STA 208: Homework 4(Do not distribute)

Due Monday 6/2/2019 at midnight

Instructions: Submit it on canvas. The canvas should include all of your code either in this notebook file, or a separate python file that is imported and ran in this notebook. We should be able to open this notebook and run everything here by running the cells in sequence. The written portions can be either done in markdown and TeX in new cells or written clearly by hand when you hand it in. Submit each file separately.

- · Code should be well organized and documented
- All math should be clear and make sense sequentially
- When in doubt explain what is going on
- You will be graded on correctness of your math, code efficiency and succinctness, and conclusions and modelling decisions

Exercise 1 (20 pts)

Recall that a Mercer kernel is a similarity function k(x,x') such that the matrix $K_{ij}=(k(x_i,x_j))_{ij}$ is positive semidefinite regardless of dataset $\{x_i\}_{i=1}^n\subset\mathbb{R}^p$.

- 1. Suppose that $K=U\Lambda U^{\top}$ where U is $n\times n$ and Λ is diagonal. What do we know about Λ and why?
- 2. Let kernel ridge regression be the following minimization problem where $y \in \mathbb{R}^n$,

$$\min_{eta} rac{1}{2} \|y - eta\|_2^2 + eta^ op Keta.$$

Write the solution as a function of Λ, U, y . What does this tell you about the effect of the eigenvalues on the solution (suppose that you only increase one of the eigenvalues).

- 3. Check if the following notions of similarity are Mercer kernels,
 - A. $k(x,x')=x^{ op}x'$
 - B. $k(x,x')=x_1x_1'-x_2x_2',\quad x,x'\in\mathbb{R}^2$
 - C. $k(s,s')=1\{s,s' \text{ have the same sentence structure}\}$ where the sentence structure is the sequence of parts-of-speech for each word/phrase. For example, "The cat ran around the house." is "noun verb preposition noun" has the same structure as "Steve forgot about the homework".

Exercise 2 (40 pts)

Load the poses.csv dataset, which is a concatenation of other datasets to form a larger dataset. I want you to act like the dataset is from the same experiment.

- 1. Apply 1 time lag difference of the dataset, so that each variable is the difference of the time point and the previous time point. Standardize the dataset and remove any variables that do not make sense. Run the PCA decomposition with 2 principal components. Plot the 2 principal components. Which variables have the most loading on the principal components (look at .components_)?
- 2. Also on the 1 lagged dataset. Run K-means clustering (with 6 clusters), how much does the cluster overlap with the 'task' variable. Look at the confusion matrix of the cluster against the 'task'. Is there a clear mapping from clusters to task?
- 3. Standardize the data and train an HMM with an appropriately chosen emission distribution. How much does the hidden state overlap with the 'task' variable?

```
In [4]: import numpy as np import re import pandas as pd
```

Some lines in the data was in wrong format and the below two blocks should be able to fix that problem. The idea is that each rwo has 8 numbers in total so we can extract the numbers with regular expression and then put them into the data frame.

Out[5]:

| | Unnamed: 0 | # Columns: time | avg_rss12 | var_rss12 | avg_rss13 | var_rss13 | avg_rss23 | var_rss23 | task | filename |
|---|---------------|-----------------------|-----------|-----------|-----------|-----------|-----------|-----------|---------|---------------|
| 0 | 0 | 0 | 42 | 0 | 11.5 | 4.56 | 18.5 | 0.87 | sitting | dataset14.csv |
| 1 | 1 | 250 | 41.75 | 0.43 | 21.5 | 1.8 | 11.75 | 1.48 | sitting | dataset14.csv |
| 2 | 2 | 500 | 41.67 | 0.47 | 9.33 | 5.44 | 12 | 2.83 | sitting | dataset14.csv |
| 3 | 3 | 750 | 40 | 0.82 | 12.67 | 0.94 | 17.75 | 1.09 | sitting | dataset14.csv |
| 4 | 4 | 1000 | 40.25 | 0.83 | 12.25 | 0.43 | 18.25 | 0.43 | sitting | dataset14.csv |

```
In [6]: #Here I only want to select columns easier in the future, you can igonre this bloc
   k.
   col_names = poses.columns.values
   poses.rename(columns={col_names[0]:'Id', col_names[1]:'Time'}, inplace=True)
   poses.head()
```

Out[6]:

| | ld | Time | avg_rss12 | var_rss12 | avg_rss13 | var_rss13 | avg_rss23 | var_rss23 | task | filename |
|---|----|------|-----------|-----------|-----------|-----------|-----------|-----------|---------|---------------|
| 0 | 0 | 0 | 42 | 0 | 11.5 | 4.56 | 18.5 | 0.87 | sitting | dataset14.csv |
| 1 | 1 | 250 | 41.75 | 0.43 | 21.5 | 1.8 | 11.75 | 1.48 | sitting | dataset14.csv |
| 2 | 2 | 500 | 41.67 | 0.47 | 9.33 | 5.44 | 12 | 2.83 | sitting | dataset14.csv |
| 3 | 3 | 750 | 40 | 0.82 | 12.67 | 0.94 | 17.75 | 1.09 | sitting | dataset14.csv |
| 4 | 4 | 1000 | 40.25 | 0.83 | 12.25 | 0.43 | 18.25 | 0.43 | sitting | dataset14.csv |

Besides, we also noticed that there are seven tasks in the dataset, including 'bending1' and 'bending2'. Here we replace both with 'bending' so we would have 6 tasks in total, which equals to the number of clusters in K-means.

