Chapter 3

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Linear Equations and Inequalities

An unknown variable is critical in solving some real-world problems because the unknown is what you are trying to figure out. You can use variables to stand for anything you don't know when you use an equation to represent and solve a real-world problem. A variable stands in for the answer until you can use the order of operations and number properties to determine the answer. Once you understand variables, you can represent and solve more complex algebraic problems.

Lesson 3.1
Evaluate Linear Expressions

How can you use math to describe a situation? How can you figure out whether a situation can be modeled by addition, subtraction, or another operation? Learn how to identify key words in a real-world problem and translate it to a linear expression that represents the situation in the problem.

Lesson 3.2
Solve Linear Equations

When you travel to a new place, you can find your way home by following the directions in reverse. You are essentially "undoing" each turn. When you solve a linear equation you want to "undo" every operation to solve for the variable. Learn how to solve linear equations.

Lesson 3.3
Solve Linear Inequalities

What happens when a problem has more than one solution? Linear inequalities have a range of values as their solution. Because they have more than one answer, their solutions are often graphed on number lines. Learn how to solve and graph linear inequalities.

Lesson 3.4
Use Expressions, Equations, and Inequalities to Solve Real-World Problems

Where do you start when you need to solve a real-world problem? How can you write a real-world problem as an equation or inequality that you can solve? Learn how to translate and solve real-world problems using linear equations and inequalities.

Goal Setting

Think about the last time you had to decide between two options. It might have been choosing between cell-phone plans or comparing two different housing options. What information did you know? What information was unknown? What did you need in order to make the decision? How could you use algebra to represent the two options mathematically and help you make your decision?

How else can you use algebra to help you find solutions to problems you encounter at work or at home? The world is full of unknowns. Algebra can help you make sense of what you know and what you don't know but need to find out.

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LESSON 3.1 Evaluate Linear Expressions

LESSON OBJECTIVES
- Use algebraic symbols to represent unknown quantities
- Perform operations on linear expressions
- Evaluate linear expressions

CORE SKILLS & PRACTICES
- Perform Operations
- Evaluate Expressions

Key Terms
variable
a letter that is used to represent an unknown value

constant
an expression that stays the same

coefficient
a number that is multiplied by a variable

algebraic expression
a mathematical statement containing letters and numbers organized as terms but with no equal sign

Vocabulary
evaluate
to substitute values for variables

distribute
to use multiplication over addition or subtraction

Key Concept
There are a lot of unknowns around us. In math we do not always know the total we are solving for or the values we are calculating. These unknowns are expressions. Evaluating linear expressions means substituting values (numbers) for variables (letters).

Algebraic Expressions
If you earn $10.50 an hour, how long will it take you to earn $600? What about $800? Questions like this come up all the time. As situations change we need to calculate differently.

Expressions and Variables
An **algebraic expression** is a mathematical statement containing letters and numbers organized as terms. Addition and subtraction separate terms. Terms can be numbers, letters, or letters that are multiplied or divided by numbers. The letters you see in algebraic expressions are called **variables**. The variable represents different values, so the value of an expression can change depending on the value assigned to the variable.

Algebraic Expressions:

1 Term:	$2q$	q is the variable.
2 Terms:	$5n + 4$	n is the variable.
3 Terms:	$6w + 21r - 10$	w and r are both variables.

Parts of an Algebraic Expression
In an algebraic expression, numbers that stand alone as a term are called **constants**. They are constants because their value stays the same.

Algebraic Expressions: $5n + 4$ 4 is "alone." It is a constant.

Numbers that are multiplied by variables are called **coefficients**. The coefficient of a variable is usually written in front of the variable. If a variable does not have a number in front of it, the coefficient is 1.

Algebraic Expressions:
$6w + 21r - 10$ The coefficient of w is 6. The coefficient of r is 21.

Algebraic expressions must contain at least one variable and one number. They do not contain an equal sign.

Translating Between Phrases and Expressions
Translating words to algebraic expressions is an important skill when solving real-world problems. You can begin to practice this skill by translating simple phrases. To do so, use the meanings of the operations.

Mathematical Operation	Key Phrases
Addition +	Sum, Increase, Add, All together, Total
Subtraction −	Subtract, Decrease, Difference, Minus, Fewer
Multiplication × or •	Times, Multiply, Product
Division ÷ or /	Divide, Per, Quotient

The following examples show how phrases are translated to algebraic expressions and how algebraic expressions are translated to words.

Example 1: Translating Phrases to Algebraic Expressions

Phrases		Expressions
5 less than twice a number	⟶	$2n - 5$
A number increased by 10	⟶	$x + 10$
The product of two and a number	⟶	$2r$
Six times the sum of a number and five	⟶	$6(w + 5)$

Example 2: Translating Algebraic Expressions to Words

Expressions		Phrases
$50 + w$	⟶	"the sum of 50 and w" "50 increased by w"
$20 - x$	⟶	"the difference of 20 and x" "x less than 20"
$6n$	⟶	"the product of a number and 6" "6 times n"
$\frac{10}{r}$	⟶	"the quotient of 10 and r" "10 divided by r"

Think about Math
Directions: Fill in the blank with the correct expression. Use n to represent an unknown.

1. The phrase "5 less than a number" can be written as ___________.
2. The phrase "the sum of −7 and 2 times a number" can be written as ___________.
3. The phrase "the product of 10 and the sum of 2 and a number" can be written as ___________.

WORKPLACE SKILL
Use Reasoning
In business, algebraic expressions are used to determine many things, including weekly pay, costs, discounts, and benefits. If you are paid on a commission basis, the amount you earn depends on the amount of total sales you produce. Some employers offer salaries that pay a set amount in addition to a commission. In this case, total sales represent a value that varies, and the set amount is a constant. When calculating total pay, the commission percentage is multiplied by total sales, then the set amount is added.

For example, Hilary, a clothing retail saleswoman, earns $250 per week plus a 10% commission on her sales. Write an expression, using s to represent sales, that can be used to calculate the amount of money Hilary earns each week.

Linear Expressions

Linear expressions are used to represent many different situations. For example, cell phone plans are represented by linear expressions because there is a constant monthly charge and additional charges for taxes or overage charges.

Identifying Linear Expressions

A linear expression is one type of algebraic expression. In linear expressions, no term can have two or more variables, nor can they have square roots or exponents.

These are examples of linear expressions:

$x + 7$	$3x + 7$	$3x + 7y$

These are <u>not</u> linear expressions:

x^2	No exponents on variables
$3xy + 5$	Can't multiply two variables
$\frac{x}{y} + 4$	Can't divide two variables
$2\sqrt{x}$	No square root sign on variables

In linear expressions you can only add or subtract like terms. Like terms have the same variables but may have different constants. You can simplify linear expressions by combining like terms.

Example 3: Combining Like Terms

Simplify the expression: $7w - 2 + 3w + 5$

Step 1 Rearrange the expression so like terms are next to each other.

$7w - 2 + 3w + 5$
$7w + 3w - 2 + 5$

Step 2 Combine the whole numbers.

$7w + 3w - 2 + 5$
$7w + 3w + 3$

Step 3 Combine the like terms with variables by combining their coefficients.

$7w + 3w + 3$
$10w + 3$

Adding Linear Expressions

When adding two linear expressions, like terms need to be combined in one expression.

Example 4: Adding Linear Expressions

Add: $(5x - 3) + 3(-x + 2)$

Step 1 First, we **distribute** the coefficients to remove the parentheses. To distribute means to multiply over addition or subtraction. In this case, we will multiply 1 by $(5x - 3)$, and 3 by $(-x + 2)$.

$(5x - 3) + 3(-x + 2)$
$1(5x - 3) + 3(-x + 2)$
$1(5x) + 1(-3) + 3(-x) + 3(2)$
$5x + (-3) + (-3x) + 6$

Step 2 Rearrange the expression so like terms are near each other.

$5x + (-3) + (-3x) + 6$
$5x + (-3x) + (-3) + 6$

Step 3 Combine the coefficients for like terms and simplify the expression.

$5x + (-3x) + (-3) + 6$
The answer is $2x + 3$.

Subtracting Linear Expressions

Subtracting linear expressions is similar to adding them, except that you will have to multiply a negative, or distribute the minus sign, before combining like terms.

Example 5: Subtracting Linear Expressions

Subtract: $(7r - 1) - (2r + 6)$.

Step 1 Distribute.

$(7r - 1) - 1(2r + 6)$
$7r - 1 - 1(2r) - 1(6)$
$7r - 1 - 2r - 6$

Step 2 Rearrange the expression so like terms are near each other.

$7r - 1 - 2r - 6$
$7r - 2r - 1 - 6$

Step 3 Combine like terms and simplify the expression.

$7r - 2r - 1 - 6$
$5r - 7$

CORE SKILL

Perform Operations

You will often be asked to perform operations on expressions. Every operation has a different method to follow. Those methods could involve multiplying expressions through parentheses using the Distributive Property, simplifying expressions by combining like terms, or evaluating the expression by substituting a value into the variable. The first step always is to identify what operations you are performing and then follow the steps for that particular operation, paying attention to the order of operations while you simplify. Using your knowledge of the operations as well as simplifying expressions, add the linear expressions: $2(3n - 1)$ and $5(3n + 2)$.

