Chapter 2

Ratio, Proportion, and Probability

To attract new customers and to get more business, stores will often advertise special sales. They may use newspapers, billboards, commercials, and other media outlets to get the word out about their sale. Discounts may be described as a percentage such as "75% off original prices." Stores may also use fractions such as "half-off." Understanding how to calculate using ratios and percentages will help you be a more informed shopper and will let you know how much you can really save by buying items during sales.

Lesson 2.1

Apply Ratios and Proportions

Ratios compare two different values, such as distance and time. The speed limit sign you see on the highway is an example of a ratio of number of miles per hour. Learn how to simplify ratios and set up proportions to solve real-world problems.

Lesson 2.2

Calculate Real-World Percentages

Percentages describe values as parts of a whole. Thinking about your household budget and finances in terms of percentages can help you manage how you spend your money. Categories with a larger percentage represent your biggest area of expenses and help you budget accordingly. Learn how to solve problems using percentages and convert between ratios and percentages.

Lesson 2.3

Use Counting Techniques

If you wanted to know how many eggs you had left in the fridge, you could quickly count. What happens when you have to count larger numbers of items? How would you count in such scenarios without individually counting each item? Learn how to use counting techniques to quickly calculate the number of items in a set.

Lesson 2.4

Determine Probability

How do you determine the likelihood of something happening? Probability tells you how likely it is that something will happen and is usually described as a ratio or percent. When you listen to the weather forecast and hear the chance of rain reported as a percent, that is a probability. Learn how to calculate probabilities and apply counting techniques to solve probability problems.

Goal Setting

Understanding ratios, proportions, and percentages will help you solve probability problems, which in turn will tell you how likely something is to happen. Think of situations where you encounter probability in your everyday life. Words like *chance* and *likelihood* are good indications that you are dealing with probability. For each situation, list how the probability is indicated—as a percentage, a fraction, or some other way. Based on your list, what is the most common way of indicating probability?

In what situations would a percentage probability make more sense than a fractional probability? How could you use the lessons in this chapter to help you solve problems involving probability?

LESSON 2.1 Apply Ratios and Proportions

LESSON OBJECTIVES

- Compute unit rates
- Use scale factors
- Apply ratios and proportions to solve real-world problems

CORE SKILLS & PRACTICES

- Compare Unit Rates
- Use Ratio Reasoning

Key Terms

proportion
an equation stating that two ratios are equal

ratio
a comparison of two values

scale factor
a ratio of corresponding parts of similar figures

unit rate
a ratio that compares a quantity to a single unit

Vocabulary

equivalent
equal; having the same value

similar
having the same shape, but not necessarily the same size

Key Concept

A ratio, which is often written as a fraction, is a comparison of the relative sizes of two numbers. Operations on ratios follow the same rules as operations on fractions. When two ratios are equivalent, they are called proportional.

Ratios

How do moviemakers figure out what movies audiences want to watch? How do coffee shops know which coffee roasts to sell? Many businesses conduct surveys, or polls, of people's preferences. They use computer software to translate the results into ratios, such as 3 out of 4 people prefer nonfat milk to full-fat milk in their coffee. This information helps businesses determine what products to develop or sell.

Pixtal/AGE Fotostock

Write Ratios

A **ratio** is a comparison of the relative value of two numbers. For example, the owner of a popular coffee shop surveyed 140 customers and learned that 70 customers preferred blueberry muffins to all other choices. The ratio of customers who preferred blueberry to all customers surveyed is 70 to 140.

There are three ways to write a ratio:

70 to 140 70:140 $\frac{70}{140}$

Equivalent ratios are ratios that have the same value. The ratio $\frac{70}{140}$ can be simplified to the equivalent ratio $\frac{1}{2}$, which means that one out of every two customers surveyed preferred blueberry muffins.

Unit Rates

A **unit rate** is a ratio that compares an amount or quantity to one unit, such as a unit of weight, time, or distance. In other words, the denominator of a unit rate is always 1.

$2.00 per pound	12 gallons per minute	85 miles per hour	$0.89 per foot
$\frac{2\ dollars}{1\ pound}$	$\frac{12\ gallons}{1\ minute}$	$\frac{85\ miles}{1\ hour}$	$\frac{0.89\ dollars}{1\ foot}$

You can use division to convert any ratio into a unit rate. For example, suppose a dozen doughnuts costs $3.60, or $3.60 per 12 doughnuts. Divide the numerator by the denominator 12, the number of items in a dozen, to calculate the unit rate, or cost per doughnut.

$$\frac{3.60\ dollars}{12\ doughnuts} = 3.60 \div 12 = 0.3$$

One doughnut costs $0.30.

Think about Math

Directions: Answer the following questions.

1. A 2-lb bag of lentils costs $3.70. What is the unit cost per pound?
2. A 10-oz can of corn costs $1.88. What is the unit cost per ounce?
3. A box of 144 pencils costs $5.76. What is the unit cost per pencil?

Proportions

When painting a portrait, an artist often begins by sketching the face. All human faces have the same basic proportions. Each face can be divided into thirds, with hair and eyebrows in the top third; the eyes, nose, and ears in the middle third; and the mouth in the bottom third. Once the basic proportions are sketched, the artist can add the details that make each face unique.

CORE SKILL

Compute Unit Rates

Unit rates are helpful when determining the best price on an item. For example, when a bakery is buying flour, they can calculate the unit rates for bags of different sizes to determine the best price. A large bag will likely cost more than a small bag, but the unit cost per pound may be less for the large bag, giving the bakery more flour for their money. Suppose a 25-lb bag of flour costs $25.00, a 50-lb bag of flour costs $40, and a 5-lb bag of flour costs $10. Which bag has the least unit cost per pound?

