“On my honour as a member of the Democratic Order of Pirates International (D.O.P.I), I promise to be greedy, tricky, mean and icky…”

‘The Pirates Pledge’
from the cartoon “*The Adventures of Dr.Doolittle*” (1970)

(Watch Dr. Doolitle on YouTube)

GEH1013

Basic Ocean Navigation

For Aspiring Pirates, Privateers and other Adventures...

Chammika Udalagama

2021

Please visit:
chammika-udalagama.github.io/teaching.guest.piracy

Lesson Objectives

In this lecture we will visit the following topics and questions:

- Can we convince ourselves that the Earth is spherical?
- Are there ways to figure out where we are on the planet?
- How does the GPS system work?
- Appreciate and marvel at the significant challenges faced by the pioneers who took to the sea.
- How do the system of parallels, meridians, degrees, minutes and seconds work.
- Appreciate that living on a sphere brings about subtle challenges in measuring distances, cartography and timekeeping.
- Understand how just living on a ball affects climate.

Figure 1: The path of the sun in the heavens. (Wikipedia)

The World, She is Flat...

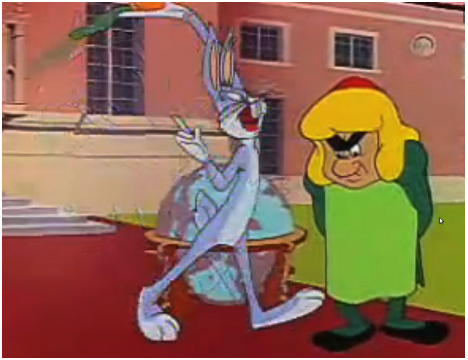


Figure 2: Bugs Bunny proves the world is round (Watch the video here)

Figure 3: Terry Practhett's 'Discworld'. (www.gopixpic.com)

How will you convince a FlatEarther that the Earth is spherical and not flat like 'Discworld'?
(Perhaps not the way Bugs does... ☺)

Did You Know that the Earth is a Ball?

Figure 4: A ship 'disappearing' over the horizon.

Figure 5: Photos of Earth's Shadow. (Anthony Ayiomamitis)

Can you think of a way to estimate the size of the Earth?

Eratosthenes

Figure 6: Portrait of Eratosthenes (Wikipedia)

- Eratosthenes of Cyrene (276 BC - 194 BC) was the second librarian of the great Library of Alexandria.
- Eratosthenes figured out that the Earth was spherical and estimated the Earth's circumference to within 8% of its real value! This was over 2000 years ago!!!
- Watch this video to figure out how he did this.

The Size of the Earth

- Eratosthenes used his keen powers of observation and a lot of ingenuity to calculate the circumference of the earth.
- He just used a deep well, the shadow of a tall pillar, and some simple geometry.
  Let's see how his method works...

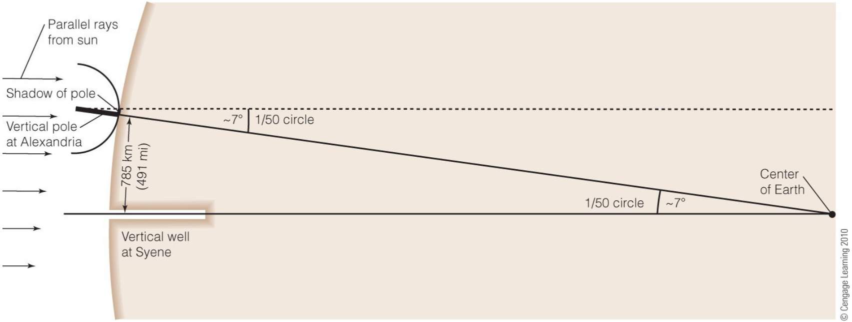


Figure 7: The idea behind Eratosthenes' method to determine the circumference of the Earth. ([Gar12])

If Eratosthenes had been in Singapore (☺), how well do you think would have his method worked?

Finding Our Way Round

- The Earth, being a ball, has no boundaries, so specifying locations is not obvious. So we use a ‘curved’ grid to help us pin-point places.
- This ‘grid’ is made up of **latitudes** (parallels) and **longitudes** (meridians).
- In this scheme the Earth is split into 180 latitudes (**parallels**) and 360 longitudes (**meridians**).

