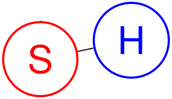
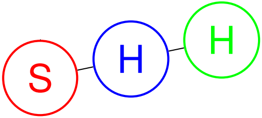
# CS 170: Algorithms

# CS 170: Algorithms

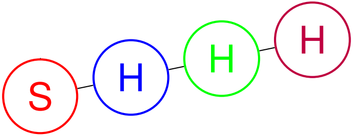
# CS 170: Algorithms



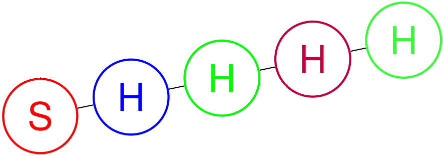
# CS 170: Algorithms



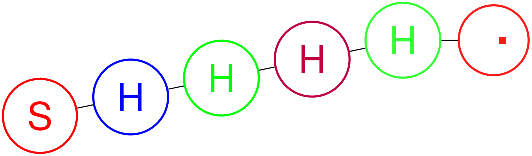
# CS 170: Algorithms



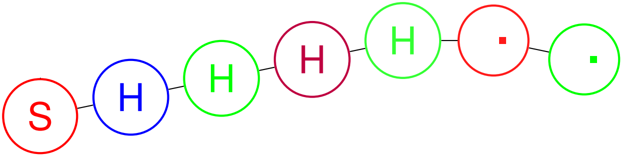
# CS 170: Algorithms



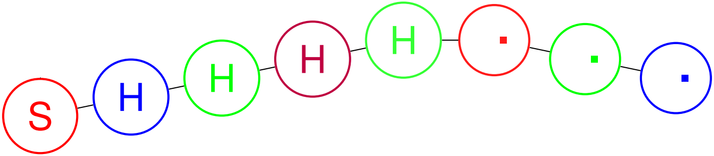
# CS 170: Algorithms



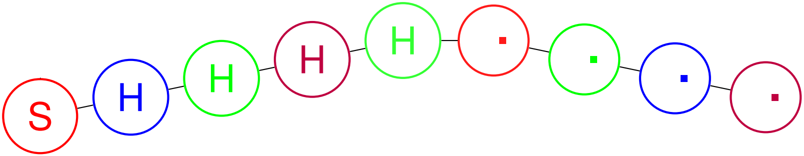
# CS 170: Algorithms



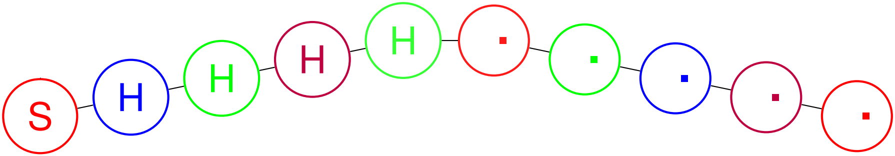
# CS 170: Algorithms



# CS 170: Algorithms



# CS 170: Algorithms



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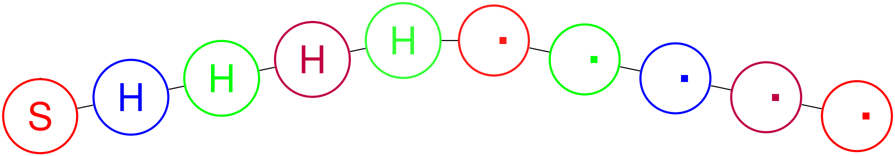
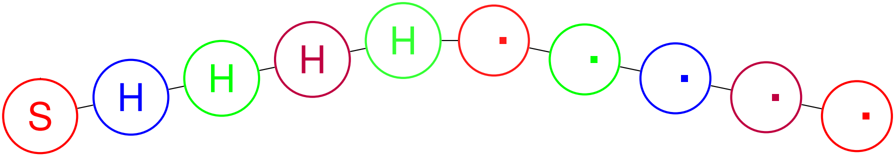



No laptops please.

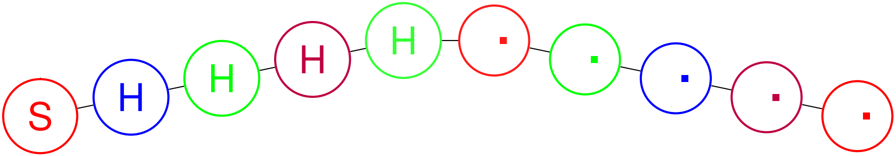
# CS 170: Algorithms




No laptops please.

Thank you

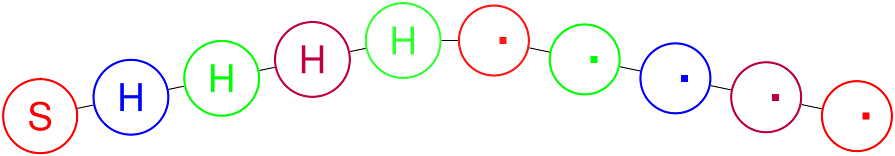
# CS 170: Algorithms




No laptops please.

Thank you !

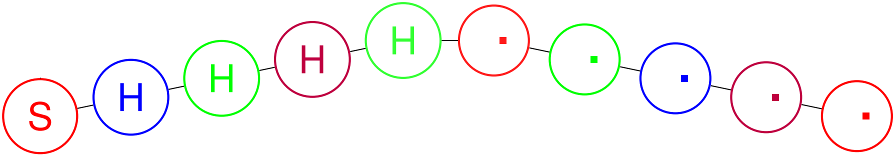
# CS 170: Algorithms




No laptops please.

Thank you ! !

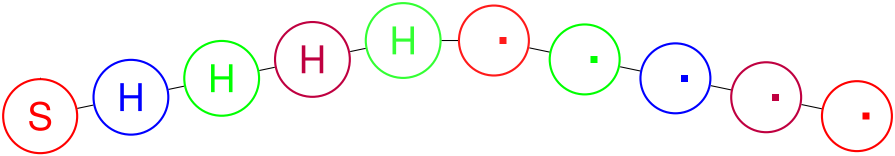
# CS 170: Algorithms




No laptops please.

Thank you ! ! !

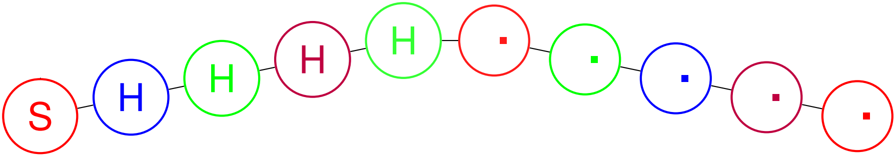
# CS 170: Algorithms




No laptops please.

Thank you ! ! ! !

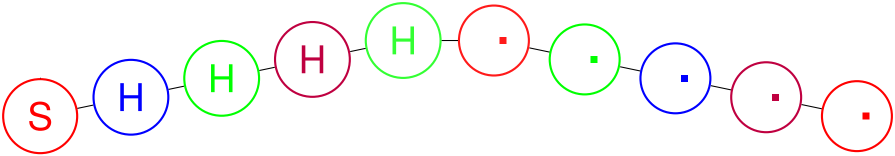
# CS 170: Algorithms




No laptops please.

Thank you ! ! ! ! !

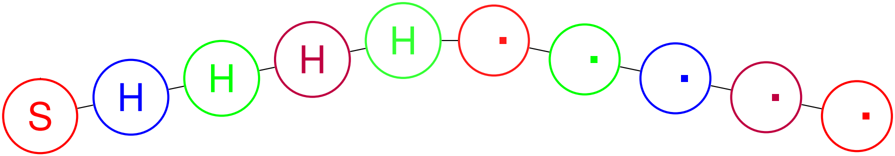
# CS 170: Algorithms




No laptops please.

Thank you ! ! ! ! ! !

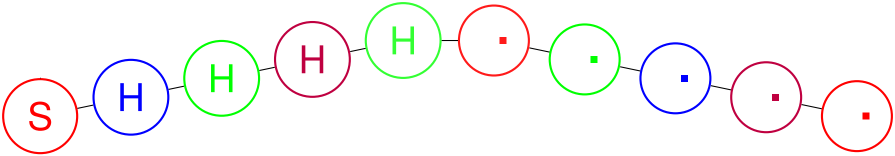
# CS 170: Algorithms




No laptops please.

Thank you ! ! ! ! ! ! !

# CS 170: Algorithms

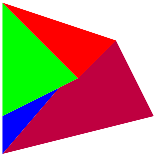


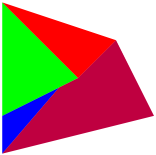

No laptops please.

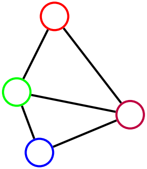
Thank you ! ! ! ! ! ! ! !

# Today

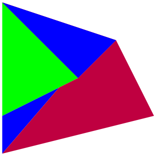
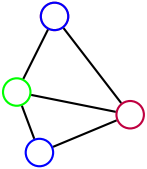
1. Graphs
2. Reachability.
3. Depth First Search



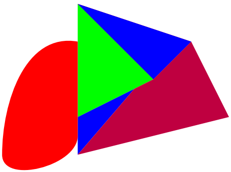


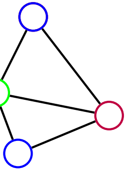


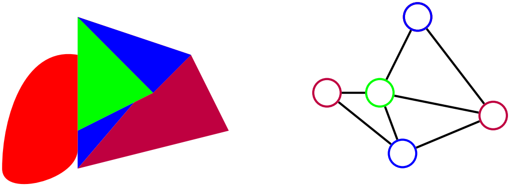
Fewer Colors?