Out[8]:

| | ld | Time | avg_rss12 | var_rss12 | avg_rss13 | var_rss13 | avg_rss23 | var_rss23 | task | filename |
|--------|---------|---------|-----------|-----------|-----------|-----------|-----------|-----------|----------|--------------|
| count | 42239.0 | 42239.0 | 42239.0 | 42239.0 | 42239.0 | 42239.0 | 42239.0 | 42239.0 | 42239 | 42239 |
| unique | 480.0 | 480.0 | 354.0 | 689.0 | 304.0 | 488.0 | 363.0 | 522.0 | 6 | |
| top | 239.0 | 32750.0 | 45.0 | 0.0 | 12.0 | 0.0 | 18.0 | 0.0 | standing | dataset3.csv |
| freq | 88.0 | 88.0 | 2969.0 | 10541.0 | 1942.0 | 8387.0 | 1189.0 | 4935.0 | 7200 | 3360 |

Drop the unnecessary columns.

```
In [9]: poseslag=poses.drop(['Id','filename'], axis=1)
    poseslagCopy = poseslag.copy()
    poseslag = poseslag.drop(['task'], axis=1)
    poseslag.head()
```

Out[9]:

| | Time | avg_rss12 | var_rss12 | avg_rss13 | var_rss13 | avg_rss23 | var_rss23 |
|---|------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0 | 0 | 42 | 0 | 11.5 | 4.56 | 18.5 | 0.87 |
| 1 | 250 | 41.75 | 0.43 | 21.5 | 1.8 | 11.75 | 1.48 |
| 2 | 500 | 41.67 | 0.47 | 9.33 | 5.44 | 12 | 2.83 |
| 3 | 750 | 40 | 0.82 | 12.67 | 0.94 | 17.75 | 1.09 |
| 4 | 1000 | 40.25 | 0.83 | 12.25 | 0.43 | 18.25 | 0.43 |

```
In [11]: from sklearn import preprocessing
    Time = poseslag['Time'].values
    print(Time)
    zeroIndex = np.array(np.where(Time==0.0)).flatten()
    print(zeroIndex)

print('shape of the frame: ',poseslag.iloc[:,:].shape)
    #Let's standedize the variables before taking the difference - standerdicing after
    the lag gave worse results
    stand = preprocessing.StandardScaler()
    stand.fit(poseslag.iloc[:,1:7])

poseslag.iloc[:,1:7] = stand.transform(poseslag.iloc[:,1:7])

[0.0 250.0 500.0 ... 119250.0 119500.0 119750.0]
    [0 480 960 1440 1919 2399 2879 3359 3839 4319 4799 5279
```

```
[ 0 480 960 1440 1919 2399 2879 3359 3839 4319 4799 5279 5759 6239 6719 7199 7679 8159 8639 9119 9599 10079 10559 11039 11519 11999 12479 12959 13439 13919 14399 14879 15359 15839 16319 16799 17279 17759 18239 18719 19199 19679 20159 20639 21119 21599 22079 22559 23039 23519 23999 24479 24959 25439 25919 26399 26879 27359 27839 28319 28799 29279 29759 30239 30719 31199 31679 32159 32639 33119 33599 34079 34559 35039 35519 35999 36479 36959 37439 37919 38399 38879 39359 39839 40319 40799 41279 41759] shape of the frame: (42239, 7)
```

Let's look at the standerdized data

```
In [12]: poseslag.head()
```

Out[12]:

```
Time avg_rss12 var_rss12 avg_rss13 var_rss13 avg_rss23 var_rss23
0
      0
           0.48645 -0.712491
                               -0.511122
                                           1.86506
                                                     0.365922
                                                               -0.458571
1
    250
          0.447001 -0.502525
                                 1.32866
                                          0.181616
                                                     -0.636165
                                                                -0.08595
2
    500
          0.434377 -0.482993
                               -0.910355
                                           2.40181
                                                     -0.599051
                                                                0.738704
          0.170854
                     -0.31209
                               -0.295867
                                         -0.342934
                                                     0.254579 -0.324183
    750
          0.210303 -0.307207 -0.373138 -0.654005
                                                    0.328808 -0.727347
4 1000
```

Out[13]:

| | avg_rss12 | var_rss12 | avg_rss13 | var_rss13 | avg_rss23 | var_rss23 |
|---|---------------------|------------|------------|-----------|-----------|-----------|
| _ | 1 -0.0394496 | 0.209966 | 1.83978 | -1.68344 | -1.00209 | 0.372621 |
| | 2 -0.0126239 | 0.0195317 | -2.23902 | 2.22019 | 0.0371143 | 0.824654 |
| | 3 -0.263523 | 0.170903 | 0.614488 | -2.74474 | 0.85363 | -1.06289 |
| | 4 0.0394496 | 0.00488293 | -0.077271 | -0.311071 | 0.0742287 | -0.403164 |
| | 5 0.0394496 | -0.161137 | -0.0919892 | 0 | -0.222686 | 0.244342 |

