MTH 401 :: Real Analysis Fall 2024

Welcome!

Before we get into the details of the course, I want to share with you a few thoughts about my general approach to teaching and learning. My main goal as the professor in this course is to help you succeed in not just learning the material but in growing as a learner. Learning takes effort, a willingness to try something that may not work, and the ability to use feedback to refine your understanding. My job is to foster an environment in which each and every one of you are supported in these aspects.

Much of the work for this course will be done within class or small group discussions for which we will rely on everyone in this class as a source for feedback and support. Because of this, it's important that we create an inclusive community that is respectful of our differences and offers space for the boundary-setting necessary for positive relationships to form. Our diversity is reflected by differences in race, gender, sexuality, ability, class, religion, nationality, and other cultural identities and material circumstances.

I am looking forward to getting to know each of you and it is my sincere goal to make our time together (in class and out) productive and engaging. I want you to feel comfortable coming to me with any questions or concerns that arise, mathematical and otherwise. If you encounter any issues that interfere with your learning, whether they are physical, mental, emotional, economic, or otherwise, or if you experience discrimination or mistreatment of any sort, please contact me immediately. Other resources for support include the Chair of the Math Department or the CAS Dean of Students.

1 At a Glance

Schedule

Tue-Thur, 9:45-11:10 DB 230

Instructor

Chris Hallstrom, PhD (he / him / his). My office is Buckley Center 270.

Email

hallstro@up.edu. Email is the best way to contact me. I will do my best to get back to you as soon as I can, but please be aware that I typically do not check my email in the evenings or on weekends.

Webpage

All course content will be posted on our class Moodle page

Zoom

In addition to stopping by my office for questions, I am also available (by prior arrangement) via zoom. Here is my personal zoom link.

Calendly

To schedule a meeting (in-person or virtual), you can check my Calendly scheduluer. If you would like to meet at time that you don't see available on Calendly, please feel free to check with me via email!

2 Course Description

In this course, we will revisit the fundamental concepts of calculus with the goal of understanding better how the basic theorems and results follow from definitions. We will consider the question of how we can make these concepts precise – as well as how we can communicate our mathematical ideas clearly. Specific topics that we will encounter include properties of the real numbers, topology of the real line, and formal treatment of limits, sequences, functions, continuity, and the derivative.

2.1 Course Learning Goals

1. I can communicate mathematics orally in a clear and complete manner
2. I can write correct and complete mathematical proofs of Real Analysis results using the conventions of mathematical writing
3. I can independently develop correct and complete proofs of Real Analysis results
4. I can demonstrate an understanding of the nature, approaches, and domain of mathematical inquiry.

2.2 Fundamental Learning Outcomes

1. I can prove results using the axioms for an ordered field.
2. I can prove results involving suprema, infima, and/or the completeness axiom.
3. I can prove results using the formal limit definition for sequences.
4. I can prove results using a formal limit defintion for functions.
5. I can prove results using the definition of continuity.
6. I can prove results using a formal limit definition of the derivative.

3 Inquiry Based Learning

We will use a method of instruction often called Inquiry Based Learning (IBL) which is designed to engage and foster skills and habits that working mathematicians use regularly; you will be asked to solve problems, make conjectures, experiment, explore, create, collaborate, and communicate your work with your peers. Rather than giving you facts to memorize or showing you clear paths to solutions, my role is to guide you via a sequence of carefully chosen problems through a journey of mathematical discovery.

Throughout the semester, you will receive lists of definitions to interpret and make sense of, as well as exercises and theorems which you and your classmates will answer or prove. There will be very little traditional lecture. Instead, class time will consist of student presentations of new material. For best results, you should come to class prepared to share your work or ideas about that day's problems. This method of inquiry does not work nearly as well if you're looking at a problem for the first time in-class.

You will be asked to share your solutions in class regularly. You will also be encouraged to critique the problems (reformulating them if needed), to generate examples and counterexamples to theorems or conjectures, to conjecture new theorems based on what you've learned, and to prove or disprove these conjectures. When observing another student's presentation, it is your responsibility to follow their argument closely and decide if they have seems reasonable. If you cannot follow their logic, or have questions about their solution, it is your responsibility to ask!

