Project 3

CHUNG,Chak Pong

20015116

Problem

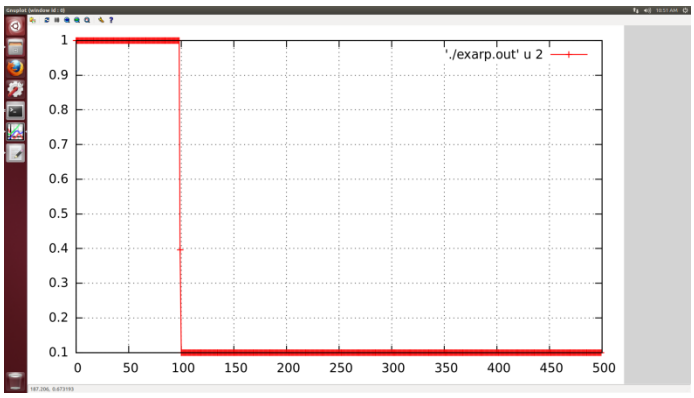
One dimentional shallow water equation with exact Riemann Solver

with Dam-Break Initial State UL=UR=0,HL=1,HR=0.1

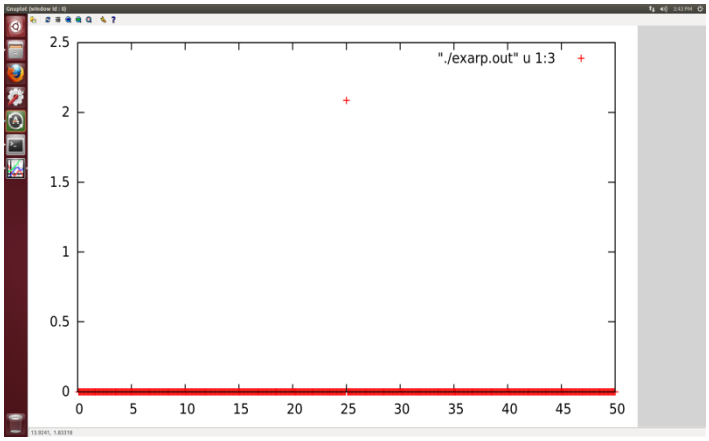
$$\begin{bmatrix} h \\ hu \end{bmatrix}_t + \begin{bmatrix} uh \\ hu^2 + \frac{1}{2}gh^2 \end{bmatrix}_x = 0.$$

