Final Year Project 2019
Department of Mechanical and Aerospace Engineering
Monash University

VISUALISATION OF PLASMA IN A FUSION REACTOR USING AUGMENTED REALITY

Chamaka Wijesinghe

Supervised by Associate Professor Tuck Ng

ABSTRACT

Plasma confinement is a vital aspect of fusion reactor design and is difficult to model and visualise. A simplified model showing how plasma behaves within a stable magnetic field is developed. It was shown that a confining magnetic field was possible to derive and the impact of having 100 particles in the reactor did not influence stability. Based on the simulation, an Augmented Reality (AR) application was developed to help visualise plasma dynamics. A flat AR marker was used to develop a simulation with a sphere representing a plasma particle and a realistically dimensioned Tokamak model. The application was designed such that the user was able to manipulate the speed of the particle and the magnetic field strength. This simulation worked well to give a qualitative understanding of how plasma confinement is affected by these two parameters. A cylindrical marker was also used with a 3D printed scaled Tokamak to add a physical element to the simulation. More work is required to produce a more realistic viewing simulation. Plasma particles seemed to be rendered on top of the Tokamak model instead of within it. Further work is also required to improve the accuracy of the model dictating plasma motion.

NOMENCLATURE

E_t	Energy
m	Mass
c	Speed of light
${}^2_1H$	Deuterium
${}^3_1H$	Tritium
${}^4_2He$	Helium
${}^1_0n$	Neutron
U_{coul}	Potential energy
ε_0	Permittivity of free space
q_i	Charge of ith particle
r	Radial distance
F	Force
F_e	Force produced from electric field
E	Electric field
B	Magnetic field
v	Velocity
π	Pi
ψ	Magnetic surface equation
z	Vertical distance
p	Plasma pressure
Fa	Arbitrary function
α	Arbitrary constant
R	Radial distance between magnetic axis and torus centre
n_ϕ	Azimuthal unit vector
$\hat{r}$	Radial unit vector
$\hat{z}$	Vertical unit vector
a	Acceleration
V_{step}	Velocity used to calculate position
V_{calc}	Velocity found from integrating acceleration
V_i	Initial velocity

INTRODUCTION

Nuclear fusion is the process of fusing light nuclei to heavier nuclei, which in the process creates large amounts of energy. The high energy production is due to the reduction in mass of the system, converting mass to energy according to $E_t = mc^2$. A promising fusion reaction with the potential to be used within a fusion reactor is Deuterium- Tritium fusion:

$$ {}^2_1H + {}^3_1H \rightarrow {}^4_2He + {}^1_0n + 17.59\ MeV\ , \tag{1}$$

this fusion reaction has an activation energy of 0.28Mev and involves deuterium and tritium combining to form Helium and a free neutron with a net kinetic energy of 17.59 MeV [1]. The relatively high activation energy required is due to overcoming the Coulomb barrier for the strong force to dominate. The Coulomb barrier is produced from the electrostatic force repelling two like charges. The governing equation is:

$$U_{coul} = \frac{1}{4\pi\varepsilon_0} * \frac{q_1 q_2}{r}. \tag{2}$$

As the distance between two particles reduce, the resulting potential energy increases, but the strong force begins to dominate when the distance between particles is lower than 1 femtometre [2]. The potential barrier around the point where the strong force dominates is described by figure 1. The ignition temperature required for fusion to occur is approximately 100 million Kelvin. This is lower than what is predicted with the coulomb barrier since quantum tunnelling affects allow particles to fuse at lower temperatures. At these temperatures particles transform state into plasma where its properties are exploited to sustain fusion [1].