CORE SKILL

Use Ratio Reasoning

Many problems can be solved by identifying key information in the problem that can be represented as ratios and then using your knowledge of ratios to solve.

An art teacher is buying classroom supplies. Twelve bottles of paint cost $38.28. How much will 36 bottles of paint cost?

$$\frac{12}{\$38.28} = \frac{36}{x}$$

$$\$38.28 \times 36 = 12x$$

$$1378.08 = 12x$$

$$x = \$114.84$$

36 bottles of paint will cost $114.84.

How much will 50 bottles of paint cost?

Testing for a Proportion

A **proportion** is a mathematical statement that two ratios are equivalent: $\frac{a}{b} = \frac{c}{d}$.

Example 1

Determine whether the ratios $\frac{4}{6}$ and $\frac{2}{3}$ form a proportion.

Step 1 Multiply the numerator of the first ratio by the denominator of the second ratio.

$\frac{4}{6} \stackrel{?}{=} \frac{2}{3}$ $\quad 4 \times 3 = 12$

Step 2 Multiply the numerator of the second ratio by the denominator of the first ratio.

$\frac{4}{6} \stackrel{?}{=} \frac{2}{3}$ $\quad 2 \times 6 = 12$

Step 3 If the two products are equal, the ratios form a proportion. Otherwise, the ratios do not form a proportion. These two ratios form a proportion.

Use Proportions to Solve Problems

Understanding the cross-multiplication strategy for determining proportionality makes it possible to solve problems in which you must determine an unknown value.

Example 2

Suppose you can buy 8 paintbrushes for $10.00. How many paintbrushes can you buy for $25.00?

Step 1 Write a proportion representing the situation.

$$\frac{8}{10} = \frac{p}{25}$$

Step 2 Cross-multiply and set the two products equal to each other.

$$8 \times 25 = 10p$$

Step 3 Solve the equation for p.

$$200 = 10p$$

$$\frac{200}{10} = \frac{10p}{10}$$

$$20 = p$$

You can buy 20 paintbrushes for $25.00.

Think about Math

Directions: Use the cross-multiplication strategy to answer the following questions.

1. A small auto company produces 220 vehicles in a 5-day workweek. How many cars will they produce in 15 days?
   A. 1,100
   B. 660
   C. 73
   D. 44

2. A car company pays $112.40 for 8 door handles. How much will they pay for 24 door handles?
   A. $2697.60
   B. $899.20
   C. $337.20
   D. $37.46

Scale

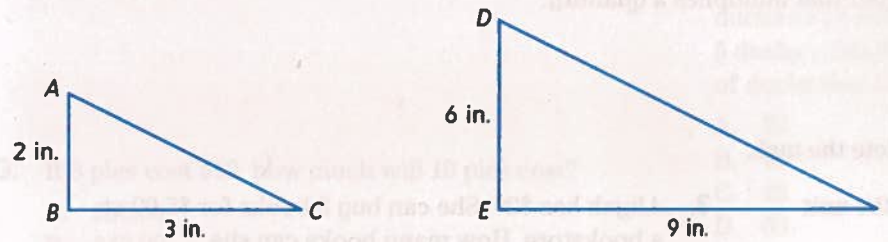
A scale model is a proportional copy of a real object. The model has the same shape as the original object, but is usually a different size. Engineers use scale models to test the performance of a new design without having to create an expensive full-sized prototype. Scale models are also used by architects to show the look of a new skyscraper and how it will fit in with the rest of a city's skyline.

Similarity and Scale Factor

Similar figures have the same shape but may have different sizes. You can use proportions to determine whether two figures or objects are similar.

Example 3

Determine whether ΔABC and ΔDEF are similar. If so, determine the scale factor from ΔABC to ΔDEF.



Step 1 Use the measures of two corresponding sides to write ratios.

$\frac{AB}{BC} \stackrel{?}{=} \frac{DE}{EF}$

$\frac{2}{3} \stackrel{?}{=} \frac{6}{9}$

Step 2 Perform cross multiplication to determine whether the ratios form a proportion.

$2(9) \stackrel{?}{=} 6(30)$

$18 = 18$

The ratios form a proportion, so the triangles are similar.

Step 3 The difference in size between similar figures is determined by a scale factor, a number which scales, or multiplies, some quantity. The dimensions of ΔDEF are 3 times the dimensions of ΔABC, so the scale factor from ΔABC to ΔDEF is 3.

Think about Math

Directions: Answer the following questions.

1. A photography studio enlarged a photograph that was 8 inches wide and 9 inches long to a similar photograph that was 36 inches long. How wide was the enlarged photograph?
2. What scale factor did the photography studio use to enlarge the photograph?

21ST CENTURY SKILL

Civic Literacy

Urban planners often rely on scale models of skyscrapers to examine existing structures and to design new ones. To use the scale of a building or map, write a proportion. Be sure to keep units in mind. For example, to solve the following problem, you need to convert inches to feet.

A model of a skyscraper is built at a scale of 1:800 and the model's height is 14.4 inches. What is the height of the actual skyscraper in feet?

Vocabulary Review

Directions: Write the missing term in the blank.

unit rate	**equivalent**	**scale factor**
proportion	**ratio**	**similar**

1. A(n) _____ is a comparison of two numbers.
2. Two figures or objects that have the same shape but may have different sizes are _____.
3. A(n) _____ compares a quantity to a single unit.
4. Two equivalent ratios form a(n) _____.
5. Two ratios that are _____ have the same value.
6. A(n) _____ is a number that multiplies a quantity.

