CS 170: Algorithms

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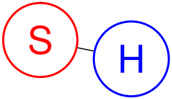
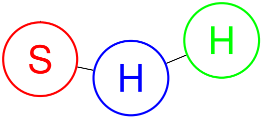
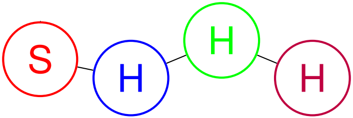


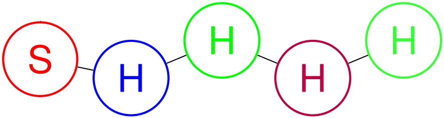
CS 170: Algorithms



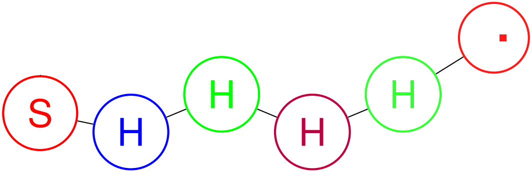
CS 170: Algorithms



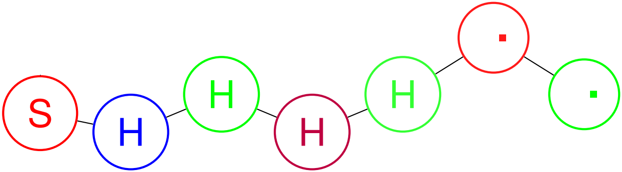
CS 170: Algorithms



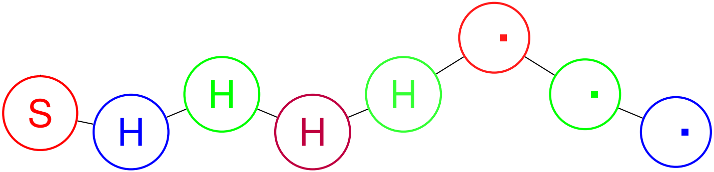
CS 170: Algorithms



CS 170: Algorithms



CS 170: Algorithms



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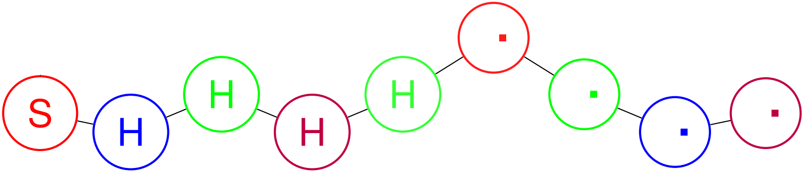

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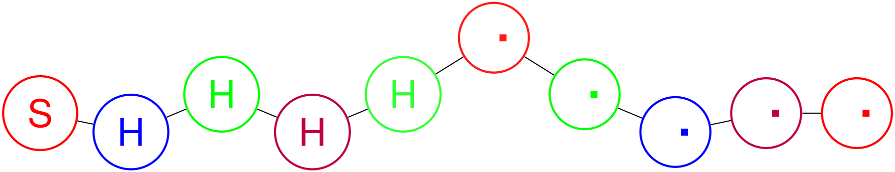

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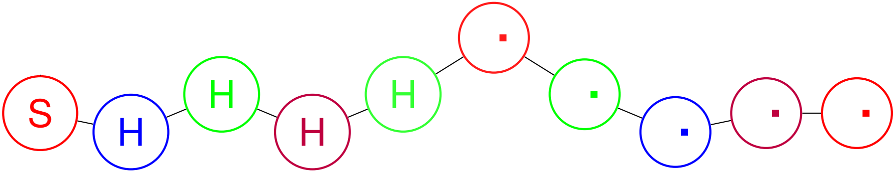

No laptops please.

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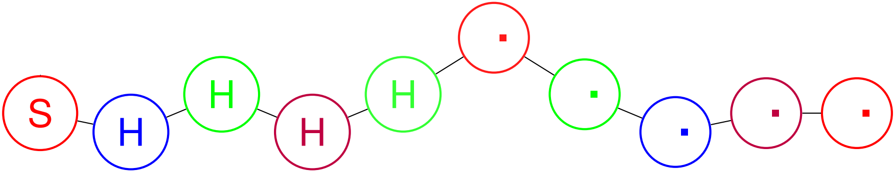

No laptops please.

