XPBD: Position-Based Simulation of Compliant Constrained Dynamics

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Reminder: PBD (Position Based Dynamics)

set of vertices	+	set of constraints

- Each vertex has
  - mass
  - position
  - velocity

- Each constraint is a function:

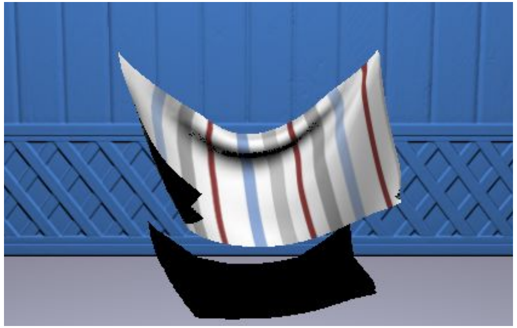
$$C_j : \mathbb{R}^{3n_j} \to \mathbb{R}$$

Reminder: PBD

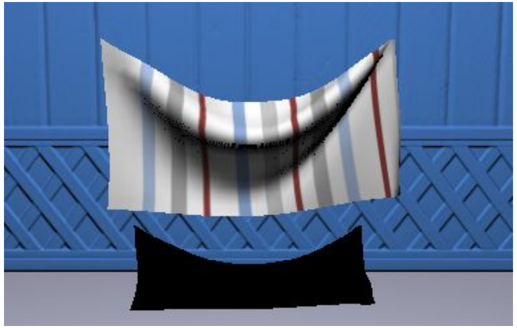
Algorithm 1 Position-based dynamics

for all vertices i **do**
  initialize $\mathbf{x}_i = \mathbf{x}_i^0$, $\mathbf{v}_i = \mathbf{v}_i^0$, $w_i = 1/m_i$
end for
loop
  for all vertices i **do** $\mathbf{v}_i \leftarrow \mathbf{v}_i + \Delta t w_i \mathbf{f}_{\text{ext}}(\mathbf{x}_i)$
  for all vertices i **do** $\mathbf{p}_i \leftarrow \mathbf{x}_i + \Delta t \mathbf{v}_i$
  for all vertices i **do** genCollConstraints($\mathbf{x}_i \rightarrow \mathbf{p}_i$)
  loop solverIteration **times**
    projectConstraints($C_1, \ldots, C_{M+M_{\text{Coll}}}, \mathbf{p}_1, \ldots, \mathbf{p}_N$)
  end loop
  for all vertices i **do**
    $\mathbf{v}_i \leftarrow (\mathbf{p}_i - \mathbf{x}_i)/\Delta t$
    $\mathbf{x}_i \leftarrow \mathbf{p}_i$
  end for
  velocityUpdate($\mathbf{v}_1, \ldots, \mathbf{v}_N$)
end loop

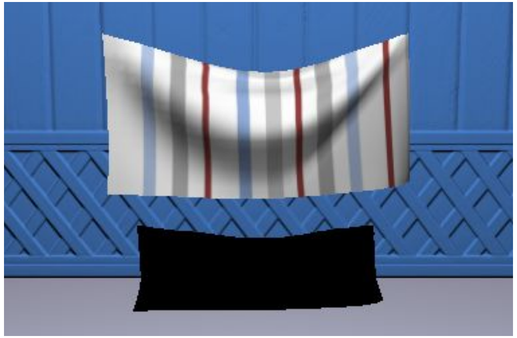
PBD



5 iterations

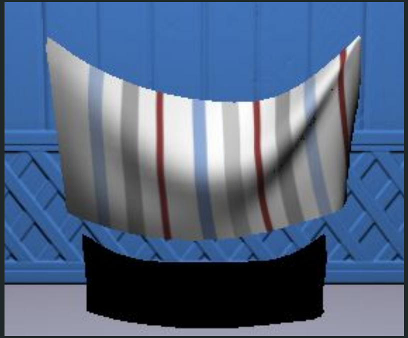


20 iterations

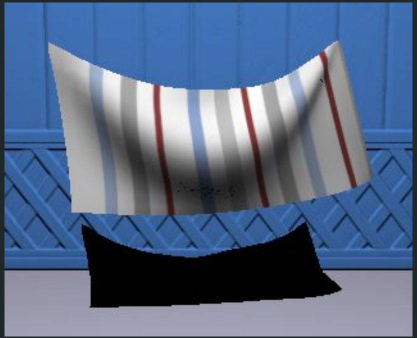


40 iterations

XPBD



20 iterations



40 iterations

What is the difference?