⚠ A key feature of the IBL method is student **discovery** and therefore **outside resources are not allowed.** This means that you should not consult texts (other than the one handed out in class), the internet, students not currently enrolled in the course, or faculty other than myself. Consulting outside resources will only deprive yourself of opportunities to engage with the material authentically.
You are encouraged to work with your classmates on the problems, but for best results you should get as far as you can on your own before collaborating. It's important that you do not feel overwhelmed – so please let me know if you're stuck on a problem and I'll be happy to give you hints.

4 Student Support

Throughout the semester, I expect that you will have questions that might not get answered in class. This could happen while doing homework or reviewing your notes, for example. Or perhaps you have a question about the material that did not get fully resolved in class. For these reasons, it's important that you know how to get help outside of class.

⚠ Because of the inquiry method of the class, it's important that you **do not** seek assistance from outside resources – including and especially the internet!

4.1 Drop-In Hours

I have set aside the following specific times during the week that I am available for drop-in help. You do not need to let me know you're coming – just stop by my office (BC 270). Many students find these drop-in hours can be particularly helpful if you are working together with classmates. You can work together at one of the tables down the hall from my office and just pop in when you have questions.

- Wed 10:00-11:00
- T 3:00-4:00
- Th 2:00-3:00

ℹ Note: since I'm setting these times before the semester actually begins, these times may change if my schedule changes. If it turns out that I need to adjust them, I will let you know!

4.2 Open Door

I have the scheduled drop-in hours simply to give you some times when you know that I'll be available – but you are **always** welcome to stop by my office **at any time**. Unless I'm in class or in a meeting, I will generally be available to meet with you!

4.3 Sign-Up Hours

I recognize that your schedule might not allow you to stop by during all the posted drop-in hours. Or you might simply find it more convenient to meet with me at a different time. If you go to my Calendly scheduler, you can sign-up for a time slot to meet, either in-person or via zoom. If there is a specific time that works for you and you don't see it available on the Calendly scheduler, please reach out to me via email and we will find a time that works for your schedule.

5 Course Structure

5.1 Daily Work

Your standing assignment in this course is to work out and prepare solutions or proofs for the exercises in the class notes. Keep track of what problems we cover in class so that you can work ahead and come to class prepared to discuss your work and share your ideas, both in small groups and as a whole class. If you get stuck on a problem, come prepared to ask questions.

I will post on Moodle a running account of what material we cover each day so that if you do happen to miss class, you can still keep track of where we are in the notes.

5.2 Written Homework

Roughly once per week, I will ask you to hand in write-ups of selected problems. The goal of these written assignments is to demonstrate your understanding of the material as well as your abilty to communicate that understanding in writing. I will provide feedback on your work after which you are welcome to revise and resubmit for further feedback if you wish. You may choose to use some of these problems as evidence of your progress in the course. Guidelines for written work can be found in Section 7.

You are welcome to write your homework assignments by hand, but you might also choose use this opportunity to learn to use latex (pronounced LAY-TEK). This is the typesetting system your professors use to write documents that have math notation. There are several free ways to use latex, the easiest of which is probably the web-based Overleaf. Since it's web-based, there's nothing to install. Look for the free student version.

5.3 Synthesis Problems

Throughout the semester, we will encounter problems whose solutions require the synthesis of multiple concepts and results from the course. These will be more varied and more challenging and will typically require deep thought over an extended period of time. You will likely find that you will hit some dead ends along the way. As with all mathematical research, the key is

to learn from what doesn't work and to persist through the difficulties. The aha moment that will eventually come will be worth the wait!

5.4 Generative AI Policy

Given the IBL nature of the course, no AI assistance of any kind should be used in this course. This is also true regarding the proof writing aspect of the course – your goal is to develop skill in communicating mathematics through the mechanism of feedback from human readers of your work. You should not use AI for any narrative submissions either - I'm interested in your thoughts in your own words.