Thank you

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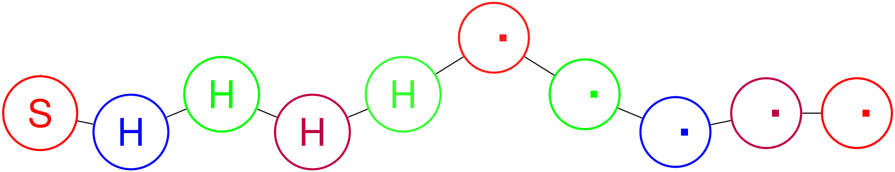

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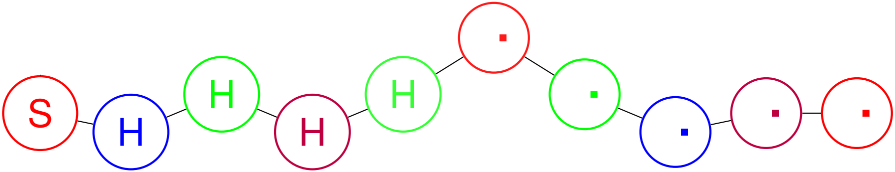

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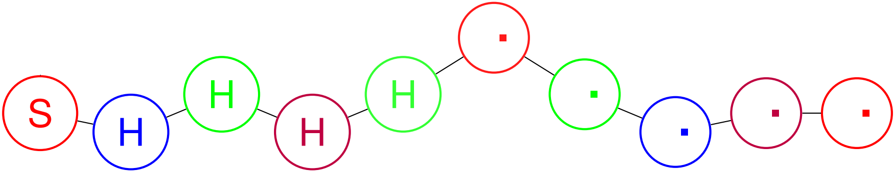

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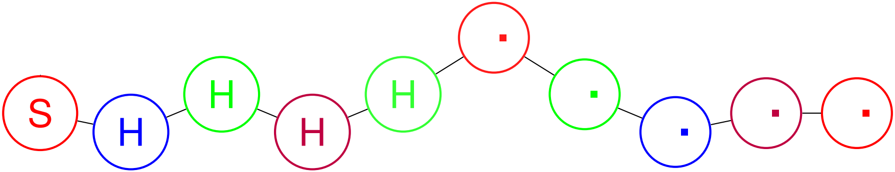

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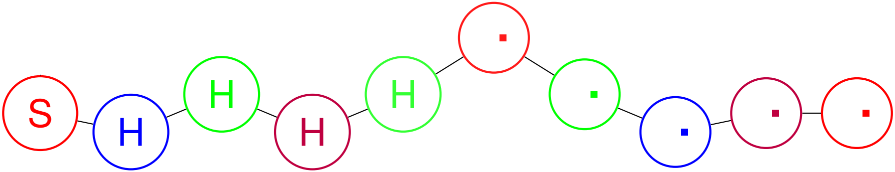

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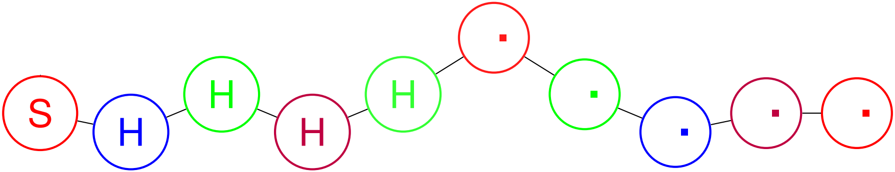

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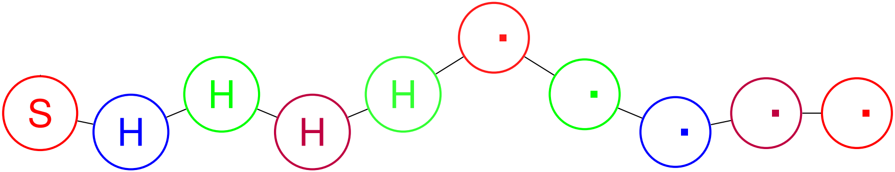

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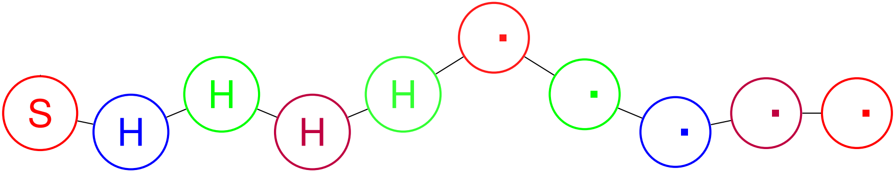

No laptops please.

