CS 170: Algorithms

CS 170: Algorithms

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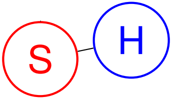
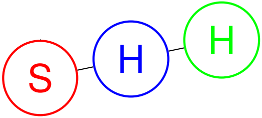
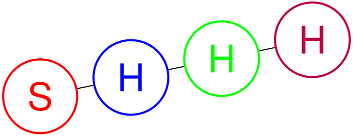


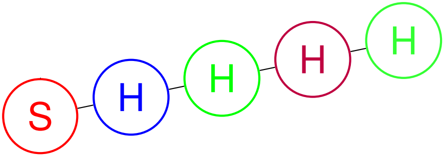
CS 170: Algorithms



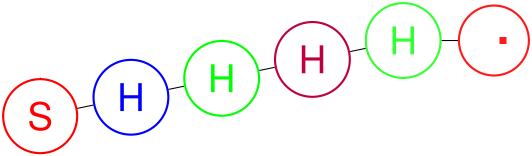
CS 170: Algorithms



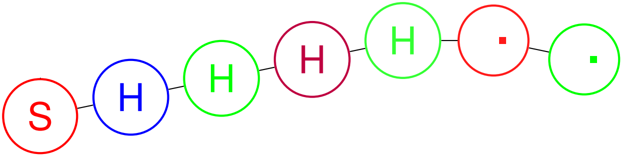
CS 170: Algorithms



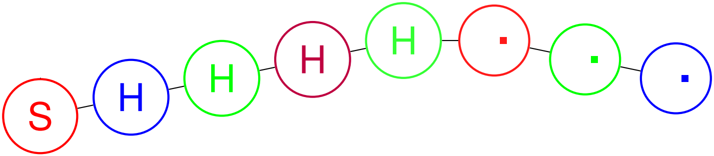
CS 170: Algorithms



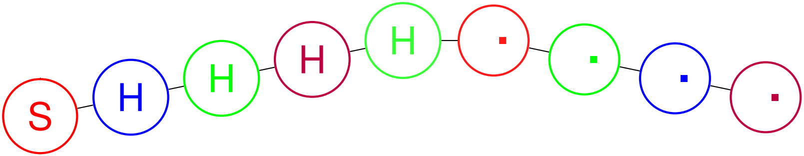
CS 170: Algorithms



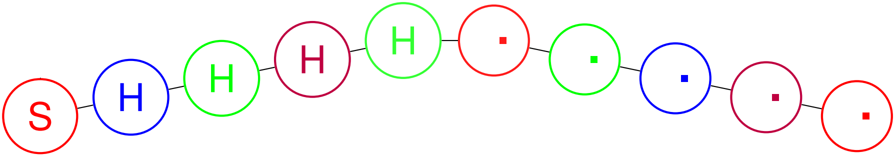
CS 170: Algorithms



CS 170: Algorithms



CS 170: Algorithms



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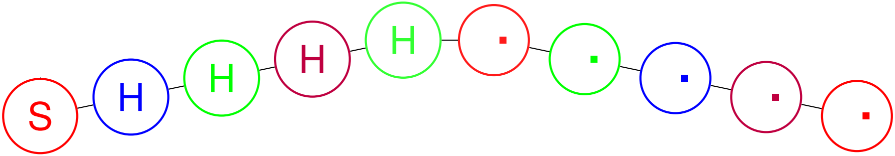

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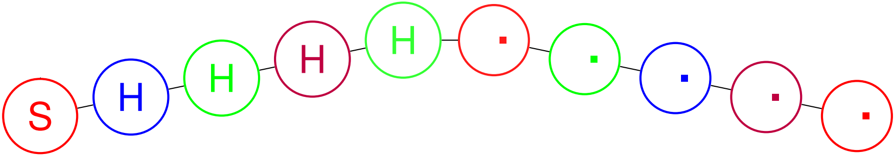
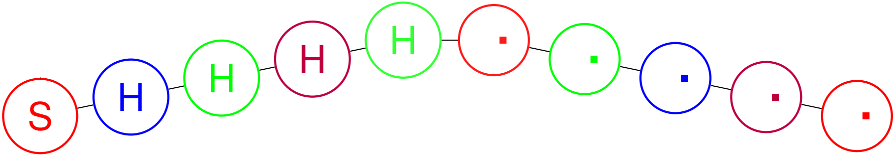
CS 170: Algorithms



No laptops please.

