Lecture 11

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CSCI 2270 Data Structures

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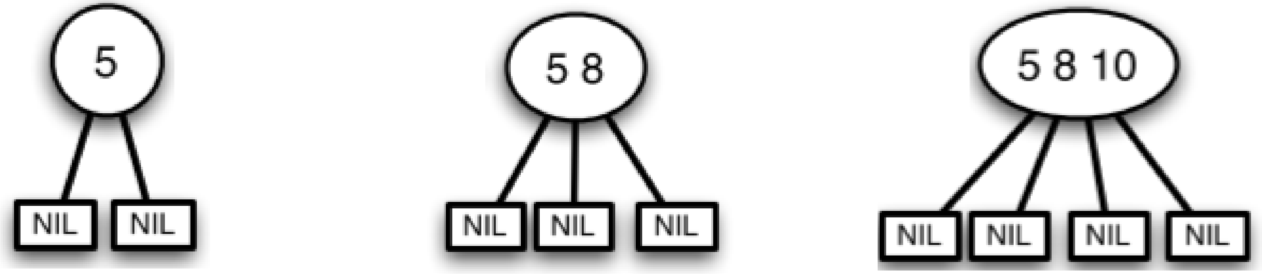
Slides generated from textbook & Dr. Cochran's (WSUV) 234/RB-Tree Lecture

Red Black Trees

- A valid red-black tree holds to the following properties
  1. A node is either red or black
  2. The root node is black
  3. Every leaf (NULL) node is black
  4. If a node is red, then both of its children must be black
  5. For each node in the tree, all paths from that node to the leaf nodes contain the same number of black nodes

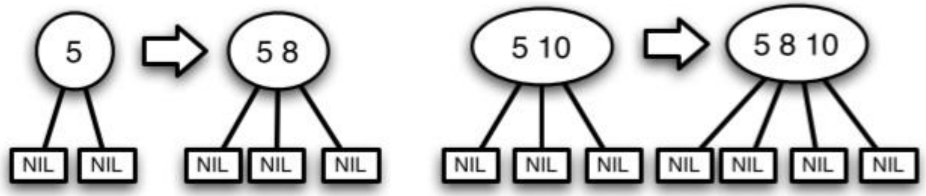
234 Trees

- A 234 tree is named after each type of node it contains
  - 2-node has 2 children
  - 3-node has 3 children
  - 4-node has 4 children

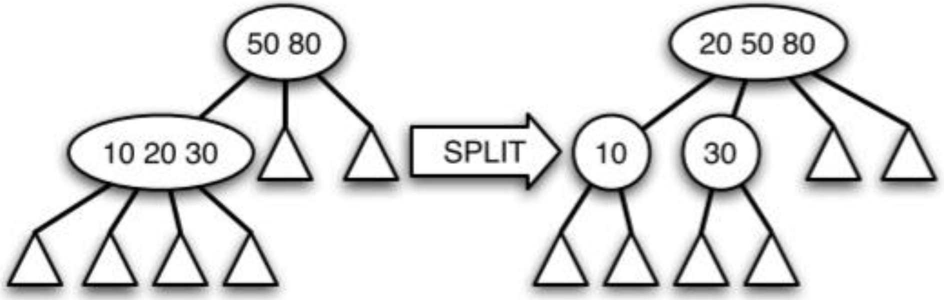


234 Trees

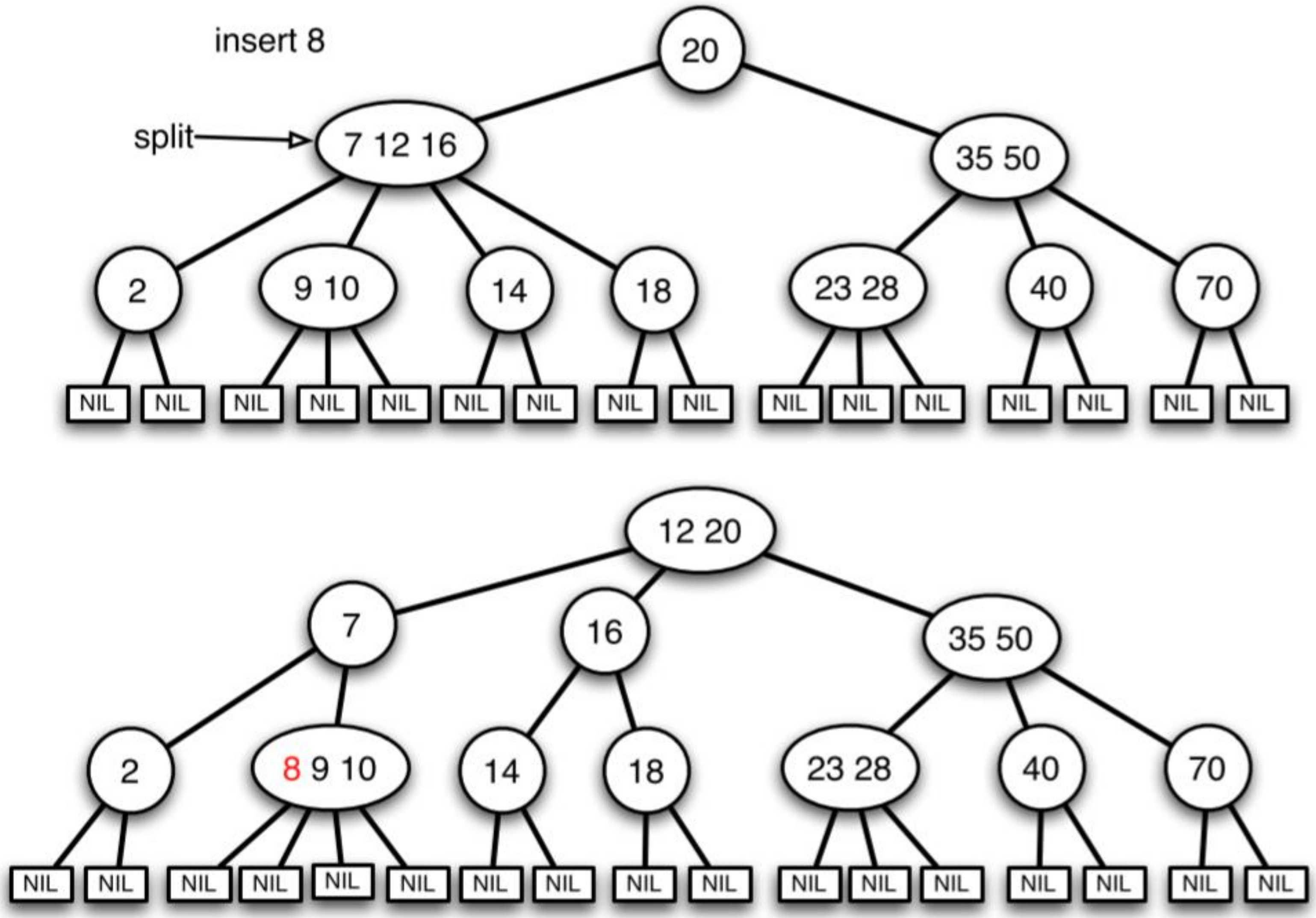
- To insert:
  - Search and insert into lowest internal node
  - 2-nodes become 3-nodes, 3-nodes become 4-nodes



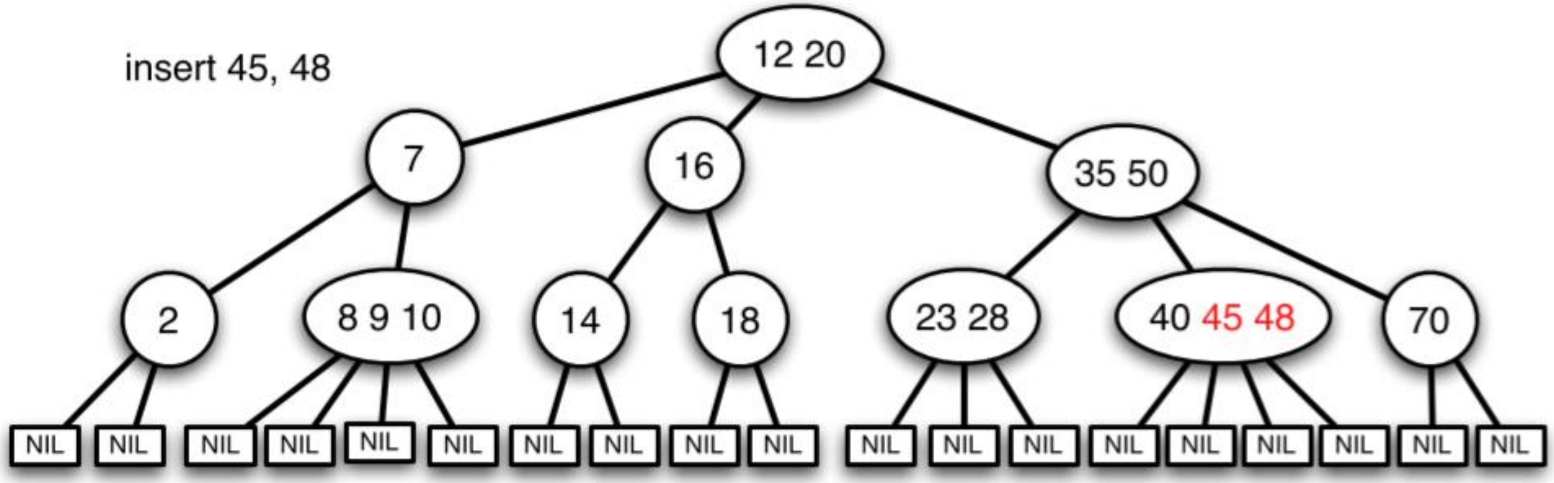
  - As we descend the tree, we split 4-nodes into two 2-nodes and move the middle element to the parent