Contruct X and y variables

Now we are done with data preparation.

```
In [15]: from sklearn.neighbors import KNeighborsRegressor from sklearn.linear_model import Ridge, LinearRegression from sklearn.model_selection import KFold from sklearn.metrics import mean_squared_error from sklearn import preprocessing from sklearn.decomposition import PCA
```

```
In [16]: #normaliza X
         #standerdize x values
         #stand = preprocessing.StandardScaler()
         #stand.fit(X)
         Xnew = X#stand.transform(X)
         print(Xnew)
         #PCA
         pca poses=PCA(2, whiten=False)
         X proj = pca poses.fit transform(Xnew)
         X proj.shape
         [[-0.03944956609486683 0.20996618889278673 1.8397846631271664
           -1.683440621753161 -1.0020870473087864 0.3726213395681395
          [-0.01262386115035713 \ 0.019531738501654572 \ -2.2390179350257617
           2.2201898055005462 0.03711433508551054 0.8246537842901448]
          [-0.2635231015137106 \ 0.17090271188947764 \ 0.6144880774844736
           -2.7447401441627632 0.8536297069667441 -1.0628870997517421]
          [0.03944956609486683 - 0.1611368426386503 \ 0.1379838497345375
           1.1710891281761122 0.0 0.22601622236100255]
          [0.03944956609486683 -0.03418054237789553 0.5059407823599708
           -0.26227516933110856 \ -0.14845734034204217 \ -0.22601622236100255]
          [-0.07889913218973366\ 0.19531738501654583\ 0.9198923315635834
           -0.9088139588450037 -0.07422867017102108 0.1282794775562447]]
Out[16]: (42151, 2)
```

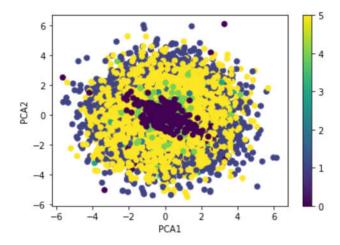
plot the 2 PCA projections

```
In [17]: import numpy as np import matplotlib.pyplot as plt
```

```
In [18]: #convert y values to numbers
    from sklearn import preprocessing
    le = preprocessing.LabelEncoder()
    le.fit(y.reshape((y.shape[0])))
    ynew=le.transform(y.reshape((y.shape[0])))
    print(le.inverse_transform([0,1,2,3,4,5]))
    print(ynew)

plt.xlabel('PCA1')
    plt.ylabel('PCA2')
    plt.scatter(X_proj[:,0], X_proj[:,1], c=ynew)
    plt.colorbar()
    plt.show()
    #following atr the labels for each number mapping.
```

```
['bending' 'cycling' 'lying' 'sitting' 'standing' 'walking']
[3 3 3 ... 0 0 0]
```



calculate loadings

```
In [19]: print(pca_poses.components_)
    print(pca_poses.components_.shape)
    print(pca_poses.explained_variance_ratio_)

[[-0.04163243    0.49197157  -0.18068327    0.73196364  -0.06540286    0.42841099]
    [    0.00546743  -0.04225479    0.14011905  -0.4562811  -0.147734    0.86517831]]
    (2, 6)
    [0.29513513    0.26763138]
```

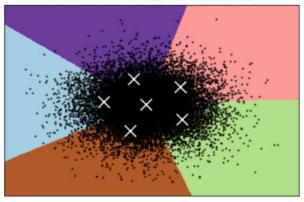
PCA-1 has the most loading from var_rss13 and PCA-2 has the most loading from var_rss23