Thank you

CS 170: Algorithms



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Thank you !

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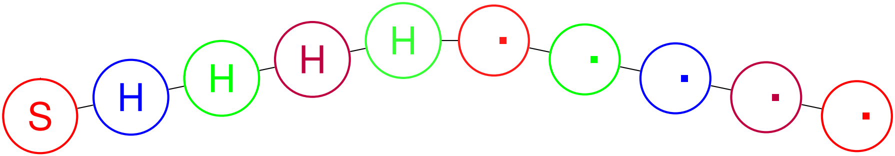

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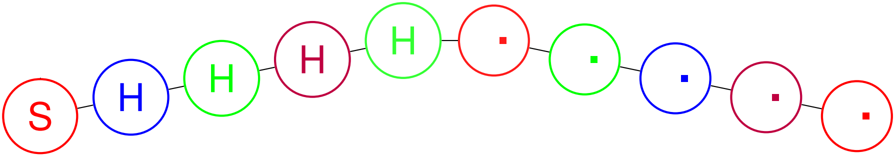

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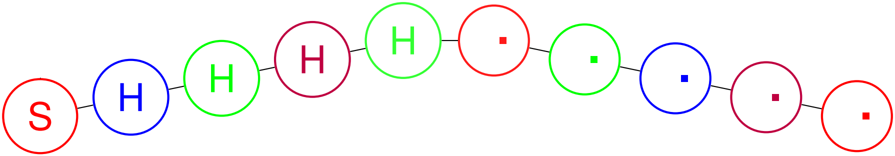

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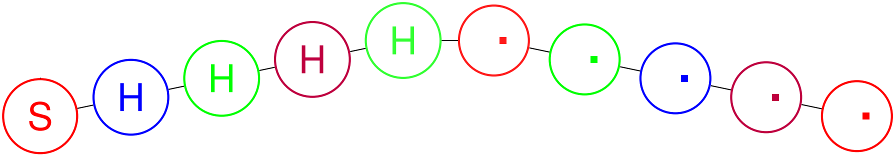

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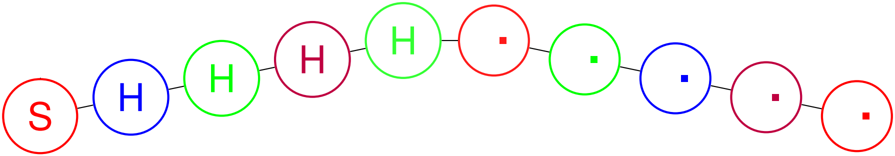

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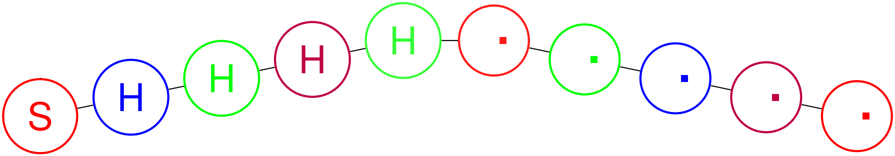

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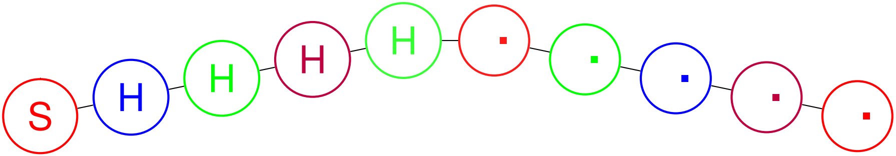

No laptops please.

Thank you ! ! ! ! ! ! ! !

Dijkstra's Algorithm.

foreach v: $d(v) = \infty$.

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foreach v: $d(v) = \infty$.
$d(s) = 0$.

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While u = Q.DeleteMin():
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    if $d(v) > d(u) + l(u, v)$:
      $d(v) = d(u) + l(u, v)$
      Q.InsertOrDecreaseKey(v,d(v))

Dijkstra's Algorithm.

foreach v: $d(v) = \infty$.
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Runtime:

Dijkstra's Algorithm.

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While u = Q.DeleteMin():
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Runtime:
$|V|$ DeleteMins.

Dijkstra's Algorithm.