5.5 Due Dates

Here is how due dates work in the "real world" – they exist and they're usually there for a reason, but they're usually somewhat flexible. In this class, they are meant to help you organize your time and to help me from getting overwhelmed. But if you need a little more time (no more than a few days) to work on something, that is usually fine. Just send me an email to let me know that your work will be late. **You do not need to provide a reason.** If you need more than a few days, you should come talk with me about it.

5.6 Proof Portfolio

As we proceed through the course material, I will ask you to select examples of your work that exemplify your understanding and engagement with the class content. At the end of the semester, this portfolio will provide evidence of your learning throughout the semester. To give me time to review your portfolio, please plan on submitting your portfolio to me by the last day of classes, Dec. 5th. There is no prescribed format for your portfolio – many students create a Google Doc that they can then share with me, but if you have other ideas, just let me know!

5.7 Class Proof Journal

Over the course of the semester, we will collaboratively create a proof journal consisting of solutions for each problem that have been presented in class. You can think of this as a textbook for the class, and you may refer to these solutions (with proper attribution) in your own work.

5.8 Exams

We will have a mid-semester take home exam the week of Oct. 8th which will give you a chance to demonstrate understanding of the material that we've covered up to that point. Whether or not you include this in your portfolio is up to you.

We will not have a traditional final exam. Instead, we will have a final reflection assignment to be completed during finals week which will give you an opportunity to reflect on and synthesize some of the main themes of the course. You will also be asked to self-assess your learning over the course of the semester.

6 Honors

If you are taking this course as part of Honors Program then there is an additional requirement of a **course project** that involves a deeper dive into a topic related to the course material. This project will include:

- Selecting a topic that either extends an idea or result from class or explores a new topic that connects with course material. I will provide you with a list of possible project ideas, but you're also encouraged to come up with your own ideas.
- Investigating and preparing a report detailing your exploration. This work should include at least one proof that uses results or ideas from the course content.
- Presenting a summary of your work in a talk. This could either be a presentation to the class or else some other public facing talk such as the mathematics colloquium or other appropriate setting.

Note: Engaging with an honors project is not limited to students in the honors program. All students can choose to complete this project as a way to explore and engage in course content at a deeper level.

7 Writing Proofs

One of the goals of this course is to hone your skill in communicating your mathematical ideas, particularly in writing. In creating and writing proofs, your goal is not to simply come up with a correct argument but you must also convince someone else that you know what you're talking about!

To best achieve this goal, you might think of your proof as a short essay that effectively integrates mathematical work with explanation and reasoning. Like any piece of writing, you should expect to write multiple drafts, proofread, and edit your work. Here is an incomplete set of guidelines you may find helpful:

- Work is clearly written using complete sentences. In particular, all mathematical notation and expressions are part of a sentence.
- Writing should be neat and legible. If your handwriting is not up to the task, you should consider using a typesetting enviornment such as latex.
- It should probably contain more words than symbols.
- It is written at a level that is appropriate for a MTH 401 audience, namely, members of our class who are familiar with the content of the course but who may not have worked on the particular problem whose solution you are presenting.
- The proof is complete, meaning that a member of our MTH 401 audience (see above) can trace your reasoning from beginning to end and be persuaded that your proof is correct without having to reconstruct or guess at significant portions of your thinking.
- The proposition or claim to be proven is clearly stated as such.
- The proof is correct and the steps in the proof are also correct.
- Your writing is almost free, if not entirely free, of spelling errors.
- Your writing is almost free, if not entirely free, of basic grammatical errors such as incomplete sentences, subject-verb disagreement, and misuse of punctuation.

8 Grades

> Extrinsic motivation, which includes a desire to get better grades, is not only different from, but often undermines, intrinsic motivation, a desire to learn for its own sake.
>
> – Alfie Kohn, "The Case Against Grades"

Grades, as they are traditionally thought of, are inherently imprecise and don't represent a full picture of your growth and learning over the course of a semester. Worse than that, research suggests that grades undermine the learning process in several ways:

- Grades tend to diminish interest in what you're learning.
- Grades create a preference for the easiest task. In other words, students tend to do what they need to get a certain grade, but no more.
- Grades tend to reduce the quality of student thinking. The moment we ask "**how** am I doing?'' we lose track of **what** we're doing.