1. K-means clustering - to map the out put labels to ground truths, I went through all the permutations of y label encoding and select the one with lowest hinge loss of the difference of labelling. This take time. I burrowed the labelling from HMM (next section which I ran completely and found to be the best.)

```
In [38]: from sklearn.cluster import KMeans
                  from time import time
                  from sklearn import metrics
                  import itertools
                  print(82 * ' ')
                  print('init\t\ttime\tinertia\thomo\tcompl\tv-meas\tARI\tAMI\tsilhouette')
                  def createy(seq, y):
                          yseq = np.zeros(y.shape)
                          for i in range(0, y.shape[0]):
                                   yseq[i] = seq[list(poseslist.flatten()).index(y[i])]
                                   #print(yseq[i])
                          return yseq
                  def genLabels(y_pred,y):
                          print('in genLabels')
                          minMSE = 500000
                          minMapping = None
                          perms=list(itertools.permutations([0, 1, 2,3,4,5]))
                          for seq in perms:
                                   yseq = createy(seq,y).astype('int')
                                  MSE = np.sum(y pred != yseq)
                                   if (minMSE>MSE):
                                          minMSE = MSE
                                          minMapping = yseq
                          print('selected')
                          return minMapping
                   def bench k means(estimator, name, data, labels):
                          t0 = time()
                          estimator.fit(data)
                           #we need a function to map fitted labels to ground truths
                           #ymap=genLabels(estimator.labels, labels)
                           #with the seq burrowed from next section,
                          ymap = createy(minseq,y)
                          ymap = ymap.astype('int')
                          print('%-9s\t%.2fs\t%i\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t%.3f\t
                                       % (name, (time() - t0), estimator.inertia,
                                            metrics.homogeneity score(ymap, estimator.labels),
                                            metrics.completeness_score(ymap, estimator.labels_),
                                            metrics.v measure score(ymap, estimator.labels),
                                            metrics.adjusted rand score (ymap, estimator.labels),
                                            metrics.adjusted mutual info score(ymap, estimator.labels,
                                                                                                                   average method='arithmetic'),
                                            metrics.silhouette score(data, estimator.labels,
                                                                                              metric='euclidean',
                                                                                              sample size=300)))
                          return estimator.labels ,ymap
                   y pred, y true = bench k means (KMeans (init='k-means++', n clusters=6, n init=10),
                                              name="k-means++", data=X,labels=ynew)
                  bench k means(KMeans(init='random', n clusters=6, n init=10),
                                              name="random", data=X, labels=ynew)
                   kmeans = KMeans(init='k-means++', n clusters=6, n init=10)
                  kmeans.fit(X proj)
                  reduced_data = X_proj
                   # Step size of the mesh. Decrease to increase the quality of the VQ.
                  h = .02
                                        # point in the mesh [x_min, x_max]x[y_min, y_max].
                   # Plot the decision boundary. For that, we will assign a color to each
                   x_{min}, x_{max} = reduced_data[:, 0].min() - 1, <math>reduced_data[:, 0].max() + 1
```

| <u>in</u> it | time | inertia homo | compl | v-meas | ARI | AMI | silhouet |
|--------------|-------|---------------|-------|--------|-------|-------|----------|
| te | | | | | | | |
| k-means++ | 1.90s | 1836093 0.367 | 0.404 | 0.384 | 0.284 | 0.384 | 0.262 |
| random | 1.79s | 1836094 0.367 | 0.405 | 0.385 | 0.285 | 0.385 | 0.264 |

K-means clustering on the poses dataset (PCA-reduced data) Centroids are marked with white cross



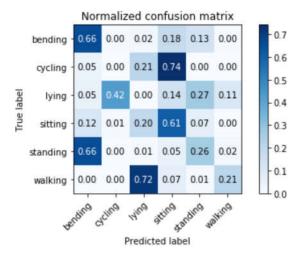
We can measure the overlapping of the clusters as mutual information, which is 0.385 for k-means. This means classes are not seperable and classes members are completely split across different clusters. hard to cluter.

Plot confusion matrix

```
In [39]: from sklearn.metrics import confusion matrix
         from sklearn.utils.multiclass import unique labels
         def plot_confusion_matrix(y_true, y_pred, classes,
                                    normalize=False,
                                    title=None,
                                    cmap=plt.cm.Blues):
             This function prints and plots the confusion matrix.
             Normalization can be applied by setting `normalize=True`.
             if not title:
                 if normalize:
                     title = 'Normalized confusion matrix'
                     title = 'Confusion matrix, without normalization'
             # Compute confusion matrix
             cm = confusion_matrix(y_true, y_pred)
             # Only use the labels that appear in the data
             classes = classes[unique_labels(y_true, y_pred)]
             if normalize:
                 cm = cm.astype('float') / cm.sum(axis=1)[:, np.newaxis]
                 print("Normalized confusion matrix")
             else:
                 print('Confusion matrix, without normalization')
             print(cm)
             fig, ax = plt.subplots()
             im = ax.imshow(cm, interpolation='nearest', cmap=cmap)
             ax.figure.colorbar(im, ax=ax)
             # We want to show all ticks...
             ax.set(xticks=np.arange(cm.shape[1]),
                    yticks=np.arange(cm.shape[0]),
                    # ... and label them with the respective list entries
                    xticklabels=classes, yticklabels=classes,
                    title=title,
                    ylabel='True label',
                    xlabel='Predicted label')
             # Rotate the tick labels and set their alignment.
             plt.setp(ax.get xticklabels(), rotation=45, ha="right",
                      rotation mode="anchor")
             # Loop over data dimensions and create text annotations.
             fmt = '.2f' if normalize else 'd'
             thresh = cm.max() / 2.
             for i in range(cm.shape[0]):
                 for j in range(cm.shape[1]):
                     ax.text(j, i, format(cm[i, j], fmt),
                             ha="center", va="center",
                              color="white" if cm[i, j] > thresh else "black")
             fig.tight layout()
             return ax
```

```
In [40]: #plot confution mat
         # Plot normalized confusion matrix
         plot_confusion_matrix(y_true, y_pred, classes=le.inverse_transform([0, 1,
         2,3,4,5]), normalize=True,
                               title='Normalized confusion matrix')
         Normalized confusion matrix
         [[6.58333333e-01 3.05555556e-03 2.09722222e-02 1.80138889e-01
           1.34027778e-01 3.47222222e-03]
          [4.58333333e-02 0.00000000e+00 2.11111111e-01 7.41805556e-01
           2.77777778e-04 9.7222222e-04]
          [5.08012821e-02 4.24679487e-01 1.60256410e-03 1.43750000e-01
           2.69070513e-01 1.10096154e-01]
          [1.15432699e-01 9.30684817e-03 1.95443812e-01 6.10084734e-01
           6.69537436e-02 2.77816363e-03]
          [6.59722222e-01 0.00000000e+00 1.09722222e-02 4.97222222e-02
           2.61527778e-01 1.80555556e-02]
          [1.52777778e-03 0.00000000e+00 7.17777778e-01 6.72222222e-02
           6.6666667e-03 2.06805556e-01]]
```

