MTH 304
Complex Functions

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Chapter 1

Welcome!

I'm looking forward to getting to know each of you this semester and it is my sincere goal to make our time together (in class and out) productive and engaging. I am here to help you succeed, not just in learning the material but in growing as a learner. Learning takes effort, a willingness to try something that may not work, and an openness to learning from feedback. My role is to help create an environment in which all of you are supported in these aspects.

I want you to feel comfortable coming to me with any questions or concerns that arise, mathematical and otherwise. If you encounter any issues that interfere with your learning, whether they are physical, mental, emotional, economic, or otherwise, or if you experience discrimination or mistreatment of any sort, please contact me immediately.

Chapter 2

Course at a Glance

2.1 Class Schedule

Tue-Thu, 2:30-3:55 DB 233

2.2 Instructor Info

Chris Hallstrom, PhD (he/him)

- **BC 270**
  Feel free to stop by at any time or check the Section 4.1 schedule.

- hallstro@up.edu
  Email is the best way to contact me. I will do my best to get back to you as soon as I can, but please be aware that I do not check my email regularly in the evenings or on weekends.

- **Calendly**
  If you would like to schedule a meeting (in-person or virtual), you can use my Calendly scheduluer. If you'd like to meet at time that you don't see available on Calendly, feel free to check with me via email!

2.3 Syllabus and Class Materials

Additional course information can be found on this webpage. All course materials and assignments will be posted on our class Moodle page..

2.4 Text

We will be using a set of inquiry-oriented notes *Complex Functions* by Joshua Bowman. These notes are posted on our Moodle page.

Inquiry Method

Becuase we are using an inquiry method in this class, I will ask you to refrain from using any outside resources for this class – at least as they pertain to the material. You're welcome to look up facts and identities from prior math courses - e.g. calculus - if you think that would help. For more information about inquiry-based pedagogy, see Section 3.

2.5 Technology and Other Resources

Some of the work we will do in this class will benefit from the use of technology such as Desmos. I highly recommend that you create a (free) Desmos account so that you can save your work. If you have questions about how to use any of these tools, please let me know. If you do use technology in your work, please indicate this in your written solutions.

A word on search engines

Search engines (e.g. Google) use algorithms to determine what content on the internet is likely to be relevant to your search query. They cannot find information that is not on the internet, and search results may be affected by hidden parameters introduced by the companies that run them. Whether this content actually is relevant (or correct) is up to you to figure out.

A word on GenAI

AI text generators (ChatGPT, Copilot, Gemini, etc.) use Large Language Models (LLMs) to produce text that is likely to resemble a human-produced response to a given prompt. They are most likely to produce a meaningful output on topics where a great deal of human-produced text has already been written (which happens to include most of undergraduate mathematics). This output, however, is based on statistical predictions from contextual clues, not logical deductions or expert knowledge and there's no guarantee that it's mathematically correct!

My general feeling about the use of AI tools in the context of this course is that they fundamentally get in the way of your learning in several ways and should **not** be used in any capacity.

- They are prone to both plagiarizing as well as making things up. Since their responses are produced from an amalgamation of many sources (sometimes without the permission of the original authors) they invariably use definitions, notation, assumptions, and concepts that we have not introduced in our class.
- AI tools undermine the collaborative learning community that we would like to develop. A significant way in which we will be building understanding in the class is through discussion with each other.
- Using AI tools means that you are cheating yourself out of an opportunity to learn. The whole point of being in this class is to learn how to use certain ideas and tools to solve problems. Note that I didn't say that the point is to solve problems. This is a very much like the difference between going to a restaurant vs. learning how to cook.

- Using AI tools undermines the relationship between student and instructor. When a student gives me work that seems AI generated, it creates an adversarial dynamic and now I'm looking at everything that student gives me with suspicion. If you ever feel tempted to use AI tools to shortcut your learning, please pause and come talk to me about finding a different strategy.
- Apart from doing mathematics, students sometimes use AI tools to help with writing (and here I'm also thinking of tools like Grammerly). Here's a secret – AI-assisted writing is not good writing, period. It might be gramatically "correct" but that's not the same thing. Good writing involves actual thinking by a unique individual person.