Thank you ! ! ! ! ! ! ! !

Midterm.

Thursday.

Midterm.

Thursday.

Rooms: will give algorithm shortly.

Midterm.

Thursday.

Rooms: will give algorithm shortly.

Midterm style: see midterm 1 last fall.

Midterm.

Thursday.

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- True/false.

Midterm.

Thursday.

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Midterm style: see midterm 1 last fall.
 True/false.
 Short/and pretty short answers.

Midterm.

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- True/false.
- Short/and pretty short answers.
- Two/Three longer questions.

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Why?

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- Two/Three longer questions.

Why? Coverage.

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Why? Coverage. Fairness.

Midterm.

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   Short/and pretty short answers.
   Two/Three longer questions.

Why? Coverage. Fairness. Smoothness.

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One cheat sheet:

Midterm.

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One cheat sheet: one side!

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- 8.5×11, No mobius strips,

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Must be questions.

Negative edges.

Notice: argument for Dijkstra breaks for negative edges.

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For example.

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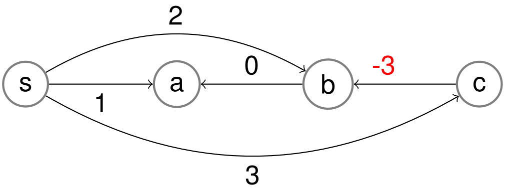

Dijkstra:

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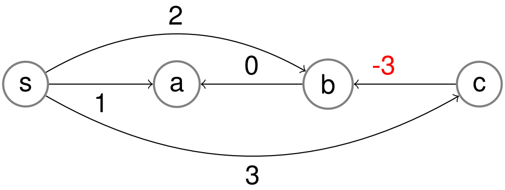
For example.



Dijkstra:
Process s:

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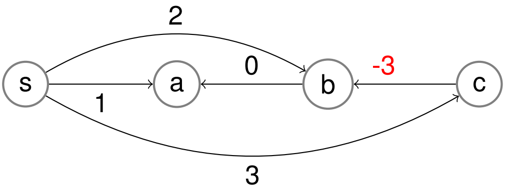
For example.

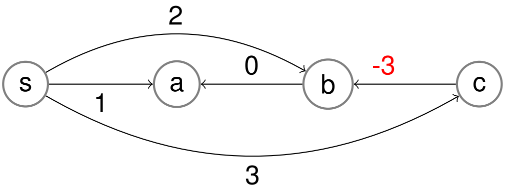


Dijkstra:
Process s: Set $d(a) = 1, d(b) = 2, d(c) = 3$.

Negative edges.

Notice: argument for Dijkstra breaks for negative edges.

For example.



Dijkstra:
Process s: Set $d(a) = 1, d(b) = 2, d(c) = 3$.
Process a, $d(a) = 1$:

Negative edges.

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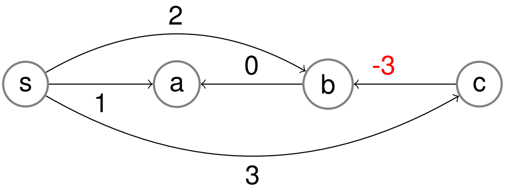
For example.



Dijkstra:
Process s: Set $d(a) = 1, d(b) = 2, d(c) = 3$.
Process a, $d(a) = 1$: No outgoing edges.

Negative edges.

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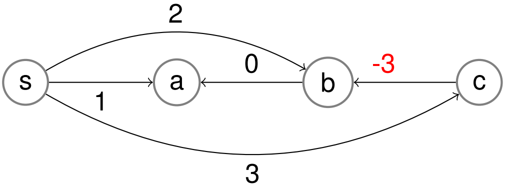
For example.