Out[40]: <matplotlib.axes._subplots.AxesSubplot at 0x232a8bc04e0>



This is because the classes are not seperable.

HMM

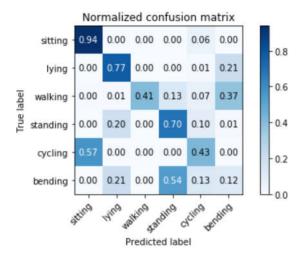
```
In [27]: #standerdize x values
         stand = preprocessing.StandardScaler()
         stand.fit(X)
         Xnew2 = X#stand.transform(X)
         from hmmlearn import hmm
         model= hmm.GaussianHMM(n components=6).fit(Xnew2)
         y_pred=model.predict(Xnew2)
         print(y pred)
         [0 4 4 ... 2 2 2]
In [34]: #create labels for
         import statistics
         from statistics import mode
         from sklearn.metrics import mean squared error,log loss
         import itertools
         def createy(seq):
             yseq = np.zeros(y.shape)
             for i in range(0, y.shape[0]):
                 yseq[i] = seq[list(poseslist.flatten()).index(y[i])]
                 #print(yseq[i])
             return yseq
         minMSE = 500000
         minMapping = None
         minseq = None
         perms=list(itertools.permutations([0, 1, 2, 3, 4, 5]))
         for seq in perms:
             yseq = createy(seq).astype('int')
             MSE = np.sum(y pred != yseq)
             if(minMSE>MSE):
                 minMSE = MSE
                 minMapping = yseq
                 minseq = seq
         print('hinge loss of the classification of Gaussian HMM',minMSE)
```

hinge loss of the classification of Gaussian ${\tt HMM}$ 22441

confusion matrix

```
In [31]: plot_confusion_matrix(minMapping.astype('int'), y_pred, classes=poseslist.flatten
         (), normalize=True,
                               title='Normalized confusion matrix')
         Normalized confusion matrix
         [[9.38750000e-01 0.00000000e+00 0.00000000e+00 0.00000000e+00
           6.12500000e-02 0.00000000e+00]
          [6.9444444e-04 7.7444444e-01 0.00000000e+00 6.9444444e-04
           1.31944444e-02 2.10972222e-01]
          [4.96794872e-03 8.81410256e-03 4.12339744e-01 1.31250000e-01
           6.82692308e-02 3.74358974e-01]
          [2.7777778e-04 2.00555556e-01 0.00000000e+00 6.95972222e-01
           9.5555556e-02 7.63888889e-03]
          [5.70694444e-01 4.16666667e-04 0.0000000e+00 5.55555556e-04
           4.28333333e-01 0.00000000e+00]
          [1.80580636e-03 2.11418253e-01 3.47270454e-03 5.36602306e-01
           1.31407140e-01 1.15293791e-01]]
```

Out[31]: <matplotlib.axes. subplots.AxesSubplot at 0x232abcb5d30>



This is better than the previous case. Let's calculate the mutual info for the classses to calculate the overlapping.

classes members are less split across different clusters which is favourable.

Exercise 3 (40 pts)

Load the housing.csv dataset, your task is to predict the Sale price. Deal with the missing data by simple imputation and by creating missingness indicator variables. Train random forests, gradient tree boosting (XGBoost), K-nearest neighbors, and kernel SVMs. Compare them using appropriate cross-validation.

```
In [23]: #read the data frames
    data_mat = pd.read_csv('housing.csv')
    data_mat.head()
```

Out[23]:

| | ld | MSSubClass | MSZoning | LotFrontage | LotArea | Street | Alley | LotShape | LandContour | Utilities | PoolArea |
|---|----|------------|----------|-------------|---------|--------|-------|----------|-------------|-----------|--------------|
| 0 | 1 | 60 | RL | 65.0 | 8450 | Pave | NaN | Reg | Lvl | AllPub | |
| 1 | 2 | 20 | RL | 80.0 | 9600 | Pave | NaN | Reg | Lvl | AllPub | |
| 2 | 3 | 60 | RL | 68.0 | 11250 | Pave | NaN | IR1 | Lvl | AllPub | |
| 3 | 4 | 70 | RL | 60.0 | 9550 | Pave | NaN | IR1 | Lvl | AllPub | |
| 4 | 5 | 60 | RL | 84.0 | 14260 | Pave | NaN | IR1 | Lvl | AllPub | |