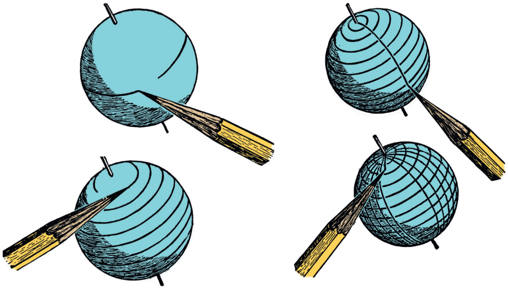
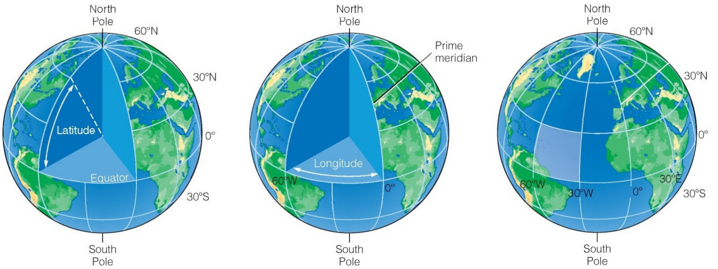


Figure 8: There are 180 latitudes (also called parallels) and 360 longitudes (also called meridians). ([Gar12])

Where is Zero?

- An obvious choice for zero latitude (parallel) is the **equator**.
- There is no obvious choice of zero for longitudes (meridians). It is agreed that the **prime meridian** (i.e. zero longitude) is the one through Greenwich, England[1].



a Latitude is measured as the angle between a line from Earth's center to the equator and a line from Earth's center to the measurement point.

b Longitude is measured as the angle between a line from Earth's center to the measurement point and a line from Earth's center to the prime (or Greenwich) meridian, which is a line drawn from the North Pole to the South Pole passing through Greenwich, England.

c Lines of latitude are always the same distance apart, but the distance between two lines of longitude varies with latitude.

Figure 9: How latitudes and longitudes are specified. ([Gar12])

[1] This was not always the case (see this Wikipedia page).

Parallels & Meridians

- Latitudes and longitudes are given in degrees.
- We also use **N** or **S** for latitudes and **E** or **W** for longitudes.
- E.g. Singapore is approximately at:

$$1^\circ \text{N } 103^\circ \text{E}$$

- Further refinement is obtained by using minutes ($'$) and seconds ($''$).
- $1^\circ = 60'$ and $1' = 60''$
- E.g. Singapore is 'exactly' at:

$$1^\circ 17' \text{N } 103^\circ 50' \text{E}$$

Where is this 'exact' point in Singapore? Google to find out!

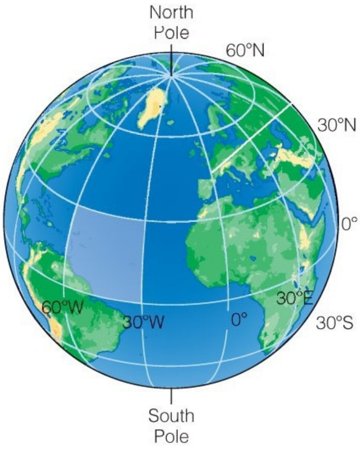


Figure 10: Location can be specified using angles. ([Gar12])

This Minute is Not the Same as that Minute...

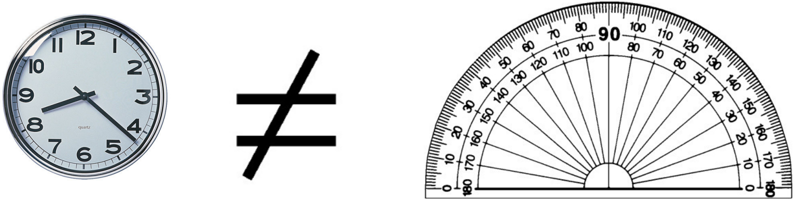


Figure 11: Minutes (′) and Seconds (′′) are measures of angles, not time! (
)

- Degrees (°), Minutes (′) and Seconds (′′) are use to measure **angles**.
- If we used the more familiar decimal system:

$$1' = \left(\frac{1}{60}\right)^{\circ} = 0.017^{\circ}$$

$$1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{60} \times \frac{1}{60}\right)^{\circ} = \left(\frac{1}{3,600}\right)^{\circ} = 0.00028^{\circ}$$

$$10^{\circ}30' = 10.5^{\circ}$$

- Notice that it is easier to use degrees, minutes and seconds that its decimal equivalent.

Where in the World Are We?!

- The North Star (Polaris), can be used to determine latitude of a location in the **northern** hemisphere.