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Runtime:
$|V|$ DeleteMins.
$|V|$ Inserts.

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Runtime:
$|V|$ DeleteMins.
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$\leq |E|$ DecreaseKeys.

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      Q.InsertOrDecreaseKey(v,d(v))

Runtime:
$|V|$ DeleteMins.
$|V|$ Inserts.
$\leq |E|$ DecreaseKeys.

Binary heap: $O((|V| + |E|)\log|V|)$

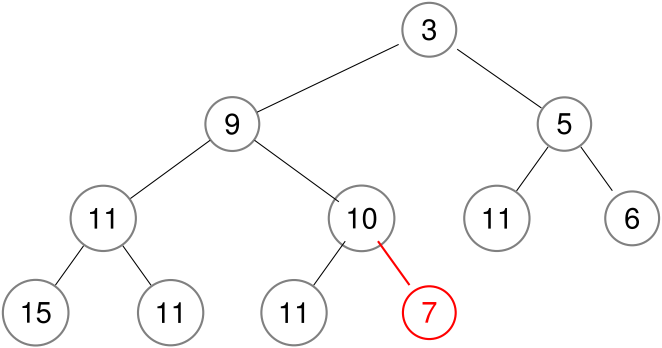
Binary Heap.

Heap[1]: bigger children.

[1] values only

Binary Heap.

Heap[1]: bigger children.
$\Longrightarrow$ smallest at root.

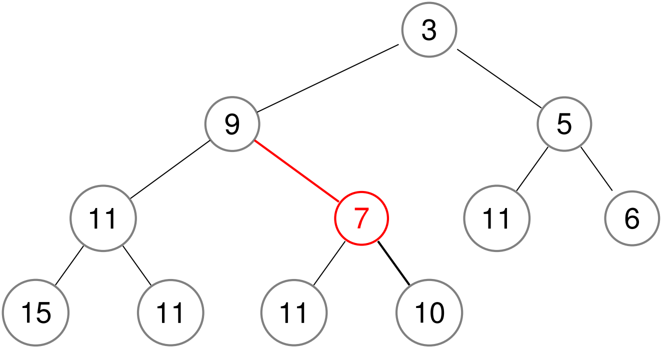


[1] values only

Binary Heap.

Heap[1]: bigger children.

$\implies$ smallest at root.



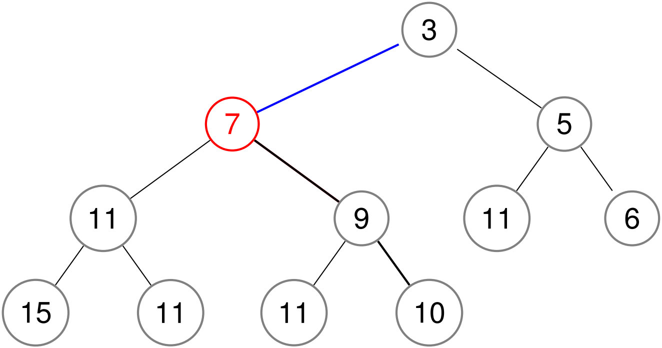
Insert(7):

[1] values only

Binary Heap.

Heap[1]: bigger children.

$\Longrightarrow$ smallest at root.



Insert(7): Bubble up: check parent.

[1] values only

Binary Heap.

Heap[1]: bigger children.

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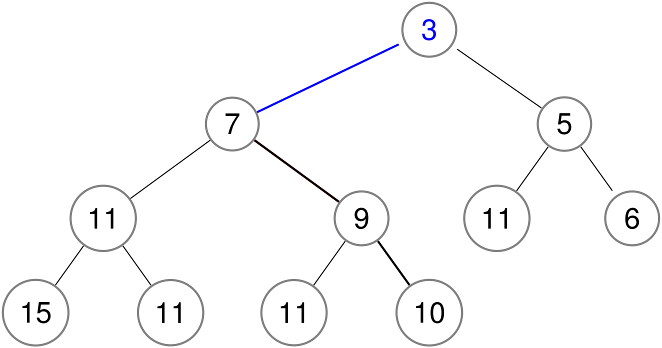

Insert(7): Bubble up: check parent. . **depth** comp.