5 rows × 81 columns

In [24]: data_mat.describe()

Out[24]:

| | ld | MSSubClass | LotFrontage | LotArea | OverallQual | OverallCond | YearBuilt | YearRemodAdd |
|-------|-------------|-------------|-------------|---------------|-------------|-------------|-------------|--------------|
| count | 1460.000000 | 1460.000000 | 1201.000000 | 1460.000000 | 1460.000000 | 1460.000000 | 1460.000000 | 1460.000000 |
| mean | 730.500000 | 56.897260 | 70.049958 | 10516.828082 | 6.099315 | 5.575342 | 1971.267808 | 1984.865753 |
| std | 421.610009 | 42.300571 | 24.284752 | 9981.264932 | 1.382997 | 1.112799 | 30.202904 | 20.645407 |
| min | 1.000000 | 20.000000 | 21.000000 | 1300.000000 | 1.000000 | 1.000000 | 1872.000000 | 1950.000000 |
| 25% | 365.750000 | 20.000000 | 59.000000 | 7553.500000 | 5.000000 | 5.000000 | 1954.000000 | 1967.000000 |
| 50% | 730.500000 | 50.000000 | 69.000000 | 9478.500000 | 6.000000 | 5.000000 | 1973.000000 | 1994.000000 |
| 75% | 1095.250000 | 70.000000 | 80.000000 | 11601.500000 | 7.000000 | 6.000000 | 2000.000000 | 2004.000000 |
| max | 1460.000000 | 190.000000 | 313.000000 | 215245.000000 | 10.000000 | 9.000000 | 2010.000000 | 2010.000000 |

8 rows × 38 columns

```
In [25]: #create a missing variables:
    #first drop id and sales price
    data_matforfill = data_mat.drop(['Id','SalePrice'],axis=1)
    data_matforfill.describe()

#create missingdataframe
    data_null = data_matforfill.isnull()
    data_null.head()

#change column names
for i in data_null.columns.values:
        data_null.rename(columns={i:i+'m'}, inplace=True)

data_null = data_null.astype('int')
    data_null.head()
```

Out[25]:

| | MSSubClassm | MSZoningm | LotFrontagem | LotAream | Streetm | Alleym | LotShapem | LandContourm | Utilitiesm |
|---|-------------|-----------|--------------|----------|---------|--------|-----------|--------------|------------|
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | |
| 2 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | |
| 3 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | |
| 4 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | |

5 rows × 79 columns

create one hot encoding for categorical variables

```
In [26]: #imputate nan values.
    data_mat = data_mat.fillna(data_mat.mode().iloc[0])

#select cat columns
    cat_columns = data_mat.select_dtypes(['object']).columns
#print('cat columns',cat_columns)
    dataCat = data_mat.filter(cat_columns)
    dataCat.head()
```

Out[26]:

| | MSZoning | Street | Alley | LotShape | LandContour | Utilities | LotConfig | LandSlope | Neighborhood | Condition1 |
|---|----------|--------|-------|----------|-------------|-----------|-----------|-----------|--------------|------------|
| 0 | RL | Pave | Grvl | Reg | Lvl | AllPub | Inside | Gtl | CollgCr | Norm |
| 1 | RL | Pave | Grvl | Reg | Lvl | AllPub | FR2 | Gtl | Veenker | Feedr |
| 2 | RL | Pave | Grvl | IR1 | LvI | AllPub | Inside | Gtl | CollgCr | Norm |
| 3 | RL | Pave | Grvl | IR1 | LvI | AllPub | Corner | Gtl | Crawfor | Norm |
| 4 | RL | Pave | Grvl | IR1 | LvI | AllPub | FR2 | Gtl | NoRidge | Norm |

5 rows × 43 columns

```
In [27]: #one hot encoding for each column
    from sklearn.preprocessing import OneHotEncoder
    enc = OneHotEncoder(handle_unknown='ignore')
    enc.fit(dataCat)
    Onehotarray=enc.transform(dataCat).toarray()
```

form X and y

```
In [34]: datanum=data mat.drop(cat columns,axis=1)
         #concat missing values
         datanum = pd.concat([datanum, data_null], axis=1)
         datanum.head()
         X = datanum.drop(['Id', 'SalePrice'], axis=1).values
         y = datanum['SalePrice'].values
         #print('shapes',X.shape,y.shape)
         #concatenate one hot matrix
         X = np.concatenate((X,Onehotarray),axis=1)
         #print('shapes', X.shape, y.shape)
         X = X.astype('float32')
         y = y.astype('float32')
         #center y
         y = y-np.mean(y)
         X.shape, y.shape
Out[34]: ((1460, 367), (1460,))
```

