MATH5350

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The project is to solve the burgers Equation：

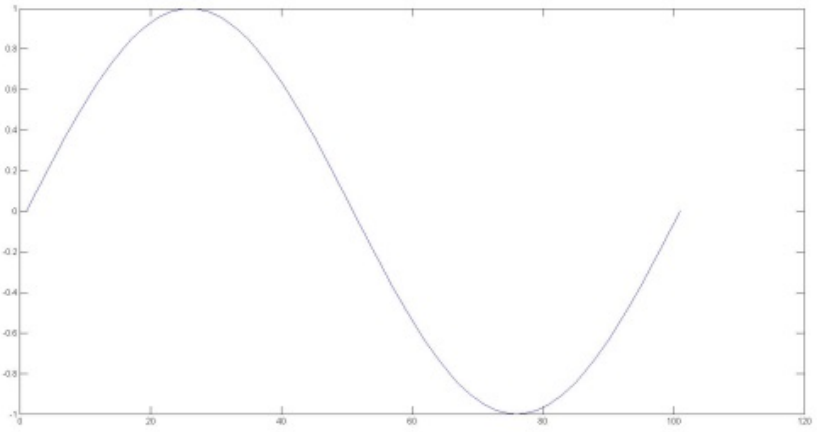
$$\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x}(u^2) = 0.$$

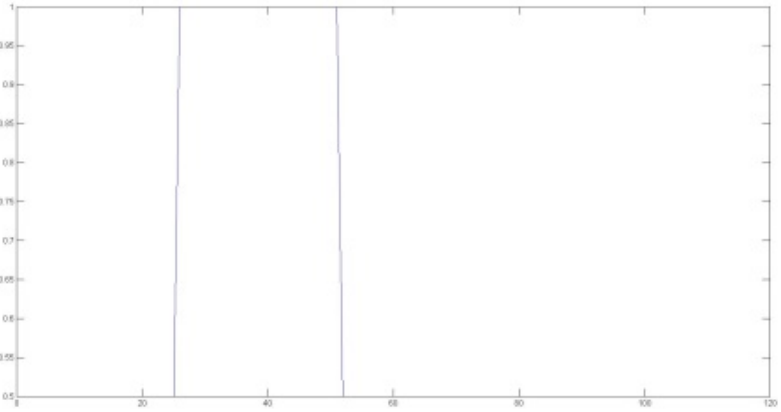
Using the final volume method:

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}(F_{i+1/2}^n - F_{i-1/2}^n),$$

With Initial condition:

1.Sine wave 2.Square Pulse


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
20
40
60
80
100
120

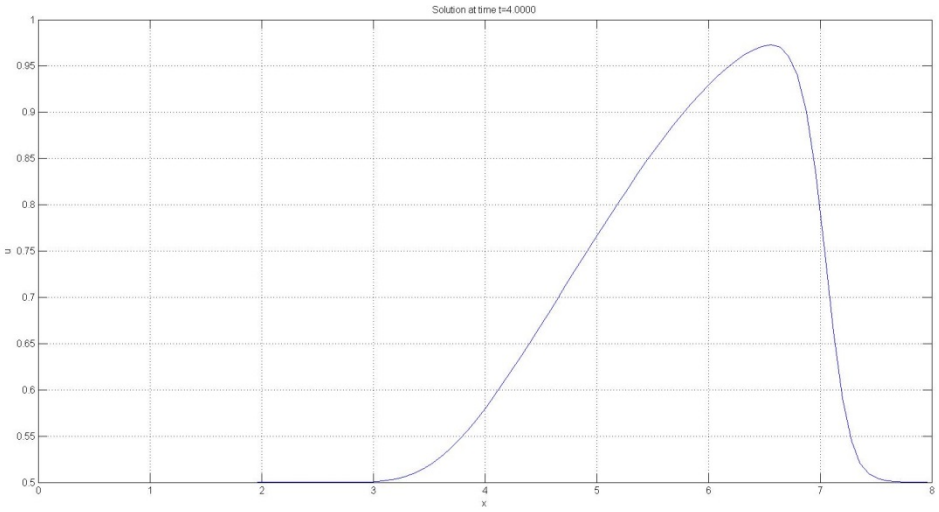

1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0
20
40
60
80
100
120

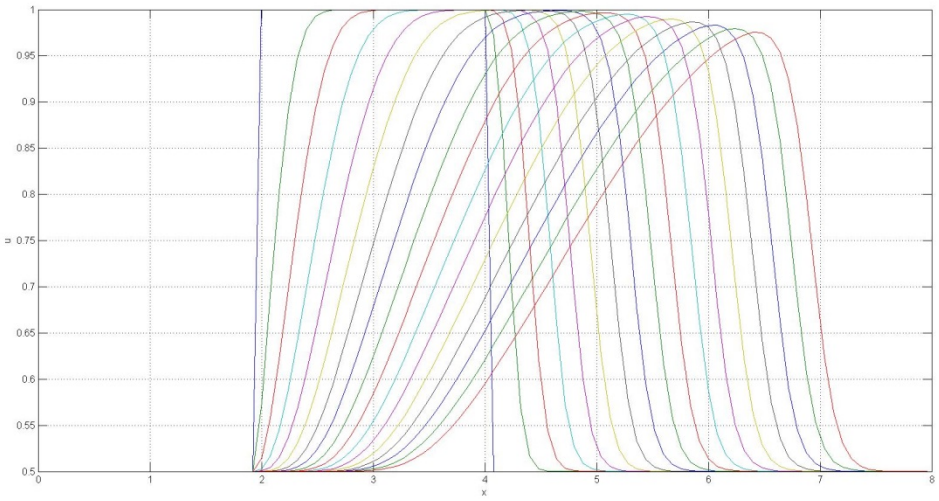
Flux function:

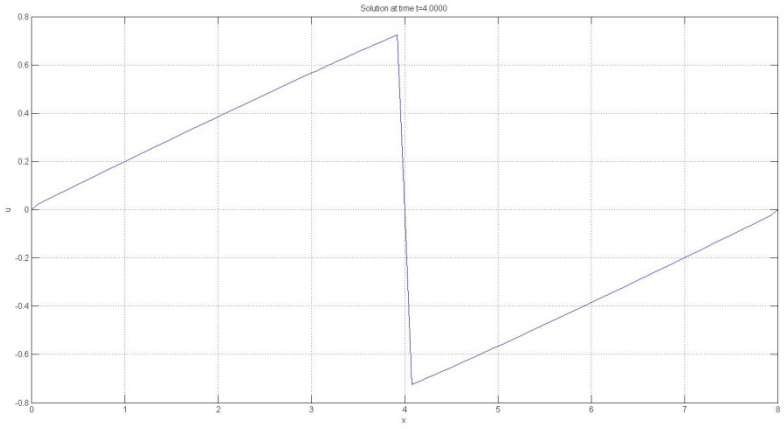
```
function ret = rpbu2( uL, uR )

s = 0.5 * (uL + uR);
if uL <= uR,
  if uR <= 0,
    ret = uR;
  else

    if uL >= 0,
      ret = uL;
    else
      ret = 0;
    end
  end
else
  if s > 0,
    ret = uL;
  else
    ret = uR;
  end
end
```

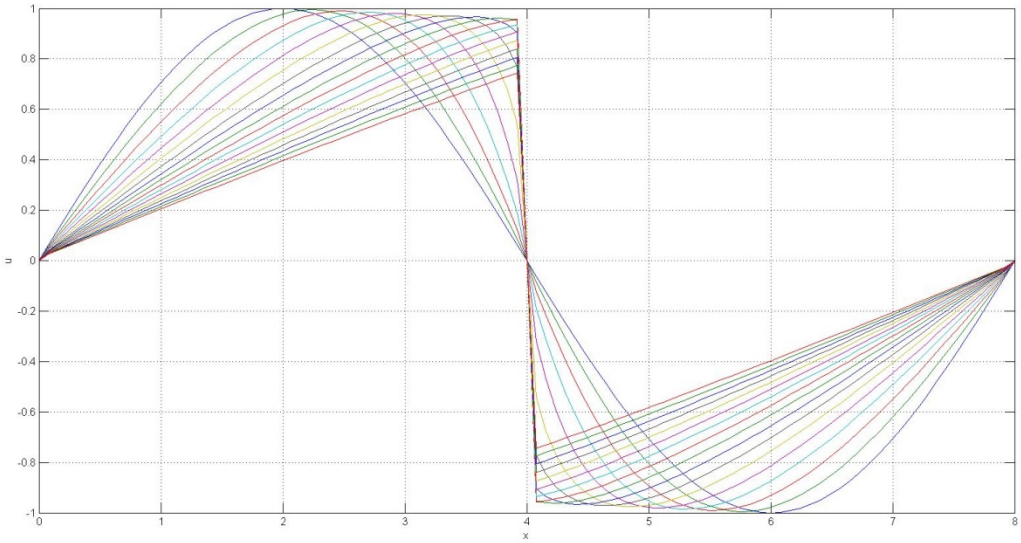
Result:

For the two schemes of flux,the result for the square wave initial condition is the same,which is shown below






Solution at time t=4.0000
u
x


u
x

The matlab code is shown below:

```
clear;

nx    = 100;
 dt    = 0.01;
 ictype= 5;     % 1 = shock; 2 = expansion;
                                  % 3 = sonic expansion; 4 = square pulse;5 =
sine

 tend = 4;     % end time
xmax = 8;     % domain length [0,xmax]
dx = xmax/nx; % mesh spacing (constant)
x  = [0 : dx : xmax];
nt = floor(tend/dt);
dt = tend / nt;
ntprint = 50; % for printing

u0   = uinit(x,ictype);
u    = u0;
unew = 0*u;
us   = unew(1:end-1);

disp( ['   dx = ', num2str(dx)] );
disp( ['   dt = ', num2str(dt)] );

ntprint = min(nt, ntprint);
dtprint = tend / ntprint;

uall = zeros(ntprint+1,nx+1);

uall(1,:) = u0;

ip = 1;
figure(1)
for i = 1 : nt,
  t = i*dt;

  us = rpbu2(u(1:end-1), u(2:end));
  unew(2:end-1) = u(2:end-1) + dt/dx * (f(us(1:end-1)) - f(us(2:end)));
  unew(1)   = u(1);
  unew(end) = u(end);

  % Plot the solution profiles.
  if t >= ip*dtprint,
    plot(x, unew)
    xlabel('x'), ylabel('u')
    title( ['Solution at time t=', num2str(t,'%9.4f')] )
    grid on, shg
    pause(0.1)
    ip = ip + 1;

    uall(ip,:) = unew;
  end
  u = unew;
end

figure(2)
nskip = 3;
plot(x,uall(1:nskip:end,:));
xlabel('x'), ylabel('u')
grid on, shg
```