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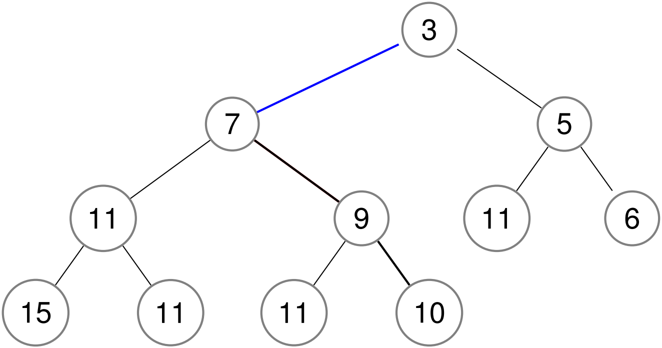

Insert(7): Bubble up: check parent. . **depth** comp.

DeleteMin:

[1] values only

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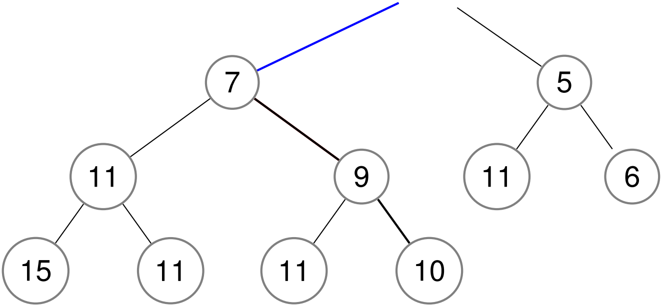

Insert(7): Bubble up: check parent. . **depth** comp.

DeleteMin:

[1] values only

Binary Heap.

Heap[1]: bigger children.

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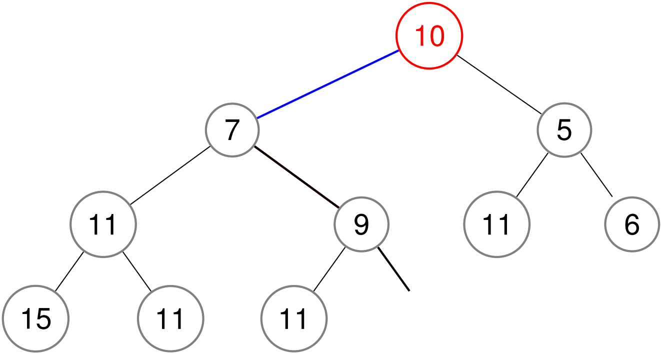


Insert(7): Bubble up: check parent. . **depth** comp.

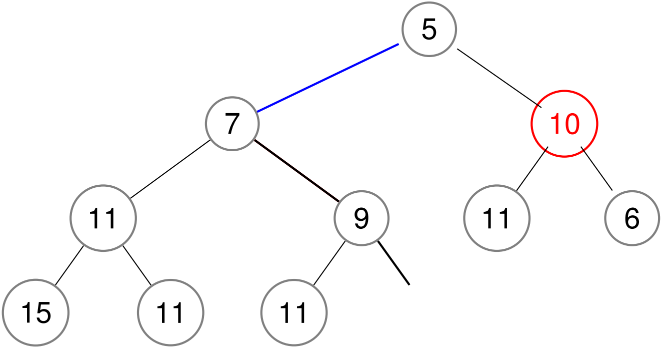
DeleteMin: Replace.

[1]values only

Binary Heap.

Heap[1]: bigger children.

$\Longrightarrow$ smallest at root.



Insert(7): Bubble up: check parent. . **depth** comp.

DeleteMin: Replace. Bubble down: check **both** children..

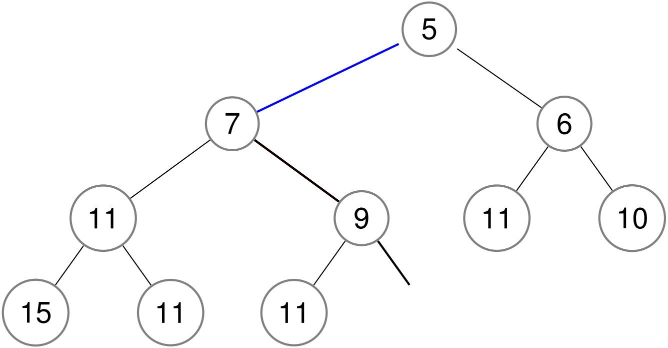
---

[1] values only

Binary Heap.

