Tokamak startup model in DREAM

by Mathias Hoppe January 26, 2022

In this document we review the requirements for DREAM to function as a basic tokamak start-up code, similar to DYON [1]. The code DYON seems in turn to have been inspired by previous work, such as [2].

Although DREAM provides the ability to simulate a spatially homogeneous (1D) plasma, the model used in DYON is 0D and does not admit a straightforward generalization to 1D. Hence, we will here assume that DREAM is run in 0D mode and focus on which models are currently missing from the code.

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1 Circuit equation for plasma current

DYON uses a two-ring model for the the plasma current. In addition to the plasma ring, a significant eddy current is expected to be induced in the so-called MK2 ring (a divertor mechanical support structure), which has the lowest electrical resistance of the various vessel components. The model in DYON therefore is

$$V_{\mathrm{loop}} = I_{\mathrm{p}} R_{\mathrm{p}} + L_{\mathrm{p}} \frac{\mathrm{d}I_{\mathrm{p}}}{\mathrm{d}t} + M \frac{\mathrm{d}I_{\mathrm{MK2}}}{\mathrm{d}t}, \tag{1a}$$

$$V_{\mathrm{loop}} = I_{\mathrm{MK2}} R_{\mathrm{MK2}} + L_{\mathrm{MK2}} \frac{\mathrm{d}I_{\mathrm{MK2}}}{\mathrm{d}t} + M \frac{\mathrm{d}I_{\mathrm{p}}}{\mathrm{d}t}, \tag{1b}$$

where V_{loop} is the loop voltage, R_{p} the plasma resistance, M the mutual inductance and the self-inductance

$$L_{\text{p}} = \mu_0 R \left(\ln \frac{8R}{a} + \frac{l_i}{2} - 2 \right), \tag{2}$$

with internal inductance

$$l_i = \frac{2 \int_0^a B_\theta^2 r \, \mathrm{d}r}{a^2 B_{\theta a}^2}. \tag{3}$$

The self-inductance L_{MK2} of the MK2 ring is not specified.

1.1 Current formulation

The current model in DREAM consists of the following equations:

$$2\pi\mu_0 \left\langle \boldsymbol{B} \cdot \nabla\phi \right\rangle \frac{j_{\text{tot}}}{B} = \frac{1}{V'} \frac{\partial}{\partial r} \left[V' \left\langle \frac{|\nabla r|^2}{R^2} \right\rangle \frac{\partial \psi}{\partial r} \right], \tag{4a}$$

$$\psi_{\text{edge}} = \psi_{\text{wall}} - M_{\text{ew}} I_{\text{p}}, \tag{4b}$$

$$\psi_{\text{wall}} = -L_{\text{ext}} \left(I_{\text{p}} + I_{\text{wall}} \right), \tag{4c}$$

$$V_{\text{loop}} = R_{\text{p}} I_{\text{p}}, \tag{4d}$$

$$V_{\text{loop}}^{(\text{wall})} = R_{\text{wall}} I_{\text{wall}}, \tag{4e}$$

$$\frac{\partial \psi}{\partial t} = V_{\text{loop}}, \tag{4f}$$

We begin by showing that this model yields equations (1), with $V_{\text{loop}} = 0$. To do so, we first note that the time derivative of the current diffusion equation (4a) can be regarded as an equation for the voltage difference ΔV between $V_{\text{loop}}^{(\text{edge})}$ and V_{loop} inside the plasma:

$$\Delta V = V_{\text{loop}}^{(\text{edge})} - V_{\text{loop}} = L \frac{\mathrm{d}I_{\text{p}}}{\mathrm{d}t}. \tag{5}$$

(for a derivation of this, see Appendix A). The edge loop voltage can be expressed in terms of the loop voltage on the wall using (4b), which can in turn be substituted for (4c):

$$\begin{gathered} V_{\text{loop}}^{(\text{wall})} - M_{\text{ew}} \frac{\mathrm{d}I_{\text{p}}}{\mathrm{d}t} = R_{\text{p}} I_{\text{p}} + L \frac{\mathrm{d}I_{\text{p}}}{\mathrm{d}t}, \\ \Longleftrightarrow \\ -L_{\text{ext}} \left(\frac{\mathrm{d}I_{\text{p}}}{\mathrm{d}t} + \frac{\mathrm{d}I_{\text{wall}}}{\mathrm{d}t} \right) - M_{\text{ew}} \frac{\mathrm{d}I_{\text{p}}}{\mathrm{d}t} = R_{\text{p}} I_{\text{p}} + L \frac{\mathrm{d}I_{\text{p}}}{\mathrm{d}t}. \end{gathered} \tag{6}$$

Collecting terms, we arrive at

$$R_{\text{p}} I_{\text{p}} + \left(L + L_{\text{ext}} + M_{\text{ew}} \right) \frac{\mathrm{d}I_{\text{p}}}{\mathrm{d}t} + L_{\text{ext}} \frac{\mathrm{d}I_{\text{wall}}}{\mathrm{d}t} = 0, \tag{7}$$

which, although not identical to (1a), has the same structure, with $V_{\text{loop}} = 0$. Similarly, we can combine (4c) and (4e) to obtain

$$R_{\text{wall}} I_{\text{wall}} + L_{\text{ext}} \left(\frac{\mathrm{d}I_{\text{wall}}}{\mathrm{d}t} + \frac{\mathrm{d}I_{\text{p}}}{\mathrm{d}t} \right) = 0, \tag{8}$$

which has the same structure as (1b).