K-fold cross validation with K=5

```
In [38]: from sklearn.neighbors import KNeighborsRegressor
from sklearn.linear_model import Ridge, LinearRegression
from sklearn.model_selection import KFold,train_test_split
from sklearn.metrics import mean_squared_error
from sklearn.preprocessing import normalize,StandardScaler
from sklearn.decomposition import PCA
from sklearn import linear_model,svm
from matplotlib import pyplot as plt
```

split the dataset

```
In [39]: #standerdise X
    stand = StandardScaler()
    X_new = stand.fit_transform(X)

X_tr, X_te, y_tr, y_te = train_test_split(X_new,y,test_size=0.1,random_state=42)

In [40]: kf = KFold(n_splits=5, shuffle=True, random_state=17) #Fix the results
    kf.get_n_splits(X_tr)
    train_indices = []
    test_indices = []
    for train_index, test_index in kf.split(X_tr):
        train_indices.append(train_index)
        test_indices.append(test_index)
```

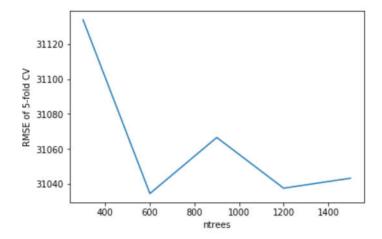
train a linear regression as a benchmark

```
In [43]: LRlosses=[]
for k in range(5):
    lm = linear_model.LinearRegression(fit_intercept=False)
    lm.fit(X_tr[train_indices[k]], y_tr[train_indices[k]])
    y_pred = lm.predict(X_tr[test_indices[k]])
    LRlosses.append(mean_squared_error(y_tr[test_indices[k]],y_pred))
print('CV loss of linear regression:', np.sqrt(np.mean(LRlosses)))
CV loss of linear regression: 141667860.0
```

Train a random forest

```
In [45]: from sklearn.ensemble import RandomForestRegressor
         RFlosses= np.zeros((5,5))
         # Instantiate model with 1000 decision trees
         for i in range(5):
             print(i)
             for k in range(5):
                 rf = RandomForestRegressor(n_estimators = i*300 +300, random_state = 42)
                 rf.fit(X_tr[train_indices[k]], y_tr[train_indices[k]])
                 y pred = rf.predict(X tr[test indices[k]])
                 RFlosses[i,k] = mean squared error(y tr[test indices[k]],y pred)**0.5
         print('CV loss of random forests:', np.mean(RFlosses,axis=1))
         print('CV loss of Xgboost:', np.mean(RFlosses,axis=1))
         plt.plot(range(300, 1800,300), np.mean(RFlosses, axis=1))
         plt.xlabel('ntrees')
         plt.ylabel('RMSE of 5-fold CV')
         plt.show()
```

```
0
1
2
3
4
CV loss of random forests: [31133.89492345 31034.48469363 31066.5773047
31037.56611726
31043.25479958]
CV loss of Xgboost: [31133.89492345 31034.48469363 31066.5773047 31037.56611726
31043.25479958]
```

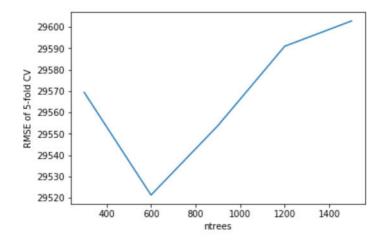


CV loss for the tandom forest is RMSE = 31034 \$ at 600 trees

Train Xgboost

```
In [46]: XGBlosses = np.zeros((5,5))
         import xgboost as xgb
         for i in range(5):
             print(i)
             for k in range(5):
                 xg reg = xgb.XGBRegressor(objective ='reg:squarederror', colsample bytree =
         0.3, learning_rate = 0.1, max_depth = 3, alpha = 10, n_estimators = i*300+300)
                 xg_reg.fit(X_tr[train_indices[k]], y_tr[train_indices[k]])
                 y pred = xg reg.predict(X tr[test indices[k]])
                 XGBlosses[i,k] = mean squared error(y tr[test indices[k]],y pred) **0.5
         print('CV loss of Xgboost:', np.mean(XGBlosses,axis=1))
         plt.plot(range(300, 1800,300), np.mean(XGBlosses, axis=1))
         plt.xlabel('ntrees')
         plt.ylabel('RMSE of 5-fold CV')
         plt.show()
         0
         1
         2
```

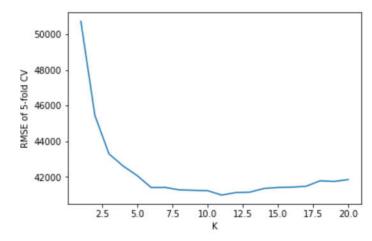
29602.85134813]



CV error for Xgboost is RMSE = 29521\$ at 600 trees

Train K nearest neighbors

```
In [47]: KNNlosses = np.zeros((5,20))
         for k in range(20):
             regressor = KNeighborsRegressor(n_neighbors = k*1+1)
             for index in range(5):
                 regressor.fit(X_tr[train_indices[index]], y_tr[train_indices[index]])
                 predictions = regressor.predict(X tr[test indices[index]])
                 KNNlosses[index, k] = mean squared error(predictions, y tr[test indices[ind
         ex]])**0.5
         print('CV loss of KNNlosses:', np.mean(KNNlosses,axis=0))
         plt.plot(range(1,21), np.mean(KNNlosses, axis=0))
         plt.xlabel('K')
         plt.ylabel('RMSE of 5-fold CV')
         plt.show()
         CV loss of KNNlosses: [50727.82510177 45440.87280858 43285.10672824
         42608.41535175
          42069.47429465 41396.79390145 41402.57623926 41265.68664991
          41242.33564043 41222.77327505 40977.51906625 41113.65516264
          41137.79483345 41342.97866879 41399.87067262 41419.95426065
          41467.52606592 41774.68710704 41738.47898334 41844.96686238]
```

