CS 170: Algorithms

CS 170: Algorithms

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Today:

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Today:

Review Inverse FFT

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No laptops please.

Thank you ! ! ! ! ! ! ! !

Today:

Review Inverse FFT

FFT network.

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No laptops please.

Thank you ! ! ! ! ! ! ! !

Today:

Review Inverse FFT

FFT network.

Other parallel systems.

Multiplying polynomials?

Coefficient representation:
$a_0, a_1, \ldots, a_{n-1}$

Value representation:
$A(x_0), A(x_1), \ldots, A(x_{n-1})$.

Multiplying polynomials?

Evaluation: $O(n \log n)$ if choose $1, \omega, \omega^2, \ldots, \omega^{n-1}$.

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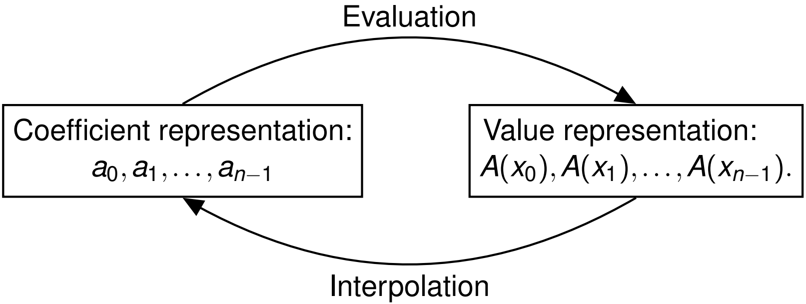

Interpolation: From points $A(x_0), \ldots, A(x_{n-1})$ to "function".

Multiplying polynomials?

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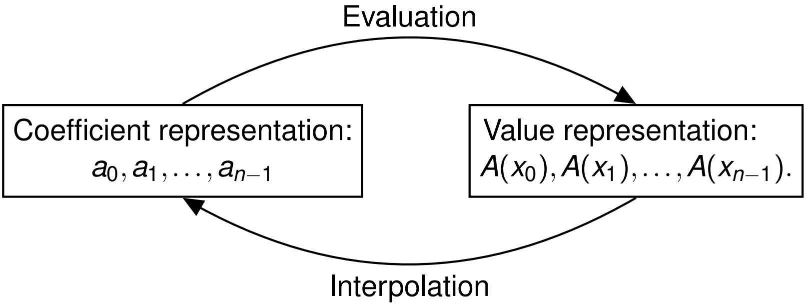

Interpolation: From points $A(x_0), \ldots, A(x_{n-1})$ to "function".
How?

Polynomial Evaluation and Matrices

Compute $A(\cdot)$ from a_i's:

$$\begin{bmatrix} A(x_0) \\ A(x_1) \\ \vdots \\ A(x_{n-1}) \end{bmatrix} = \begin{bmatrix} 1 & x_0 & x_0^2 & \cdots x_0^{n-1} \\ 1 & x_1 & x_1^2 & \cdots x_1^{n-1} \\ & & \vdots & \cdots \\ 1 & x_{n-1} & x_{n-1}^2 & \cdots x_{n-1}^{n-1} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{bmatrix}$$

Compute inverse of matrix above.

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Compute inverse of matrix above.
Multiply.

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Compute inverse of matrix above.
Multiply. $O(n^2)$!

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Doh!!

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Multiply. $O(n^2)$!

Doh!!

Also, computing inverse not even easy.

FFT Matrix

FFT: ω is complex nth root of unity

FFT Matrix

FFT: ω is complex nth root of unity
and matrix is ...

$$M_n(\omega) = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \cdots & \omega^{(n-1)} \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

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Compute inverse of $M_n(\omega)$?

Algebraically.

Inversion formula: $(M_n(\omega))^{-1} = \frac{1}{n} M_n(\omega^{-1})$.

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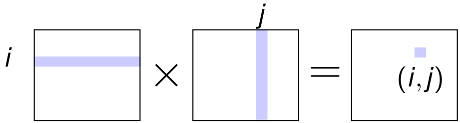
$C = M_n(\omega) \times M_n(\omega^{-1})$?



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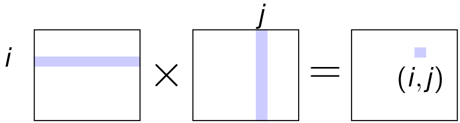

Recall: $\omega = e^{2\pi/n}$.

$$c_{ij} = \sum_k \omega^{ik} \omega^{-kj}$$

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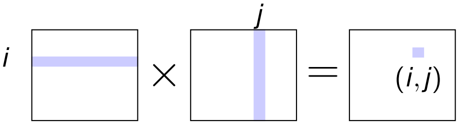

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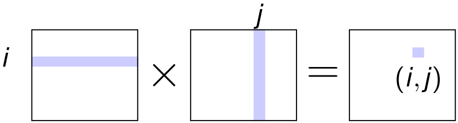

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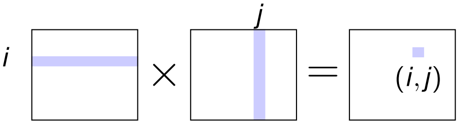

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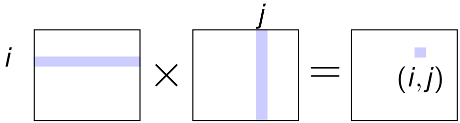

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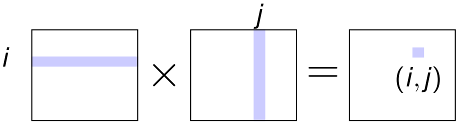

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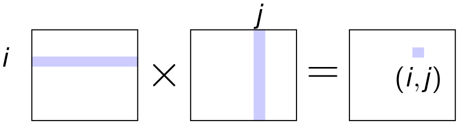

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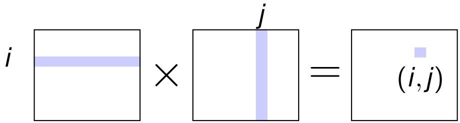