Skill Review

Directions: Read each problem and complete the task.

1. A 3-lb bag of rice costs $3.87. What is the unit cost per pound?
   A. $1.19
   B. $1.29
   C. $1.33
   D. $11.61

2. A company is packaging bags of sugar for sale. The unit rate for a pound of sugar is $0.74/lb. What will be the cost of a 5-lb bag of sugar?
   A. $0.15
   B. $1.48
   C. $3.70
   D. $5.74

3. Aliyah has $20. She can buy 3 books for $5.00 at a bookstore. How many books can she buy with her $20?

4. A model airplane has a scale factor of 1:48. The wingspan of the model is 8 inches. What is the wingspan of the actual airplane?
   A. 6 ft
   B. 32 ft
   C. 48 ft
   D. 384 ft

Skill Practice

Directions: Read each problem and complete the task.

1. Explain the difference between a ratio and a proportion. Use examples in your explanation.

2. A school can purchase paper in boxes of 5 reams per box. Each box costs $11.20. How much will the school pay for 20 reams of paper?
   A. $6.20
   B. $44.80
   C. $56.00
   D. $224.00

3. If 3 pies cost $12, how much will 10 pies cost?
   A. $30.00
   B. $40.00
   C. $48.00
   D. $60.00

4. Ginnie has two rectangular vegetable gardens that are similar to one another. The smaller garden has a length of 3 feet and a width of 2 feet. The larger garden has a length of 15 feet. What is the width of the larger garden?

5. Samantha is comparing two boxes of cereal. One box contains 12 ounces of cereal and costs $2.88. The other box contains 16 ounces of cereal and costs $3.52. Which box of cereal is the better value? Compare unit rates to explain.

6. An architectural firm is creating a scale model of a new building. The actual building will be 125 feet tall, and the model will have a scale of 1:400 inches. How tall should the model be?
   A. 0.3125 inches
   B. 3.13 inches
   C. 3.75 inches
   D. 4.5 inches

7. Derek has $24. He wants to buy as many plastic ducks as possible for a carnival. A package of 5 ducks costs $2.50. What is the greatest number of ducks that Derek can buy?
   A. 29
   B. 45
   C. 48
   D. 60

8. The length of a rectangular swimming pool is 18 feet. The width is half the length. A company is building a similar pool with a width of 4.5 feet. What is the length of the similar pool? What scale factor did the company use to determine the dimensions of the similar pool?

LESSON 2.2 Calculate Real-World Percentages

LESSON OBJECTIVES

- Relate fractions, decimals, and percents
- Compute percent change
- Find a discount
- Calculate simple interest
- Use percent to solve real-world problems

CORE SKILLS & PRACTICES

- Use Tools Strategically
- Use Percent

Key Terms

discount
a decrease or reduction in price

percent
a ratio of a number to 100

simple interest
a charge paid on an original principal

Vocabulary

benchmark
a point of reference from which other measurements or estimates can be made

interest rate
the amount that is earned or charged during a certain amount of time

principal
an amount of money invested or borrowed

Key Concept

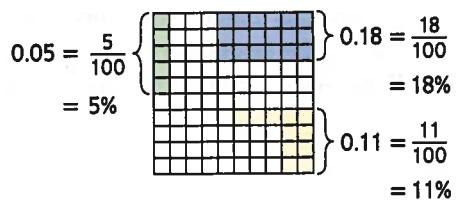
A percent is a ratio of a number to 100. In fact, the word *percent* comes from the Latin term *per centum*, meaning "by the hundred," and it is represented by the symbol %. Fractions and decimals are also ratios, and they are related to percents.

Percent of a Number

The U.S. Department of Labor includes a Bureau of Labor Statistics. This organization collects workforce data. One area of focus is the activity of recent high-school graduates. Recently, the bureau analyzed information regarding the post-high school activities of 3.2 million graduates. Among those graduates, 66.2 percent were enrolled in colleges or universities.

The Meaning of Percent

A ratio, often written as a fraction, describes a part of a whole. A **percent** is a ratio of a number to 100.



Graph paper is a useful tool for modeling percents. A block of 10 squares by 10 squares is 100 total squares and represents 100%. You can shade squares to represent any percent, or part of the whole.

Just like a fraction and a percent, a decimal represents a part of a whole. The "whole" in a decimal is 1.0. Therefore, a 10-by-10 block of graph paper squares is a useful model for representing decimals, and for representing the relationships between fractions, decimals, and percents.

Author's Image/PunchStock

Percents as Decimals

The term "e-commerce" describes any business conducted electronically, especially over the Internet. Statisticians collect e-commerce data to determine how and why people shop online. Businesses use this information to identify and market to potential customers.

Example 1: Use a Decimal to Find Percent of a Number

In a survey of 500 shoppers, 73% reported doing at least half of their shopping online. How many of the surveyed shoppers does this represent?

Step 1 Change the percent to a decimal. $73\% = 0.73$

Step 2 Multiply the decimal by the total number of shoppers. $0.73 \times 500 = 365$

Of the 500 shoppers surveyed, 365 reported doing at least half of their shopping online.

Percents as Fractions

In the same e-commerce survey, 4 out of 5 shoppers reported shopping online because they can find a larger selection of items.