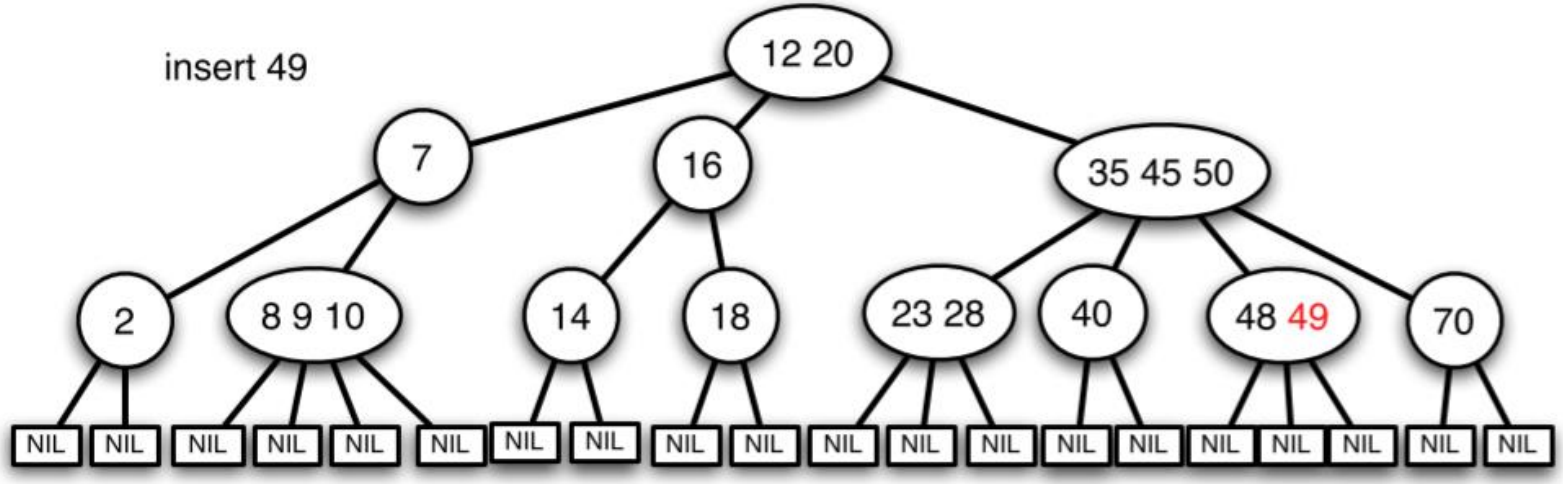


234 Trees: Example

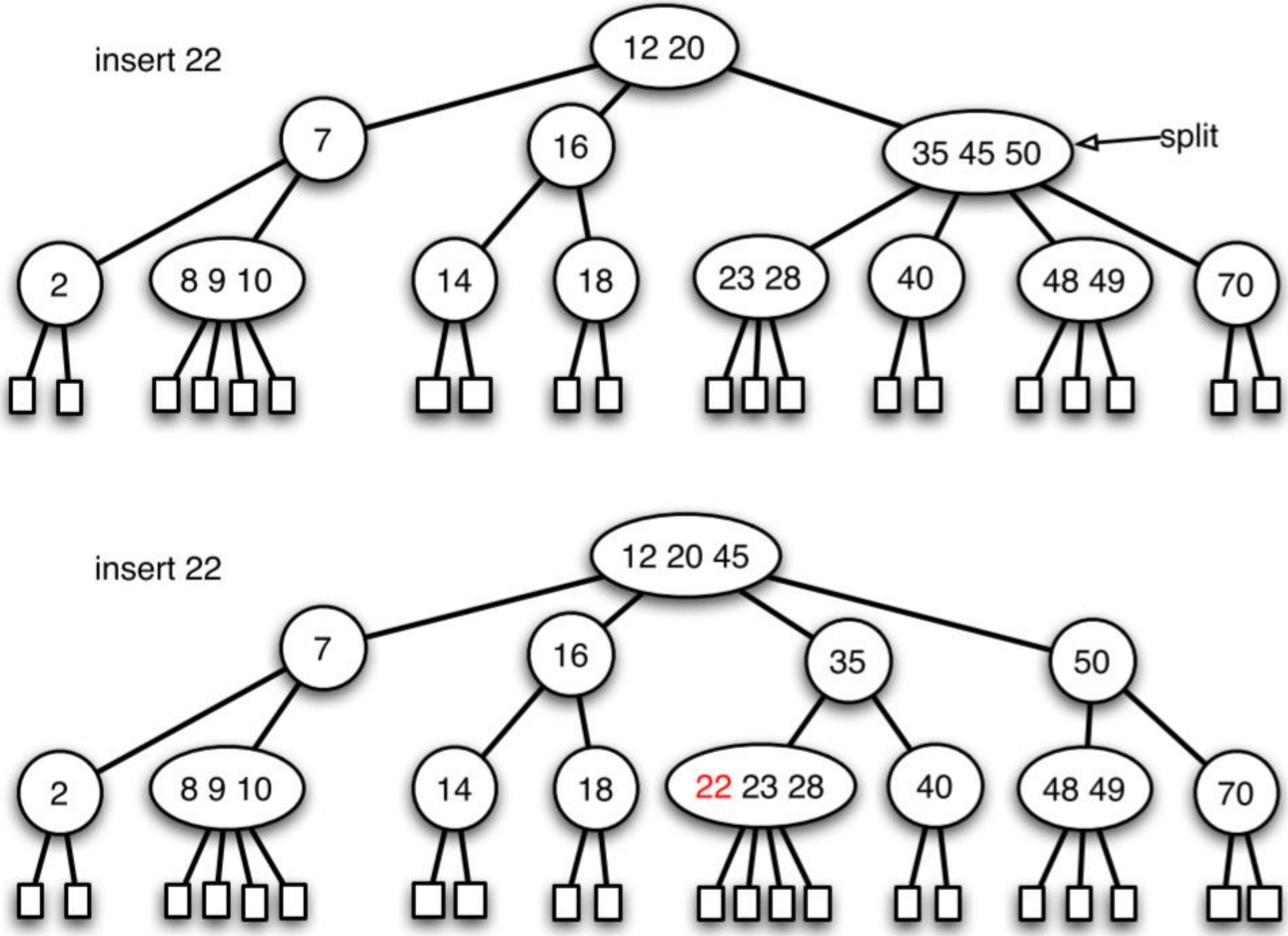


234 Trees: Example

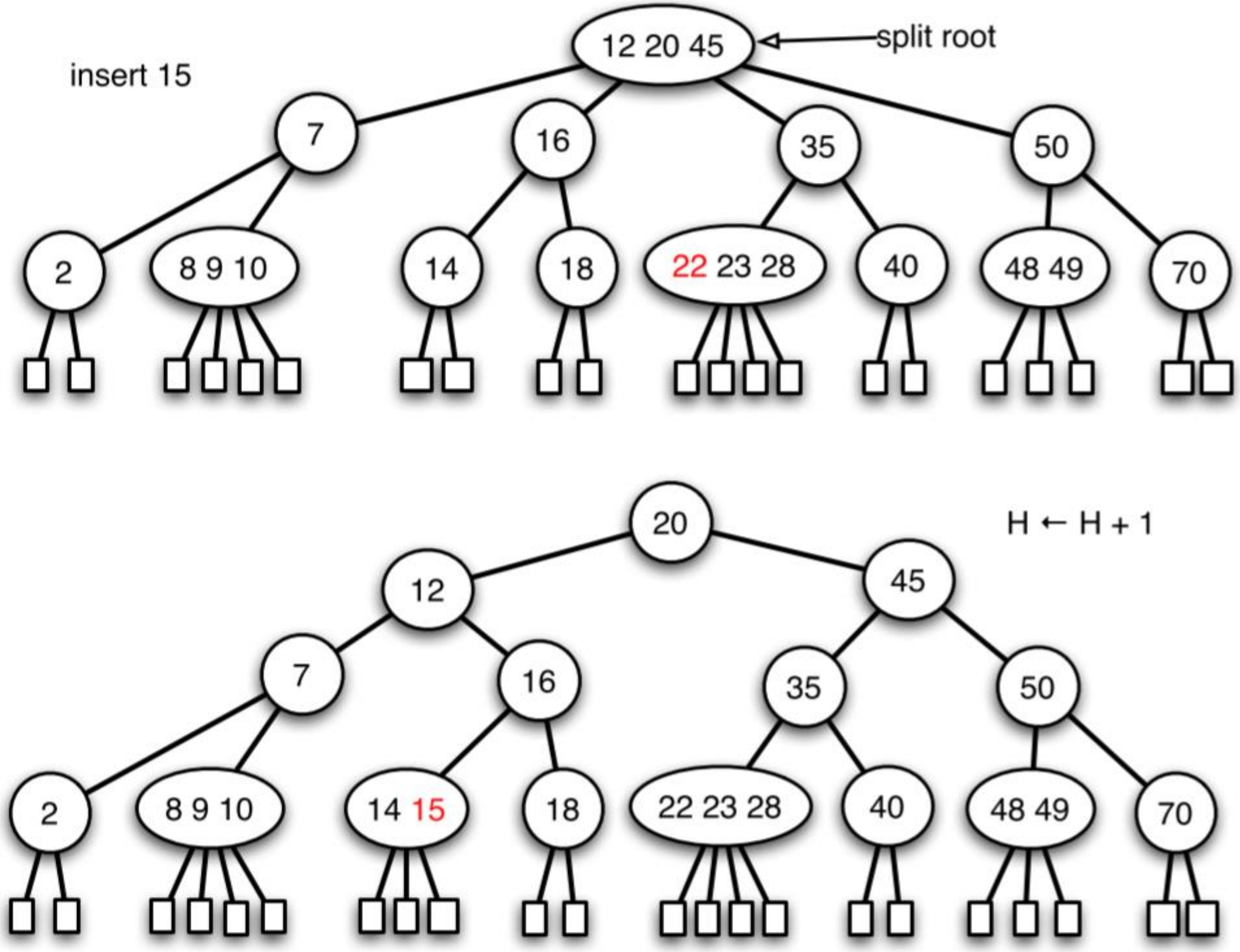




234 Trees: Example



234 Trees: Example



234 Trees: Analysis

- Pros
  - Balanced
    - All subtrees have the same height!
  - O(log(N)) search, insert, (delete?)
- Cons
  - Complex node structure

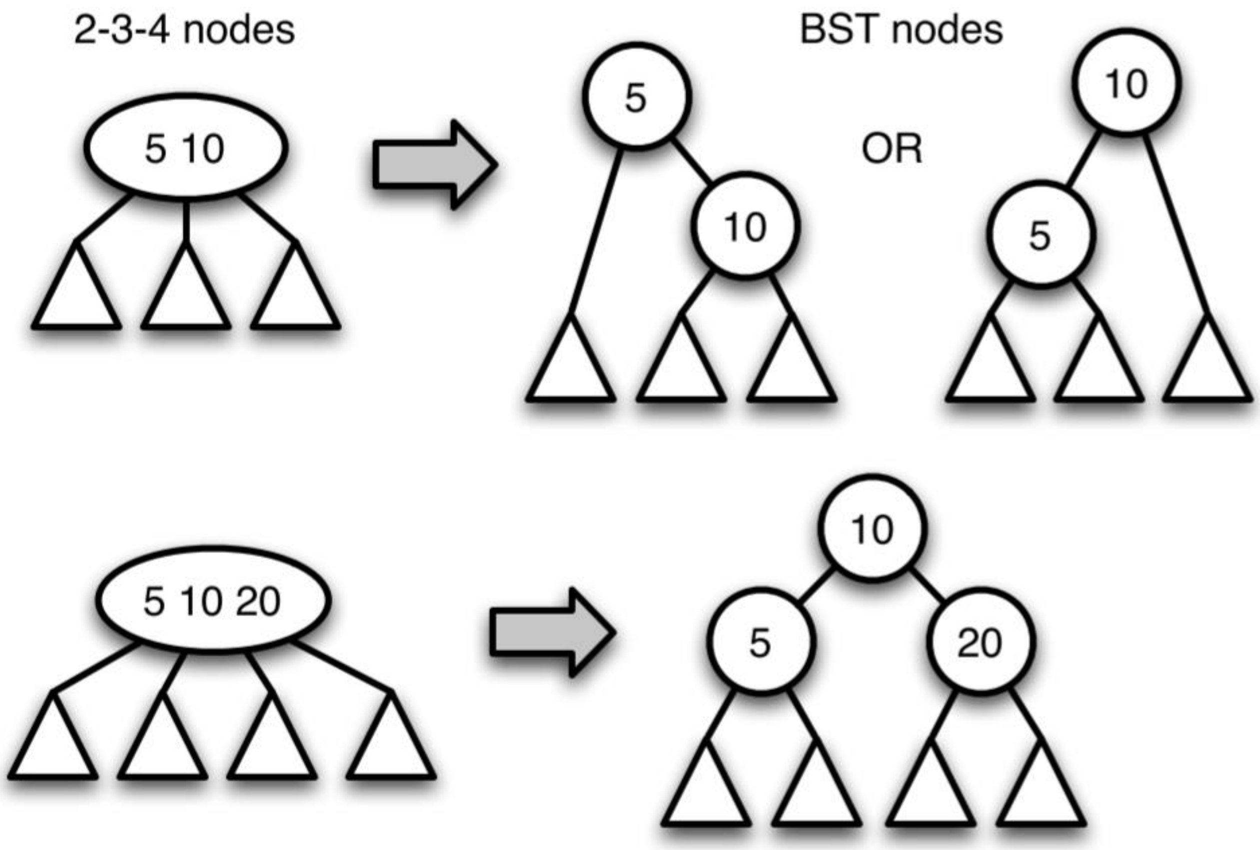
```
class Node {  // yuck!
  int n;
  K[] key;
  V[] val;
  Node[] children;
}
```