So where does that leave us? I really do not want to lay down a policy of "THOU SHALL NOT..." because I'd much rather simply trust you. So that's what I'll do - you now know how I feel about using AI tools in this class and I will trust that you will respect me, your classmates, and our goals for being here together enough that you'll honor that trust.

Chapter 3

Inquiry Based Learning

We will use a method of instruction sometimes called **Inquiry Based Learning** (IBL) which is designed to engage and foster skills and habits that working mathematicians use regularly; you will be asked to solve problems, make conjectures, experiment, explore, create, collaborate, and communicate your work with your peers. Rather than giving you facts to memorize or showing you clear paths to solutions, my role is to guide you via a sequence of carefully chosen problems through a journey of mathematical discovery.

Throughout the semester, I will give you lists of definitions to interpret and make sense of, as well as a sequence of Tasks which you and your classmates will complete. These tasks are carefully chosen to build up your understanding of the course material. There will be very little traditional lecture. Instead, class time will consist largely of student presentations of new material. For best results, you should come to class prepared to share your work or ideas about that day's problems. This method of inquiry does not work very well if you're looking at a problem for the first time in class.

I will ask you to share your solutions in class regularly, typically in smaller groups. You are encouraged to critique the problems (reformulating them if needed), to generate examples and counterexamples to theorems or conjectures, to formulate new theorems based on what you've learned, and to prove or disprove these conjectures. When observing another student's presentation, it is your responsibility to follow their argument closely and decide if they have seems reasonable. If you cannot follow their logic, or have questions about their solution, it is your responsibility to ask!

A key feature of the IBL method is student **discovery** and for that reason **outside resources are not allowed.** This means that you should not consult texts (other than the one handed out in class), the internet, students not currently enrolled in the course, or faculty other than myself. Consulting outside resources will only deprive yourself of opportunities to engage with the material authentically.
You are encouraged to work with your classmates on the problems, but for best results you should get as far as you can on your own before collaborating. It's important that you do not feel overwhelmed – so please let me know if you're stuck on a problem and I'll be happy to give you hints.

Chapter 4

Student Support

Throughout the semester, I expect that you will have questions. This might happen while doing homework or reviewing your notes, or perhaps a question came up during class that didn't get fully resolved.

You might also find that you don't have a specific question but just feel generally confused or uncertain about the stuff we're doing in class. That's okay too - that's what learning is all about!

For these reasons, it's important that you know how to get help outside of class.

> Pro Tip
>
> If you **don't** have questions, let me know that also – the whole point of the class is to engage with the material at a deep enough level that there **should** be questions! That's how learning works!

4.1 Drop-In Hours

These are specific times that I've set aside during the week when I'm available for drop-in help. You do not need to let me know you're coming – just show up at my office (BC 270).

MW	10:00 - 11:00
TR	11:30 - 1:00

Many students find these drop-in hours can be particularly helpful if you are working together with classmates. You can work together at one of the tables down the hall from my office and just pop in when you have questions.

> Note
>
> Since I'm setting these times before the semester actually begins, these times may change if my schedule changes. If it turns out that I need to adjust them, I will let you know!

4.2 Sign-Up Hours

I recognize that your schedule might not allow you to stop by during the posted drop-in hours. Or you might simply find it more convenient to meet with me at a different time. No problem! If you go to my Calendly scheduler, you can sign-up for a different time slot to meet, either in-person or via zoom. If there is a specific time that works for you and you don't see it available on the Calendly scheduler, please reach out to me via email and we will find a time that works for your schedule.

4.3 Open Door Policy

I have the scheduled drop-in hours simply to give you some times when you know that I'll be available – but you are **always** welcome to stop by my office **at any time**. Unless I'm in class or in a meeting, I am generally available to meet with you!

Chapter 5

Course Content

MTH 304 introduces the fundamental ideas and tool of complex variables and functions. A large part of this course is devoted to extending basic concepts from calculus to this new framework. Along the way, we'll get a glimpse how connecting the geometry of complex functions to ideas of calculus has far reaching consequences. Indeed, the study of complex variables opens up whole new avenues of research in a huge array of topics including algebraic geometry, analytic number theory, dynamical systems, signal analysis, and fluid mechanics just to name a few.