Example 2: Write a Fraction as a Percent

What percent of the surveyed shoppers reported shopping online because they can find a larger selection of items?

Step 1 Write "4 out of 5" as a fraction. $4 \text{ out of } 5 = \frac{4}{5}$

Step 2 Write an equivalent fraction with a denominator of 100. $\frac{4 \times 20}{5 \times 20} = (80/100) = 80\%$

80% of the shoppers surveyed reported shopping online because they can find a larger selection of items.

CALCULATOR SKILL

You can change a percent to a decimal on many calculators by using the % button.

For example, to convert 18% to a decimal, press 18 followed by the % button.

CORE PRACTICE

Use Tools Strategically

In mathematics, there are a number of tools you can use to solve a problem. There are physical tools, such as protractors or calculators, and there are mental tools.

Benchmarks are useful mental tools when working with percents. A **benchmark** is a point of reference from which other measurements or estimates can be made. For example, you can estimate 33% of 25 by using a benchmark fraction or decimal in place of 33%. You know that 30% = 0.3, so 33% of 25 is about 0.3 × 25 = 7.5. Because 33% is greater than 30%, 33% of 25 is greater than 7.5. A good estimate is 8.

Use a benchmark to estimate 48% of 170. What benchmark did you use?

Percent as Proportion

You can write and solve a proportion to find a percent of a number.

$$\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}}$$

Example 3: Use a Proportion to Find Percent of a Number

Imagine that you and your friends celebrate a special event by eating dinner at your favorite restaurant. The amount of the bill before tax is $54.00. You want to leave your server a 15% tip. How much should you leave?

Step 1 Write a proportion.

$$\frac{\text{percent}}{100} = \frac{\text{part}}{\text{whole}} \qquad \frac{15}{100} = \frac{p}{54}$$

Step 2 Use cross products to solve the proportion for p.

$$15(54) = 100p$$
$$810 = 100p$$
$$\frac{810}{100} = \frac{100p}{100}$$
$$8.1 = p$$

You should leave $8.10 for a tip.

Think about Math

Directions: Find the percent of each number.

1. 18% of 50
2. 63% of 1,000
3. 90% of 150
4. 45% of 500
5. 32% of 250
6. 75% of 900

Percent Change

Many people invest their money in the stock market. Each day, the prices of stocks fluctuate. This change represents a change in percent.

Find Percent Change

Gyms and fitness centers often offer a variety of payment plans, including biweekly plans, monthly plans, and yearly plans.

Example 4: Percent Increase

When a local gym first opened last year, the biweekly fee was $17.99. After a year of operation, the biweekly fee is now $24.95. What is the percent change in cost?

Step 1 Subtract the original fee from the new fee.	$24.95 − $17.99 = $6.96
Step 2 Divide the difference by the original fee.	$6.96 divided by $17.99 is about 0.39.
Step 3 To write the quotient as a percent, multiply by 100.	0.39 × 100 = 39%

The percent change in cost is about 39%. Notice that the percent change is positive. This is because the new fee is greater than the original fee.

Example 5: Percent Decrease

Each year, the gym holds a New Year's Resolution promotion. For members who join during the month of January, the gym lowers its monthly fee from $32.99 to $29.99. What is the percent change in the monthly fee during the month of January?

Step 1 Subtract the original fee from the new fee.	$29.99 − $32.99 = −$3.00
Step 2 Divide the difference by the original fee.	−$3.00 divided by $32.99 is about −0.09.
Step 3 To write the quotient as a percent, multiply by 100.	−0.09 × 100 = −9%

The percent change in cost is about −9%. Notice that the percent change is negative. This is because the new fee is less than the original fee.

CORE SKILL

Use Percent

Whether a change is an increase or a decrease, the method used to calculate percent change is the same.

$$\text{percent change} = \frac{\text{new amount} - \text{original amount}}{\text{original amount}}$$

In the month of April, a residential customer's water bill was $26.30. In the month of August, the same customer's bill was $39.83. This customer said that the percent change in cost from April to August was about −51%. How can you tell immediately that this is incorrect? What error did the customer make and what is the correct percent change?

Discounts

A **discount** is a decrease, or reduction, in price. Sometimes advertisers describe a discount by stating the new, reduced price. Other advertisers indicate the discount as a percent instead, leaving it to you to calculate how much money you will save.

Example 6: Discounts

The original cost of a bike helmet was $24.99, but the price has now been reduced to $19.99. What is the amount of the discount? What is the discount as a percent?

Step 1 Find the amount of the discount by subtracting the original price from the new, reduced price.

$\$19.99 - \$24.99 = -\$5.00$ The amount of the discount is $5.00.

Step 2 Find the percent of the discount by dividing the amount of the discount by the original price and writing the quotient as a percent.

$\frac{-\$5.00}{\$24.99} \approx -0.20 = -20\%$

The change in price represents a 20% discount.

Think about Math

Directions: Answer the following questions.

1. A company's stock started the day priced at $18.99 per share. The company's stock price ended the day at $17.25 per share. What is the percent change in price per share for this particular day?
   A. 9.2%
   B. 0.9%
   C. −0.9%
   D. −9.2%

2. A student studying computer programming printed an online coupon for a software program. At the store, the student saved $6 on the purchase of the software originally priced at $39.99. What was the percent discount?
   A. −84%
   B. −15%
   C. 15%
   D. 84%

Simple Interest

The College Board, the organization that offers the SAT and Advanced Placement, monitors the costs of higher education and trends in student financial aid. Financial aid comes from a variety of sources, including loans. Because of their fixed interest rates, federal loans can be one of the least expensive ways to pay for college.