Red Black Trees

- 234 trees are isometrically the same as red black trees
- We can change the way we’ve looked at the previous trees to make them red black trees

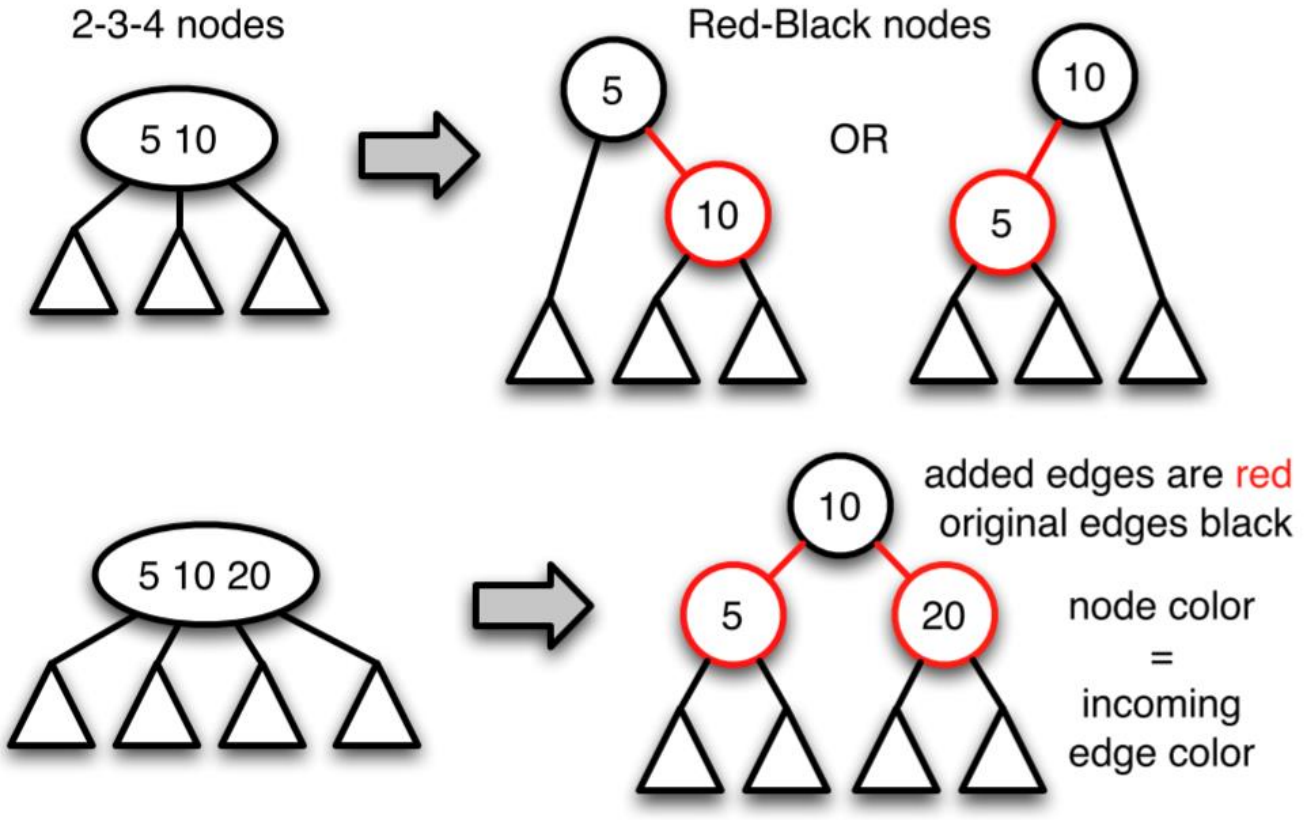
Red Black Trees

- Representing 2-3-4 nodes with BST nodes



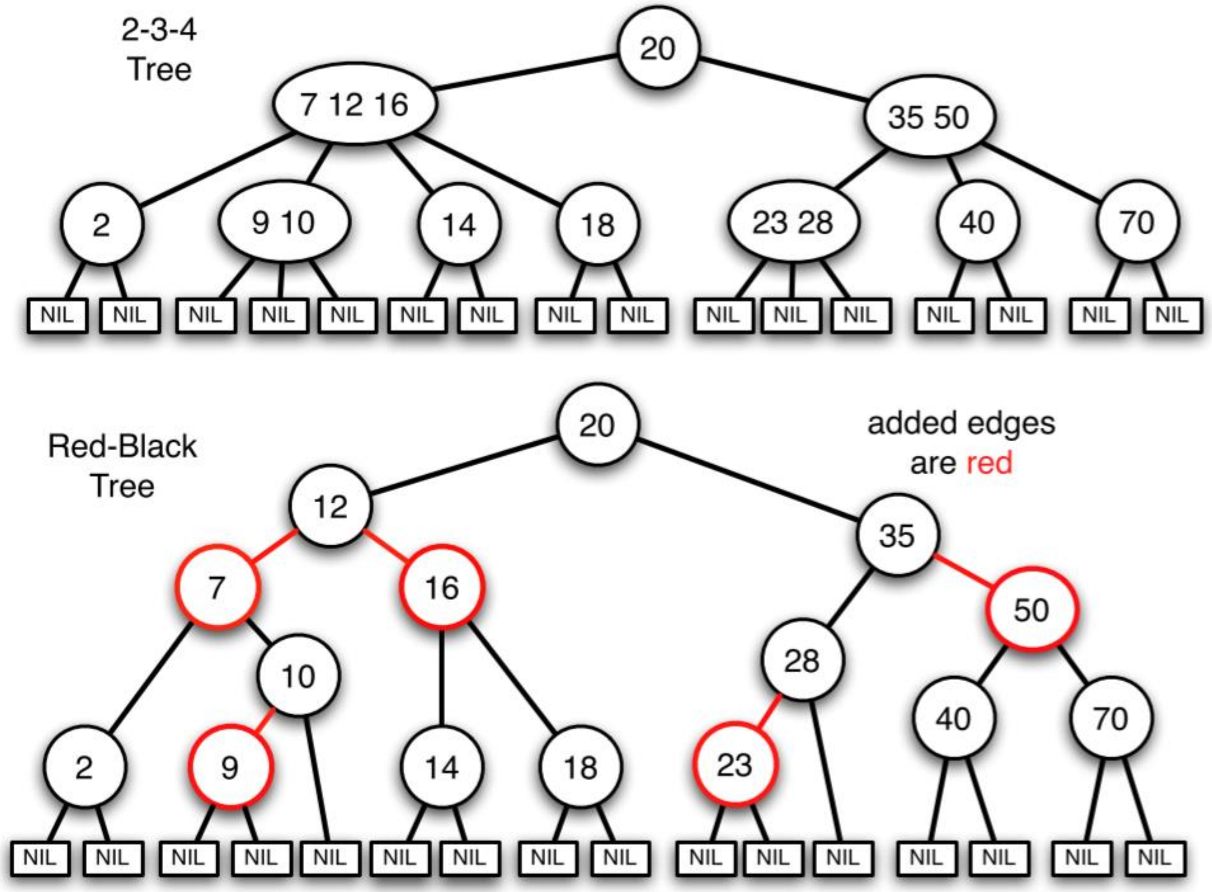
Red Black Trees

- Representing 2-3-4 nodes with Red-Black nodes



Red Black Trees

- Representing 2-3-4 Tree with Red-Black Tree

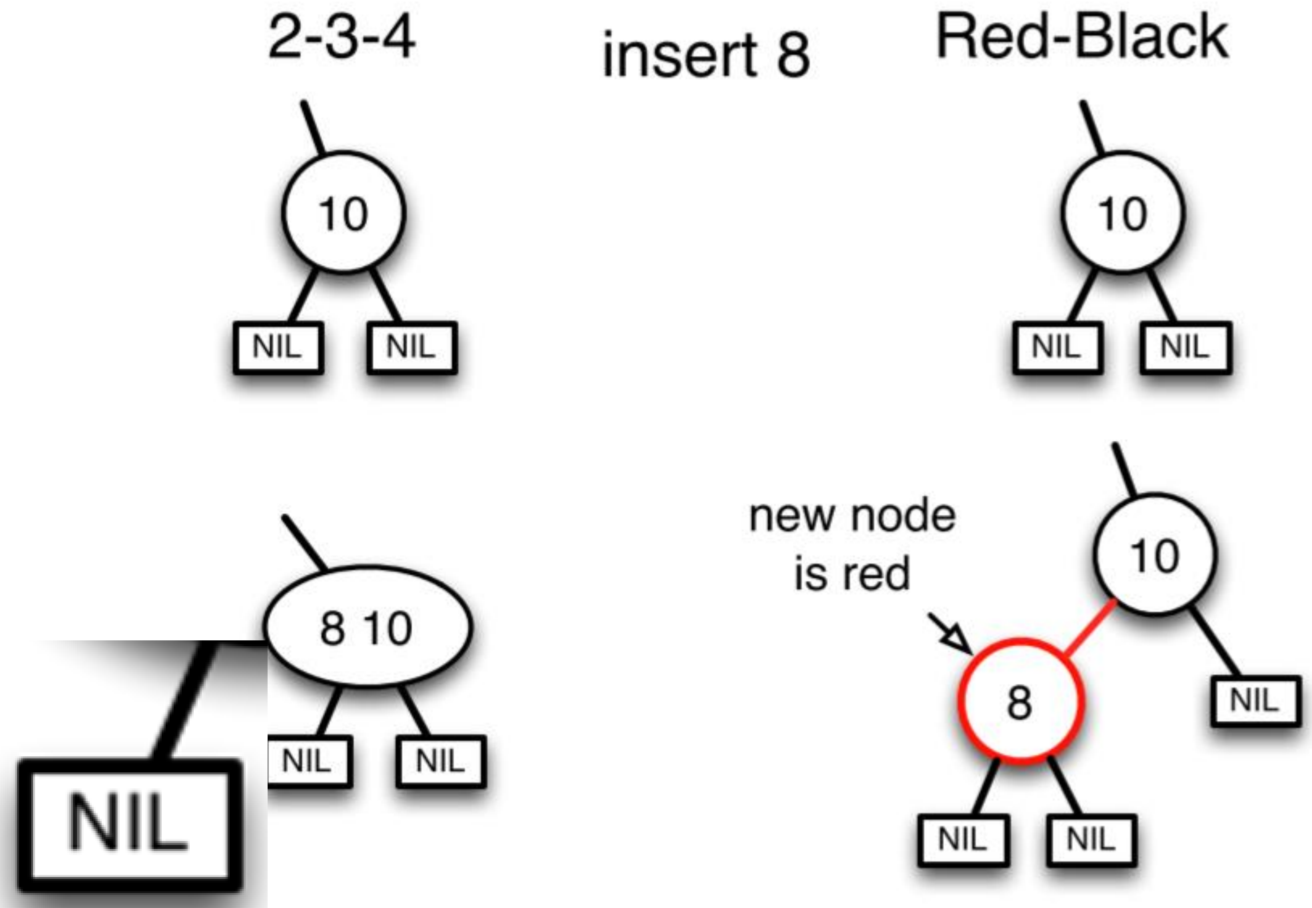


Red Black Trees

- A valid red-black tree holds to the following properties
    1. Edges (nodes) is either red or black
    2. The root node is black
    3. Every leaf (NULL) node is black
    4. If a node is red, then both of its children must be black
        1. There should never be two consecutive red edges in a path
    5. For each node in the tree, all paths from that node to the leaf nodes contain the same number of black nodes
        1. This is known as B = “black height”