Think about Math

Directions: Select the most appropriate answers.

1. Multiply the rational coefficient by the linear expression:
   $-12(3b - 4)$
   A. $36b + 4$
   B. $-36b - 4$
   C. $-36b - 48$
   D. $-36b + 48$

2. Multiply the rational coefficient by the linear expression:
   $-6(2x + 10)$
   A. $-12x - 60$
   B. $-12x + 60$
   C. $-12x + 10$
   D. $-8x - 16$

Evaluating Linear Expressions

Ever wonder how many calories you burn while exercising? The number will actually depend on the type of exercise and the amount of time you spend exercising. When you find out this information, you can write and evaluate a linear expression to find out how many calories you are actually burning.

Evaluate Linear Expressions

When you **evaluate** linear expressions, you are substituting a number for a variable in the expression then simplifying the expression.

Example 6: Evaluate a Linear Expression

Evaluate: $9x + 10$ for $x = 5$

Step 1 First, determine what number will replace the variable. In this instance, 5 will be replacing x.

$9x + 10$ for $x = 5$
$9(5) + 10$

Step 2 Next, multiply 9 by 5.

$9(5) + 10$
$45 + 10$

Step 3 Finally, add the terms together. The value of the expression when $x = 5$ is 55.

$45 + 10 = 55$

Problem Solving Practice

Compare the following taxi cab charges. Based on the information given, which company is best for someone who needs to travel 30 miles?

Example 7: Evaluate a Linear Expression

Taxi Company #1: \$0.25 per mile plus \$3.00 service fee
Taxi Company #2: \$0.30 per mile, no service fee

Step 1 Write an expression that represents Taxi Company #1.
Let m represent the number of miles.

$0.25m + 3.00$

Step 2 Write an expression that represents Taxi Company #2.
Let m represent the number of miles.

$0.30m$

Step 3 Evaluate each expression when $m = 30$.

Taxi Company #1:
$0.25m + 3.00 = 0.25(30) + 3.00 = 7.50 + 3.00 = 10.50$

Taxi Company #2:
$0.30m = 0.30(30) = 9.00$

In the case of someone who travels 30 miles, Taxi Company #2 is a better option. This plan saves the customer \$1.50 compared to Taxi Company #1.

Think about Math

Directions: Select all appropriate answers.

1. When evaluated at $p = 3$, which of the following have a value greater than 9?
   A. $3p$
   B. $2p + 4$
   C. $-2p + 7$
   D. $5p - 4$

2. Which expression has a value of 15 when evaluated at $q = 7$?
   A. $2(q + 1) + 5q - 8$
   B. $3(q + 1) - 17 + q$
   C. $4(q - 2) - q + 2$
   D. $q + 12 - (q - 4)$

CORE SKILL

Evaluate Expressions

Some linear expressions have more than one variable. Evaluating these expressions still follows the same process as for evaluating expressions with only one variable. The only difference occurs at the beginning; you must first substitute both values for their respective variables and then simplify. Be sure to substitute accordingly. If you substitute an incorrect value for a variable or a value for the wrong variable, the resulting expression will be incorrect. What is the value of the expression, $3g + 6r$, when $g = -2$ and $r = 8$?

CALCULATOR SKILL

Entering complex calculations into a calculator can seem rather hard, especially when parentheses must be entered to guarantee the correct expression is calculated. While using the TI-30XS MultiView™, press the (and) buttons to place an expression in parentheses. Simplify the expression $2.25(2 + 1.758 - 6.4)$.

Vocabulary Review

Directions: Match the terms with their description.

variable	**constant**	**distribute**
coefficient	**evaluate**	**algebraic expression**

1. the number that is multiplied by a variable ________________

2. to use multiplication over addition or subtraction ________________

3. an expression that stays the same ________________

4. to substitute values for variables ________________

5. a letter that is used to represent an unknown ________________

6. a mathematical statement containing letters and numbers organized as terms ________________

Skill Review

Directions: Write expressions that describe the following phrases.

1. Seven less than twice a number
2. The product of -3 and a number

Directions: Follow the prompt for each of the linear expressions.

3. Add: $(3n - 4) + (2n - 5)$
4. Subtract: $(9w - 3) - (6w - 3)$
5. Multiply: $-4(5p - 1)$
6. Evaluate: $4n + 3$, when $n = 6$

Directions: Answer the following questions.

7. When evaluated at $r = 3$, which of the following expressions result in an even number?
   A. $2(r - 1)$
   B. $r + 2 - 3r$
   C. $3(r + 1) + 1$
   D. $(r - 2) + 7r - 1$

8. Which statement represents the expression $3n + 7$?
   A. The product of a number and the quantity 3 plus 7.
   B. The sum of a number and 7 times 3.
   C. Seven plus the product of 3 and a number.
   D. Three times the sum of a number and 7.

Skill Practice

Directions: Read each problem and complete the task.

1. Susan had to write an algebraic expression for the following phrase: "the difference between two times a number and negative 5," using n to represent the unknown. She then had to evaluate the expression when $n = -5$. Her final answer was -5. Was Susan correct?

2. A volunteer group is planning a celebration for its second anniversary. The local hall charges \$200 for rent of the space and \$15 per person for food and beverages. Write an expression that represents this situation. Let p represent people.

3. Perform the operations.

$$(3x - 4) + (-2x - 1) - 5(x - 3)$$

4. A couple is saving money to purchase a new car. They do not want to finance any amount. The couple has already saved \$12,000, and they are saving an additional \$500 each month. The expression that represents this situation is: $500m + 12{,}000$, where m is the number of months. If they save for one year, how much will they be able to spend total?

5. A local gym has a special promotion offering new members three free months if they sign a one-year contract. After the first three months, the gym membership will cost \$20 per month. A new gym has just opened and is offering one-month free membership and \$18 per month thereafter. Compare the cost of both gyms after one year of membership. Which is more expensive?

6. When evaluated at $p = 6$ and $q = -3$, which of the following expressions have a value greater than -6?
   A. $3p + 2q$
   B. $2p - 3q$
   C. $-p + q$
   D. $p - 4q$

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LESSON 3.2 Solve Linear Equations

LESSON OBJECTIVES

- Write and solve one-step equations
- Solve multi-step equations

CORE SKILLS & PRACTICES

- Solve Simple Equations by Inspection
- Solve Linear Equations

Key Terms

equation
a mathematical statement that two expressions are equal

solution of an equation
a value for the variable that makes the equation true

Vocabulary

expression
a mathematical statement that contains numbers, operations, and/or variables but no equal sign

inverse operations
operations that undo each other

reciprocal
the number that has a product of 1 when multiplied by the original number

variable
an unknown quantity

Key Concept

You can solve an equation by performing inverse operations on both sides of the equation. The solution can be checked using substitution.

One-Step Equations

Ask anyone who has ever taken algebra what he or she remembers most from the class. Most people will probably mention solving equations. Solving equations is a fundamental concept that is used in almost all branches of mathematics as well as in various other fields, such as science and economics.

Equations and Solutions

A **variable** is a letter or symbol that represents an unspecified number. An **expression** is a mathematical phrase that contains numbers, operations, and/or variables. An **equation** is a mathematical statement that two expressions are equal. It is important to recognize the difference between an expression and an equation. An equation always has an equal sign.

Equations	Expressions
$16 - 9 = 7$	$2q$
$5 + x = 25$	$5n + 4$
$2w = 10x$	$6w + 21r - 10$

An equation that contains a variable may be true or false depending on the replacement value for the variable. A **solution of an equation** with one variable is a value that, when substituted for the variable, makes the equation true. You solve an equation when you find the solution for the variable.

Equation	Solution
$r + 20 = 30$	The solution is $r = 10$ because $10 + 20 = 30$.
$6 - t = 2$	The solution is $t = 4$ because $6 - 4 = 2$.

Writing Equations

It is important to be able to write equations from verbal descriptions. Real-world problems are stated using verbal descriptions, and being able to translate these situations into equations creates an opportunity to find mathematical solutions. The correct solution can be found only if the equation is written correctly.

Translating from Words to Equations

Look for key words that indicate the operation being performed. Also look for words and phrases that mean "equals," such as *is* and *is equal to*. Represent an unknown number by using any letter as a variable.

Words	Equations
A number increased by 32 is equal to 40.	$n + 32 = 40$
Four times a number is 36.	$4x = 36$
Seven less than what number equals 15?	$k - 7 = 15$
The product of a number and 3 is –15.	$3w = -15$
6 divided by a number is equal to 2.	$\frac{6}{a} = 2$

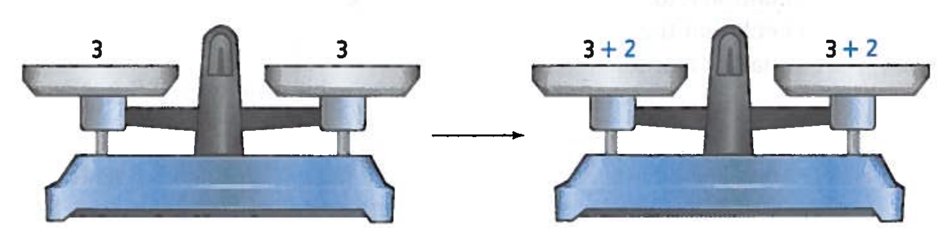
Equations can be written by breaking up the words and translating each part.