1.2 Required modifications

Based on the derivation of the DYON equations above, one could notice that only equation (4c) appears in the derivation of both circuit equations. By modifying this term appropriately, we could therefore cause a prescribed loop voltage to appear in both circuit equations (7) and (8). But how do we justify such a term?

As a matter of fact, equation (4c) should actually be written

$$\psi_{\text{wall}} = \psi_{\text{sym}} - L_{\text{ext}} \left(I_{\text{p}} + I_{\text{wall}} \right), \tag{9}$$

where ψ_{sym} is the poloidal flux at the symmetry axis, $R = 0$. Since the poloidal flux is typically defined as the integral

$$\psi(r) = \int_0^{R(r)} \boldsymbol{B} \cdot \hat{\boldsymbol{z}} \, \mathrm{d}R, \tag{10}$$

$\psi_{\text{sym}} = \psi(R = 0)$ is usually taken to as zero. The poloidal flux can however be defined with an offset ψ_0, and this offset can have a time dependence. We can therefore include this term explicitly in equation (4c) and evolve it according to a prescribed loop voltage:

$$\psi_{\text{sym}} = V_{\text{loop}}^{(\text{appl})}. \tag{11}$$

Physically, we can think of this as a transformer passing through $R = 0$ applying a loop voltage $V_{\text{loop}}^{(\text{appl})}$.

As for the MK2 structure, we can use the regular "self-consistent" poloidal flux boundary conditions and simply think of "wall" as "MK2". The wall radius must then be chosen in order to obtain the appropriate inductance.

With these modifications, the corresponding circuit equations in DREAM are

$$\begin{aligned} V_{\text{loop}} &= R_{\text{p}} I_{\text{p}} + (L + L_{\text{ext}} + M_{\text{ew}}) \frac{\mathrm{d}I_{\text{p}}}{\mathrm{d}t} + L_{\text{ext}} \frac{\mathrm{d}I_{\text{wall}}}{\mathrm{d}t}, \\ V_{\text{loop}} &= R_{\text{wall}} I_{\text{wall}} + L_{\text{ext}} \frac{\mathrm{d}I_{\text{wall}}}{\mathrm{d}t} + L_{\text{ext}} \frac{\mathrm{d}I_{\text{p}}}{\mathrm{d}t} \end{aligned} \tag{12}$$

Summary — Loop voltage

In summary, an applied loop voltage would require the following modifications to DREAM:

1. Add an unknown ψ_{sym}, representing the poloidal flux at the symmetry axis.
2. Add ψ_{sym} to the RHS of equation (4c).
3. Add the equation $\partial\psi_{\text{sym}}/\partial t = V_{\text{loop}}^{(\text{appl})}$ to the system of equations.

2 Neutral particles

Before we get into the details of the energy and particle balance equations, we should note the special treatment granted to neutral particles in DYON. In the code, it is imagined that the central part of the plasma volume is fully ionized and screens out neutral particles. As such, the plasma consists mutliple overlapping volumes (also illustrated i figure 1):

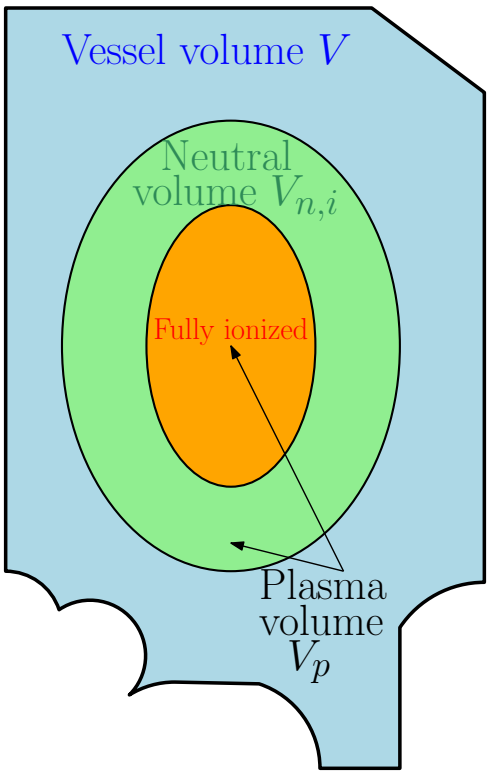

Figure 1: Illustration of how the various volumes are defined in DYON. The vacuum vessel volume is denoted V and is the combination of the blue, green and orange regions. The plasma volume V_p consists of both the neutral (green) and fully ionized (orange) regions, while the neutral volume for ion species i, denoted $V_{n,i}$, is just the green region.