PBD

Correction of an individual point:

$$\Delta p_i = -skw_i \nabla_i C(p_1, .., p_n)$$

$$s = \frac{C(p_1, .., p_n)}{\sum_j w_j |\nabla_j C(p_1, .., p_n))|^2}$$

- k - stiffness
- w_i - inverse mass

XPBD

$$\Delta p_i = \Delta\lambda \cdot w_i \cdot \nabla_i C(p_1, .., p_n)$$

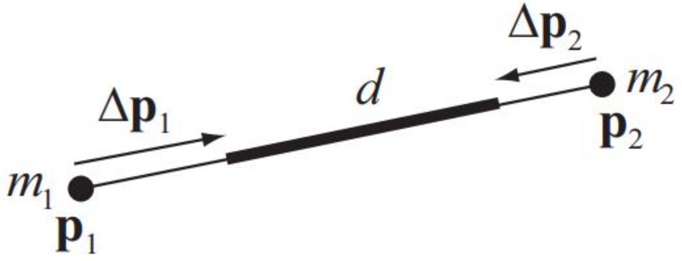
$$\Delta\lambda = \frac{-C(p_1, .., p_n) - \tilde{\alpha}\lambda}{\sum_j w_j |\nabla_j C(p_1, .., p_n))|^2 + \tilde{\alpha}}$$

$$\lambda_{i+1} = \lambda_i + \Delta\lambda$$
$$x_{i+1} = x_i + \Delta x$$

- $\tilde{\alpha} = \alpha / \Delta t^2$
- α - compliance, inverse stiffness

Stretching constraint

$$C(x_1, x_2) = |x_1 - x_2| - d$$



$$\Delta\lambda = \frac{d - |x_1 - x_2| - \tilde{\alpha}\lambda}{w_1 + w_2 + \tilde{\alpha}}$$

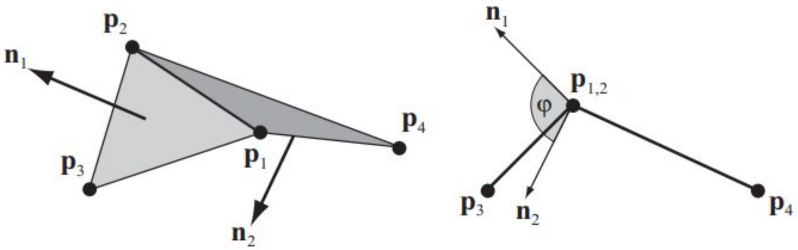
$$\Delta x_i = \pm w_i \frac{x_1 - x_2}{|x_1 - x_2|} \Delta\lambda$$

Bending constraint

$$C(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) = \arccos\left(\frac{\mathbf{p}_{21} \times \mathbf{p}_{31}}{|\mathbf{p}_{21} \times \mathbf{p}_{31}|} \cdot \frac{\mathbf{p}_{21} \times \mathbf{p}_{41}}{|\mathbf{p}_{21} \times \mathbf{p}_{41}|}\right) - \varphi_0$$

