CS 170: Algorithms

Account forms after class!

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Static Course Webpage. (inst.cs.berkeley.edu/c̃s170)

CS 170: Algorithms

Account forms after class!

Static Course Webpage. (inst.cs.berkeley.edu/c̃s170)

Will mostly use piazza. Should have/get invitation soon.

CS 170: Algorithms

Account forms after class!

Static Course Webpage. (inst.cs.berkeley.edu/c̃s170)

Will mostly use piazza. Should have/get invitation soon.

Homeworks will be turned in electronically.

CS 170: Algorithms

Account forms after class!

Static Course Webpage. (inst.cs.berkeley.edu/c̃s170)

Will mostly use piazza. Should have/get invitation soon.

Homeworks will be turned in electronically.
Latex ok, formatting ok, scanning handwritten stuff ok.

CS 170: Algorithms

Account forms after class!

Static Course Webpage. (inst.cs.berkeley.edu/c̃s170)

Will mostly use piazza. Should have/get invitation soon.

Homeworks will be turned in electronically.
Latex ok, formatting ok, scanning handwritten stuff ok.
Instructions on piazza.

Instructor.

Prof. Satish Rao

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Algorithms, graph algorithms, optimization, approximation.

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School: ..a long time ago, far far away..

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(also Microsoft, Bell Labs, ICAD,

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... Berkeley!

(also Microsoft, Bell Labs, ICAD, ..., paperboy, sold newspapers door to door.)

GSIs

Rudy Dai
Berkeley Undergrad

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Berkeley Undergrad

Lewin Gan

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PhD student
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Di Wang PhD Student
Theoretical Computer Science

Puzzler.

Given a set of strings, is there a pair which has the same letters permuted?

Puzzler.

Given a set of strings, is there a pair which has the same letters permuted?

ATE, CAT, CATEGORY, GORY, ALOOF,

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Given a set of strings, is there a pair which has the same letters permuted?

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Given a set of strings, is there a pair which has the same letters permuted?

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Try all permutations of each word, check for duplicates

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Try all permutations of each word, check for duplicates ?

Can you do better?

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Can you do better?

Idea: make a canonical representation of each word.

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Try all permutations of each word, check for duplicates ?

Can you do better?

Idea: make a canonical representation of each word.

Sort letters of each word, check for duplicates.

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AET, ACT, ACEGORTY, GORY,ALFOO, AET...

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ATE, CAT, CATEGORY, GORY, ALOOF, EAT, ...?

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Can you do better?

Idea: make a canonical representation of each word.

Sort letters of each word, check for duplicates.

AET, ACT, ACEGORTY, GORY,ALFOO, AET...

```
def find˙permutations(words):
   stash =
   for w in words:
      s = str(sorted(w))
      if s in stash:
         return [w,stash[s]]
      else:
         stash[s] = w
   return False
```

Another problem.

Does a list have a cyle?

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Mark first node, see if one comes back to it.

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Stupid Pet Quiz: Does this work?

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Mark first node, see if one comes back to it.

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a) Yes. b) No.

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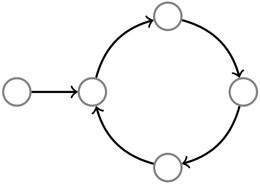
a) Yes. b) No.



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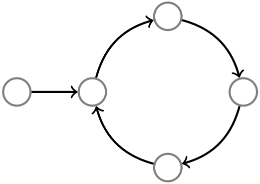
a) Yes. b) No.



First node not in cylce!

Another problem.

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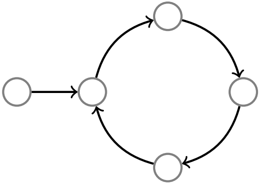
Stupid Pet Quiz: Does this work?

a) Yes. b) No.



First node not in cylce!

Answer is no.

Does a list have a cyle?

Does a list have a cyle?

Two ptrs:

Does a list have a cyle?

Two ptrs: advance ptr 1 twice

Does a list have a cyle?

Two ptrs: advance ptr 1 twice
advance ptr 2 once.