- **Plasma volume V_p:** This is the typical plasma volume in DREAM.
- **Neutral volume for species i, $V_{n,i}$:** This volume is equal to V_p, minus the volume of the plasma which is fully ionized.
- **Tokamak volume V:** This volume typically is not used in DREAM. For the sake of this startup model, it could be provided as an input parameter.

Based on these definitions, we always have $V_{n,i} \leq V_\mathrm{p} < V$.

The neutral volume enters in many terms of the particle and energy balance equations through a factor which we call $\alpha_i = V_{n,i}/V_\mathrm{p}$, i.e. the ratio of neutral to plasma volume for species i. In addition to this factor, the neutral volume coefficient

$$\gamma_{n,i} = 1 - \frac{V_\mathrm{p} - V_{n,i}}{V}, \tag{13}$$

also appears in a few places. In particular, we note that $\gamma_{n,i}V$ is the total neutral volume, i.e. the volume occupied by neutrals, including inside the plasma (except for the fully ionized volume).

The neutral volume is expressed in terms of the mean free path λ_i of neutrals of species i:

$$V_{n,i} = \begin{cases} 2\pi R\left[\pi\kappa a^2 - \pi\kappa\left(a-\lambda_i\right)^2\right], & \text{if } \lambda_i \leq a, \\ V_\mathrm{p}, & \text{if } \lambda_i > a. \end{cases} \tag{14}$$

DYON uses elliptical flux surfaces with elongation κ. In DREAM, however, we also allow for a finite triangularity δ which we could try to account for.

While λ_i is not given in the DYON paper [1], it is given in Ref. [2] as

$$\lambda_i = \frac{v_i}{n_e I_i^{(0)}}, \tag{15}$$

where the ionization rate for species i is $I_i^{(0)} = I_i^{(0)}(n_e, T_e)$ and $v_i = \sqrt{2T_i/m}$. (In [2] they actually only specify λ_i for deuterium, but I think it should generalize like this to arbitrary ion species)

3 Energy balance

DYON considers the energy of both electrons and the various ion species present in the plasma. The equations used for the energy balance are mostly similar to those in DREAM, with some deviations.

3.1 DYON electron equations

The model in DYON consists of the following equations:

$$\frac{\mathrm{d}W_e}{\mathrm{d}t} = P_\Omega - (P_\mathrm{ioniz} + P_\mathrm{rad}) - P_\mathrm{equi} - P^e_\mathrm{conv}, \tag{16}$$

where

$$P_\Omega = \frac{I_\mathrm{p}^2 R_\mathrm{p}}{V_\mathrm{p}}, \qquad \text{(Ohmic heating)}$$

$$P_\mathrm{ioniz} + P_\mathrm{rad} = \sum_i \left\{ \frac{V_{n,i}}{V_\mathrm{p}} \left[L_\mathrm{line} + \Delta W_i^{(0)} I_i^{(0)} \right] + \right.$$
$$\left. + \sum_{j\geq 1} \left[L_\mathrm{line} + L_\mathrm{free} + \Delta W_i^{(j)} \left(I_i^{(j)} - R_i^{(j)} \right) \right] \right\}, \qquad \text{(Ionization \& radiation)}$$

$$P_\mathrm{equi} = \mathrm{const} \times n_e \ln\Lambda \frac{T_e - T_i}{T_e^{3/2}} \sum_{ij} \frac{n_i^{(j)} \left(Z_i^{(j)} \right)^2}{m_i}, \qquad \text{(Electron-ion collisions)}$$

$$P^e_\mathrm{conv} = \frac{1}{\tau_e} W_e, \qquad \text{(Convective transport)}$$

We first note that although written slightly differently, the ohmic heating and equilibration terms P_Ω and P_equi are identical to those appearing in DREAM. Convective transport can also be modelled in DREAM, and the only complication arising from P^e_conv is the determination of the electron confinement time τ_e (taken equal to the deuterium confinement time) which will be discussed in section 5. The only term looking somewhat different from what we have in DREAM is the ionization and radiation loss term, $P_\mathrm{ioniz} + P_\mathrm{rad}$, which differs by a factor $V_{n,i}/V_\mathrm{p}$ on the neutral loss term. This is due to the neutral screening effect which is modelled in DYON (but currently not in DREAM), and should be taken into account.

