ECE 484
Digital Control Systems
Ball and Beam Lab Manual

CONTENTS

1. NOTES ON GROUP WORK AND PLAGIARISM

Occasionally there has been confusion about what is allowed and what is not allowed in labs. To make things clear, the following rules apply to ECE 484. These rules have been created for one simple reason: so that each of you *learns* something about control engineering, and you will best learn by doing the work yourself. It is farcical having a lab where the only person who actually did the work is some guy from 2010, while everyone else copies his report. So, please note that:

1. You are responsible for knowing what constitutes an "academic offense" according to Policy 71 of the university. In particular, according to the policy:

    a. Cheating is an academic offence. Cheating includes copying from another student's work or allowing another student to copy from one's own work, submitting another person's work as one's own, fabrication of data, and use of unauthorized aids.

    b. Plagiarism (the act of presenting the ideas, words or other intellectual property of another as one's own) is an academic offence. The use of other people's work must be completely and unambiguously acknowledged and referenced in all written material, including laboratory reports and computer programs.

2. In this course labs should be done in groups of two students, unless the class has an odd number of students, in which case, a group of one student will exist. Both group partners must contribute to contribute to all aspects and steps of the project. However:

    a. You may talk with the lab instructor, the lab teaching assistants, and the course instructor about any aspect of the lab.

    b. You are allowed to consult with other students in the class to share only high-level ideas and approaches, but not to share detailed analysis or detailed design results.

You may not share with other students (whether they are current students or former students of ECE 484/481) computer code or Simulink diagrams in any way.

3. You may not obtain or look at lab reports (either in hardcopy or softcopy) written by other students, whether they are current students or former students of ECE 484/481. You may not let any other student access any part of your lab report (either in hardcopy or softcopy).
4. In your report, you must completely and unambiguously acknowledge and reference any person, website, report, book, or notes that you used to help you with your work. You should reference this lab manual and the course notes, for example.
5. You must include in each lab report, and sign, the following statement:

> We acknowledge and promise that:
>
> (a) We are the sole authors of this lab report and associated simulation files/code.
>
> (b) This work represents our original work.
>
> (c) We have not shared detailed analysis or detailed design results, computer code, or Simulink diagrams with any other student.
>
> (d) We have not obtained or looked at lab reports from any other current or former student of ECE 484/481, and we have not let any other student access any part of our lab work.
>
> (e) We have completely and unambiguously acknowledged and referenced all persons and aids used to help us with our work.
>
> Signed ______________________________ and ______________________________

2. LAB OVERVIEW

This project deals with a classical experiment where the objective is to control the position a ball on a beam by appropriately inclining the beam. Figure 1 shows the ball and beam apparatus. Because of the pandemic, this term we will not be able to work directly with the hardware, but rather we will control a simulated model of the plant.

THE BALL AND BEAM SYSTEM

As seen in Figure 1, a direct-current (dc) motor (connected to the smallest of the three gears) is used to control a lever arm, which in turn is used to raise or lower the right side of the beam to cause the ball to move. The beam itself consists of two parallel rods, which together with the ball form a potentiometer used to measure the ball's position. One of the rods has a resistive film glued onto it, while the other rod and the ball form the wiper of the potentiometer. The left side of the beam is pivoted on a fixed support.

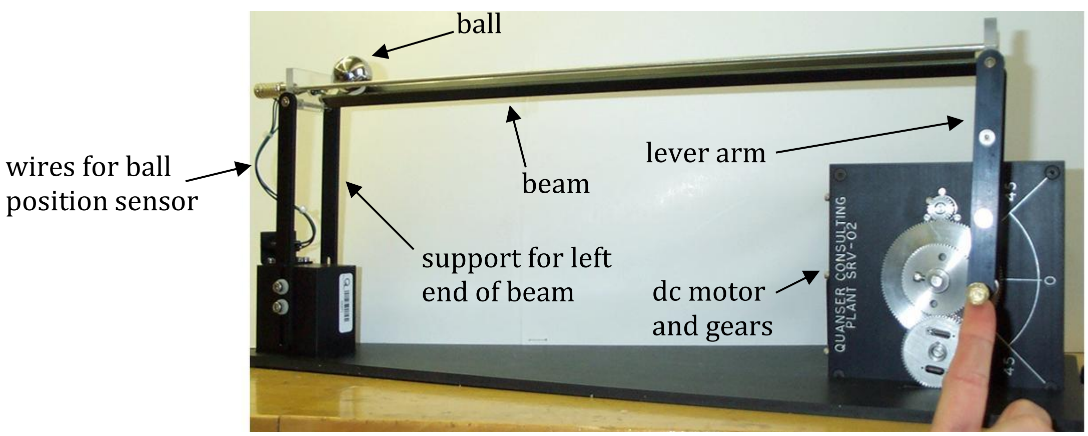


FIGURE 1: The plant, a "ball and beam" apparatus.

The input to the ball and beam apparatus is the motor voltage; both the ball position and the motor gear angle are measurable outputs. Hence, the ball and beam system has:

- one actuator: the dc motor
- two sensors: one for the angular position of the motor shaft, and one for the ball position.

