Fluid Flow Pattern in Upflow Reactors for Anaerobic Treatment of Beet Sugar Factory Wastewater

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Summary

Residence-time-distribution experiments for the fluid in a 30-m^3 pilot plant and a 200-m^3 prototype upflow reactor were performed by means of continuous injection of an LiCl solution as a tracer in the influent of the reactor and measurement of the response of this stimulus on several locations in the reactor and in the effluent. In a similar way as described in an article published earlier, models have been developed by use of the measured data of the fluid flow pattern which consisted of regions of ideal mixing, plug flow, dead space, and short circuiting. It appeared that the fluid flow patterns in the two reactors were to a large extent analogous. For the pilot plant, three-mixer models appeared to be appropriate while for the prototype reactor two-mixer models have been found. This difference was a result of the difference in the heights of the sludge beds in the reactors: 2–3 m in the pilot plant and only 0.4 m in the prototype reactor, a result of too small an amount of sludge. Another difference was that, due to a large amount of mud in the prototype reactor, a region of dead space occurred in the models for the fluid flow pattern in this reactor. The dimensions of the prototype reactor have been chosen according to several recommendations obtained from work with the pilot plant (e.g., scale-up should be done by increasing the cross section of the reactor; one influent point should be applied per 5 m^2 bottom surface). The results presented here clearly show the value of these recommendations.

INTRODUCTION

In an earlier article the fluid flow pattern in a 30-m^3 upflow reactor has been described.[1] This reactor was used for anaerobic treatment of wastewater of a beet sugar factory. The method of examination of the fluid flow pattern was based on measurements of the residence time distribution (RTD) of tracer material that was injected into the influent of the reactor. This study was part of a large project on anaerobic treatment of wastewater performed in the Netherlands,[2-4] in which several research groups cooperated: Centrale Suiker Maatschappij, C.S.M.; Delft University of Technology; Agricultural University, Wageningen; University of Amsterdam. It was financially supported by the Governmental Department for Public Health and Environmental Hygiene of the Netherlands. The aim of this project was to develop an efficient full-scale reactor.

Biotechnology and Bioengineering, Vol. XXIV, Pp. 443–459 (1982)
 CCC 0006-3592/82/020443-17$01.70

From the experiments with the 30-m^3 reactor, it could be deduced that for scaling-up, only the horizontal dimensions should be enlarged and that the height of the sludge bed (the concentrated suspension of 80–100 kg suspended solids/m^3 in the lower part of the reactor) should be kept constant, with a value of 2–3-m. A 200-m^3 reactor was constructed according to these suggestions. The residence time distribution experiments performed in this reactor will be described in this article. They have been performed in the same way as those in the 30-m^3 reactor.[1,5] The residence time distribution curves found were used to develop—via trial and error—model descriptions of the fluid flow pattern. These models consisted of combinations of regions with different idealized flow regimes.[6] The mathematical responses of these combinations to a simulated tracer signal had to fit with the measured residence time distribution curves. By comparison of the models, conclusions about the influences of the process parameters on the fluid flow pattern were obtained.

REACTORS USED

The 30-m^3 reactor has already been described earlier.[1-4] It is an upflow reactor with a height of 6.5 m and a diameter of 2.5 m. The influent enters the reactor through one influent point in the bottom; the effluent leaves the reactor via a settler at the top. The gas produced is collected below the settler and leaves the system there through a gas outlet. The sludge is retained in the system by means of the settler from where it can fall back into the reactor. After the experiments described in ref. 1 a new influent distribution system was installed. It had been found with the system used that, in spite of the expected radial distribution by means of the horizontal plate above the influent opening, preference flow occurred. In the new system that consisted of a simple, horizontal pipe (1.5 in. diam), preference flow occurred as well but in a tangential direction which is completely specified via the design of the construction. The results of one RTD experiment with this unidirectional influent system in the 30-m^3 pilot plant are presented in this article. As the results with the two influent distribution systems in the 30-m^3 reactor indicated that both systems produced roughly identical fluid flow patterns, it was decided to apply the unidirectional system in the 200-m^3 prototype.

The 200-m^3 reactor is presented schematically in Figures 1, 2, and 3, giving, respectively, cross section and front, top, and bottom of the reactor. In Figure 3 the locations and main influent directions of the ten influent points used have been shown. To obtain for each influent point the same influent flow rate, they have been constructed in such a way that the pressure drop between the large main pipe and the influent outlet was approximately the same for each point. In these figures the locations of the sample points have also been indicated. The most important design data are given in Table I.

The unusual form of the reactor resulted from the fact that a rectangular settler had been constructed on top of a circular reactor tank. This had been done because it was supposed that the surface area of the settler had to be

