

$$\sum_{l=0}^{\infty} \sum_{m=-l}^l Y_{l1}^{m1} Y_{l2}^{m2} = \sum_{l=0}^{\infty} \sum_{m=-l}^l a_l^m Y_l^m$$

$$\int_0^{2\pi} \int_0^\pi Y_{l1}^{m1} Y_{l2}^{m2} \bar{Y}_{l'}^{m'} \sin\theta d\theta d\phi = \int_0^{2\pi} \int_0^\pi \sum_{l=0}^{\infty} \sum_{m=-l}^l a_l^m Y_l^m \bar{Y}_{l'}^{m'} \sin\theta d\theta d\phi$$

$$\int_0^{2\pi} \int_0^\pi Y_{l1}^{m1} Y_{l2}^{m2} (-1)^{m'} \bar{Y}_{l'}^{m'} \sin\theta d\theta d\phi = a_{l'}^{m'} \quad \text{--- using (7)}$$

$$\Rightarrow (-1)^{m'} \left[\frac{(2l_1+1)(2l_2+1)(2l'+1)}{4\pi} \right]^{1/2} \begin{pmatrix} l_1 l_2 l' \\ 0 0 0 \end{pmatrix} \begin{pmatrix} l_1 l_2 l' \\ m_1 m_2 -m' \end{pmatrix} = a_{l'}^{m'} \quad \text{--- using (13)} \\ = 0$$

$$Y_{l1}^{m1} Y_{l2}^{m2} Y_{l3}^{m3} = \sum_{l^*=0}^{\infty} \sum_{m^*=-l^*}^{l^*} b_{l^*}^{m^*} Y_{l^*}^{m^*}$$

$$\sum_{l=0}^{\infty} \sum_{m=-l}^l a_l^m Y_l^m Y_{l3}^{m3} = \sum_{l^*=0}^{\infty} \sum_{m^*=-l^*}^{l^*} b_{l^*}^{m^*} Y_{l^*}^{m^*}, \quad \text{multiplied by both sides by } \bar{Y}_{l'}^{m'}, \text{ then } \int \int \sin\theta d\theta d\phi$$

$$\int_0^{2\pi} \int_0^\pi \sum_{l=0}^{\infty} \sum_{m=-l}^l a_l^m Y_l^m Y_{l3}^{m3} (-1)^{m'} \bar{Y}_{l'}^{m'} \sin\theta d\theta d\phi = b_{l'}^{m'} \quad \text{--- using (7)}$$

$$\sum_{l=0}^{\infty} \sum_{m=-l}^l a_l^m (-1)^{m'} \left[\frac{(2l+1)(2l_3+1)(2l'+1)}{4\pi} \right]^{1/2} \begin{pmatrix} l l_3 l' \\ 0 0 0 \end{pmatrix} \begin{pmatrix} l l_3 l' \\ m m_3 -m' \end{pmatrix} = b_{l'}^{m'} \quad \text{--- using (13)}$$

where a_l^m is shown in the first part ①