Project 2   MATH5350

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Problem

Use Riemann solver to code Linearised gas dynamics equation

$$\mathbf{U}_t + \mathbf{A}\mathbf{U}_x = \mathbf{0} \, ,$$

with

$$\mathbf{U} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \equiv \begin{bmatrix} \rho \\ u \end{bmatrix} \, , \quad \mathbf{A} = \begin{bmatrix} 0 & \rho_0 \\ a^2/\rho_0 & 0 \end{bmatrix} \, .$$

Finite volume scheme is used in this project:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x_i}(F^*_{i+1/2} - F^*_{i-1/2})$$

The flux F is A*U,where U is

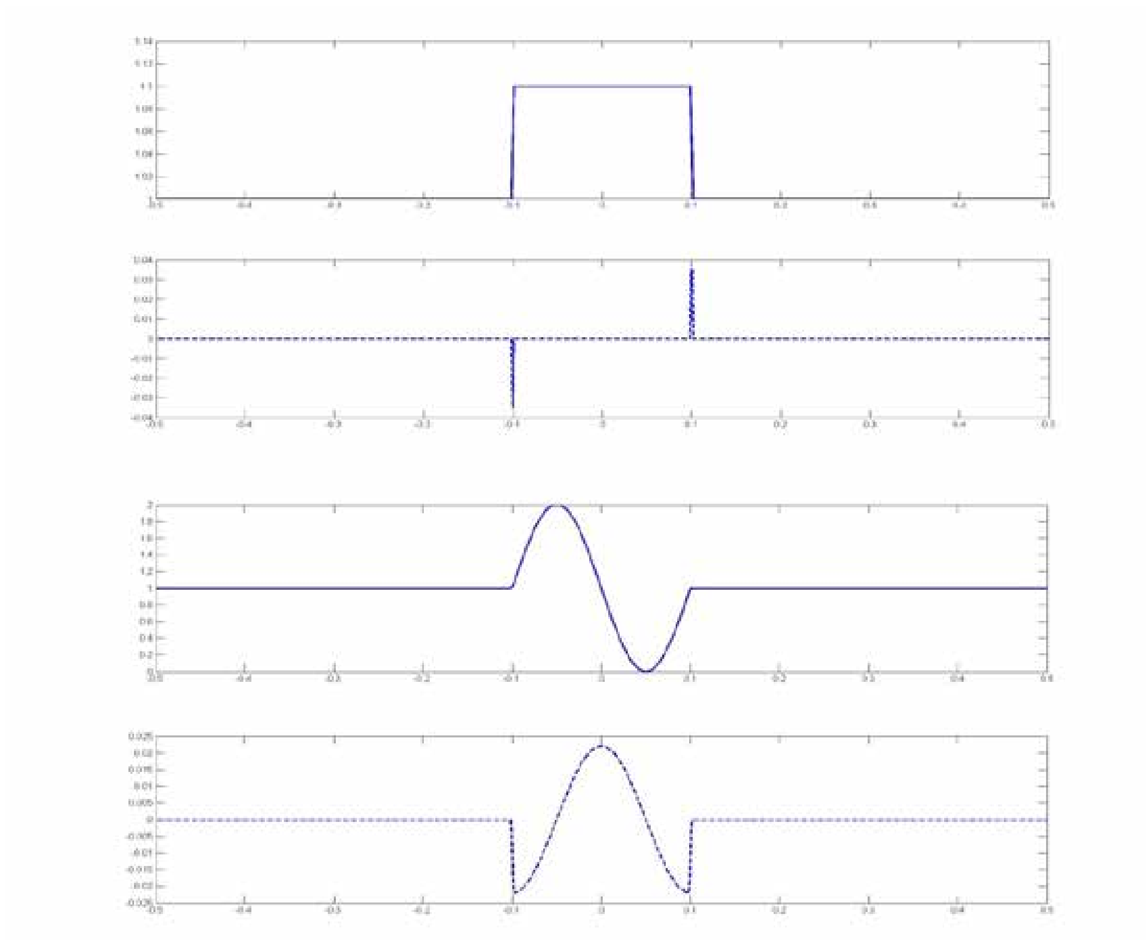
$$\mathbf{U}^* = \begin{bmatrix} \rho^* \\ u^* \end{bmatrix} = \beta_1 \begin{bmatrix} \rho_0 \\ -a \end{bmatrix} + \alpha_2 \begin{bmatrix} \rho_0 \\ a \end{bmatrix} \, .$$