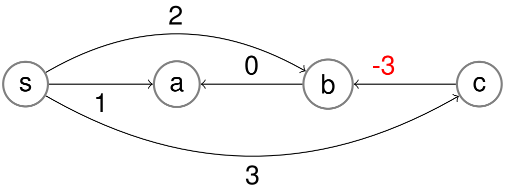


Dijkstra:
Process s: Set $d(a) = 1, d(b) = 2, d(c) = 3$.
Process a, $d(a) = 1$: No outgoing edges.
Process b, $d(b) = 2$:

Negative edges.

Notice: argument for Dijkstra breaks for negative edges.

For example.

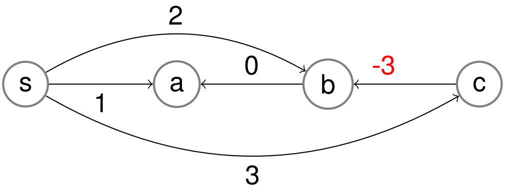


Dijkstra:
Process s: Set $d(a) = 1, d(b) = 2, d(c) = 3$.
Process a, $d(a) = 1$: No outgoing edges.
Process b, $d(b) = 2$: $d(a)$ still set to 1.

Negative edges.

Notice: argument for Dijkstra breaks for negative edges.

For example.



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Process c, $d(c) = 3$:

Negative edges.

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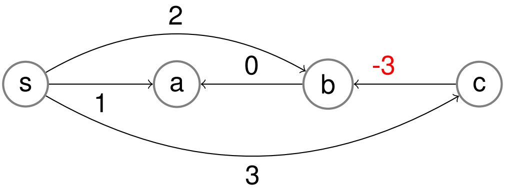
For example.



Dijkstra:
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Process c, $d(c) = 3$: Set $d(b) = 0$.

Negative edges.

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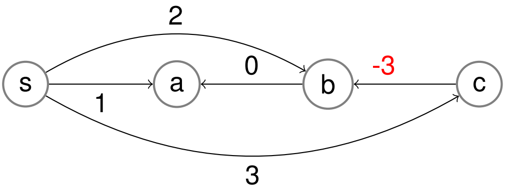
For example.



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Process s: Set $d(a) = 1, d(b) = 2, d(c) = 3$.
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Process c, $d(c) = 3$: Set $d(b) = 0$.
But, can't process b, again!!!

Negative edges.

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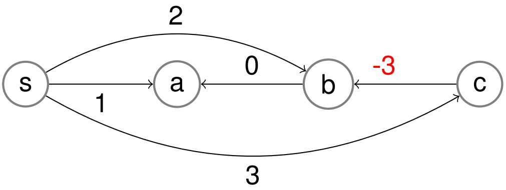

Dijkstra:
Process s: Set $d(a) = 1, d(b) = 2, d(c) = 3$.
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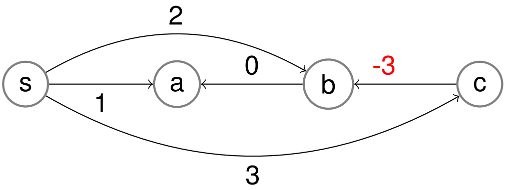

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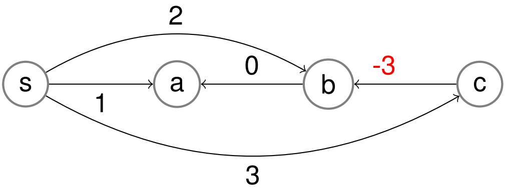

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Process c, $d(c) = 3$: Set $d(b) = 0$.
But, can't process b, again!!!
$d(a)$ still 1. Should be 0
Problem: $d(b)$ was incorrect when processed due to negative edge.

Update/Bellman-Ford.

def update $((u, v))$:

dist(v) = min (dist(v), dist(u) + l (u,v)).