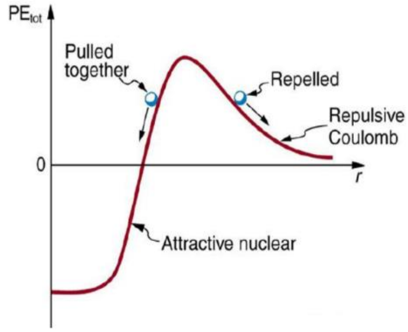

Figure 1: Potential barrier of a nuclei [2]

Plasma is a hot ionized gas consisting of charged particles that can be influenced by electric and magnetic fields. Magnetic confinement is used to contain plasma inside the reactor without any physical contact. However, due to the high temperature and curved path particles follow, drift always occurs, and plasma will touch the walls of the reactor. If plasma contacts the walls of the reactor it introduces impurities into the plasma reducing temperature and fusion reaction rate as well as damaging the reactor. A twisted helically shaped magnetic field have been used to limit drifting [3]. Two most promising designs are Tokamaks and Stellarator. A Tokamak uses a torus shaped magnetic field lines to confine the plasma and has been the most experimentally researched due to its simpler design. A Stellarator uses a helically shaped magnetic field lines to confine the plasma, but this option is not as popular due to the complexity of the design [4]. Figure 2 highlights the difference between the two designs.

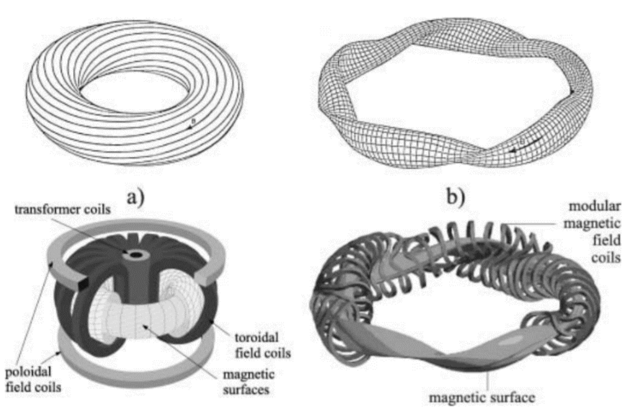

Figure 2: a) Torus magnetic field lines used in a Tokamak and Tokamak fusion reactor design. b) Helically warped magnetic field lines used in Stellarator and design of a Stellarator fusion reactor [3]

The simplest approach to modelling plasma is to use the Guiding Centre model. This model identifies the behaviour of particles within a magnetic and electric field. This is based on the Lorentz force acting on a charged particle within a magnetic and electric field:

$$F = q(E + v \times B). \tag{3}$$

This is the governing equation used by basic particle-in-cell (PIC) modelling which tracks every particle in the system.

METHODOLOGY

Modelling stable magnetic fields using MATLAB

The model developed required several simplifications to be made. The point particle model is used with the Lorentz force (equation 3) acting as the governing equation for modelling the motion of each particle. Protons are only considered in the system and the system is assumed to be in equilibrium meaning the kinetic energy of each particle is constant. The electric field force is produced by all other particles in the system. The force arising from the electric field is governed by:

$$F_e = \frac{1}{4\pi\varepsilon_0} * \frac{q_1 q_2}{r^2}. \tag{4}$$

A stable magnetic field is derived from the Grad-Shafranov equation [5]. For a torus this is:

$$r\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial r} + \frac{\partial^2\psi}{\partial z^2} + r^2 p'(\psi) + FaFa'(\psi) = 0. \tag{5}$$

Solving for the magnetic surface equation (ψ) in equation 5, a general solution is found of the form [5]:

$$\psi = r^2(2R^2 - r^2 - 4\alpha^2 z^2), \tag{6}$$

The stable magnetic field is then derived using equation 7 [5]:

$$B = \frac{1}{r}\left[n_\phi \times \nabla\psi(r,z) + n_\phi Fa(r,z)\right]. \tag{7}$$

Equation 3 is then used to determine the force on the particle. The acceleration is then solved using $F = m * a$. The result is then integrated twice using Euler method to determine the final position of the particle. The velocity of the particle is normalised with every step using equation 8:

$$V_{step} = V_{calc}/V_i, \tag{8}$$

this ensures a constant kinetic energy is maintained since magnetic fields only accelerate particles perpendicular to direction of motion. This also is used to correct errors associated propagated using Euler method. The algorithm is summarised in figure 3.