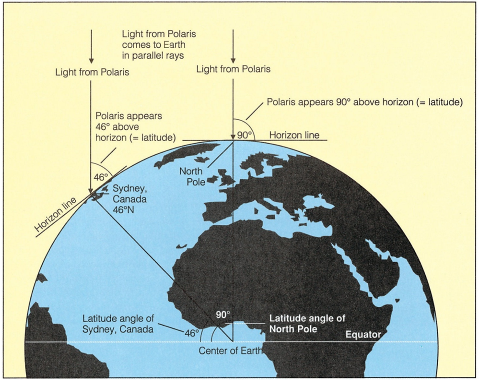


Figure 12: The North Star can be used to determine latitude. ([Gar12])

Figure 13: The Earth's axis of rotation points at the North Star. ([Gar12])

Using the North Star for Latitude

- Its all in the angles...

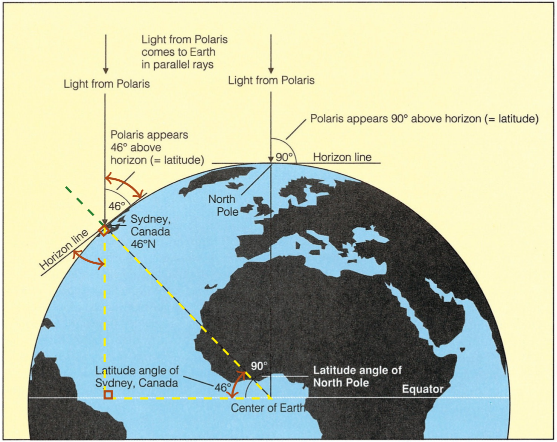


Figure 14: The angles that help us to determine **latitude** using Polaris ([Gar12])

What do we do in the southern hemisphere? Is there a 'South' star?

Importance of Longitude

- Even if you knew north/south and latitude, travelling the ocean without knowing your longitude is dangerous.

  > *"For lack of a practical method of determining longitude, every great captain in the Age of Exploration became lost at sea despite the best available charts and compasses. From Vasco da Gama to Vasco Núñez de Balboa, from Ferdinand Magellan to Sir Francis Drake–they all got where they were going willy-nilly, by forces attributed to good luck or the grace of God."*

  From Dave Sobel's 'Longitude...' [Sob95]

- A storm or current can easily make you lose your bearing.
- The 'longitude problem' cost governments lots of lives and money that in 1714 the British government passed an act of parliament that offered a (staggering) prize of £20,000 for an accurate solution.

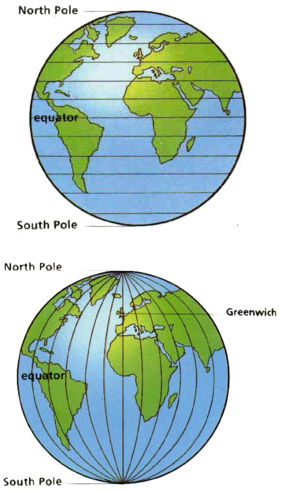


Figure 15: The heavenly bodies can help with latitude but not with longitude.

Time & Longitude

- Since there are 360° meridians and the Earth rotates through all these meridians in 24 hours:

Time for the Sun 'to go' from one meridian (1°) to the next

$$= \frac{24\ \text{h}}{360^\circ} = \frac{24 \times 60\ \text{min}}{360^\circ} = 4\ \text{min}/^\circ$$

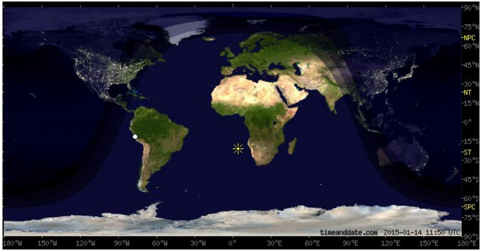


Figure 16: The time of day (usually) depends on the position of the Sun. Look here to see where the Sun is now! (Day and Night World Map)

- So, if we know the time at our location, we can figure out[2] the time at another location if we know its longitude.
- Also, if we know the times[1] at two locations, then we can figure out the difference in longitude between these two locations.

[2] Assuming the time is based on the Sun.

Noon is Not Just Only for Lunch...

Figure 17: The position of the Sun when it casts the shortest shadow signifies 'noon'.

- Noon marks the point when the Sun is at its peak, for that day.
- Noon is when the shadow cast by the Sun is the shortest (not necessarily zero; why?).
- This unique position of the Sun can be used to standardize what time is.

Can 'sunrise' or 'sunset' be used for time-keeping?

Longitude in the Days Past

- When you are at sea, you can use the Sun and the stars to determine your latitude, but not longitude.
- Complicated methods involving the Sun, the Moon and the stars were proposed and even used.
- The most reliable way was to use ‘noon’ at the present location, along with an accurate clock.
- The problem was designing and manufacturing a clock that can withstand the vicissitudes of ocean travel.

