1 Short test (Solution) (40)

(1) (20) Solve the linear system of equations using Gauss-Jordan elimination.

$$\begin{aligned} 3x + 3z &= 0 \\ 2x + 2y + 6z &= 2 \\ x + y + 3z &= 2 \end{aligned}$$

Solution. The augmented matrix of the system is

$$\left[\begin{array}{ccc|c} 3 & 0 & 3 & 0 \\ 2 & 2 & 6 & 2 \\ 1 & 1 & 3 & 2 \end{array}\right].$$

Since the 2nd row of the coefficient matrix A is a multiple of the 3rd row of A the determinant is 0. Hence A is not invertible and we expect either infinitely many solutions or no solution. By Gauss-Jordan elimination we obtain that

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

and thus the system has no solution. □

(2) (20) Calculate the determinants. Think first calculate less, time is money!

(a) $\begin{vmatrix} 2 & -3 & 1 & 0 & 4 \\ 0 & 3 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 8 & 2 & 3 & 1 \end{vmatrix}$

Solution. Using cofactor expansion along first column and then along first row we obtain subdeterminants of 3x3 matrices which are easy to calculate. Namely,

$$\begin{vmatrix} 2 & -3 & 1 & 0 & 4 \\ 0 & 3 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 8 & 2 & 3 & 1 \end{vmatrix} = 2\begin{vmatrix} 3 & 0 & 2 & 0 \\ 0 & 1 & 3 & 2 \\ 1 & 0 & 1 & 0 \\ 8 & 2 & 3 & 1 \end{vmatrix} = 2\cdot 3\begin{vmatrix} 1 & 3 & 2 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{vmatrix} + 2\cdot 2\begin{vmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 8 & 2 & 1 \end{vmatrix} = -6.$$

Alternatively, we could use row reduction as follows

$$\begin{vmatrix} 2 & -3 & 1 & 0 & 4 \\ 0 & 3 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 8 & 2 & 3 & 1 \end{vmatrix} = -\begin{vmatrix} 2 & -3 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 2 & -5 & 1 \end{vmatrix} = -\begin{vmatrix} 2 & -3 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{vmatrix} = -6.$$ □

(b) *Solution.* After short inspection we notice that the matrix in (b) can be obtained from the matrix in (a) by interchanging 3rd and 5th row. Hence using the properties of determinants we conclude that the determinant is 6. □