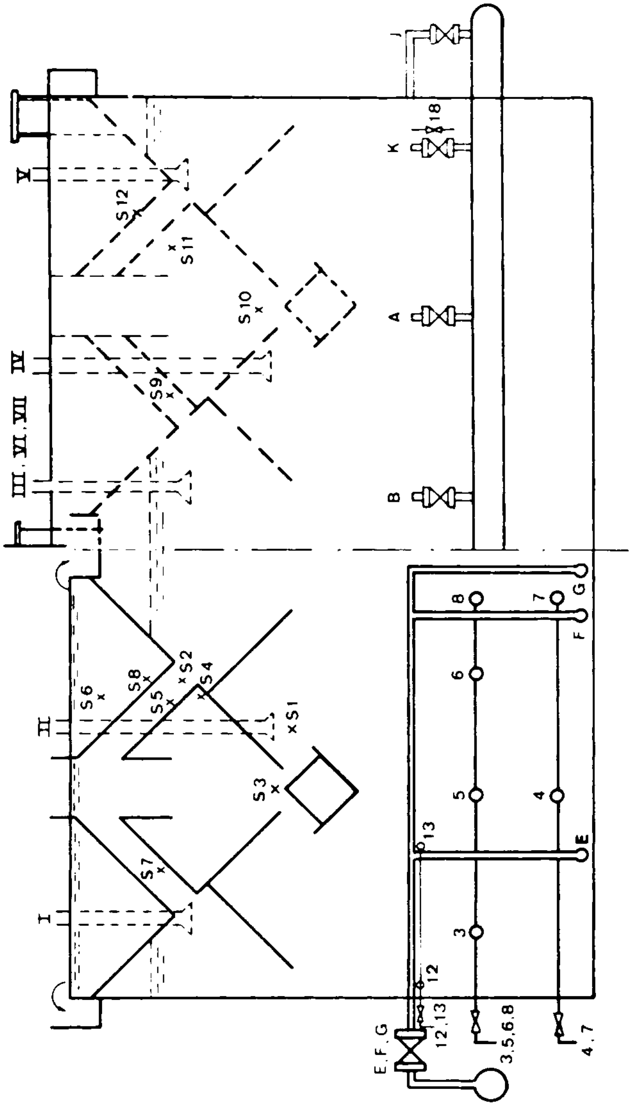


Fig. 1. Outline of the cross section (A–A) and front of the 200-m^3 full-scale reactor.

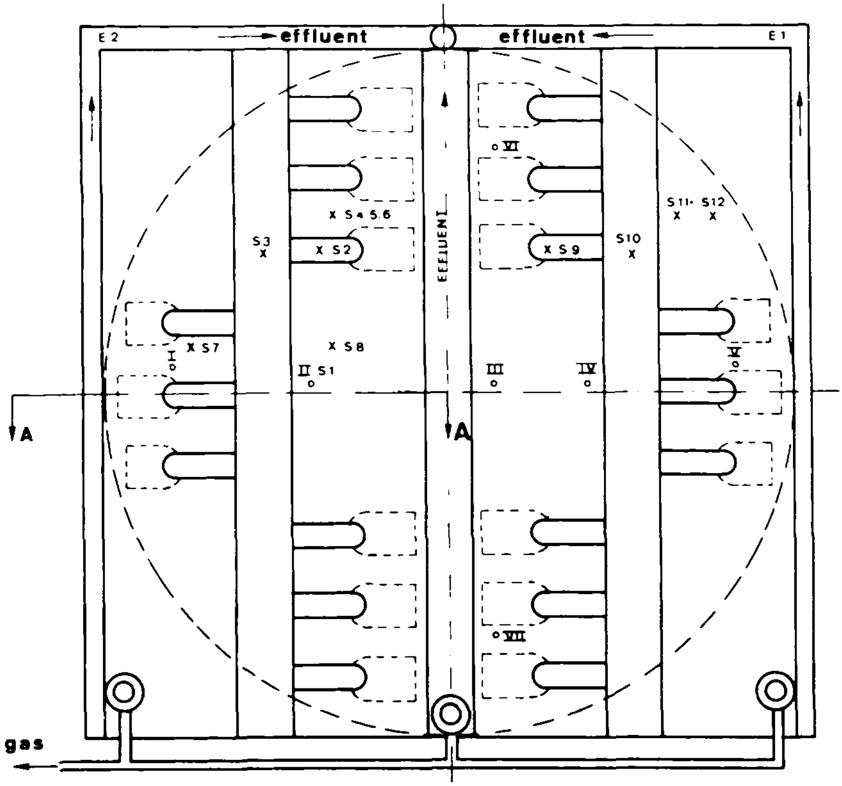


Fig. 2. Outline of the top of the 200-m^3 full-scale reactor.

larger than that of the reactor, to give a linear fluid velocity at the surface of the settler smaller than in the reactor itself. After it was found that this was not necessary, later constructions used rectangular reactors and settlers. In fact, the design of the reactor (shown in Figs. 1–3) can be seen as an intermediate between a circular and a rectangular reactor design.

The reactor was part of the wastewater circuit of a sugar factory (the C.S.M. factory of Halfweg, near Amsterdam, as shown in the flow diagram of Fig. 4). About one-quarter of the effluent of the factory could be treated. Centrifuged samples of this water had a chemical oxygen demand of 500–5000 ppm, which was dependent on the time after the start of the beet sugar campaign, and contained as main pollutants salts of acetic and propionic acid. Because this reactor was operated as part of the wastewater circuit, a high purification efficiency was of primary importance. To attain this, the process operation (i.e., in most cases the organic load of the reactor) was adapted to the situation in the wastewater circuit. As a consequence, the process conditions for the experiments could not be chosen independently. Fortunately, the organic load and the gas production changed during the cam-