Figure 18: Using a ‘Jacob’s Staff’ to determine the position of the Sun. Lots of people ended up blind in one eye with this instrument. (Wikipedia: "Jacobstaff" by John Seller (1603-1697) - Scan from the original book Practical navigation (1st edition 1669) p. 200)

H-4

- John Harrison's fantastic chronometer, that won him the £20,000 prize in 1773.
- This is the fourth and the most accurate that Harrison manufactured.

  > *"I think I may make bold to say, that there is neither any other Mechanical or Mathematical thing in the World that is more beautiful or curious in texture than this my watch or Timekeeper for the Longitude . . . and I heartily thank Almighty God that I have lived so long, as in some measure to complete it."*
  >
  > – John Harrison –
  > From 'Longitude' [Sob95]

- There is a very rich historical backstory to this topic involving a lot of great names such as Galileo, Euler, Newton, Halley…(again; see 'Longitude' [Sob95]).

Figure 19: Harrison's fourth timepiece: H-4. ([Gar12])

‘Naive’ & Real Time Differences

City	Longitude (′ ″ °)	Longitude (°)	Longitude ‘Distance’	Time Difference (h) ‘Naive’	Time Difference (h) ‘Real’ (UTC)
Singapore	$103°55'$ E	103.9 E	0	0	0
New York	$73°56'$ W	73.9 W	177.8	-11.9	-13
San Francisco	$122°25'$ W	122.4 W	226.3	-15.1	-16
London	$0°15'$ E	0.3 E	103.6	-6.9	-8
Shanghai	$121°28'$ E	121.5 E	17.6	1.2	0

(UTC) Time Differences in Real Life

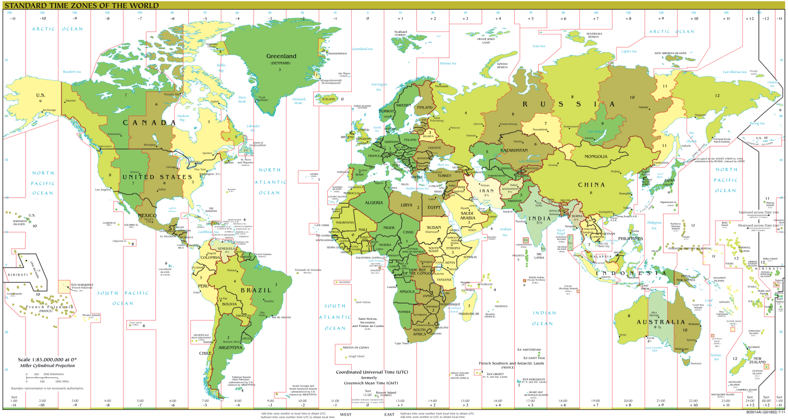


Figure 20: The world has been separated into 26 different time zones. Visit `http://www.timeanddate.com/time/map/` for an interactive map.

- UTC (*Coordinated Universal Time*) times zones extends from -12 to +14 from Greenwich.

Where is a ‘Day’ born?

- A ‘new day’ is born when midnight passes the **International Date Line**.
- The date line should ideally follow the 180°meridian (it does not).
- If you cross this line right $\rightarrow$ left (according to the image) you essentially travel into ‘tomorrow’. So you need to add a day to your watch[1].
- If you cross this line left $\rightarrow$ right (according to the image) you essentially travel into ‘yesterday’. So you need to subtract a day from your watch.
- First place to experience a new day: Kiritimati or Chritmas Island ($157^\circ 24"$ W).

3

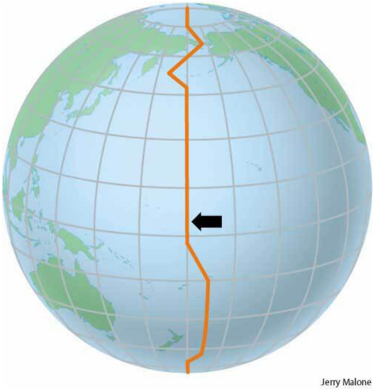


Figure 21: The ‘International Date Line’: where a day is born.

[3] Recall what happens in ‘Around the World in 80 days’?

The Date Line is NOT Straight!

- Kiritimati or Christmas Island is in the **Western** hemisphere ($157°24''$ W)!
- This results in a large irregularity in the international date line.
- The UTC zones +13 and +14 are in this region.
- Notice how there is a difference of a day between Kiritimati and Hawaii, which is almost on the same meridian!