Use a Formula

Saving and borrowing money both involve interest. If you save money in a bank account, the bank pays you interest for the use of your money. If you borrow money from a bank, you pay the bank interest for the use of its money.

The amount of money that is initially borrowed or saved is called the **principal**. The **interest rate** is a percentage earned or charged during a certain time period. **Simple interest** is the amount of interest charged or earned after the interest rate is applied to the principal.

Simple interest (I) is the product of three values: the principal (P), the interest rate written as a decimal (r), and time (t).

$$I = P \times r \times t$$

Example 7: Simple Interest

A bank offers simple interest loans at an interest rate of 6.5% per year. How much interest will you pay if you borrow $1,500 for 2 years?

Step 1 Write the interest rate as a decimal by dividing the percent by 100. $6.5\% \div 100 = 0.065$

Step 2 Substitute the values into the simple interest formula and multiply.

$I = P \times r \times t$
$= 1{,}500 \times 0.065 \times 2$
$= 195$

You will pay $195 in interest.

Think about Math

Directions: Answer the following questions.

1. A car dealership finances simple interest car loans at a fixed rate of 4.2% per year for 3 years. How much interest will a buyer pay if she borrows $13,999.00 for a new car?
   A. $1,763.87
   B. $1,399.90
   C. $587.96
   D. $5,879.58

2. A student borrows $3,000.00 at a fixed 5.7% simple interest rate. The loan period is 2 years. How much interest will the student pay on the loan?
   A. $171.00
   B. $342.00
   C. $300.00
   D. $150.00

21ST CENTURY SKILL

Financial, Economic, Business, and Entrepreneurial Literacy

Three friends invested money in accounts earning simple interest.

- Carlos invested $1,000 for 5 years at a 5% interest rate.
- Molly invested $1,500 for 5 years at a 3% interest rate.
- Jin invested $900 for 10 years at a 3% interest rate.

Carlos says that he will earn the most interest because he has the greatest interest rate. Molly says that she will earn the most interest because she has the greatest principal. Jin says that she will earn the most interest because her investment is for the greatest amount of time. Who is correct?

Vocabulary Review

Directions: Write the missing term in the blank.

benchmark	**discount**	**interest rate**
percent	**principal**	**simple interest**

1. You can write a decimal as a(n) _____ by multiplying by 100.
2. A(n) _____ is a decrease, or reduction, in price.
3. When investing, the original amount of money invested is the _____.
4. _____ is the amount of interest charged or earned after an interest rate is applied to the principal.
5. The _____ is the percentage that is applied to a principal and paid by borrowers for use of the money they have borrowed.
6. A(n) _____ is a tool you can use to estimate a percent.

Skill Review

Directions: Read each problem and complete the task.

1. Write the decimal, percent, and fraction that this model represents.

_____

2. A bank offers simple interest loans at an interest rate of 6% per year. How much interest will you pay if you borrow $1,200 for 3 years?
   A. $21.60
   B. $216.00
   C. $2,160
   D. $21,600

3. Janice and her friend have lunch at a restaurant. The amount of the bill is $25.00, and the friends add an 18% tip. What is the total amount that Janice and her friend spent on lunch?
   A. $4.50
   B. $9.00
   C. $29.50
   D. $43.00

4. In 2011, the yearly fee for a fruit-of-the-month club was $59.99. In 2012, the yearly fee was $64.99. What is the percent change in cost?
5. The original cost of a wireless keyboard was $45.50, but the price has now been reduced to $38.68. What is the amount of the discount? What is the discount as a percent?
6. In a survey of 600 shoppers, 55% reported using fabric softener. How many of the surveyed shoppers does this percent represent?
7. In a survey of 200 computer shoppers, 3 out of 4 shoppers reported purchasing a laptop computer. How many of the surveyed shoppers does this represent?

Skill Practice

Directions: Read each problem and complete the task.

1. A day-care center charges $50 per day for each child. The center plans on increasing the cost to $60 per day. What will the percent change in cost be?
2. Elena wants to borrow $2,200 from the bank. They have two simple-interest loans she can choose from. One is a 3-year loan at an interest rate of 6.3%, and the other is a 4-year loan at an interest rate of 5.5%. Elena thinks she will pay less interest on the 4-year loan because the interest rate is lower. Do you agree with Elena? Why or why not?
3. Alex and a friend order meals at a restaurant that each cost $9. There is a 6.5% sales tax. They would like to leave a 20% tip in addition to the taxed amount for the waitress. How much money will the two friends spend in all?
4. A group of 500 people were surveyed about their grocery shopping habits. Of the people surveyed, 6 out of 10 paid with a credit card and 30% paid with cash. How many of the people surveyed paid with some other method?
   A. 10 people
   B. 50 people
   C. 180 people
   D. 320 people
5. Emma plans to buy a new computer monitor. She has two coupons that she could use, but the store only allows you to use one coupon per purchase. The first coupon is for a 15% discount. The second coupon is for $25 off any purchase over $125. Do you need any additional information to help Emma decide which coupon to use? If so, what else do you need to know? If not, which coupon should Emma use and why?
6. A fabric store regularly mails coupons to its customers. The store has collected data on coupon use over the last three months.