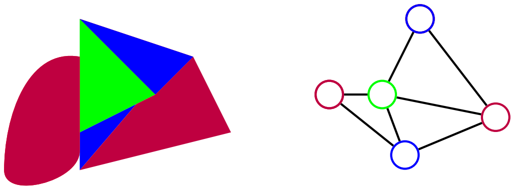
 

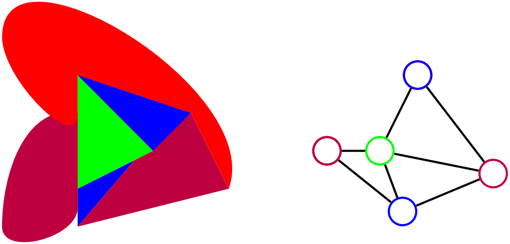
Yes! Three colors.

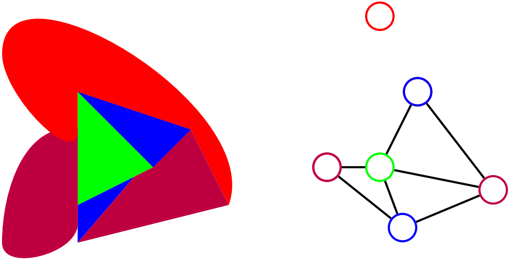


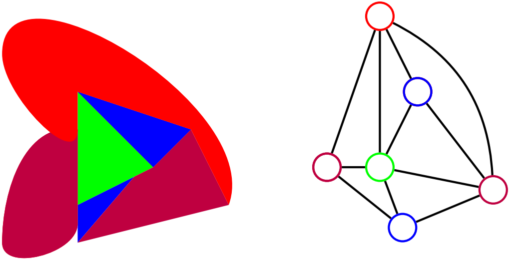


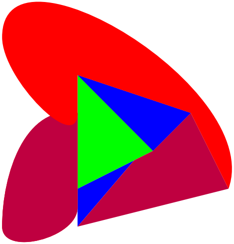


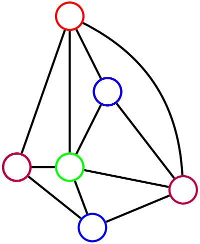




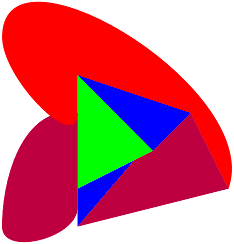


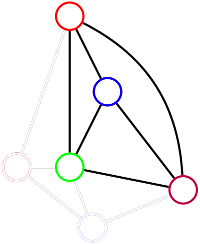


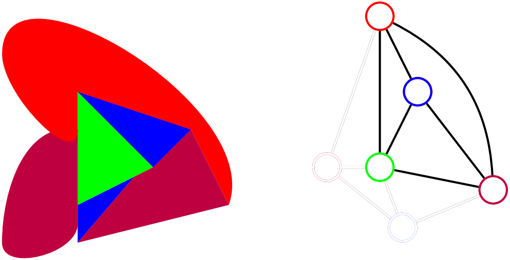




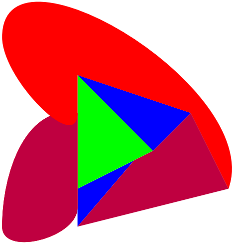
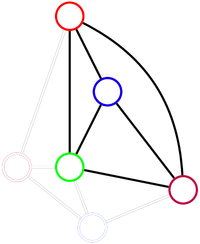
Fewer Colors?







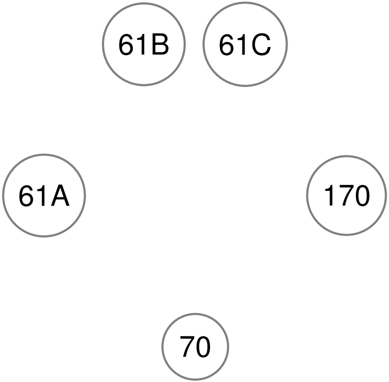
Four colors required!

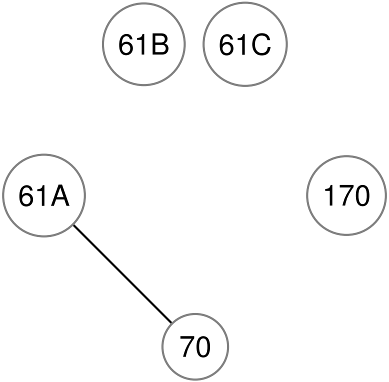
Four colors required!

Theorem: Four colors enough.

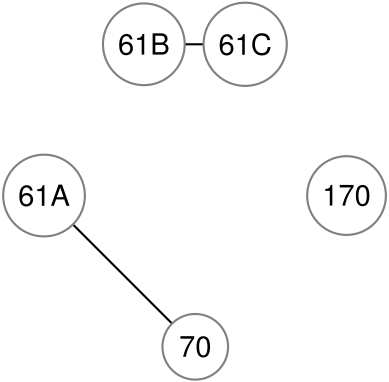
# Scheduling: coloring.



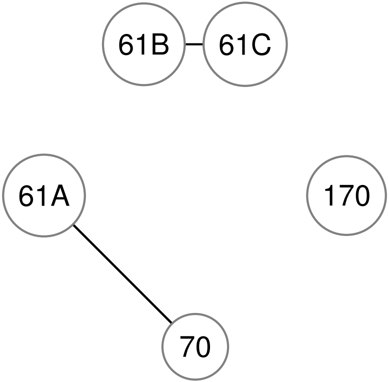
# Scheduling: coloring.



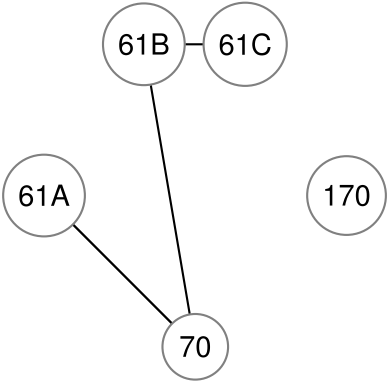
# Scheduling: coloring.



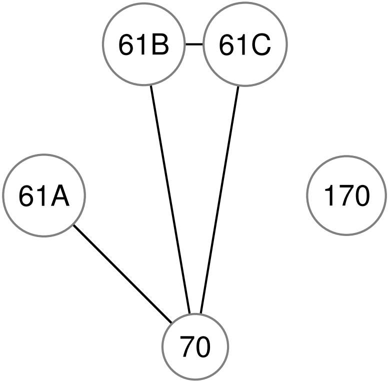
# Scheduling: coloring.



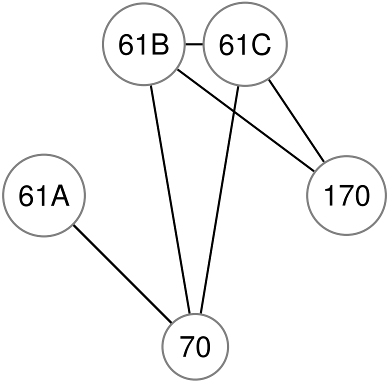
# Scheduling: coloring.



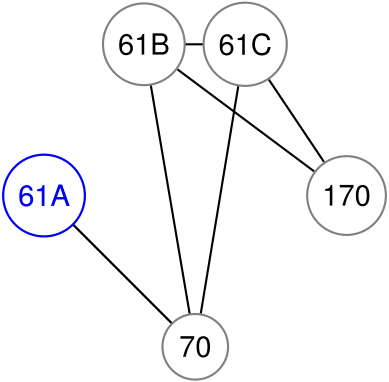
# Scheduling: coloring.



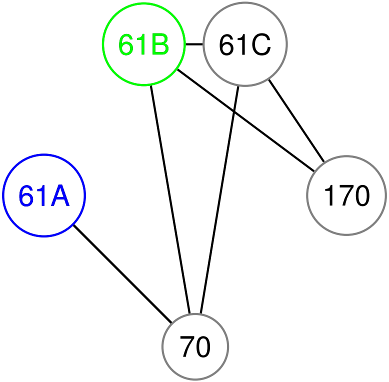
# Scheduling: coloring.



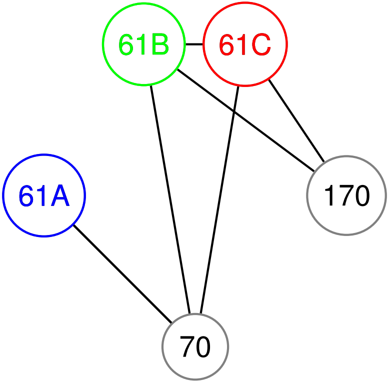
# Scheduling: coloring.



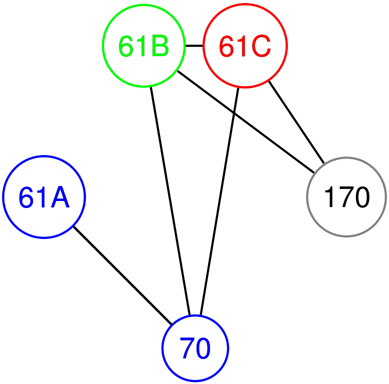
# Scheduling: coloring.