Specific topics that we'll cover in this course include

- Representations of complex numbers
- Fundamental complex-valued functions
- Topology and limits in the complex plane
- Derivatives and holomorphic functions
- Series and analyticity
- Contour integrals
- Cauchy-Goursat theorem
- Cauchy integral formula
- Residue theorem

In addition to these specific *mathematical* topics, our learning goals also include:

- communicating mathematics effectively
- developing strategies for solving challenging, multi-step problems
- developing a practice of self-reflection on one's own learning.

Certain affective traits (i.e., dispositions and habits that are useful for all learners, regardless of topic) are also valued, such as persistence and growth mindset, and the development of these qualities will be fostered by the course.

Chapter 6

Course Structure

In broad strokes, this course is designed to engage you with rich problems in a collaborative setting. We will work in small groups on activities that engage the material – solving problems and sharing your learning with your classmates. Outside of class, you will have Tasks to practice what we've learned and to stretch your problem solving skills. You will also be asked to reflect on your learning – mathematics is as much about the problem solving process as it is about finding a correct answer and so reflecting and writing about your work is an important part of what we'll do.

6.1 Attendence

Since we will be doing much of our learning through collaborative in-class activities, it's important that you to come to class prepared to engage and participate. That said, I also recognize that for many reasons, this will not always be possible. If you need to miss class **for any reason** just let me know – there is no penalty apart from missing out on that day's activity. If you miss class often, you can expect me to reach out to see how I can support you coming to class.

Poem 013: Did I Miss Anything? by Tom Wyman

I will do my best to post on Moodle a short summary of what we do in class, so if you do miss class, you should look there to see what we did that day. You may also find it helpful if you're able to check-in with a classmate to see what we covered. Finally, I'm always available in office hours to discuss anything you missed.

6.2 Daily Tasks

Your standing assignment is to do the Tasks in the notes. You should work ahead so that you come to class having completed - or at least worked on - the Tasks for that day, ready to discuss or present your solutions.

While there is no requirement that everyone present their work, it is highly encouraged. The act of communicating your ideas to your classmates is a highly effective way to solidify and deepen your understanding of the material.

While my hope is that this class is a safe and supportive learning environment in which everyone feels welcome to share their ideas, I recognize that some students may have some anxiety around sharing their work. If this is the case, please let me know and we can find alternative ways for you to participate.

6.3 Written Homework

Most every week, I will assign a short set of problems to write up and hand in. These might be new problems or they might be problems that we've seen in class already (i.e. daily Tasks).

This written homework serves a few different purposes. In many cases, it's an opportunity to refine or fill in any gaps of solutions that came up in class. In other cases, it gives you a chance to extend or apply ideas from class to new problem. Finally, homework is a place to practice communicating your understanding to others. Your goal in completing the homework is not just to find a solution, but to convince your reader that your solution is reasonable.

Homework is an opportunity to be challenged, make mistakes, struggle productively, and learn how to proceed in the face of uncertainty. Overall, the benefit lies as much in the process as it does in producing a correct answer.

Feedback and Revisions

Weekly homework will not be graded in the usual sense - instead, I will provide feedback that addresses your progress towards learning the material, as well as your effectiveness in communicating your ideas. If you would like to revise any of your homework problems after getting feedback, you may hand it back in for further feedback **within one week**.

6.4 Quizzes

We will likely have a few short in-class quizzes, generally at the end of each chapter to help you assess your progress.

6.5 Due Dates

Just like in the "real world", due dates in this class exist and are important but there's usually a certain amount of flexibility. Life happens and sometimes circumstances may prevent you from completing an assignment on time. Or maybe you simply want a little more time to finish. For these reasons, you can (almost) always request an extension – just send me an email. Note that if you ask for lots of extensions, you can expect that I will reach out to see if we can work together to find ways for you to keep up with the work in the course.

That said, there are reasons for having due dates. For one thing, they help you organize your life by helping you know how to budget your time. For another, they help me organize my life – in particular, they help me keep up on getting your work back to you. If you hand in work after the due date, I cannot guarantee that I will get it back to you with useful feedback.

