multivariate polynomial $P(x_1, x_2, x_3, \ldots, x_n)$ with integer coefficients, specified as a sum of monomials.

OUTPUT: Integers $a_1, a_2, \ldots, a_n$ such that $P(a_1, a_2, a_3, \ldots, a_n) = 0$.

Show that 3-SAT reduces in polynomial time to Integer-Zeros. (INTEGER ZEROS IS NOT IN NP)

Continued on Page 6

Extra space for Problem 1

Continued from Page 1

Extra space for Problem 2

Continued from Page 2

Extra space for Problem 3

Continued from Page 3

Extra space for Problem 4

Continued from Page 4