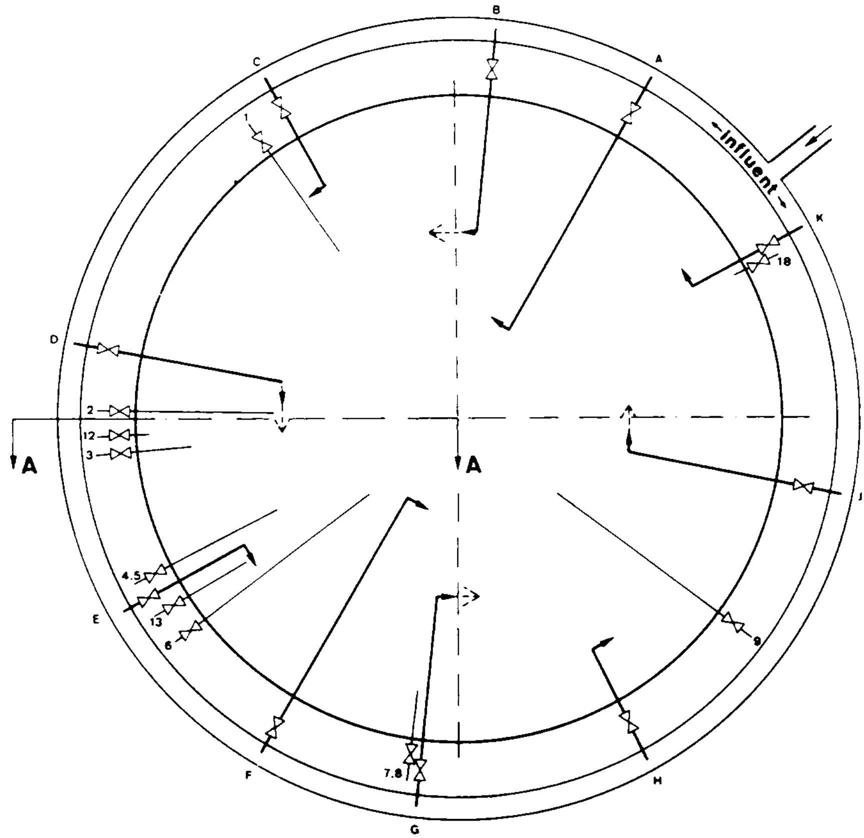


Fig. 3. Outline of the bottom of the 200-m^3 full-scale reactor.

paign so that their influences on the fluid flow pattern could still be examined. The settling properties of the sludge particles as such did not change significantly, but due to a wrong operation of the wastewater circuit about three weeks after the start of the campaign, a large amount (ca. 6000 kg) of mud had accumulated in the reactor. Then the properties of the mixture of sludge and mud altered sharply.[2] As a result of this change the influence of the solids concentration on the fluid flow pattern could be examined.

EXPERIMENTS

The procedure followed for the tracer experiments in the 30-m^3 reactor has already been presented.[1] For the experiments in the 200-m^3 reactor an identical procedure was followed. A difference was that because of the long duration of the latter experiments (up to 35 h) during the night an automatic sampling apparatus has been used. Also the concentrations of the Li^+ solutions

TABLE I

Dimensions of the 200-m^3 Reactor, Locations of the Sample Points (SP)

V_R = 204 m^3, d_R = 7.5 m, h_{st} = 1.9 m, l_{st} = 7.5 m
V_{st} = 53.5 m^3, h_R = 4.5 m, b_{st} = 3.5 m

SP	1	2	3	4	5	6	7	8	9	12	13	18	
h (m)	0.8	0.8	1.0	0.3	1.0	1.0	0.3	1.0	0.8	1.5	1.5	1.5	
SP		S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
Depth (m)		2.0	0.9	1.7	1.1	0.8	0.4	0.8	0.6	0.9	1.7	0.8	0.6