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Properties:
1) $d(v)$ is correct if u is second to last node on shortest path, and $d(u)$ correct.
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$$s \rightarrow u_1 \rightarrow u_2 \rightarrow \cdots \rightarrow u_k \rightarrow t$$

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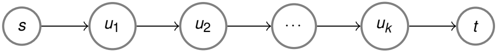

If update (s, u_1),

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$$s \longrightarrow u_1 \longrightarrow u_2 \longrightarrow \cdots \longrightarrow u_k \longrightarrow t$$

If update (s, u_1), then (u_1, u_2),

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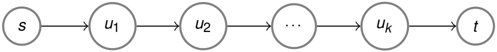

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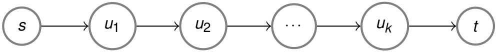

If update (s, u_1), then (u_1, u_2), ... and finally (u_k, t).

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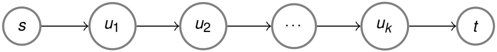

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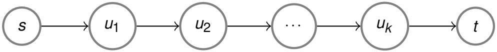

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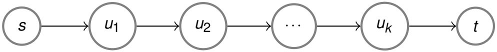

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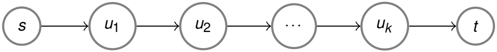

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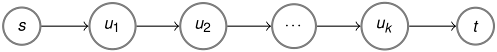

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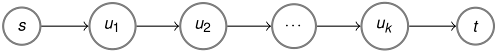

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Time: $O(|V||E|)$

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Bellman-Ford: $d(v) \leq$ length of i edge shortest path.

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Update will change a distance!

DAG

Dijkstra:

DAG

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$O((m+n)\log n)$ time

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Shortest path for DAG:

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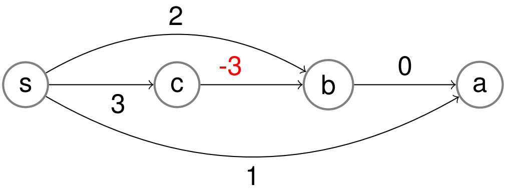
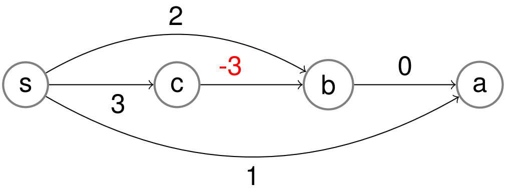
Shortest path for DAG:
linearize
process nodes (and update neighbors in order.)

DAG

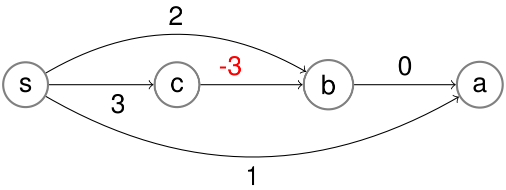


DAG



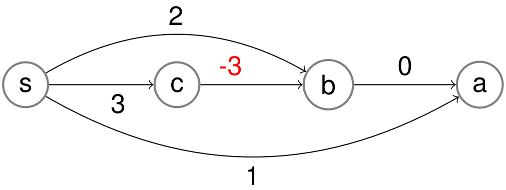
Process s, $d(s) = 0$:

DAG



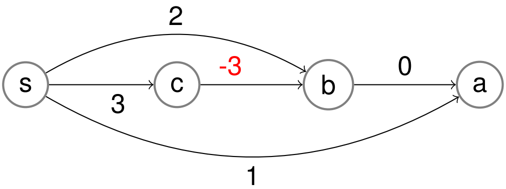
Process s, $d(s) = 0$: Updates $d(c) = 3$,

DAG



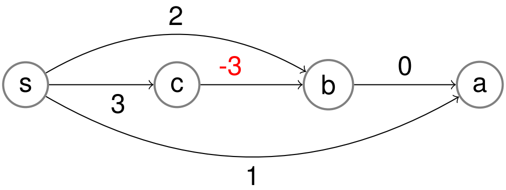
Process s, $d(s) = 0$: Updates $d(c) = 3, d(b) = 2$,

DAG



Process s, $d(s) = 0$: Updates $d(c) = 3, d(b) = 2, d(a) = 1$.