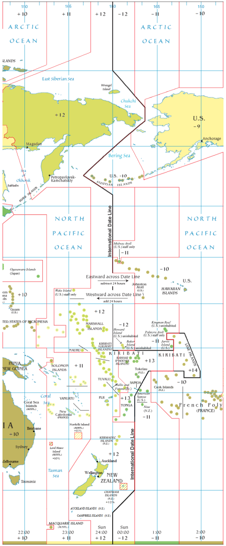


Figure 22: The not-so-straight date line. (Image from the Wikipedia article on the International Date Line.)

What is the 'Real' Shape of the Earth

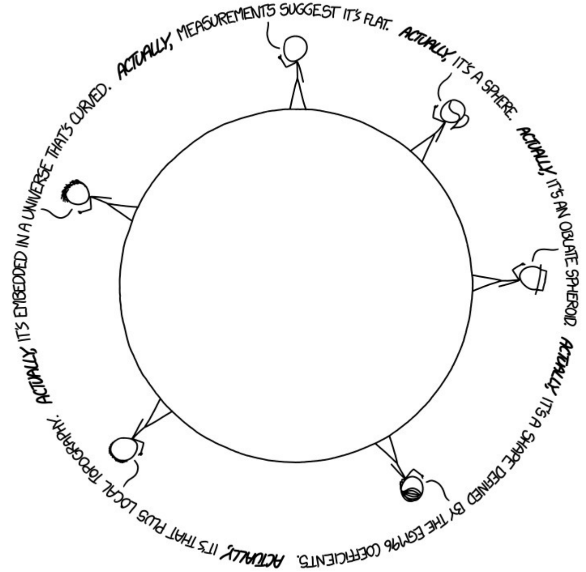


Figure 23: The shape of the Earth according to xkcd (xkcd)

Map worth $10,000,000!

Figure 24: The Waldseemüller map (1507). (See Wikipedia for more information.) ([Gar12])

Looks can be Deceiving

- Which is bigger, Africa or Greenland?

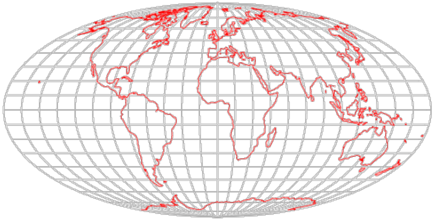


Figure 25: Mollweide Projection

Figure 26: Mercator Projection

Beware of Greeks Bearing Maps

- A (3D) sphere **cannot** be perfectly projected onto a (flat) plane.
- Each projection is optimised for a specific use (e.g. **directions** for navigation or **area** for surveying).

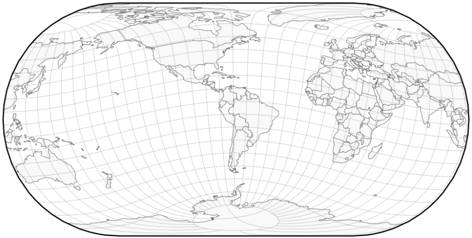


Figure 27: There many projections possible. Click here to explore.

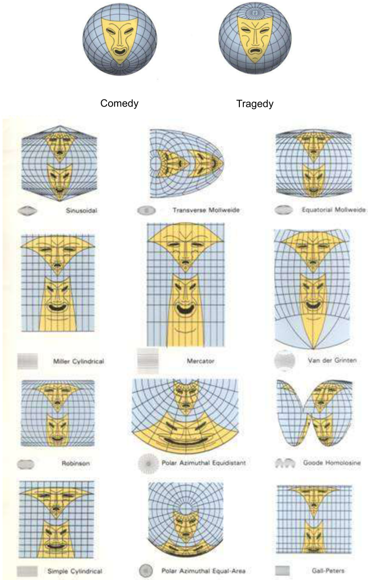


Figure 28: Effect of different projections on the **comedy** and **tragedy** faces.

What is the 'Real' Shape of the Earth

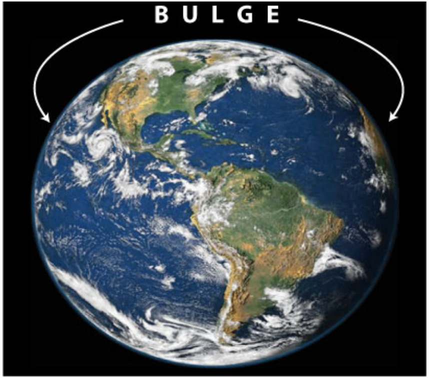


Figure 29: The Earth is a bit bulging at the waist ☺.