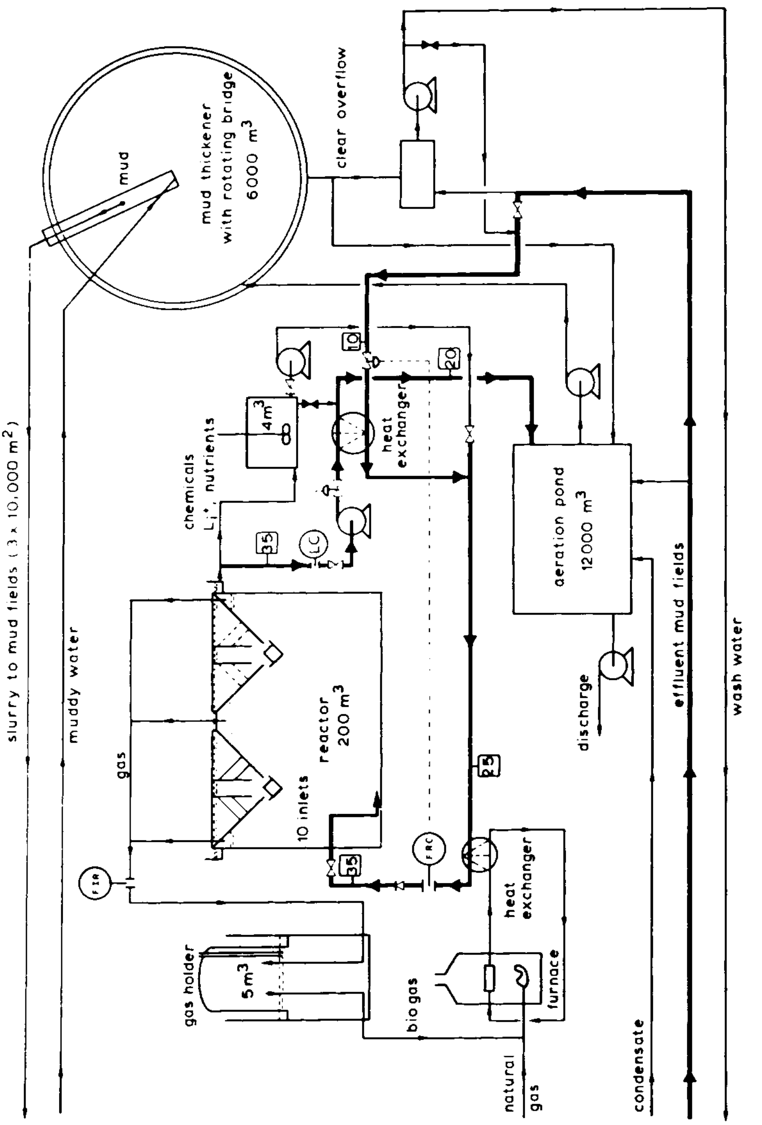


Fig. 4. Flow sheet of the purification system for the wastewater of the beet sugar factory of C.S.M. in Halfweg.

used and the flow rates of these concentrated solutions, to be injected into the influent of the 200-m^3 reactor, were different from the values applied for the experiments in the 30-m^3 reactor.

The trial and error procedure used to find the model whose responses best fit with the measured curves has been automated. For this purpose, an iterating, curve-fitting computer program has been used that was derived from a standard IBM program.[7]

One tracer experiment that has been performed in the 30-m^3 reactor and four in the 200-m^3 will be presented. In these experiments, the process conditions such as the sludge concentration and the gas production were different, the number of influent points (in the 200-m^3 reactor) has been varied. The tracer experiments are numbered 1-5. Each experiment has been evaluated via the derivation of the best fitting model, and accordingly the five models obtained have been numbered 1-5. By comparison of the five models the influences of the gas production, the sludge concentration, the number of influent points, and the scale of the reactor on the fluid flow pattern can be qualified (see Table II for the process conditions during the experiments). As can be seen in Table II, many other process conditions were also different in the experiments. They have been taken up in Table II because in total they give an overall impression of the process performance during the experiment. The influence of most of these parameters on the fluid flow pattern in the reactor was considered to be negligible.

In Figures 5-9, the data points give the measured dimensionless concentration–time curves, to wit the measured local Li^+ concentration, divided by the influent Li^+ concentration as a function of the time after the start of the tracer injection divided by the liquid residence time. The five models obtained by evaluation of these results are presented in Table III and the Figures 10 and 11. In Table III the quantitative values of the parameters of the models have been collected, together with the starting values for the trial-and-error curve-fitting procedures. In the figures the block diagrams have been presented that represent the fluid flow pattern in the 30-m^3 reactor during experiment No. 1 and in the 200-m^3 reactor during the experiment Nos. 2-5, respectively. Generalization of the four models found for the experiments in the 200-m^3 reactor in the form of one figure appeared to be possible since all four models derived contained the same subunits and differed only in the quantitative values of the parameters. The block diagrams and the relations given in Figures 10 and 11 together form the models used. (It should be noted that for the flow rate the more generally accepted symbol F has been used instead of Q that was used in the earlier article.)

Because the balances over the ideal mixers in the models are coupled differential equations, a digital computer was used to calculate the responses. This made it also possible to use an iterating program to find the model whose responses are in best agreement with the measured curves. The dimensionless concentration–time curve for the model's effluent is obtained by shifting the calculated C_d/C_0–θ curve horizontally over the reduced residence

TABLE II
Process Conditions of the Residence Time Distribution Experiments

	Experiment No.				
	1 March 1977	2 October 1977	3 October 1977	4 December 1977	5 December 1977
Number of influent points	1	10	10	10	4
X (kg SS)	1150	1795	3510	6465	5565
% VSS	84	73.5	53.5	38.5	38.5
V_b (m^3)	12	17.3	15.5	20.8	17.7
h_b (m)	2.2	0.39	0.35	0.47	0.40
C_{md} (kg SS/m^3)	18	5.74	17.3	33.6	29.9
q (d^{-1})	~0.5	0.65	0.53	0.73	0.85
F (m^3/h)	7.6	35.5	20.9	24.5	30.5
τ (h)	4.4	5.7	9.8	8.3	6.7
C_{so} (kg C/m^3)	~1.1	0.50	0.94	1.43	1.18
$C_{s,e}$ (kg C/m^3)	~0.05	0.19	0.15	0.10	0.09
$C_{fa,e}$ (kg C/m^3)		0.05	0.05	0.05	0.06
Percent conversion (TOC, centrifuged samples)	~95	61.5	84.0	93.0	93.0
Percent conversion (fa)		91.0	95.0	97.0	95.0
ϕ_s''' (kg C/m^3 d)	~5.8	1.3	1.9	3.8	3.9
ϕ_g (m^3/h)	6.5	11.8	18.8	46.0	40.5
ϕ_g''' (h^{-1})	0.20	0.06	0.09	0.23	0.20
$V_{L,o}$ (m/s)		0.9	0.5	0.6	1.9
$C_{Li,o}$ (calc. 10^{-3} kg/m^3)	11.5	...	...	5.64	8.0
$C_{Li,o}$ (meas. 10^{-3} kg/m^3)	11.76	5.78	9.87	5.61	7.92