"A number is multiplied by three, and then nine is subtracted to give a result of twelve."

A number → n
is multiplied by three → $3n$
and then nine is subtracted → $3n - 9$
to give a result of → $3n - 9 =$
twelve → $3n - 9 = 12$

Solving One-Step Equations

An equation is a perfect balance between what is on the left side and what is on the right side of the equal sign. If you make any changes on the left side, you must make the same changes on the right side. Think of an equation as a balanced scale.



Inverse operations are operations that undo each other. For example, the inverse operation of adding 5 is subtracting 5, and the inverse operation of dividing by 10 is multiplying by 10. To solve an equation, perform inverse operations to isolate the variable. This means that the variable is by itself on one side of the equation.

CORE SKILL

Solve Simple Equations by Inspection

To solve an equation by inspection means to find the solution mentally, without using pencil and paper or a calculator. Simple equations like $5x = 20$ can be solved by inspection because only one operation is involved and because the numbers are familiar.

The solution of this equation can be found by remembering the multiples of 5. What multiple of 5 is equal to 20?

$5 \times 1 = 5$
$5 \times 2 = 10$
$5 \times 3 = 15$
$5 \times 4 = 20$

The solution of $5x = 20$ is $x = 4$.

Solve each of the following equations by inspection.

- $2n = 6$
- $10 = 5r$
- $q + 6 = 12$
- $\frac{6}{g} = 2$

Example 1: Equations Involving Addition or Subtraction

Solve each equation.

	$y + 12 = 18$	$n - 7 = 13$
Step 1 Identify the operation performed on the variable.	12 is added to y.	7 is subtracted from n.
Step 2 Perform the inverse operation on both sides of the equation.	Subtract 12 from both sides. $\begin{aligned} y + 12 &= 18 \\ -12 &= -12 \\ \hline y \quad &= \quad 6 \end{aligned}$	Add 7 to both sides. $\begin{aligned} n - 7 &= 13 \\ +7 &= +7 \\ n &= 20 \end{aligned}$
Step 3 Substitute the answer back into the original equation and check that the equation is true.	$y + 12 = 18$ $6 + 12 = 18$ $18 = 18$ ✓	$n - 7 = 13$ $20 - 7 = 13$ $13 = 13$ ✓

Example 2: Equations Involving Multiplication or Division

Solve each equation.

	$4r = 20$	$\frac{x}{2} = 9$
Step 1 Identify the operation performed on the variable.	r is multiplied by 4.	x is divided by 2.
Step 2 Perform the inverse operation on both sides of the equation.	Divide both sides by 4. $4r = 20$ $\frac{4r}{4} = \frac{20}{4}$ $r = 5$	Multiply both sides by 2. $\frac{x}{2} = -9$ $2 \times \frac{x}{2} = 2 \times (-9)$ $x = -18$
Step 3 Substitute the answer back into the original equation and check that the equation is true.	$4r = 20$ $4(5) = 20$ $20 = 20$ ✓	$\frac{x}{2} = -9$ $\frac{-18}{2} = -9$ $-9 = -9$ ✓

Think about Math

Directions: Write an equation to represent each statement. Use the variable n to represent unknown numbers. Then solve your equation.

1. Ten less than a number is 62.
2. A number increased by 20 is equal to 30.
3. A number is multiplied by 3 and the result is 12.
4. The sum of -7 and a number is 2.
5. A number divided by 4 is equal to -5.

Multi-Step Equations

Equations are useful tools not only in mathematics but also in the sciences. Scientists balance chemical equations following the concept of performing the same actions on both sides.

Two-Step Equations

When solving equations that contain more than one operation, the goal is still to find the value of the variable that makes the equation true. Work backwards using inverse operations to undo each operation until the variable is isolated.

Solving Equations Step by Step

- Simplify expressions on both sides of the equations.
- Use addition and/or subtraction to gather all of the variable terms on the left side of the equation.
- Use inverse operations to undo addition and subtraction on the variable term.
- Use inverse operations to undo multiplication and division on the variable term.

Example 3: Two-Step Equations

Solve $5n - 4 = 6$.

Step 1 Add 4 to both sides of the equation.

$$\begin{aligned} 5n - 4 &= 6 \\ +4 \quad &\; +4 \\ \hline 5n &= 10 \end{aligned}$$

Step 2 Divide both sides of the equation by 5.

$$\frac{5n}{5} = \frac{10}{5}$$
$$n = 2$$

Step 3 Substitute the answer back into the original equation and check that the equation is true.

$$5n - 4 = 6$$
$$5(2) - 4 = 6$$
$$10 - 4 = 6$$
$$6 = 6 ✓$$

Using Reciprocals

The inverse operation of multiplying by a fraction is multiplying by the reciprocal of that fraction. A reciprocal is a number that has a product of 1 when multiplied by the original number The reciprocal of a fraction can be found by interchanging the numerator and the denominator. For example, the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$ because $\frac{3}{5} \times \frac{5}{3} = \frac{3 \times 5}{5 \times 3} = \frac{15}{15} = 1$.

CALCULATOR SKILL

You can use a calculator to check your solutions to equations. A calculator may be especially useful for equations that require several steps to solve. For example, suppose you solved $3(2d + 7) + 5(d - 8) = 80$ and found the solution $d = 9$. To check whether $d = 9$ is correct, substitute 9 for d on the left side of the equation and evaluate. Press the following sequence of buttons on the TI-30XS MultiView™ calculator,

$3 \times (2 \times 9 + 7) + 5 \times (9 - 8)$

then press equals/enter. The display should read 80, which means that $d = 9$ is the correct solution.

Use a calculator to check whether $x = 20$ is the solution of $3(x + 4) - 3(x - 1) = 114$.

Example 4: **Multiplication by the Reciprocal**

Solve $16 = \frac{1}{2}x + 10$.

Step 1 Subtract 10 from both sides of the equation.

$$\begin{array}{r} 16 = \frac{1}{2}x + 10 \\ \underline{-10 \qquad -10} \\ 6 = \frac{1}{2}x \end{array}$$

Step 2 The variable is multiplied by a fraction. To undo this operation, multiply both sides of the equation by the reciprocal of the fraction. The reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$, or 2.

$$2 \times 6 = 2 \times \frac{1}{2}x$$
$$12 = x$$

Step 3 Substitute the answer back into the original equation and check that the equation is true.

$$16 = \frac{1}{2}x + 10$$
$$16 = \frac{1}{2}(12) + 10$$
$$16 = 6 + 10$$
$$16 = 16 \checkmark$$

Simplifying Before Solving

Some equations may require you to simplify one or both sides of the equation before performing inverse operations. For example, you may need to use the Distributive Property.

Example 5: **Distributive Property**

Solve $5(r - 3) = 21$.

Step 1 Distribute 5 to each term in parentheses.

$$5(r - 3) = 21$$
$$5(r) - 5(3) = 21$$
$$5r - 15 = 21$$

Step 2 Add 15 to both sides of the equation.

$$\begin{array}{r} 5r - 15 = 21 \\ \underline{+15 \quad +15} \\ 5r = 36 \end{array}$$

Step 3 Divide both sides of the equation by 5.

$$\frac{5r}{5} = \frac{36}{5}$$
$$r = 7.2$$

Step 4 Substitute the answer back into the original equation and check that the equation is true.

$$5(r - 3) + 21$$
$$5(7.2 - 3) = 21$$
$$5(4.2) = 21$$
$$21 = 21 \checkmark$$

Example 6: **Combining Like Terms**

Solve $2x + 5 - 4x = 15$.

Step 1 Identify and combine like terms on the left side of the equation.

$$2x + 5 - 4x = 15$$
$$-2x + 5 = 15$$

Step 2 Subtract 5 from both sides of the equation.

$$\begin{array}{r} -2x + 5 = 15 \\ \underline{-5 \quad -5} \\ -2x = 10 \end{array}$$

Step 3 Divide both sides of the equation by -2.

$$\frac{-2x}{-2} = \frac{10}{-2}$$
$$x = -5$$

Step 4 Substitute the answer back into the original equation and check that the equation is true.

$$2x + 5 - 4x = 15$$
$$2(-5) + 5 - 4(-5) = 15$$
$$-10 + 5 - (-20) = 15$$
$$-5 + 20 = 15$$
$$15 = 15 \checkmark$$

Think about Math

Directions: Solve each equation.

1. $\frac{1}{3}x + 6 = 2$
2. $4(r + 2) = -16$
3. $5y - 2y + 1 = 13$

CORE SKILL

Solve Linear Equations

Some equations, like $5x + 4 = 3x - 2$, have variables on both sides of the equal sign. Just as with all other equations, the goal when solving these equations is to isolate the variable. In order to do this, you will need to first collect all of the variable terms on one side of the equation:

$$\begin{array}{r} 5x + 4 = 3x - 2 \\ \underline{-3x \qquad -3x} \end{array}$$ Subtract $3x$ from both sides.

$$2x + 4 = \quad -2$$ Combine like terms on the left side: $5x - 3x = 2x$.

Once you have collected the variable terms on one side of the equation, continue to solve the equation using inverse operations until the variable is isolated. Check your answer by substituting back into the original equation.