An 8-minute video showing the apparatus in action is available on LEARN under Content, in the Laboratory folder.

OVERVIEW OF THE CONTROL TOPOLOGY

Before attempting to control the ball and beam system, it is important to consider the control topology; that is, we need to decide if a single control loop or a double inner/outer-loop control structure is best, and consider what type of compensation is reasonable. As we will show later, the essential linearized ball and beam dynamics are summarized in three equations:

$$\frac{\theta(s)}{V(s)} = \frac{K_1}{s(\tau s + 1)} \qquad \text{... (1)}$$

$$\frac{\phi(s)}{\theta(s)} = K_2 \qquad \text{... (2)}$$

$$\frac{Y(s)}{\phi(s)} = \frac{K_3}{s^2} \qquad \text{... (3)}$$

The signals $V(t)$, $\theta(t)$, $\phi(t)$, and $y(t)$ as well as some key geometric variables are shown in Figure 2. The parameters K_1, K_2, K_3 and τ will be determined later using system identification methods.

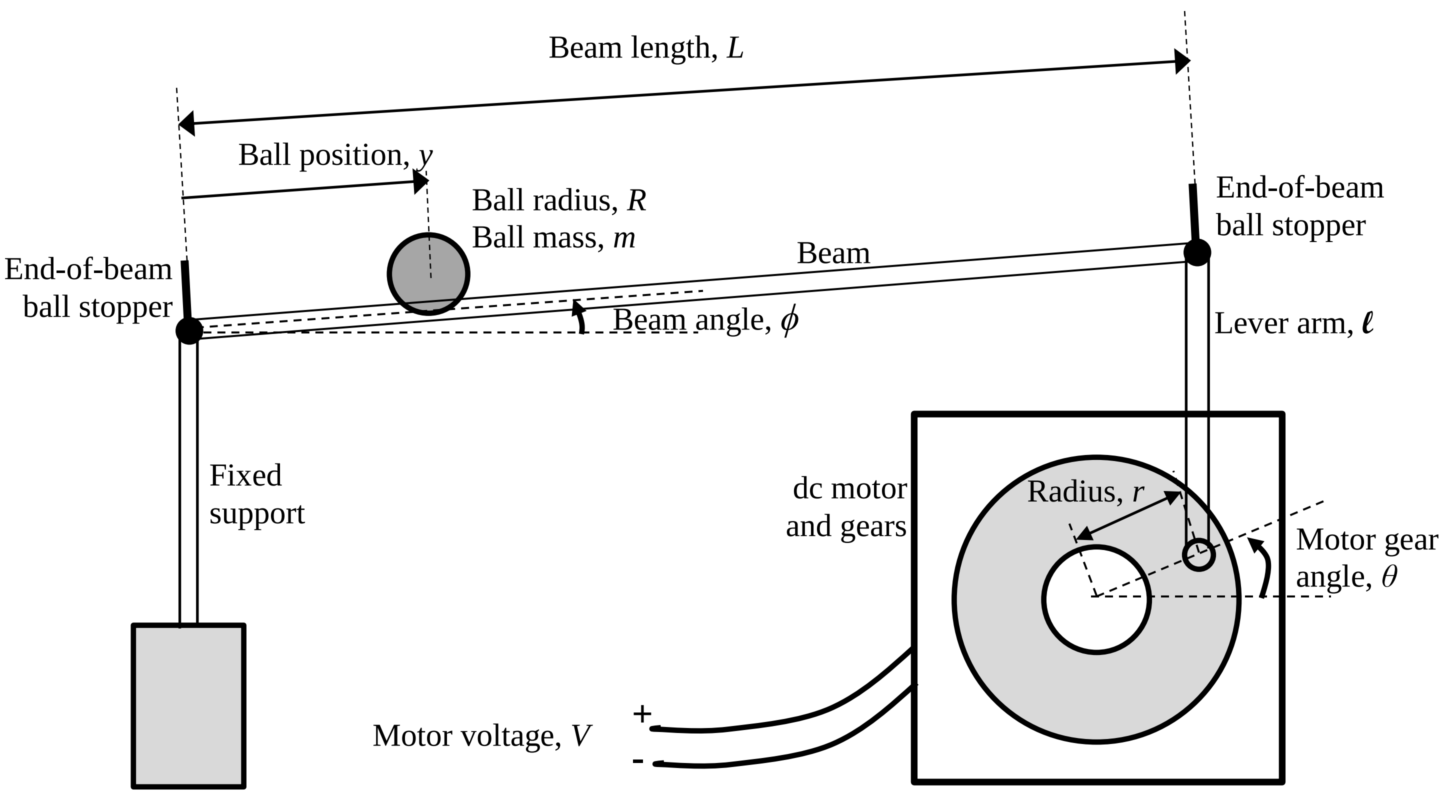


FIGURE 2: Geometry of the ball and beam system.