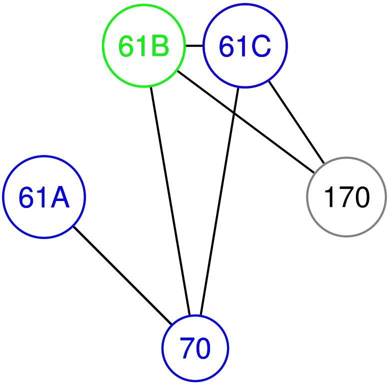
# Scheduling: coloring.



# Scheduling: coloring.



# Scheduling: coloring.



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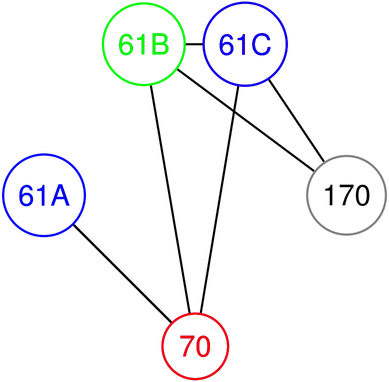
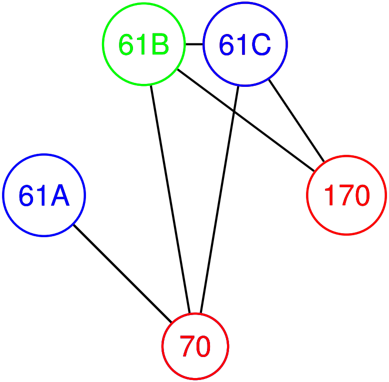


# Scheduling: coloring.



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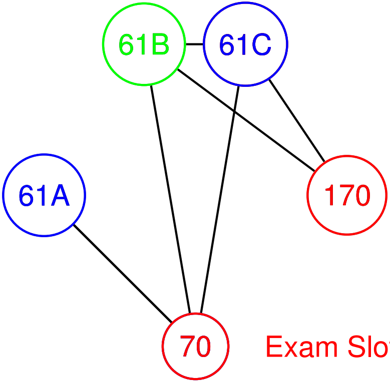


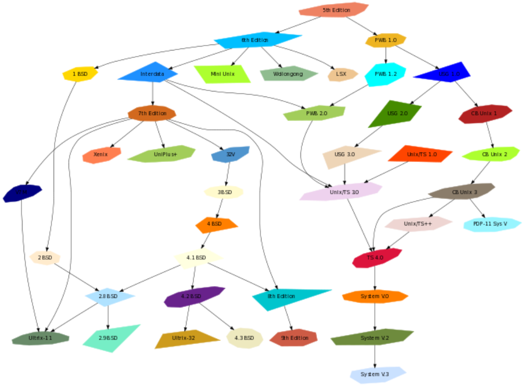

Exam Slot 1.

Exam Slot 2.

Exam Slot 3.

# Directed acyclic graphs.

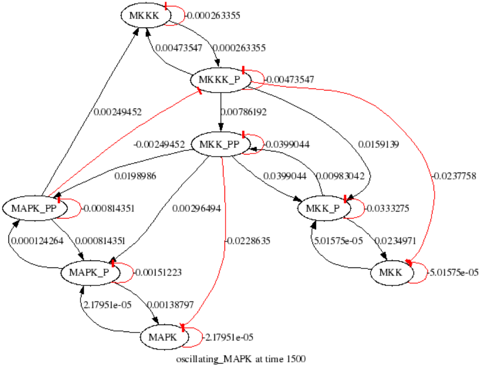
Heritage of Unix.




*Object Oriented Graphs*
*Stephen North, 3/19/93*

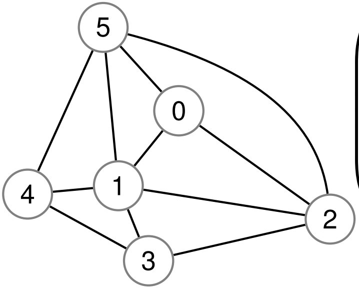
From http://www.graphviz.org/content/crazy.

# Chemical networks.




From http://www.tbi.univie.ac.at/ raim/odeSolver/doc/app.html.

Graph Implementations.



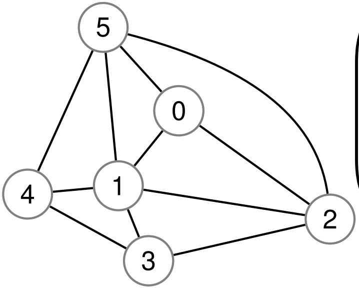

Matrix Representation.

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$V = \{0, 1, 2, 3, 4, 5\}$$

$$E = \{(0,1), (0,2), (0,5), (1,3) \ldots\}$$

Graph Implementations.




Matrix Representation.

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

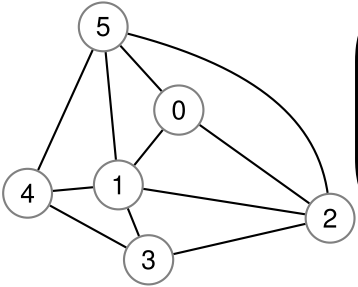
$$V = \{0,1,2,3,4,5\}$$

$$E = \{(0,1),(0,2),(0,5),(1,3)\dots\}$$

Adjacency List

$$\begin{array}{ll} 0: & 1,2,5 \\ 1: & 0,2,3,4,5 \\ 2: & 0,1,3 \\ 3: & 1,2,4 \\ 4: & 1,3,5 \\ 5: & 0,1,2,4 \end{array}$$

## Graph Implementations.




### Matrix Representation.

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$V = \{0,1,2,3,4,5\}$$

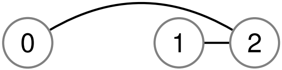
$$E = \{(0,1),(0,2),(0,5),(1,3)\ldots\}$$

### Adjacency List

0 : 1, 2, 5
1 : 0, 2, 3, 4, 5
2 : 0, 1, 3
3 : 1, 2, 4
4 : 1, 3, 5
5 : 0, 1, 2, 4

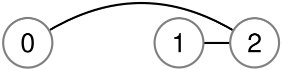
| | Matrix | Adj. List |
|---|---|---|
| Edge $(u,v)$? | $O(1)$ | $O(\|V\|)$ |
| Neighbors of $u$ | $O(\|V\|)$ | $O(d)$ |
| Space | $O(\|V\|^2)$ | $O(\|E\|)$ |

# Test your understanding..




Adjacency list of node 0?

# Test your understanding..



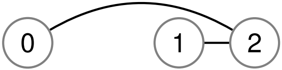

Adjacency list of node 0?

(A) $0:1$

(B) $0:1,2$

(C) $0:2$

# Test your understanding..




Adjacency list of node 0?

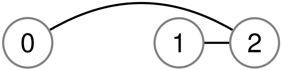
(A) $0 : 1$

(B) $0 : 1, 2$

(C) $0 : 2$

(C)

# Test your understanding..




Adjacency list of node 0?

(A) $0 : 1$

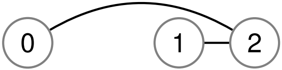
(B) $0 : 1, 2$

(C) $0 : 2$

(C)
How many edges?

(A) 2

# Test your understanding..




Adjacency list of node 0?

(A) $0:1$

(B) $0:1,2$

(C) $0:2$

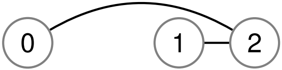
(C)

How many edges?

(A) 2

Total length of adacency lists?

# Test your understanding..




Adjacency list of node 0?

(A) $0:1$

(B) $0:1,2$

(C) $0:2$

(C)

How many edges?

(A) 2

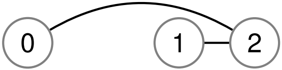
Total length of adacency lists?

(A) 2

(B) 3

(C) 4

# Test your understanding..

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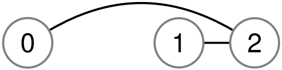


Adjacency list of node 0?

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(B) $0:1,2$

(C) $0:2$

(C)

How many edges?

(A) 2

Total length of adacency lists?

(A) 2

(B) 3

(C) 4

(C) 2 entries for each edge!

# Exploring a maze.

Theseus: ...

# Exploring a maze.

Theseus: ...gotta kill the minatour

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
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Gives Theseus Ball of Thread and Chalk!

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
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Gives Theseus Ball of Thread and Chalk!

Explore a room:

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..fortunately ..she's smart.

Gives Theseus Ball of Thread and Chalk!

Explore a room:
Mark room with chalk.

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..fortunately ..she's smart.

Gives Theseus Ball of Thread and Chalk!

Explore a room:
Mark room with chalk.
For each exit.

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..fortunately ..she's smart.

Gives Theseus Ball of Thread and Chalk!

Explore a room:
Mark room with chalk.
For each exit.
  Look through exit. If marked, next exit.

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..fortunately ..she's smart.

Gives Theseus Ball of Thread and Chalk!

Explore a room:
Mark room with chalk.
For each exit.
  Look through exit. If marked, next exit.
  Otherwise go in room unwind thread.

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..fortunately ..she's smart.

Gives Theseus Ball of Thread and Chalk!

Explore a room:
Mark room with chalk.
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  Otherwise go in room unwind thread.
    Explore that room.