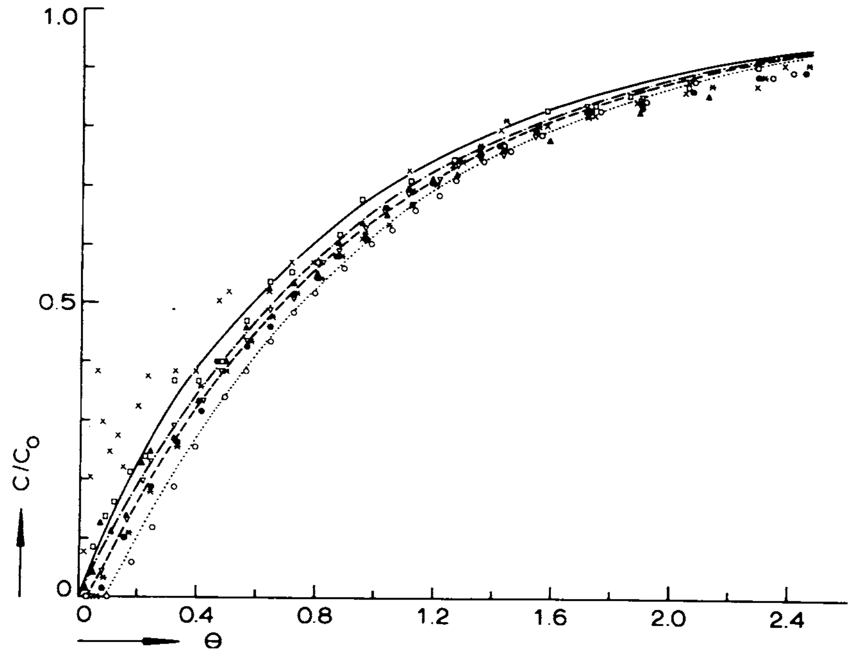


Fig. 5. Results of stimulus-response experiment No. 1 (30-m^3 pilot plant) and the calculated responses of model No. 1. (x, □) sludge bed; (▲, ▿) transitional area; (●, *) sludge blanket; (○) effluent; (—) v_b; (-·-) v_{ta}; (---) v_d; (...) v_{pf}.

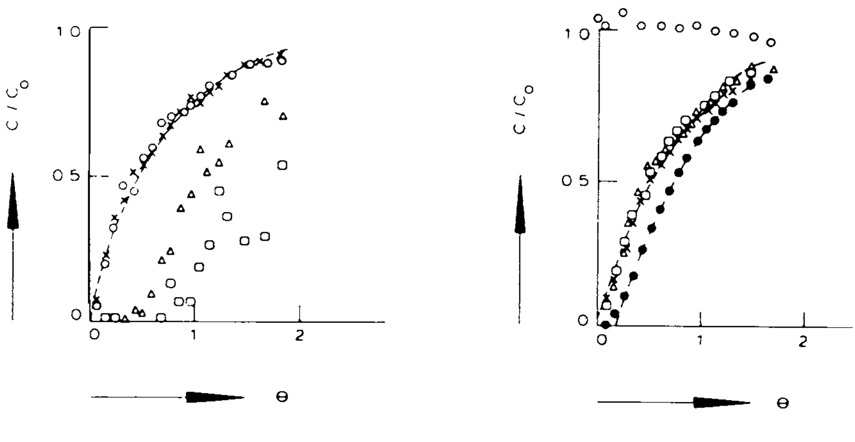


Fig. 6. Results of stimulus-response experiment No. 2 (200-m^3 reactor) and the calculated responses of model No. 2. Figure at the left: (x) SP1; (○) SP2; (□) SP4; (△) SP7; (---) v_b. Figure at the right: (○) influent; (●) effluent; (△) SP3, 6; (□) SP12; (x) SP18; (---) v_b, v_d; (-·-) v_{pf}.

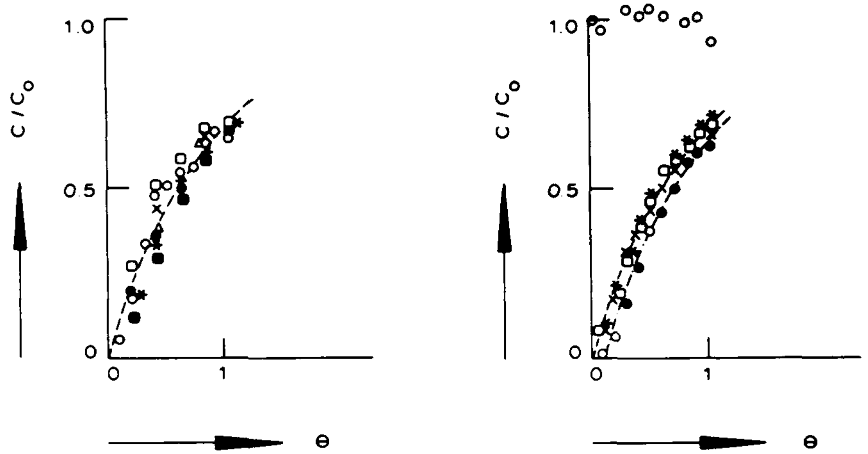


Fig. 7. Results of stimulus-response experiment No. 3 (200-m^3 reactor) and the calculated responses of model No. 3. Figure at the left: (○) SP4; (□) SP7; (●) SPI; (x) SPII; (*) SPIII; (△) SPIV; (■) SPV (all SPs in the sludge bed); (---) ν_b. Figure at the right: (○) influent; (●) effluent; (x) SP5; (*) SP1, 2; (□) SP12, 13; (---) ν_b, ν_d; (-·-) ν_{pf}.

time of the plug flow region ($\theta_{pf} = V_{pf}/V_R$). The responses obtained with five models have been drawn in Figures 5-9.