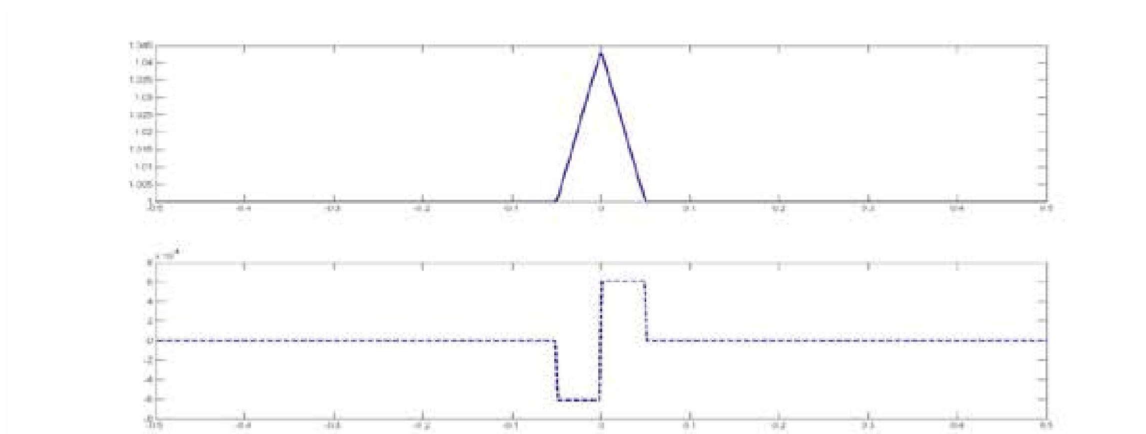
Where beta and alpha are given by

$$\beta_1 = \frac{a\rho_\mathrm{R} - \rho_0 u_\mathrm{R}}{2a\rho_0} \, , \quad \beta_2 = \frac{a\rho_\mathrm{R} + \rho_0 u_\mathrm{R}}{2a\rho_0} \, .$$

$$\alpha_1 = \frac{a\rho_\mathrm{L} - \rho_0 u_\mathrm{L}}{2a\rho_0} \, , \quad \alpha_2 = \frac{a\rho_\mathrm{L} + \rho_0 u_\mathrm{L}}{2a\rho_0} \, .$$

