Assignment #2

1. EKF

a) Define EKF

The following procedure is used to define an EKF to estimate the robot state:

Step 1: Determine G_t

The non-linear motion model from HW#1 is shown below:

State $x_t = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$ Initial $x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$x_t = g(x_{t-1}, u_t) + \varepsilon_t \quad ; \quad \varepsilon_t \sim N(0, R_t)$$

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + P^{-1}(\theta) \begin{bmatrix} r\omega_1 \\ r\omega_2 \\ r\omega_3 \end{bmatrix} dt + \begin{bmatrix} N(0, 0.01) \\ N(0, 0.01) \\ N(0, 0.1 * \frac{\pi}{180}) \end{bmatrix}$$

Where $$\mathrm{P}(\theta) = \begin{bmatrix} -\sin\theta & -\cos\theta & l \\ -\sin\left(\frac{\pi}{3} - \theta\right) & -\cos\left(\frac{\pi}{3} - \theta\right) & l \\ \sin\left(\frac{\pi}{3} + \theta\right) & \cos\left(\frac{\pi}{3} + \theta\right) & l \end{bmatrix}$$

The non-linear motion model, $g(x_{t-1}, u_t)$, must be linearized about the previous mean, μ_{t-1}:

$$g(x_{t-1}, u_t) \approx g(\mu_{t-1}, u_t) + \frac{\partial}{\partial x_{t-1}} g(x_{t-1}, u_t)|_{x_{t-1}=\mu_{t-1}} (x_{t-1} - \mu_{t-1})$$
$$(x_{t-1}, u_t) \approx g(\mu_{t-1}, u_t) + G_t \cdot (x_{t-1} - \mu_{t-1})) + \varepsilon_t$$

Determine G_t:

$$G_t = \frac{\partial}{\partial x_{t-1}} g(x_{t-1}, u_t)|_{\mathrm{x_{t\text{-}1}}=\mu_{t-1}}$$

The linearization is performed in MATLAB:

```
%% linearizing g(x_t-1, u_t)
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```
syms x1 x2 x3 u1 u2 u3
r = 0.25;
L = 0.30;
dt = 0.1;
x_arr = [x1;x2;x3];
v_x = (r*2/3) * (-u1*cos(x3) + u2*cos(pi/3-x3) + u3*cos(pi/3+x3));
v_y = (r*2/3) * (u1*sin(x3) + u2*sin(pi/3-x3) - u3*sin(pi/3+x3));
omega = r/(3*L) * (u1+u2+u3);
g = x_arr + [ v_x * dt; v_y * dt; omega * dt];
G = jacobian(g, [x1; x2; x3])
```

The result of the linearization is shown below:

$$G = \begin{bmatrix} 1 & 0 & \frac{u_1 \sin(x_3)}{60} - \frac{u_2 \sin\left(x_3 - \frac{\pi}{3}\right)}{60} - \frac{u_3 \sin\left(x_3 + \frac{\pi}{3}\right)}{60} \\ 0 & 1 & \frac{u_1 \cos(x_3)}{60} - \frac{u_2 \cos\left(x_3 - \frac{\pi}{3}\right)}{60} - \frac{u_3 \cos\left(x_3 + \frac{\pi}{3}\right)}{60} \\ 0 & 0 & 1 \end{bmatrix}$$

Step 2: Determine R_t

R_t is the process noise covariance matrix. Its values are based on the additive Gaussian disturbances provided in HW#1 for the motion model. Standard deviations were provided, so those values must be squared to get variance.

$$R_t = \begin{bmatrix} (0.01)^2 & 0 & 0 \\ 0 & (0.01)^2 & 0 \\ 0 & 0 & \left(\frac{0.1\pi}{180}\right)^2 \end{bmatrix}$$