The volume of the region of dead space in the reactor can be estimated by means of the measured **F** curve, because

$$\int_0^\infty [1 - \mathbf{F}(\theta)]\, d\theta = 1$$

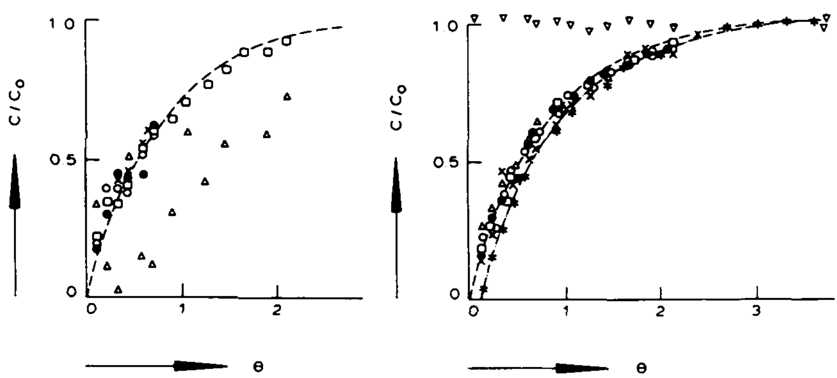


Fig. 8. Results of the stimulus-response experiment No. 4 (200-m^3 reactor) and the calculated responses of model No. 4. Figure at the left: (△) SP4; (□) SP7; (●) SPI; (○) SPII; (x) SPIII (SPs in the sludge bed); (---) ν_b. Figure at the right: (▽) influent; (*) effluent; (●) SP1; (○) SP2, 3; (□) SP6; (△) SP12; (x) SP13, 18; (---) ν_b, ν_d; (-·-) ν_{pf}.

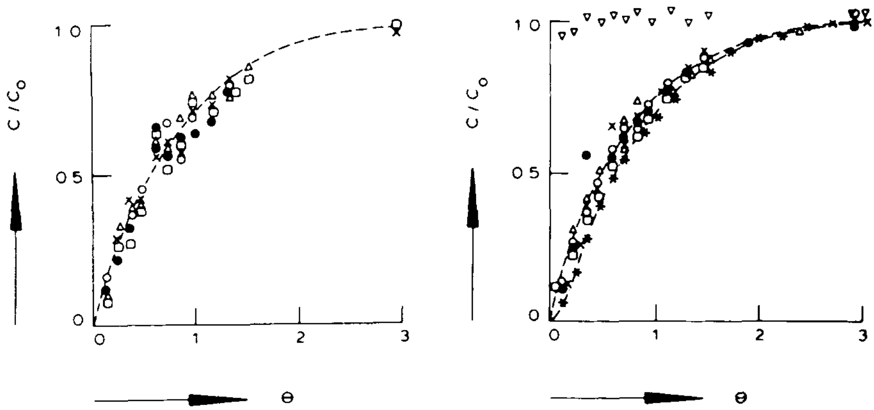
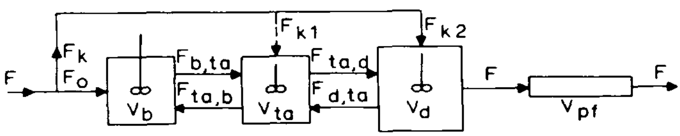


Fig. 9. Results of stimulus-response experiment No. 5 (200-m^3 reactor) and the calculated responses of model No. 5. Figure at the left: (△) SP4; (□) SP7; (●) SPI; (○) SPII; (x) SPIII (SPs in the sludge bed); (---) v_b. Figure at the right: (▽) influent; (∗) effluent; (●) SP1, 2; (○) SP3, 6; (□) SP12, 18; (x) SP13; (△) SP14; (---) v_b, v_d; (-··) v_{pf}.

and this integral is equal to the area between the **F** curve and the line $C/C_0 = 1$ in the C/C_0-θ diagram.[5] If in the experimental results this area turns out to be smaller than unity, the reactor volume occupied by the flowing liquid must have been smaller than the real volume of the reactor. The difference between these volumes is the volume of the region of dead space.