- The actual shape of the Earth is complicated due to its features (valleys, mountains, trenched, ...).
- The Earth is not really a ball but an oblate ('fatter' in the middle) spheroid.

	Equator	Poles
Diameter	12,756 km	12,713 km

- This difference of ≈ 43 km(!) is a consequence of the Earth's rotation.
- Nowadays, the absolute position on Earth is expressed using the World Geodetic System (WGS).
- WGS uses fixed points (datums) and gravitational measurements to establish it reference.
- GPS plays a prominent role in modern geodacy (see NGS and CORS).

Global Positioning System (GPS)

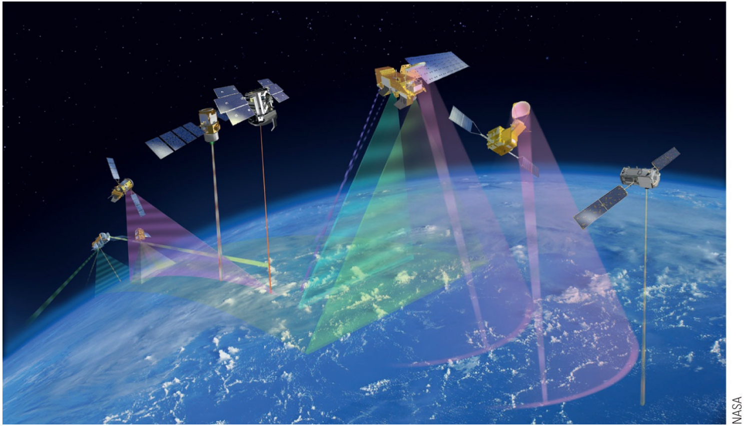


Figure 30: GPS uses a constellation of satellites.

- **GPS** or **Global Positioning System** was originally called **NAVSTAR** - *Navigation System for Timing and Ranging.*

GPS - xkcd

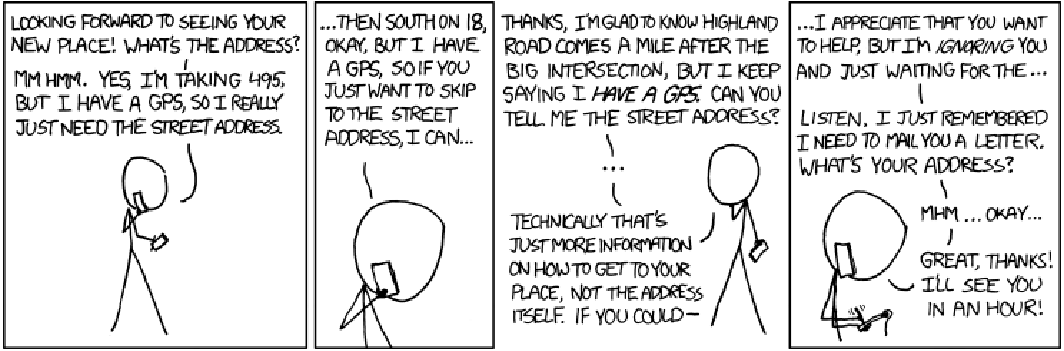


Figure 31: ☺ (xkcd)

Some Facts about GPS

- GPS consisted of 24 satellites (now around 30) and was setup by the US in 1973 for defense purposes.
- GPS is free but requires a GPS receiver and line-of-sight to the satellites.
- The orbits of the satellites ensure that there is **always** at least four in line-of-sight from any point on the globe! (check out WinOrbit)
- GPS can usually work in all types of weather including heavy cloud cover but not underwater or underground.

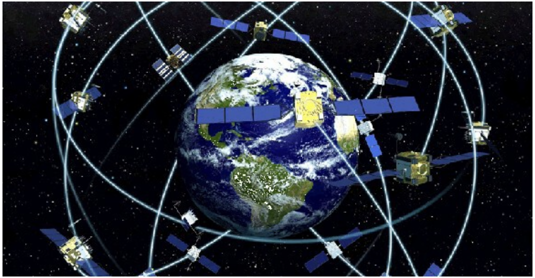


Figure 32: You can always see at least four satellites in the NAVSTAR system!

How GPS Works I

- The GPS consists of three segments:
  1. **Space segment**: This is the constellation of satellites, each carrying an atomic clock!
  2. **Control segment**: Ground stations around the globe for monitoring/correcting the orbits. The main control station is in the Schriever Air Force Base, Colorado.
  3. **User segment**: This is your GPS receiver.

