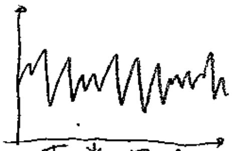
AR-process (Auto Regression) 自己回帰過程

- AR(1) process

$$x(t) = \underbrace{\alpha_1 x(t-1)}_{\text{1つ前のデータ}} + \underbrace{\mu}_{\text{定数項}} + \underbrace{\varepsilon_t}_{\text{誤差項}}$$

ex) $x(t) = 0.8x(t-1) + 1 + \varepsilon_t \qquad (\varepsilon_t \sim N(0,1))$



定常過程

非定常過程

- AR(p) process

$$x(t) = \alpha_1 x(t-1) + \alpha_2 x(t-2) + \cdots + \alpha_p x(t-p) + \mu + \varepsilon_t$$

MA-process (Moving Average) 移動平均過程

- MA(1) process

$$x(t) = \mu + \varepsilon_t + \underbrace{\theta_1 \varepsilon_{t-1}}_{\text{1つ前の誤差項}}$$

ex) $x(t) = 10 + \varepsilon(t) + 0.5\,\varepsilon(t-1)$

- MA(q) process

$$x(t) = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

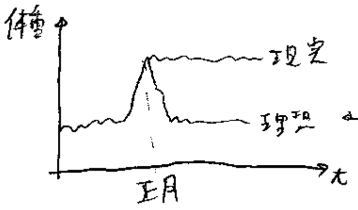
ARMA-process

- ARMA(p, q)

$$x(t) = \mu + \alpha_1 x(t-1) + \cdots + \alpha_p x(t-p) + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$$

誤差項の意味

誤差項 ε_t はただの誤差ではない



← ただの誤差ならすぐに元に戻る

→ 時系列データの場合, 誤差が将来に尾を引くことがある.

AR-process, MA-process の特徴

AR(1) Process

$$x_t = \phi_0 + \phi_1 x_{t-1} + \varepsilon_t$$

定常条件

$|\phi_1| < 1$ のとき AR(1) process は定常性をもつ

$|\phi_1| \geq 1$ のとき発散する

統計量

$E[\varepsilon_t] = 0,\ V[\varepsilon_t] = \sigma^2,\ |\phi_1| < 1$ とする.

このとき

$$\begin{aligned} E[x_t] &= E[\phi_1 x_{t-1} + \phi_0 + \varepsilon_t] \\ &= \phi_1 E[x_{t-1}] + E[\phi_0] + E[\varepsilon_t] \\ &= \phi_1 E[x_{t-1}] + \phi_0 \end{aligned}$$

$|\phi_1| < 1$ より定常であるから $E[x_t] = E[x_{t-1}]$ より

$$E[x_t] = \frac{\phi_0}{1-\phi_1}$$

$$\begin{aligned} V[x_t] &= V[\phi_1 x_{t-1} + \phi_0 + \varepsilon_t] \\ &= \phi_1^2 V[x_{t-1}] + \sigma^2 \end{aligned}$$

同様に

$$V[x_t] = \frac{\sigma^2}{1-\phi_1^2}$$

自己共分散

$\gamma_j = Cov[x_t, x_{t-j}] = Cov[\phi_0 + \phi_1 x_{t-1} + \varepsilon_t, x_{t-j}]$

$Cov[x_{t-j}, \varepsilon_t] = 0$ より

$= \phi_1 Cov[x_{t-1}, x_{t-j}] = \phi_1 \gamma_{j-1}$

$\gamma_0 = V[y_t]$ より

$$\gamma_j = \phi_1 \gamma_{j-1} = \phi^2 \gamma_{j-2} = \cdots = \frac{\sigma^2}{1-\phi_1^2} \phi_1^j$$

自己相関

$$\rho_j = \frac{\gamma_j}{\gamma_0} = \phi_1^j$$

AR(n) process

$x_t = \phi_0 + \phi_1 x_{t-1} + \cdots + \phi_n x_{t-n} + \varepsilon_t$

統計量(定常のとき)

$E[x_t] = E[x_{t-1}] = \cdots = E[x_{t-n}]$ だから

$E[x_t] = \phi_0 + \phi_1 E[x_{t-1}] + \cdots + \phi_n E[x_{t-n}]$

$$E[x_t] = \frac{\phi_0}{\phi_1 + \phi_2 + \cdots + \phi_n}$$

$V[x_t] = \phi_1^2 V[x_{t-1}] + \cdots + \phi_n^2 V[x_{t-n}] + \varepsilon^2$

定常のとき $^\forall t$ に対して $V[x_t] = \gamma_0$ より

$$V[x_t] = \frac{\varepsilon^2}{\phi_1^2 + \cdots + \phi_n^2}$$

$$\begin{aligned}\gamma_j &= Cov[x_t, x_{t-j}] \\ &= \phi_1 Cov[x_{t-1}, x_{t-j}] + \phi_2 Cov[x_{t-2}, x_{t-j}] + \cdots + \phi_n Cov[x_{t-n}, x_{t-j}] \\ &= \phi_1 \gamma_{j-1} + \cdots + \phi_n \gamma_{j-n}\end{aligned}$$

$$\begin{aligned}\rho_j &= \frac{1}{\gamma_0} \gamma_j \\ &= \phi_1 \rho_{j-1} + \cdots + \phi_n \rho_{j-n}\end{aligned}$$

MA(n) process … 常に定常

$$x_t = \theta_0 + \theta_1 \varepsilon_{t-1} + \cdots + \theta_n \varepsilon_{t-n} + \varepsilon_t$$

統計量

$$E[x_t] = E[\theta_0 + \theta_1 \varepsilon_{t-1} + \cdots + \theta_n \varepsilon_{t-n} + \varepsilon_t]$$

$E[\varepsilon_t] = 0$ より

$$= \theta_0 \qquad \therefore E[x_t] = \theta_0$$

$$V[x_t] = V[\theta_0 + \theta_1 \varepsilon_{t-1} + \cdots + \theta_n \varepsilon_{t-n} + \varepsilon_t]$$
$$= \theta_1^2 \sigma^2 + \cdots + \theta_n^2 \sigma^2 + \sigma^2$$
$$= (1 + \theta_1^2 + \cdots + \theta_n^2) \sigma^2$$

$$\gamma_j = Cov[x_t, x_{t-j}]$$
$$= Cov[\theta_0 + \theta_1 \varepsilon_{t-1} + \cdots + \theta_n \varepsilon_{t-n} + \varepsilon_t,\ \theta_0 + \theta_1 \varepsilon_{t-j-1} + \cdots + \theta_n \varepsilon_{t-j-n} + \varepsilon_{t-j}]$$

$0 \leq j \leq n$ のとき

$$\gamma_j = (\theta_j + \theta_{j+1}\theta_1 + \cdots + \theta_n \theta_{n-j}) \sigma^2$$

$n < j$ のとき

$$\gamma_j = 0$$

$$\rho_j = \frac{\theta_j + \theta_{j+1}\theta_1 + \cdots + \theta_n \theta_{n-j}}{1 + \theta_1^2 + \cdots + \theta_n^2} \qquad (0 \leq j \leq n)$$

$$\rho_j = 0 \qquad (n < j)$$