CS 170 Algorithms
Fall 2013 Satish Rao — HW 3

Due September 20, 6:00pm

1. (11 pts.) Problem 2.5 Solve the following recurrence relations and give a Θ bound for each of them

(a) $T(n) = 2T(n/3) + 1$

(b) $T(n) = 5T(n/4) + n$

(c) $T(n) = 7T(n/7) + n$

(d) $T(n) = 9T(n/3) + n^2$

(e) $T(n) = 8T(n/2) + n^3$

(f) $T(n) = 49T(n/25) + n^{3/2}\log n$

(g) $T(n) = T(n-1) + 2$

(h) $T(n) = T(n-1) + n^c$, where $c \geq 1$ is a constant

(i) $T(n) = T(n-1) + c^n$, where $c > 1$ is a constant

(j) $T(n) = 2T(n-1) + 1$

(k) $T(n) = T(\sqrt{n}) + 1$

2. (15 pts.) Problem 2.23 (majority element)

3. (20 pts.) Problem 2.32 (closest pair of points)

4. (15 pts.) Practice with polynomials and complex numbers
The purpose of this problem is to familiarize you with computation using polynomials and complex numbers, to help you follow the lectures on the fast Fourier transform. In the following, let $\omega = e^{2\pi i/8} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$.

(a) Calculate $\omega + \omega^7$, to 2 significant digits. (or using radicals.)

(b) Let $p(x) = x^2 + 1$. Evaluate $p(1)$, $p(\omega)$, $p(\omega^2)$, and $p(\omega^3)$. Simplify your answer as much as possible.

(c) Find the unique polynomial $q(x)$ of degree at most 3 (with complex coefficients) that satisfies $q(1) = 3$, $q(\omega) = 1 + \sqrt{2}i$, $q(\omega^2) = 1$, and $q(\omega^3) = 1 + \sqrt{2}i$.

Feel free to do this computation either by hand or using software; if you use software, be sure to show the setup and explain what you did.

5. (20 pts.) Finding the missing integer An array A of length N contains all the integers from 0 to N except one (in some random order). In this problem, we cannot access an entire integer in A with a single operation. The elements of A are represented in binary, and the only operation we can use to access them is ”fetch the jth bit of $A[i]$”. Using only this operation to access A, give an algorithm that determines the missing integer by looking at only $O(N)$ bits (you may take up to $O(N\log N)$ time and $O(N\log N)$ space). (Note that there are $O(N\log N)$ bits total in A, so we can't even look at all the bits).

6. (20 pts.) **Pareto points** Given a set of *distinct* points S, in the form $\{(x_1, y_1), \ldots, (x_n, y_n)\}$, a Pareto optimal point is a point $p = (x, y)$ where for all other points, $(x', y') \in S$ *either* $x' < x$ or $y' < y$. For example, for the set $\{(1,1), (2,4), (4,2), (3,3)\}$, the points $(2,4), (4,2)$ and $(3,3)$ are Pareto optimal.

Give an efficient algorithm to find the set of Pareto optimal points. (An $O(n^2)$ algorithm is easy, and not the answer we are seeking.)

Week 3 Fun Fact

The famous economist, Vilfredo Pareto, devised the notion Pareto-optimal allocation of resources which is achieved when it is not possible to make anyone better off without making someone else worse off.
In his later years Pareto shifted from economics to sociology in response to his own change in beliefs about how humans act. He came to believe that humans act nonlogically, but they make believe they are acting logically.