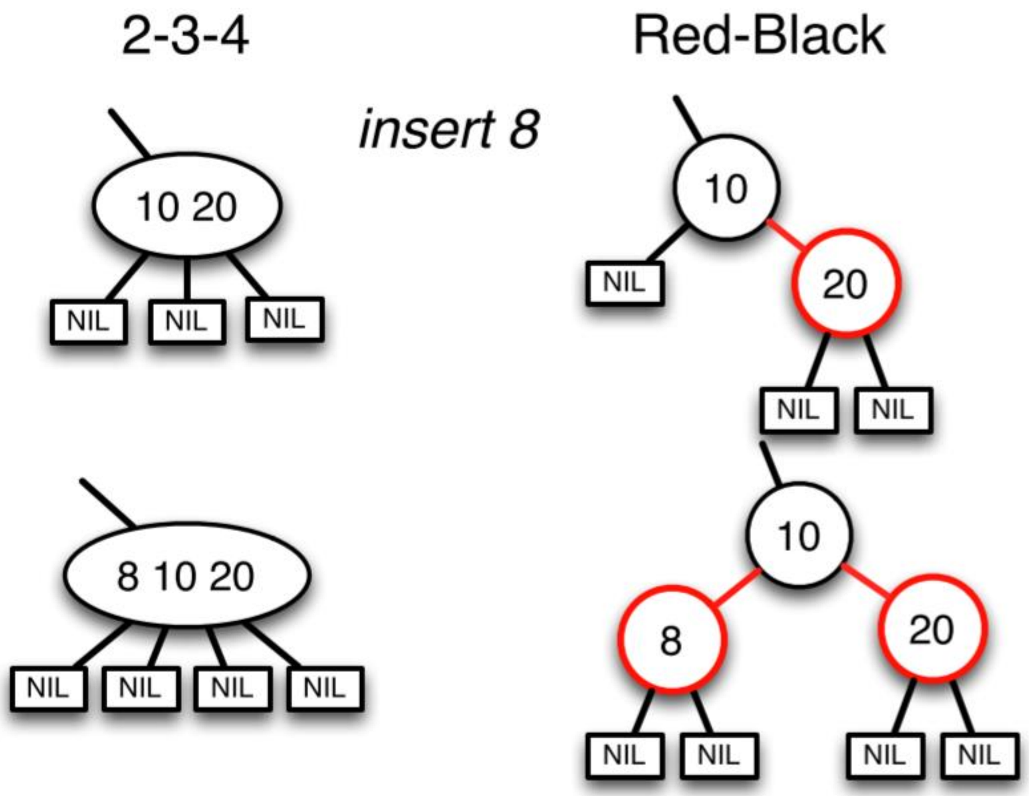
Red Black Trees

- Inserting into a 2-Node



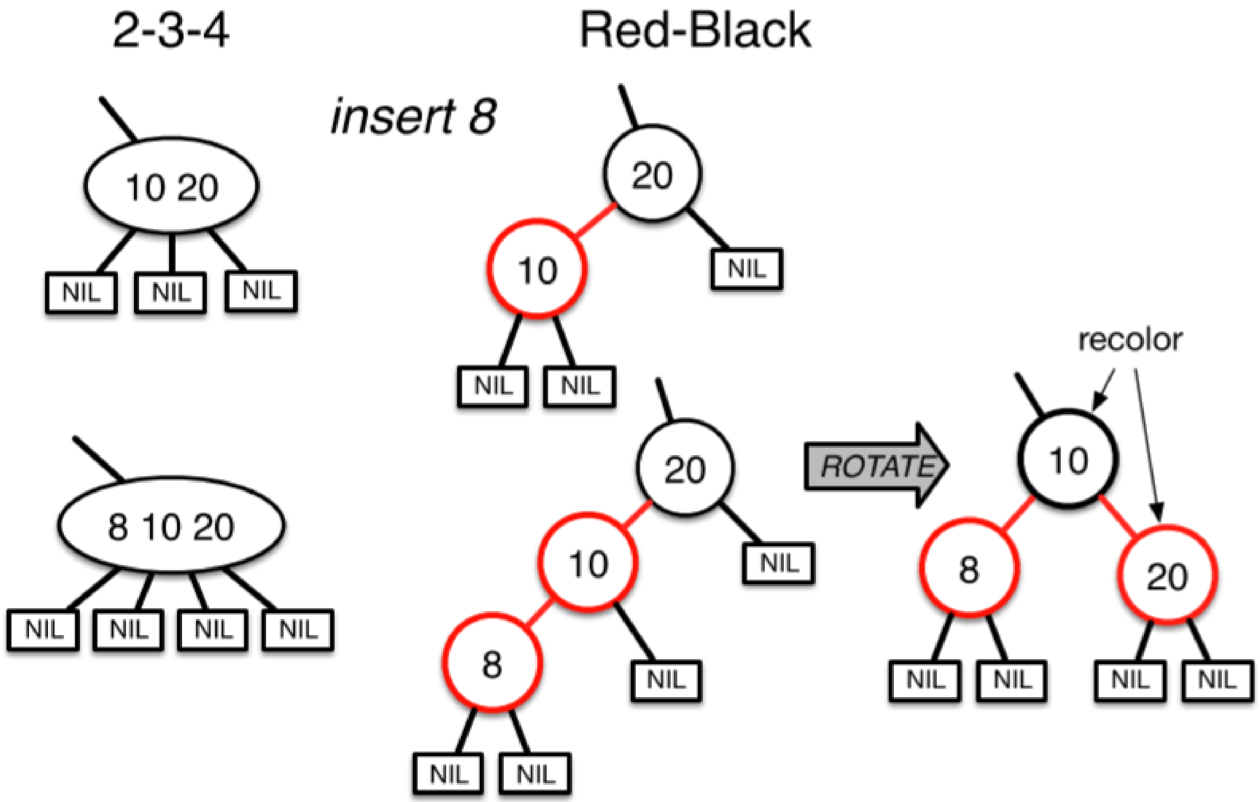
Red Black Trees

- Inserting LEFT key into 3-Node (Case 1)



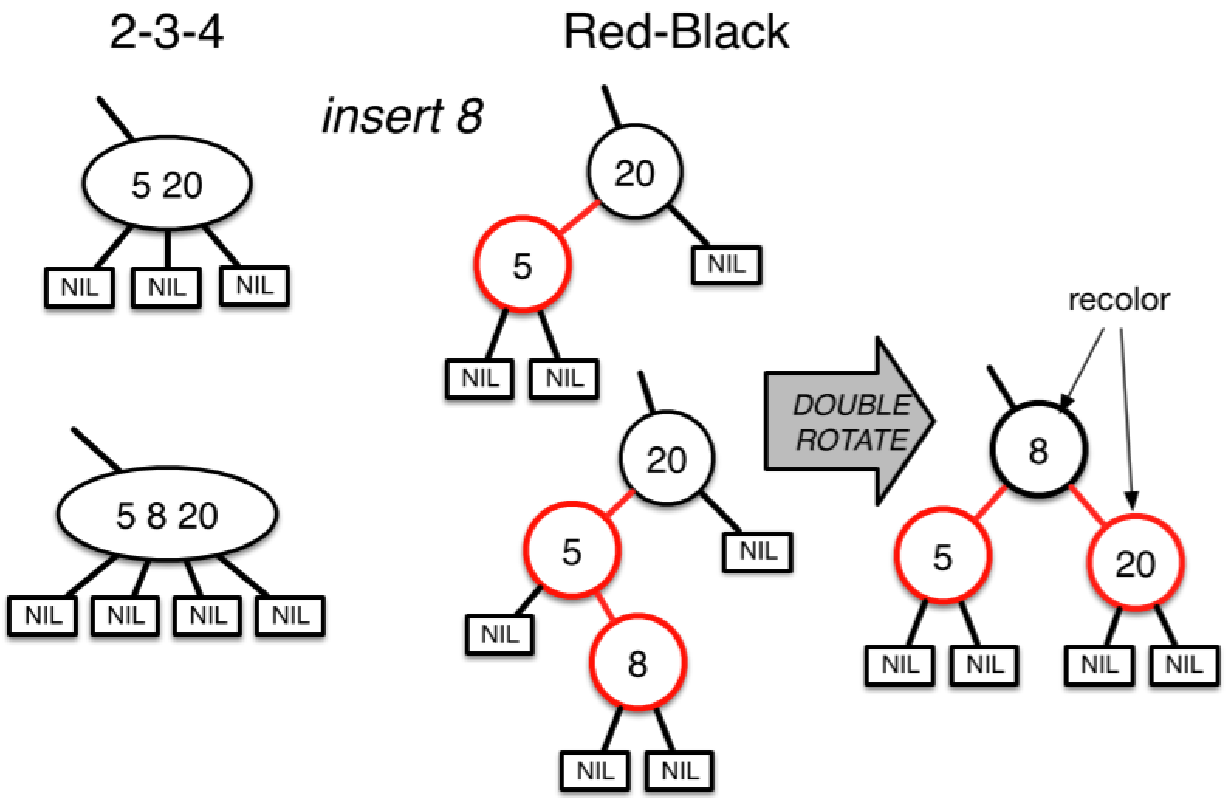
Red Black Trees

- Inserting LEFT key into 3-Node (Case 2)