Does a list have a cyle?

Two ptrs: advance ptr 1 twice
advance ptr 2 once.
If ever at the same place, report cycle.

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Two ptrs: advance ptr 1 twice
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Correctness:

Does a list have a cyle?

Two ptrs: advance ptr 1 twice
advance ptr 2 once.
If ever at the same place, report cycle.

Correctness:
If no cycle, slow pointer never catches fast one.

Does a list have a cyle?

Two ptrs: advance ptr 1 twice
advance ptr 2 once.
If ever at the same place, report cycle.

Correctness:
If no cycle, slow pointer never catches fast one.
If cycle, both pointers will enter cycle at some time.

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If ever at the same place, report cycle.

Correctness:
If no cycle, slow pointer never catches fast one.
If cycle, both pointers will enter cycle at some time.
d - distance from fast ptr to slow ptr.

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Correctness:
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d - distance from fast ptr to slow ptr.

d decreases every step.

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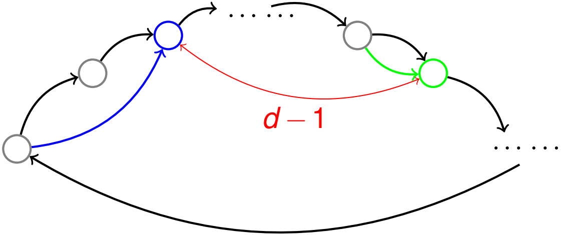

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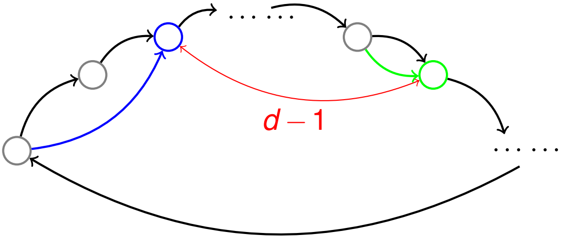

Runtime: n steps to cycle

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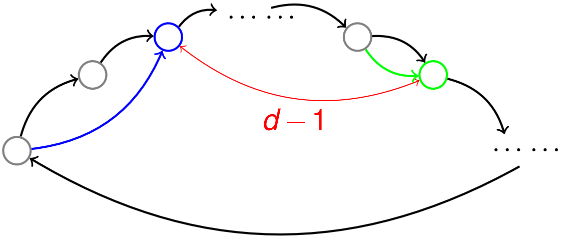

Runtime: *n* steps to cycle *n* steps to catch up.

Does a list have a cyle?

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Correctness:
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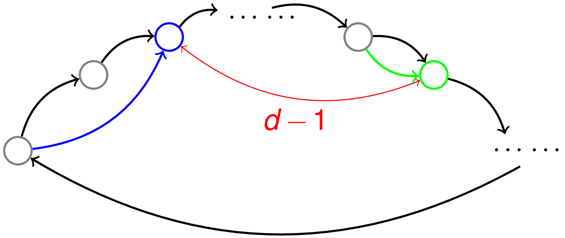

Runtime: n steps to cycle n steps to catch up. $O(n)$

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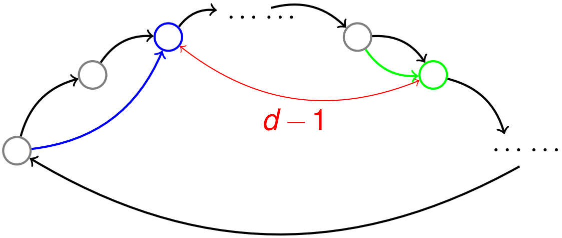

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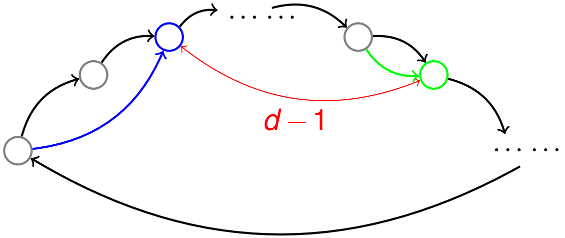

Runtime: n steps to cycle n steps to catch up. $O(n)$
Additional storage: two pointers. $O(1)$

Puzzles ..