Three different initial conditions

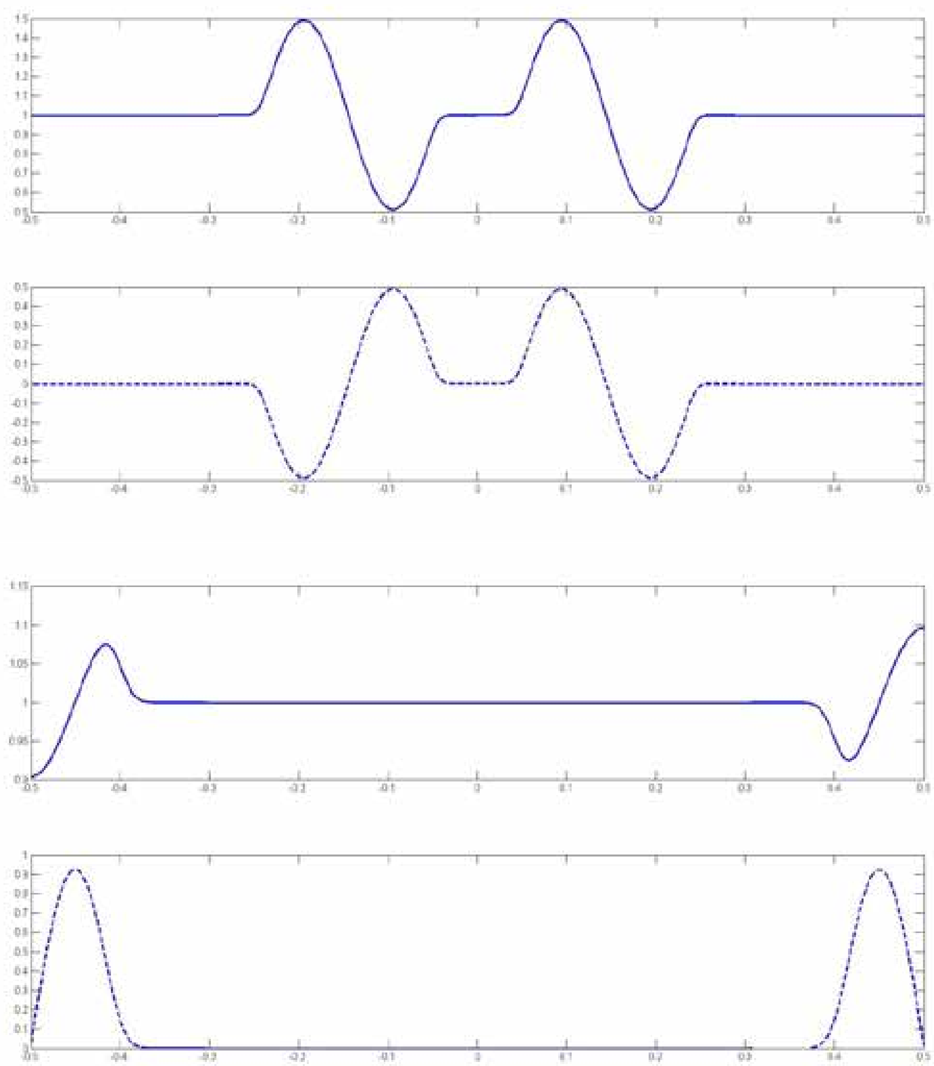




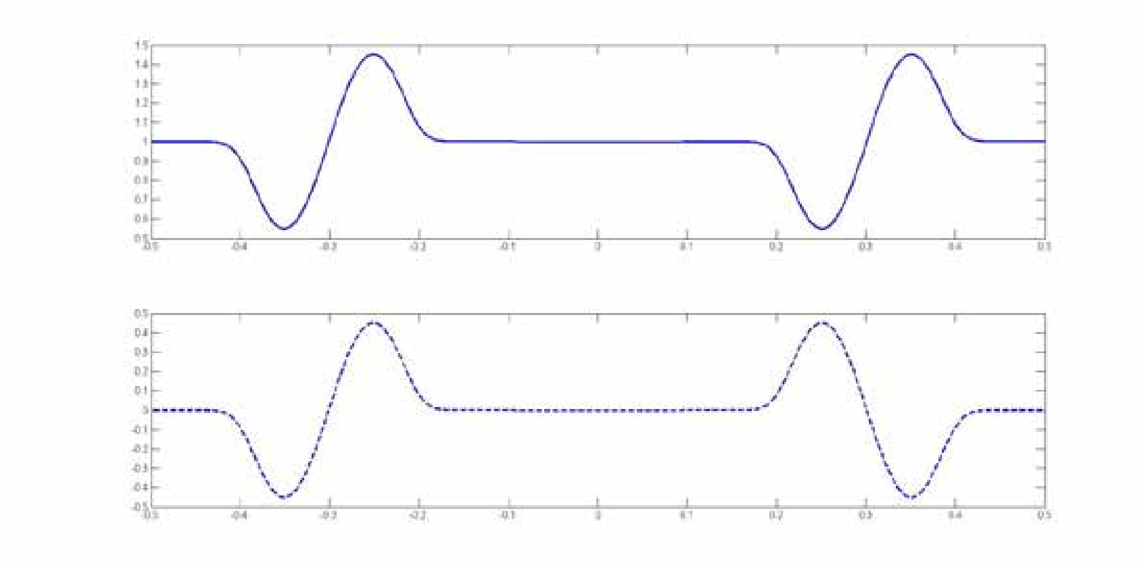
Boundary condition

The difference between **open boundary** and **reflection boundary** lies in the **two far end** of the grid. Only two ghost points are used for this Riemann solver used. But 4 ghost points will be used if the flux depends on neighboring four points. For open boundary, we need to copy the value of the point next to the ghost point to the ghost point. For reflection boundary, we can fill in the ghost point in a way that there is a wave coming in opposite direction.