Unfortunately, I am required to submit a grade for each student at the end of the semester – but I will do what I can to de-emphasize the role of grades in this course so that as much as possible our focus is on learning.

Collaborative Grading

Rather than giving you marks on individual assignments, I will instead give you extensive feedback on your work. After addressing that feedback, you're welcome to resubmit for further feedback if you wish. Throughout the semester, I will periodically ask you to reflect carefully on your work and to evaluate your progress. You will collect evidence of your understanding of the course content and based on that evidence, you will be asked to suggest a final course grade. In this way, we will determine your your grade collaboratively.

The intention here is to help you focus on learning in a way that is more organic, as opposed to simply working as you think you're expected to. If this process causes more anxiety than it alleviates, please see me at any point to confer about your progress in the course – I'm always happy to talk with you about your learning!

Here some of the many ways that you might demonstrate your understanding of the course material:

- Submit weekly homework, including revisions that incorporate instructor feedback that reflects understanding of specific topics.
- Successfully complete challenging synthesis problems.
- Submit a proof portfolio that demonstrates your understanding through:
  - Finding and demonstrating connections between ideas.
  - Constructing examples and non-examples that demonstrate understanding of definitions.
  - Correctly using and explaining the role of axioms and definitions.
- Submit solutions to the course journal.
- Provide complete solutions on quizzes or exams.
- When presenting, give helpful answers to questions.
- When listening to presentations, give presenters helpful feedback through your questions, suggestions, etc.
- Read others' class journal submissions and use them to improve your own proof writing.

Qualitative Descriptions of Grades

Here are some qualitative descriptions that I find helpful in thinking about student grades. You might find them helpful as well.

A This grade generally indicates superior work that demonstrates a deep understanding of the material such that you could apply the material in unfamiliar or especially complex situations. You should should consistently demonstrate this deep understanding of the material using a wide variety of methods described above.

B This grade indicates good work that is eminently satisfactory. You should be able to use and extend this knowledge in many situations although you might have difficulty with particularly challenging or unfamilar problems. You should consistently demonstrate your understanding of the material using some of the methods described above.

C This grade indicates competent work that demonstrates an basic understanding of the course material. You should be able to handle most of the more straightforward problems encountered but might struggle with more challenging problems. You might consistently meet only a few of the criteria listed above or else meet several criteria but less often.

D/F These grades represent a fundamental breakdown of expectations. A D represents a meaningful but unsuccessful attempt at earning a C or above. An F represents such a severe lack of engagement, effort, or understanding that there is no evidence of meaningful progress.

Engagement

Although your course grade should be based on your **understanding** of course content and not on course engagement, in my experience these typically go hand in hand. So while engagement in the course is not itself evidence of understanding, it does usually help us achieve that goal.

Here are some ways that you can engage with the class:

- Attend class regularly
- Work ahead on new problems and come to class prepared to discuss
- Work to make sense of new definitions or axioms.
- Ask questions - either in class or in drop-in hours.
- Volunteer to present your work. If you're not comfortable sharing your work with the class, you can share with me during drop-in hours.
- Actively participate in discussions and group work. This can mean sharing your work but it can also mean asking questions or helping to facilitate the discussion.
- Support your classmates and help them succeed.

In discussing your course grade together, we may opt to add a modifier to your grade to acccount for engagement.

- A $-$ modifier might be added to your grade if you've met the standard for the base grade but lack of engagment prevented you from doing more in the course.
- A $+$ modifier might be added to your grade if you've met the standards for a particular grand and you've engaged in the course in particularly significant ways.

9 University Policies

9.1 Academic Integrity

The University of Portland is a diverse academic community of learners and scholars who are dedicated to freely sharing ideas and engaging in respectful discussion of those ideas to discover truth. Such pursuits require each person, whether student or faculty, to present truthfully our own ideas and give credit to others for the ideas that they generate. Thus, cheating on exams, copying another student's assignment, including homework, or using the work of others without proper citation are some examples of violating academic integrity.