Month	Number of Shoppers	Percent Who Used Coupon
1	375	32%
2	200	55%
3	310	40%

Which shows the months in order from greatest number of shoppers who used a coupon to least number of shoppers who used a coupon?
   A. Month 2, Month 3, Month 1
   B. Month 2, Month 1, Month 3
   C. Month 3, Month 1, Month 2
   D. Month 1, Month 3, Month 2

LESSON 2.3 Use Counting Techniques

LESSON OBJECTIVES

- Apply the Fundamental Counting Principle
- Recognize and calculate factorials
- Determine permutations and combinations

CORE SKILLS & PRACTICES

- Use Counting Techniques
- Model with Mathematics

Key Terms

combination
a selection of objects or values in which order is not important

experiment
an activity or situation in which the results are uncertain

factorial
the product of a series of all descending consecutive positive integers from a given starting point

outcome
a result of an experiment or activity that involves uncertainty

permutation
a selection of objects or values in which order is important

Vocabulary

tree diagram
a branching diagram that shows possible outcomes of an experiment

Key Concept

Certain events can allow for uncertainty. When this occurs, it can be possible to determine the number of possible outcomes by using permutations and combinations.

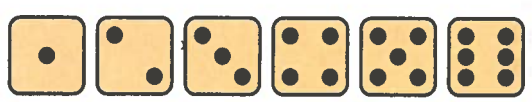
Factorials

Sometimes you make yourself a list of errands that you must complete and you have to make certain chores a priority. By ordering the list, you are finding one of the many possible ways that the errands can be ordered. Factorials are used to determine how many ways ordering can be done.

The Language of Counting

Learning to use counting techniques is easier when you know the meanings of special terms. An **experiment**, for example, is an activity or situation in which the results are uncertain and an **outcome** is a result of an experiment or activity that involves uncertainty.

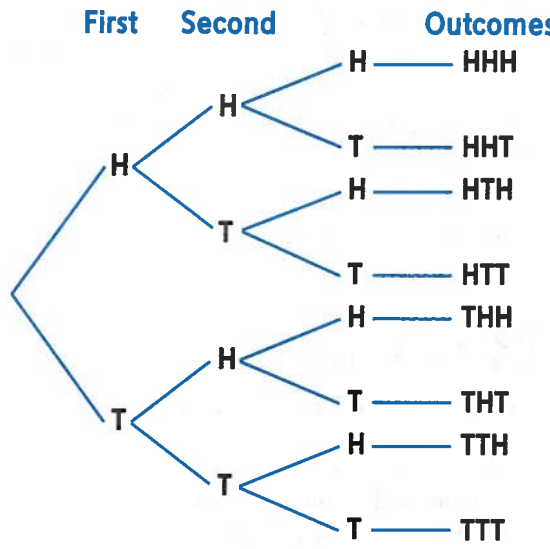
Now let's put those terms in the context of a simple experiment. Say you're playing a board game with friends, and it's your turn to roll a die. Rolling the die is an experiment with an uncertain outcome because six possible outcomes exist: 1, 2, 3, 4, 5, or 6.



Say that you roll the die, and the number 4 appears. The number 4 is an outcome, or result of the experiment.

Tree Diagrams

You can use a branching diagram, called a **tree diagram**, that shows possible outcomes of an experiment. Take, for example, the toss of a coin. There are two possible outcomes—heads or tails. Say you want to know all of the possible outcomes of tossing a coin 3 times.



The branches of a tree diagram show all of the possible outcomes after each toss. To the right of the diagram is a list of all possible outcomes after 3 tosses. There are 8 possible outcomes.

Another way to look at the outcomes of the tree diagram is by putting them into a chart. Notice there are 8 total results listed on the chart, just like in the tree diagram.

Toss		
1st	2nd	3rd
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

The Factorial

Suppose you wanted to organize your music collection. How many ways could you list your songs? If you have a large collection of music, this number is also very large. But how would you determine it?

Let's look at the problem on a smaller scale. Suppose you had 4 songs (A, B, C, and D) you wanted to organize. How many options are there for the first song in the order? There are 4 options since there are 4 songs. What about the second song? How many choices are there? Since one song has already been chosen, that leaves 3 songs. The third song? 2 choices, leaving 1 choice for the 4th song.

Song A
Song B
Song C
Song D

1st Song	2nd Song	3rd Song	4th Song
4 options	3 options	2 options	1 option

CORE SKILL

Use Counting Techniques

In more complex situations, you may want to use the Fundamental Counting Principle. This involves multiplying to find the total number of possible outcomes. So, if there are n outcomes for event N and m choices for event M, then there are nm outcomes for the event where both N and M occur. This can be generalized to more than 2 events.

For example, the choices at a frozen yogurt shop are shown below. Customers must select a yogurt type, 1 fruit, and 1 syrup. How many possible mixtures of yogurt, fruit, and syrup can the shop advertise?

Yogurt	Fruit	Syrup
Vanilla	Cherries	Caramel
Chocolate	Blueberries	Marshmallow
Coffee	Pineapples	Hot Fudge
Peach	Mango	Peanut Butter
	Strawberries	

Using the Fundamental Counting Principle, you can determine that there are $4 \times 3 \times 2 \times 1 = 24$ possible outcomes. A product of this kind has a special name. The **factorial** of a number is the product of the series of all descending consecutive integers from the number. It is represented by using the exclamation point (!). Therefore, in our music scenario, $4! = 4 \times 3 \times 2 \times 1 = 24$.

If you had 5 songs to organize, the total possible ways to order the songs would be $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

Think about Math

Directions: Answer the following questions.