Heap[1]: bigger children.

$\Longrightarrow$ smallest at root.



Insert(7): Bubble up: check parent. . **depth** comp.

DeleteMin: Replace. Bubble down: check **both** children..

2$\times$ **depth** – comparisons.

[1] values only

d-ary heap

Degree – d, Depth – $\log_d n$.

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n / \log d$.

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n/\log d$.
DeleteMin – $d \log n/\log d$. (Check all children.)

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n / \log d$.
DeleteMin – $d \log n / \log d$. (Check all children.)

Dijkstra:

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n / \log d$.
DeleteMin – $d \log n / \log d$. (Check all children.)

Dijkstra:
$O(|V|)$ deletemins. $O(d \log n / \log d)$ each.

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n / \log d$.
DeleteMin – $d \log n / \log d$. (Check all children.)

Dijkstra:
$O(|V|)$ deletemins. $O(d \log n / \log d)$ each.
$O(|E|)$ insert/decrease-keys. $O(\log n / \log d)$ each.

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n / \log d$.
DeleteMin – $d \log n / \log d$. (Check all children.)

Dijkstra:
$O(|V|)$ deletemins. $O(d \log n / \log d)$ each.
$O(|E|)$ insert/decrease-keys. $O(\log n / \log d)$ each.

$O(|V| d \log n / \log d + |E| \log n / \log d)$.

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n / \log d$.
DeleteMin – $d \log n / \log d$. (Check all children.)

Dijkstra:
$O(|V|)$ deletemins. $O(d \log n / \log d)$ each.
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Optimal Choice:

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n/\log d$.
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$O(|V|)$ deletemins. $O(d \log n/\log d)$ each.
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$O(|V| d \log n/\log d + |E| \log n/\log d)$.

Optimal Choice: Choose $d = |E|/|V|$ (average degree/2)

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n / \log d$.
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$O(|E| \log n/\log d)$

For dense graphs it approaches linear.

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n/\log d$.
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Optimal Choice: Choose $d = |E|/|V|$ (average degree/2)
$O(|E| \log n/\log d)$

For dense graphs it approaches linear.

Fibonacci Heaps:

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n / \log d$.
DeleteMin – $d \log n / \log d$. (Check all children.)

Dijkstra:
$O(|V|)$ deletemins. $O(d \log n / \log d)$ each.
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$O(|V| d \log n / \log d + |E| \log n / \log d)$.

Optimal Choice: Choose $d = |E|/|V|$ (average degree/2)
$O(|E| \log n / \log d)$

For dense graphs it approaches linear.

Fibonacci Heaps:
$O(\log n)$ per delete.

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n / \log d$.
DeleteMin – $d \log n / \log d$. (Check all children.)

Dijkstra:
$O(|V|)$ deletemins. $O(d \log n / \log d)$ each.
$O(|E|)$ insert/decrease-keys. $O(\log n / \log d)$ each.

$O(|V| d \log n / \log d + |E| \log n / \log d)$.

Optimal Choice: Choose $d = |E|/|V|$ (average degree/2)
$O(|E| \log n / \log d)$

For dense graphs it approaches linear.

Fibonacci Heaps:
$O(\log n)$ per delete.
$O(1)$ average decrease-key.

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n / \log d$.
DeleteMin – $d \log n / \log d$. (Check all children.)

Dijkstra:
$O(|V|)$ deletemins. $O(d \log n / \log d)$ each.
$O(|E|)$ insert/decrease-keys. $O(\log n / \log d)$ each.

$O(|V| d \log n / \log d + |E| \log n / \log d)$.

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For dense graphs it approaches linear.

Fibonacci Heaps:
$O(\log n)$ per delete.
$O(1)$ average decrease-key.

$O(|V| \log |V| + |E|)$.

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n / \log d$.
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Dijkstra:
$O(|V|)$ deletemins. $O(d \log n / \log d)$ each.
$O(|E|)$ insert/decrease-keys. $O(\log n / \log d)$ each.

$O(|V| d \log n / \log d + |E| \log n / \log d)$.

Optimal Choice: Choose $d = |E|/|V|$ (average degree/2)
$O(|E| \log n / \log d)$

For dense graphs it approaches linear.

Fibonacci Heaps:
$O(\log n)$ per delete.
$O(1)$ average decrease-key.

$O(|V| \log |V| + |E|)$.
Linear for moderately dense graphs!

