Under the harmonic expansion (2) of the function f,

$$\begin{aligned}\mathbf{mm}:\langle\mathbf{mm}\rangle &= \frac{4\pi}{3}a_{0,0}\,Y_0^0+\frac{8\pi}{15}a_{2,-2}\,Y_2^{-2}+\frac{8\pi}{15}a_{2,-1}\,Y_1^{-1}\\ &\quad+\frac{8\pi}{15}a_{2,0}\,Y_2^0+\frac{8\pi}{15}a_{2,1}\,Y_2^1+\frac{8\pi}{15}a_{2,2}\,Y_2^2\\ &=: e_{0,0}Y_0^0+\sum_{p=-2}^{2}e_{2,p}Y_2^p \end{aligned}\tag{32}$$

where,

$$\begin{array}{ll} e_{0,0}=\frac{4\pi}{3}a_{0,0}, & e_{2,-2}=\frac{8\pi}{15}a_{2,-2}\\ e_{2,-1}=\frac{8\pi}{15}a_{2,-1}, & e_{2,0}=\frac{8\pi}{15}a_{2,0}\\ e_{2,1}=\frac{8\pi}{15}a_{2,1}, & e_{2,2}=\frac{8\pi}{15}a_{2,2}\end{array}\tag{33}$$

The expression of the potential V, without external effects, is

$$V=-\frac{3}{2}N\,kT\,\mathbf{mm}:\langle\mathbf{mm}\rangle\tag{34}$$

It can be expanded in the spherical harmonics:

$$V=-\frac{3}{2}N\,kT\left(e_{0,0}Y_0^0+\sum_{p=-2}^{2}e_{2,p}Y_2^p\right).\tag{35}$$

Appendix B

The rotational gradient operator $\mathcal{R}$ satisfies

$$\mathcal{R}_\alpha=i\,L_\alpha,\qquad \alpha=x,y,z\tag{36}$$

where, L_α are angular momentum operators. Define

$$L_\pm=L_x\pm L_y\,,\tag{37}$$

Then

$$\boxed{\begin{aligned} L_+Y_l^m &= c_{l,m,1}\,Y_l^{m+1} &(38)\\ L_-Y_l^m &= c_{l,m,2}\,Y_l^{m-1} &(39)\\ L_zY_l^m &= m\,Y_l^m &(40)\end{aligned}}$$

with

$$\boxed{\begin{aligned} c_{l,m,1} &= \sqrt{(l-m)(l+m+1)} &(41)\\ c_{l,m,2} &= \sqrt{(l+m)(l-m+1)} &(42)\end{aligned}}$$