$$V_R = V_b + V_{ta} + V_d + V_{pf}$$

$$F = F_o + F_k$$

$$F_k = F_{k_1} + F_{k_2}$$

$$F_{b,ta} = F_o + F_{ta,b}$$

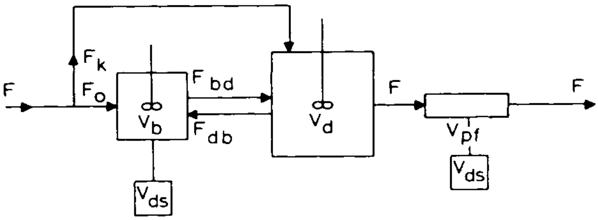
$$F_{ta,d} = F_{b,ta} + F_{k_1} + F_{d,ta} - F_{ta,b}$$

$$\frac{dC_b}{dt} = \{F_oC_o + F_{ta,b}C_{ta} - F_{b,ta}C_b\}/V_b$$

$$\frac{dC_{ta}}{dt} = \{F_{k_1}C_o + F_{b,ta}C_b + F_{d,ta}C_d - F_{ta,b}C_{ta} - F_{ta,d}C_{ta}\}/V_{ta}$$

$$\frac{dC_d}{dt} = \{F_{k_2}C_o + F_{ta,d}C_{ta} - F_{d,ta}C_d - F\,C_d\}/V_d$$

Fig. 10. Block diagram and relations of model No. 1.



$$V_R = V_b + V_d + V_{pf} + V_{ds}$$

$$F = F_o + F_k$$

$$F_{bd} = F_o + F_{db}$$

$$\frac{dC_b}{dt} = \{F_oC_o + F_{db}C_d - F_{bd}C_b\}/V_b$$

$$\frac{dC_d}{dt} = \{F_kC_o + F_{bd}C_b - F_{db}C_d - F\,C_d\}/V_d$$

Fig. 11. Generalized block diagram and relations of model Nos. 2-5.

TABLE III
The Models Representing the Fluid Flow Patterns

	Experiment No.				
	1	2	3	4	5
V_b (% of V_R)	25 ± 5[a]	18 ± 2.5	22 ± 2.5	11 ± 2.5	12 ± 2.5
V_{ta} (% of V_R)	20 ± 5	...	...	...	...
V_d (% of V_R)	50 ± 5	59 ± 5	66 ± 5	66 ± 5	67 ± 5
V_{pf} (% of V_R)	5 ± 2.5	23 ± 2.5	12 ± 2.5	8 ± 2.5	10 ± 2.5
V_{ds} (% of V_R)	0 ± 2.5	0 ± 2.5	0 ± 2.5	16 ± 2.5	12 ± 2.5
F_k (% of F)	60 ± 5	61 ± 5	71 ± 5	86 ± 5	83 ± 5
F_{k1} (% of F)	7 ± 5	...	...	...	...
F_{k2} (% of F)	53 ± 5	...	...	...	...
$F_{ta.b}$ (% of F)	160 ± 50	...	...	...	...
$F_{d.ta}$ (% of F)	160 ± 50	...	...	...	...
F_{db} (% of F)		160 ± 50	160 ± 50	160 ± 50	160 ± 50
Starting values[b]					
V_b (% of V_R)	32	8.5	7.4	10.3	8.6
V_d (% of V_R)	51	65.3	66.4	62.5	65.2
V_{st} (pf) (% of V_R)	17	26.2	26.2	26.2	26.2

[a]The intervals presented are based on the sensitivities of the iterations.
[b]The starting values of the independent parameters in the iterations are also presented in this table, so that they can be compared with the "model values" finally found.

DISCUSSION OF THE RESULTS

The influence of the gas production (ϕ_g and ϕ_g''', Table II) on the fluid flow pattern can be studied by comparison of model Nos. 2 and 3. Model No. 2 is obtained from experiment No. 2 that was performed when the 200-m^3 reactor still was in a start-up situation. The gas production was low because both the organic load and the percent conversion of total organic carbon (TOC) were low. During experiment No. 3 both the organic load and the TOC conversion were higher (compare 1.3 and 1.9 kg C/m^3 d, 61.5 and 84.0%), which resulted in a large production of gas. In the models the effect of the increased gas production can be determined. The volume of the plug flow region (in the settler) decreased from 23% of V_R (model No. 2) to 12% of V_R (model No. 3). This decrease has to be ascribed to the increase of the volumes of the perfectly mixed regions in model No. 2. This leads to the conclusion that, although even at the low gas production per m^3 reactor volume $\phi_g''' = 0.06\ h^{-1}$ (experiment No. 2), the system is already well mixed, mixing improves considerably at higher gas productions. The decrease of the plug flow volume in the settler at higher gas productions will be a result of the turbulent fluid flow in the reactor just underneath the settler. This leads to the recommendation that the settler has to be separated efficiently from the rest of the reactor.

The process conditions during experiment No. 4 differ in two main points from those of experiment No. 3: the gas production is more than twice as high, and the suspended solids concentration in the blanket has increased likewise. The first difference was caused by the high organic load (3.8 kg C/m^3 d) that could be applied at a high TOC conversion (93%), the second difference was caused by the presence of a large amount of mud in the reactor. Comparison of model Nos. 4 and 3 shows that the increased gas production did not have a pronounced effect on mixing: the volumes of the ideally mixed regions had not increased. An important difference is the increase of the bypassing stream F_k in model No. 4. This will have been a result of the larger gas production, but the high sludge concentration may also be an important cause of this difference. This conclusion follows from the observation that in model No. 4 a region of dead space is present (16% of V_R) that did not occur in model No. 3. Dead space must have been caused by the increased solids concentration (increased blockage of fluid flow) and since dead spaces always are more or less accompanied by bypassing flows, the increase of F_k will partly be caused by the increased concentration of suspended solids. If one considers the bypassing flow in model No. 2, one sees that it was smaller than in model No. 3. This difference will be due to the smaller gas production in experiment No. 2, giving a smaller upward transport of fluid behind the rising gas bubbles. It can thus be concluded that an increased gas production gives improved mixing but also increased bypassing and that increased concentration of suspended solids leads to the presence of dead space and therefore also to increased bypassing.