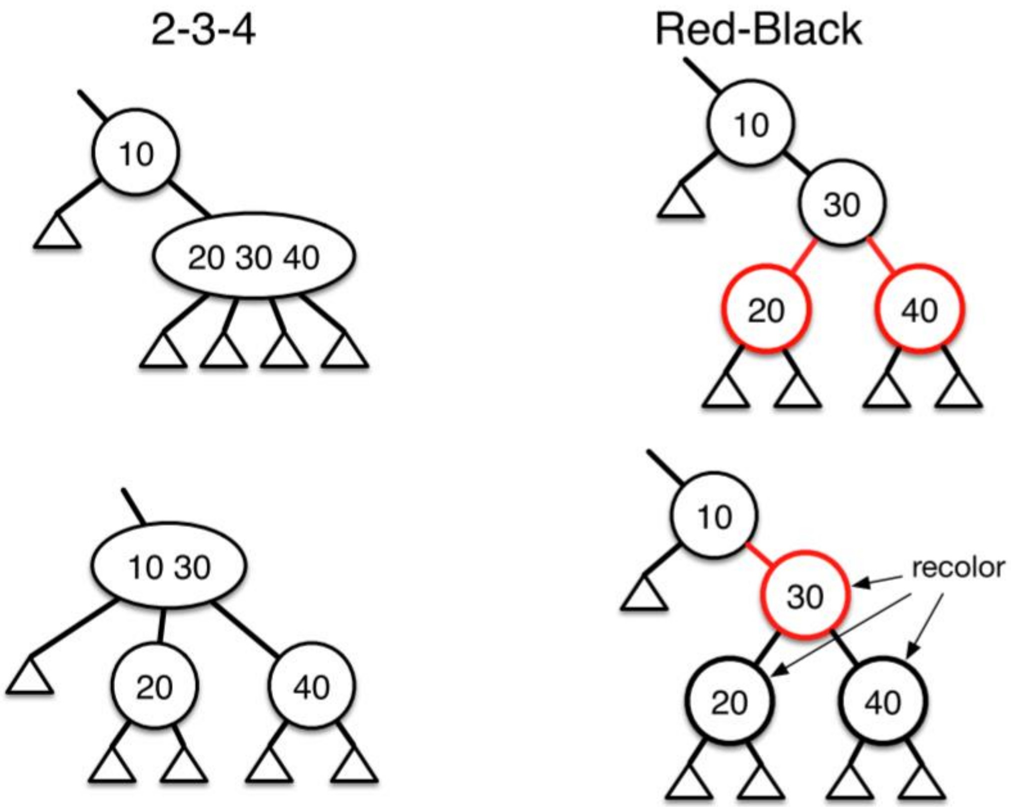
Red Black Trees

- Inserting MIDDLE key into 3-Node



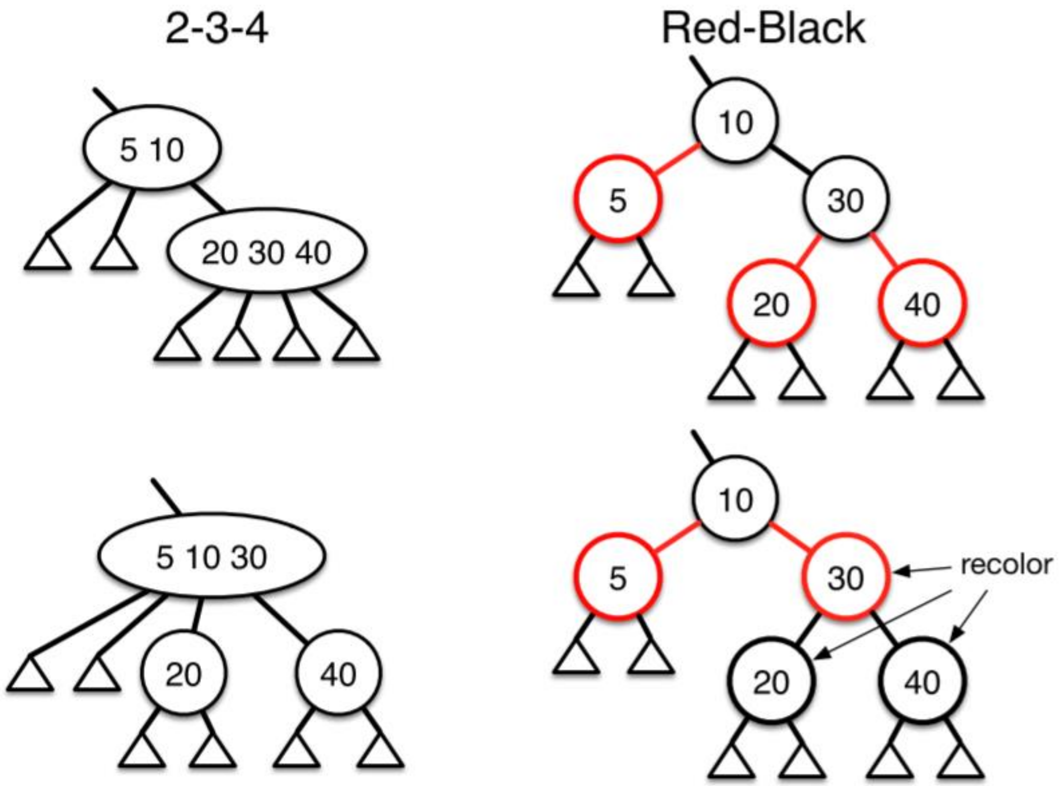
Red Black Trees

- Split 4-Node that is the child of a 2-Node



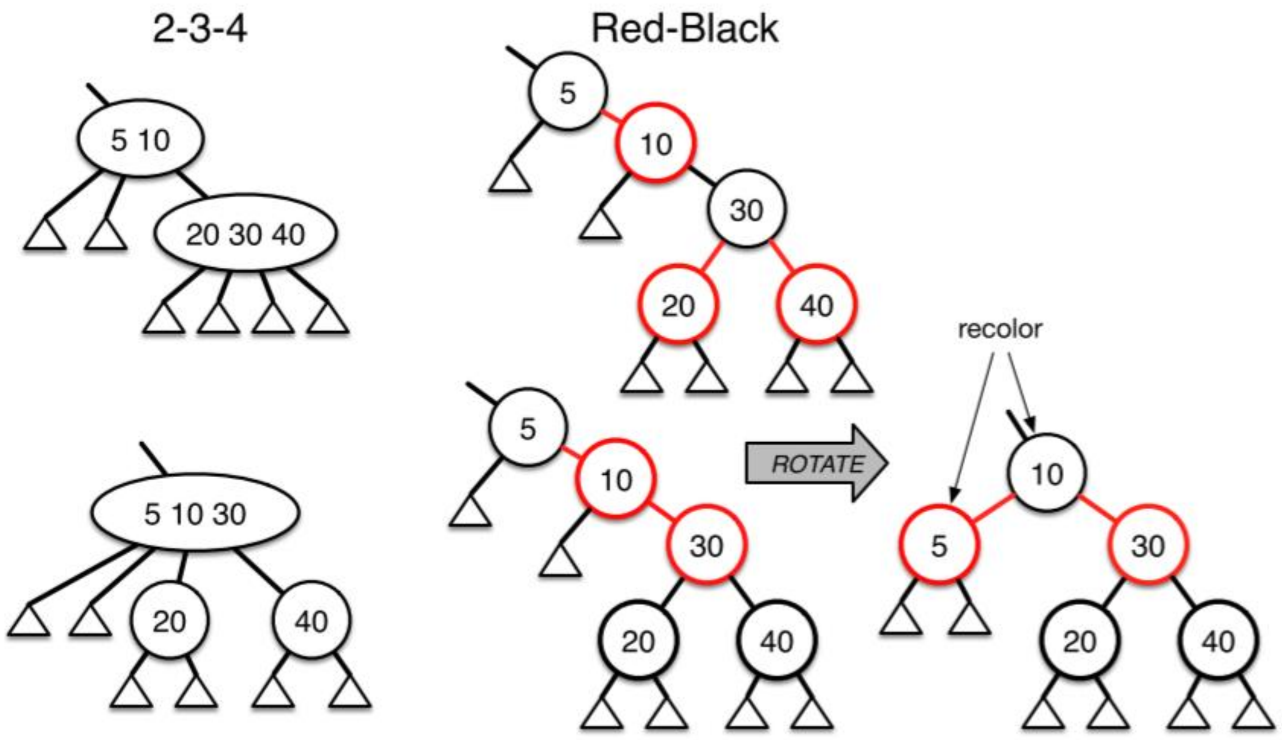
Red Black Trees

- Split 4-Node that is the rightmost child of 3-Node (Case 1)



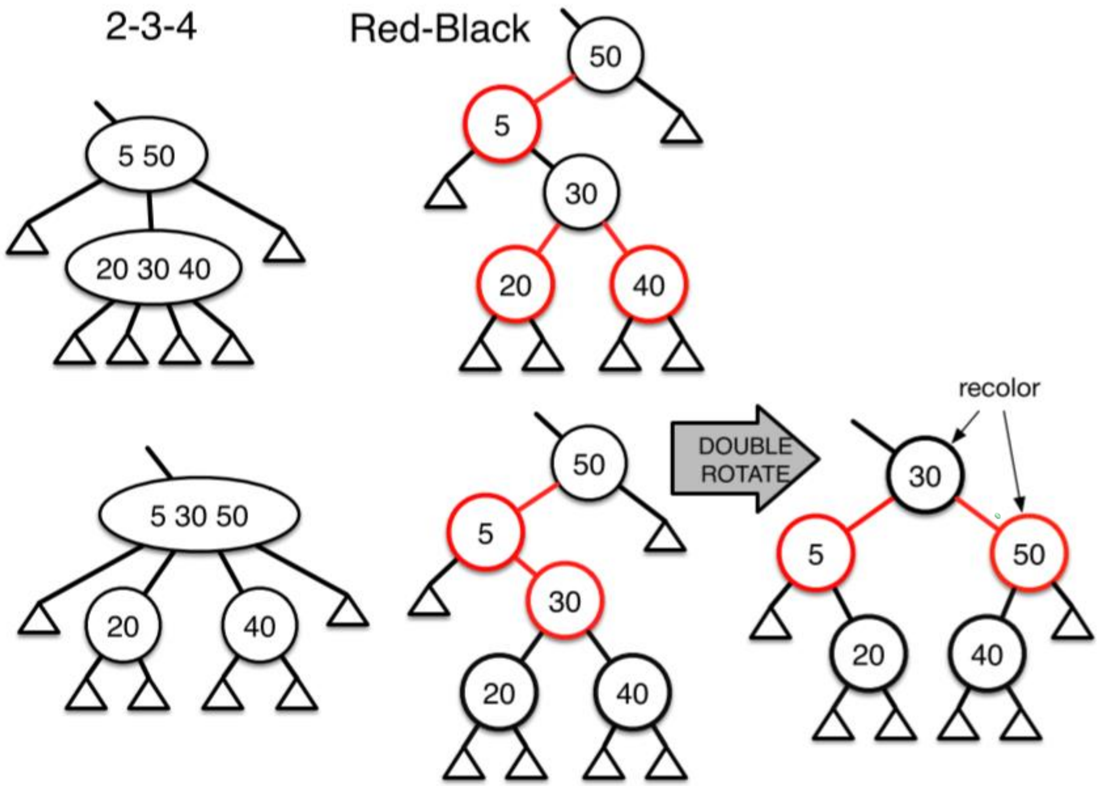
Red Black Trees

- Split 4-Node that is the rightmost child of 3-Node (Case 2)