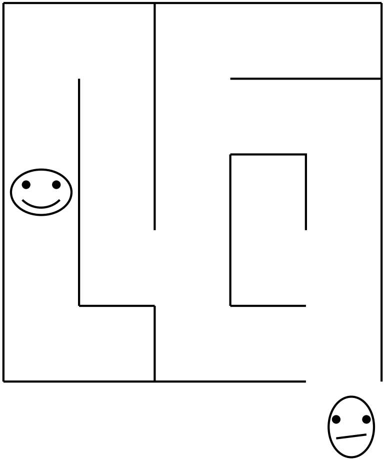
# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..fortunately ..she's smart.

Gives Theseus Ball of Thread and Chalk!

Explore a room:
Mark room with chalk.
For each exit.
  Look through exit. If marked, next exit.
  Otherwise go in room unwind thread.
    Explore that room.
Wind thread to go back to "previous" room.

Where is the minatour?



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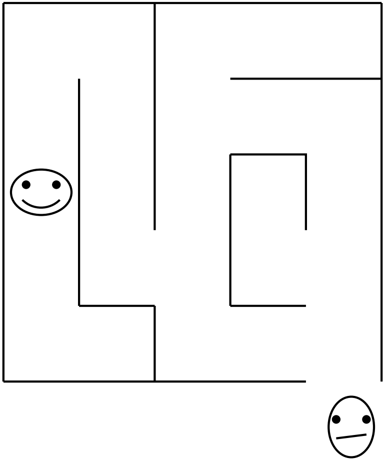

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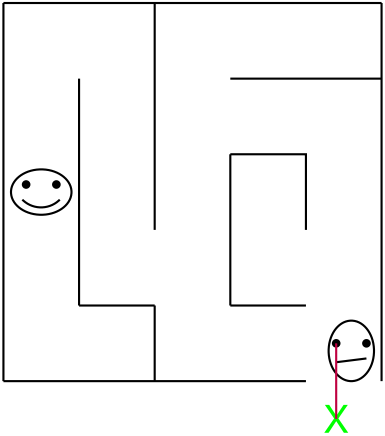

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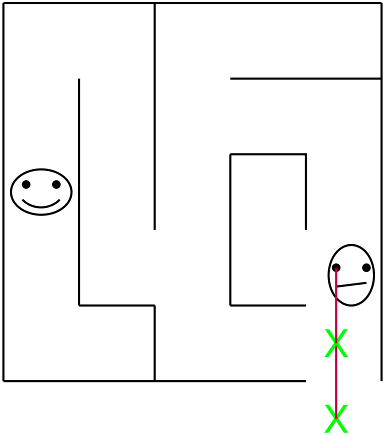

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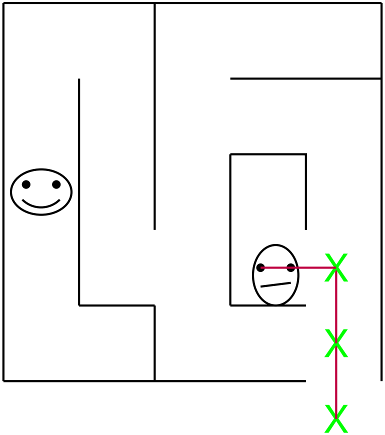

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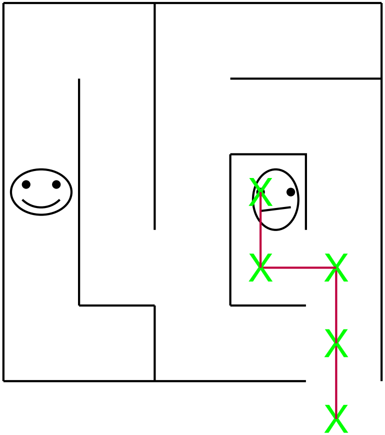

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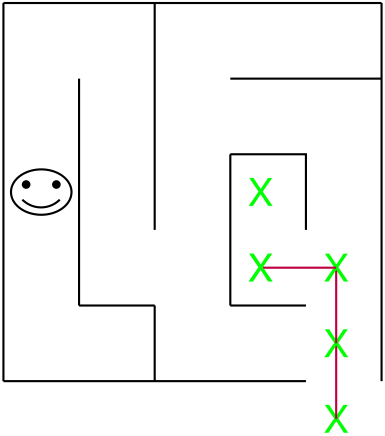

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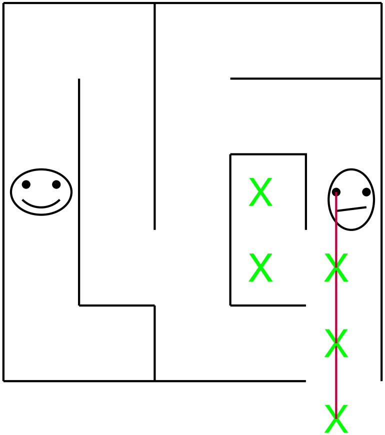

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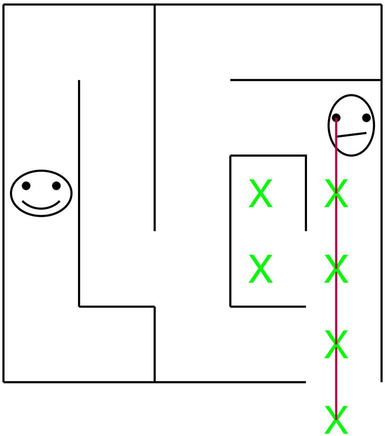

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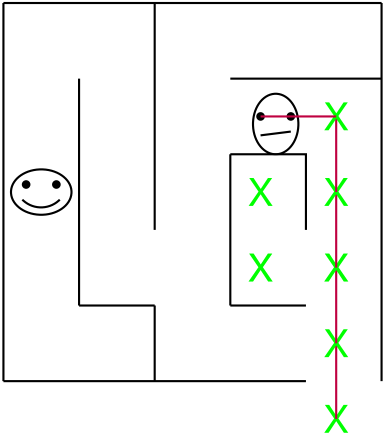

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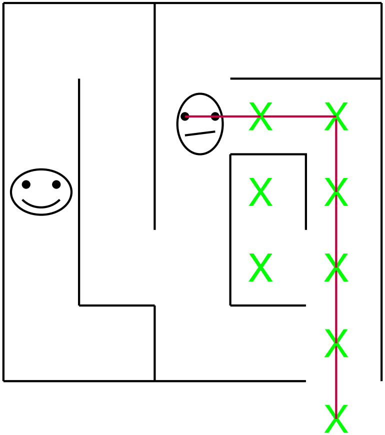

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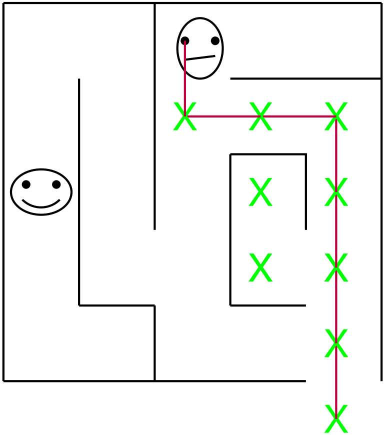

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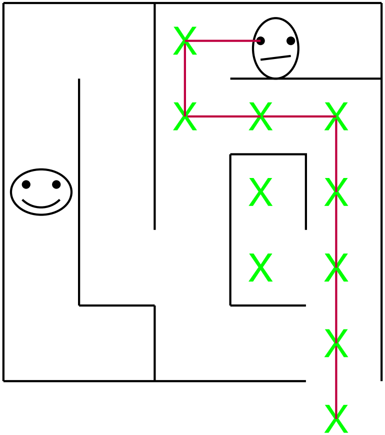

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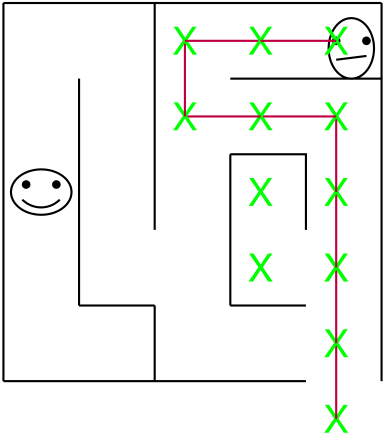

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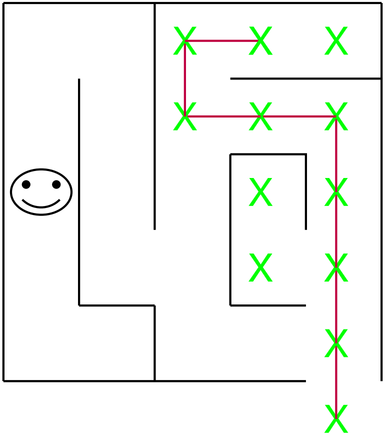

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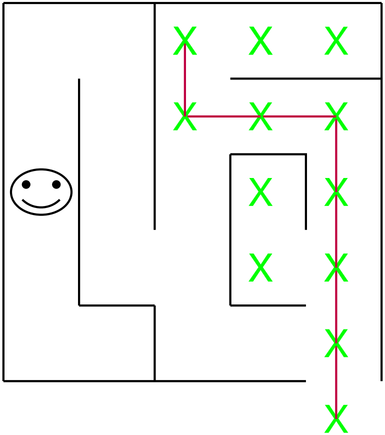