$$\nabla_i C(p_1, p_2, p_3, p_4) = \frac{-q_i}{\sqrt{1-d^2}}$$

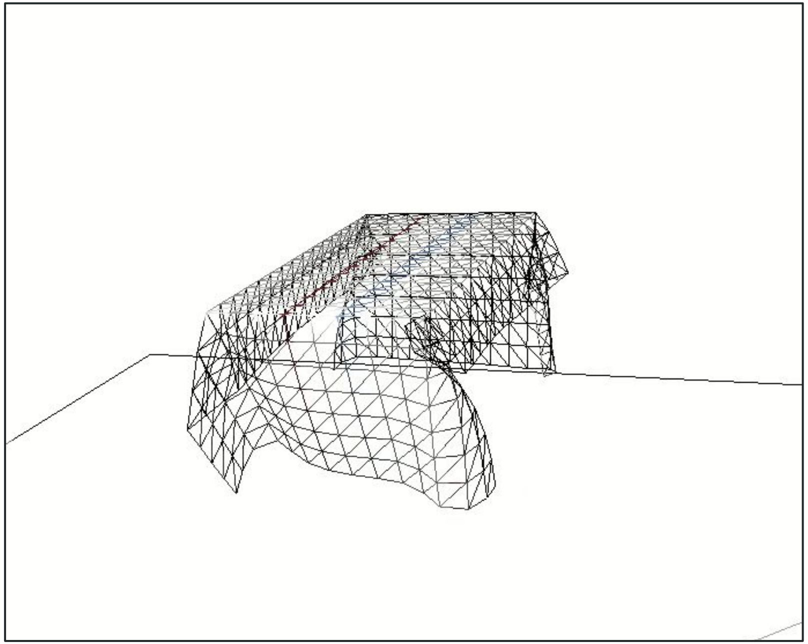
$$\Delta\lambda = \frac{\varphi_0 - arccos(d) - \tilde{\alpha}\lambda}{\frac{1}{1-d^2}\sum_j w_j |q_i|^2 + \tilde{\alpha}}$$

