Input Data & Preprocessing

Collect observed demand (y_t) and stock data (s_t)

Label each data point:

- Uncensored: Stock available and issued
- Partially Censored: Demand exists, but limited by stock
- Fully Censored: No stock issued, even though demand may exist

Structure data into time series (per store-product)

Initialize the State-Space Model

Define latent state:

 $X_t = [level \ell_t, trend \tau_t, seasonality \gamma_t]$

Initialize using STL decomposition:

- ℓ_0 : last trend value
- τ_0 : recent trend slope
- γ_0 : final or mean seasonal pattern

Set initial uncertainty: P_0 = diagonal of STL variances

Set model parameters: Q_0 , R, ϕ , λ (Use MLE to optimise parameters)

State Prediction (Kalman Filter)

Predict next state:

- $X_t|_{t-1} = F * X_{t-1}|_{t-1}$
- $P_t|_{t-1} = F * P_{t-1}|_{t-1} * F^T + Q_t$
- F: Transition Matrix

If censored, increase Q_t to reflect rising uncertainty.

Observation Update (If Uncensored)

Compute residual: $\tilde{y}_t = y_t - H * X_t|_{t-1}$

Compute Kalman gain and update:

- $K_t = P_t|_{t-1} * H^T * (H * P_t|_{t-1} * H^T + R)^{-1}$
- $|X_t|_t = |X_t|_{t-1} + |X_t|^* \tilde{y}_t$
- $P_t|_t = (I K_t * H) * P_t|_{t-1}$

If censored: skip update

Quantify Uncertainty (Conformal Prediction)

Use only uncensored time points to compute residuals:

• $r_t = |y_t - \hat{y}_t|$

Collect all residuals R

Calculate conformal quantile:

• $q_{\alpha} = Quantile_{1-\alpha}(R)$

Create prediction interval:

• $\hat{y}_t \pm q_{\alpha}$, bounded below by 0

Output

True demand estimate

• (\hat{y}_t)

Prediction interval

• $[\max(0, \hat{y}_t - q_{\alpha}), \hat{y}_t + q_{\alpha}]$