DAG



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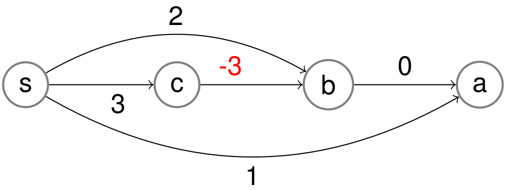
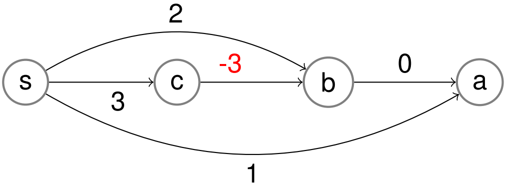
Process c, $d(c) = 3$:

DAG



Process s, $d(s) = 0$: Updates $d(c) = 3, d(b) = 2, d(a) = 1$.
Process c, $d(c) = 3$: Update $d(b) = 0$.

DAG



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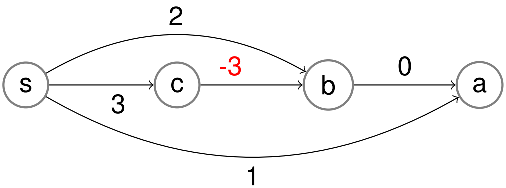
Process b, $d(b) = 0$:

DAG



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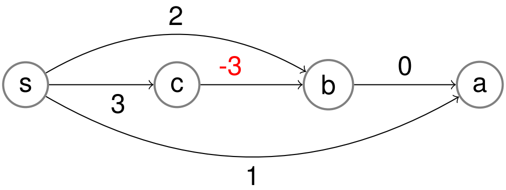
DAG



Process s, $d(s) = 0$: Updates $d(c) = 3, d(b) = 2, d(a) = 1$.
Process c, $d(c) = 3$: Update $d(b) = 0$.
Process b, $d(b) = 0$: $d(a) = 0$.
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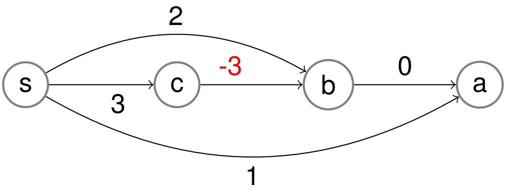

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Done.

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Property: A tree has $n - 1$ edges.

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Start with empty graph with n components.

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Adding some edge reduces components by one.

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Given a graph, $G = (V, E)$, edge weights w_e, find the cheapest possible connected subgraph.

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If negative edges, then restrict to tree.

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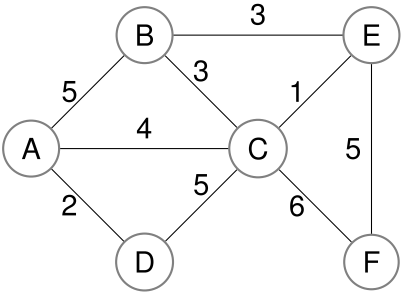
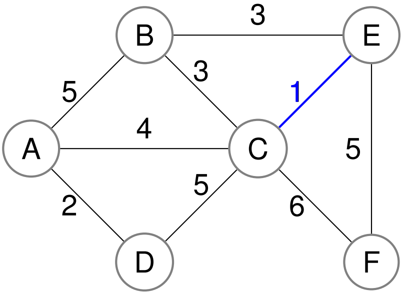
Given a graph, $G = (V, E)$, edge weights w_e, find the lowest weight spanning tree.

Example and Algorithm



Example and Algorithm



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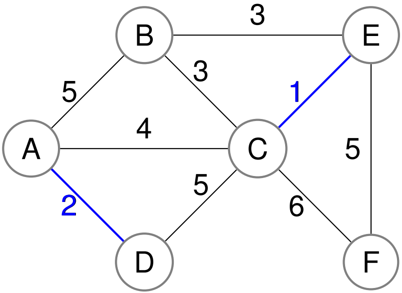

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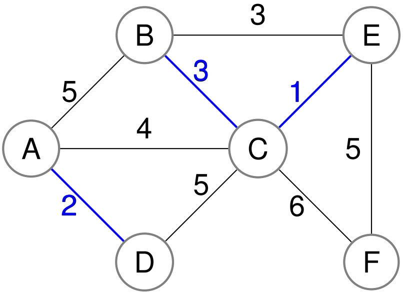