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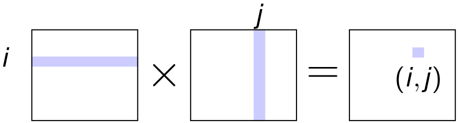

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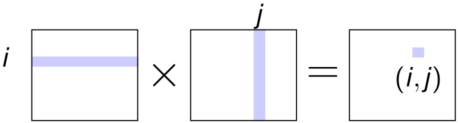

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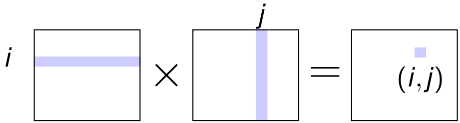

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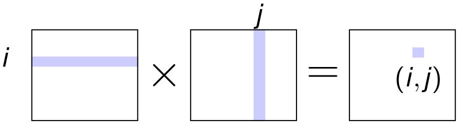

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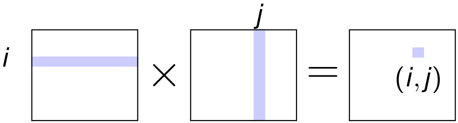

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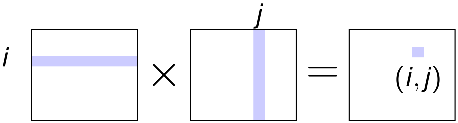

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For C – diagonals are n and the off-diagonals are 0.

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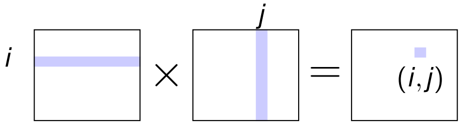

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For C – diagonals are n and the off-diagonals are 0.
Divide by n get identity!

Algebraically.

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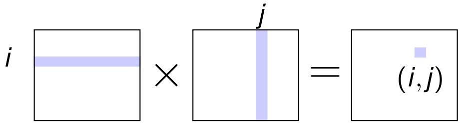

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For C – diagonals are n and the off-diagonals are 0.
Divide by n get identity!

Inversion formula: $M_n(\omega)^{-1} = \frac{1}{n}M_n(\omega^{-1})$.

Computing inverse.

FFT works with points with basic root of unity: ω or ω^{-1}

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Evaluation: FFT(A,ω).

Computing inverse.

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Fast convolution!

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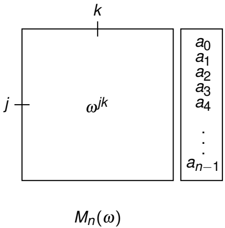
Fast convolution! Digital signal processing. Many other things.

Cool!!

FFT: a closer look.



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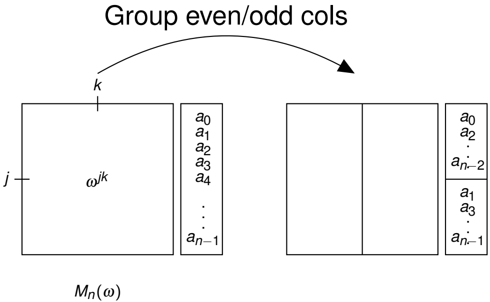

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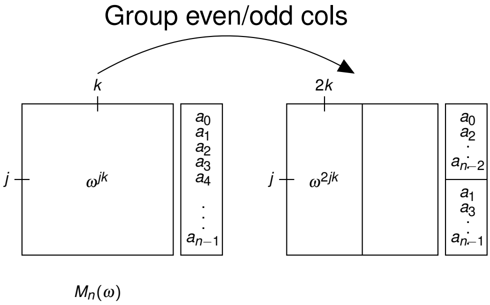

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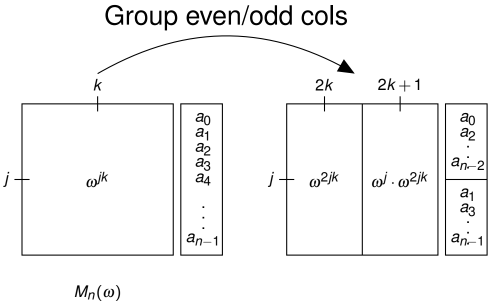

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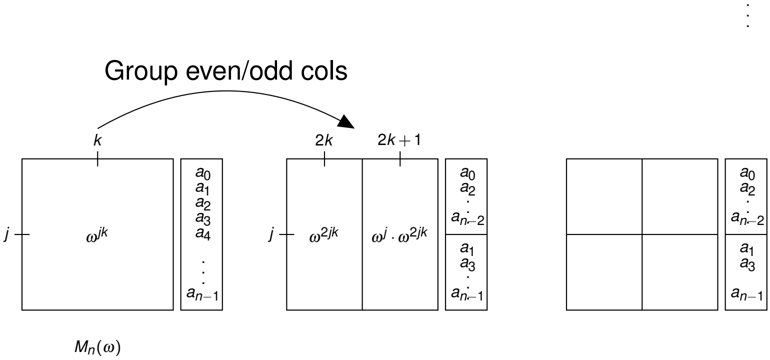

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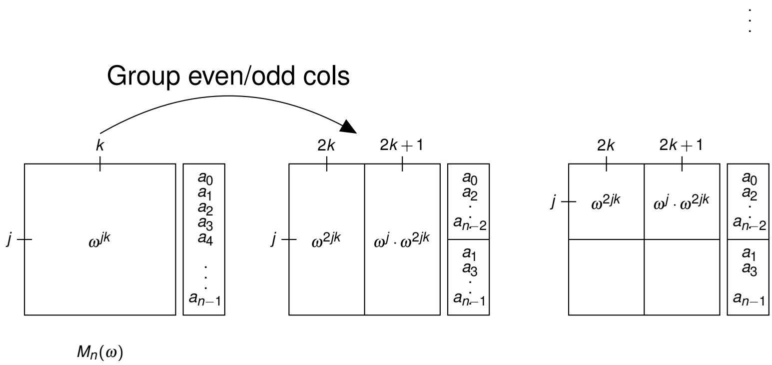