Finish the solution of the equation above. Then solve $10w - 10 = 5w + 15$.

Vocabulary Review

Directions: Write the missing term in the blank.

equation	**variable**	**solution of an equation**
reciprocal	**expression**	**inverse operation**

1. A mathematical phrase is a(n) ________________.
2. $\frac{2}{3}$ is the ________________ of $\frac{3}{2}$.
3. The ________________ of division is multiplication.
4. A(n) ________________ contains an equal sign.
5. A letter that is used to represent an unknown number is called a(n) ________________.
6. A ________________ that contains one variable is a value for the variable that makes the equation true.

Skill Review

Directions: Read each problem and complete the task.

1. Tell whether each item below is an expression or an equation.
   - $-3x + 94$
   - $12 = 9 + 3$
   - $-2 - (-10)$
   - 1.55
   - $2x - 3y = 16$
   - 8 more than y
   - The sum of a and -13 is b.
   - $-5n$

Directions: Write an equation to represent each verbal description.

2. Four less than twice a number is equal to 7.
3. Two more than 6 times a number is equal to 6.

Directions: Solve each equation.

4. $x + 3 = 7$
5. $-4x = 32$
6. $\frac{3}{4}x + 3 = 30$
7. $7x + 14 = 35$
8. Martin likes to run. He has been training to run a race next month. He is able to run 5 miles in 35 minutes. Assuming he can run as many miles as he wants at the same pace, write an equation that models the number of minutes, y, it takes Martin to run x miles. How long will it take him to run 8 miles?
9. Louis called a plumber to fix his broken sink. In addition to a $50 fee for the visit, the plumber charges $22 per hour to fix Louis's sink. Write an equation that models this situation and determine how many hours the plumber took if Louis's total bill was $116.
10. If $7x = -28$, what is the value of $x - 8$?
    A. -29
    B. -12
    C. 27
    D. 188
11. If $-3(n + 2) = 6$, what is the value of $12n$?
    A. -48
    B. -16
    C. 0
    D. 9

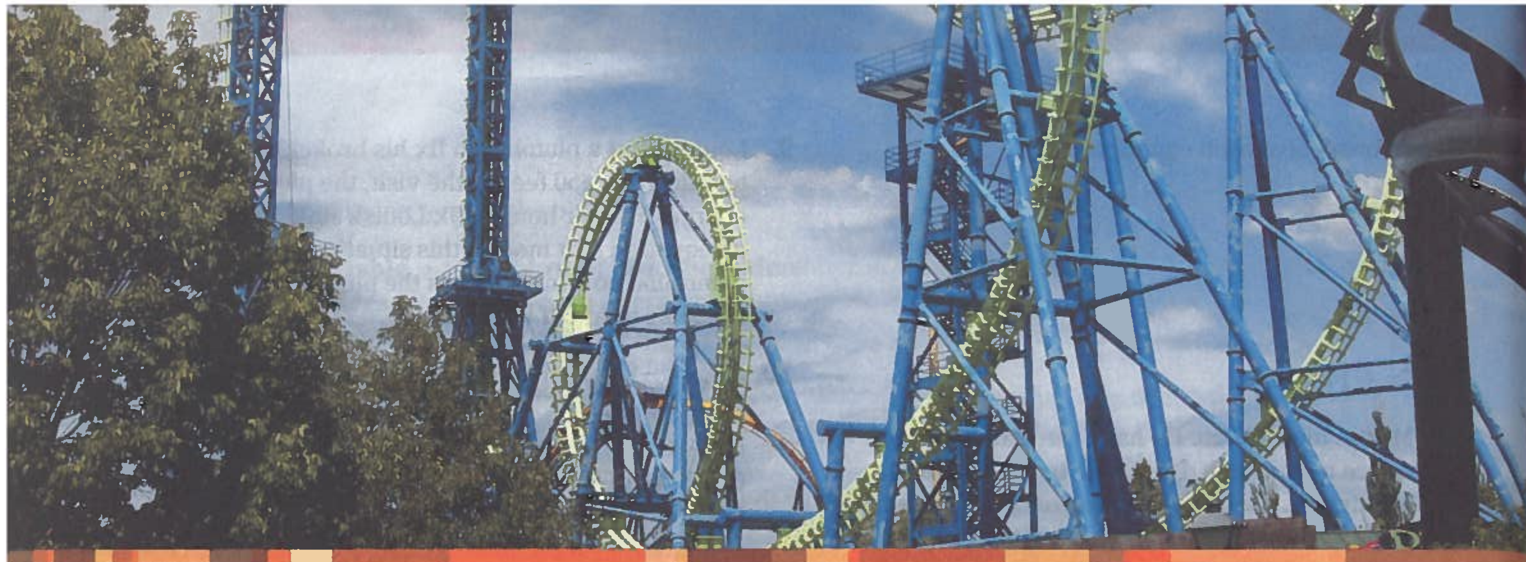
Skill Practice

Directions: Read each problem and complete the task.

1. The sum of a number and 4 is multiplied by -2 and the result is -6. What is the number?
   A. -7
   B. -1
   C. 1
   D. 2
2. When 10 is added to 3 times a number, the result is 100. Find the number.
   A. 8
   B. 30
   C. 36
   D. 270
3. Solve the equation $(3x - 1) + (-2x - 1) = 2$.
4. Silvia needs $2,100 for a vacation next summer. She plans to save $350 per month. The equation $350m = 2{,}100$ represents this situation, where m is the number of months Silvia saves. Solve the equation to determine the number of months it will take Silvia to save enough for her vacation.
5. Andrew had a gift card worth $10 to his favorite clothing store. He bought one shirt, and his total cost after using the gift card was $18.05, which included $2.55 in sales tax. The equation $s - 10 + 2.55 = 18.05$ represents this situation, where s is the original cost of the shirt. Solve the equation to find the original cost of the shirt.
6. Jermaine solved the equation $2r - 4 = -7$ as shown below. Identify Jermaine's error. What is the correct solution?

$$2r - 4 = -7$$
$$2r = -3$$
$$r = -6$$

7. In the equation $x - c = 100$, c is a positive number. Is the solution of the equation greater than 100 or less than 100? Explain your reasoning.



LESSON 3.3 Solve Linear Inequalities

LESSON OBJECTIVES

- Solve linear inequalities
- Represent solutions of linear inequalities on a number line

CORE SKILLS & PRACTICES

- Represent Real-World Problems
- Solve Inequalities

Key Terms

inequality
a mathematical statement showing that two quantities are not equal

inequality signs
symbols used to show the relationship between the expressions in an inequality ($<$, $>$, $\leq$, or $\geq$)

solution of an inequality
the numbers that, when substituted for the variable in an inequality, make the inequality statement true

Vocabulary

equation
a mathematical statement showing that two quantities are equal

inverse operations
operations that reverse the effect of other operations

variable
a symbol used to represent an unknown value

Key Concept

Solving linear inequalities is very similar to solving linear equations, except the solution to a linear inequality will include a range of values, called the solution set. The solution set can be graphed on a number line.

Inequalities

Roller coasters require riders to be a minimum height. Getting a B on the final exam requires a minimum percentage of questions answered correctly. Auditoriums have a maximum capacity. These situations can be described with inequalities because they require numerical values to fall within a certain range.

Inequalities and Signs

An **inequality** is a statement that two expressions are not equal. In an inequality one expression can be compared to another expression as greater, greater than or equal, less than, or less than or equal. **Inequality signs** are symbols used to show the relationship between the expressions in the inequality. You read an inequality from left to right as indicated in the table.

$x < y$	$x \leq y$	$x > y$	$x \geq y$
x is less than y	x is less than or equal to y	x is greater than y	x is greater than or equal to y

Solutions of Inequalities

The **solutions of an inequality,** also called the solution set, are the numbers that, when substituted for the variable in an inequality, make the inequality true. Recall that a **variable** is a symbol used to represent an unknown value.

Checking Solutions

We will review how to find the solution of an inequality in the next sections. For now, consider this inequality and the particular values of the variable. You can test if the values make the inequality true.

Chuck Eckert/Alamy

Consider the inequality $x + 1 > 2$ and its solution, $x > 1$ (x is greater than 1).

Substituting any number greater than 1 will make the inequality true.

$x = 2$	$x = 1.5$
$2 + 1 > 2$	$1.5 + 1 > 2$
$3 > 2$ ✓	$2.5 > 2$ ✓

Substituting any number that is less than 1 will make the inequality false.

$x = 0$	$x = -1$
$0 + 1 > 2$	$-1 + 1 > 2$
$1 > 2$ ✗	$0 > 2$ ✗

Graphing Solutions

The solution of an inequality that contains one variable can be graphed on a number line. For example, here n is a variable and c is a value on the number line. Notice the circles, or end points, are filled in when the value is part of the solution set.

Graphing Inequalities

$n < c$	$n > c$
c The value of n is less than c. Shaded arrow points left.	c The value of n is greater than c. Shaded arrow points right.
$n \leq c$	$n \geq c$
c The value of n is less than or equal to c. Shaded arrow points left and the circle is filled in.	c The value of n is greater than or equal to c. Shaded arrow points right and the circle is filled in.