3.2 DYON ion equations

The ion energy balance equation for species i in DYON takes the form

$$\frac{\mathrm{d}W_\mathrm{ions}}{\mathrm{d}t} = P_\mathrm{equi} - P_\mathrm{CX} - P^\mathrm{ions}_\mathrm{conv}, \tag{17}$$

where P_{equi} is the same as in the electron energy balance equation and

$$P_{\text{CX}} = \frac{V_{n,D}}{V_{\text{p}}}\left[\frac{3}{2}n_D^{(0)}\left(T_{\text{ions}} - T_0\right)\sum_i R_{i,\text{cx}}^{(1)} n_i^{(1)}\right], \qquad \text{(Charge exchange)}$$

$$P_{\text{conv}}^{\text{ions}} = \sum_i \sum_{j\geq 1} \frac{3}{2}\frac{n_i^{(j)} T_{\text{ions}}}{\tau_D}. \qquad \text{(Convective transport)}$$

where $T_0 = 0.026\,\text{eV}$ corresponds to the temperature of the lower-energy deuterium atom in the charge exchange reaction, assumed to be at room temperature, and $R_{i,\text{cx}}^{(1)}$ is the charge exchange recombination coefficient (the one called CCD in Open-ADAS). It is assumed that only deuterium is available for donating electrons in a charge exchange reaction, and that all ions are transported at the same rate τ_D (although only ionized particles, with $Z_0 > 0$, are transported).

Further, we note that DYON assumes that all ions have the same temperature. In DREAM, we only assume that the temperature is the same in every charge state, but allow for different ion species to have different temperatures.

3.3 Required modifications

Based on the discussion above, five modifications will be required in DREAM to match the energy balance model in DYON:

Summary — Energy balance

1. Introduce the neutral volume for species i, $V_{n,i}$, as an unknown quantity and multiply the $Z_0 = 0$ term by the ratio of neutral to plasma volume in the `RadiatedPowerTerm` in DREAM.
2. Add the particle confinement time τ_p as an unknown quantity that is solved for. The details of how to solve for this quantity are discussed in section 5.
3. Introduce an electron transport term utilizing the particle confinement time τ_p. Although this term should model convective particle transport, it is numerically more convenient to model it using a diffusion operator. Due to the 0D approach, the physics are not affected.
4. Add a charge exchange energy loss term.
5. Introduce an ion transport term utilizing the particle confinement time τ_p. As for the electron transport term, this ion transport term should be taken as a diffusion operator rather than a convection operator.

4 Particle balance

The electron density is determined in DYON by requiring quasi-neutrality—exactly as in DREAM—i.e.

$$n_e = \sum_{ij} Z_{0i}^{(j)} n_i^{(j)}. \tag{18}$$

Table 1: Table

	$i = \mathrm{D}$	$i = \mathrm{C}$	$i = \mathrm{O}$
$k = \mathrm{D}$	Eq. (21)	$Y_C^D = 0$	$Y_O^D = 0$
$k = \mathrm{C}$	$Y_D^C = 0.03$	$Y_C^C = 0$	$Y_O^C = 1$
$k = \mathrm{O}$	$Y_D^O = 0$	$Y_C^O = 0$	$Y_O^O = 1$

The ion densities are evolved separately depending on whether the ion is of the main deuterium species, or is an impurity. For deuterium, DYON uses

$$\begin{aligned}
\frac{\mathrm{d}n_D^{(0)}}{\mathrm{d}t} &= \frac{V_\mathrm{p}}{\gamma_{n,D}V} R_D^{(1)} n_e n_D^{(1)} - \frac{V_{n,D}}{\gamma_{n,D}V} I_D^{(0)} n_e n_D^{(0)} - \frac{V_{n,D}}{\gamma_{n,D}V} \sum_{i,j\geq 1} R_{i,\mathrm{cx}}^{(j)} n_D^{(0)} n_i^{(j)} + \frac{\Gamma_{D,\mathrm{in}}^{\mathrm{tot}}}{\gamma_{n,D}V}, \\
\frac{\mathrm{d}n_D^{(1)}}{\mathrm{d}t} &= \frac{V_{n,D}}{V_\mathrm{p}} I_D^{(0)} n_e n_D^{(0)} - R_D^{(1)} n_e n_D^{(1)} + \frac{V_{n,D}}{V_\mathrm{p}} \sum_{i,j\geq 1} R_{i,\mathrm{cx}}^{(j)} n_D^{(0)} n_i^{(j)} - \frac{n_D^{(1)}}{\tau_D},
\end{aligned} \tag{19}$$

where the total influx of neutral deuterium particles from the wall is

$$\Gamma_{D,\mathrm{in}}^{\mathrm{tot}} = V_\mathrm{p} \frac{Y_D^D n_D^{(1)}}{\tau_D}, \tag{20}$$

with the deuterium recycling coefficient

$$Y_D^D(t) = c_1 - c_2 \left(1 - \mathrm{e}^{-t/c_3}\right). \tag{21}$$

The constant coefficients c_1, c_2 and c_3 are chosen based on the scenario. In section 3.2 of [1], they are chosen as $c_1 = 1.1$, $c_2 = 0.09$ and $c_3 = 0.1$. The deuterium confinement time τ_D will be discussed in section 5.