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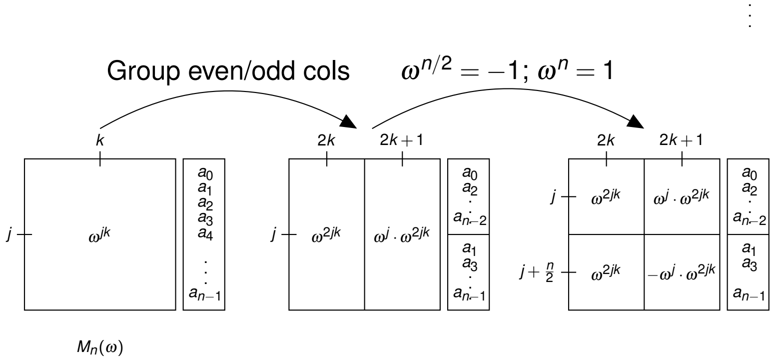

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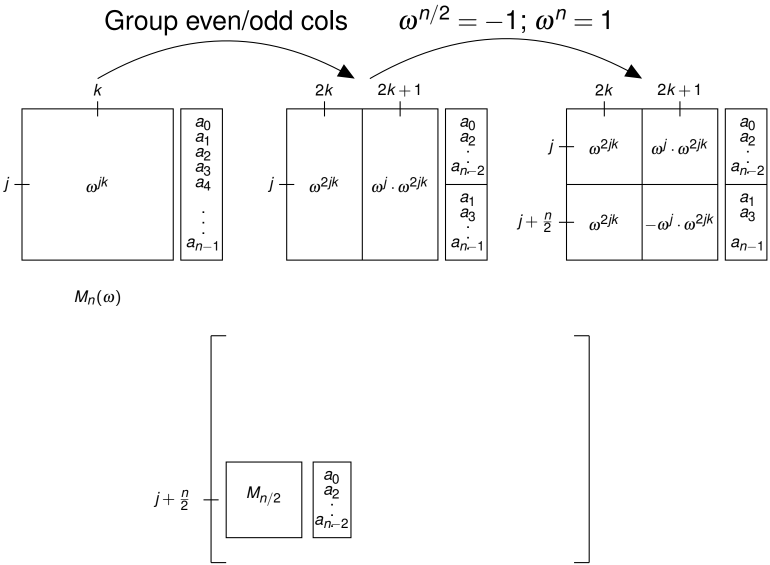

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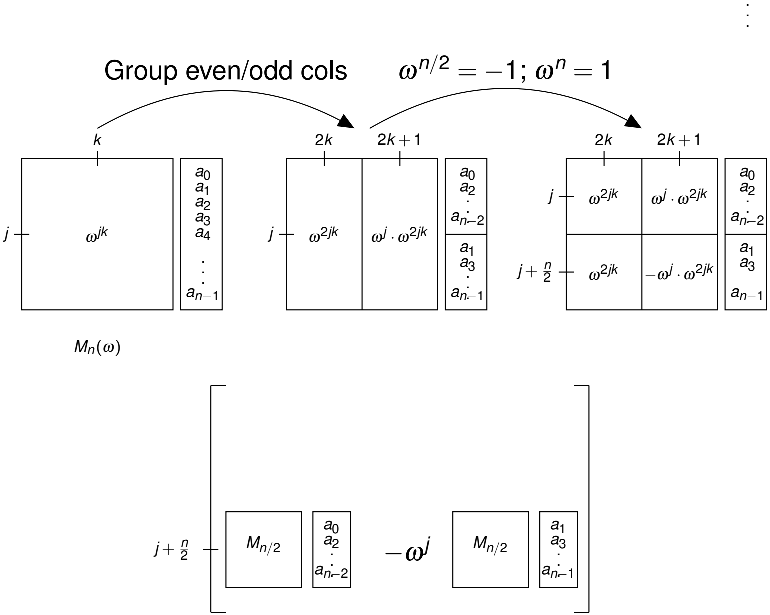

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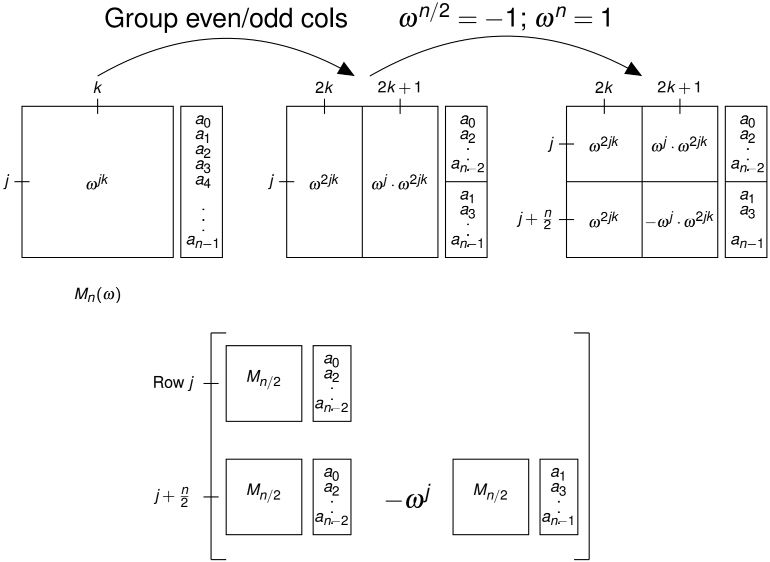