The simplest control topology is the single feedback control loop shown in Figure 3, where the plant is treated as the cascade connection of (1), (2), and (3). Although it is possible to design the compensator $C(s)$ such that the closed-loop system is stable, the triple integrator in the plant contributes 270° phase lag to the loop gain, making it difficult to obtain good closed-loop performance. Another practical difficulty with this setup, which will also be considered in simulation, is that it is not clear how one can ensure that the motor only rotates as much as to keep the gear angle in the range

$$-\frac{\pi}{4}\ \text{rad} < \theta(t) < \frac{\pi}{4}\ \text{rad}.$$

This restriction is necessary to ensure that the apparatus is not damaged and to ensure that the use of linearization is a reasonable approximation of the nonlinear dynamics.

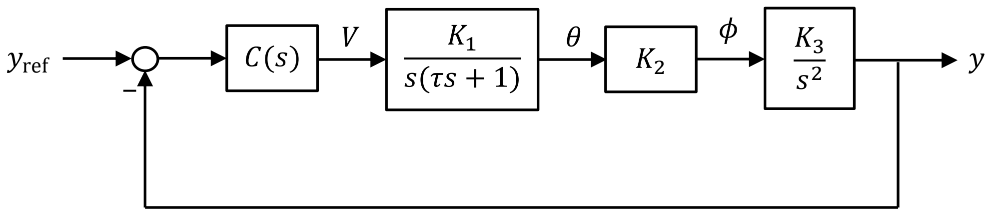


FIGURE 3: A single feedback control loop for the linearized plant. Note that a continuous-time controller is used in this diagram even though a sampled-data controller would be used in the end.

An alternative control arrangement is shown in Figure 4. In this setup, two feedback loops are used. The purpose of the inner loop is to control the motor gear angle position; controller $C_1(s)$ should be designed so that $\theta(t)$ tracks the reference signal $\theta_{\text{ref}}(t)$. The outer loop uses the inner feedback loop to control the ball position. From the point of view of controller $C_2(s)$, the "plant" is composed of everything enclosed by the dotted box in Figure 4. Clearly the inner-loop controller must be designed before the outer-loop controller can be designed. We will also see that the inner/outer loop control setup in Figure 4 avoids the two problems associated with the single-loop control configuration in Figure 3.

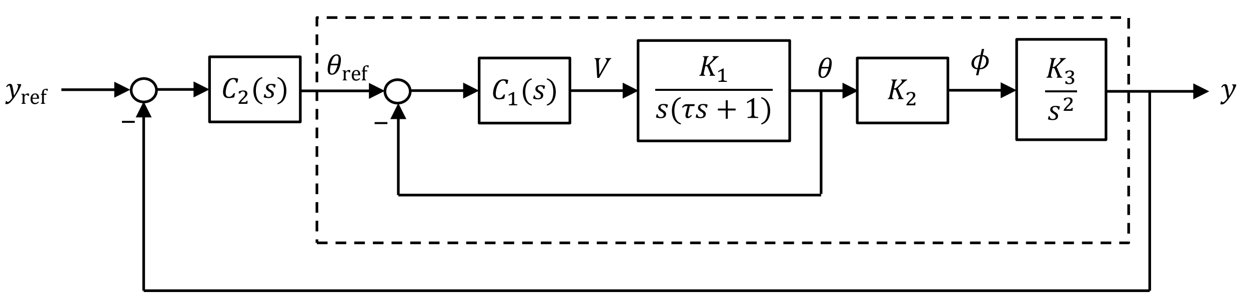


FIGURE 4: Inner/outer loop control configuration for the linearized plant. As in Figure 3, continuous-time controllers are used in this diagram.

GENERAL REPORT REQUIREMENTS

Write up your reports applying the following requirements:

- Submit your report, in pdf format, to the appropriate drop-box on LEARN.
- Include and sign the statement from page 2 of this lab manual.
- Include introduction and conclusions sections.
- Your report should be self-contained and complete. Be sure to comment on every lettered section from the lab manual, except for Lab 1(a).
- Be sure to include enough data and information so that your results are reproducible.
- The report should use a font no smaller than 12pt Times New Roman with single line spacing.
- Your report must be typed, although you can write in equations if your handwriting is very neat. Handwriting the entire analysis is not acceptable.
- Always point out unexpected results and provide possible explanations.
- Discuss the validity of any assumptions you made.
- Clearly define all variables in your report.
- Always indicate units when giving numerical data.
- All plots must be fully and appropriately labelled, including pertinent performance measurements (e.g., explicitly label settling time, overshoot, steady-state values).
- For your controller design in Labs 2 and 3, you should demonstrate appropriate use and understanding of control tools (e.g., root locus plots, Bode plots), and you should concisely explain and justify design decisions. Show that you know what you are doing and that is not merely a fluke that you got your controller working. For example, simply saying you used an “automatic tuning algorithm” or a “trial and error” approach is not enough.
- Formatting and presentation are important. Make your report readable and easy to follow. For example, do not put all figures at the end of your report since this is very tedious for the reader.
- Quality matters more than quantity. Do not put “filler” in your report.

3. LAB 1: MODELING OF THE MOTOR SYSTEM

This lab manual is written at a level appropriate for fourth-year students. The lab requires students to exercise engineering judgement and creativity. Design objectives must be met, but how various tasks are completed can vary from one student to another. Hints are offered at various points, instead of detailed step-by-step instructions. This approach is purposeful and intended to provide an enriching engineering exploration.