Step 3: Determine H_t

The measurement model given in the problem is below (note: I changed y to h):

$$y_t = h_t = h(x_t) + \delta_t\,; \qquad \delta_t \sim N(0, Q_t)$$

$$h_t = x_t + \begin{bmatrix} 0 \\ 0 \\ -9.7 * \frac{\pi}{180} \end{bmatrix} + \begin{bmatrix} N(0, 0.5) \\ N(0, 0.5) \\ N(0, 10 * \frac{\pi}{180}) \end{bmatrix}$$

The measurement model, $h(x_t)$, must be linearized about the predicted mean, $\bar{\mu}_t$:

$$h(x_t) \approx h(\bar{\mu}_t) + \frac{\partial}{\partial x_t} h(x_t)|_{x_t=\bar{\mu}_t}(x_t - \bar{\mu}_t)$$
$$h(x_t) \approx h(\bar{\mu}_t) + H_t \cdot (x_t - \bar{\mu}_t)$$

Determine H_t:

$$H_t = \frac{\partial}{\partial x_t} h(x_t)|_{x_t=\bar{\mu}_t}$$

The measurement model in already linear. The partial derivatives of $\bar{\mu}_t$ are equal to 1 for each state variable, therefore, H_t is simply a 3x3 identity matrix:

$$H_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 4: Determine Q_t

Q_t is the measurement noise covariance matrix. Its values are based on the additive Gaussian disturbances provided in HW#2 for the measurement model. Standard deviations were provided, so those values must be squared to get variance.

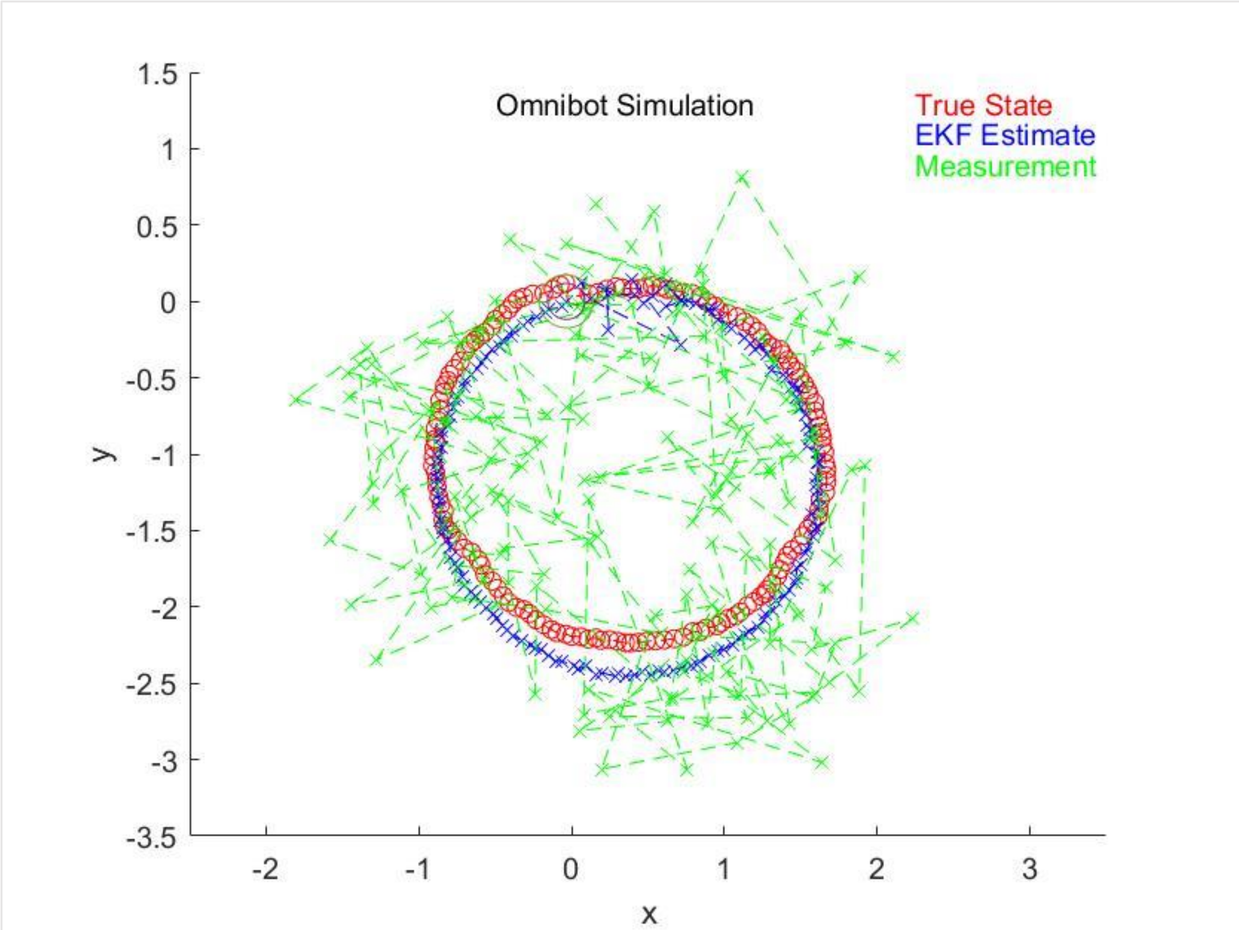
$$Q_t = \begin{bmatrix} (0.5)^2 & 0 & 0 \\ 0 & (0.5)^2 & 0 \\ 0 & 0 & \left(\frac{10\pi}{180}\right)^2 \end{bmatrix}$$