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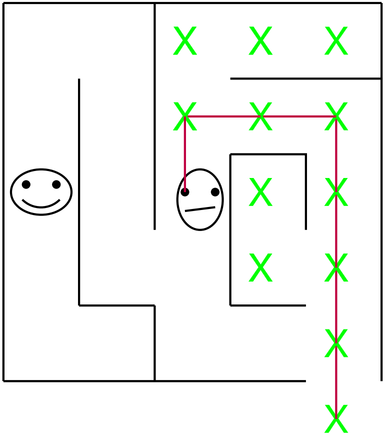

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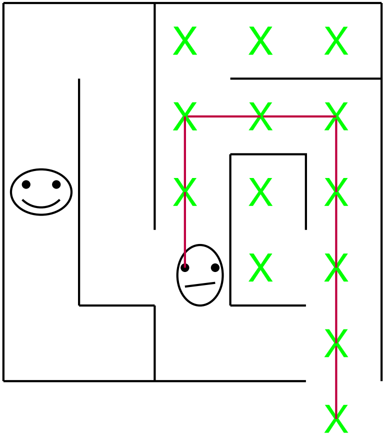

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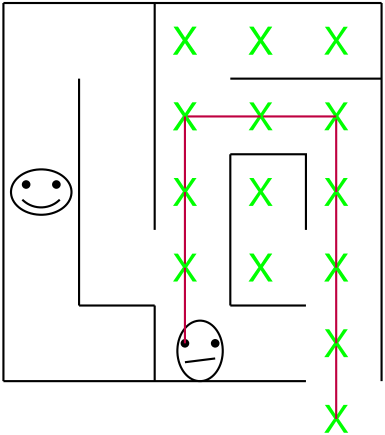

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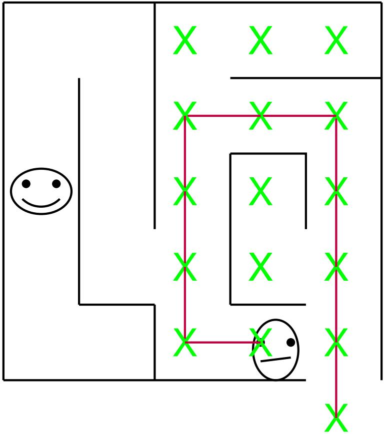

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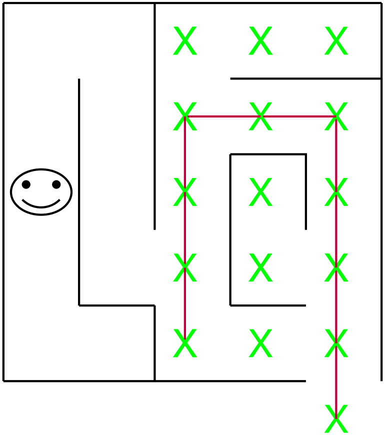

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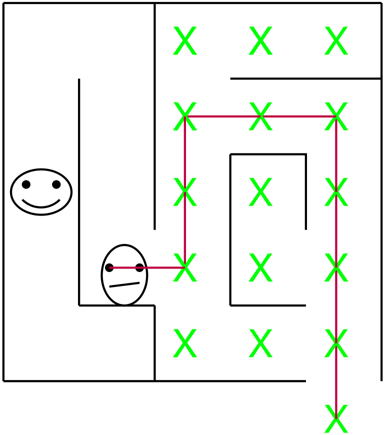

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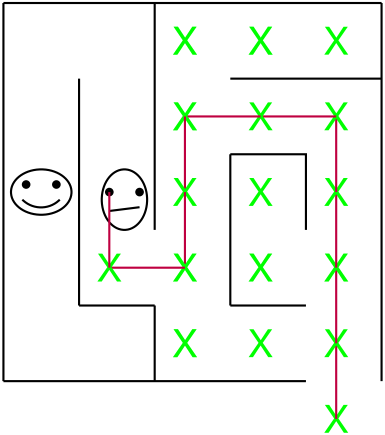

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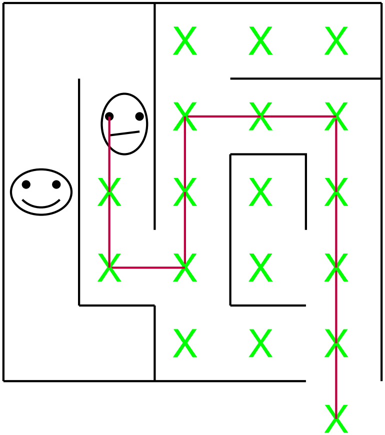

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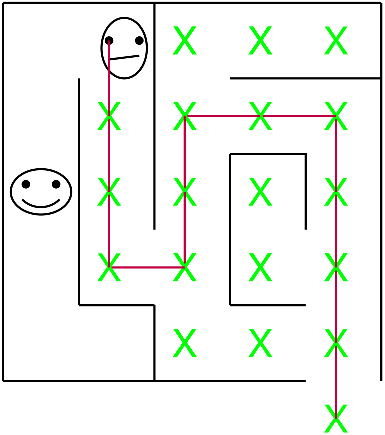

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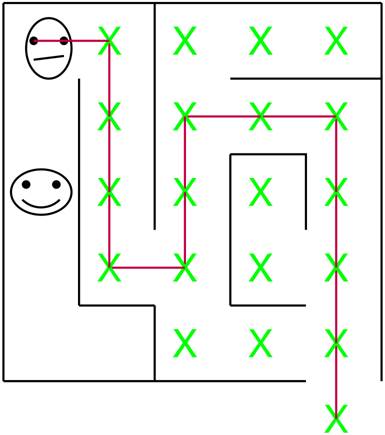

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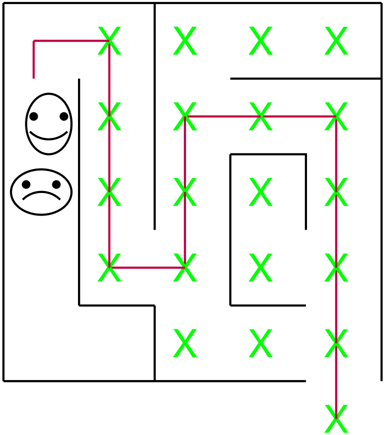

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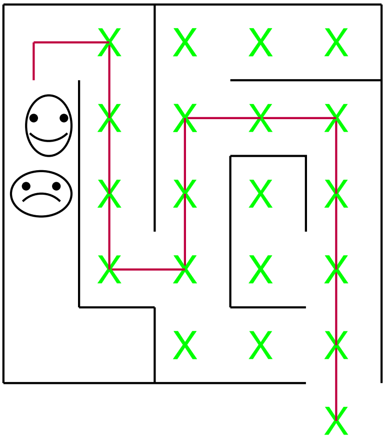

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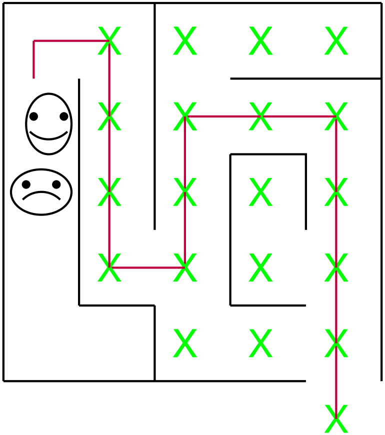

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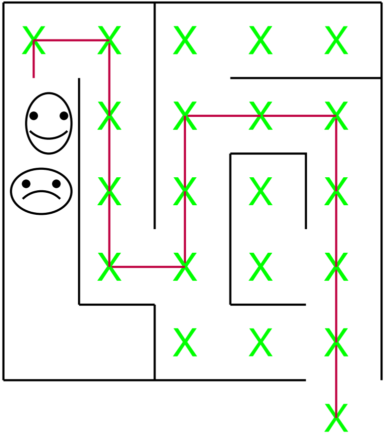


# Searching

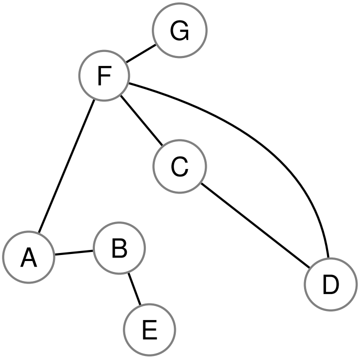
Find a minatour!

# Searching

Find a minatour!

Find out which nodes are reachable from $A$.

# Explore.



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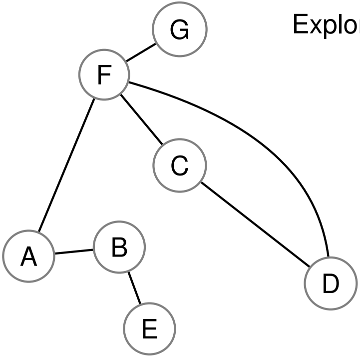
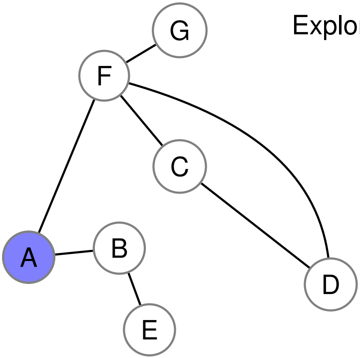



Explore(v):

1. Set visited[v] := true
2. for each edge (v,w) in E
3. if not visited[w]: Explore(w).

Chalk.
Stack is Thread.