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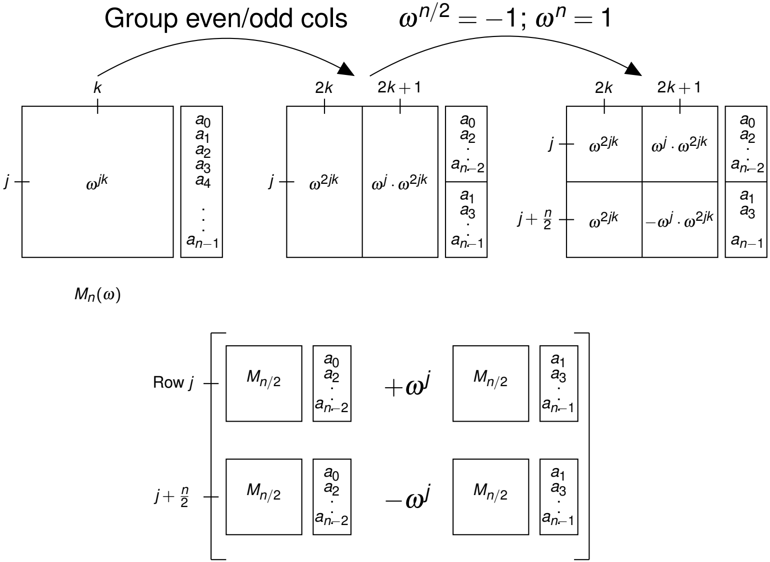
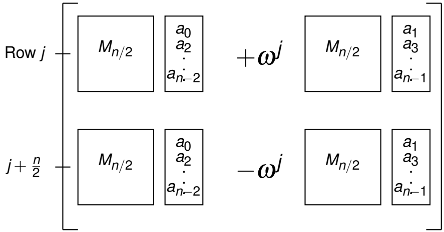


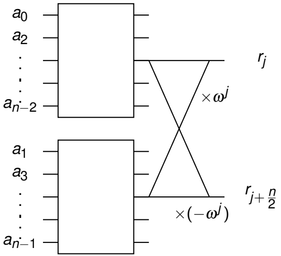
Unfolding FFT.

$$\begin{array}{c} \text{Row } j \\ \\ j+\frac{n}{2} \end{array} \begin{bmatrix} \boxed{M_{n/2}} \begin{bmatrix} a_0 \\ a_2 \\ \vdots \\ a_{n-2} \end{bmatrix} + \omega^j \boxed{M_{n/2}} \begin{bmatrix} a_1 \\ a_3 \\ \vdots \\ a_{n-1} \end{bmatrix} \\ \\ \boxed{M_{n/2}} \begin{bmatrix} a_0 \\ a_2 \\ \vdots \\ a_{n-2} \end{bmatrix} - \omega^j \boxed{M_{n/2}} \begin{bmatrix} a_1 \\ a_3 \\ \vdots \\ a_{n-1} \end{bmatrix} \end{bmatrix}$$

Butterfly switches!

Unfolding FFT.





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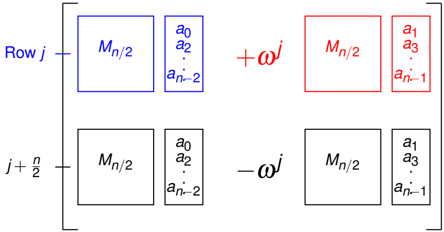

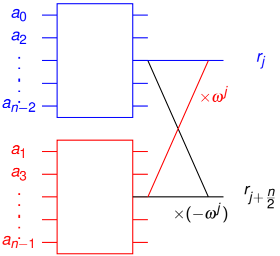

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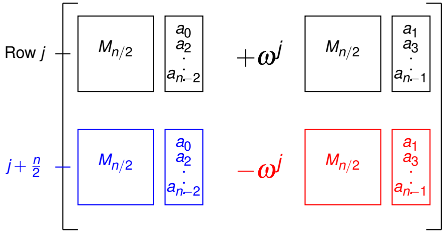
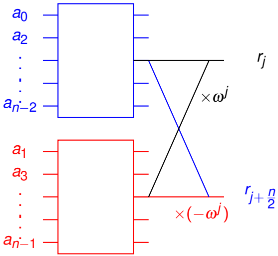
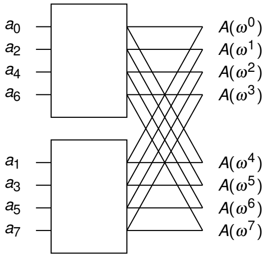




Butterfly switches!

Expanding FFT...



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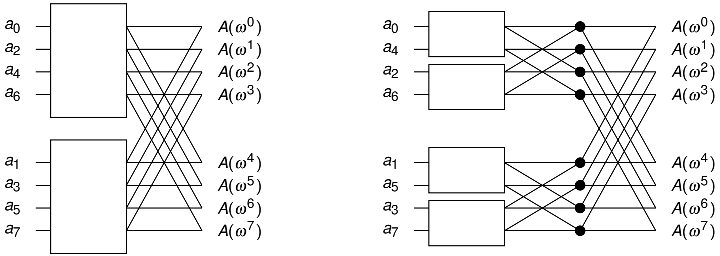

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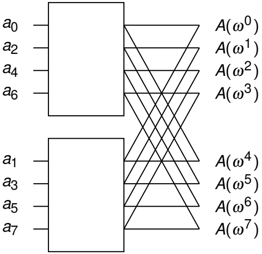