Limitations of this method are that tracking all particles is not an efficient method to simulate a full-size reactor. This method is computationally heavy and inefficient and would take too long to model a full-size reactor. The most advanced plasma code can only track 1 billion particles [6]. Techniques like clumping many particles into a single particle used in the Blob-in-cell technique could be used to find the influence of large numbers of particles in the system. The plasma dynamics model is limited due to not accounting for how electrons effect the magnetic confinement of plasma. It is also not considering how particle trajectories change with collisions or from the gain in kinetic energy from undergoing a fusion reaction. Effects such as magnetic mirror effect or quantum effects have not been considered and could be investigated in future work.

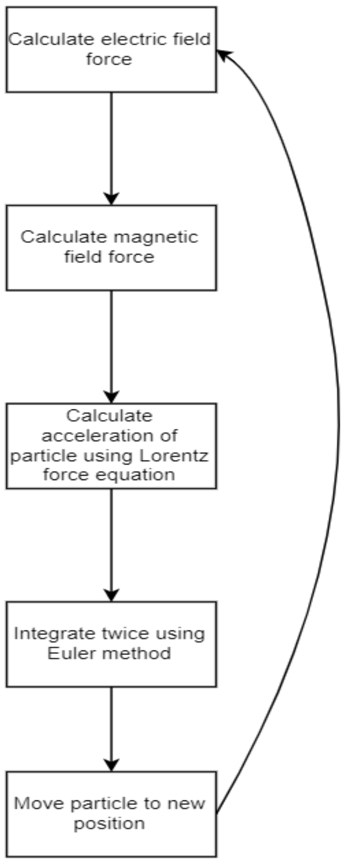

Figure 3: Summary of algorithm used in MATLAB implementation of plasma simulation

Visualising plasma model using Unity

Unity is used to develop an AR application. The script that models the particle movement is the same as the MATLAB script but has been converted to C# to run on Unity. A solid sphere was used to represent a plasma particle. A representative Tokamak split in two was produced using Solidworks. This allowed the user to see how the particle moved within the torus. A virtual simulation was first created shown in figure 4. The trail renderer function was used to draw the path line of the plasma particle.

After the virtual simulation was set up and working, augmented reality components were incorporated. A flat AR marker was first used. The marker was sourced online using [7] and was created with high lines, high triangles and no colour specified. This is recommended by Vuforia, the AR target management software, for optimal marker detection [8]. An example of a marker used is shown in figure 4.

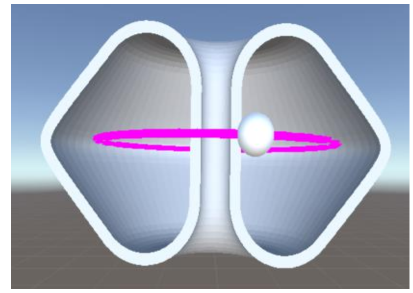

Figure 4: Virtual simulation using Unity

Appropriate scaling of dimensions was required to allow the marker to display the Tokamak simulation within the screen. Additional features to display and change parameters of the simulation were added to allow the user to interact with the simulation. Parameters that were able to change were speed of the plasma particle and the magnitude of the magnetic field. The user was able to vary the parameters using a slider button. Simulation data of both parameters were displayed in the top left corner of the screen. The simulation prompts the user to restart the simulation if the particle exceeds the boundary of the Tokamak. The canvas interface is shown in figure 6.

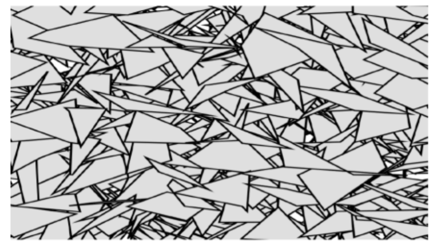

Figure 5: AR marker with high lines, high triangles and no colour specified

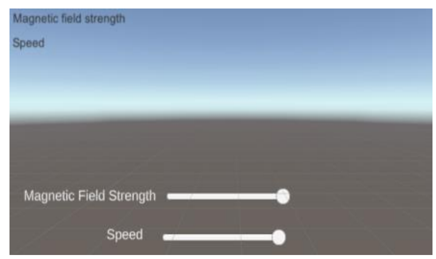

Figure 6: Canvas interface of AR application

A cylindrical marker was then incorporated to enable the use of a 3D printed Tokamak. The scale model Tokamak was approximately 15cm tall and 20cm in diameter. One half was glued to a cylindrical pipe 9cm in diameter and 15 cm tall shown

