And finally we have

$$\begin{aligned}
&\int_{\|\mathbf{m}\|=1} [\mathbf{m} \times \dot{\mathbf{m}}]\, Y_n^{k*} \mathcal{R} Y_l^m \, d\mathbf{m} \\
&= i\frac{1}{4}\sqrt{\frac{8\pi}{15}}\,\mu \Big\{ [c_{l,m,1}|l,m+1;n,k;2,-1\rangle + a\, c_{l,m,1}|l,m+1;n,k;2,1\rangle \\
&\qquad -a\, c_{l,m,2}|l,m-1;n,k;2,-1\rangle - c_{l,m,2}|l,m-1;n,k;2,1\rangle] \\
&\qquad -2\,m\left[\sqrt{\frac{10}{3}}|l,m;n,k;0,0\rangle - \sqrt{\frac{2}{3}}|l,m;n,k;2,0\rangle \right.\\
&\qquad \left. -a|l,m;n,k;2,2\rangle - a|l,m;n,k;2,-2\rangle\right]\Big\}
\end{aligned} \tag{60}$$

# Appendix D

$$L_+ = e^{i\phi}\left(\frac{\partial}{\partial\theta} + i\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\phi}\right) \tag{61}$$

$$L_- = e^{-i\phi}\left(-\frac{\partial}{\partial\theta} + i\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\phi}\right) \tag{62}$$

$$L_z = \frac{1}{i}\frac{\partial}{\partial\phi} \tag{63}$$

$$\begin{aligned}
L_x &= \frac{1}{2}(L_+ + L_-) \\
&= i\left(\cos\phi\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\phi} + \sin\phi\frac{\partial}{\partial\theta}\right)
\end{aligned} \tag{64}$$

$$\begin{aligned}
L_y &= \frac{1}{2i}(L_+ - L_-) \\
&= -i\left(-\sin\phi\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\phi} + \cos\phi\frac{\partial}{\partial\theta}\right)
\end{aligned} \tag{65}$$

$$\mathcal{R}_x = i\,L_x, \qquad \mathcal{R}_y = i\,L_y \tag{66}$$

$$\left\{\begin{aligned}
\mathcal{R}_x &= -\frac{\cos\theta}{\sin\theta}\cos\phi\frac{\partial}{\partial\phi} - \sin\phi\frac{\partial}{\partial\theta} && (67)\\
\mathcal{R}_y &= -\frac{\cos\theta}{\sin\theta}\sin\phi\frac{\partial}{\partial\phi} + \cos\phi\frac{\partial}{\partial\theta} && (68)\\
\mathcal{R}_z &= \frac{\partial}{\partial\phi} && (69)
\end{aligned}\right.$$