For Lab 1, we focus on the inner loop in Figure 4, where the motor gear angle is controlled. The loop is duplicated in Figure 5(a). Figure 5(b) shows the later addition of a saturator to limit the motor operating angle range needed when connecting the beam. Figure 5(c) shows the discretized controller acting on the continuous-time plant.

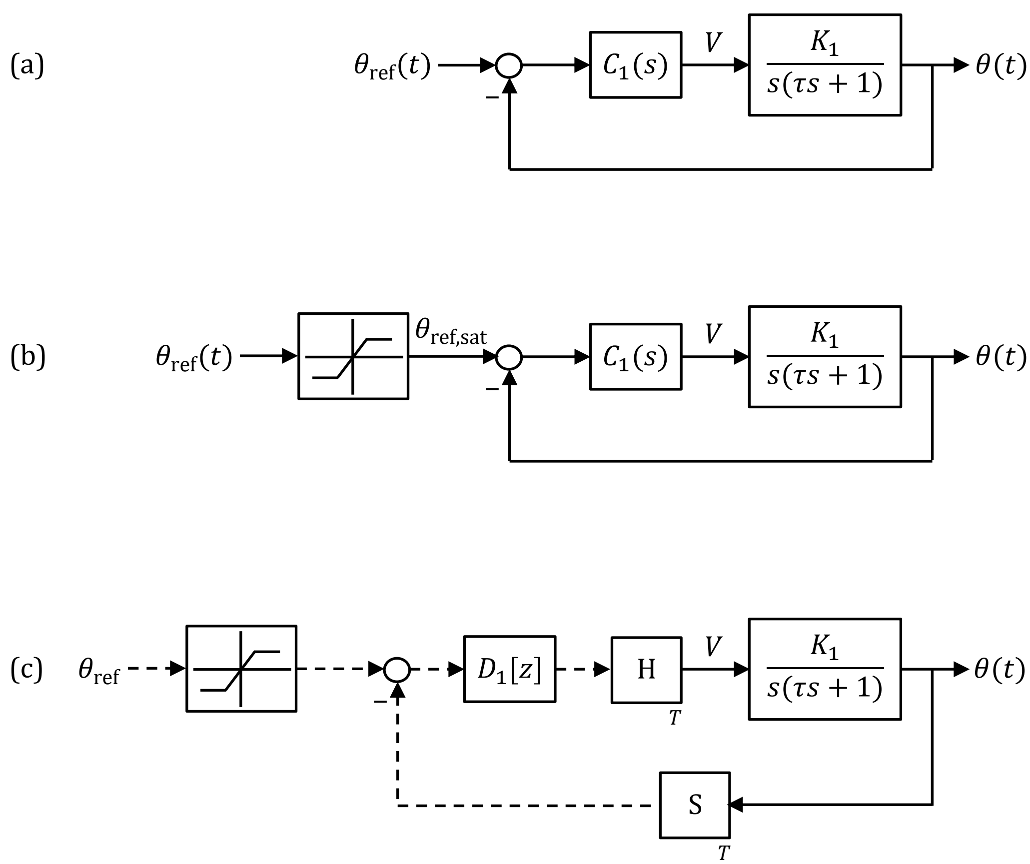
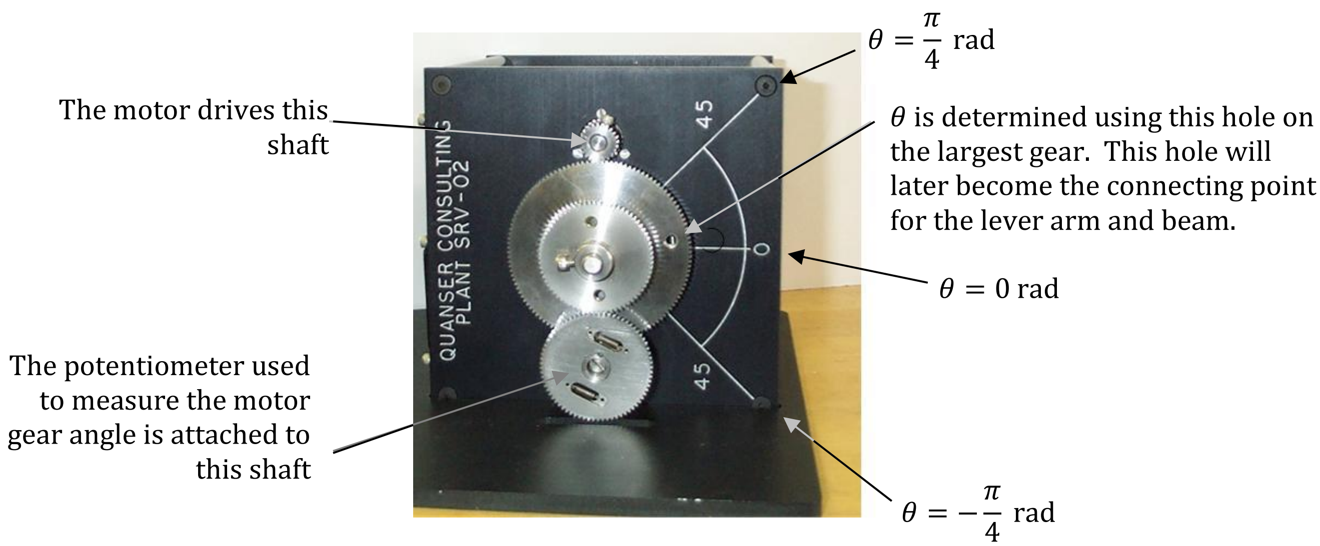
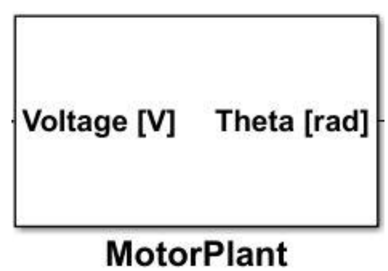