For a better knowledge of the influence of the influent distribution on the fluid flow, two comparisons can be made. First, the results of the experiment with the 30-m^3 reactor in which a unidirectional influent distribution system was used as reported here can be compared with the results with radial influent distribution reported earlier.[1] If this is done, there appears to be no essential difference between the two models derived for these experiments. This leads to the observation that the radial influent distribution system and the unidirectional system give quite similar fluid flow patterns. From this comparison, no preference for one of the two systems could be derived.

Comparison of the results of experiment Nos. 4 and 5 should give an idea of the number of influent points that could best be used in the 200-m^3 reactor. Since the models differ only in details, it can be stated that only four influent points (B, D, G, and J; see Fig. 3) instead of ten influent points can be used without significant distribution problems.

The last effect to be discussed, the effect of the reactor scale on the fluid flow pattern, can be studied by comparing the results of the experiments in the 200-m^3 reactor with those of the experiments in the 30-m^3 reactor. The model found for the fluid flow pattern in the 30-m^3 reactor (model No. 1) contains one ideal mixer for the bed, one for the blanket, and one for the area of transition between the bed and the blanket. For the fluid flow pattern in the 200-m^3 reactor, "two mixer models" have been found: one ideal mixer for the bed and one for the sludge blanket. Another important difference is the magnitude of the bypassing stream in the 200-m^3 reactor that was much larger than in the 30-m^3 reactor. Both differences stem from the low height of the sludge bed in the 200-m^2 reactor. It might well be that if this reactor had contained sufficient sludge so that the bed height would have been 1.5–2.5 m, for the fluid flow pattern in this reactor "three mixer models" might have been found also. With respect to the difference in magnitude of the bypassing streams, it will be clear that at the influent points in the 200-m^3 reactor the entering fluid streams will break easily through a sludge bed of only 0.4 m high.

Close examination of the tracer curves measured in the 200-m^3 reactor shows that quite some scattering existed between the data measured at different places in the bed. This indicates that, rather than one ideal mixer for the bed, a combination of several ideal mixers would give a more appropriate description of the physical reality with respect to the fluid flow pattern in the bed. Of course, if the distribution of the influent over the bottom of the reactor would have been uniform, no distinction should have been found between the tracer curves measured in different places in the bed. However, instead of several parallel mixers and bypassing streams one ideal mixer and one bypassing stream apparently seemed to be sufficient for the description of the fluid flow pattern in the bed. From the scattering found in the 200-m^3 reactor, it can be concluded that the influent was not completely uniformly distributed over the bottom. This fact, and the similarities and differences between the models for the fluid flow in both reactors that have been discussed, lead to the indication that scaling-up of an upflow reactor for anaerobic treatment of wastewater in horizontal directions is quite feasible. Provided that the influent is well distributed inside the reactor (e.g., by means of several influent points), scaling-up is not restricted to a maximum.

CONCLUSIONS

1) At the influent concentration and liquid residence times applied (respectively 0.50–1.43 kg C/m^3 and 4.4–9.8 h), the gas produced by the anaerobic bacteria provides good mixing of the fluid inside the reactor. In order to prevent turbulence in the settler on top of the reactor, this region has to be shielded effectively from the region in the reactor where the anaerobic conversions take place. This could be effected by means of plates placed at an inclination of 45° (see Fig. 1).

2) High concentrations of mud in the blanket ($C_{md} > 30$ kg SS/m^3) may cause extra thickening of the suspended solids in the bed, resulting in the occurrence of dead space regions.

3) Upflow reactors for anaerobic treatment of wastewater should contain so much sludge that the height of the sludge bed is 1.5–2.5 m. At bed heights of 0.4 m most of the influent bypasses the bed.

4) Scaling-up of upflow reactors in horizontal directions is no problem if the influent distribution has been given special attention. With the present knowledge the optimum reactor height seems to lie between 4 and 6 m.

Nomenclature

b	width (m)
C	concentration (kg m^{-3})
d	diameter (m)
F	flow (m^3 h^{-1})
$\mathbf{F}$	response on an up-step stimulus (dimensionless)
h	height (m)
l	length (m)
q	specific substrate conversion rate (s^{-1})
t	time (s)
v	velocity (m s^{-1})
V	volume (m^3)
X	amount of sludge (kg)
ϕ_g	volume of gas per unit of time (m^3 h^{-1})
ϕ_g'''	volume of gas/m^3/time (h^{-1})
θ	relative time (dimensionless)
τ	residence time (h)

Subscripts

b	sludge bed
bd	from bed to blanket
d	blanket
db	from blanket to bed
ds	dead space

e	effluent
fa	fatty acids
g	gas
k	bypassing
L	linear
Li	lithium
m	solids; sludge
mb	sludge from the bed
md	sludge from the blanket
o	influent
pf	plug flow
R	reactor
s	substrate
st	settler
ta	area of transition

The authors are thankful to the "Centrale Suiker Maatschappy" and the Department for Public Health and Environmental Hygiene for the help and finances that made this study possible, and to G. J. Keilman and Y. H. G. Lok for their preliminary work.

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Accepted for Publication July 14, 1981