

Wiener deconvolution exploration

Status report
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Problem statement

Find the solution for the signal restoration problem, if the data model is formulated as following:

$$y = hx + n,$$

Where y is the detected signal, x is the real signal, h is the point spread function and n is noise.

Progress

- Created preprocessing pipeline:
 - Image from the “The Mouse hematopoietic stem cells in hydrogel microwells” dataset is loaded
 - PSF is generated, the kernel is a gaussian kernel with user defined parameters, for testing filter of size 5 and $\sigma = 10$ was used
 - The image is blurred using generated PSF
 - Two types of noise are added to the blurred image
 - The poisson noise is a nonparametric noise, that depends on the intensity of the original signal
 - Gaussian noise is generated via user-defined parameters, for testing $\sigma = [0.01, 0.11, 0.3]$ was used
- Created the environment for testing Richardson-Lucy and Wiener deconvolution algorithms
 - To evaluate the effectiveness of the algorithms two metrics were used: SSIM and PSNR

Wiener Filtering

The Wiener filter in the frequency domain can be defined as:

$$G(f) = \frac{H^*(f) S(f)}{|H(f)|^2 S(f) + N(f)},$$

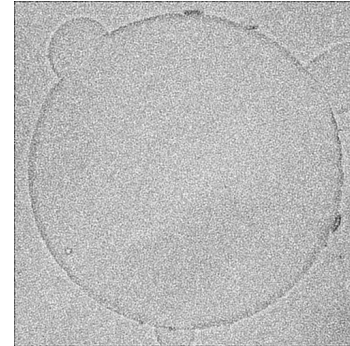
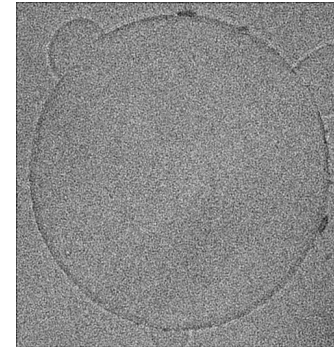
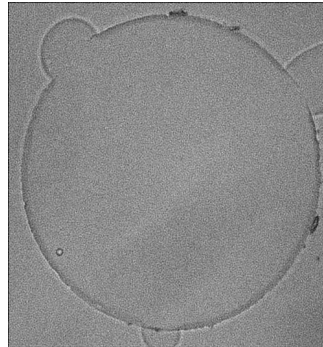
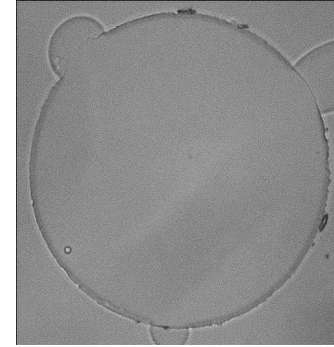
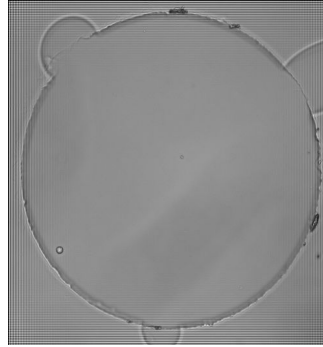
Where:

- $H^*(f)$ is the complex conjugate of the Fourier Transform of the PSF
- $H(f)$ and $G(f)$ are Fourier Transforms of the PSF and the Wiener Filter respectively
- $S(f)$ is the mean power spectral density of the original signal
- $N(f)$ is the mean power spectral density of the noise

Results of Wiener filtering

The resulting values of SSIM and PSNR are:

	SSIM	PSNR
Only blur	0.6798	18.0324
Blur + $G(\sigma = 0.01)$	0.7064	25.3072
Blur + $G(\sigma = 0.11)$	0.5456	23.3830
Blur + $G(\sigma = 0.3)$		
Blur + Poisson noise	0.3611	13.4614



Richardson-Lucy

Richardson-Lucy iterative algorithm is defined as follows:

$$\hat{u}^{(t+1)} = \frac{\hat{u}^{(t)}}{1 - \lambda V} \left(\frac{d}{\hat{u}^{(t)} \otimes P} \otimes P^T \right)$$

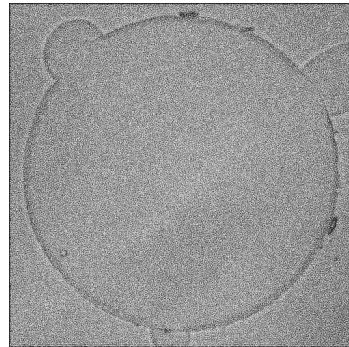
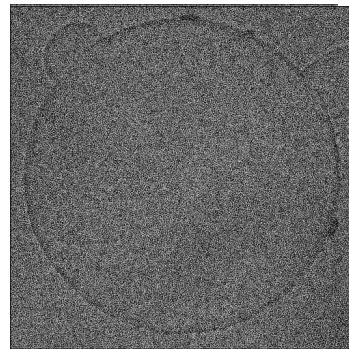
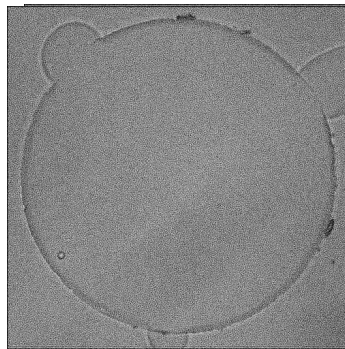
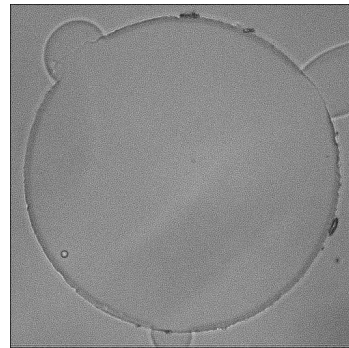
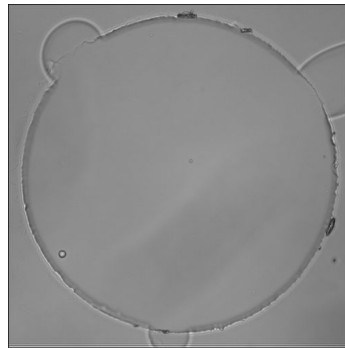
Where:

- $u^{(t)}$ denotes signal on t-th iteration
- λ is a parameter of Total Variation Loss
- V is Total Variation
- d is the detected signal
- P is the point spread function
- \otimes is the convolution operation

Results of Richardson-Lucy deconvolution

The resulting values of SSIM and PSNR are:

	SSIM	PSNR
Only blur	0.9463	20.0621
Blur + $G(\sigma = 0.01)$	0.1808	16.7633
Blur + $G(\sigma = 0.11)$	0.0583	11.9206
Blur + $G(\sigma = 0.3)$	0.026	8.0828
Blur + Poisson noise	0.0355	9.7349



Conclusion

- Both filters are highly susceptible to noise, however, Wiener filter amplifies it to a somewhat smaller degree
- Wiener filter introduces edge ringing artifacts that are not as prevalent on the Richardson-Lucy deconvolution
- Both metrics for Wiener Filtering are better than for Richardson-Lucy deconvolution

The code and a deeper report are available on the Github page:

<https://github.com/chameleon-lizard/Deconvolutions>