Hard Deadlines

To prevent work from backing up, there are two **hard** deadlines in this course - one is February 28th (the Friday before spring break) and the other is April 21st (the last day of classes). Late work will not be accepeted after those dates, except by prior arrangment.

6.6 Portfolio

As we proceed through the course material, I will periodically ask you to select examples of your work that demonstrate your understanding of the material. By the end of the semester, you should have a carefully curated collection of work that tells the story of you as a mathematician and student of complex variables and which provides evidence of your learning throughout the semester. Assembling your portfolio is also a way to reflect on the course as a whole.

- **Due date** Your portfolio should be complete by the last day of classes, April 25th. If you need more time, please let me know ahead of time.
- **What do I include in the portfolio?**
  - Table of contents. List all items in the portfolio in the order they are included. Include the number and name of each Task or Homework Problem, or a title for any other items (e.g. Presentation of Task 54 on 2/23).
  - Artifacts that provide evidence of your learning. These will likely include many of your homework problems or daily Tasks, but can include other items too. For example:
    - Did you have great work on a daily prep, but didn't present it? Share your notes!
    - Did you give a presentation of a particularly significant problem? Include it!
    - Did you work on optional problems, or Tasks that we skipped or didn't get to?
    - Did you research an application or theorem that we didn't cover?
  - Include with each artifact a brief explanation of how it reflects your learning (e.g. what particular topics are represented) and why you selected that particular piece of work.
- **Revisions** You can revise any homework or other artifacts again before including them in the portfolio. Your goal is to show how you've met the criteria for each grade by the end of the semester. If something confused you early on, but you've figured it out, that's great – show me!

6.7 Final Reflection

Instead of a final exam, I will ask you to complete a final self-evaluation of your learning in this course. The goal is give you an opportunity to summarize what you've learned and how your understanding of the material has evolved over the course of the semester.

Chapter 7

Grades

> Extrinsic motivation, which includes a desire to get better grades, is not only different from, but often undermines, intrinsic motivation, a desire to learn for its own sake.
>
> – Alfie Kohn, "The Case Against Grades"

Grades, as they are traditionally thought of, are inherently imprecise and don't represent a full picture of your growth and learning over the course of a semester. Worse than that, research suggests that grades undermine the learning process in several ways:

- Grades tend to diminish interest in what you're learning.
- Grades create a preference for the easiest task. In other words, students tend to do what they need to get a certain grade, but no more.
- Grades tend to reduce the quality of student thinking. The moment we ask "**how** am I doing?'' we lose track of **what** we're doing.

7.1 Qualitative Assessment

For these reasons, we will focus on **qualitative** not quantitative assessment. I will not be grading individual assignments but will instead give you feedback and comments that engage your work rather than simply evaluate it. After addressing that feedback, you're encouraged to resubmit for further feedback.

Throughout the semester, I will periodically ask you to reflect carefully on your work and to evaluate your progress. You will collect evidence of your understanding of the course content and based on that evidence, you will be asked to suggest a final course grade. In this way, we will determine your your grade collaboratively.

The intention here is to help you focus on learning in a way that is more organic, as opposed to simply working as you think you're expected to. If this process causes more anxiety than it alleviates, please see me at any point to confer about your progress in the course – I'm always happy to talk with you about your learning!

Here some of the many ways that you might demonstrate your understanding of the course material:

- Keep up with the daily Tasks and come to class ready to discuss and/or present your work.

- Submit weekly homework, including revisions that incorporate feedback if needed, that reflects understanding of the class content.
- Submit a portfolio that demonstrates your understanding of the course content as a whole body of knowledge, making connections between topics covered over the semester.

While I am required to submit a grade for each student at the end of the semester – I will do what I can to de-emphasize the role of grades in this course so that as much as possible our focus is on learning.

7.2 Grade Guidelines

If find it helpful to start with a qualitative description of what particular grades represent.

A

This grade generally indicates superior work that demonstrates a deep understanding of all of the course the material such that you could apply the material to new or unfamiliar situations. You can consistently demonstrate this understanding in a wide variety of ways.
A very good way to demonstrate an ability to extend your understanding to new topics is work beyond the material in the course notes. There are lots of applications of complex functions relevant as well as many interesting mathematical topics with connections to complex analysis. If you would like to explore additional topics, let me know and I can point you to additional resources.