# Explore.

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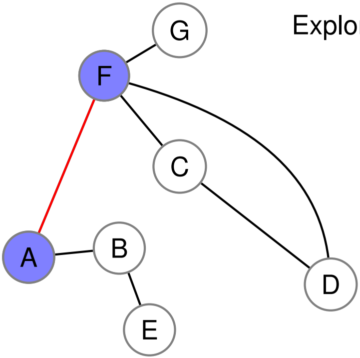


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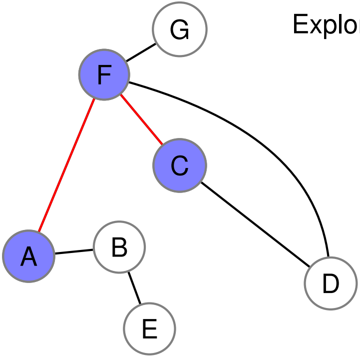


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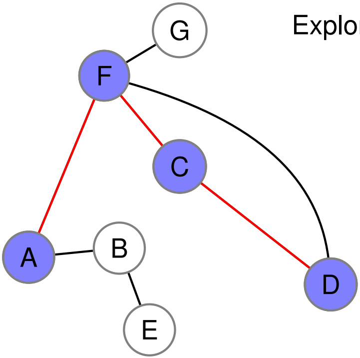


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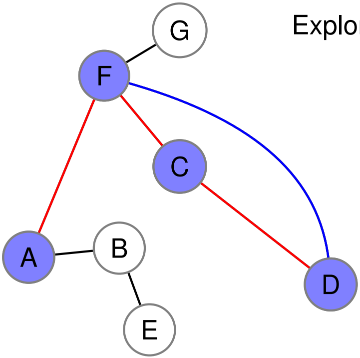


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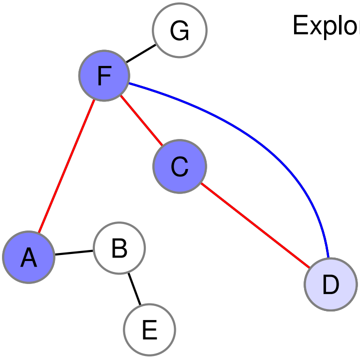


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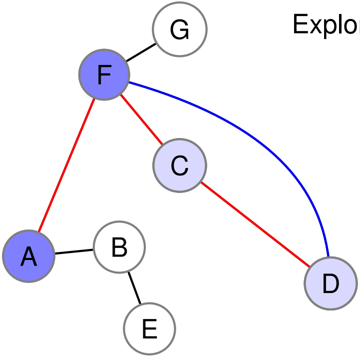


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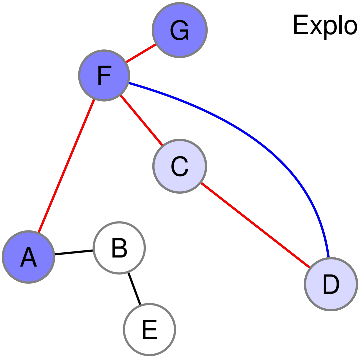


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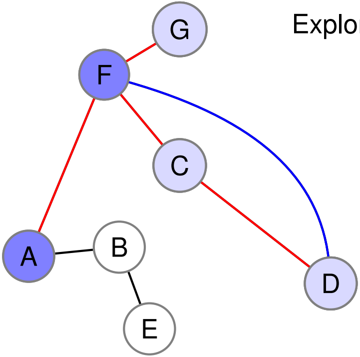


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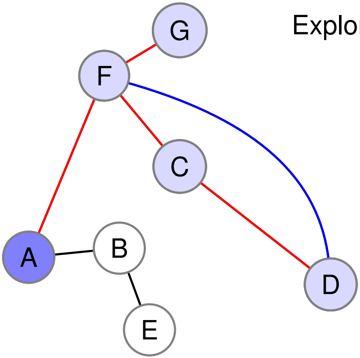


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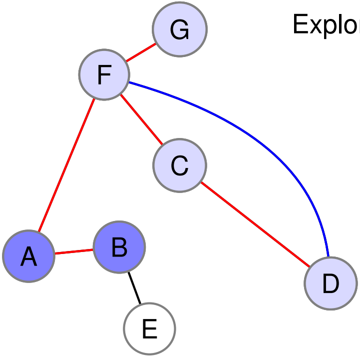


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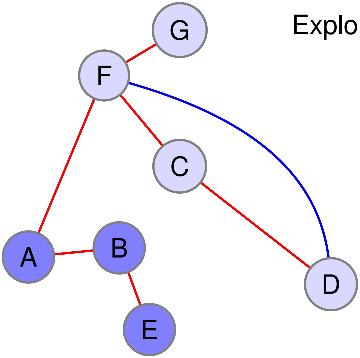


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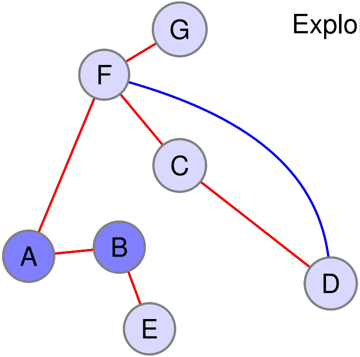


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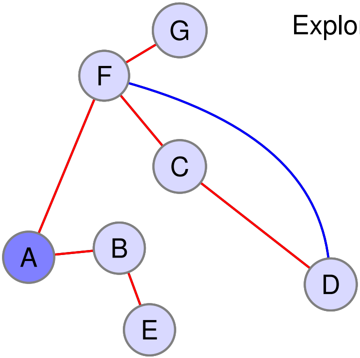


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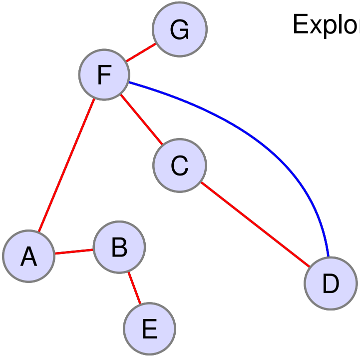


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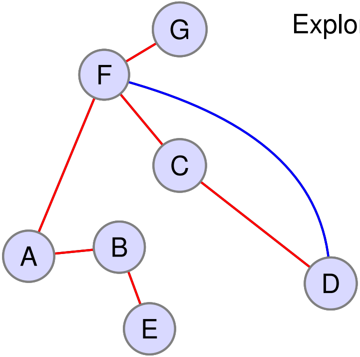


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Chalk.
Stack is Thread.

Explore builds tree.

*Tree* and *back* edges.

# Correctness.

**Explore(v):**

1. Set visited[v] := **true**.
2. For each edge (v,w) in E
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**Property:**

All and only nodes reachable from $A$ are reached by explore.

# Correctness.

**Explore(v):**

1. Set visited[v] := **true**.
2. For each edge (v,w) in E
3.     if not visited[w]: Explore(w)

**Property:**

All and only nodes reachable from $A$ are reached by explore.

Only: when $u$ visited.

# Correctness.

**Explore(v):**

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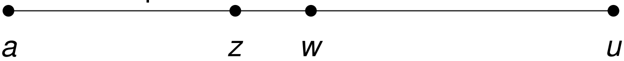

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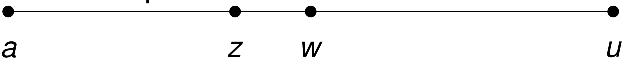


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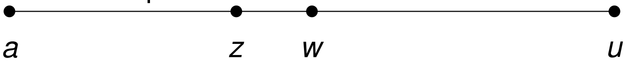


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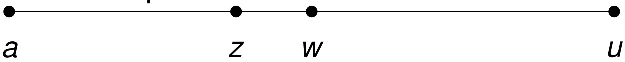


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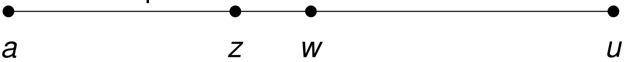


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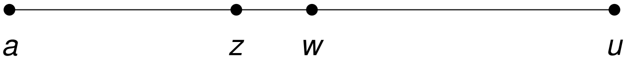


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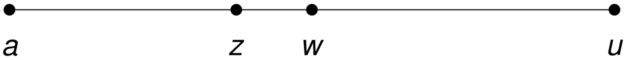


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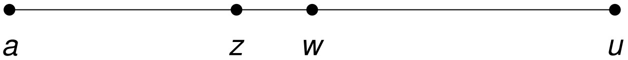


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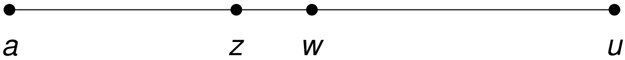


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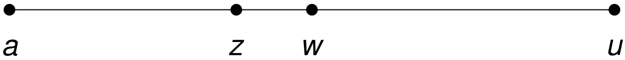


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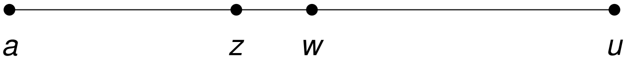


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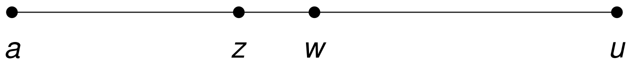


