Mathematics for Computing - IT1030

Lecture 02
Boolean Algebra

Boolean Algebra

- A variable used in an **algebraic formula** so far, is assumed to take *a set of numerical values*.
- All variables in **boolean equations** can take only one of *two possible values*.
- Used symbols for the two values are **0** and **1**.
- Rules first defined for logic by George Boole (1854), were adapted for the use in designing electronic circuits.
- The circuits in computers and other electronic devices have inputs, each of which is either a 0 or a 1.

Boolean Algebra (cont'd.)

- One major advantage in using these rules is to simplify an electronic circuit.
- Boolean algebra provides the operations and the rules for working with boolean variables.
- Three (3) boolean operators are discussed.
  - Complement
  - Boolean sum
  - Boolean product
- Ten (10) rules are also discussed (aka Boolean Identities).

Boolean Operators

- **Complement**
  - Defined as the opposite of the value that a boolean variable takes.
  - Denoted with a bar (E.g.: $\overline{\mathrm{A}}$).
  - $\overline{0} = 1$ and $\overline{1} = 0$.
- **Boolean Sum**
  - Defined as the output to be 1 if at least one variable is 1.
  - Denoted with the symbol $+$ or by OR.
  - $0 + 0 = 0$, $0 + 1 = 1$, $1 + 0 = 1$ and $1 + 1 = 1$.

Boolean Operators (cont'd.)

- **Boolean Product**
  - Defined as the output to be $\mathbf{0}$ if at least one variable is $\mathbf{0}$.
  - Denoted with the symbol ($\cdot$) or by $\mathbf{AND}$.
  - $\mathbf{0 \cdot 0 = 0}$, $\mathbf{0 \cdot 1 = 0}$, $\mathbf{1 \cdot 0 = 0}$ and $\mathbf{1 \cdot 1 = 1}$.
  - When there is no danger of confusion, the symbol $\cdot$ can be omitted.

- Order of boolean operators,
  1. Complement.
  2. Boolean products.
  3. Boolean sums.

Boolean Identities

1. Law of Double Complement
   - $\overline{\overline{A}} = A$
2. Idempotent Laws
   - $A + A = A$
   - $A \cdot A = A$
3. Identity Laws
   - $A + 0 = A$
   - $A \cdot 1 = A$
4. Domination/Null/Universal Bound Laws
   - $A + 1 = 1$
   - $A \cdot 0 = 0$

Boolean Identities (cont'd.)

5. Commutative Laws
   - $A + B = B + A$
   - $A \cdot B = B \cdot A$
6. Associative Laws
   - $A + (B + C) = (A + B) + C$
   - $A \cdot (B \cdot C) = (A \cdot B) \cdot C$
7. Distributive Laws
   - $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$
   - $A + B \cdot C = (A + B) \cdot (A + C)$

Boolean Identities (cont'd.)

8. De Morgan's Laws
   - $\overline{(A \cdot B)} = \overline{A} + \overline{B}$
   - $\overline{(A + B)} = \overline{A} \cdot \overline{B}$
9. Absorption Laws
   - $A \cdot (A + B) = A$
   - $A + A \cdot B = A$
10. Inverse Laws / Unit & Zero Properties
    - $A + \overline{A} = 1$
    - $A \cdot \overline{A} = 0$

Examples

1. Find the values of the following expressions.
   i. $1 \cdot \bar{0}$
   ii. $1 + \bar{1}$
   iii. $\overline{(1 + 0)}$
2. Prove both variants of the absorption law using other boolean identities.
3. Simplify the following expressions.
   i. $A\bar{B}D + A\bar{B}\bar{D}$
   ii. $(\bar{A} + B)(A + B)$
   iii. $M = \bar{W}\bar{X}\bar{Y}\bar{Z} + \bar{W}X\bar{Y}\bar{Z} + WX\bar{Y}\bar{Z} + W\bar{X}\bar{Y}\bar{Z}$

Truth Tables

- To verify the above rules, a **truth table** can be used.
- It's also known as a **Table of Combinations**.
- It's a table displaying all possible values for the variables and the outcomes for a boolean expression.
- If there are $\mathbf{n}$ number of variables, there will be 2^n number of rows in the truth table.
- If the truth table for two boolean expressions shows the same outcomes for the same values for the variables, it can be concluded that the expressions are the same/equal.

Examples

1. Use a table to express the values of each of these Boolean functions.
   i. $\overline{A}B$
   ii. $M = x\bar{y} + \overline{(xyz)}$
   iii. $F(x, y, z) = \bar{y}(xz + \bar{x}\bar{z})$

2. Using a truth table, show that,

$$x\bar{y} + y\bar{z} + \bar{x}z = \bar{x}y + \bar{y}z + x\bar{z}$$

Sum of Products (SoP)

- In some cases, the truth table might be known and we might want to know the expression that gives the truth table.
- This can be done by representing as a **Sum of Products (SoP)** of the variables and their complements.
- Steps:-
  1. Select the rows in the truth table that gives **1** as the outcome.
  2. Write how we can obtain **1** for the first selected row by using the **product** of the variables.
  3. Repeat step two for all selected rows and use the **sum** to combine all results.