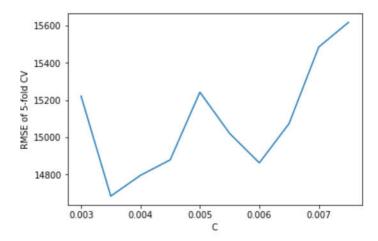


CV error for KNN is RMSE = 40978\$ at 11 neighbors

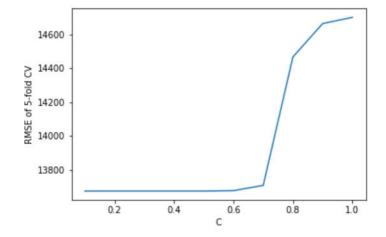
Train KernelSVD

```
In [49]: #Linear
         SVMlossesl = np.zeros((5,10))
         for k in range(10):
             print(k)
             svm_sim = svm.SVC(kernel="linear", C=k*0.0005+0.003, gamma='auto')
             for index in range(5):
                 svm sim.fit(X tr[train indices[index]], y tr[train indices[index]].astype('
         int'))
                 predictions = svm sim.predict(X tr[test indices[index]])
                 SVMlossesl[index, k] = mean squared error(predictions, y tr[test indices[in
         dex]].astype('int'))**0.5
         print('CV loss of Linear SVM:', np.mean(SVMlossesl,axis=0))
         plt.plot(np.arange(10).astype('float32')*0.0005+0.003, np.mean(SVMlosses1, axis=0))
         plt.xlabel('C')
         plt.ylabel('RMSE of 5-fold CV')
         plt.show()
         0
```

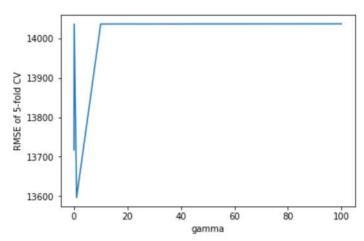
1 2 3 4 5 6 7 8 9 CV loss of Linear SVM: [15221.52471191 14684.66574958 14796.08072802 14879.27878785 15242.34083338 15021.13814618 14862.98352764 15073.5285543 15484.71847919 15617.03649246]



```
In [50]: #rbf - not sensitive to c after 0.6
         SVMlossesr = np.zeros((5,10))
         for k in range(10):
             print(k)
             svm sim = svm.SVC(kernel="rbf", C=k*0.1+0.1, gamma='auto')
             for index in range(5):
                 svm sim.fit(X tr[train indices[index]], y tr[train indices[index]].astype('
         int'))
                 predictions = svm sim.predict(X tr[test indices[index]])
                 SVMlossesr[index, k] = mean squared error(predictions, y tr[test indices[in
         dex]].astype('int'))**0.5
         print('CV loss of RBF SVM:', np.mean(SVMlossesr,axis=0))
         plt.plot(np.arange(10)*0.1+0.1, np.mean(SVMlossesr, axis=0))
         plt.xlabel('C')
         plt.ylabel('RMSE of 5-fold CV')
         plt.show()
```



```
In [56]: \#rbf - tuning for when c=0.4
         SVMlossesrg = np.zeros((5,5))
         gamma = np.array([1e-2,1e-1,1,10,100]).astype('float32')
         for k in range(5):
             print(k)
             svm sim = svm.SVC(kernel="rbf", C=0.4, gamma=gamma[k])
             for index in range(5):
                 svm sim.fit(X tr[train indices[index]], y tr[train indices[index]].astype('
         int'))
                 predictions = svm sim.predict(X tr[test indices[index]])
                 SVMlossesrg[index, k] = mean_squared_error(predictions, y_tr[test_indices[i
         ndex]].astype('int'))**0.5
         print('CV loss of RBF SVM:', np.mean(SVMlossesrg,axis=0))
         plt.plot(gamma, np.mean(SVMlossesrg, axis=0))
         plt.xlabel('gamma')
         plt.ylabel('RMSE of 5-fold CV')
         plt.show()
         0
         2
         3
         CV loss of RBF SVM: [13716.98355805 14036.66720102 13596.16840847 14036.66720102
          14037.11511149]
```



RBF kernels perform better than linear kenels for the following tuned values.

Summary of the results I obtained the following results.

| Method | 5 - fold CV RMSE | parameter |
|-------------------|------------------|--------------------|
| Linear regression | 141667860.0 | with intercept |
| Random forest | 31034 | ntrees = 900 |
| Xgboost | 29521 | ntrees = 900 |
| KNN | 40977 | K = 11 |
| SVM - Linear | 14684 | C = 0.0035 |
| SVM - RBF | 13596 | C = 0.4 ,gamma = 1 |

RGF Kernel SVM performs the best. Let's calculate the test error for that.