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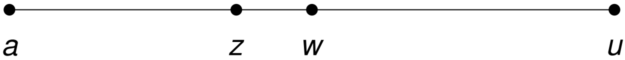


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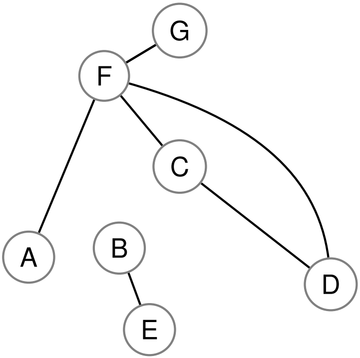

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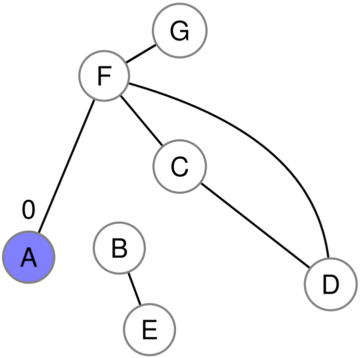

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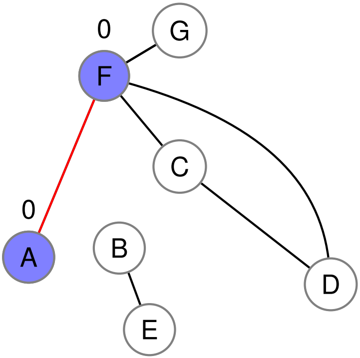

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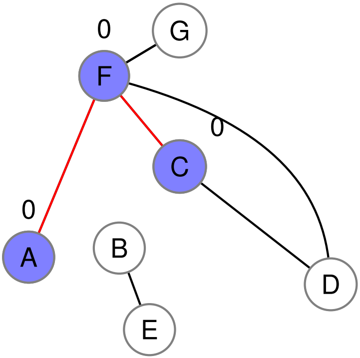

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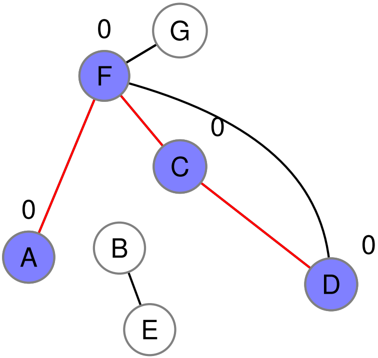

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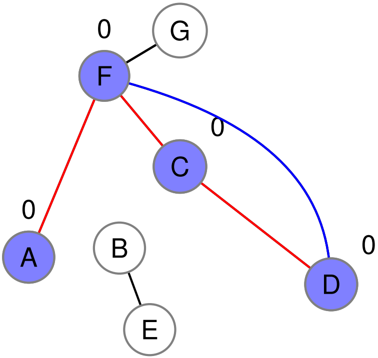

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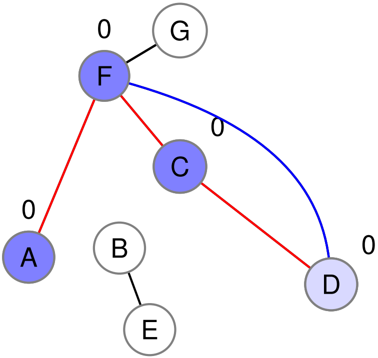

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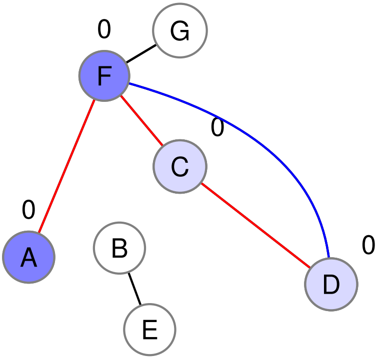

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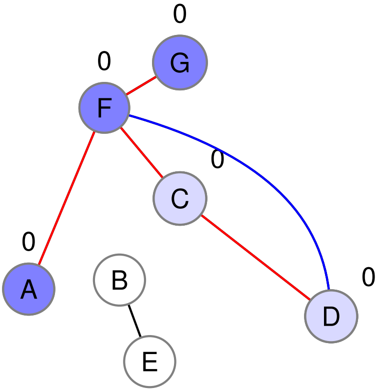

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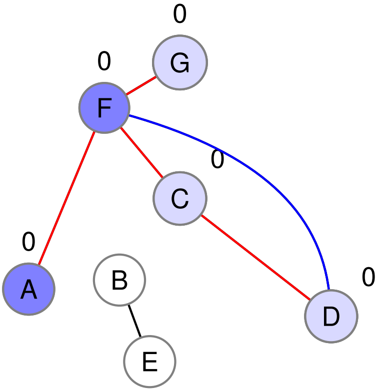

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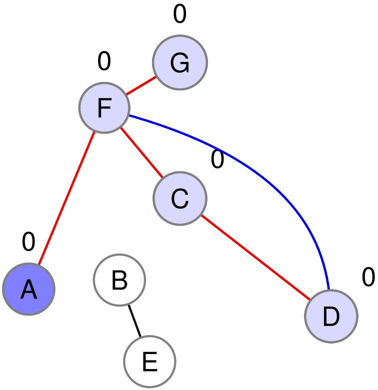

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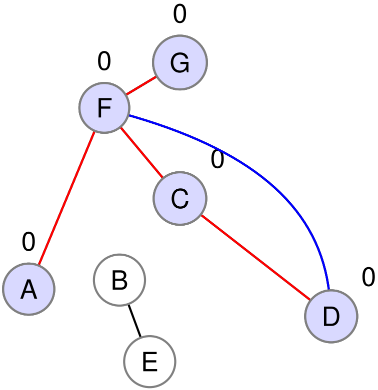

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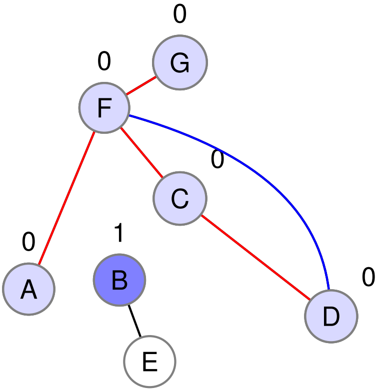

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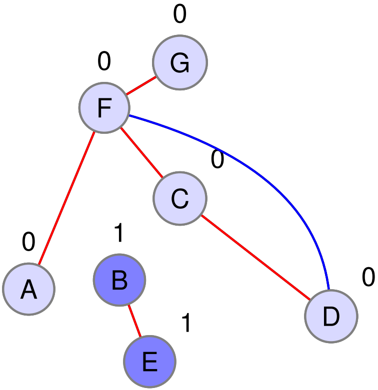

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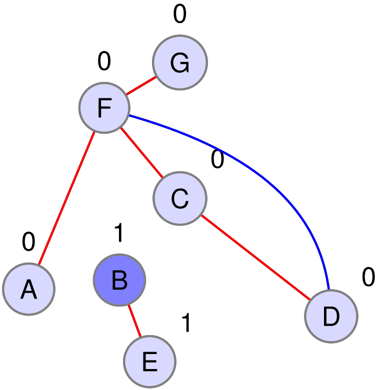

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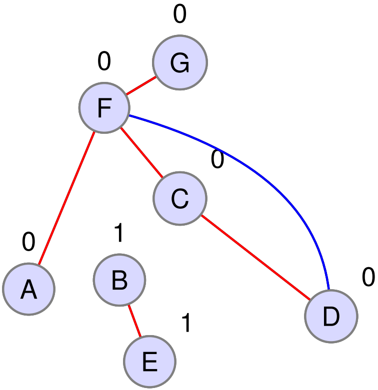

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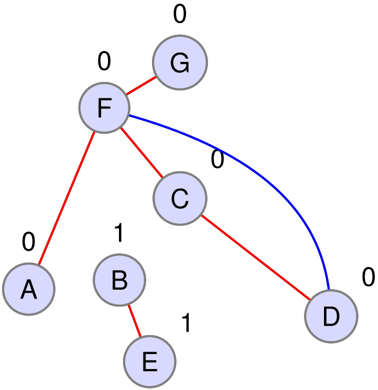

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**DFS(G):**

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**Postvisit(v):**

1. Set post[v] := clock.
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**DFS(G):**

0. Set clock := 0.

⋮

Clock: goes up to

# Introspection: pre/post.

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$\vdots$

Clock: goes up to 2 times number of tree edges.

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**Property:** For any two nodes, $u$ and $v$, $[pre(u), post(u)]$ and $[pre(v), post(v)]$ are either disjoint or one is contained in other.

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Either both on stack at some point (contained) or not (disjoint.)

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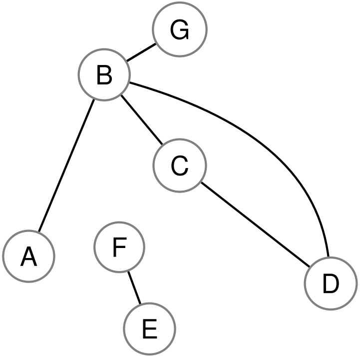
Either both on stack at some point (contained) or not (disjoint.)

Let's just watch it work!

