$$\mathbf{m}\times\dot{\mathbf{m}} = \begin{pmatrix} \frac{1}{4}\sin 2\theta(2\,a\,p_1\sin\phi - (p_2(a-1)+p_3(a+1))\cos\phi) \\ \frac{1}{4}\sin 2\theta(2\,a\,p_1\cos\phi + (p_2(a+1)+p_3(a-1))\sin\phi) \\ \frac{1}{2}\sin^2\theta(-2\,a\,p_1\sin 2\phi - (p_2-p_3) + a(p_2+p_3)\cos 2\phi) \end{pmatrix} \tag{83}$$

And then

$$[\mathbf{m}\times\dot{\mathbf{m}}]_x = i\frac{a}{2}p_1\sqrt{\frac{8\pi}{15}}(Y_2^{-1}+Y_2^1) - \frac{1}{4}(p_2(a-1)+p_3(a+1))\sqrt{\frac{8\pi}{15}}(Y_2^{-1}-Y_2^1) \tag{84}$$

$$[\mathbf{m}\times\dot{\mathbf{m}}]_y = \frac{a}{2}p_1\sqrt{\frac{8\pi}{15}}(Y_2^{-1}+Y_2^1) + i\frac{1}{4}(p_2(a+1)+p_3(a-1))\sqrt{\frac{8\pi}{15}}(Y_2^{-1}+Y_2^1) \tag{85}$$

$$\begin{aligned}[\mathbf{m}\times\dot{\mathbf{m}}]_z \triangleq\; & -iap_1\sqrt{\frac{8\pi}{15}}(Y_2^{-2}-Y_2^2) + \frac{a}{2}(p_2+p_3)\sqrt{\frac{8\pi}{15}}(Y_2^{-2}+Y_2^2) \\ & -\frac{1}{3}(p_2-p_3)(\sqrt{4\pi}Y_0^0 - \sqrt{\frac{4\pi}{5}}Y_2^0)\end{aligned} \tag{86}$$

Since we know(from last appendix):

$$\boxed{\mathcal{R} = \begin{pmatrix}\mathcal{R}_x\\ \mathcal{R}_y\\ \mathcal{R}_z\end{pmatrix} = \begin{pmatrix} iL_x\\ iL_y\\ iL_z\end{pmatrix} = \begin{pmatrix}\frac{i}{2}(L_+ + L_-)\\ \frac{1}{2}(L_+ - L_-)\\ iL_z\end{pmatrix}} \tag{87}$$

Finally we got the expansion

$$\begin{aligned}&[\mathbf{m}\times\dot{\mathbf{m}}]\cdot(\mathcal{R}Y_l^m)\\ =&c_{l,m,1}Y_l^{m+1}Y_2^{-1}\frac{i}{4}\sqrt{\frac{8\pi}{15}}(p_2-p_3) - c_{l,m,1}Y_l^{m+1}Y_2^1\frac{a}{2}\sqrt{\frac{8\pi}{15}}(p_1-\frac{i}{2}(p_2+p_3))\\ &- c_{l,m,2}Y_l^{m-1}Y_2^{-1}\frac{a}{2}\sqrt{\frac{8\pi}{15}}(p_1+\frac{i}{2}(p_2+p_3)) - c_{l,m,2}Y_l^{m-1}Y_2^1\frac{i}{2}\sqrt{\frac{8\pi}{15}}(p_2-p_3)\\ &+ imY_l^m\sqrt{\frac{8\pi}{15}}\Big[\frac{a}{2}(p_2+p_3)(Y_2^{-2}+Y_2^2) - ap_1i(Y_2^{-2}-Y_2^2)\\ &-(p_2-p_3)\sqrt{\frac{5}{6}}Y_0^0 + (p_2-p_3)\sqrt{\frac{1}{6}}Y_2^0\Big]\end{aligned} \tag{88}$$

RY_j^p