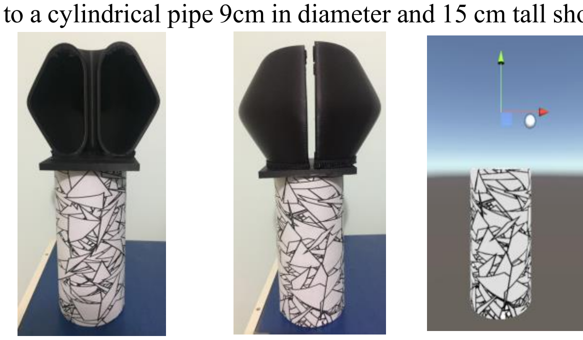

Figure 7: a) (left) Marker and Tokamak model setup. b) (middle) Full Tokamak model setup. c) (right). Origin offset relative to the cylindrical marker.

in figure 7a and the full setup is shown in figure 7b. The marker was required to be resized to fit the cylindrical pipe the Tokamak is glued to. The simulation is then transformed such that the global origin is adjusted to above the cylindrical marker shown in figure 7c. Adjustments to scaling in the simulation was required to fit the dimensioning of the 3D printed Tokamak.

The limitations of this setup are that it only enables the user to test one type of fusion reactor. For future work a flexible mount could be incorporated to allow different types of fusion reactors to sit on top of the cylinder. Parameters such as the timescale of the simulation or multiple particles entering the Tokamak could also be investigated in the future.

RESULTS

MATLAB simulation results

For the simulations tested the function Fa defined in equation 7 was set to 0 and parameters for the magnetic surface equation (ψ) from equation 6 were R = 2m and α = 0.5. The resultant magnetic surfaces are shown in figure 8.

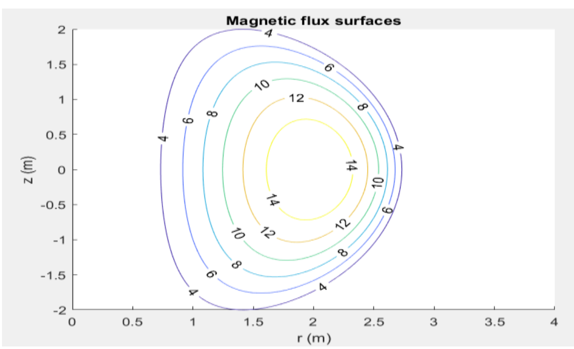

Figure 8: Magnetic surface contour plot

Using equation 7 a stable magnetic field was solved as:

$$B = (1/r - 2zr^2)\,\hat{r} + (-1/r * (4r - 4r^3 - 2rz^2))\,\hat{z} \quad (8)$$

A step size of $1 * 10^{-9}$ seconds was used and was simulated until particles displayed steady state trajectories. Initial velocity of the particle is set at a typical fusion reactor beam velocity which is 0.03c [9] and positioned within the torus. Path lines of each particle were plotted to visualise trajectories and the inner and outer edges of the torus was plotted in red to show the volume particles were required to be confined to.

To verify the model a single particle was positioned at the magnetic axis and the trajectory was analysed. The path line of the particle shown in figure 9 indicates a corkscrew motion which is the expected trajectory of a charged particle in a magnetic field [6].

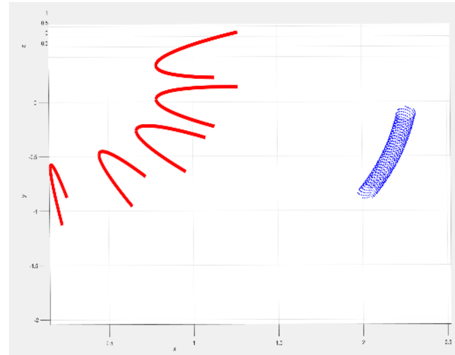

Figure 9: Simulation results of a single particle positioned at the magnetic axis. Path line indicates a corkscrewing motion which is the expected theoretical motion.