The densities of all other ion species are

$$\begin{aligned}
\frac{\mathrm{d}n_i^{(0)}}{\mathrm{d}t} &= -\frac{V_{n,i}}{\gamma_{n,i}V} I_i^{(0)} n_e n_i^{(0)} + \frac{V_\mathrm{p}}{\gamma_{n,i}V} R_i^{(1)} n_e n_i^{(1)} + \frac{V_{n,D}}{\gamma_{n,i}V} R_{i,\mathrm{cx}}^{(1)} n_D^{(0)} n_i^{(1)} + \frac{\Gamma_{i,\mathrm{in}}^{(0)}}{\gamma_{n,i}V}, \\
\frac{\mathrm{d}n_i^{(1)}}{\mathrm{d}t} &= \frac{V_{n,i}}{V_\mathrm{p}} I_i^{(0)} n_e n_i^{(0)} - I_i^{(1)} n_e n_i^{(1)} + R_i^{(2)} n_e n_i^{(2)} - R_i^{(1)} n_e n_i^{(1)} \\
&\quad + \frac{V_{n,D}}{V_\mathrm{p}} R_{i,\mathrm{cx}}^{(2)} n_D^{(0)} n_i^{(2)} - \frac{V_{n,D}}{V_\mathrm{p}} R_{i,\mathrm{cx}}^{(1)} n_D^{(0)} n_i^{(1)} - \frac{n_i^{(1)}}{\tau_i}, \\
\frac{\mathrm{d}n_i^{(j)}}{\mathrm{d}t} &= I_i^{(j-1)} n_e n_i^{(j-1)} - I_i^{(j)} n_e n_i^{(j)} + R_i^{(j+1)} n_e n_i^{(j+1)} - R_i^{(j)} n_e n_i^{(j)} \\
&\quad + \frac{V_{n,D}}{V_\mathrm{p}} R_{i,\mathrm{cx}}^{(j+1)} n_D^{(0)} n_i^{(j+1)} - \frac{V_{n,D}}{V_\mathrm{p}} R_{i,\mathrm{cx}}^{(j)} n_D^{(0)} n_i^{(j)} - \frac{n_i^{(j)}}{\tau_i},
\end{aligned} \tag{22}$$

where the confinement time for species i is $\tau_i = \tau_D$, and the influx of neutral atoms of species i is

$$\Gamma_{i,\mathrm{in}}^{(0)} = V_\mathrm{p} \sum_k \sum_{j\geq 1} \frac{Y_k^i n_k^{(j)}}{\tau_k}, \tag{23}$$

where Y_k^i is the sputter yield (or recycling coefficient) of species i due to the bombardment of incident ion k. In DYON, these coefficients are chosen according to table 1

The impurity density equations (22) differ from the deuterium equations mainly in that the charge-exchange is assumed to occur between deuterium and impurities only, not

between impurities and other impurities. However, I suspect that the reason for this is that impurity-impurity terms are assumed negligible, either because $n_i/n_D \ll 1$ for all $i \neq D$, or because the coefficients $R^{(j)}_{i,\mathrm{cx}}$ are defined for interactions between species i and hydrogen only.

4.1 Generalizations for DREAM

In contrast to DYON, DREAM evolves all ions according to the same equations, regardless of the ion species (of course with different atomic coefficients). Since we would like to maintain the simple formulation used in DREAM, we should attempt to reformulate equations (19) and (22) as a single equation instead.

First, we introduce the volumes $V_i^{(j)}$ and $\hat{V}_i^{(j)}$ which are

$$V_i^{(j)} = \begin{cases} \gamma_{n,i} V, & j = 0, \\ V_\mathrm{p}, & j \geq 1, \end{cases} \qquad \hat{V}_i^{(j)} = \begin{cases} V_{n,i}, & j = 0, \\ V_\mathrm{p}, & j \geq 1 \end{cases}, \tag{24}$$

i.e. $V_i^{(j)}$ denotes the volume occupied by ions (or neutrals) of species i in charge state j, while $\hat{V}_i^{(j)}$ denotes the volume occupied by ions (or neutrals) *inside the plasma* of species i in charge state j. Due to the neutral screening effect, neutrals will not be present in the fully ionized region.