B

This grade indicates good work that is eminently satisfactory. You will be able to use and extend this knowledge in many situations. To earn a B, you should consistently and/or frequently meet many criteria from the list above.

C

This grade indicates competent work that demonstrates an acceptable level of knowledge relevant to the course. To earn a C, you might consistently meet only a few of the criteria listed above or else meet several criteria but less often. In other words, maybe you do a few things very well and consistently, or else do many things fairly well but less often.

D/F

These grades represent a fundamental breakdown of expectations. A D represents a meaningful but unsuccessful attempt at earning a C or above. An F represents such a severe lack of engagement, effort, or understanding that there is no evidence of meaningful progress.

7.3 Engagement

Although your course grade will be based primarily on your **understanding** of course content and not on course engagement, in my experience these typically go hand in hand. So while engagement in the course is not itself evidence of understanding, it does usually help us achieve that goal.

Here are some ways that you can engage with the class:

- Come to class regularly, prepared to discuss that day's Tasks. Work to make sense of new definitions or theorems (even if incorrrect!). Try multiple approaches; include errors and comments on your thought process.
- Volunteer to present your work. If you're not comfortable sharing your work with the whole class, you can share it in groups or during drop-in hours.
- Actively participate in class discussions and group work. This can take many forms – e.g. sharing your own work, asking questions, or helping to facilitate the discussion.
- Complete assignments on time. Actively seek out resources to catch up if you miss class or if you have questions.
- Support classmates and help them succeed.

Chapter 8

Writing Mathematics

One of the goals of this course is to hone your skill in communicating complex technical ideas, particularly in writing. Your aim is not simply to come up with a correct solution, but you must also convince someone else that you know what you're talking about!

A good solution is one that effectively communicates not only the mathematical ideas needed, but an explanation of how you got there. Like any piece of writing, you should expect to write, proofread, and edit multiple drafts of your work.

8.1 General Guidelines

The following are some general suggestions for effective communication in mathematics.

- Write using complete sentences. In particular, all mathematical notation and expressions should be part of a sentence. You can tell that you've done this adequately if you can read what you've written aloud - verbatim - and it makes sense.
- Don't rely on symbols and notations alone, use words to explain how you're getting from one step to the next.
- Think about who your audience is and include an appropriate amount of detail. Your reader should be convinced that your solution is correct without having to fill in any missing steps.
- Respect your reader. This means that your solution should clear, easy to read, and free of spelling and grammatical errors, even if the mathematical steps are correct. Don't ask them to put in more work than necessary to understand your solution!
- You should first work out the solutions on scratch paper and then neatly write up your solutions. Do not hand in your scratch paper.

8.2 Specific Instructions

Here are some specific instructions regarding how you should submit your work.

- Papers with multiple pages should be stapled in the upper left hand corner. If your paper has been torn out of a spiral notebook, remove the fringes.

- Writing should be neat, legible, and not too small. If your handwriting is not up to the task, you might consider using a typesetting enviornment such as LaTeX or Typst. I can help you get started with these tools.
- Include your name, the date, the class and section number, and the name of the assignment.
- Problems should be clearly labeled and numbered on the left side of the paper. Leave a visible separation between problems. Write the problems in order.
- Begin your solution with the original problem statement (or a paraphrasing of such). Do not assume that your reader has the problem statement in front of them.
- Use whitespace on the margins and between problems to make your solutions easier to read. Leave room in the margins for comments.
- If you worked with other students on a problem, acknowledge their contributions.
- If you used technology on a problem, acknowledge where and how.
- Assignments submitted electronically should be submitted as a single PDF file. If scanned, make sure your images are clear and not cropped. Take care that pages are in the correct order and are rotated to the proper orientation. File names should include some identifying information such as your last name and what assignment this is (imagine this file in a folder with submissions from other students).

Chapter 9

University Policies

9.1 Academic Integrity

The University of Portland is a diverse academic community of learners and scholars who are dedicated to freely sharing ideas and engaging in respectful discussion of those ideas to discover truth. Such pursuits require each person, whether student or faculty, to present truthfully our own ideas and give credit to others for the ideas that they generate. Thus, cheating on exams, copying another student's assignment, including homework, or using the work of others without proper citation are some examples of violating academic integrity.

