CS 170 Fall 2013 — Algorithms, Satish Rao — HW 6

Due October 18, 6:00pm

1. **(10+5 pts.) Graph construction**
   A farmer(F) wants to cross the river with his wolf(W), sheep(S) and cabbage(C). F can carry at most one of W, S or C in the boat. Without the presence of F, W will eat S, and S will eat C. F can go back and forth across the river, and he wants to safely move all W, S, and C from the east side of the river (E) to the west side (W). One river crossing means the farmer goes from E to W, or W to E.

   (a) find a solution involving the least number of river crossings for the farmer.

   (b) How many different such solutions (i.e. with least number of crossings) are there?

   Hint: Think about how we can cast this problem as finding a path in a graph. What should the graph be?

2. **(15 pts.)** In a country with n cities, we have two methods of travel: roads and planes. (more formally, our graph is $G = (V, R \cup P)$, where V is our set of cities, R is the set of roads, and P are the set of planes). Roads i is specified as an **undirected** edge (u_i, v_i, C_i) that connects the two cities u_i, v_i with a **non-negative** cost C_i. Planes j specified as **directed** edges (u_j, v_j, C_j) which connects city u_j to v_j (one way) with cost C_j (Due to some weird airline award program, the cost **can possibly be negative!**) with the additional constraint that there is no sequence of roads and planes that we can follow that takes us back from v_j to u_j.
   Assume we have the the adjacency list representation of roads and planes (two separate data structures), and we can freely swap between the two methods of travel (i.e. we can take an arbitrary amount of roads, then planes, then roads, and so on...).
   Given two cites s, t, give an efficient algorithm to compute the shortest path from s to t (should have same running time as Dijkstra).

3. **(10+5 pts.) Shortest path in currency trading**
   Problem 4.21 (online version)

4. **(5+5+5+5 pts.) Cycle property and another MST algorithm**
   Problem 5.21 (online version)

5. **(5+5+5+5 pts.) Update MST after changing one edge**
   Problem 5.22 (online version)

Week 6 Fun Fact

One of Edsger W. Dijkstra's sidelines was serving as Chairman of the Board of the fictional Mathematics Inc., a company that he imagined having commercialized the production of mathematical theorems in the same way that software companies had commercialized the production of computer programs. He invented a number of activities and challenges of Mathematics Inc. and documented them in several papers in the EWD series. The imaginary company had produced a proof of the Riemann Hypothesis but then had great difficulties collecting royalties from mathematicians who had proved results assuming the Riemann Hypothesis. The proof itself was a trade secret.