

Due September 6, 6:00pm

Instructions: Get an EECS instructional computer account if you don't have one already. Register with the grading system.

Please write your name, the username for your instructional account, your TA's name, your discussion section time (e.g., Wed 3pm) prominently on the first page of your homework. Also list your study partners for this homework, or "none" if you had no partners.

You are welcome to form small groups (up to four people) to work through the homework, but you **must** write up all your solutions strictly by yourself, and you must acknowledge any ideas you got from others (including from books, papers, web pages, etc.). Please read the collaboration policy on the course web page.

This homework is due Friday, September 6, at 6:00pm electronically. You need to submit it via glookup with your instructional computer account. See piazza for details on glookup and format. Please turn in all problems, we may only grade a subset of the problems due to resource limitations.

1. (10 pts.) Getting started

Please read the course policies on the web page, especially the course policies on collaboration. If you have any questions, contact the instructors via piazza. Once you have done this, please write "I understand the course policies." on your homework to get credit for this problem.

2. (20 pts.) Compare Growth Rates. In each of the following situations, indicate whether $f = O(g)$, or $f = \Omega(g)$, or both (in which case $f = \Theta(g)$). Give a one sentence justification for each of your answers.

- | | $f(n)$ | $g(n)$ |
|-----|---------------------|---------------------|
| (a) | $n^{3.75}$ | $n^{2.72}$ |
| (b) | $10n - (\log n)^2$ | $n + \log n$ |
| (c) | $n^3 + 1000$ | $n^3 - 1000$ |
| (d) | $\log 500n$ | $\log 8n$ |
| (e) | $5 \log n$ | $\log(n^7)$ |
| (f) | $5n \log 5n$ | $n \log n$ |
| (g) | $n^3 / \log n$ | $n^4 (\log n)^3$ |
| (h) | $n(\log n)^5$ | $n^{1.01}$ |
| (i) | $(\log n)^{100}$ | $n^{0.01}$ |
| (j) | $\sqrt[3]{n}$ | $(\log n)^2$ |
| (k) | $n \log n$ | $(\log n)^{\log n}$ |
| (l) | $n^{1/3}$ | $3^{\log_2 n}$ |
| (m) | 5^n | $n4^n$ |
| (n) | 2^n | 2^{3n} |
| (o) | 7^n | 7^{n-1} |
| (p) | $(\log n)^{\log n}$ | $7^{(\log n)^2}$ |
| (q) | 8^n | $n!$ |

3. (12 pts.) Prove Order of Growth.

1. Show that, for any constant k ,

$$\sum_{i=1}^n i^k = \Theta(n^{k+1}).$$

(Hint: To show an upper bound, compare i^k with n^k . To show a lower bound, compare i^k with $(n/2)^k$ for $i \geq n/2$.)

2. Show that

$$\log(n!) = \Theta(n \log n).$$

(Hint: To show an upper bound, compare $n!$ with n^n . To show a lower bound, compare it with $(n/2)^{n/2}$.)

3. Show that

$$\sum_{i=1}^n \frac{1}{i} = \Theta(\log n).$$

(Hint: To show an upper bound, compare $\frac{1}{i}$ with $\frac{1}{t}$, where t is a power of two just smaller than i . To show a lower bound, compare $\frac{1}{i}$ with $\frac{1}{t}$, where t is a power of two just greater than i .)

4. (10 pts.) Problem 0.2 (online version)

5. (10 pts.) Problem 1.19 (online version)

6. (5 pts.) Problem 1.22 (online version)