Especially for written and oral assignments, students have an ethical responsibility to properly cite the authors of any books, articles, or other sources that they use. Students should expect to submit assignments to Turnitin, a database that ensures assignments are original work of the student submitting. Each discipline has guidelines for how to give appropriate credit, and instructors will communicate the specific guidelines for their discipline. The Clark Library also maintains a webpage that provides citation guidelines at libguides.up.edu/cite.

The misuse of AI to shortcut course learning outcomes will be treated as a violation of academic integrity comparable to plagiarism or cheating. Faculty are responsible for including a written "Course AI Policy" in their syllabi that clearly states what they consider appropriate and inappropriate uses of AI in the context of their courses. Students are responsible for using AI in ways that do not detract from the established learning outcomes of the course. All members of the scholarly community are responsible for demonstrating sound judgment in discerning when and how to utilize AI in their work, upholding standards of citation, originality, and integrity.

9.2 Assessment Disclosure

Student work products for this course may be used by the University for educational quality assurance purposes. For reasons of confidentiality, such examples will not include student names.

9.3 Accessibility

The University of Portland strives to make its courses and services fully accessible to all students. Students are encouraged to discuss with their instructors what might be most helpful in enabling them

to meet the learning goals of the course. Students who experience a disability are encouraged to use the services of the Office for Accessible Education Services (AES), located in the Shepard Academic Resource Center (503-943- 8985). If you have an AES Accommodation Plan, you should meet with your instructor to discuss how to implement your plan in this class. Requests for alternate location for exams and/or extended exam time should, where possible, be made two weeks in advance of an exam, and must be made at least one week in advance of an exam. Also, if applicable, you should meet with your instructor to discuss emergency medical information or how best to ensure your safe evacuation from the building in case of fire or other emergency. All information that students provide regarding disability or accommodation is confidential. All students are responsible for completing the required coursework and are held to the same evaluation standards specified in the course syllabus.

9.4 Mental Health

Anyone can experience problems with their mental health that interfere with academic experiences and negatively impact daily life. If you or someone you know experiences mental health challenges at UP, please contact the University of Portland Counseling Center in the upper level of Orrico Hall (down the hill from Franz Hall and near Mehling Hall) at 503-943-7134 or hcc@up.edu. Their services are free and confidential. In addition, mental health consultation and support is available through the Pilot Helpline by calling 503-943-7134 and pressing 3. The University of Portland Campus Safety Department (503-943-4444) also has personnel trained to respond sensitively to mental health emergencies at all hours. Remember that getting help is a smart and courageous thing to do – for yourself, for those you care about, and for those who care about you. For more information on health and wellness resources at UP go to www.linktr.ee/wellnessUP.

9.5 Non-Violence

The University of Portland is committed to fostering a safe and respectful community free from all forms of violence. Violence of any kind, and in particular acts of power- based personal violence, are inconsistent with our mission. Together, all UP community members must take a stand against violence. Learn more about what interpersonal violence looks like, campus and community resources, UP's prevention strategy, and what we as individuals can do to assist on the Green Dot website. Further information and reporting options may be found on the Title IX website.

9.6 Ethics of Information

The University of Portland is a community dedicated to the investigation and discovery of processes for thinking ethically and encouraging the development of ethical reasoning in the formation of the whole person. Using information ethically, as an element in open and honest scholarly endeavors, involves moral reasoning to determine the right way to access, create, distribute, and employ information, including: considerations of intellectual property rights, fair use, information bias, censorship, and privacy. More information can be found in the Clark Library's guide to the Ethical Use of Information.

9.7 The Learning Commons

Students may receive academic assistance through Learning Commons tutoring services and workshops. The Co-Pilot peer tutoring program provides students with opportunities to work with other students to get help in writing, math, group projects, and many other courses. Schedule an appointment to meet with a Co-Pilot (tutor) by visiting the Learning Commons website. Students can also meet with a Co-Pilot during drop-in hours. Check the Learning Commons website or stop by the Learning Commons in BC 163 to learn more about their services. Co-Pilots are a wonderful support along your academic journey.