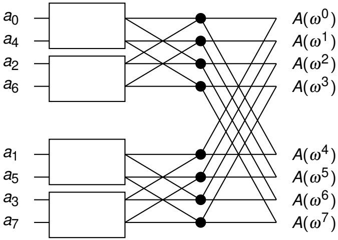

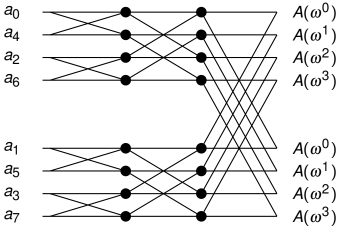

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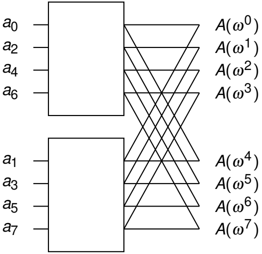
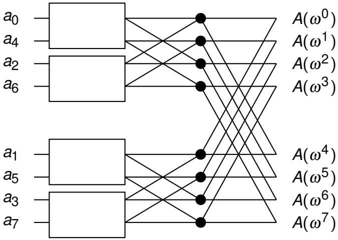
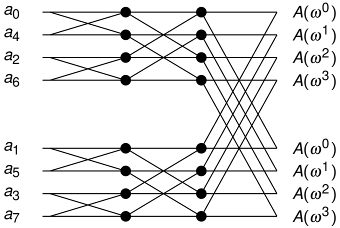
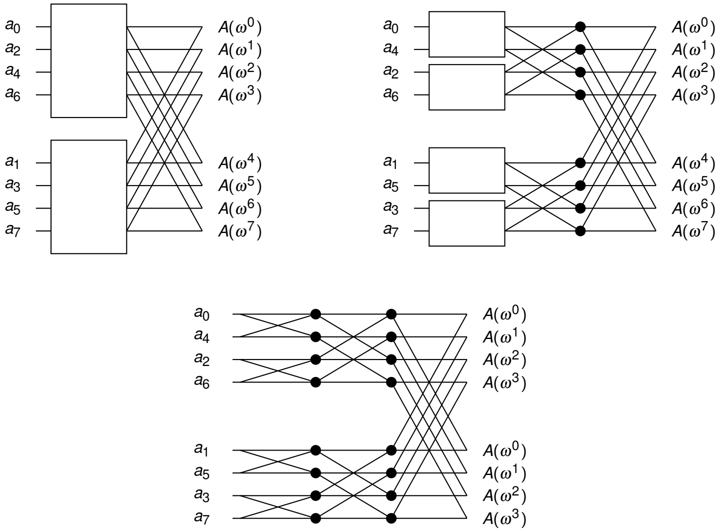






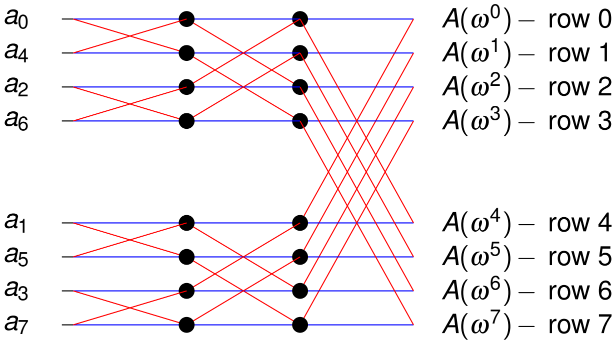
Edges from lower half of FFT have multipliers!

Expanding FFT...



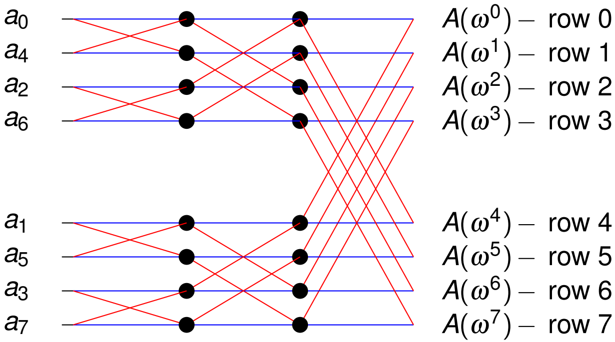
Edges from lower half of FFT have multipliers!

FFT Network.



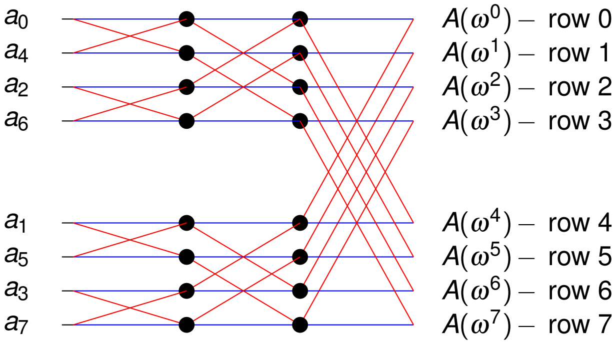
$\log N$ - levels.

FFT Network.



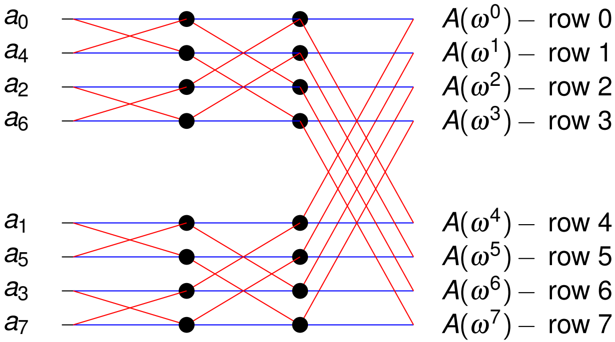
$\log N$ - levels.
N - rows.

FFT Network.



$\log N$ - levels.
N - rows.
In level i:

FFT Network.



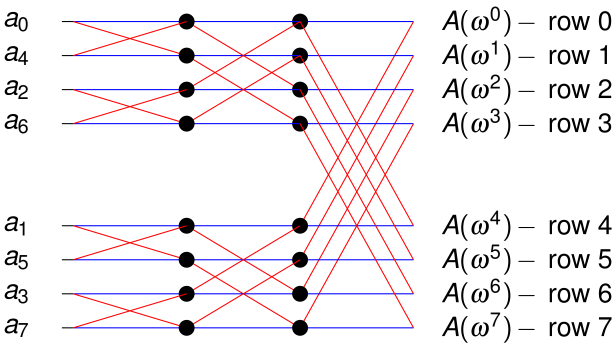
$\log N$ - levels.

N - rows.

In level i:

Row r node is connected to row r node in level $i+1$.

FFT Network.



$\log N$ - levels.