Writing Inequalities

It is important to be able to write inequality statements from verbal descriptions. This is easier to do if you recognize that certain phrases indicate inequalities.

$<$	$\leq$	$>$	$\geq$
• less than • fewer than • smaller than	• less than or equal • no more than • at most	• greater than • more than • larger than	• greater than or equal • no less than • at least

When writing inequalities, look for phrases that indicate which symbol to use. Also, look for key words that indicate any operation between quantities.

"A number increased by 4 is greater than 10."
a number increased by 4 ⟶ $n + 4$
is greater than 10 ⟶ $n + 4 > 10$

"Two times a number is no more than that number plus 12."
two times a number ⟶ $2q$
is no more than ⟶ $2q \leq$
that number plus 12 ⟶ $2q \leq q + 12$

"Seven less than a number is at least 50."
seven less than a number ⟶ $r - 7$
is at least 50 ⟶ $r - 7 \geq 50$

CORE SKILL

Represent Real-World Problems

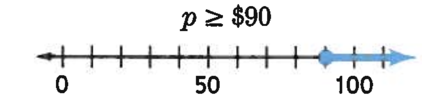
Inequalities can be used to represent many situations. Consider the following scenario. A department store credit card monthly payment must be at least 15% of the account balance. A shopper has charged a total of $600 to her credit card. Use a number line to show payment amounts that are at least 15% of the credit card balance.

To use a number line that represents the solution to a real-world problem, consider the situation and think of how the number line will look. Begin by calculating the payment amount.

- credit card balance is $600
- the payment must be at least 15% of $600
  ($600)(0.15) = $90

The solution will show a circle filled in at 90 with an arrow going to the right. This means the payment must be at least $90.

$p \geq \$90$



Now you try. Tasha has a monthly maximum spending budget of 20% of her take-home pay. Last month her take-home pay totaled $2,600. Graph the solution set for her spending budget.

Think about Math

Directions: Answer the following questions.

1. Write an inequality that represents this situation: "The thermostat is always set at a temperature that is below 75 degrees."
2. Is the number 10 in the solution set for $x - 20 \le -10$?

One-Step Inequalities

Major highways often have safety weigh stations for large trucks on the road. To pass inspection, the weight of the truck plus the weight of the cargo cannot exceed the maximum allowed weight. Because the sum of the weights must not exceed a certain number, then using the truck weight and cargo weights as variables, a linear inequality can be written.

Solve Using Addition and Subtraction

An **equation** is a mathematical statement showing that two quantities are equal. To solve an equation, **inverse operations** are applied to reverse the effect of other operations. When solving inequalities, inverse operations are performed in the same manner as when solving equations. Also, what you do to one side, you will have to do to the other. For example, if $a < b$ and c is any number, then $a + c < b + c$ and $a - c < b - c$. This means that you can add or subtract any number to both sides of the inequality and not change the inequality.

For example, you know that $3 < 7$, so the following are also true.

$$3 + 8 < 7 + 8 \qquad \text{or} \qquad 3 - 8 < 7 - 8$$
$$11 < 15 \qquad\qquad\qquad -5 < -1$$

Example 1: Inequalities Involving Addition or Subtraction

Solve these inequalities using inverse operations.

$y + 4 \ge 5$	$p - 8 < -4$

Step 1 Identify the operation being used on the variable.

$y + 4 \ge 5$ 4 is added to y	$p - 8 < -4$ 8 is subtracted from p

Step 2 Perform the inverse operation on both sides of the inequality.

Subtract 4 from both sides $y + 4 \ge 5$ $-4 \quad -4$ $y \ge 1$	Add 8 to both sides $p - 8 < -4$ $+8 \quad +8$ $p < 4$

Check: Test values that are in the solution set. Any number in the solution set will make the inequality true.

Substitute values greater than or equal to 1. $y + 4 \ge 5$ $1 + 4 \ge 5$ $5 \ge 5$ ✓	Substitute values less than 4. $p - 8 < -4$ $3 - 8 < -4$ $-5 < -4$ ✓
$2 + 4 \ge 5$ $6 \ge 5$ ✓	$0 - 8 < -4$ $-8 < -4$ ✓

Solve Using Multiplication and Division

Inequalities that contain multiplication or division are also solved using inverse operations. Again, what you do to one side, you will have to do to the other. For example, if $a < b$ and $c > 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$. This means that multiplication or division by a positive number does not change the inequality.

For example, you know that $2 < 9$ so the following are true.

$$(2)(5) < (9)(5) \qquad \text{or} \qquad \frac{2}{5} < \frac{9}{5}$$
$$10 < 45 \qquad\qquad\qquad 0.4 < 1.8$$

Example 2: Inequalities Involving Multiplication or Division

Step 1 Identify the operation being performed on the variable.

$9q \le 72$ q is multiplied by 9.	$\frac{b}{4} > 12$ b is divided by 4.

Step 2 Perform the inverse operation on both sides of the inequality.

Divide both sides by 9. $\frac{9q}{9} \le \frac{72}{9}$ $q \le 8$	Multiply both sides by 4. $(4)\frac{b}{4} > 12(4)$ $b > 48$

Check: Test values that are in the solution set. Any number in the solution set will make the inequality true.

Substitute values less than or equal to 8. $9q \le 72$ $9(8) \le 72$ $72 \le 72$ ✓	Substitute values greater than 48. $\frac{b}{4} > 12$ $\frac{50}{4} > 12$ $12.5 > 12$ ✓
$9(0) \le 72$ $0 \le 72$ ✓	$\frac{100}{4} > 12$ $25 > 12$ ✓

Solve Using Multiplication and Division with Negatives

When the inverse operation includes multiplying or dividing by a negative number, the inequality symbol must be reversed for the inequality to be true. For example, if c is negative, the direction of the inequality must be reversed when we multiply or divide both sides of the inequality by c. If $a < b$ and $c < 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$. The following illustrates this: $6 < 13$ and $-2 < 0$, so $-12 = -2(6) > -2(13) = -26$. Also, $-3 = \frac{6}{-2} > \frac{13}{-2} = -6.5$.

Example 3: Inequalities Involving Multiplication or Division and Negatives

Step 1 Identify the operation being performed on the variable.

$-10q \le 150$ q is multiplied by -10.	$\frac{b}{-2} > 7$ b is divided by -2.

Step 2 Perform the inverse operation on both sides of the inequality and reverse the direction of the inequality.

Divide both sides by -10. $-10q \le 150$ $\frac{-10q}{-10} \le \frac{150}{-10}$ $q \ge -15$ Inequality is reversed!	Multiply both sides by -2. $\frac{b}{-2} > 7$ $-2\left(\frac{b}{-2}\right) > -2(7)$ $b < -14$ Inequality is reversed!

CORE SKILL

Solve Inequalities

A student determined that the number 2 was a solution to the inequality $-2r \le -2$. Was the student correct? In order to answer the question, you must use inverse operations to solve the inequality.

Begin by dividing by -2. Note that since you are dividing by a negative number you will need to reverse the direction of the inequality sign.

$$\frac{-2r}{-2} \le \frac{-2}{-2}$$
$$r \ge 1$$

The answer is $r \ge 1$. The student was correct in saying that 2 was a solution to the inequality because 2 is greater than or equal to 1.

Now you try. Determine if -10 is a solution to the inequality $-10n > 5$.

CALCULATOR SKILL

Many scientific calculators do not have the inequality symbols. Instead, you have to change the inequality into an equation, replacing the inequality sign with an equal sign. You can then use your calculator to find the solution to the equation. Then, use your reasoning to decide which direction the inequality must face based on the operations used to solve the equation.

Use the TI-30XS MultiView™ calculator to solve the inequality $-1.5x + 6.725 \geq 4.25$. First, rewrite as $-1.5x + 6.725 = 4.25$. When you find the solution and need to write it as an inequality, note that you have divided by the negative number -1.5. What is the solution to the inequality?

Check: Test values in the solution set. Any number in the solution set will make the inequality true.

Substitute values greater than or equal to −15.	Substitute values less than −14.
$-10(-15) \leq 150$ $150 \leq 150$	$\frac{-20}{-2} > 7$ $10 > 7$
$-10(-10) \leq 150$ $100 \leq 150$	$\frac{-15}{-2} > 7$ $7.5 > 7$

Multi-Step Inequalities

Inequalities can model situations that involve personal budgets and money. Using them can help you manage your finances and bring you closer to reaching financial goals, such as buying a car. These types of inequalities might extend beyond the one-step process.

Two-Step Inequalities

When solving inequalities that contain more than one operation, you must isolate the variable on one side of the inequality to find the solution set. Isolate variables using inverse operations.

Example 4: Two-Step Inequality

Solve the inequality, $2x + 4 > 6$.

Step 1 Subtract 4 from both sides of the inequality.
$$\begin{aligned} 2x + 4 &> 6 \\ -4 \quad &\;\; -4 \end{aligned}$$

Step 2 Divide both sides of the inequality by 2.
$$\frac{2x}{2} > \frac{2}{2}$$
$$x > 1$$

Check: Test values that are in the solution set. Any number in the solution set will make the inequality true. Since the answer is x is greater than 1, check values greater than 1.