Solutions

Puzzles ..

Solutions

..are Algorithms...

Puzzles ..

Solutions

..are Algorithms...

which...

Puzzles ..

Solutions

..are Algorithms...

which...

..are correct...

Puzzles ..

Solutions

..are Algorithms...

which...

..are correct...and efficient.

Puzzles ..

Solutions

..are Algorithms...

which...

..are correct...and efficient.

Is this a useful process?

Algorithms for the Human Genome Project

Reconstruct DNA...

Algorithms for the Human Genome Project

Reconstruct DNA...

ACTGAAACTGAGTAGATA....

Algorithms for the Human Genome Project

Reconstruct DNA...

ACTGAAACTGAGTAGATA....

Read first, then next, then next, ...3.1 billion times...
.. slow... error prone...

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Parallel sequencing yields chunks of overlapping DNA.

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AGTAG, AGATA, TGAGT , ACTGAA , CTGAA , AAACTG

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Assemble into a consistent string?

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```
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```

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    AAACTG
        TGAGT
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          AGTAG
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    AAACTG
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```
ACTGAA
    AAACTG
        TGAGT
          AGTAG
             AGATA
-------------------
ACTGAAACTGAGTAGATA
```

Page Rank.

Problem: What is good on the web?

Page Rank.

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Website that pays search engine most?

Page Rank.

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Website that pays search engine most? Goto.com.

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Random Surfer Model (Brin-Page):

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Sort search results by PageRank!

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Sort search results by PageRank!

Made us happy.

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Sort search results by PageRank!

Made us happy.

Google.

Page Rank.

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Sort search results by PageRank!

Made us happy.

Google. (300 million dollars for license for Stanford!)

Algorithms...

Driving Directions

Algorithms...

Driving Directions
Airline Scheduling

Algorithms...

Driving Directions
Airline Scheduling
Compiling

Algorithms...

Driving Directions
Airline Scheduling
Compiling
Compression

Algorithms...

Driving Directions
Airline Scheduling
Compiling
Compression
Cryptography

Algorithms...

Driving Directions
Airline Scheduling
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Compression
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Algorithms...

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...(at Akamai, finding fastest, cheapest least loaded caches)

Algorithms...

Driving Directions
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...(at Akamai, finding fastest, cheapest least loaded caches)
.
.
.

Doing well in this class..

The Bloom 2-sigma effect.

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One on one instruction and mastery learning leads to a two sigma improvement in performance.

Doing well in this class..

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One on one instruction and mastery learning leads to a two sigma improvement in performance.

50th percentile to the 98th percentile.

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The median Harvard applicant

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The median Harvard applicant would get in!

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One on one instruction ....for 300 students

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Resources: office hours, discussion, piazza,...

and .. you!

You have the book, homeworks, exams, slides, and the web...

The book.

More e.e. cummings than Danielle Steele.

The book.

More e.e. cummings than Danielle Steele.

Close reading.

The book.

More e.e. cummings than Danielle Steele.

Close reading.

Work through examples.

The book.

More e.e. cummings than Danielle Steele.

Close reading.

Work through examples.

Take care to understand arguments.

Homeworks.

Do them, in groups (up to four.)

Homeworks.

Do them, in groups (up to four.)

Write up solutions, on your own, in your own words.

Homeworks.

Do them, in groups (up to four.)

Write up solutions, on your own, in your own words.

Uh...

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To be clear... extremely important for exams

Homeworks.

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To be clear... extremely important for exams ...fun too!!

Quiz.

According to the Bloom effect, with one on one tutoring and mastery learning, a 50th percentile student could reasonably expect to

Quiz.

According to the Bloom effect, with one on one tutoring and mastery learning, a 50th percentile student could reasonably expect to

a) perform at the 50th percentile.

Quiz.

According to the Bloom effect, with one on one tutoring and mastery learning, a 50th percentile student could reasonably expect to

a) perform at the 50th percentile.

b) get into Harvard!

Quiz.

According to the Bloom effect, with one on one tutoring and mastery learning, a 50th percentile student could reasonably expect to