With the volumes above, a unified ion rate equation can be written as

$$\begin{aligned} \frac{\mathrm{d}n_i^{(j)}}{\mathrm{d}t} = \frac{1}{V_i^{(j)}} (& \hat{V}_i^{(j-1)} I_i^{(j-1)} n_e n_i^{(j-1)} - \hat{V}_i^{(j)} I_i^{(j)} n_e n_i^{(j)} + \hat{V}_i^{(j+1)} R_i^{(j+1)} n_e n_i^{(j+1)} \\ & - \hat{V}_i^{(j)} R_i^{(j)} n_e n_i^{(j)} + \hat{V}_D^{(0)} n_D^{(0)} A_{i,\mathrm{cx}}^{(j)}) + \Gamma_i^{(j)}. \end{aligned} \tag{25}$$

Here the coefficient $A_{i,\mathrm{cx}}^{(j)}$ is given by

$$A_{i,\mathrm{cx}}^{(j)} = \begin{cases} (-1)^{(j+1)} \sum_{k,l \geq 1} R_{ik,\mathrm{cx}}^{(l)} n_k^{(l)}, & i = D, k \neq D, \\ R_{ik,\mathrm{cx}}^{(j+1)} n_i^{(j+1)} - R_{ik,\mathrm{cx}}^{(j)} n_i^{(j)}, & i \neq D, k = D, \end{cases} \tag{26}$$

where we note that there are no charge-exchange contributions for $i = k = D$ and $i, k \neq D$, i.e. exactly one of the ion species involved in the interaction is deuterium, and the particle flux is given by

$$\Gamma_i^{(j)} = \begin{cases} \Gamma_{i,\mathrm{in}}^{(0)} / V_i^{(j)}, & j = 0, \\ -n_i^{(j)} / \tau_i, & j \geq 1, \end{cases} \tag{27}$$

where $\Gamma_{i,\mathrm{in}}^{(0)}$ is given by equation (20) if $i = D$ and by equation (23) if $i \neq D$.

4.2 Required modifications

Based on the above discussion, the following modifications will be required to DREAM:

Summary — Particle balance

1. Implementation of the factor $\hat{V}_i^{(j)}/V_i^{(j)}$ in the ionization/recombination operator.
2. Addition of a charge-exchange term.
3. Introduction of the particle flux term $\Gamma_i^{(j)}/V_i^{(j)}$ (probably as two separate terms, corresponding to one plasma-wall interaction term for neutrals, and one transport term for ions).

5 Particle confinement

Particle confinement is characterized in DYON by the deuterium confinement time

$$\frac{1}{\tau_D} = \frac{1}{\tau_{D,\parallel}} + \frac{1}{\tau_{D,\perp}}, \tag{28}$$

where $1/\tau_{D,\parallel}$ and $1/\tau_{D,\perp}$ are the transport rates parallel and perpendicular to magnetic field lines, respectively. The confinement time is assumed to be the same for all other ion species.

The perpendicular confinement time $\tau_{D,\perp}$, which dominates later during the startup process when magnetic field lines are closed, is assumed to be governed by Bohm diffusion [3]:

$$\tau_{D,\perp} = \frac{a(t)}{v_{\mathrm{Bohm}}(t)} = \frac{a^2(t)}{2D_{\mathrm{Bohm}}(t)}, \tag{29}$$

with

$$D_{\mathrm{Bohm}}(t) = \frac{1}{16}\frac{T_e\,[\mathrm{eV}]}{B_\phi}, \tag{30}$$

where B_ϕ is the toroidal magnetic field strength. The time dependence of the minor radius a is indicated to emphasize that the minor radius may evolve in time. In Ref. [1], no analytical model is used for $a(t)$, but instead it is taken as input from EFIT magnetic equilibrium reconstructions.

The parallel confinement time, which is dominant during the early phase of burn-through before closed flux surfaces have formed, is given by

$$\tau_{D,\parallel} = \frac{L_\mathrm{f}}{C_\mathrm{s}}, \tag{31}$$

where C_s is the ion sound speed

$$C_\mathrm{s} = \sqrt{\frac{T_e + T_i}{m_D}}. \tag{32}$$

The effective connection length L_f is modelled as

$$L_\mathrm{f} = \frac{a(t)}{4}\frac{B_\phi}{B_z(t)}\exp\left(\frac{I_\mathrm{p}}{I_\mathrm{ref}}\right), \tag{33}$$

with $I_\mathrm{ref} = 100\,\mathrm{kA}$. The stray field $B_z(t)$ is composed of the vertical magnetic field $B_\mathrm{v} \approx 1\times10^{-3}\,\mathrm{T}$ and the magnetic field $B_\mathrm{eddy}(t)$ created by the eddy current running through the MK2 structure:

$$B_\mathrm{eddy}(t) = \frac{\mu_0}{\pi l_\mathrm{MK2}} I_\mathrm{MK2}, \tag{34}$$

where l_{MK2} is the distance between the centre of the plasma and the MK2 structure (unspecified).