Initial Condition

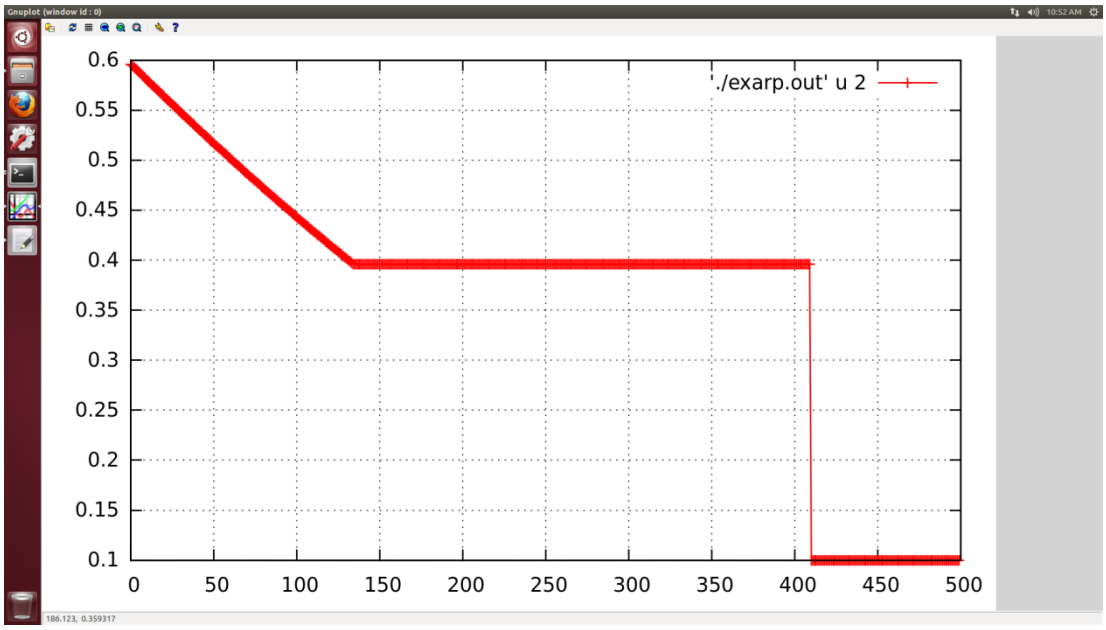
Initial H



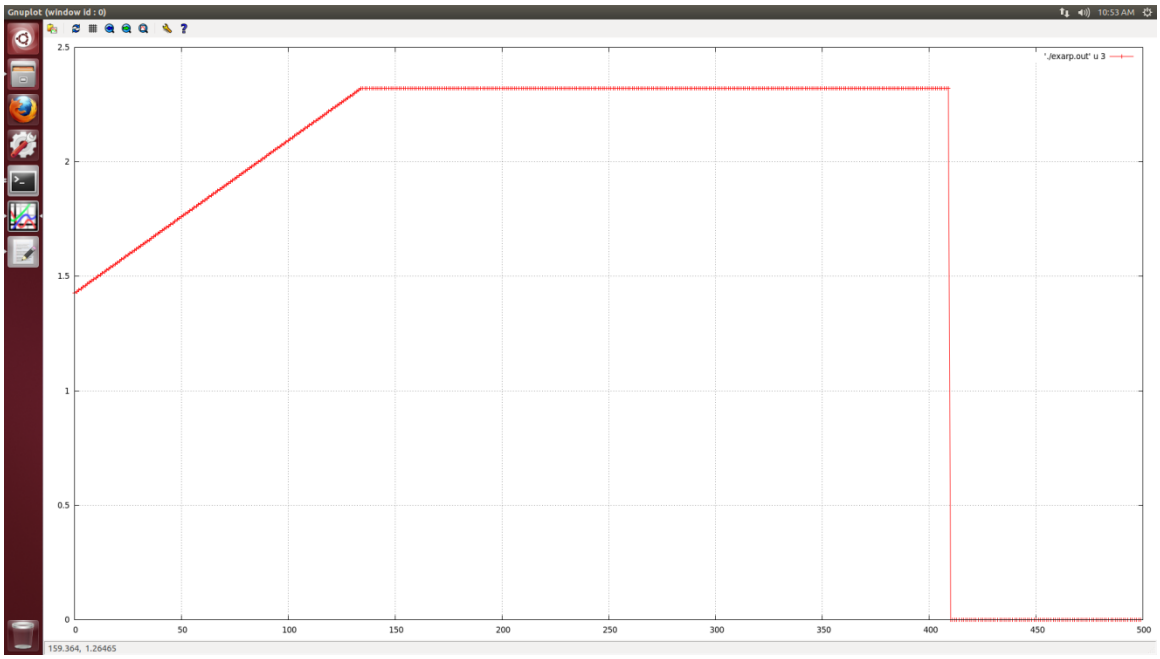
Initial HU



H at 10s



HU at 10s



Key steps in determing the Flux at star region

- Find $h_\star$ such that $\Phi(h_\star) = \Phi_r(h_\star) - \Phi_\ell(h_\star) = 0,$ where

$$\Phi_\ell(h_\star) := \begin{cases} u_\ell - (h_\star - h_\ell)\sqrt{g\left(\frac{1}{2h_\star} + \frac{1}{2h_\ell}\right)} & \text{if} \quad h_\star > h_\ell \\ u_\ell + 2\left(\sqrt{gh_\ell} - \sqrt{gh_\star}\right) & \text{if} \quad h_\star \le h_\ell \end{cases}$$

$$\Phi_r(h_\star) := \begin{cases} u_r + (h_\star - h_r)\sqrt{g\left(\frac{1}{2h_\star} + \frac{1}{2h_r}\right)} & \text{if} \quad h_\star > h_r \\ u_r - 2\left(\sqrt{gh_r} - \sqrt{gh_\star}\right) & \text{if} \quad h_\star \le h_r \end{cases}$$

- Newton iteration: $h_\star^{k+1} = h_\star^k - \frac{\Phi(h_\star^k)}{\Phi'(h_\star^k)}$

u* is determined by

US = 0.5*(UL + UR) + 0.5*(FR - FL)

A more detailed derivation can be found in Toro ‘s book *Shock-Capturing Methods for Free-Surface Shallow Flows*, section 5.3.,from equation 5.5 to equation 5.12

The code used is attached.