Inference for Single Proportion - part 2

Chapter 16

Medical consultant example...

A consultant tried to attract patients by noting the average complication rate for liver donor surgeries in the US is about 10%, but her clients have had only 3 complications in the 62 liver donor surgeries she has facilitated. She claims this is strong evidence that her work meaningfully contributes to reducing complications (and therefore she should be hired!).

$$\hat{p} = \frac{3}{62} = 0.048$$

State the null hypothesis

Claim is that $\hat{p}$ is less than then national average of 0.1

- $H_0 : p = 0.1$
- $H_A : p < 0.1$

Discernment level?

- Type I error: mistakenly reject H_0 when it's true
  - Misled by false advertising!
  - Make α smaller to avoid
- Type II error: mistakenly fail to reject H_0 when it's false
  - Being cautious leads to missing out on above average care.
  - Make α larger to avoid

Check conditions for CLT

- Independent? yes
- Large enough sample size? no!

  $np_0 = 62 \cdot 0.1 = 6.2$

  $n(1 - p_0) = 62 \cdot 0.9 = 55.8$

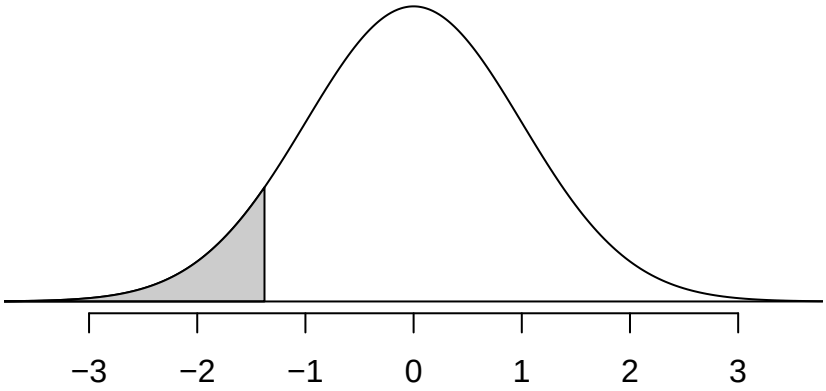
What happens if we ignore this?

Calculate Z

$$SE = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.1 \cdot 0.9}{62}} = 0.038$$

$$Z = \frac{\hat{p} - p_0}{SE} = \frac{0.048 - 0.1}{0.038} = -1.37$$

Find p-value for Z = -1.37




```
pnorm(-1.37)
```

```
[1] 0.08534345
```

Compare to discernment level

Suppose $\alpha = 0.1$ (trying to avoid Type II errors)

Since p-value (0.085) is less than α, we do have evidence to reject the null hypothesis.

We think that our consultant's complication rate is significantly below the national average.

BUT WAIT!

CLT isn't valid!!

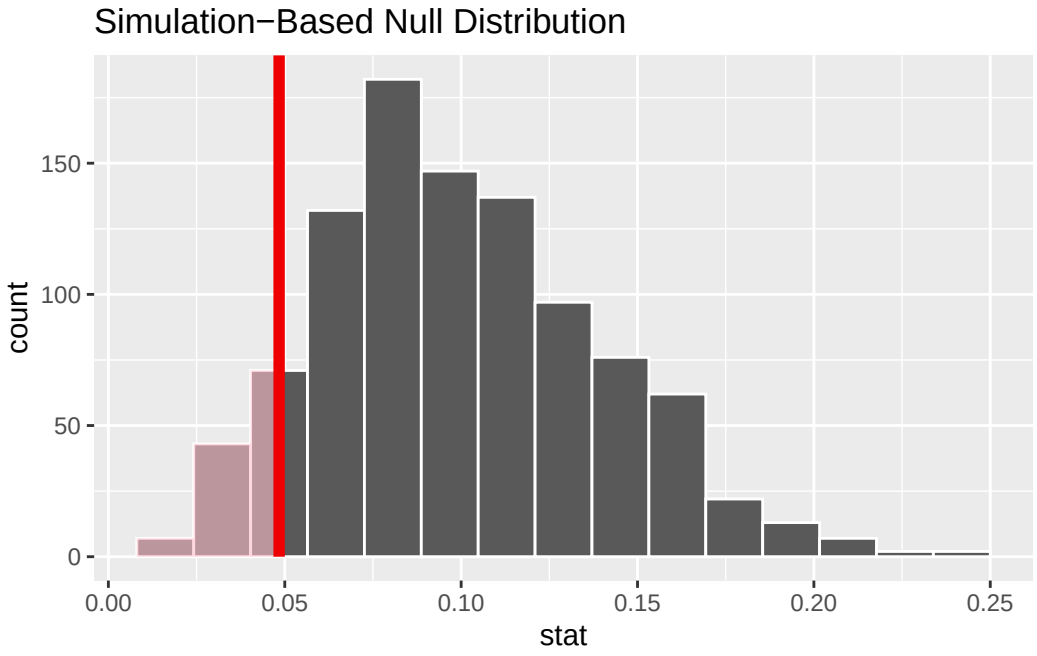
Do a simulation!!

```
set.seed(2024)

null_dist <- organ_donor |>
  specify(response = outcome, success = "complication") |>
  hypothesize(null = "point", p = 0.1) |>
  generate(reps = 1000, type = "draw") |>
  calculate(stat = "prop")
```

Simulated Null-Distribution

```
visualize(null_dist) +
  shade_p_value(obs_stat = 0.0484, direction = "less")
```



Actual p-value?

```
get_p_value(null_dist, obs_stat = 0.0484, direction = "less")
```

```
# A tibble: 1 x 1
  p_value
    <dbl>
1   0.121
```

- Not less than $\alpha = 0.1$
- Previously had found $p = 0.085$
- conclusion depends on discernment level as well as how we found p-value