5.1 Runaway electron confinement

A topic not treated in DYON, but which is important for the study of runaway electron dynamics is the confinement time for runaway electrons. Some thoughts on this can be found in internal ITER reports, most notably the reports [4–6], with the most complete discussion found in the latter.

Early during the discharge, before flux surfaces have formed completely, the electron transport will be dominated by parallel transport to the wall, just as in the case of thermal electrons and ions. The time scale for this transport is set by how fast an electron traverses a distance comparable to the effective connection length L_{f} (33) of the plasma. Assuming that the electron is being freely accelerated by the electric field E, which is further assumed to vary slowly on the transport time scale (so that it is effectively constant), we have that

$$\frac{1}{2}a_{\mathrm{RE}}\tau_{\mathrm{RE},1}^2 = L_{\mathrm{f}}, \tag{35}$$

where we can solve for the runaway electron confinement time $\tau_{\mathrm{RE},1}$, given that the acceleration $a_{\mathrm{RE},1} = eE/m_e$. We thus obtain

$$\boxed{\tau_{\mathrm{RE},1} = \sqrt{\frac{2m_e L_{\mathrm{f}}}{eE}}.} \tag{36}$$

Later during the discharge, once $I_{\mathrm{p}} \gtrsim I_{\mathrm{ref}}$ and flux surfaces have formed, runaway electron losses are hypothesized to be dominated by particle orbits being shifted due to drifts. As the energy of the electron increases, the particle orbit will be shifted further out from the plasma. Eventually the electron reaches an energy γ_{max} such that the orbit intersects the tokamak wall, causing the particle to be lost. The value of this maximum energy was estimated in ref. [4] and found to be

$$\gamma_{\mathrm{max}} \approx \frac{56R_0}{a} I_{\mathrm{p}}\,[\mathrm{MA}], \tag{37}$$

for passing electrons. If an electron is trapped, the maximum energy scaling changes, but since trapped runaways are expected to make up a negligible fraction of all electrons we prefer the above estimate.

An alternative scaling was derived in ref. [7], where the maximum runaway energy was given as

$$\gamma_{\mathrm{max}} = \sqrt{1 + \left(2R_0\left(1 - \frac{R}{R_l}\right)\frac{I_{\mathrm{p}}[\mathrm{A}]}{17000R_l}\right)^2}, \tag{38}$$

with R denoting the major radius at which the runaway electron was initially generated, R_l the major radius of the limiter (/wall), and R_0 the plasma major radius. We will however use (37) due to its simpler form, and since both expressions are anyway rough estimates.

Knowing the maximum runaway energy, we can calculate the time it takes for an electron to reach that energy from the relativistic form of Newton's second law:

$$\frac{\mathrm{d}\left(\gamma m_e v\right)}{\mathrm{d}t} = eE. \tag{39}$$

(Note that we neglect the direction of the electric field here; we are merely interested in the energies attained). Assuming that $\gamma_{\max} \gg 2$, so that $v \approx c$ is reached on a very short time scale, we can write

$$\gamma_{\max} = \frac{e}{m_e c} \int_0^{\tau_{\mathrm{RE},2}} E \, \mathrm{d}t. \tag{40}$$

Assuming further that $E = \text{const.}$, the integral becomes trivial and we find

$$\gamma_{\max} = \frac{eE}{m_e c} \tau_{\mathrm{RE},2}. \tag{41}$$

Now, inserting the estimate (37) for the runaway energy, we can solve for $\tau_{\mathrm{RE},2}$ and obtain

$$\boxed{\tau_{\mathrm{RE},2} = \frac{m_e c \gamma_{\max}}{eE} \approx \frac{R_0}{10a} \frac{I_\mathrm{p}\,[\mathrm{MA}]}{E\,[\mathrm{V/m}]}.} \tag{42}$$

Finally, as suggested in ref. [6], we interpolate between the two timescales $\tau_{\mathrm{RE},1}$ and $\tau_{\mathrm{RE},2}$. Since $\tau_{\mathrm{RE},1}$ is expected to dominate early during the discharge, when $I_\mathrm{p} \lesssim I_\mathrm{ref}$, and vice versa, we choose the interpolation

$$\boxed{\frac{1}{\tau_\mathrm{RE}} = \frac{\exp\left(-I_\mathrm{p}/I_\mathrm{ref}\right)}{\tau_{\mathrm{RE},1}} + \frac{1 - \exp\left(-I_\mathrm{p}/I_\mathrm{ref}\right)}{\tau_{\mathrm{RE},2}}.} \tag{43}$$

5.2 Required modifications

One way of implementing the particle confinement model of DYON into DREAM would be to introduce the particle confinement time τ_D as an unknown quantity in DREAM and solve for it (since it depends on other unknowns—I_p and T_e—which are solved for implicitly by DREAM). However, since τ_D can be expressed explicitly in terms of I_p and T_e, it is better to implement the transport terms in sections 3 and 4 using that explicit expression (which can be obtained after some algebra using the details of this section).