Alt Proof.

Dijkstra:

Alt Proof.

Dijkstra:
  ”Know distance to processed nodes, R.”

Alt Proof.

Dijkstra:

"Know distance to processed nodes, R."
"Add node v closest to s outside of R."

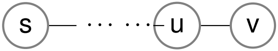
Alt Proof.

Dijkstra:

"Know distance to processed nodes, R."

"Add node v closest to s outside of R."

Closest node v has path...

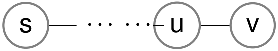


Alt Proof.

Dijkstra:

"Know distance to processed nodes, *R*."
"Add node *v* closest to *s* outside of *R*."

Closest node *v* has path...



u in *R*.

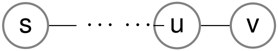
Alt Proof.

Dijkstra:

"Know distance to processed nodes, R."

"Add node v closest to s outside of R."

Closest node v has path...

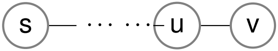


u in R. Since v is closest node not in R.

Alt Proof.

Dijkstra:
”Know distance to processed nodes, R.”
”Add node v closest to s outside of R.”

Closest node v has path...

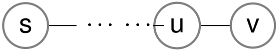


u in R. Since v is closest node not in R.
$d(u)$ correct by induction.

Alt Proof.

Dijkstra:
"Know distance to processed nodes, R."
"Add node v closest to s outside of R."

Closest node v has path...



u in R. Since v is closest node not in R.
$d(u)$ correct by induction.
$d(u)$ corresponds to the length of a shortest path.
$d(v) \leq d(u) + l(u, v)$.

Alt Proof.

Dijkstra:
  "Know distance to processed nodes, R."
  "Add node v closest to s outside of R."

Closest node v has path...

s — $\cdots$ $\cdots$ u — v

u in R. Since v is closest node not in R.
$d(u)$ correct by induction.
  $d(u)$ corresponds to the length of a shortest path.
$d(v) \leq d(u) + l(u,v)$. Since u was processed by Algorithm.

Alt Proof.

Dijkstra:

”Know distance to processed nodes, R.”
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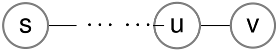

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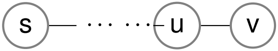

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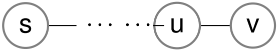

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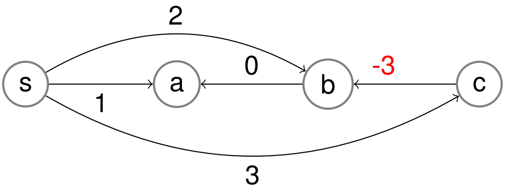

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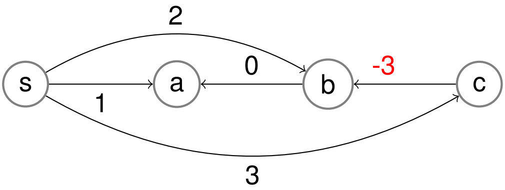

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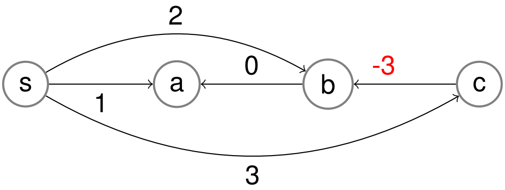

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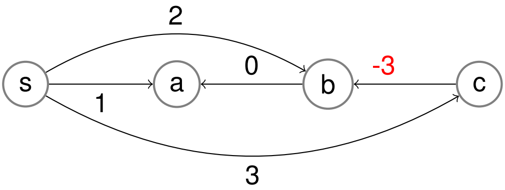

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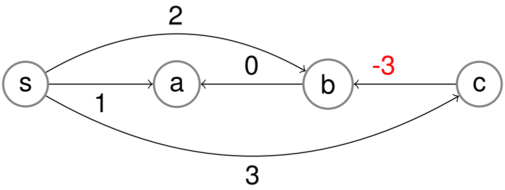

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Process a, $d(a) = 1$: No outgoing edges.