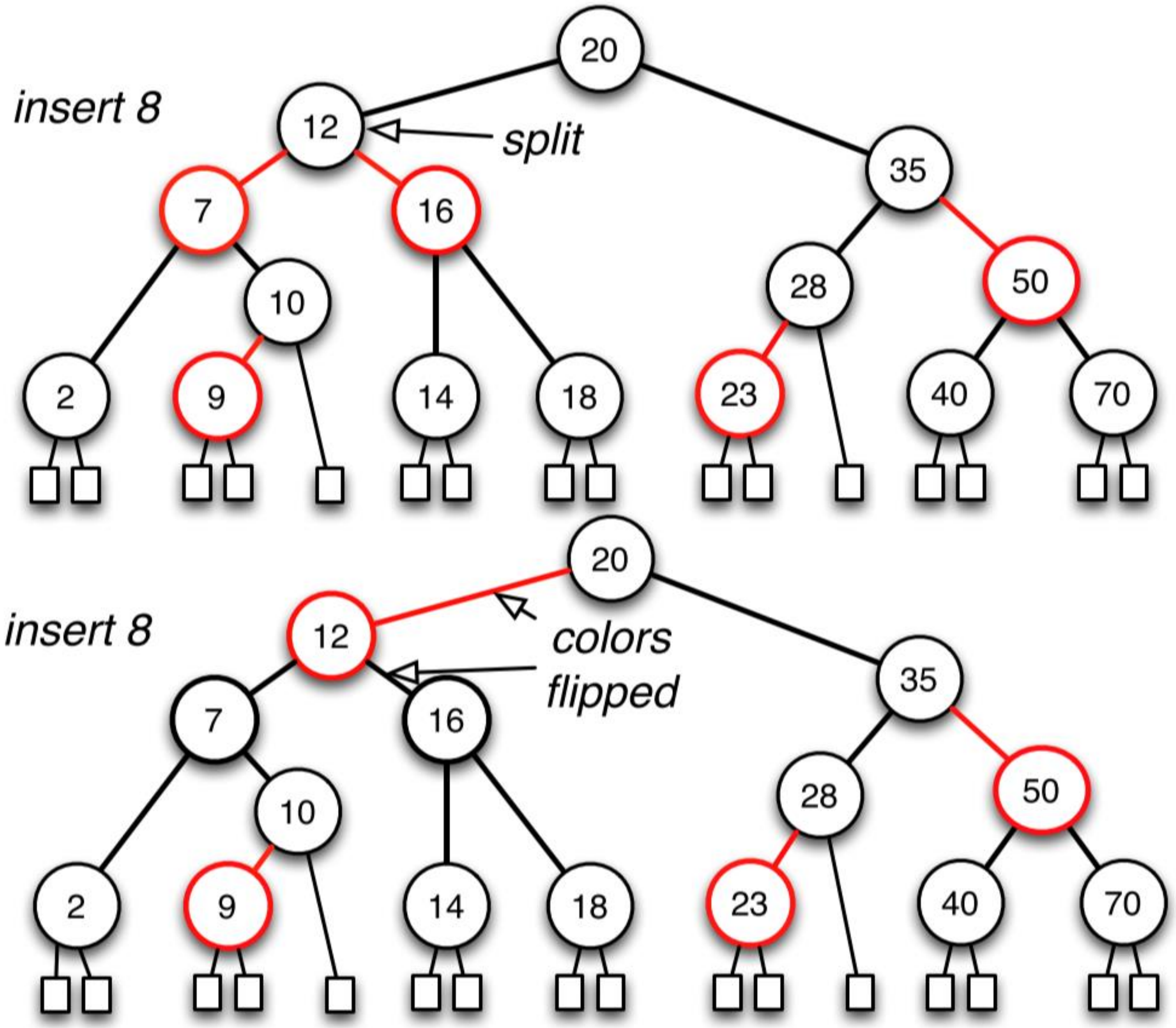


Red Black Trees

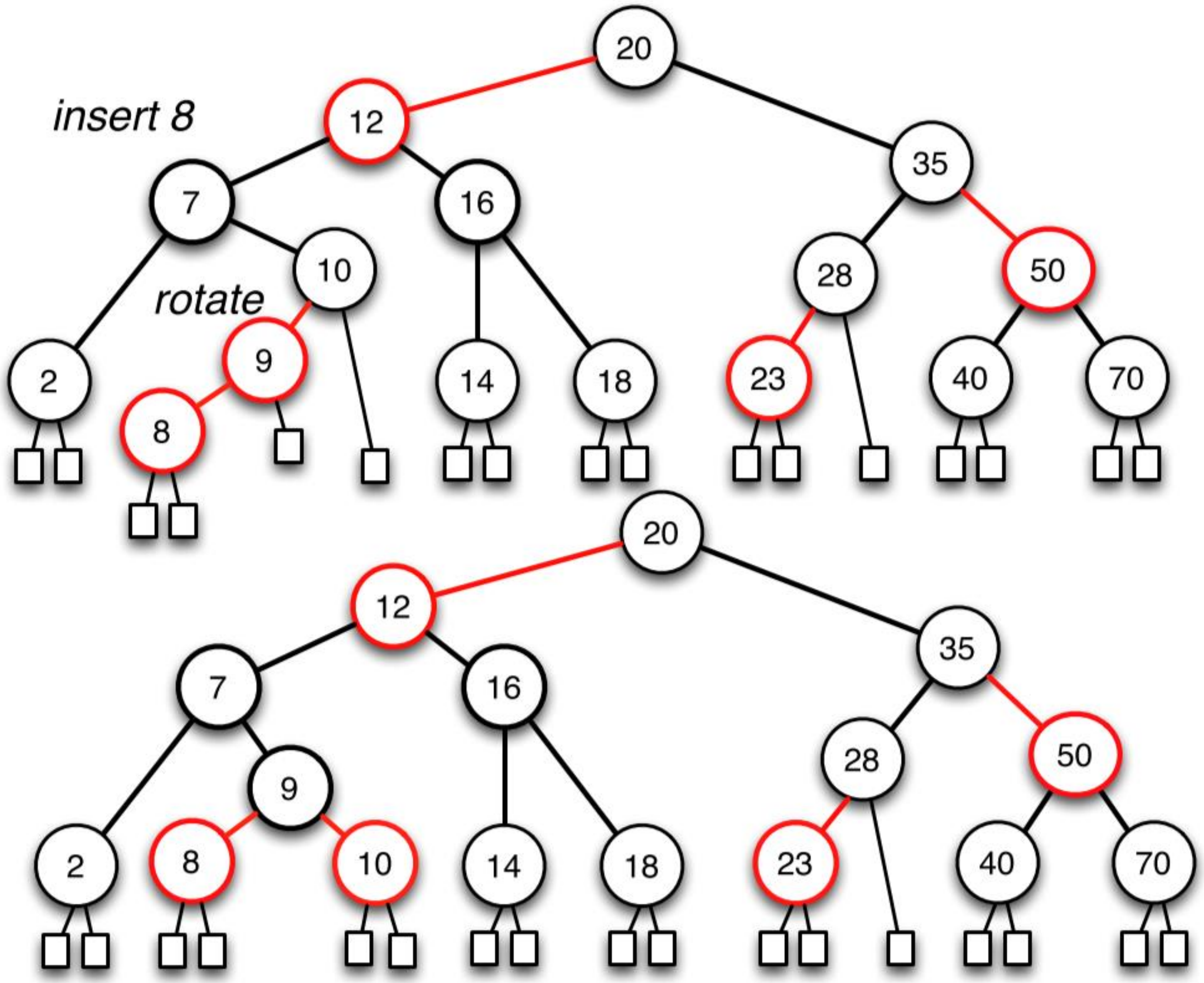
- Split 4-Node that is the MIDDLE child of 3-Node