- The signal sent by the satellite usually consists of its:
  `[name, position` (x, y, z)`, time` (t_s)`]`
- The distance between you and the satellite can be determined by comparing the time t_{sat} and the time (t_{you}) in the clock in your GPS receiver.

$$\text{distance} = (t_{you} - t_{sat}) \times c \tag{1}$$

$c = 3.0 \times 10^8$ m s^{-1} is the speed of the (electromagnetic) signal.

- Because c is so large, even a small inaccuracy in time will result in a big error in distance.

Figure 33: Accurate timing is crucial for GPS.

- Actually, our watches needs to be as good as an **atomic clock**(!) which is a problem. Luckily there is an ingenious solution!

HOW THE OCEAN WORKS

Ingenuity of GPS I

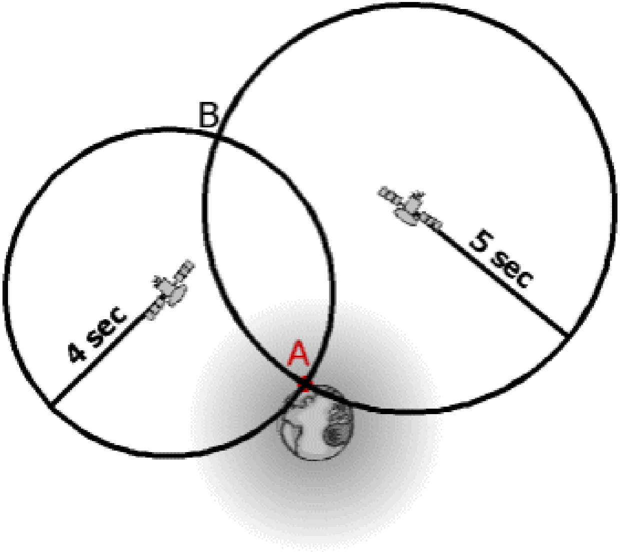


Figure 34: If you have a ‘good’ watch all you need is two satellites.

Ingenuity of GPS II

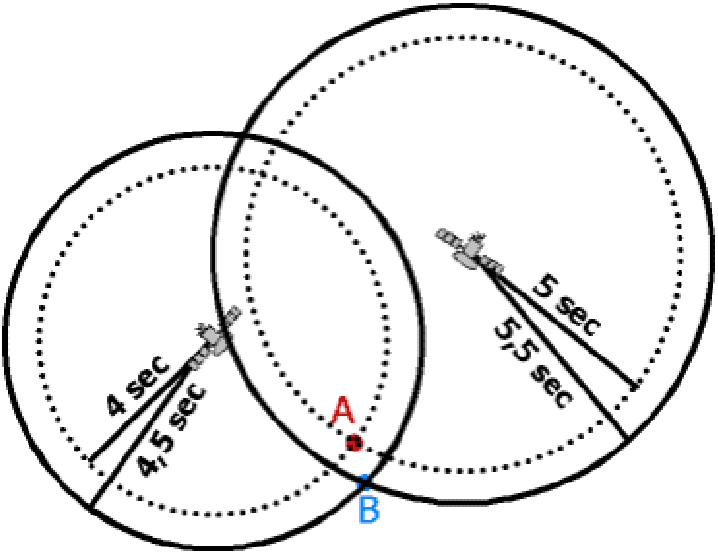


Figure 35: If your watch is ‘off’ you will be at A but think you are at B.

Ingenuity of GPS III

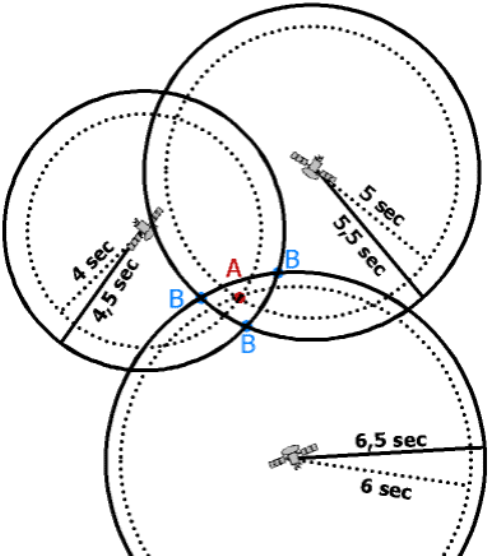


Figure 36: With three satellites, even if your watch is 'off' you can correct your time until the B's coincide with A!