FIGURE 5: (a) continuous-time inner loop, (b) continuous-time inner loop with a saturator, (c) sampled-data inner loop with a saturator (the sampling period is T).

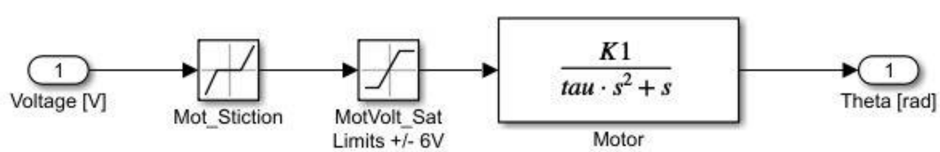
(a) Assume that the lever arm and beam (recall Figure 1) are not attached to the gear at this point. Figure 6 shows a detail of the motor system plant, indicating where the gear angle θ is measured. More detail about the hardware system simulated in this project is available in Appendix 1.



(a)



(b)



(c)

FIGURE 6: (a) The motor system plant for Lab 1, (b) The equivalent motor plant Simulink model – high level; (c) The equivalent motor plant Simulink plant - detail inside the block; this content is protected.

The dc motor is connected to the smallest of the gears (top gear), and so the position of the shaft is mapped to the largest of the gears (middle gear), where the right-end of the lever arm will later be attached. In simulation, sensors are "transparent", i.e., the output of the Simulink model shown in Figure 6(b) is the gear angle θ (Theta) measured in radians.

In the physical lab, each ball and beam apparatus has different motor system characteristics. To replicate this variation, each group in the class will be provided with their own unique Simulink subsystem (Figure 6(b)). Because you are not able to access the physical motor system, you should treat the provided Simulink block as if it was the actual plant.

(b) Any motor has some amount of static friction, often called stiction. From a control point of view, stiction is undesirable since it can make fine motor motions very difficult. One strategy to deal with stiction is to cancel it out.

The plant, provided as the Simulink subsystem in Figure 6(b), includes a model for the motor, gears, and stiction. The protected content of this block is shown in Figure 6(c); users should treat this block as a black-box.

In Simulink, determine the motor voltage required to overcome stiction in both directions: clockwise and counter-clockwise. Explain briefly in your report how you determined suitable stiction offset values.

Implement in Simulink a stiction-compensator scheme so that the plant formed by the motor, gears, and stiction-compensation scheme behaves like a linear system. From now on, it will be assumed that the stiction-compensation scheme is implemented in your simulation setup.

(c) If you are not familiar with dc motor models, consult an undergraduate control textbook to verify that (1) is a standard model.

In Simulink, use system identification methods to determine the specific values of K_1 and τ for your plant.

Note the following:

- The plant is open-loop unstable, and therefore it is difficult to perform system identification directly on the plant. Instead, come up with a $C_1(s)$ in Figure 5(a) to stabilize the plant (how?), and use system identification techniques (e.g., use step response overshoot and time to first-peak to deduce the unknown system parameters, or perhaps least squares) to determine a model of the closed-loop system. Then use these results to deduce K_1 and τ. Include units with your results.
- The sample rate in the plant distributed to each group, has been set to 1 ms. For system identification, in general, we would want a step size sufficiently small, such that the response captures all essential data-points, rather than a crude plot that looks like connected line segments. This setting is available in Simulink, Simulation tab, Model Settings/Configuration Parameters, under the Solver properties.
- Use several sets of data and different system setups, e.g., different controller gains and/or different step sizes, to increase the probability that your parameter estimates are accurate.

- Similar to dealing with the experimental system, be sure that you do not apply to the motor a voltage outside the range [-6V, +6V]. Include in your report a plot of the motor voltage, along with the plot of signals used for system modeling, as evidence that you have avoided saturation (and, doing so, kept the system linear) during the system identification tests you performed.

(d) Validate the model obtained in (c), i.e., the parameter values parameters K_1 and τ, by using a different method. Ensure it is a different method you are using, and not just the same method as in (c) with different values. For example, if the identification in part (c) relied on measuring overshoot and time to first-peak in the step response, perhaps try least squares or Bode plot fitting for validation.