Especially for written and oral assignments, students have an ethical responsibility to properly cite the authors of any books, articles, or other sources that they use. Students should expect to submit assignments to Turnitin, a database that ensures assignments are original work of the student submitting. Each discipline has guidelines for how to give appropriate credit, and instructors will communicate the specific guidelines for their discipline. The Clark Library also maintains a webpage that provides citation guidelines at libguides.up.edu/cite.

The misuse of AI to shortcut course learning outcomes will be treated as a violation of academic integrity comparable to plagiarism or cheating. Faculty are responsible for including a written "Course AI Policy" in their syllabi that clearly states what they consider appropriate and inappropriate uses of AI in the context of their courses. Students are responsible for using AI in ways that do not detract from the established learning outcomes of the course. All members of the scholarly community are responsible for demonstrating sound judgment in discerning when and how to utilize AI in their work, upholding standards of citation, originality, and integrity.

9.2 Assessment Disclosure

Student work products for this course may be used by the University for educational quality assurance purposes. For reasons of confidentiality, such examples will not include student names.

9.3 Accessibility

The University of Portland strives to make its courses and services fully accessible to all students. Students are encouraged to discuss with their instructors what might be most helpful in enabling them to meet the learning goals of the course. Students who experience a disability are encouraged to use the services of the Office for Accessible Education Services (AES), located in the Shepard Academic Resource Center (503-943- 8985). If you have an AES Accommodation Plan, you should meet with your instructor to discuss how to implement your plan in this class. Requests for alternate location for exams and/or extended exam time should, where possible, be made two weeks in advance of an exam, and must be made at least one week in advance of an exam. Also, if applicable, you should meet with your instructor to discuss emergency medical information or how best to ensure your safe evacuation from the building in case of fire or other emergency. All information that students provide regarding disability or accommodation is confidential. All students are responsible for completing the required coursework and are held to the same evaluation standards specified in the course syllabus.

9.4 Mental Health

Anyone can experience problems with their mental health that interfere with academic experiences and negatively impact daily life. If you or someone you know experiences mental health challenges at UP, please contact the University of Portland Counseling Center in the upper level of Orrico Hall (down the hill from Franz Hall and near Mehling Hall) at 503-943-7134 or hcc@up.edu. Their services are free and confidential. In addition, mental health consultation and support is available through the Pilot Helpline by calling 503-943-7134 and pressing 3. The University of Portland Campus Safety Department (503-943-4444) also has personnel trained to respond sensitively to mental health emergencies at all hours. Remember that getting help is a smart and courageous thing to do – for yourself, for those you care about, and for those who care about you. For more information on health and wellness resources at UP go to www.linktr.ee/wellnessUP.

9.5 Non-Violence

The University of Portland is committed to fostering a safe and respectful community free from all forms of violence. Violence of any kind, and in particular acts of power- based personal violence, are inconsistent with our mission. Together, all UP community members must take a stand against violence. Learn more about what interpersonal violence looks like, campus and community resources, UP's prevention strategy, and what we as individuals can do to assist on the Green Dot website. Further information and reporting options may be found on the Title IX website.

9.6 Ethics of Information

The University of Portland is a community dedicated to the investigation and discovery of processes for thinking ethically and encouraging the development of ethical reasoning in the formation of the whole person. Using information ethically, as an element in open and honest scholarly endeavors, involves moral reasoning to determine the right way to access, create, distribute, and employ information, including: considerations of intellectual property rights, fair use, information bias, censorship, and privacy. More information can be found in the Clark Library's guide to the Ethical Use of Information.

9.7 The Learning Commons

Students may receive academic assistance through Learning Commons tutoring services and workshops. The Co-Pilot peer tutoring program provides students with opportunities to work with other students to get help in writing, math, group projects, and many other courses. Schedule an appointment to meet with a Co-Pilot (tutor) by visiting the Learning Commons website. Students can also meet with a Co-Pilot during drop-in hours. Check the Learning Commons website or stop by the Learning Commons in BC 163 to learn more about their services. Co-Pilots are a wonderful support along your academic journey.