Living on a Ball…is Different

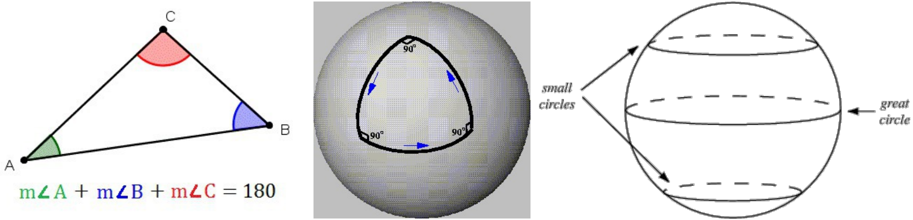


Figure 37: The usual rules of geometry does not seem to work on spheres!

- Triangles are different, ‘parallel’ is different & the shortest distance is **not** straight.
- The shortest distance between two points is along a ‘**great circle**’
- A ‘great circle’ shares its centre with the centre of the sphere.
- This is partly why planes seem to fly ‘funny’ (see here for a great simulation).

If all this is true, why don’t we notice these weird things in everyday life?

Seasons in the Sun

- The position of the Sun in the sky changes throughout the year.

Figure 38: The path of the sun in the heavens. (Wikipedia)

- The Sun is directly over the equator **twice** a year. These times are called **equinoxes**.

	Spring	Fall
Equinox	≈ 20 Mar	≈ 20 Sep

- The Sun 'turns around' once it is over the **Tropic of Cancer** (23.5°N) or **Tropic of Capricorn** (23.5°S). These times are called **solstices**.

	Summer	Winter
Solstices	≈ 20 Jun	≈ 20 Dec

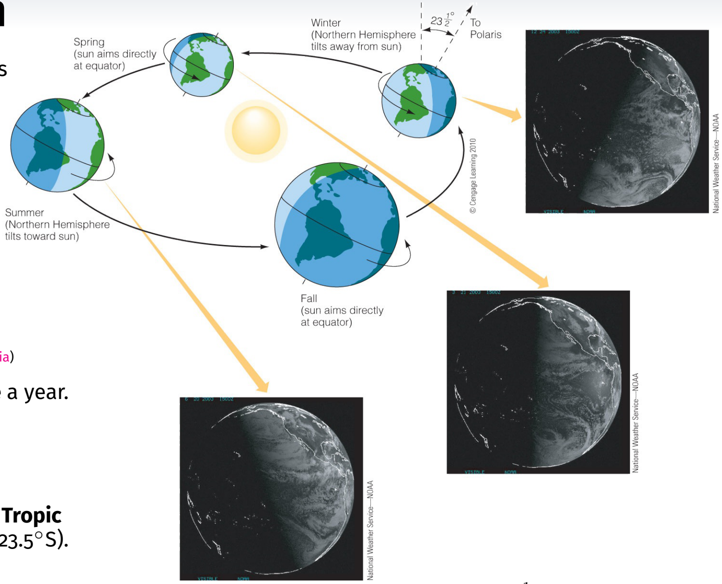


Figure 39: Seasons are caused due to the Earth's $23\frac{1}{2}°$ tilt. ([Gar12])

Angles Are Important

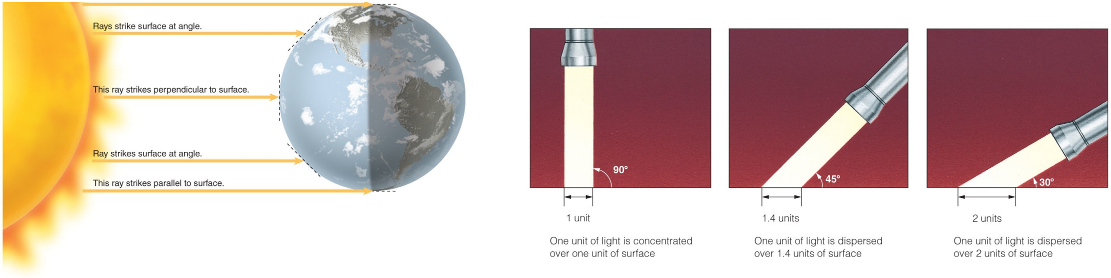


Figure 40: The angle of incidence of sunlight determines how much light is available for absorption. ([Gar12])

- There are many other factors (e.g. Milankovitch cycles[4]) that affect the amount of light that we receive from the Sun.

[4]Look here for a cool animation of Milankovitch cycles.

List of Resources

- Tom Garrison, *Essentials of oceanography*, 6th ed., Brooks/Cole, Cengage Learning, Belmont, CA, 2012.
- Dava Sobel, *Longitude: the true story of a lone genius who solved the greatest scientific problem of his time*, Walker, New York, 1995.