Design and Analysis of Algorithm

Tutorial - 4

Ans-1) $T(n) = 3T(n/2) + n^2$

$a = 3,\ b = 2,\ f(n) = n^2$

$n^{\log_b^a} = n^{\log_2^3}$

Comparing $n^{\log_2^3}$ and n^2

$n^{\log_2^3} < n^2$ (case-3)

$\therefore$ according to master thearem

$T(n) = \theta(n^2)$

Ans-2) $T(n) = 4T(n/2) + n^2$

$a = 4,\ b = 2$

$n^{\log_b^a} = n^{\log_2^4} = n^2 = f(n)$ (case 2)

$\therefore$ according to master's thearem

$T(n) = \theta(n^2 \log n)$

Ans-3) $T(n) = T(n/2) + 2^n$

$a = 1,\ b = 2$

$n^{\log_2^1} = n^0 = 1$

$1 < 2^n$ (case 3)

$\therefore$ according to master's thearem

$T(n) = \theta(2^n)$

Ans-4) $T(n) = 2^n T(n/2) + n^2$

$\therefore$ Master's Theorem is not applicable as 'a' is function of n

Ans-5) $T(n) = 16T(n/4) + n$

$a=16 \quad b=4 \quad f(n)=n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 > f(n)$ (case-1)

$T(n) = \Theta(n^2)$

Ans-6) $T(n) = 2T(n/2) + n\log n$

$a=2, \quad b=2, f(n)=n\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

Now $f(n) > n$

$\therefore$ According to master's $T(n) = \Theta(n\log n)$

Ans-7) $T(n) = 2T(n/2) + n/\log n$

$a=2, \quad b=2, \quad f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

$n > f(n)$

$\therefore$ According to master's Theorem

$T(n) = \Theta(n)$

Ans-8) $T(n) = 2T(n/4) + n^{0.51}$

$a=2, \quad b=4, \quad f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < f(n)$

$\therefore$ according to master's theorem

$T(n) = \Theta(n^{0.51})$

Ans-9) $T(n) = 0.5T(n/2) + 1/n$

$\therefore$ Master's Theorem Not applicable as $a < 1$.

Ans-10) $T(n) = 16T(n/4) + n!$

$a = 16, \quad b = 4, \quad f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 < n!$

According to masters, $T(n) = \theta(n!)$

Ans-11) $T(n) = 4T(\frac{n}{2}) + \log n$

$a = 4, b = 2, \quad f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

$\therefore$ According to Master's, $T(n) = \theta(n^2)$.

Ans-12) $T(n) = sqrt(n)\, T(n/2) + \log n$

$\therefore$ Master's Not applicable as a is not constant.

Ans-13) $T(n) = 3T(n/2) + n$

$a = 3, \ b = 2. \quad f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

$\therefore$ According to Master's theorem, $T(n) = O(n^{\log_2 3})$.

Ans-14) $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3, \ b = 3, \quad f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

$\therefore$ According to Master theorem, $T(n) = \theta(n)$.

Ans-15) $T(n) = 4T(n/2) + cn$

$a = 4, \quad b = 2, \quad f(n) = c * n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > c * n$

$\therefore$ According to master's theorem, $T(n) = \theta(n^2)$.

Ans-16)

$T(n) = 3T(n/4) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$

$\therefore$ According to master's theorem,

$T(n) = \Theta(n\log n)$

Ans-17)

$T(n) = 3T(n/3) + n/2$

$a = 3,\ b = 3,\ f(n) = \frac{n}{2}$

$n^{\log_b a} = n^{\log_3 3} = n$

$\Theta(n) = \Theta(n/2)$

$\therefore$ According to master's theorem.

$T(n) = \Theta(n\log n)$

Ans-18)

$T(n) = 6T(n/3) + n^2\log n$

$a = 6,\ b = 3,\ f(n) = n^2\log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$n^{1.63} < n^2\log n$

$\therefore$ According to Master's theorem

$T(n) = \Theta(n^2\log n)$

Ans-19)

$T(n) = 4T(n/2) + n/\log n$

$a = 4,\ b = 2,\ f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n/\log n$

$\therefore$ According to Master's theorem

$T(n) = \Theta(n^2)$

Ans-20) $T(n) = 64T(n/8) - n^2 \log n$

Master's thearem is not applicable as $f(n)$ is not increasing function.

Ans-21) $T(n) = 7T(n/3) + n^2$

$a = 7$, $b = 3$, $f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$

$\therefore$ According to Master's, $T(n) = \Theta(n^2)$

Ans-22) $T(n) = T(n/2) + n(2 - \cos n)$

Master's thearem isn't applicable since regularly condition is isolated in case 3.