(e) As mentioned earlier, it is important that the motor gear angle satisfies at all times:

$$-\frac{\pi}{4}\ \text{rad} < \theta(t) < \frac{\pi}{4}\ \text{rad}. \qquad \text{... (4)}$$

To help ensure this condition is satisfied, prefilter θ_{ref} using a saturator as shown in Figure 5(b). In Lab 2(a) you will design a controller to achieve an overshoot no higher than 5%; when this controller is operating, the condition $\mathbf{-0.7\ rad < \boldsymbol{\theta}_{ref,sat} < 0.7\ rad}$ will ensure that (4) holds.

Use Simulink with a meaningful input to the system in Figure 5(b) to verify that the saturator works as expected. Include in your report the Simulink diagram used and the plot showing the effect of the saturator.

For your Lab 1 report, follow the General Report Requirements given on page 6. In addition:

- Attempt, the best you can, to fit your Lab 1 report within 15 pages from the front page to the end, including any appendices.
- **Submit, in addition to your report, the final Simulink diagram and any Matlab scripts you used in Lab 1** so that these files can be tested by the marker. Ensure that values needed for testing are set as default values.

4. LAB 2: DESIGN OF THE INNER-LOOP CONTROLLER AND MODELING OF THE BALL & BEAM

INNER-LOOP CONTROLLER DESIGN

In Lab 2 we will be using the emulation design approach to control the motor angle.

(a) Design a ***dynamic*** controller $C_1(s)$ in Figure 5(a) to satisfy the following specifications:

- The (linearized) feedback loop is stable.
- When the reference signal steps from $\theta_{\text{ref}} = -0.7$ rad to $\theta_{\text{ref}} = +0.7$ rad:
  - The step response steady-state tracking error is zero.
  - The step response 2% settling time is no more than the specification given in the document **Plant Assignment ECE.484.F20 and Ts for C1**.pdf posted on LEARN.
  - The step response overshoot is no more than 5%.
  - The motor voltage does not saturate.

A classical compensator design, or a method of your choice may be used. Even if a proportional controller is able to satisfy the specifications, *a dynamic controller is required* so that parts (b) and (c) are not trivial to do. The following controllers, both practically equivalent to a proportional controller, are *not acceptable as $C_1(s)$*: a low-pass filter with a fast pole or an active controller with the pole/zero very close to each other.

Verify the effectiveness of your compensator in simulation. Include in your Lab 2 report details of how you performed your controller design so that it is reproducible. If more than one iteration was necessary, include details about the changes you made.

(b) Discretize your controller $C_1(s)$ to obtain the discrete-time controller $D_1[z]$ in Figure 5(c):

- Choose a sampling period suitable for the emulation approach and provide justification.
- Use any reasonable discretization method to find the discretized controller $D_1[z]$. In Simulink run the sampled-data system shown in Figure 5(c). Confirm that the closed-loop performance is similar to that of the system in Figure 5(b). Note that if you make $D_1[z]$ a "discrete transfer function" block, then Simulink will automatically insert the sample and hold operators shown explicitly in Figure 5(c). Do not insert your own sample and hold, and be sure to correctly set the sampling period in the discrete transfer function.

(c) A guide on discrete-time controller implementation can be found in Appendix 2. Write out the time-domain equation for the controller $D_1[z]$ that would be needed if you were to implement it on the hardware experiment. In your controller equation, use notation consistent with the signals present on your Simulink diagram.

Caution: It is important to put as many decimals as you can on your controller coefficients; in Matlab, the command `format long` will give you 15 digits. Numerical errors associated with rounded coefficients may cause your controlled system to go unstable. To extract the coefficients of a controller, `SYS`, you may use the Matlab command `[NUM,DEN] = tfdata(SYS,'v')`.

MODELLING OF THE BALL AND BEAM

Proceed with the project as if the right-end of the beam and the lever arm are now attached to the dc motor gear as shown in Figure 1. This change in configuration has two consequences on the actual (non-simulated) apparatus:

- Attaching the lever arm and beam to the motor changes the dynamics of the inner-loop model in Figure 5(a). Technically, the inner-loop should be remodeled, with the beam attached, and a new controller should be designed. However, due to the high-ratio gearing of the motor, the dynamics do not change significantly. So for simplicity, we will assume that the inner-loop model has not changed.
- Because the weight of the beam pushes down on the gear, the stiction characteristics are likely to change. This additional change is not incorporated into the Simulink subsystem in Figure 6(b). So maintain the same stiction offset values that you determined in Lab 1(b).

(d) Use an analytical approach to model the lever arm and the beam. Then linearize your equations about an operating point corresponding to $\phi = 0$; you should get equations with the structure of equations (2) and (3) on page 4. The following hints may be useful:

- The ball position, expressed in meters, has its origin as shown in Figure 2.
- Geometric parameters, some of them depicted in Figure 2, are available in Table 1.