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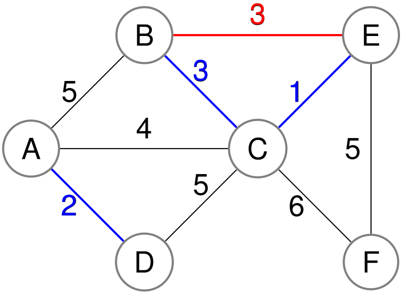

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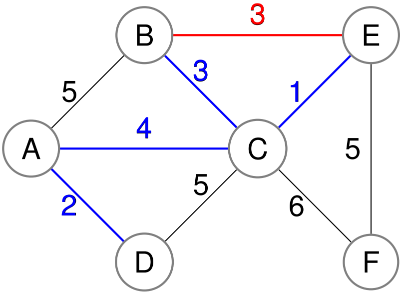

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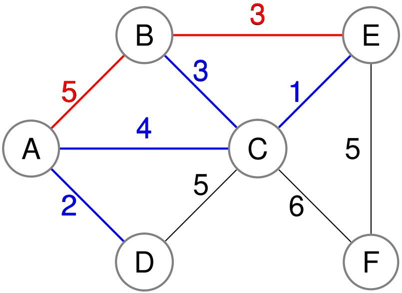

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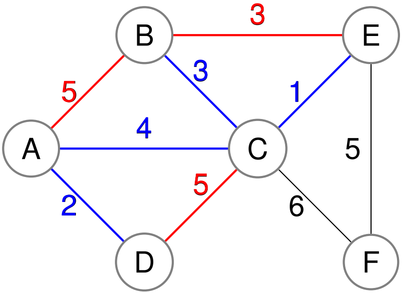

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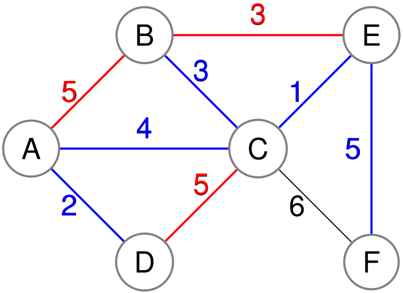

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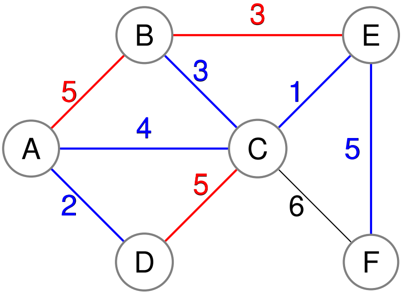

MST: total cost is 2+4+3 +1 + 5 = 15.

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Smallest edge across any cut is in some MST.

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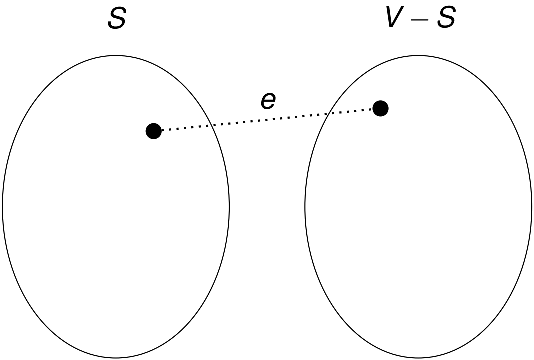

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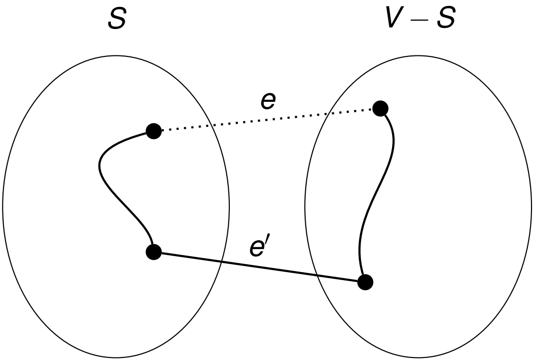