Summary — Particle confinement

- Implement a runaway transport term with confinement time $\tau_\mathrm{re} \propto (v/c)\tau_D$, where v is the ion sound speed C_s for $\tau_{D,\parallel}$, and the Bohm velocity v_Bohm for $\tau_{D,\perp}$.

6 Simulation roadmap

Once the tool described above has been fully implemented, it is time for simulations. Since we have little experience with self-consistent burn-through simulations, I suggest we start by reproducing and understanding the runaway-free simulations of Ref. [1] before attacking any more novel physics question.

I think that merely ("merely") reproducing experimental results of a plasma containing runaways using the code would be a significant step forward in the simulation of tokamak. The most accessible experimental data is that of JET, which has been previously studied, primarily by de Vries *et al.* [8]. Data should also be available from at least Alcator C-Mod and COMPASS, but likely also other European tokamaks.

Tasks — Self-consistent burn-through simulations

1. Learn to run startup simulations by reproducing DYON results in Ref. [1].
2. Consider the scenarios studied by de Vries *et al.* (2020), but run the simulations fully self-consistently.
3. Model other experimental scenarios from some nice tokamak.
4. Consider the evolution of the runaway electron distribution function in one of the above scenarios. Is it close to Maxwellian or does a significant fraction of superthermal electrons appear (as speculated in Ref. [9])? If so, this could indicate the need for a non-linear collision operator.

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A Circuit induction equation

The current diffusion equation in DREAM reads

$$2\pi\mu_0 \left\langle \boldsymbol{B}\cdot\nabla\phi\right\rangle \frac{j_{\mathrm{tot}}}{B} = \frac{1}{V'}\frac{\partial}{\partial r}\left[V'\left\langle\frac{|\nabla r|^2}{R^2}\right\rangle\frac{\partial\psi}{\partial r}\right]. \tag{44}$$

To obtain the corresponding circuit equation, we first multiply both sides with the Jacobian V' and integrate both sides over the plasma radius:

$$2\pi\mu_0\int_0^r \left\langle \boldsymbol{B}\cdot\nabla\phi\right\rangle \frac{j_{\mathrm{tot}}}{B}\,V'\,\mathrm{d}r' = \int_0^r \mathrm{d}r'\,\frac{\partial}{\partial r'}\left[V'\left\langle\frac{|\nabla r'|^2}{R^2}\right\rangle\frac{\partial\psi}{\partial r'}\right]. \tag{45}$$

The integral on the LHS is 2π times the total plasma current enclosed within the flux surface r, here denoted $I(r)$, while on the RHS the integral simply cancels the outer derivative (since $V'=0$ in $r=0$, causing the entire expression to vanish there). We thus obtain

$$\left(2\pi\right)^2\mu_0 I(r) = V'\left\langle\frac{|\nabla r|^2}{R^2}\right\rangle\frac{\partial\psi}{\partial r}. \tag{46}$$

This equation yields the solution

$$\psi(r) = \psi\left(r_0\right) + \left(2\pi\right)^2\mu_0\int_{r_0}^r \frac{I(r')}{V'\left\langle|\nabla r'|^2/R^2\right\rangle}\,\mathrm{d}r', \tag{47}$$

where r_0 is an arbitrary radius. The integral can roughly be considered as the plasma inductance times the current flowing between r_0 and r. If we now consider the entire plasma as a circuit, and only the poloidal flux *outside* the plasma, we may let $r_0 = a$ and $r > a$. This gives

$$\psi(r) = \psi(a) + \left(2\pi\right)^2\mu_0 I_{\mathrm{p}}\int_a^r \frac{\mathrm{d}r'}{V'\left\langle|\nabla r'|^2/R^2\right\rangle}, \tag{48}$$

where the total plasma current $I_{\mathrm{p}} = I(a)$. After differentiating this expression with respect to time, we obtain the circuit induction equation

$$V_{\mathrm{loop}}(r) - V_{\mathrm{loop}}^{(\mathrm{edge})} = L\frac{\mathrm{d}I_{\mathrm{p}}}{\mathrm{d}t}, \tag{49}$$

with

$$L = \left(2\pi\right)^2\mu_0\int_a^r \frac{\mathrm{d}r'}{V'\left\langle|\nabla r'|^2/R^2\right\rangle}. \tag{50}$$

In cylindrical geometry, $\langle|\nabla r|^2/R^2\rangle = 1/R_0^2$, and $V' = \left(2\pi\right)^2 r R_0$, so that the inductance becomes

$$L = \mu_0 R_0\int_a^r \frac{\mathrm{d}r'}{r} = \mu_0 R_0\ln\left(\frac{r}{a}\right). \tag{51}$$