$2x + 4 > 6$ $2(1.1) + 4 > 6$ $2.2 + 4 > 6$ $6.2 > 6$	$2x + 4 > 6$ $2(4) + 4 > 6$ $8 + 4 > 6$ $12 > 6$

Inequalities in which the variable has a negative coefficient will require multiplication or division by a negative number in order to solve. This means the direction of the inequality symbol will have to be reversed.

Example 5: Inequalities with a Negative Fractional Coefficient

Solve the inequality, $10 - \frac{1}{2}n \leq 4$.

Step 1 Subtract 10 from both sides of the inequality.
$$\begin{aligned} 10 - \tfrac{1}{2}n &\leq 4 \\ -10 \quad &\;\; -10 \end{aligned}$$
$$-\frac{1}{2}n \leq -6$$

Step 2 Multiply both sides by the reciprocal, −2, and turn the symbol.
$$-2\left(-\frac{1}{2}\right)n \geq (-6)(-2)$$
$$n \geq 12$$

Check: Substitute values that fall in the solutions range. Again, any number in the solution set will make the inequality true. Since the answer is n is greater than or equal to 12, check values greater than or equal to 12.

$10 - \frac{1}{2}(12) \leq 4$ $10 - 6 \leq 4$ $4 \leq 4$	$10 - \frac{1}{2}(20) \leq 4$ $10 - 10 \leq 4$ $0 \leq 4$

Simplify Before Solving

Some inequalities need to be simplified on each side before solving.

Example 6: Distribution and Variables on Both Sides

Solve the inequality $5(g + 1) \leq 3(g + 2)$.

Step 1 Distribute the 5 on the left and the 3 on the right.
$$5(g + 1) \leq 3(g + 2)$$
$$5g + 5 \leq 3g + 6$$

Step 2 Subtract $3g$ from both sides.
$$\begin{aligned} -3g \quad &\;\; -3g \\ 2g + 5 &\leq 6 \end{aligned}$$

Step 3 Subtract 5 from both sides.
$$-5 \;\; -5$$

Step 4 Divide both sides by 2.
$$\frac{2g}{2} \leq \frac{1}{2}$$
$$g \leq \frac{1}{2}$$

Check: Test values that are in the solution set. Any number in the solution set will make the inequality true. In this case, we will check values less than or equal to $\frac{1}{2}$.

$5(g + 1) \leq 3(g + 2)$ $5(0 + 1) \leq 3(0 + 2)$ $5(1) \leq 3(2)$ $5 \leq 6$	$5(g + 1) \leq 3(g + 2)$ $5(0.5 + 1) \leq 3(0.5 + 2)$ $5(1.5) \leq 3(2.5)$ $7.5 \leq 7.5$

Think about Math

Directions: Solve the following inequalities.

1. $2b + 6 < -7$
2. $-\frac{2}{3}q - 3 \leq 6$
3. $8(q + 1) > 2(q - 2)$
4. $-3x < 3(5x + 2)$

21ST CENTURY SKILL

Financial Literacy

When buying a home, most people obtain loans called mortgages to help pay for the home. Lenders usually require the homebuyer to have a 20% down payment that goes toward the purchase of the property. Other factors that a lender will consider before giving a loan include credit history, job history, and total debt. Consider the following situation:

A young couple wishes to purchase a home. Both have good credit, a good job history, low debt, and are focused on saving money for a 20% down payment. They are budgeting to buy a home that costs $80,000. They are able to save $400 a month and already have $8,000 saved. How many months will it be until they have enough money saved for the down payment?

To solve, begin by calculating the down payment needed:

$(0.20)(80{,}000) = 16{,}000$
20% of $80,000 = $16,000

Use the inequality to answer the question:

m represents months

$$400m + 8{,}000 \geq 16{,}000$$

How many months will it take until the couple has enough money for the down payment?

Vocabulary Review

Directions: Write the missing term in the blank.

inequality	**solution of an inequality**	**inequality sign**
equation	**inverse operations**	**variable**

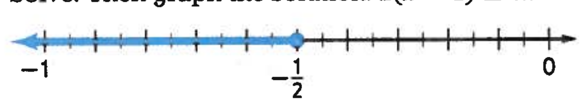
1. A mathematical statement showing that two quantities are equal is a(n) ________________.
2. A(n) ________________ is a symbol used to write an inequality.
3. ________________ are operations that reverse the effect of other operations.
4. A(n) ________________ is a number that, when substituted for the variable in an inequality, makes the inequality statement true.
5. A letter that represents an unknown quantity is a(n) ________________.
6. A(n) ________________ is a mathematical statement showing that two quantities are not equal.

Skill Review

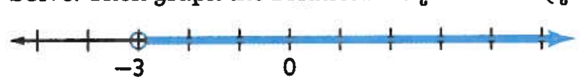
Directions: Read each problem and complete the task.

1. Translate the following: "A number increased by 4 is less than 6."
2. Translate the following: "Three times a number is at least that number plus 1."
3. Solve. Then graph the solution: $-x + 3 < 9$

4. Solve. Then graph the solution: $2(x - 1) \geq 6x$



5. Solve. Then graph the solution: $-3q + 1 < -2(q - 2)$



6. Is 0 a solution of $-x \geq 0$?
7. Is 10 a solution of $-2(x - 4) \leq -10$?
8. Is -2 a solution of $\frac{1}{2}x + 6 \geq 4$

Skill Practice

Directions: Read each problem and complete the task.

1. A major credit card company requires monthly payments equal to or greater than 8% of the total balance. A consumer has a credit card balance of \$320. Write an inequality that represents acceptable payment amounts.
2. Solve the inequality:
   $-3(n + 1) - 2(n + 4) > 6n$
3. Is 0.4 a solution to the inequality?
   $-\frac{1}{2}(x + 4) + 2 \leq 4x - 2$
4. Martin's summer allowance of \$400, and his spending per week of \$25 can be represented by the expression, $400 - 25w$. Martin needs to have at least \$175 at the end of the summer. Using the expression, write an inequality that can be solved to determine how many weeks Martin can spend money and still have \$175 at the end of the summer. Then solve the inequality.
5. Create a situation involving saving money that can be represented by the inequality:
   $2{,}000 + 30x \geq 9{,}000$.
6. Michael solved the inequality $5(r + 2) < -10r$ as shown below. Identify Michael's error. What is the correct solution?

$$5(r + 2) < -10r$$
$$5r + 10 < -10r$$
$$15r < -10$$
$$r > -\frac{2}{3}$$

7. To pass a state nursing exam, students must answer at least 70% of all the questions correctly. This year, the exam has 150 questions. Write an inequality that can be solved to determine all the possible numbers of incorrect answers a student can get and still pass the this year's exam. Then solve the inequality and state the answer in a complete sentence.
8. Write the correct symbols to complete each property of inequalities.
   a. If $a < b$ and c is a positive number, then ac ____ bc and $\frac{a}{c}$ ____ $\frac{b}{c}$.
   b. If $a < b$ and c is a negative number, then ac ____ bc and $\frac{a}{c}$ ____ $\frac{b}{c}$.
9. Solve the inequality $5x - 4 > -16x + 3$. Then describe how the graph of the solution should look on a number line.
10. Emily is conducting an experiment. She starts with a solution that has a temperature of 44°F. She lowers the temperature by 6°F each hour. The temperature of the solution cannot go below 20°F. Write an inequality that can be solved to determine the maximum number of hours Emily can lower the temperature. Then solve the inequality and state the answer in a complete sentence.
11. Why do you need to reverse the direction of an inequality symbol when you multiply or divide both sides by a negative number?
12. Jane solved the inequality $bx < 5b$ as shown below. Explain the error that Jane made.

$$bx < 5b$$
$$\frac{bx}{3b} < \frac{5b}{b}$$
$$x < 5$$

Digital Vision/Getty Images

LESSON 3.4 Use Expressions, Equations, and Inequalities to Solve Real-World Problems

LESSON OBJECTIVES

- Write algebraic expressions to represent real-world situations
- Solve real-world problems involving linear equations
- Write linear equations to represent real-world problems
- Solve real-world problems using inequalities

CORE SKILLS & PRACTICES

- Evaluate Expressions
- Solve Real-World Problems

Key Terms

algebraic expression
an expression that contains at least one variable

Vocabulary

equation
a mathematical statement showing that two quantities are equal

inverse operations
operations that undo each other

inequality
a mathematical statement showing that two quantities are not equal

Key Concept

Real-world problems can be translated into algebraic expressions, equations, and inequalities. Mathematical methods can then be used to find real-world solutions.

Expressions and Equations

Expressions and equations are used to model real-world problems all the time. For example, when painting a room, it is important to know how much paint to buy. If you purchase too much, you will waste money. If you don't purchase enough, you will waste time returning to the store for more. You can use expressions and equations to calculate the amount of paint you need.

Real-World Expressions

A real-world situation can be translated into an **algebraic expression**, an expression that contains at least one variable.

Example 1: Rental Car Charges

A car rental agency charges $29.99 plus $0.39 per mile to rent a compact car. A customer rented a compact car and drove 220 miles. What will be the total charge?

Step 1 Identify the variable quantity and assign a variable. — Let m = number of miles.

The number of miles driven will vary.

Step 2 Use the variable and the information given in the problem to write an expression for the total charge.