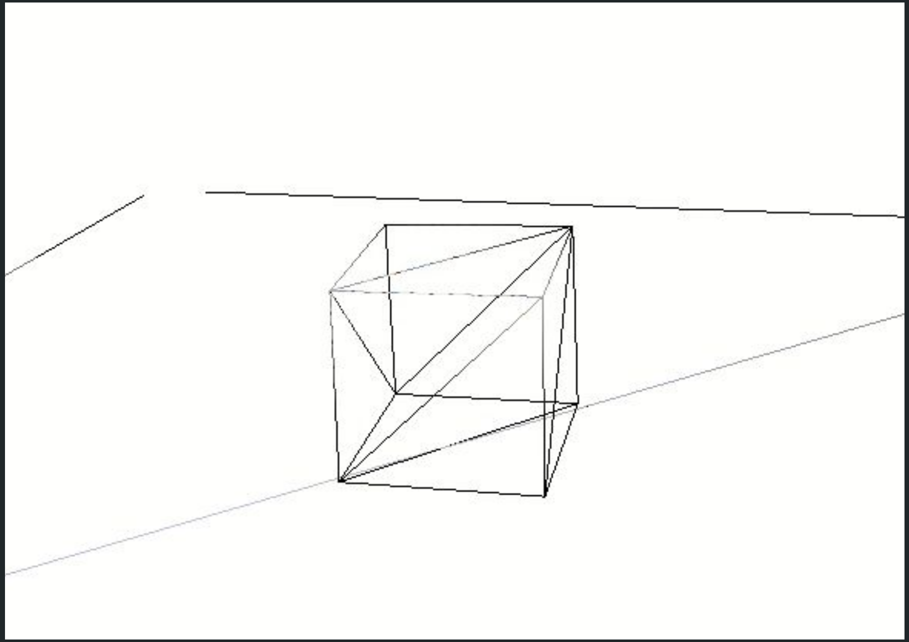


$$\Delta p_i = \Delta\lambda w_i \frac{q_i}{\sqrt{1-d^2}}$$

here q_i and d are notations from the PBD paper

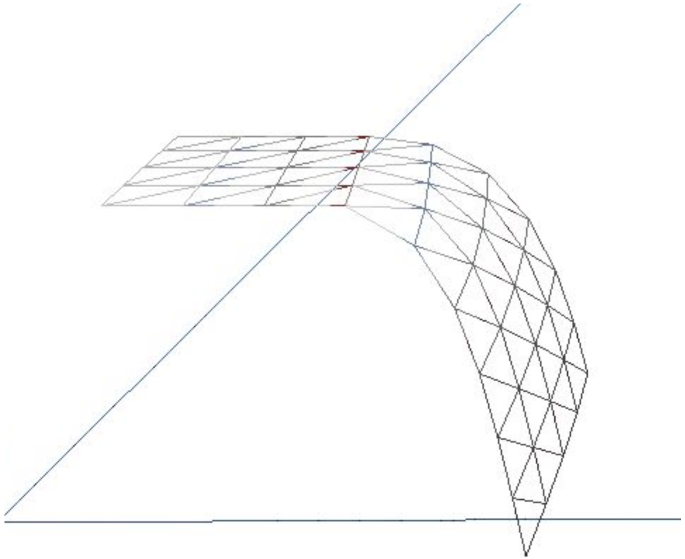
Results





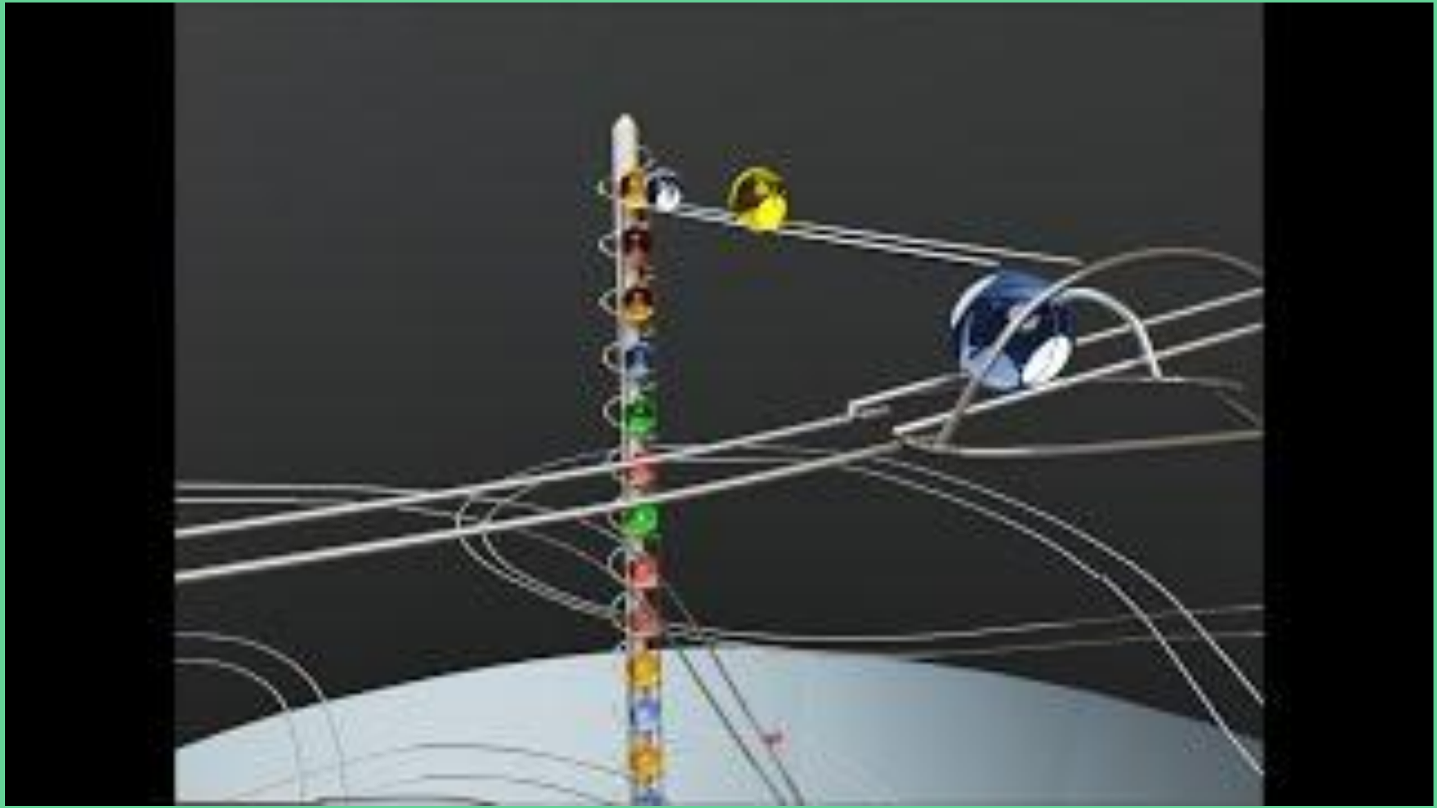
Limitations

1. May be difficult to derive formulas for some constraints
2. Difficulties with arccos() function
3. Tuning of the parameters (compliance)
4. The result is mesh-dependent



Recent work

Detailed Rigid Body Simulation using XPBD



Bibliography

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