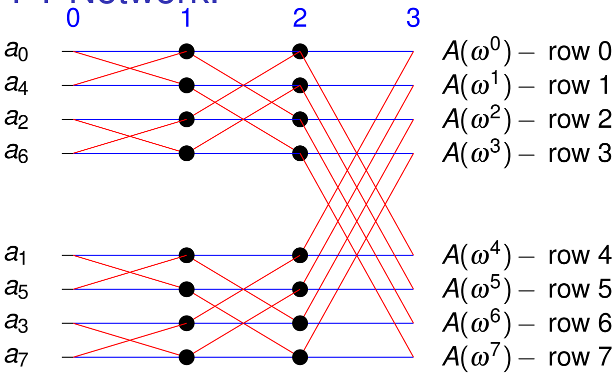
N - rows.

In level i:

Row r node is connected to row r node in level $i+1$.

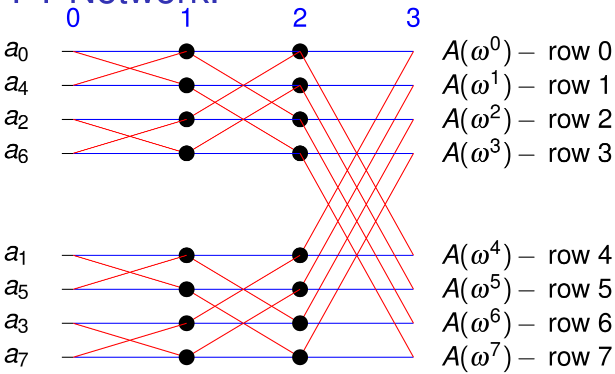
Row r node connected to row $r \pm 2^i$ node in level $i+1$

FFT Network.



Row r node connected to row $r \pm 2^i$ node in level $i+1$

FFT Network.



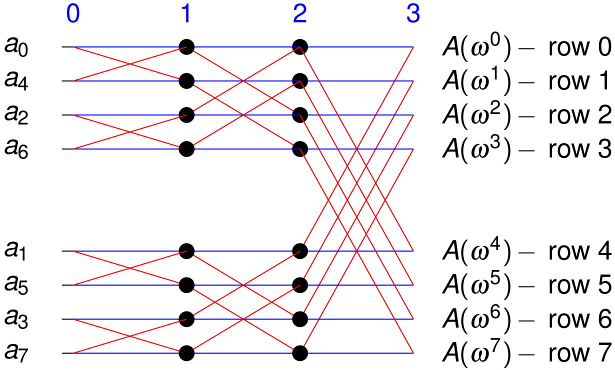
Row r node connected to row $r \pm 2^i$ node in level $i+1$

When is it $r + 2^i$?

(A) When $\lfloor r/2^i \rfloor$ is odd.

(B) When $\lfloor r/2^i \rfloor$ is even.

FFT Network.



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(B).

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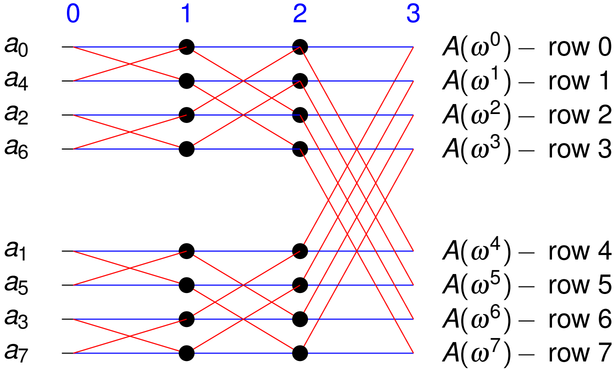


Row r node connected to row $r \pm 2^i$ node in level $i+1$

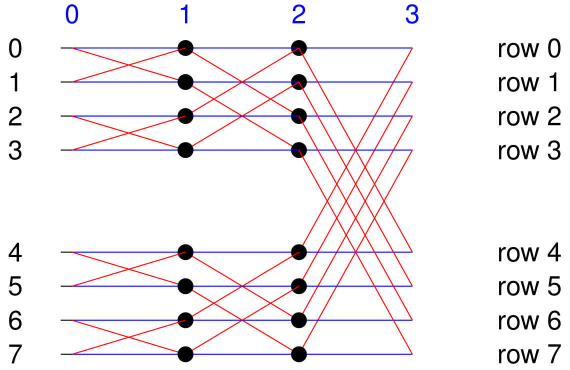
When is it $r + 2^i$?

(A) When $\lfloor r/2^i \rfloor$ is odd.

(B) When $\lfloor r/2^i \rfloor$ is even.

(B). Red edges flip bit!

Communication Architecture.



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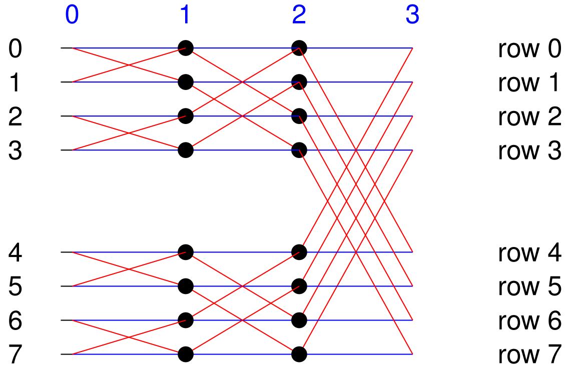
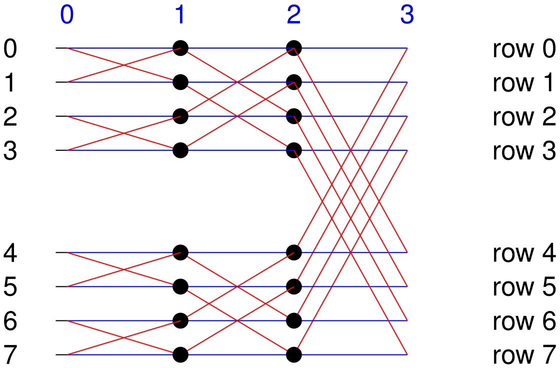


Route from input $i = 101$ to output $j = 000$?