# Example: Pre/Post numbering.



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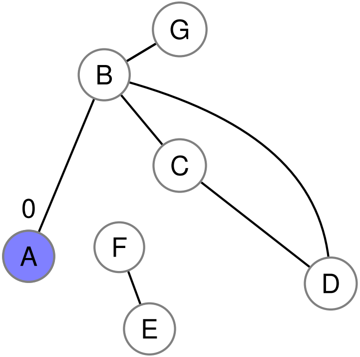

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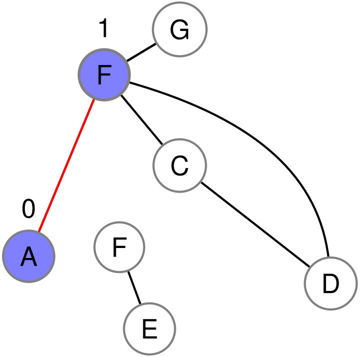

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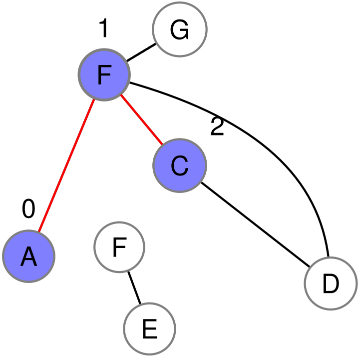

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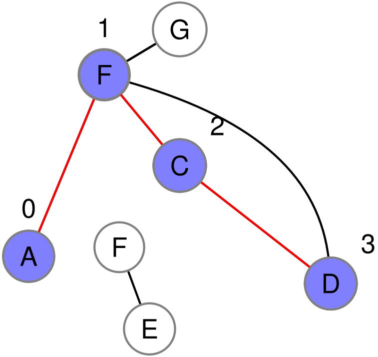

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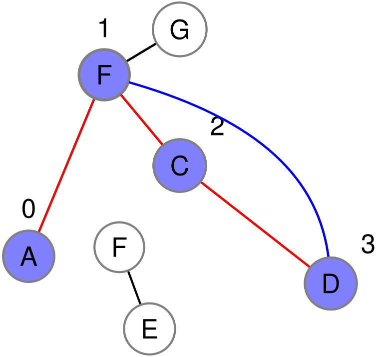

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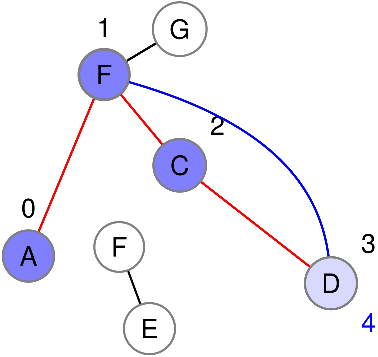

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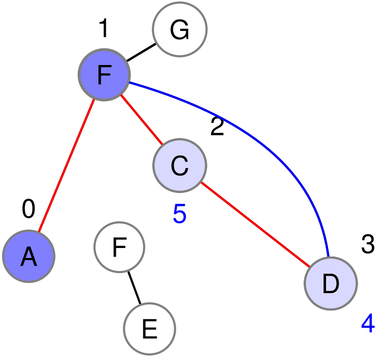

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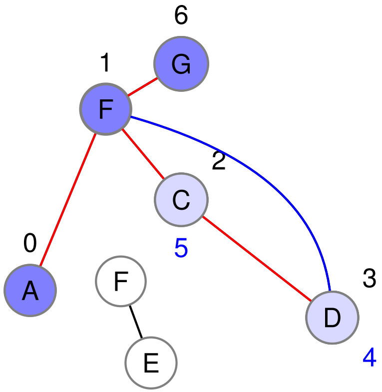

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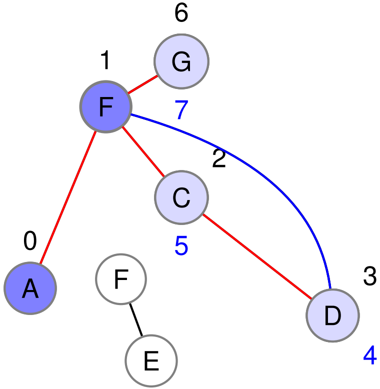

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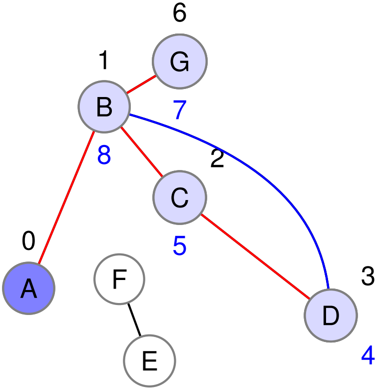

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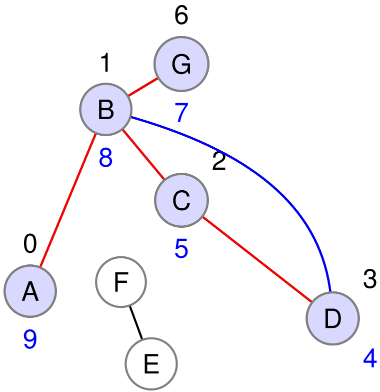

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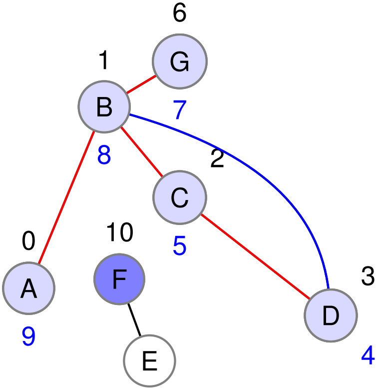

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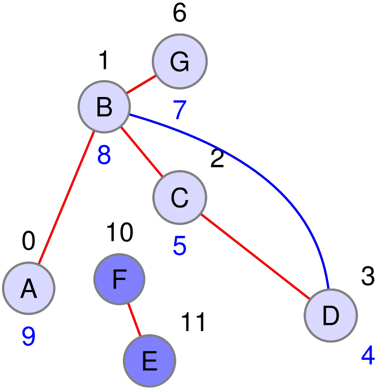

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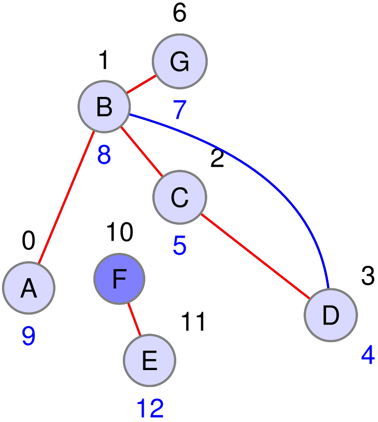

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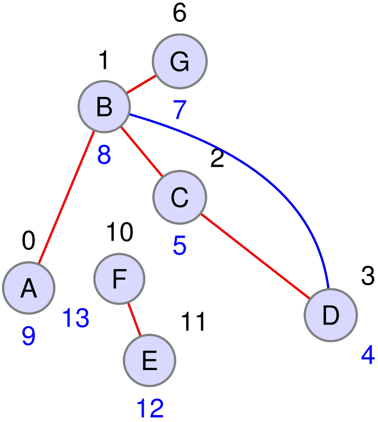

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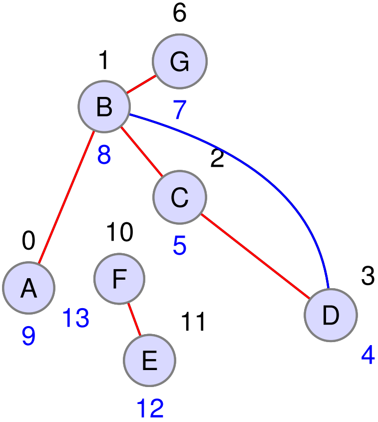


Edge $(u,v)$ is tree edge iff $[pre[v], post[v]] \in [pre[u], post[u]]$.

$u$ on stack before $v$.

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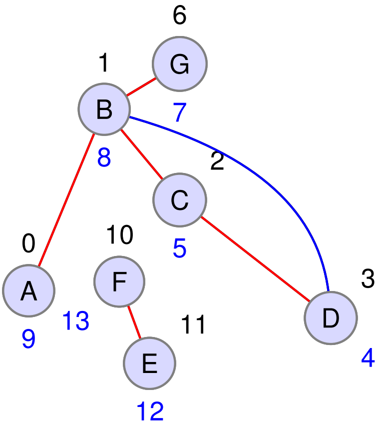


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Edge $(u, v)$ is back edge iff $[pre[u], post[u]] \in [pre[v], post[v]]$.

*v* on stack when *v* on stack. Path from *v* to *u*! Cycle!

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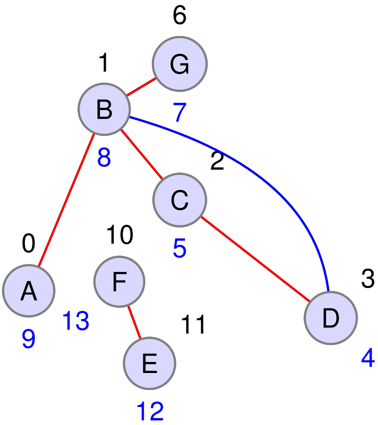


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Edge $(u, v)$ is back edge iff $[pre[u], post[u]] \in [pre[v], post[v]]$.

$v$ on stack when $v$ on stack. Path from $v$ to $u$! Cycle!

No edge between $u$ and $v$ if disjoint intervals.

# On Friday/Monday

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Christos Papadimitriou will lecture!