The first test conducted was to inject 5 particles positioned with equal spacing on the same plane shown in figure 10. The results of the simulations at different time intervals are shown in figure 11. Each snapshot is taken at the same location.

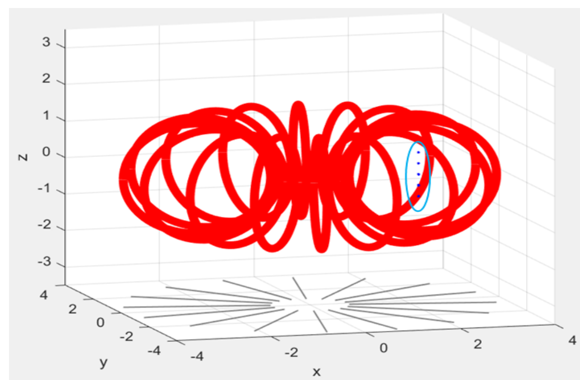

Figure 10: Simulation of 100 particles evenly spaced initial setup

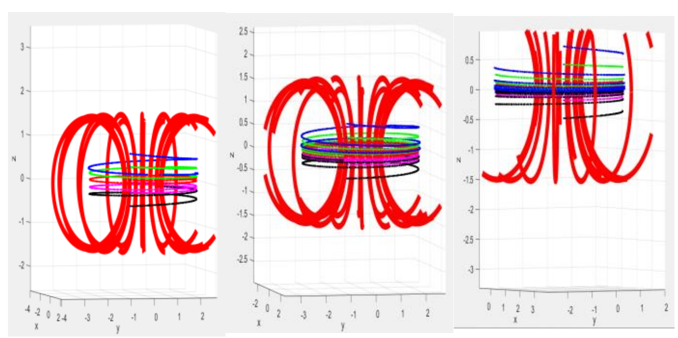

Figure 11: Snapshot of particle simulation. From left to right the time elapsed is 1ms, 2ms and 5ms

The results indicate a stable magnetic field as expected since the particles furthest from the magnetic axis shown in blue and black converge to the centre of the magnetic axis located 2m away from the global origin. Particles located centrally remain and converge to the magnetic axis.

The second test was to inject a larger particle density in the Tokamak to determine whether the electric field of other particles influence stability. 100 particles evenly spaced were positioned at the magnetic axis of the Tokamak as shown in figure 12. The result of this test is shown in figure 13.

This indicates that the influence of charge from 100 particles in the system does not affect the stability of the plasma confinement for this magnetic field since all particle remain within the torus boundary.