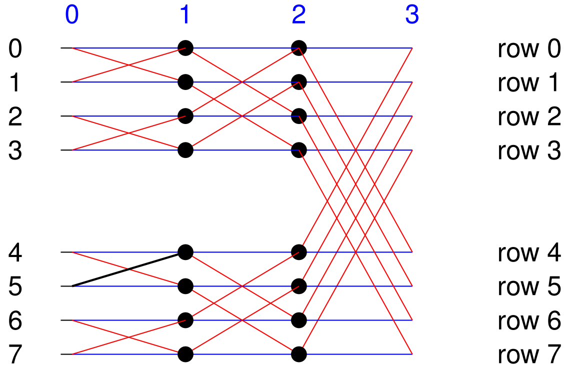
Communication Architecture.



Route from input $i = 101$ to output $j = 000$?

Flip first bit.

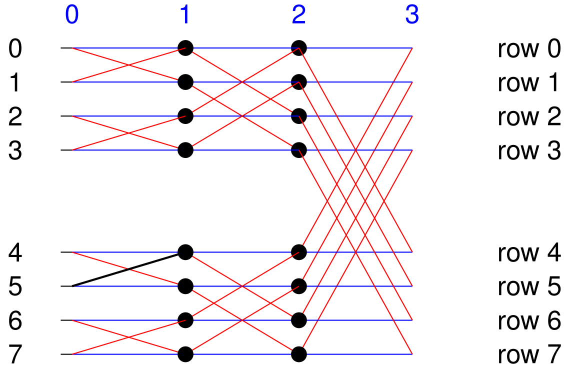
Communication Architecture.



Route from input $i = 101$ to output $j = 000$?

Flip first bit. Red (cross) edge.

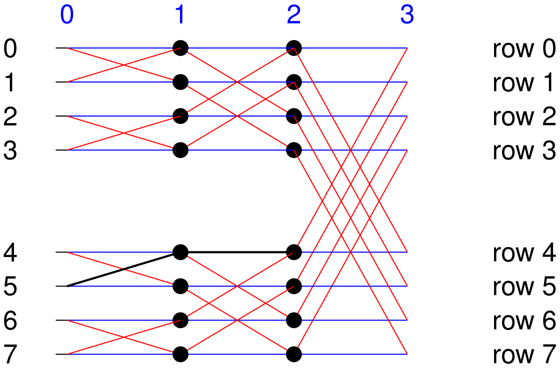
Communication Architecture.



Route from input $i = 101$ to output $j = 000$?

Flip first bit. Red (cross) edge.
Keep second bit.

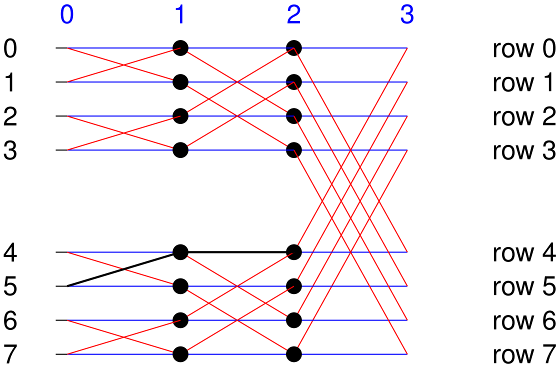
Communication Architecture.



Route from input $i = 101$ to output $j = 000$?

Flip first bit. Red (cross) edge.
Keep second bit. Blue (straight) edge.

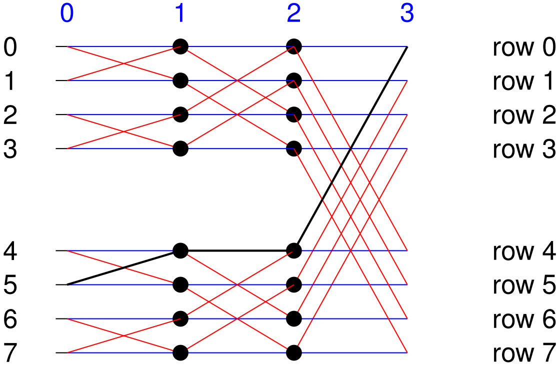
Communication Architecture.



Route from input $i = 101$ to output $j = 000$?

Flip first bit. Red (cross) edge.
Keep second bit. Blue (straight) edge.
Flip third bit.

Communication Architecture.



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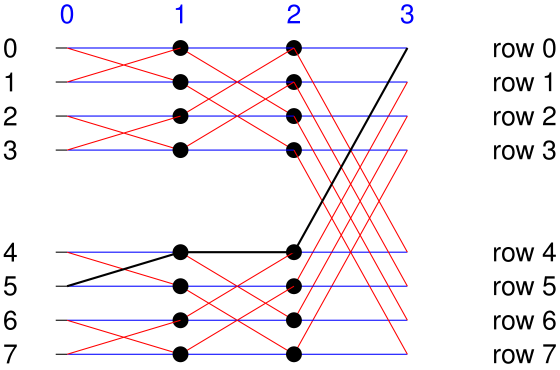

Route from input $i = 101$ to output $j = 000$?

Flip first bit. Red (cross) edge.
Keep second bit. Blue (straight) edge.
Flip third bit. Red (cross edge).

Basis of communication switches!

Algorithms, circuits, and parallelism.

Add up n numbers; $a_0, \ldots a_{n-1}$.

Algorithms, circuits, and parallelism.

Add up n numbers; $a_0, \ldots a_{n-1}$.

$O(n)$ time.

Algorithms, circuits, and parallelism.

Add up n numbers; $a_0, \ldots a_{n-1}$.

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Circuit:

Algorithms, circuits, and parallelism.

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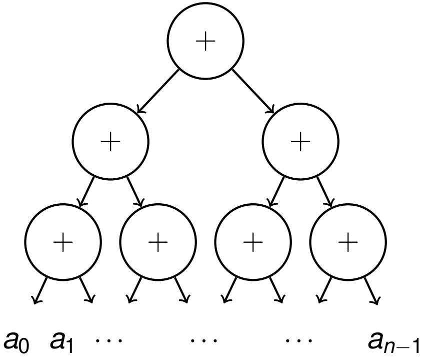
$O(n)$ time.