Example

Find the boolean expression for F from the given truth table.

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Product of Sums (PoS)

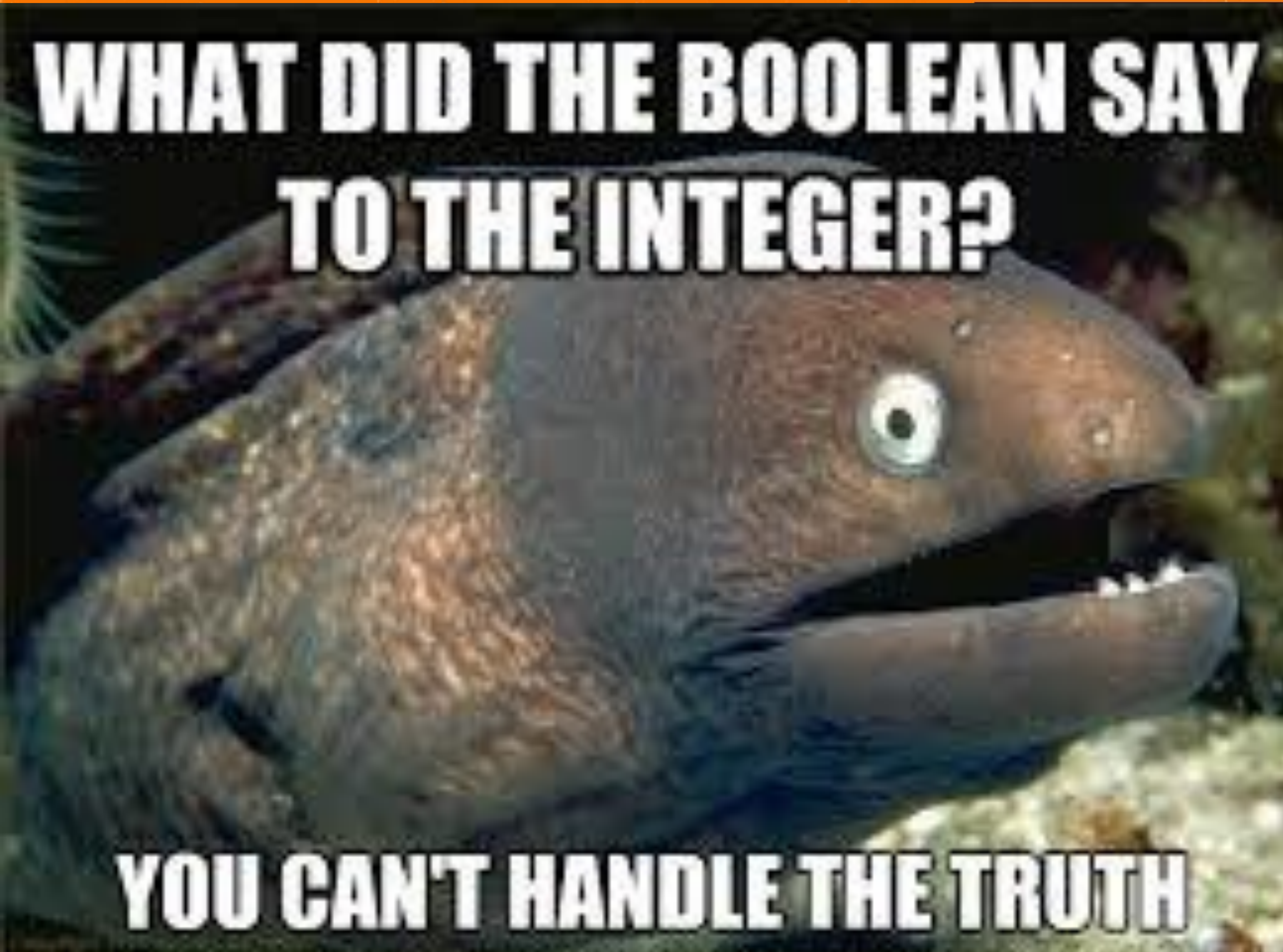
- Used for the same reason as a SoP.
- Product of Sums (PoS) has opposite steps of SoP.
- Steps:-
    1. Select the rows in the truth table that gives **0** as the outcome.
    2. Write how we can obtain **0** for the first selected row by using the **sum** of the variables.
    3. Repeat step two for all selected rows and use the **product** to combine all results.
- Conversion can be done between the two using De Morgan’s rule.

Duality Principle

- In a boolean expression, if all the sums (+) and products (·) are exchanged as well as if 1's and 0's are exchanged, the resulting expression is the opposite of the initial expression.

- This property is observed between SoP and PoS.

- The duel of the complement of one form is equal to the expression in the other form.


WHAT DID THE BOOLEAN SAY
TO THE INTEGER?
YOU CAN'T HANDLE THE TRUTH

Summary

- Students should be able to,
  - Understand the boolean expressions.
  - Learn laws and rules of boolean algebra.
  - Simplify boolean expressions using boolean identities.
  - Use Sum of Products (SoP) and Product of Sums (PoS) to find boolean expressions.
  - Understand similarities and differences between boolean variables as opposed to regular variables.

End of Lecture 02

Next Lecture:-
Logic Gates