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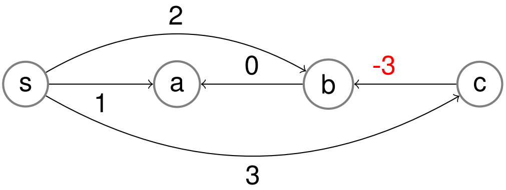

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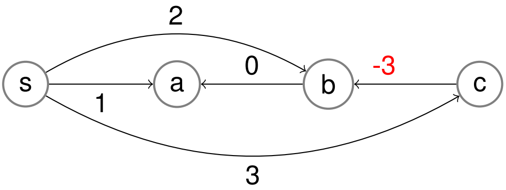

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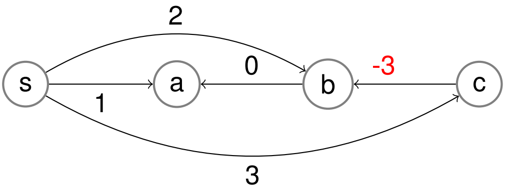

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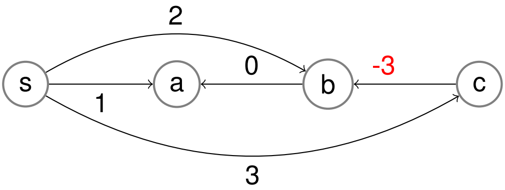

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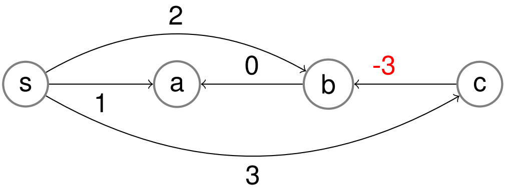

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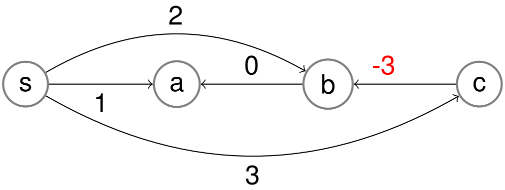

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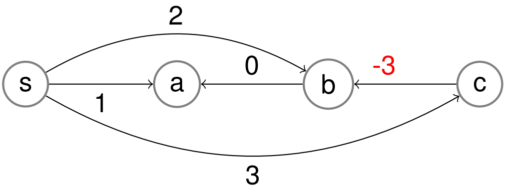

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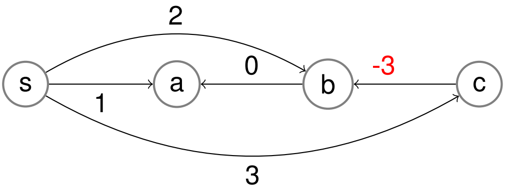

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Problem: $d(b)$ was incorrect when processed due to negative edge.

Update/Bellman-Ford.

def update $((u, v))$:
  dist(v) = min (dist(v), dist(u) + l (u,v)).

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Update/Bellman-Ford.

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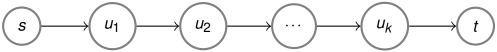

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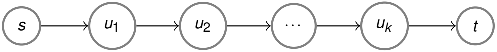

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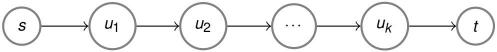

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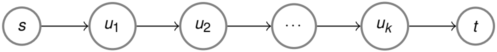

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  do $n-1$ times,
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Correctness: After ith loop, $d(v)$ is correct for v with i edge shortest paths.

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Correctness: After ith loop, $d(v)$ is correct for v with i edge shortest paths.
Time: $O(|V||E|)$

Careful: Negative Cycles.

Bellman-Ford: $d(v) \leq$ length of i edge shortest path.

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Assumes length of shortest path is at most $n-1$.

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Assumes length of shortest path is at most $n-1$.

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After n iterations, some distance changes, there must be negative cycle!

DAG

Dijkstra:

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$O((m+n)\log n)$ time

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Shortest path for DAG:

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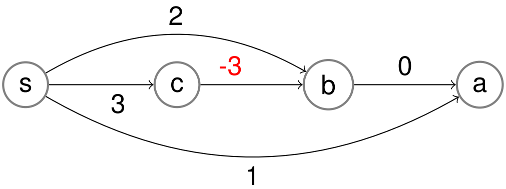
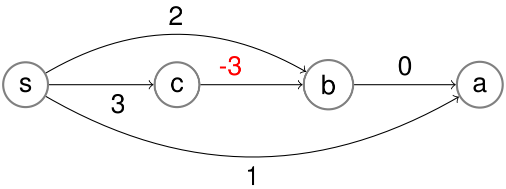
Shortest path for DAG:
linearize
process nodes (and update neighbors in order.)