1. How many possible sandwiches can be made from 3 types of bread, 5 types of cheese, and 6 types of filling, assuming each sandwich is made with 1 type of bread, 1 type of cheese, and 1 filling type?
   A. 14
   B. 80
   C. 90
   D. 104

2. How many possible ways can you order 5 people in a line?
   A. 15
   B. 120
   C. 625
   D. 3,125

Permutations

If 3 people finish a race in 1st, 2nd, and 3rd, does it matter which order they come in? Of course it does. Knowing the number of ways a set of objects can be ordered is what permutations are used for.

Order Matters

Sometimes, the order in which outcomes are arranged is important. Such a list of outcomes is called a **permutation**.

The formula for finding the number of permutations is easy to understand and can easily be calculated.

$$P(n, k) = \frac{n!}{(n-k)!}$$

P = the number of permutations
n = the number of items
k = the number of items being ordered

Say that there are 10 competitors in a math competition, but only the top 3 receive a prize, as 1st, 2nd, and 3rd place. How many different ways could the competitors finish in 1st, 2nd, and 3rd place?

We can use the Fundamental Counting Principle to determine the possibilities for 1st, 2nd, and 3rd place. How many possibilities exist for each place, starting with 1st?

(10 possibilities for 1st) × (9 possibilities for 2nd) × (8 possibilities for 3rd) $= 10 \times 9 \times 8$ possibilities $= 720$ possibilities

Another way to solve this problem is to use the permutation formula. In this case, there are 10 competitors and 3 of them are being ordered, so $n = 10$ and $k = 3$:

$$P(10, 3) = \frac{10!}{(10-3)!} = \frac{10!}{7!}$$
$$= \frac{10 \times 9 \times 8 \times \cancel{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}{\cancel{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}} = 10 \times 9 \times 8 = 720$$

Calculating Permutations

Suppose a town council needs to elect a president, vice president, treasurer, and secretary. There are 9 people on the council and no one can hold more than one office. How many ways can the council elect its officers?

Because order matters here (if someone is elected, it matters for which office they were elected), a permutation is what is being calculated.

$$P(9, 4) = \frac{9!}{(9-4)!} = \frac{9!}{5!}$$
$$= \frac{9 \times 8 \times 7 \times 6 \times \cancel{5 \times 4 \times 3 \times 2 \times 1}}{\cancel{5 \times 4 \times 3 \times 2 \times 1}} = 9 \times 8 \times 7 \times 6 = 3{,}024$$

Think about Math

Directions: Answer the following questions.

1. How many ways can 5 different positions be filled by 10 applicants?
   A. 2
   B. 120
   C. 30,240
   D. 3,628,800

2. Which represents the number of ways to order 6 people in a line?
   A. $P(6, 0)$
   B. $P(6, 6)$
   C. $P(0, 6)$
   D. $P(12, 6)$

CORE PRACTICE

Model with Mathematics

Sometimes it is helpful to use a formula to model a mathematical process, such as counting possible permutations.

There are 12 students enrolled in an advanced sculpture class. For their semester project, all students enter one sample of their work in a juried exhibit. The jury of professional artists awards one gold, one silver, and one bronze medal. How many different ways can the awards be distributed among the students?

CALCULATOR SKILL

Computing a Factorial

Some calculators have a way to evaluate factorials. Sometimes the factorial function is under the menu labeled "Probability." For example, on the TI-30XS MultiView™ calculator, the factorial symbol can be found by pressing the prb key and then the 3. But factorials can be calculated even without a specific factorial button. To calculate 5!, simply multiply 5 by every positive integer smaller than it:
$5 \times 4 \times 3 \times 2 \times 1$.
When working a problem involving a fraction that has a factorial in both the numerator and the denominator, it will help to write out both factorials and cancel all the common factors on the top and bottom of the fraction. Simplify and then compute on your calculator.

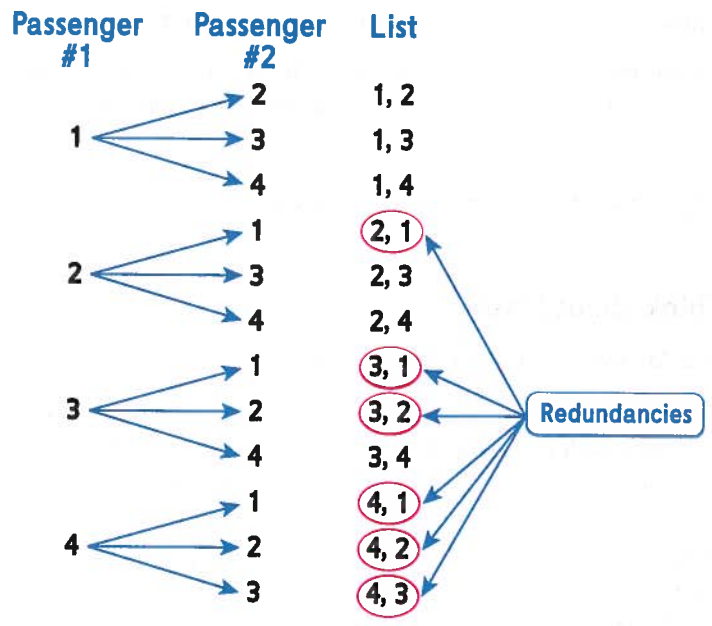
Combinations

When a large group of friends or family goes to an amusement park, they often need to figure out how to break up into smaller groups to go on rides together. It doesn't matter in what order the people are picked, just the final combinations. Likewise in math, combinations calculate the number of possible outcomes to an experiment assuming that the order that it occurs doesn't matter.