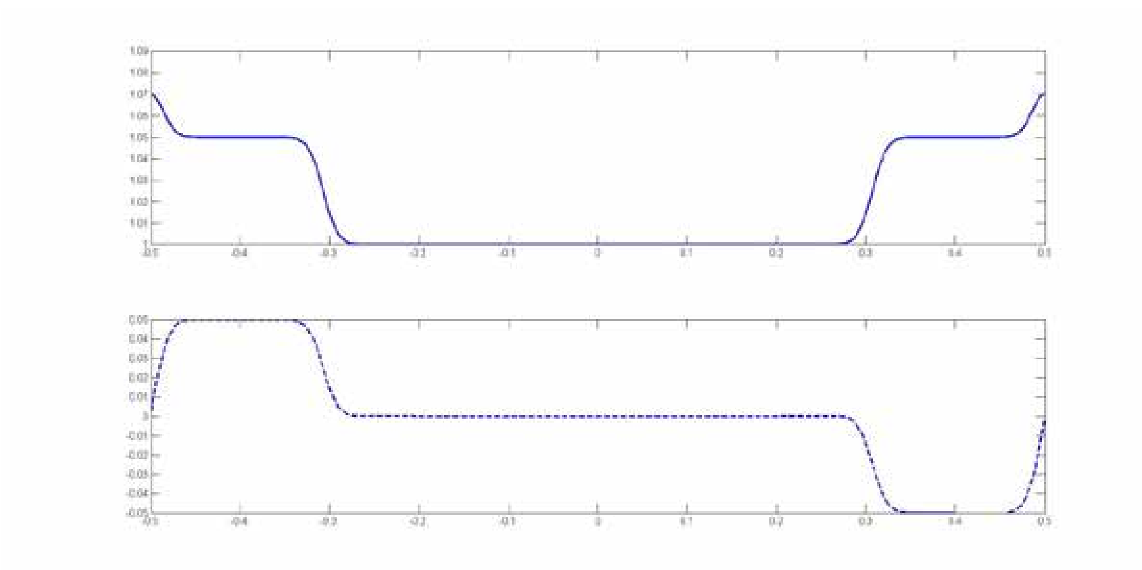
Sine:



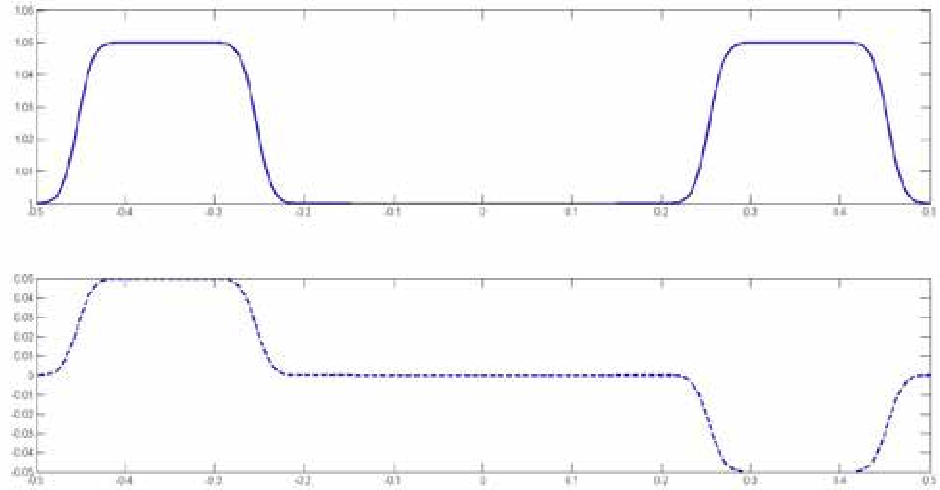
Reflection starts from here:



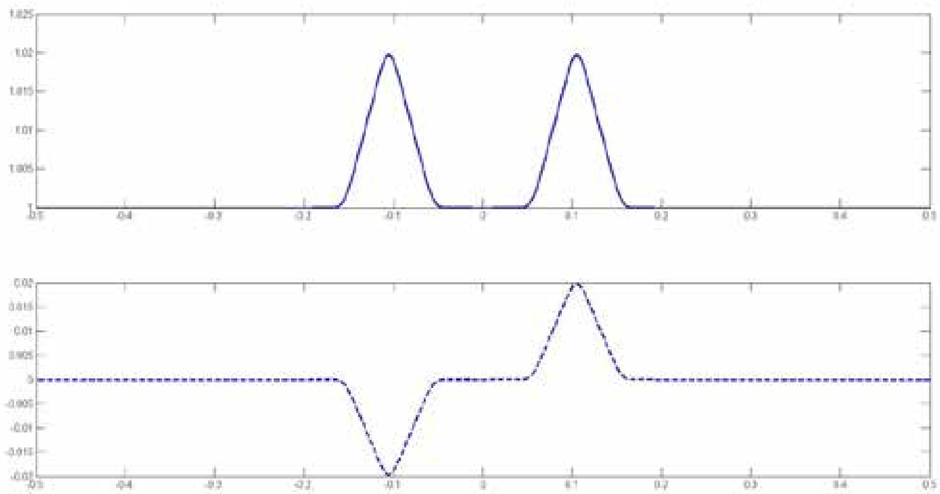
Square (spread into two half square gradually then moving to different end. Below is the graph when approaching to the end and then reflected.)



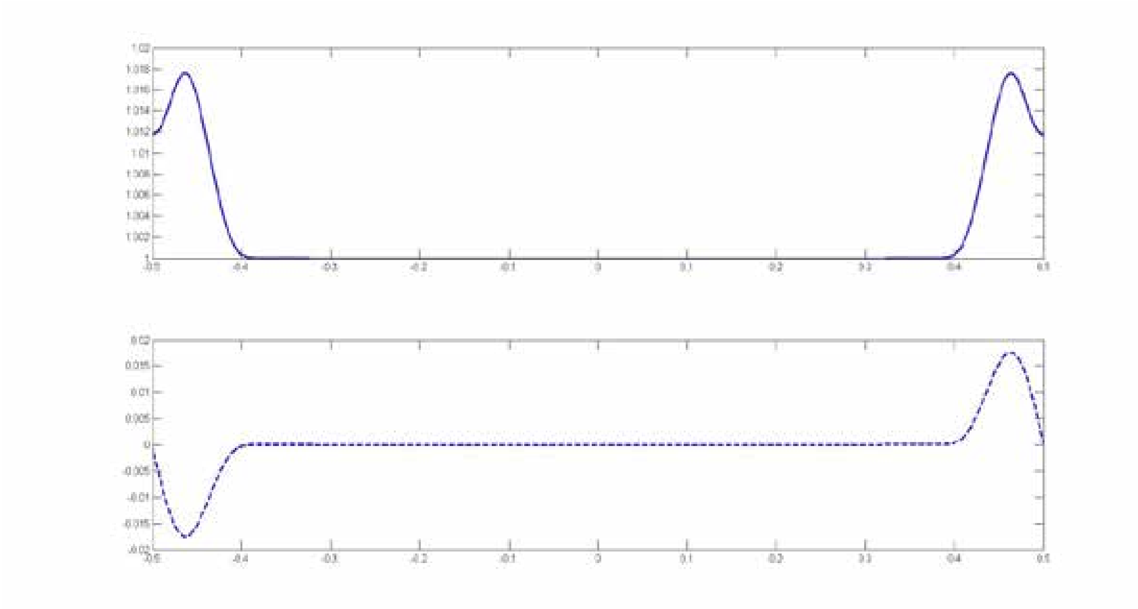
Reflection starts from here:



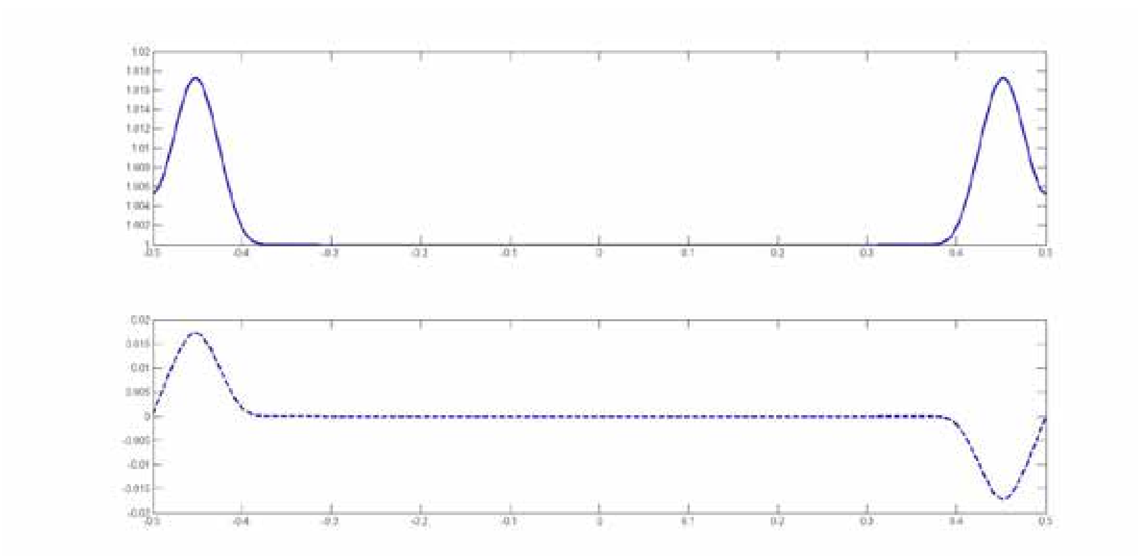
Triangle



Reflection starts from here:



Reflection starts from here:



Code

The **FORTRAN code** is attached in the end

The subroutine for drawing the graph is commented out for easy reading of the main part.

```
module datas
    real(kind=8),parameter :: PI = 4.0*atan(1.0)
    real(kind=8),parameter :: SMV = 1.0E-20
    real(kind=8),parameter :: a= 0
    real(kind=8),allocatable,dimension(:,:) :: w !solution variable
    real(kind=8),allocatable,dimension(:,:) :: flux !flux
    real(kind=8),allocatable,dimension(:) :: x
    real(kind=8),allocatable,dimension(:,:) :: u0
    real(kind=8),allocatable,dimension(:,:) :: u1
    real(kind=8) :: dx !spacing in x-direction
    real(kind=8) :: dt !time step
    real(kind=8) :: cfl !cfl number
    real(kind=8) :: lambda
    real(kind=8) :: t
    integer :: iter !iterations
end module datas

module solver
    contains
    function Riemann(ul,ur)
    real(kind=8),dimension(2) ::ul
    real(kind=8),dimension(2) ::ur
    real(kind=8) ::a=1.0
    real(kind=8) ::rho0=1.0
    real(kind=8) ::alpha2
    real(kind=8) ::beta1
    real(kind=8) ::u1
    real(kind=8) ::u2
    real(kind=8) ,dimension(2) ::Riemann

    alpha2=(a*ul(1)+rho0*ul(2))/(2*a*rho0)
    beta1=(a*ur(1)-rho0*ur(2))/(2*a*rho0)

    u1=beta1*rho0+alpha2*rho0
    u2=beta1*(-a)+alpha2*a
    Riemann(1)=rho0*u2
    Riemann(2)=a*a/(rho0)*u1
end function Riemann
end module solver

program main
    use datas
    use solver
    integer :: i,itmax,t_F,bc,flg
    real :: xmin,xmax
    integer :: m,n

    cfl = 0.7

    !geometry
    bc=2
    n =1000
    flg=1
    itmax=100
```

```
    m=1
    xmax=0.5
    xmin=-0.5
    t_F=1;bc=2;flg=1
    dx=(xmax-xmin)/(n-1)


    allocate(x(n))
    allocate(u0(n+2*m,2))
    allocate(u1(n+2*m,2))

    ! I.C.
open(unit=10,file="out.dat")
if (flg==1) then

     do i=1,n

        x(i)=-0.5+(i-1)*dx
      if (x(i)<-0.1) then
        u0(m+i,1)=0
        u0(m+i,2)=0
        elseif (x(i)<=0.1) then
            u0(m+i,1)=0
            u0(m+i,1)=0.10  !! wanring!
            u0(m+i,2)=0

        else
            u0(m+i,1)=0
            u0(m+i,2)=0
        endif
        !write(10,*) i, u0(m+i,1),u0(m+i,2)
        !write(*,*) "i,u0(m+i,1),u1(m+i,2)", i, u0(m+i,1),u0(m+i,2)
    end do
endif

! B.C.
if (bc==2) then
do i=1,m

    u0(i,1)=u0(2*m+1-i,1)
    u0(i,2)=u0(2*m+1-i,2)

    u0(m+n+i,1)= u0(m+n+1-i,1);
    u0(m+n+i,2)=-u0(m+n+1-i,2);

    enddo
endif

t=0;
dt=0.001
lambda=dt/dx    ! for constant dt,put it outside the loop

do while(t+dt<=t_F .or. it<=itmax)
```

```
      t=t+dt

    do i=1,n
        u1(m+i,:)=u0(m+i,:)-lambda*(Riemann(u0(m+i,:),u0(m+i+1,:))-Rieman
        n(u0(m+i-1,:),u0(m+i,:)))

        !write(10,*) i, u1(m+i,1),u1(m+i,2)

    enddo

if (bc==2) then
do i=1,m

    u1(i,1)=u1(2*m+1-i,1)
    u1(i,2)=u1(2*m+1-i,2)

    u1(m+n+i,1)= u1(m+n+1-i,1);
    u1(m+n+i,2)=-u1(m+n+1-i,2);

    enddo
endif

u0(:,:)=u1(:,:)

enddo

end program main


!subroutine timestep()
!    use datas
!    integer :: i
!    real(kind=8) :: umax
!
!    umax = 0.0
!    do i=1,num
!        umax=max(umax,w(i))
!    end do
!    dt = cfl*dx/umax
!end subroutine timestep
!
!subroutine calc_flux()
!    use datas
!    integer :: i
!
!    !boundary
!    flux(1) = 0.5*w(i)**2
!    flux(num+1) = 0.5*w(num)**2
!
!    !inner
!    do i=2,num
!        if (w(i-1)>=w(i)) then !form a shock
!            if (0<0.5*(w(i-1)+w(i))) then
!                flux(i) = 0.5*w(i-1)**2
```

```
!            else
!                flux(i) = 0.5*w(i)**2
!            end if
!        else !form a rarefaction wave
!            if (0<w(i-1)) then
!                flux(i) = 0.5*w(i-1)**2
!            else if (0>w(i)) then
!                flux(i) = 0.5*w(i)**2
!            else
!                flux(i) = 0.0
!            end if
!        end if
!    end do
!end subroutine calc_flux
!
!subroutine update()
!    use datas
!    integer :: i
!
!    do i=1,num
!        w(i) = w(i)+(flux(i)-flux(i+1))*dt/dx
!    end do
!end subroutine update
!
!subroutine writeout()
!    use datas
!    integer :: i
!
!    open(unit=10,file="out.dat")
!    do i=1,num
!        xpos = (-1.0+dx/2.0)+(4.0-dx)*(i-1.0)/(num-1.0+SMV)
!        write(10,*) xpos,w(i)
!    end do
!end subroutine writeout
```