b) Implement EFK

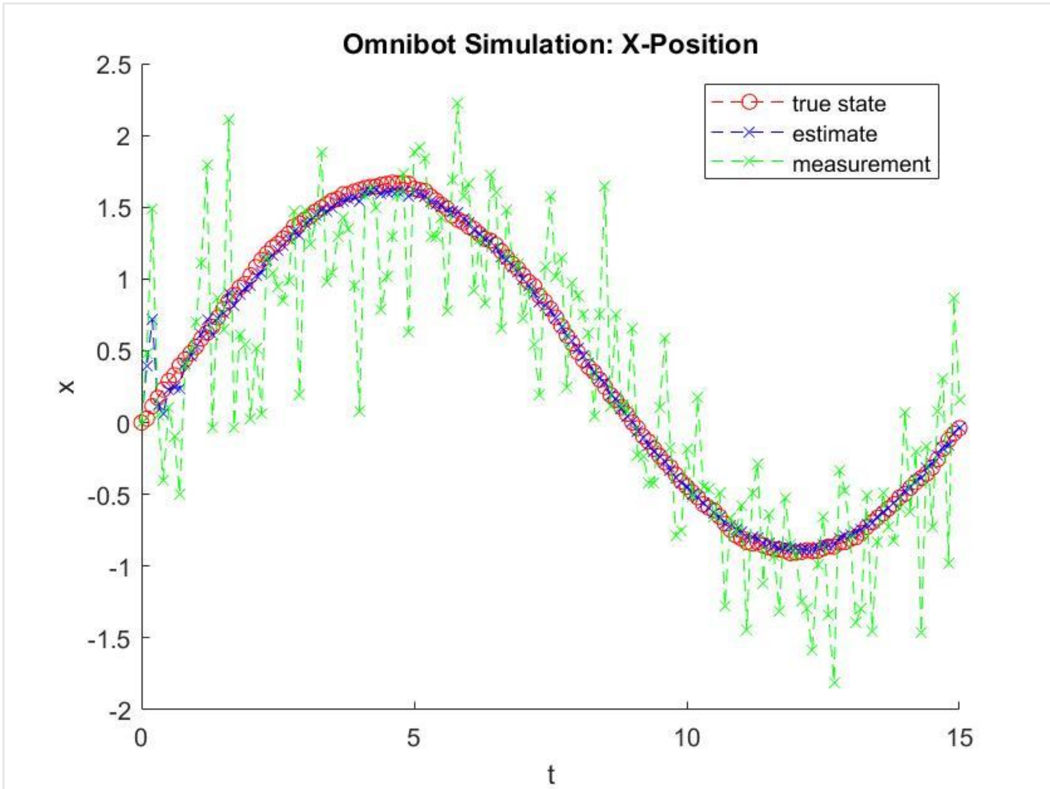
The EFK is implemented in MATLAB (files attached) with rotation inputs of:

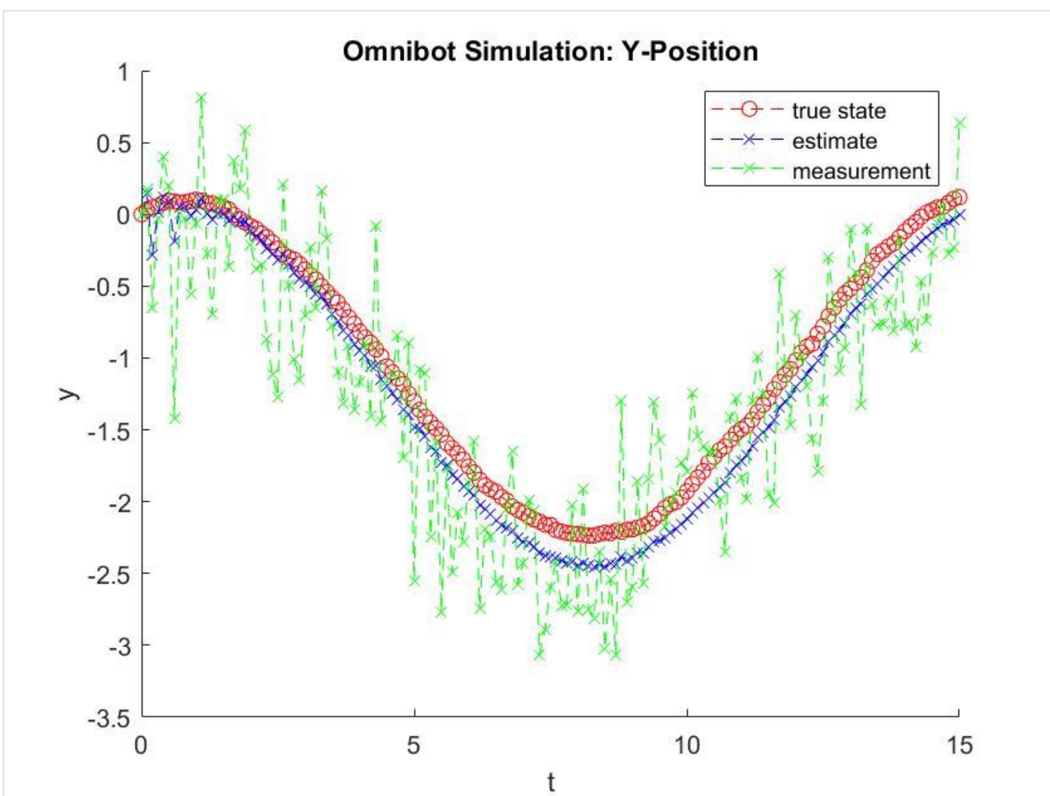
$$\mathrm{u} = \begin{bmatrix} -1.5 \\ 2 \\ 1 \end{bmatrix} \text{rad/s}$$