$29.99 plus $0.39 per mile

29.99 + 0.39 × m

$29.99 + 0.39m$

Step 3 To find the total charge for 220 miles, substitute 220 for m and simplify.

$29.99 + 0.39(220)$
$= 29.99 + 85.8$
$= 115.79$

The total charge for 220 miles is $115.79.

Example 2: Weekly Savings

Susan has saved $40 to purchase a new mountain bike. She plans to save an additional $15 each week. How much money will Susan have saved after 6 weeks?

Step 1 Identify the variable quantity and assign a variable. — Let w = number of weeks.

The number of weeks will vary.

Step 2 Use the variable and the information given in the problem to write an expression that represents the total amount saved.

$40 plus $15 each week

40 + 15 × w

$40 + 15w$

Step 3 To find the amount saved after 6 weeks, substitute 6 for w and simplify.

$40 + 15(6)$
$= 40 + 90$
$= 130$

After 6 weeks, Susan will have saved $130.

Real-World Equations

An equation is a mathematical statement that two expressions are equal to each other. For example, $2 + 3 = 4 + 1$ and $3x + 3 = 12$ are equations. Many real-world problems can be represented and solved using equations.

Example 3: Buying a Truck

Roger wants a new truck that costs $15,999 plus an additional $1,600 for taxes, title, and registration fees. Roger has saved $6,000 and plans to borrow the rest of the money he needs. How much money does Roger need to borrow?

Step 1 Identify the unknown quantity and assign a variable. The amount that Roger must borrow is unknown.

Let b = amount Roger must borrow.

Step 2 Use the variable and the information given in the problem to write an equation.

Amount saved plus amount borrowed equals cost of truck plus fees

6,000 + b = 15,999 + 1,600

Step 3 Use inverse operations to solve the equation. Inverse operations are operations that undo each other. In this case, subtract 6,000 from both sides of the equation.

$6{,}000 + b = 17{,}599$

Subtract 6,000 from both sides: $-6{,}000 \quad -6{,}000$

$b = 11{,}599$

Roger must borrow $11,599.

CORE SKILL

Evaluate Expressions

Gilbert works at a restaurant and gets paid $50 per week plus $9 per hour. The expression $50w + 9h$ represents Gilbert's pay for working h hours in w weeks. This week he is scheduled to work 35 hours. Evaluate the expression to determine Gilbert's pay this week.

When you evaluate an expression, you are finding a value. Substitute given numbers for the variables in the expression and then use the order of operations to simplify.

Expression:	$50w + 9h$
Substitute 1 for w and 35 for h:	$50(1) + 9(35)$
Multiplication first:	$50 + 315$
Addition:	365

Gilbert's pay this week will be $365.

Now you try. Jackie works at a convenience store. She earns $11 per hour, but each week $5 is deducted from her paycheck for uniform cleaning services. Write an expression that represents Jackie's pay for working h hours in w weeks. How much will Jackie earn if she works 40 hours during one week? How much will she earn if she works 40 hours during 2 weeks?

21ST CENTURY SKILL

Financial, Economic, Business, and Entrepreneurial Literacy

When you borrow money from a bank, you must pay back the amount borrowed plus an additional percentage. When you invest money, you will be paid back the amount invested plus an additional percentage. In both cases, the additional percentage is called interest. There are many methods of computing interest, and different methods result in different amounts of interest paid or earned. Recall that one way to compute interest is to use the simple interest formula.

$$I = Prt$$

I = amount of interest earned or charged

P = initial amount borrowed or invested (called the principal)

r = annual interest rate (as a decimal)

t = time that money is borrowed or invested (in years)

Keely paid 6.5% annual simple interest on a loan for 5 years. She paid a total of $308.75 in interest. Using the simple interest formula, write and solve an equation to find the amount of money that Keely borrowed.

Example 4: Prepaid Cell Phone

Cathy has a prepaid cell phone. Last month she deposited $55.00 into her cell phone account and was able to talk for 700 minutes before running out of credit. What is the charge per minute on Cathy's cell phone? Round to the nearest cent and assume no other fees apply.

Step 1 Identify the unknown quantity and assign a variable. The charge per minute is unknown.

Let m = charge per minute.

Step 2 Use the variable and the information given in the problem to write an equation.

Total minutes times charge per minute equals total cost

$$700 \times m = 55$$

Step 3 Use inverse operations to solve the equation. Round to the nearest cent.

$$700m = 55$$

Divide both sides by 700: $\frac{700m}{700} = \frac{55}{700}$

$$m \approx 0.08$$

The charge per minute is about $0.08.

Equations with Multiple Operations

For some real-world problems, you will need to write and solve an equation with more than one operation.

Example 5: Gym Membership

One year ago, Gloria joined a gym. She paid a $50 enrollment fee as well as a monthly membership fee. Her total gym expenses for the year were $410. What was Gloria's monthly membership fee?

Step 1 Identify the unknown quantity and assign a variable. The monthly membership fee is unknown.

Let m = monthly membership fee.

Step 2 Use the variable and the information given in the problem to write an equation.

Enrollment fee plus 12 times monthly fee equals total expenses

$$50 + 12 \times m = 410$$

Step 3 Use inverse operations to solve the equation.

$$50 + 12m = 410$$

Subtract 50 from both sides: $-50 \quad -50$

$$12m = 360$$

Divide both sides by 12: $\frac{12m}{12} = \frac{360}{12}$

$$m = 30$$

The monthly fee was $30.

Think about Math

Directions: Solve the following problems.

1. Roselda runs x miles three times per week. Which expression represents the number of miles that Roselda runs in 4 weeks?
   A. $3x + 4$
   B. $4x + 3$
   C. $4(3x)$
   D. $4(3) + x$

2. At a local bakery, a cupcake costs $1.25, and a cake costs $14.50. Lori bought two cakes and some cupcakes for a party. She paid $54.00 in total. How many cupcakes did Lori buy?
   A. 20
   B. 30
   C. 45
   D. 65

Inequalities

A credit card limit is the maximum amount that can be charged to a credit card. Once the limit is reached, your credit card will be denied for purchases. Inequalities can be used to determine how much you can charge to your card without exceeding the limit.

Real-World Inequalities

An **inequality** shows the relationship between two expressions that are not equal. In an inequality, one expression will be greater than, less than, greater than or equal to, or less than or equal to the other expression. The inequality symbols >, <, ≥, and ≤ are used to represent these relationships.

Inequalities

<	≤	>	≥
• less than • fewer than • smaller than • below	• less than or equal to • no more than • at most • maximum	• greater than • more than • larger than • above	• greater than or equal to • no less than • at least • minimum

Example 6: Real-World Inequalities

Write an inequality to represent each situation.

a. Drivers may drive at a speed s no greater than the posted speed limit of 70 miles per hour.

$$s \le 70$$

Speed Not greater than

b. To enlist in the United States military, a person's age a must be at least 17 years (with parental approval).

$$a \ge 17$$

Age At least

CORE SKILL

Solve Real-World Problems

To solve a real-world problem, translate the given information into numbers and mathematical symbols. When writing and solving an inequality, compare one quantity to another using inequality signs and assign variables for unknown values.

On most major highways in the United States, the weight limit for an 18-wheeler truck is 80,000 pounds. This includes the weight of the truck, the trailer, and the cargo. Trucks must stop at weigh stations located along the highway and, if a truck exceeds the weight limit, the driver can be fined and prevented from continuing his or her trip.

An 18-wheeler truck weighs 17,000 pounds and is pulling a trailer that weighs 11,000 pounds. Write and solve an inequality to find the allowable cargo weights c that this truck and trailer can carry.

Example 7: Free Delivery

A local furniture store offers free delivery of items if the total purchase amount is at least $500 before taxes. Kara went to the store and found a couch for $399. Kara is also looking for an end table to place next to the couch. How much does the end table need to cost in order for Kara to receive free delivery?

Step 1 Identify the unknown quantity and assign a variable. The cost of the end table is unknown.

Let t = cost of table

Step 2 Use the variable and the information given in the problem to write an inequality.

Cost of couch plus cost of end table must be at least $500

$$399 \quad + \quad t \quad \geq \quad 500$$

Step 3 Use inverse operations to solve the inequality.

$$399 + t \geq 500$$

Subtract 399 from both sides: $-399 \quad -399$

$$t \geq 101$$

The cost of the end table must be at least $101.

Inequalities with Multiple Operations

For some real-world problems, you will need to write and solve an inequality with more than one operation.

Example 8: Plumbing Repairs

The Wongs have budgeted $570 for some plumbing repairs. A plumber charges a $75 service fee plus $45 per hour. For how many hours can the Wongs afford to hire the plumber and stay within their budget?

Step 1 Identify the unknown quantity and assign a variable. The number of hours the Wongs can afford is unknown.

Let h = the number of hours.

Step 2 Use the variable and the information given in the problem to write an inequality.

Service fee plus $45 times number of hours cannot be more than $570

$$75 \quad + \quad 45 \quad \times \quad h \quad \leq \quad 570$$

Step 3 Use inverse operations to solve the inequality.

$$75 + 45h \leq 570$$

Subtract 75 from both sides: $-75 \quad -75$

$$45h \leq 495$$

Divide both sides by 45: $\frac{45h}{45} \leq \frac{495}{45}$

$$h \leq 11$$

The Wongs can afford to hire the plumber for no more than 11 hours.