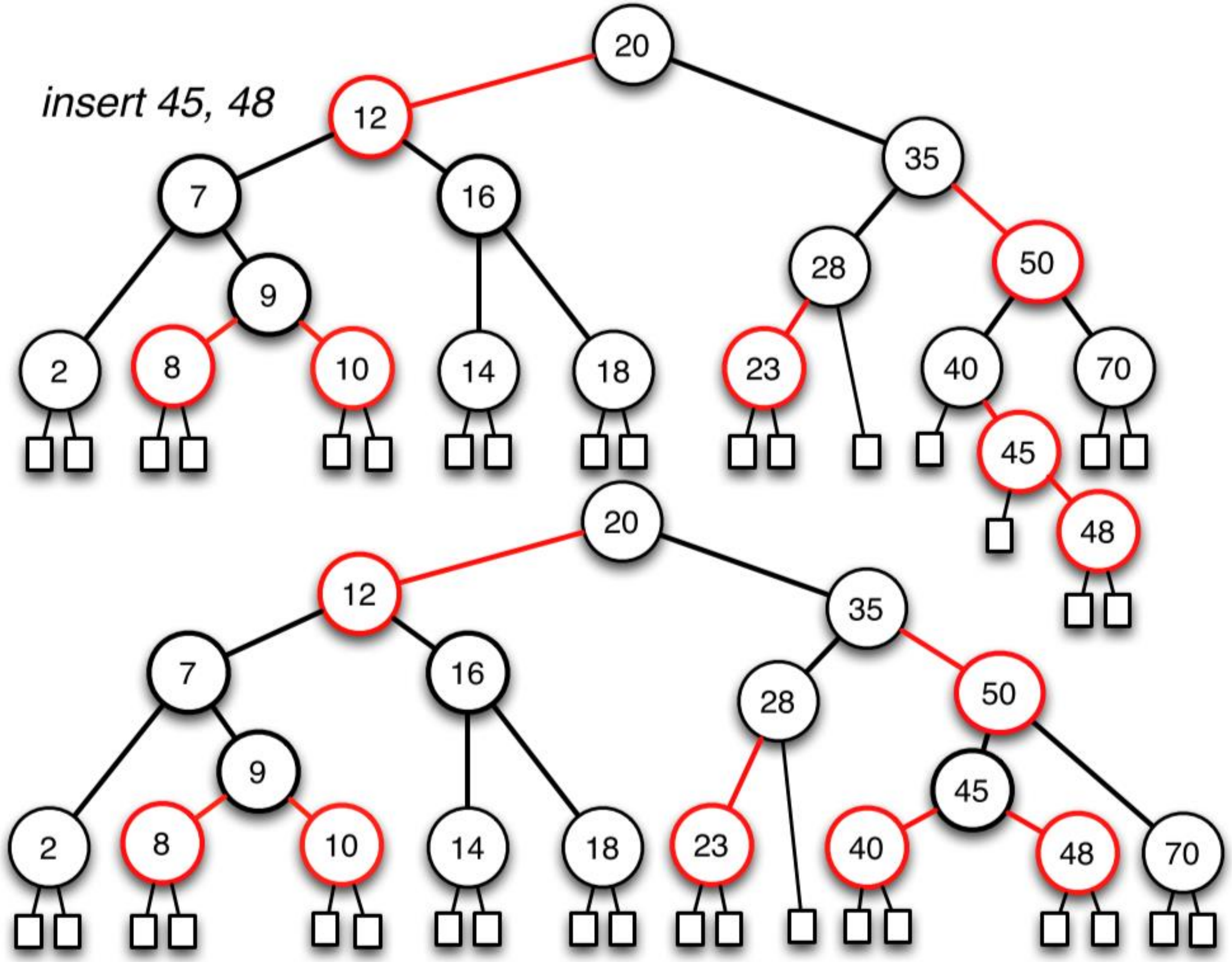
Red Black Trees



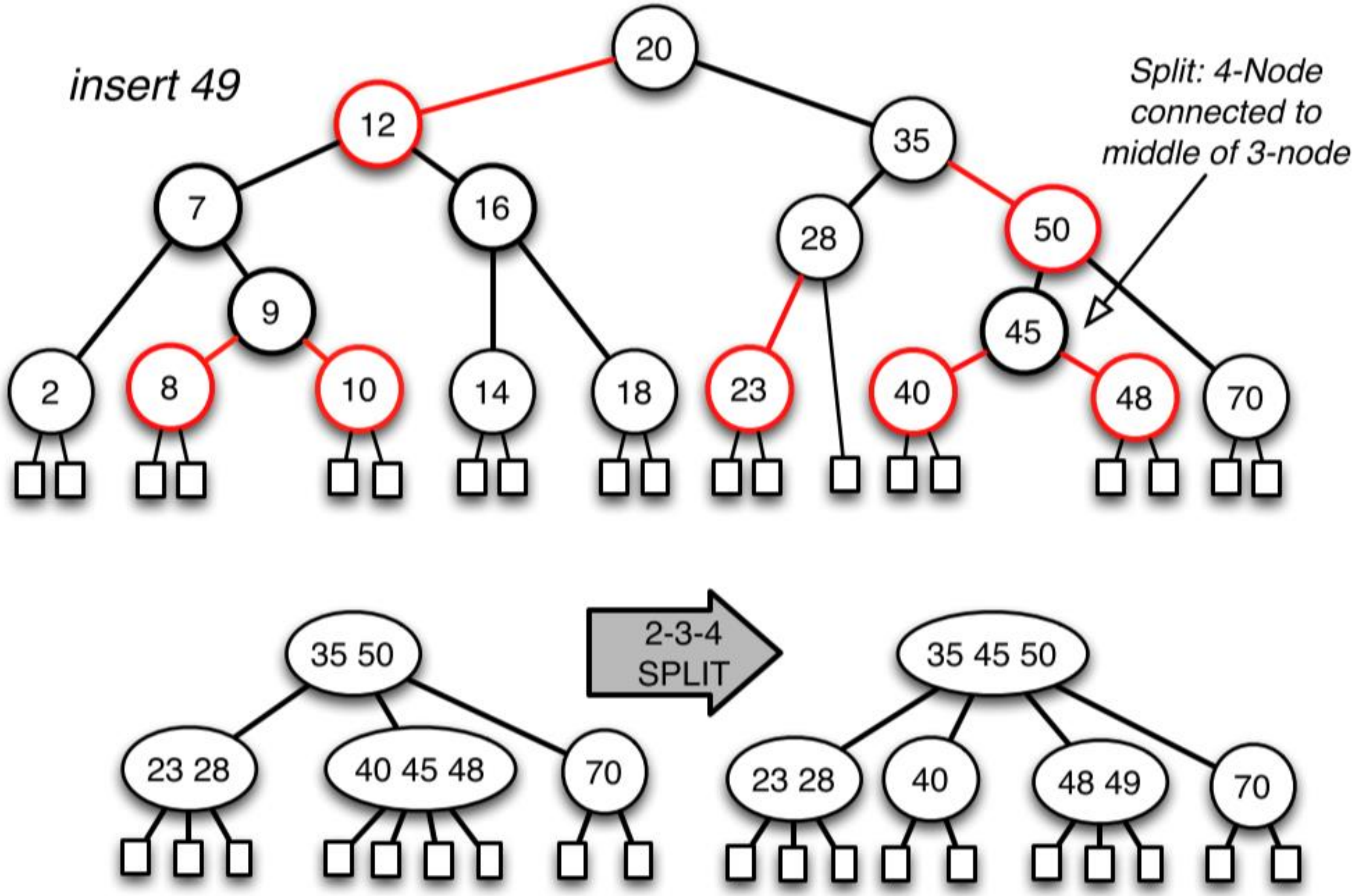
Red Black Trees



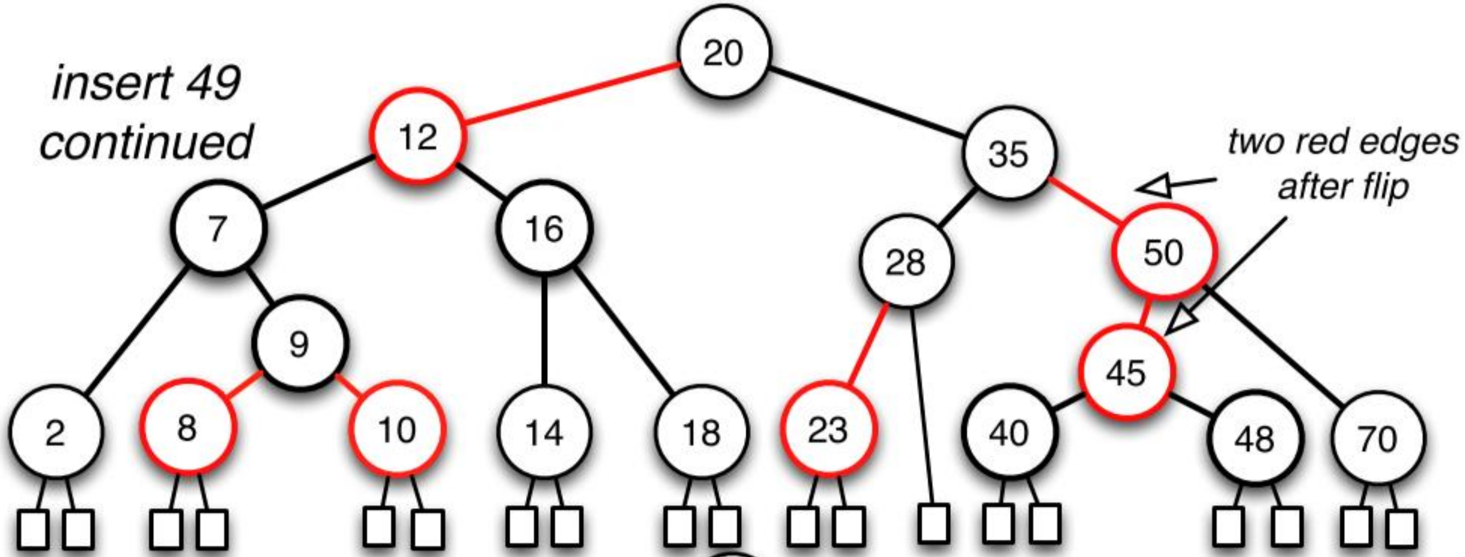
Red Black Trees

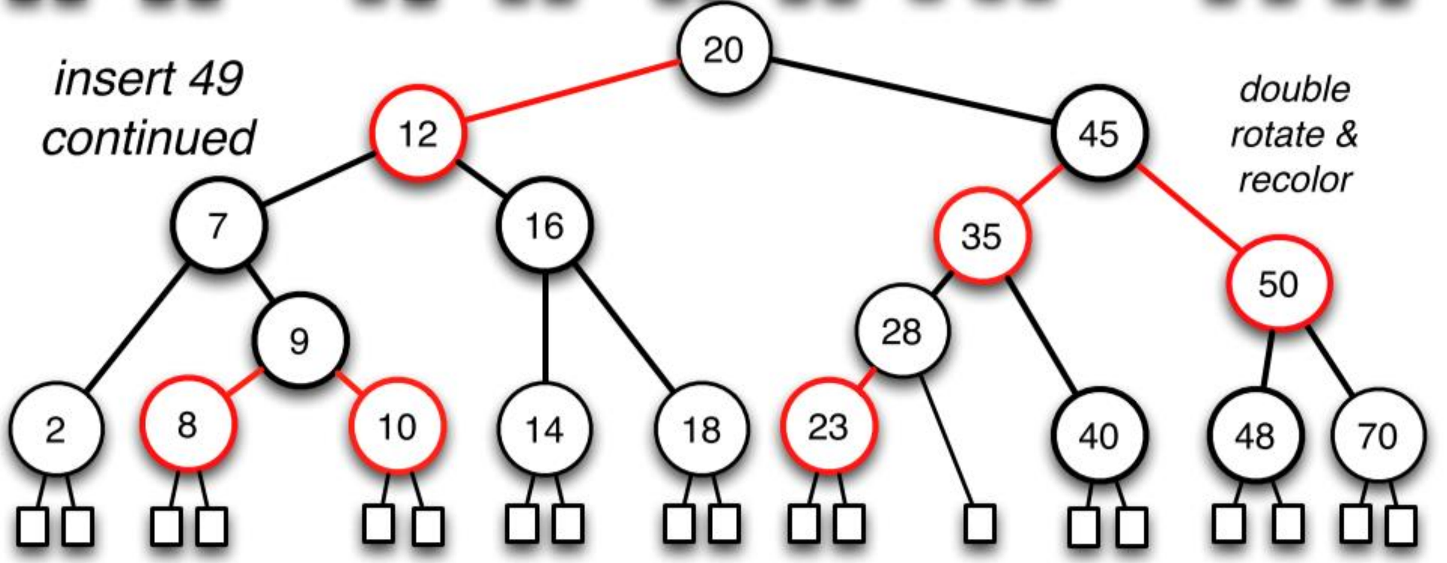


Red Black Trees

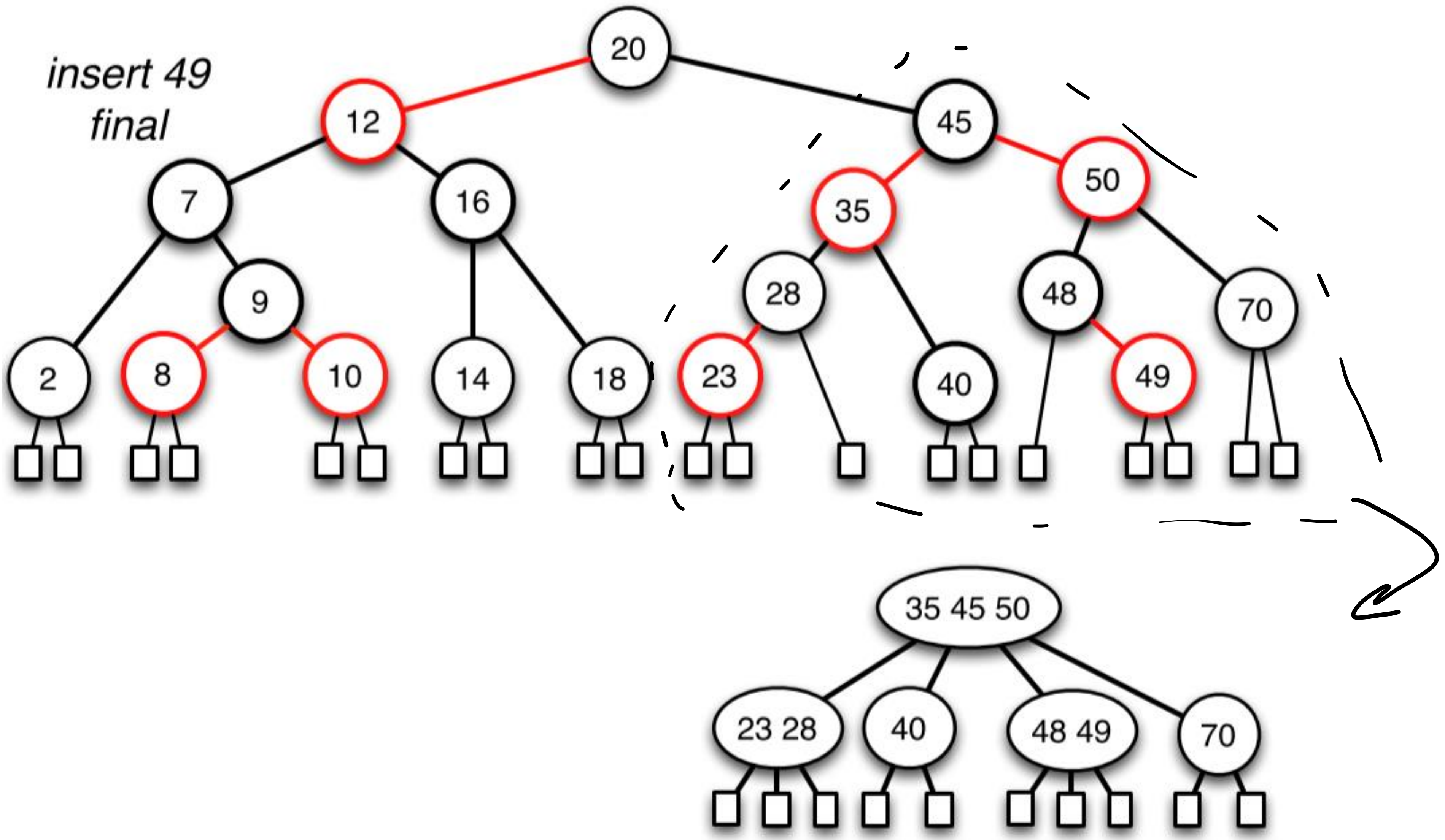


Red Black Trees