Limitations and Future Work

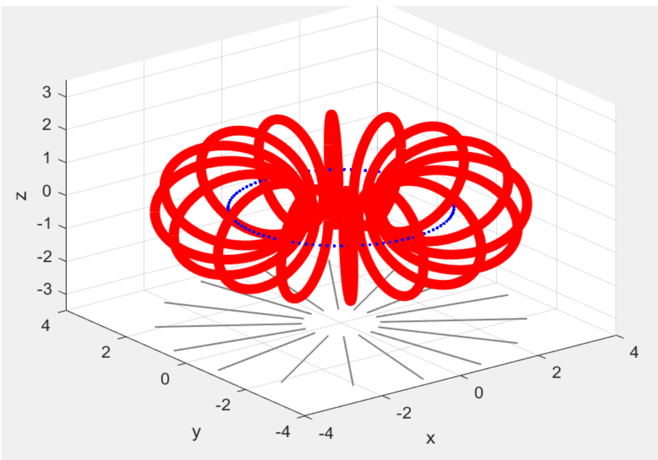

Figure 12: Simulation of 100 particles evenly spaced initial setup

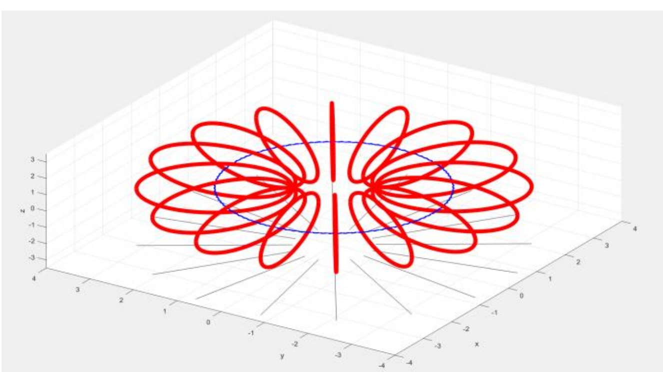

Figure 13: 100 particle simulation result

The tests that were run only simulated a relatively a small number of particles. A fusion reactor would contain in excess of billions of particles and the method developed does not efficiently solve in excess of hundreds of particles inside the reactor. Attempts were made to increase algorithm speed by increasing the step size but lead to an inaccurate result. Techniques such as Blob-in-Cell (BIC) use super particles that clump many particles into one particle to reduce the number of particles needed to be tracked. Also, the use of the Debye sphere [5] which is the maximum sphere of influence of particle motion could also be used to limit particle searching could be used to reduce computational time of the algorithm.

Another limiting factor of these simulations was that only one magnetic field was tested, and it is unknown whether the magnetic field can be practically generated. Future work may include testing practicable magnetic fields or developing magnetic fields that could be easily produced and able to confine plasma.

Computational errors are present but have been suppressed with the normalisation of velocity at each step shown in equation 8. A more in-depth error analysis could be performed to determine why kinetic energy increases without normalisation. A possible source of improvement could be to implement more accurate ODE solving techniques. Euler method has error order of 1 and was implemented due to simplicity. Techniques such as Runge-Kutta method has error order of 4 and could be used.

The algorithm developed forms a basis for other computational plasma models. Adaptation of more accurate plasma models could be investigated as well as investigation into other fusion reactor parameters such as heat output, energy generation and efficiency.

Unity simulation results

The initial virtual plasma simulation shown in figure 3 worked well. Path lines could be generated and details of the spiralling motion of the plasma particle can be observed. The limitations of using this software was that fine details of motion was difficult to observe from the path lines. This is due to the thickness of the lines that were able to be generated and the limit of the number of points that could be plotted to create the lines. Specifying a point to be plotted every 60 steps, the same specification as the MATLAB script, resulted in the program crashing. Points were plotted every 600 steps to avoid this. The use of Unity for this simulation is only useful to gain qualitative details of how plasma behaves. Finer details of motion such as the radius of gyration of the particle can be observed using the MATLAB simulation.

The use of a flat AR marker allowed for a more interactive view of how the plasma behaved. The incorporation of slider buttons to change speed and magnetic field strength allowed the user to gain a better understanding of how these parameters affect plasma stability. The motion of the particle was increased to enable the user to get a better feel for where the particle was relative to the Tokamak without the path lines. The screenshot of the simulation is shown in figure 14. If the parameters changed such that the particle was no longer within the boundaries of the Tokamak a “particle has escaped” message appears.

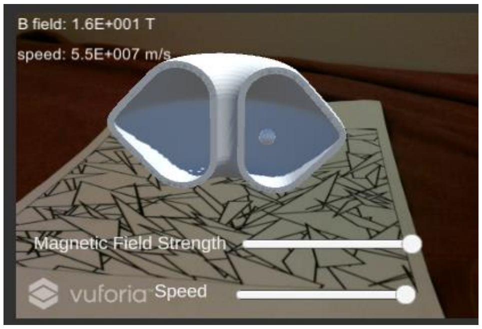

Figure 14: Simulation using flat AR marker

The limitations of this setup were that the use of path lines could not be used due to the trail renderer function not compatible in AR mode. The rendered line would only work if the camera stayed still. If the camera moved a previous rendered line would show in the perspective of the previous camera angle.