Inequalities with Negative Numbers

Remember that when you multiply or divide both sides of an inequality by a negative number, you must reverse the inequality symbol.

Example 9: Spending Money

John is at band camp for 8 weeks. His parents have given him $600 for spending money. John wants to have at least $320 left at the end of camp so he can buy a trumpet. How much money can John spend each week and still be able to buy the trumpet at the end of camp?

Step 1 Identify the unknown quantity and assign a variable. The amount of money that John can spend each week is unknown.

Let w = amount of money John can spend per week.

Step 2 Use the variable and the information given in the problem to write an inequality.

Starting amount minus 8 times amount per week must be at least $320

$$600 \quad - \quad 8 \quad \times \quad w \quad \geq \quad 320$$

$$600 - 8w \geq 320$$

Step 3 Use inverse operations to solve the inequality.

$$600 - 8w \geq 320$$

Subtract 600 from both sides: $-600 \quad -600$

$$-8w \geq -280$$

Divide both sides by −8. $\frac{-8w}{-8} \geq \frac{-280}{-8}$

Reverse the inequality symbol: $w \leq 35$

John can spend no more than $35 each week.

Think about Math

Directions: Solve the following problems.

1. R.J. has $40 to spend at a carnival. The admission price is $7, and he plans to spend $15 on souvenirs. He also plans to spend $10 to eat dinner at the carnival. Each carnival ride cost $3. Which inequality can be used to determine the number of rides r that R.J. will be able to enjoy?
   A. $7 + 15 + 10 + 3 \geq 40r$
   B. $7 + 15 + 10 + 3r \geq 40$
   C. $7 + 15 + 10 + 3r \leq 40$
   D. $7 + 15 + 10 + 3 < 40r$

2. A cleaning service company charges $15 per hour plus a traveling fee of $20 per month. Gina has $100 per month budgeted for cleaning services. The company rounds the number of hours up to the next whole hour when calculating charges. What is the greatest number of hours that Gina can afford to hire the cleaning company each month?
   A. 4
   B. 5
   C. 6
   D. 8

Vocabulary Review

Directions: Write the missing term in the blank.

algebraic expression **inverse operations**
equation **inequality**

1. To solve an equation, you must isolate the variable using ________________.
2. An example of an ________________ is $5p - 1$.
3. If a mathematical statement shows that two expressions are equal to each other, the statement is an ________________.
4. If a mathematical statement shows one expression to be greater than another, the statement is an ________________.

Skill Review

Directions: Read each problem and complete the task.

1. A book club charges $10 per book plus a $5 shipping and handling fee per order.
   a. Write an expression that represents the total cost of b books.
   b. What is the total cost of 8 books?
   c. Sally placed an order and paid $35. How many books did she order?

2. In order to save money, Yvette clips coupons each week. She finds a 20% off coupon for her favorite detergent. Let d represent the detergent's usual price. Which expression represents the amount Yvette will pay if she uses the coupon?
   A. $0.20d$
   B. $0.20d - d$
   C. $d - 0.20d$
   D. $d + 0.20d$

3. Shane has $125 in his savings account and is saving $25 each week from his paycheck. How long will it take Shane to save $400?

Directions: For Questions 4–6, write an inequality to represent each situation.

4. The minimum height h for a carnival ride is 3 feet.

5. The maximum amount s that may be charged on a credit card is $2,000.

6. Andrea, a real estate agent, wants to sell h houses this year. She would like to sell more houses this year than she sold last year. Last year, Andrea sold 16 houses.

7. Abdul borrowed $10,000 at a simple interest rate of 5% per year and paid a total of $2,500 in interest. Use the simple interest formula $I = prt$ to determine how long Abdul borrowed the money.

Skill Practice

Directions: Read each problem and complete the task.

1. Gladys is paid $4 per hour plus tips at a local restaurant. On average, Gladys earns 15% of her total sales in tips. Last Friday Gladys had total sales of $1,200 during a 6-hour work shift. How much money did Gladys earn last Friday?
   A. $204
   B. $260
   C. $720
   D. $1,224

2. Lacey begins each week with an $84 allowance for meals. So far this week, she has spent $12 per day and she has $48 left. How many days have passed since the beginning of this week?
   A. 3
   B. 4
   C. 5
   D. 7

3. Kaley has been away at college for two months and will remain there until the end of the semester. Her parents gave her $1,200 in spending money for the entire 5-month semester, but Kaley wants to save at least $100 for her summer break. She has already spent $250 each month that she has been at college. Write and solve an inequality to determine how much money Kaley can spend per month for the rest of the semester and still have at least $100 remaining at the end.

4. Cassie received a paycheck for $620 for the time that she worked last week. This amount equals her total earnings e minus 20% in deductions for taxes and benefits. Which equation represents this situation?
   A. $0.20e = 620$
   B. $e - 0.20e = 620$
   C. $(0.20)(620) = e$
   D. $e + 0.20e = 620$

5. Cal invested $10,000 in an account earning simple interest. Three years later, the total amount in his account was $11,200. Cal wrote and solved the equation $11,200 = 10,000 \times 3 \times r$ and said that the interest rate was $37\frac{1}{3}$%. Describe Cal's error. What is the correct interest rate as a percent?

6. Emilio has a board that is 52 inches long. He needs to cut the board into two pieces so that one piece is 8 inches longer than the other. Write and solve an equation to determine the length of the shorter board.

Directions: Choose the best answer to each question.

1. Allen uses an expression to represent how much a bag of apples cost with a coupon. The expression $5x - 8$ can be translated using a phrase such as ____________.

2. Abby uses an equation to represent how much she owed after a discount using this equation: $\frac{x}{10} = -12$. Which is the value of x?
   A. 120
   B. 22
   C. −2
   D. −120

3. Olivia saves \$15 each week and has \$20 in an account. She is saving for a bicycle that costs \$110. How many weeks will it take for her to have enough money for the bicycle?
   A. 145
   B. 75
   C. 8
   D. 6

4. To participate in a music class, a student needs to be older than 8 years old. This graph represents the inequality ____________.

5. This expression represents how many toys a company will make in a number of hours: $25x - 6$. So, ____________ toys will be made in 8 hours.

6. A custom jewelry manufacturer uses this inequality to help them decide how many new orders to make each month: $45 - \frac{3}{4}x \le 18$. Which represents the values for x?
   A. $x \le 36$
   B. $x \le 84$
   C. $x > 84$
   D. $x \ge 36$

7. A photographer charges \$75 for a family portrait session. It cost \$300 for her camera. Which expression represents the amount of money she will make?
   A. $300 + 75x$
   B. $75x - 300$
   C. $(75 + 300)x$
   D. $300x + 75$

8. Ryan uses the expression $15x + 50$ to represent how much he will earn in a month. Zoe uses the expression $20(x - 2)$ to represent how much she will earn in a month. In both expressions x represents the number of hours worked. How much more does Zoe earn in a month than Ryan?
   A. $5x - 90$
   B. $5x - 10$
   C. $35x + 10$
   D. $35x - 90$

9. The equation $\frac{85}{x} = 17$ can be translated using a phrase such as ____________.

10. Conner uses the equation $45 = \frac{3}{4}x + 15$ to represent how many songs he needs to sell to make \$45. What is the value of x?
    A. 80
    B. 60
    C. 45
    D. 40

11. A cab ride costs \$0.20 per mile and has a \$3.00 fee. Conner has \$15.00 to spend on the cab ride. Conner uses the equation ____________ to find how many miles he can ride and stay within his budget.

12. Use the inequality $8(4g + 2) > 24g$. Which represents the values for g?
    A. $g < -2$
    B. $g > -2$
    C. $g \le 2$
    D. $g \ge 2$

13. Brooklyn uses the expression $30x - 100$ to represent how much money she'll make in a week. Dylan uses the expression $18x + 50$ to represent how much money he'll make in a week. For both expressions, x represents the number of hours worked. How much do Brooklyn and Dylan earn in a week?
    A. $12x - 150$
    B. $48x - 50$
    C. $12x + 150$
    D. $48x + 50$

14. Use the equation $6(x - 8) = -24$. What is the value of x?
    A. −4
    B. −12
    C. 4
    D. 22

15. Use the inequality $15 + x \ge 32$. Which represents the values for x?
    A. $x \ge 17$
    B. $x \le 17$
    C. $x \le 47$
    D. $x \ge 47$

16. To pass a test, Amber needs to have a score of 70 points or better. Each question is worth 10 points. She uses the inequality ____________ to find the possible number of questions she can get right to pass the test. She found that she needs to answer 7 or more questions correctly.

Check Your Understanding

On the following chart, circle the items you missed. The last column shows pages you can review to study the content covered in the question. Review those lessons in which you missed half or more of the questions.

Lesson	Item Number(s)			Review Page(s)
	Procedural	Conceptual	Problem Solving	
3.1 Evaluate Linear Expressions	5	1, 7		82–89
3.2 Solve Linear Equations	2, 14	9	3	90–97
3.3 Solve Linear Inequalities	6, 12	4	16	98–105
3.4 Use Expressions, Equations, and Inequalities to Solve Real-World Problems	10	11	8, 13	106–113