Table 1. Ball and beam parameters [1]

Beam length (interior)	L	45	cm
Lever arm length	ℓ	12	cm
Lever end radius of rotation	r	2.54	cm
Ball radius	R	1.27	cm
Ball mass	m	0.064	kg

Note that the beam has end stoppers to prevent the ball running off the beam.

- Recognize that the relationship between θ and ϕ is purely geometric; there are no dynamics to be concerned about when deriving K_2.
- The beam that the ball is rolling or balancing on is made up of two parallel rods. One rod is a conductive strip and the other is a conductive rod. These are shown in Figure 7(b) as a rectangle and a small circle, respectively. Part of the ball hangs slightly below the points of contact with the beams. The distance between the points of contact is $AB = 2$ cm.

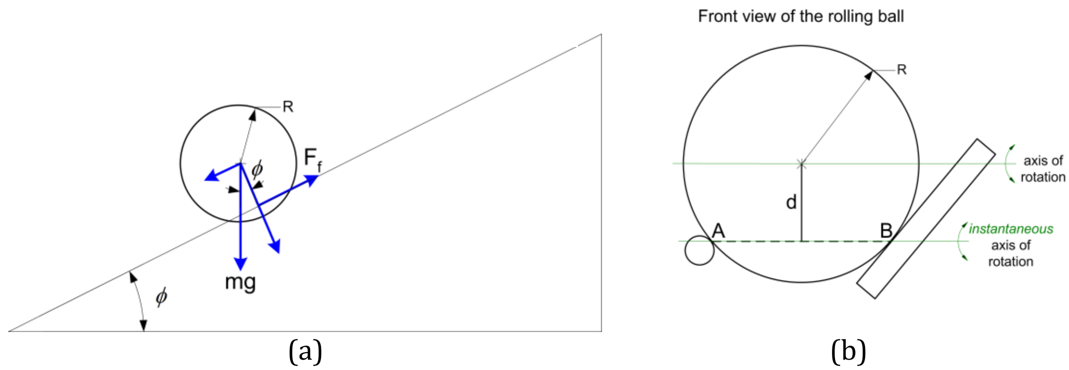


FIGURE 7: Diagram of ball on the beam: (a) side view; (b) cross-sectional front view.

- Because the beam movement is slow, you can assume that ϕ is fixed. The static friction between the ball and the beam is what causes the ball to roll (instead of just sliding down). However, it is not necessary to identify the numerical value of the coefficient of friction. After linearizing the equation(s) you will obtain an expression for K_3 in terms of known parameters.
- Present the stages of your model development in the report, not just the final result. In the end, state the numerical values that you have determined for K_2 and K_3.

(e) Implement the model of the ball and beam in your Simulink diagram. You will design a controller for this system in Lab 3.

For your Lab 2 report, follow the General Report Requirements given on page 6. In addition:

- Include the values for K_1, τ, and the stiction offsets that you obtained in Lab 1. Provide a short justification if any changes were made.
- Attempt, the best you can, to fit your Lab 2 report within 15 pages from the front page to the end, including any appendices.
- **Submit, in addition to your report, the final Simulink diagram and any Matlab scripts you used in Lab 2** so that these files can be tested by the marker. Ensure that values needed for testing are set as default values.

5. LAB 3: DESIGN OF THE OUTER-LOOP CONTROLLER

In Lab 3, we focus on controlling the position of the ball on the beam. The basic setup is the inner/outer loop control configuration shown in Figure 4. Note that, from the point of view of the outer loop controller $C_2(s)$ in Figure 4, the "plant" is the system within the dashed box.

The directions below describe a design by emulation. You may alternatively use the direct design approach, in which case you will need to include an extra step of mapping the specifications in part (b) to the z-plane.

(a) Throughout the design and simulation steps outlined below, be sure to include a saturator, as we did in Figure 5(b), to ensure that $-0.7 \text{ rad} < \theta_{\text{ref,sat}} < 0.7 \text{ rad}$. Modify Figure 4 to include the saturator.

(b) Design an outer-loop controller $C_2(s)$, that meets the following control specifications:

Think of how to intelligently simplify the "plant" transfer function before designing the controller.

The controller specifications must be demonstrated using the complete "plant".

- The (linearized) feedback system in Figure 4 is stable.
- For a square wave reference signal, switching between 0.10 m and 0.25 m:
  - The step response steady-state tracking error is zero
  - The step response 2% settling time is no more than 6 seconds
  - The step response overshoot is no more than 30%

  Note: Saturation of $\theta_{\text{ref}}(t)$ is permitted, but the saturator must be present to ensure $-0.7 \text{ rad} < \theta_{\text{ref,sat}} < 0.7 \text{ rad}$.
- The output of $C_2(s)$ is stable.

Include in your report plots to show that each of the specifications is being met.

(c) Discretize your controller $C_2(s)$ to obtain a discrete-time controller (call it $D_2[z]$):

- Choose a sampling period suitable for the emulation approach and provide justification.
- Use any reasonable discretization method to find the discretized controller $D_2[z]$. In Simulink run the appropriate sampled-data system where both the inner-loop and outer-loop controllers are discretized. Confirm that the closed-loop performance is similar to the performance achieved in (b). If the specifications are not met, tune the controller as needed.