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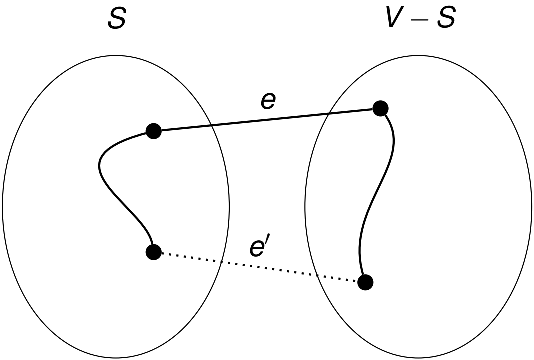

Replace e' with e.

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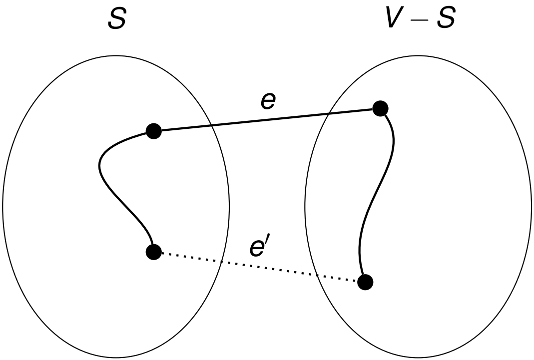

Replace e' with e.
Every pair remains connected.

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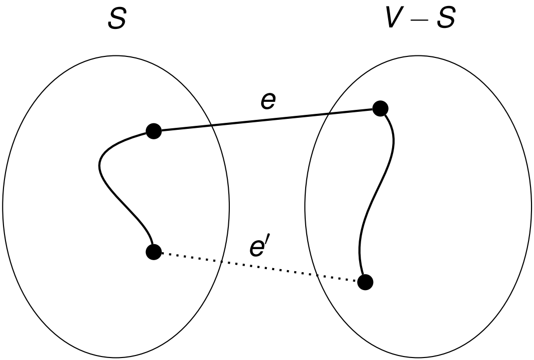

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If used e' can use path through e.

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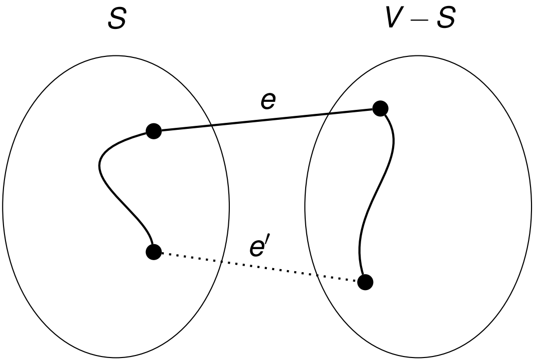

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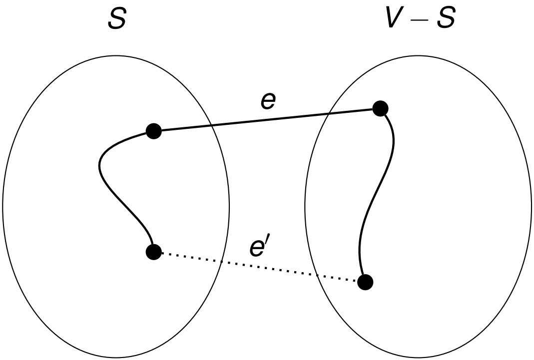

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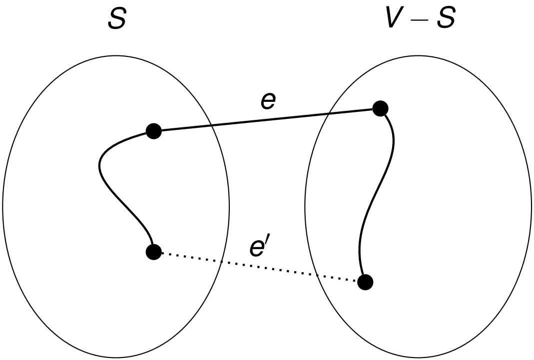

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"Add edge" $\equiv$ "**union(u,v)**"

See you

See you ..on Monday.