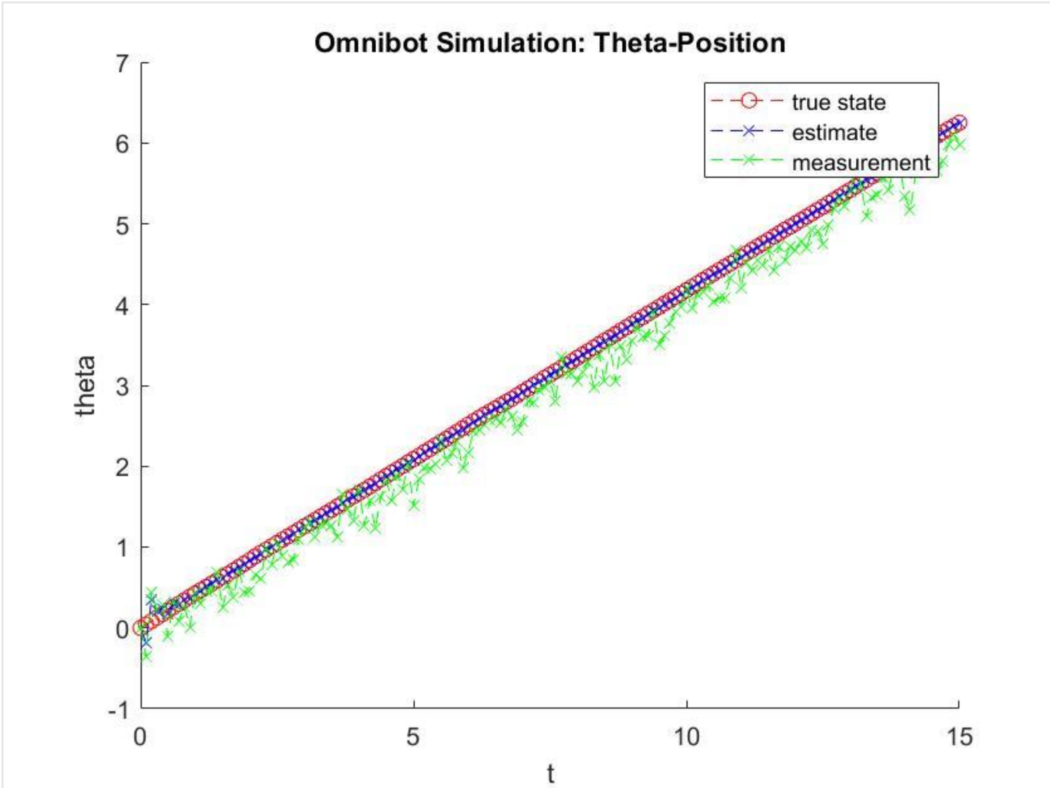
The following plot shows the results of a 15 second simulation, including the true state, estimated state, and measurement.



Below are plots of the individual states, x, y, and θ.


Omnibot Simulation: X-Position
true state
estimate
measurement
x
t
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
5
10
15


Omnibot Simulation: Y-Position
true state
estimate
measurement
y
t
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
0
5
10
15



2. PDF

Given: $y = s + n$ where $n \sim N(0,1)$

$Pr(s = 1) = p$

$Pr(s = -1) = 1 - p$

a) The PDF of y is:

$$p_y(y) = p\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(y-1)^2} + (p-1)\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(y+1)^2}$$

b) The probability that $s = 1$ give a measurement y:

$p(s) = 0.4$

$$p(s = 1|y) = \frac{p(y|s)p(s)}{p_y(y)}$$

$$= \frac{\left(\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(y-1)^2}\right)(p)}{\left(p\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(y-1)^2}+(1-p)\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(y+1)^2}\right)}$$

$$= \frac{\frac{1}{\sqrt{2\pi}}}{\frac{1}{\sqrt{2\pi}}}\left(\frac{0.4e^{-\frac{1}{2}(y-1)^2}}{0.4e^{-\frac{1}{2}(y-1)^2}+0.6e^{-\frac{1}{2}(y+1)^2}}\right)$$

$$= \frac{\frac{1}{e^{-\frac{1}{2}(y-1)^2}}}{\frac{1}{e^{-\frac{1}{2}(y-1)^2}}}\frac{0.2e^{-\frac{1}{2}(y-1)^2}}{0.2e^{-\frac{1}{2}(y-1)^2}+0.3e^{-\frac{1}{2}(y+1)^2}}$$

$$= \frac{0.2}{0.2+0.3e^{-\frac{1}{2}[(y+1)^2-(y-1)^2]}}$$

$$= \frac{1}{1+\frac{3}{2}e^{-2y}}$$

Aside: $(y+1)^2-(y-1)^2 = y^2+2y+1-(y^2-2y+1) = 4y$