The use of the 3D printed Tokamak and cylindrical marker added a physical aspect to the simulation. It enables the user to get a feel for where the plasma particle is in respect to the scaled Tokamak. The limitations of this model were that the particle was projected onto the 3D model. It did not look like the particle was within the Tokamak shown in figure 15. Possible workarounds would be to simulate the Tokamak and adjust transparency such that the 3D model is overlayed by it. This will allow the shadow of the particle to be seen to have the effect of the particle be within the Tokamak. Other possible improvements could be to use a brighter coloured 3D print material for the model since using black hides features of the Tokamak. Future work would be to use different types of Tokamaks to distinguish how well each can confine the plasma particle.

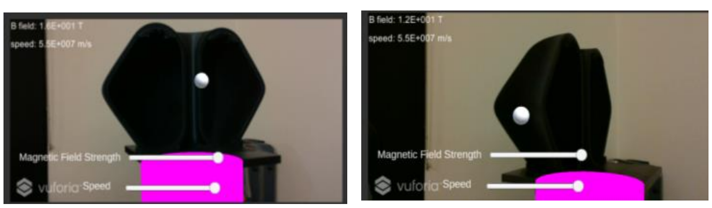

Figure 15: Simulation using a cylindrical marker. The simulation does not seem to show the particle was within the Tokamak but looked like it was rendered on top of the model

Overall the simulation using the flat AR marker worked best for understanding the motion of the plasma particle. There was no confusion as to where the particle was relative to the Tokamak and the user was able to get a sense of how the plasma particle moves within the Tokamak. Future work could be to simulate multiple particles as well as be able to observe and manipulate other parameters. Parameters such as heat or radiation would work well since the 3D aspect will enable to show how different parameters propagate within the Tokamak and how they interact with each other.

CONCLUSIONS

A simplified plasma simulation was developed and was found that the magnetic field tested was able to confine particles. The electric field force caused by the addition of 100 particles in the simulation did not influence stability.

The use of Unity to develop an AR application for this simulation was able to identify the qualitative features of the spiralling motion of plasma. The use of a flat AR marker with a virtual Tokamak and plasma particle enabled a realistic 3D view of the motion. The simulation using a 3D printed Tokamak still needs improvement to look realistic as particles seem to be rendered on top of instead of within the Tokamak.

ACKNOWLEDGEMENTS

Thank you to Tuck Wah Ng, Chin Dong Yau and Ardan Wildan for guiding me on this project.

REFERENCES

[1] Gupta, A., and Sengupta, R., "Analytical Study of the Development of Nuclear Fusion Reactors as Potential Source of Energy In the Future."
[2] D'Alessandris, P., 2019, "The Simplified Nuclear Potential Well," https://phys.libretexts.org/Bookshelves/Modern_Physics/Book%3A_Spiral_Modern_Physics_(D'Alessandris)/7%3A_Nuclear_Physics/7.1%3A_The_Simplified_Nuclear_Potential_Well. (Accessed 30/9/19)
[3] Schumacher, U., 2001, "Status and problems of fusion reactor development," Naturwissenschaften, 88(3), pp. 102-112.
[4] Xu, Y., 2018, "A general comparison between tokamak and stellarator plasmas," Matter and Radiation at Extremes, 1(4), p. 192.
[5] Taylor, J., and Newton, S., "Fundamental Concepts in Plasma Confinement."
[6] Moynihan, M., 2018, "Seven ways to model plasma," https://www.thefusionpodcast.com/blog/2018/8/26/seven-ways-to-model-plasma. (Accessed 30/9/19)
[7] http://www.brosvision.com/ar-marker-generator/. (Accessed 30/9/19)
[8] "Optimizing Target Detection and Tracking Stability," https://library.vuforia.com/articles/Solution/Optimizing-Target-Detection-and-Tracking-Stability.html. (Accessed 30/9/19)
[9] Liu, K.-F., and Chao, A. W., 2017, "Accelerator based fusion reactor," Nuclear Fusion, 57(8), p. 084002.