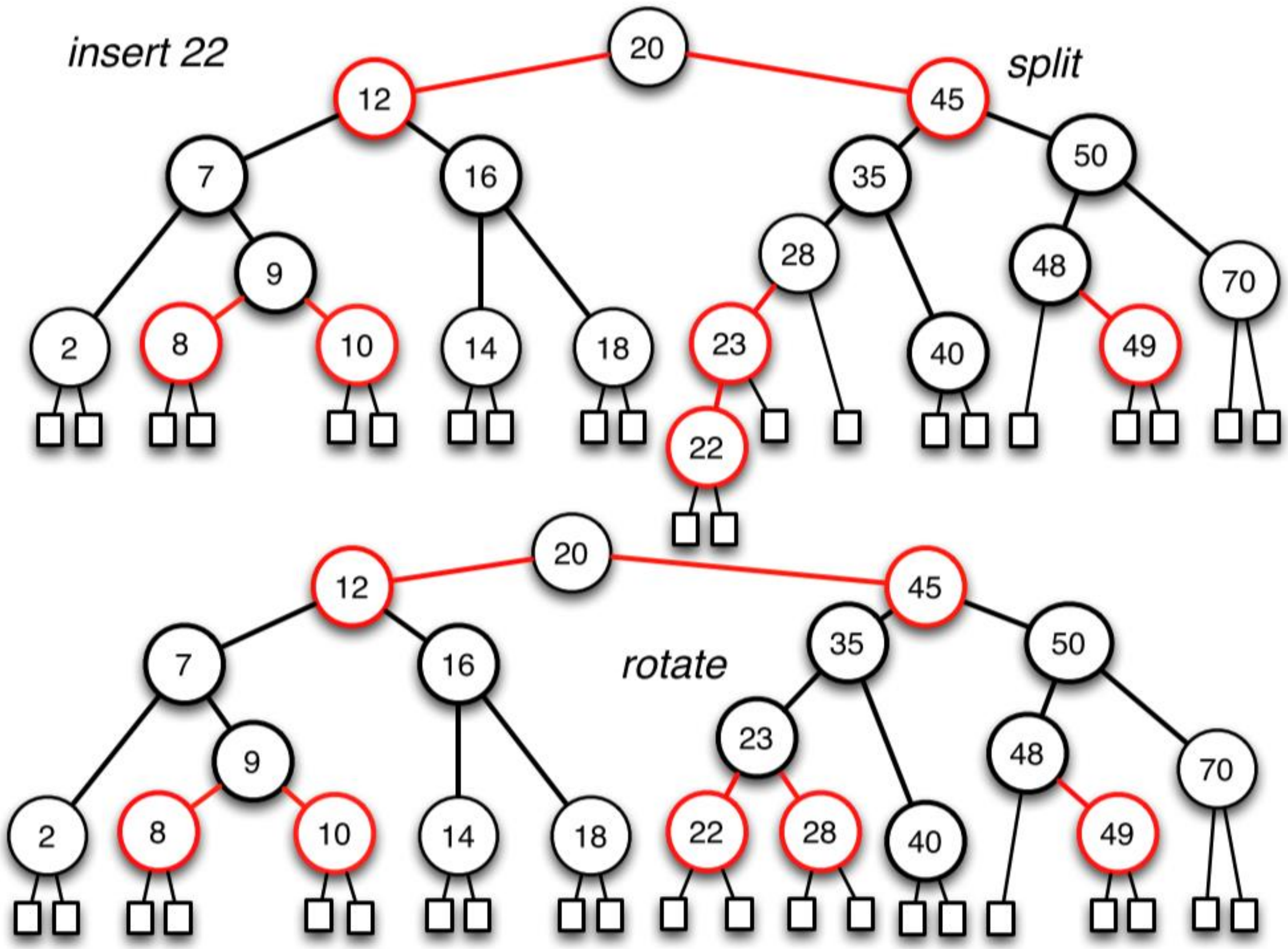




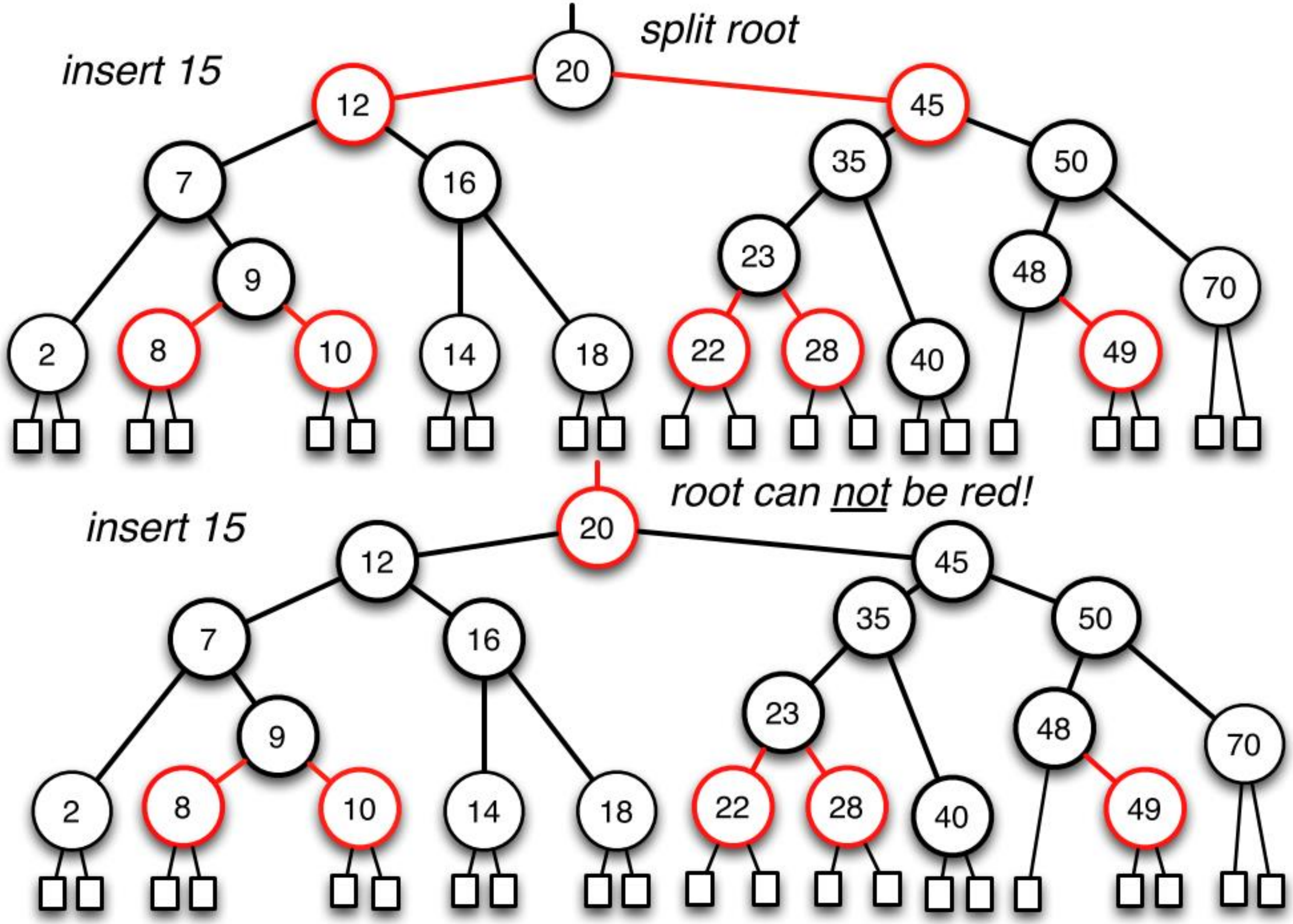
Red Black Trees



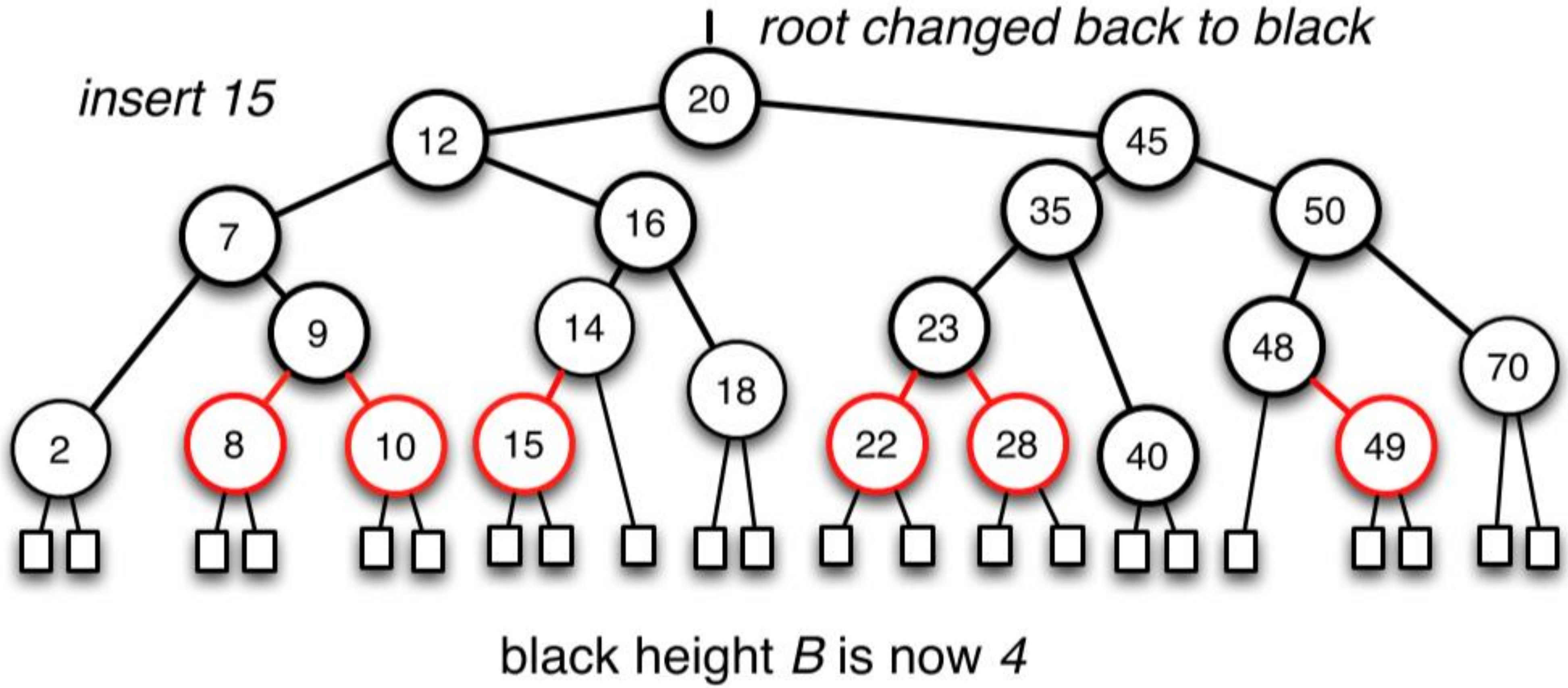
Red Black Trees



Red Black Trees



Red Black Trees



Red Black Trees

- Refer to chapter 11 in the textbook for further descriptions of
  - Red-Black Trees
  - Their algorithms
  - Examples