Circuit:



Depth $logn$.

Algorithms, circuits, and parallelism.

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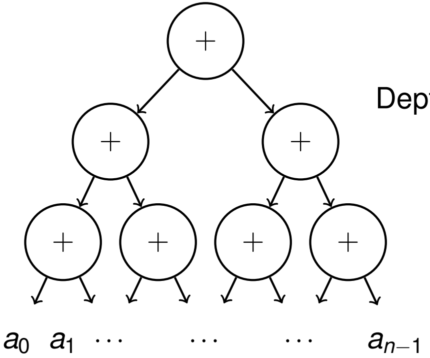
$O(n)$ time.

Circuit:



Depth $log n$.

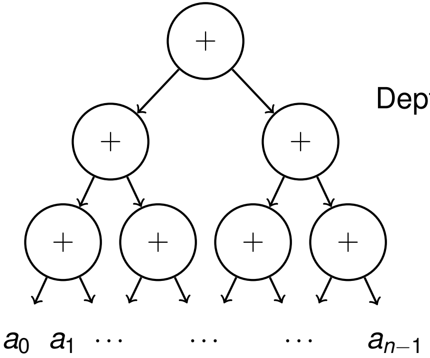
Depth $\log n \times$ (depth of "$+$").

Algorithms, circuits, and parallelism.

Add up n numbers; $a_0, \ldots a_{n-1}$.

$O(n)$ time.

Circuit:



Depth $logn$.

Depth $\log n \times$ (depth of "$+$").

Size: $O(n)$.

Other Circuits.

FFT Algorithm:

Other Circuits.

FFT Algorithm:
Sequential: $O(n \log n)$

Other Circuits.

FFT Algorithm:
Sequential: $O(n \log n)$

Depth: $O(\log n)$.

Other Circuits.

FFT Algorithm:
Sequential: $O(n \log n)$

Depth: $O(\log n)$.
Size: $O(n \log n)$.

Other Circuits.

FFT Algorithm:
Sequential: $O(n \log n)$

Depth: $O(\log n)$.
Size: $O(n \log n)$.

How to build a circuit for Matrix Multiplication?

Other Circuits.

FFT Algorithm:
Sequential: $O(n \log n)$

Depth: $O(\log n)$.
Size: $O(n \log n)$.

How to build a circuit for Matrix Multiplication?

Who makes circuits anyway!!

Other Circuits.

FFT Algorithm:
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Size: $O(n \log n)$.

How to build a circuit for Matrix Multiplication?

Who makes circuits anyway!!

Not too many people.

Other Circuits.

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How to build a circuit for Matrix Multiplication?

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Parallel?

Other Circuits.

FFT Algorithm:
Sequential: $O(n \log n)$

Depth: $O(\log n)$.
Size: $O(n \log n)$.

How to build a circuit for Matrix Multiplication?

Who makes circuits anyway!!

Not too many people.

Parallel?

Everywhere!

One model for parallel.

Shared memory model (SMM): Processors.

One model for parallel.

Shared memory model (SMM): Processors.
Any processor accesses memory location in 1 time step.

One model for parallel.

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One model for parallel.

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Any processor accesses memory location in 1 time step.

Models parallelism.

One model for parallel.

Shared memory model (SMM): Processors.
Any processor accesses memory location in 1 time step.

Models parallelism.
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Circuits $\implies$ SMM
Time on SMM = Depth of circuit.

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Any processor accesses memory location in 1 time step.

Models parallelism.
"Ignores communication cost"

Algorithm:
Time: How many steps?
Work: How much total work over processors.

Circuits $\implies$ SMM
Time on SMM = Depth of circuit.
Work on SMM = Size of circuit

Adding n numbers

$[a_0, a_1, a_2, \ldots, a_{n-1}]$

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$[(a_0 + a_1), (a_2 + a_3), \ldots, (a_{n-2} + a_{n-1})]$.

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Work: $\frac{n}{2} + \frac{n}{4} + \cdots 1$

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And so on...

Time: $\log n$

Work: $\frac{n}{2} + \frac{n}{4} + \cdots 1 = O(n)$.

Mergesort.

Algorithm:

Split into two lists.

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Make Merge faster?

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In $O(\log n)$ time using Mirrored FFT of comparators!
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Work: $O(n \log^2 n)$

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Cole's Mergesort:

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 Split into two lists.
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 Merge sublists.

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Time: $O(n) + O(n/2) + O(n/4) \cdots = O(n)$.

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In $O(\log n)$ time using Mirrored FFT of comparators!
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Time: $O(\log^2 n)$.
Work: $O(n \log^2 n)$ Worse than sequential!
Cole's Mergesort: $O(\log n)$ time; $O(n \log n)$ work. Optimal!

Large integer multiplication.

Algorithm:

Split n bit numbers into $n/2$ bit numbers.

Large integer multiplication.

Algorithm:

Split n bit numbers into $n/2$ bit numbers.

Do three (or four) multiplications of those $n/2$ bit numbers.

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Shift and Add the numbers.

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Recursion depth: $\log n$.
Addition of n-bit numbers: $O(n)$ time.

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Addition of n-bit numbers: $O(n)$ time.

Work mostly at the leaves:

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Work mostly at the leaves:
How many leaves?

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Work mostly at the leaves:
How many leaves? $O(n^{\log_2 3})$.

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Depth: $O(n) + O(n/2) + \cdots + O(1)$

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Better Sequential Algorithm as well!

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Care to guess how?

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Using the FFT!

Brent's Theorem.

What if I have fewer processors?

Brent's Theorem.

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Do complexity analysis again?

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Simulation:

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Simulation:
If some step uses P processors.

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Simulation:
If some step uses P processors.
Simulate with p processors for $\lceil \frac{P}{p} \rceil$ steps.

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Theorem:
Any work W, time T parallel algorithm can run on a p-processor shared memory machine in time $\lfloor \frac{W}{p} \rfloor + T$ and work W.

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Show Some Code.

Multiplication on my

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Multiplication on my little laptop.

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