a) perform at the 50th percentile.

b) get into Harvard!

c) be really annoyed by their tutor.

Quiz.

According to the Bloom effect, with one on one tutoring and mastery learning, a 50th percentile student could reasonably expect to

a) perform at the 50th percentile.

b) get into Harvard!

c) be really annoyed by their tutor.

d) perform at the 98th percentile.

Quiz.

According to the Bloom effect, with one on one tutoring and mastery learning, a 50th percentile student could reasonably expect to

a) perform at the 50th percentile.

b) get into Harvard!

c) be really annoyed by their tutor.

d) perform at the 98th percentile.

The best answer is

Quiz.

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The best answer is ... (d).

Calculating: 300 BC through Middle Ages in Europe.

I – one

Calculating: 300 BC through Middle Ages in Europe.

I – one
V – five

Calculating: 300 BC through Middle Ages in Europe.

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Calculating: 300 BC through Middle Ages in Europe.

I – one
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Calculating: 300 BC through Middle Ages in Europe.

I – one
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C – one hundred
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Calculating: 300 BC through Middle Ages in Europe.

I – one
V – five
X – ten
C – one hundred
D – five hundred
M – a thousand

Calculating: 300 BC through Middle Ages in Europe.

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I – one
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VIII – eight

Calculating: 300 BC through Middle Ages in Europe.

I – one
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DCLXV – five hundred plus a hundred plus fifty plus ten plus five

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Multiply roman numbers?

Modern system.

From India, via Al Khwarizmi.

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The input representation for modern computers and communication.

Writing to propagating..

Al Khwarizmi:

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..but Fibonacci popularized its use.

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So some homage...

Fibonacci numbers.

$F_0 = 0$, $F_1 = 1$.

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$T(n) \geq F_n$

Fibonacci algorithm and numbers!

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Can we do better?

Sure..much better.

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def fib(n):
  if n <= 1:
     return n
  else:
    a = [0,1]
    for i in xrange(2,n+1):
       a.append(a[i-1]+a[i-2])
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$O(n)$ operations! Maybe.

Let’s try it.

From demo: Size matters.

How many bits in F_n?

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Remember $F_n \approx 2^{0.694n}$.

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About how many bits in F_n?

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$\log_2 F_n$

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How long does it take to compute $F_{n-1} + F_{n-2}$?

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How long does Fib take?

n additions.

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How long does Fib take?

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At most $O(n^2)$.

Demo: polynomial.

Doubling size, made time grow by factor of four.

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Not true for exponential algorithms.

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Asymptotic bounds give good prediction of scaling behavior.

Doubling size, scales runtime by a constant for any polynomial time algorithm.

Not true for exponential algorithms. Could square runtime!

Asymptotic Analysis.

Used $O(n)$ for number of additions, rather than $n-2$.

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Recursive fib has faster inner loop than iterative fib.

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$2^{.694n}$ versus $O(n^2)$.

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$2^{.694n}$ versus $O(n^2)$.

For $2^{.694n}$, doubling n, squares run time.

For $O(n^2)$, doubling n, multiplies run time by four.

Asymptotic Notation.

Ignore constant factors.

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$2n^2$ asymptotically same as $4n^2$

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Upper bound.
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Upper bound.
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Formally, for positive functions g, f from integers to reals,
$g(n) = O(f(n)$, if there is a constant c where $g(n) \leq cf(n)$

More asymptotic notation.

Ω notation.

More asymptotic notation.

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A “lower bound”.

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Formally, for positive functions g, f from integers to reals,
$g(n) = \Omega(f(n))$, if there is a constant c where $g(n) \geq cf(n)$

More asymptotic notation.

Ω notation.

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Formally, for positive functions g, f from integers to reals,
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$g(n) = \Theta(f(n))$ if $g(n) = O(f(n))$ and $g(n) = \Omega(f(n))$.

...see you on Wednesday.