(d) Write out the time-domain equation for the controller $D_2[z]$ that would be needed if you were to implement it on the hardware experiment. In your controller equation, use notation consistent with the signals present on your Simulink diagram. Remember the caution provided on page 11.

For your Lab 3 report, follow the General Report Requirements given on page 6. In addition:

- Include the values for K_1, τ, the stiction offsets, the final inner-loop controller, K_2, and K_3 that you obtained in Labs 1 and 2. Provide a short justification if any changes were made.
- Include details on how you designed the outer-loop controller, providing a clear justification for the design choices you made, demonstrating how you applied course knowledge. Specifically:
  - If you used emulation, be sure to include:
    - Details of the design of $C_2(s)$, including any design iterations that were made
    - Your choice of discretization method and sampling time.
  - If you used direct design, be sure to include:
    - The mapping of the specifications to the z-plane.
    - Details of the design of $D_2[z]$, including any design iterations that were made.
- Include a final modified version of Figure 4, clearly showing how the discrete-time controllers $D_1[z]$ and $D_2[z]$ are connected. Include explicitly all sample and hold operators and the saturators.
- Attempt, the best you can, to fit your Lab 3 report into 15 pages.
- **Submit, in addition to your report, the final Simulink diagram and any Matlab scripts you used for Lab 3** so that these files can be tested by the marker. Ensure that values needed for testing are set as default values.

REFERENCES

[1] Quanser Consulting Inc., SRV02-Series Ball & Beam User Manual, Revision 1, July 2004

APPENDIX 1
SIDE-VIEW OF THE MOTOR AND GEAR PLANT

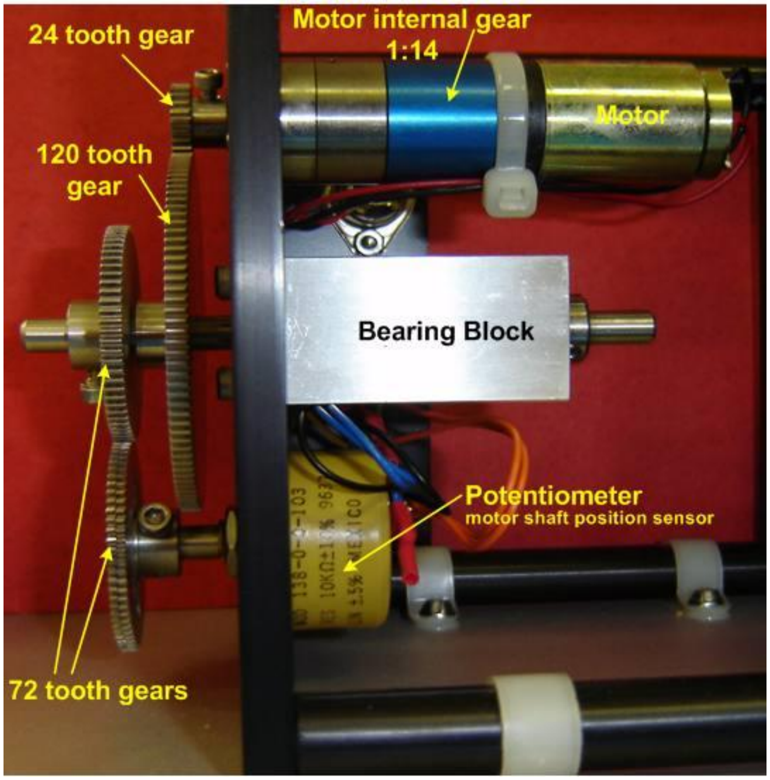


FIGURE A1: Side-view of the motor, gear, and gear sensor (potentiometer) detail

APPENDIX 2
DISCRETE-TIME CONTROLLER IMPLEMENTATION

Consider a *generic* transfer function, $D[z]$, which was obtained by discretizing the continuous transfer function $C(s)$ with a sampling period T:

$$D[z] = \frac{U[z]}{E[z]} = \frac{a_0 z^3 + a_1 z^2 + a_2 z + a_3}{b_0 z^3 + b_1 z^2 + b_2 z + b_3} = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}$$

The z-transform operator, z, is the advance operator; the notation for samples at various time points is shown in Table A2.

Table A2.

z-domain	Time domain	Description
$U[z]$	$u[k]$	Current sample
$zU[z]$	$u[k+1]$	Next sample
$z^{-1}U[z]$	$u[k-1]$	Previous sample
$z^{-2}U[z]$	$u[k-2]$	Second-previous sample

Applying the above to $D[z]$, we get:

$$U[z](b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}) = E[z](a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}).$$

The implementation of this transfer function in the time domain is as follows (for $b_0 \neq 0$):

$$\boxed{u[k] = \frac{-b_1}{b_0} u[k-1] - \frac{b_2}{b_0} u[k-2] - \frac{b_3}{b_0} u[k-3] + \frac{a_0}{b_0} e[k] + \frac{a_1}{b_0} e[k-1] + \frac{a_2}{b_0} e[k-2] + \frac{a_3}{b_0} e[k-3]}$$