DAG

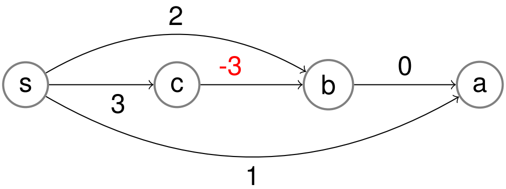


DAG



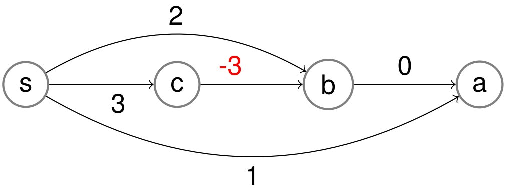
Process s, $d(s) = 0$:

DAG



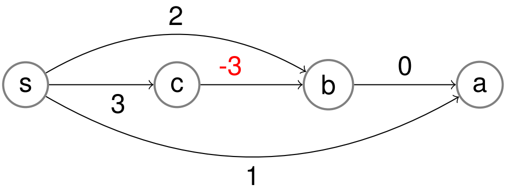
Process s, $d(s) = 0$: Updates $d(c) = 3$,

DAG



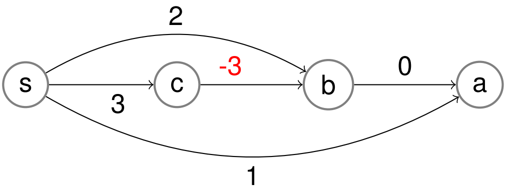
Process s, $d(s) = 0$: Updates $d(c) = 3, d(b) = 2,$

DAG



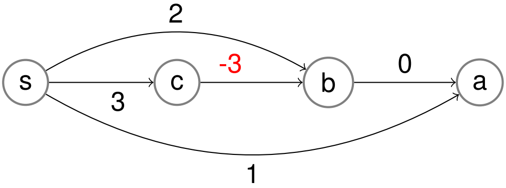
Process s, $d(s) = 0$: Updates $d(c) = 3, d(b) = 2, d(a) = 1$.

DAG



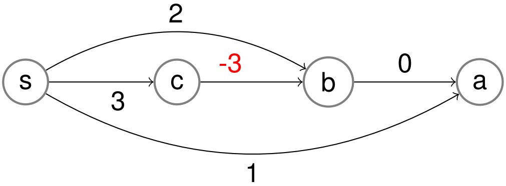
Process s, $d(s) = 0$: Updates $d(c) = 3, d(b) = 2, d(a) = 1$.
Process c, $d(c) = 3$:

DAG



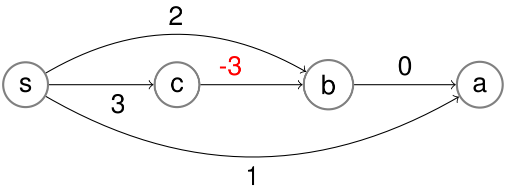
Process s, $d(s) = 0$: Updates $d(c) = 3, d(b) = 2, d(a) = 1$.
Process c, $d(c) = 3$: Update $d(b) = 0$.

DAG



Process s, $d(s) = 0$: Updates $d(c) = 3, d(b) = 2, d(a) = 1$.
Process c, $d(c) = 3$: Update $d(b) = 0$.
Process b, $d(b) = 0$:

DAG



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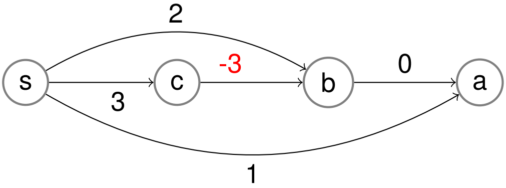

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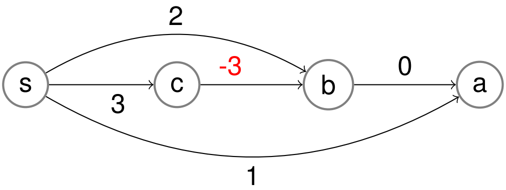

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Process a, $d(a) = 0$.
Done.

See you

See you ..on Monday.