Order Doesn't Matter

A **combination** is a selection of objects or values in which order is not important. Combinations are not like permutations in that respect, but you use permutations to find them.

Say that an airport provides pick up and delivery service from different gates at an airport terminal to remote parking lots. A shuttle arrives and picks up passengers every 5 minutes. At the last gate, a shuttle has room left for only 2 passengers, but 4 passengers are waiting to board. How many ways can 2 passengers be chosen from 4 passengers waiting to board? You can use a tree diagram to find the answer.



At first count, there are 12 permutations. However, recall that order doesn't matter in this situation. Whether you are chosen 1st or 2nd, you still get to go on the shuttle. So, some permutations in the list are redundant. In other words, they repeat existing combinations. Because each pair shows up twice (1, 2 is the same as 2,1), only half of the list is needed. By eliminating the redundancies, you are left with 6 combinations.

Calculating Combinations

You can use a formula in place of a tree diagram to find combinations. Consider the same problem, in which an airport shuttle has room for 2 passengers, but 4 passengers want to board, and the order of boarding doesn't matter.

$$C(n, k) = \frac{P(n, k)}{k!}$$

C = the number of combinations
P = the number of permutations
n = the number of passengers
k = the number of ways the passengers can be selected

To begin, insert values for n and k.

$$C(4, 2) = \frac{P(4, 2)}{2!}$$

If the order the passengers boarded mattered, it would be enough to find $P(4, 2)$. However, order doesn't matter in combinations. So, divide the value of $P(4, 2)$ by the value $k!$, or the number of ways the boarding passengers can be arranged. This eliminates the redundancies in the list.

$$P(4, 2) = \frac{4!}{2!} = \frac{24}{2} = 12$$

$$C(4, 2) = \frac{P(4, 2)}{2!} = \frac{12}{2!} = \frac{12}{2} = 6$$

There are 6 possible combinations.

Think about Math

Directions: Answer the following questions.

1. On Monday, an online movie service makes 10 new releases available for streaming. Your membership allows you to select 3 of the new releases. How many ways can this be done?
   A. 120
   B. 240
   C. 720
   D. 86,400

2. At an amusement park, there are 8 roller coasters you want to ride. Unfortunately, you only have time to ride 4 of them. How many ways can you choose the 4 coasters you want to ride?
   A. 1,680
   B. 140
   C. 70
   D. 2

TEST-TAKING SKILL

Understand the Question

Before you can understand a test question, you must understand the question's special vocabulary. Then you can rewrite the question in your own words.

> Marcus was given 4 tickets to a sporting event. He wants to take his friends but must choose who he will take from among 5 of them. Since Marcus will be using a ticket, how many ways can he pick which friends will accompany him to the sporting event?

Before you can understand the question, you need to know the meaning of the word *combinations*. A combination is a collection of objects in which the order of those object doesn't matter. Next, rewrite the question: *How many ways can 3 people be chosen from 5 people, if order doesn't matter?* Draw a tree diagram or use a formula to find the answer.

Vocabulary Review

Directions: Write the missing term in the blank.

combination	**experiment**	**factorial**
outcome	**permutation**	**tree diagram**

1. A(n) _____ is the product of a series of all descending consecutive positive integers from a given starting point.
2. A(n) _____ is an activity or situation in which the results are uncertain.
3. A(n) _____ is a result of an experiment or activity that involves uncertainty.
4. A selection of objects or values in which order is not important is a(n) _____.
5. You can use a(n) _____ to show the possible outcomes of an experiment.
6. A list of outcomes in a situation where the order of the outcomes is important is a(n) _____.

Skill Review

Directions: Read each problem and complete the task.

1. A teacher has a box of markers 12 markers that are each different colors. She chooses 1 marker for each of 2 students. How many ways can she choose the 2 markers to give to the 2 students?
   A. 66
   B. 132
   C. 1,188
   D. 1,320

2. At a sandwich shop, customers can choose white or whole wheat bread. The shop also offers 5 different vegetables, and 4 different meats. How many possible sandwiches can be made using one vegetable and one meat?
   A. 11
   B. 20
   C. 40
   D. 80

3. Six people are scheduled to speak at a conference. Because of a time constraint, only the first 4 people in line can speak. How many ways can the volunteers speak?
   A. 720
   B. 360
   C. 90
   D. 15

4. How many possible ways can you order 7 people in a line?
   A. 5,040
   B. 2,520
   C. 720
   D. 49

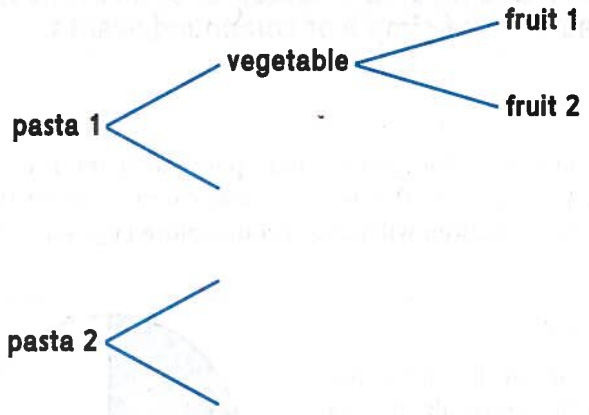
5. How many ways can 5 different positions be filled by 9 applicants?
   A. 5
   B. 20
   C. 126
   D. 15,120

6. Which number represents the number of ways to choose 5 people from a group of 15 people for a task force?
   A. $P(15, 15)$
   B. $P(15, 5)$
   C. $C(15, 5)$
   D. $C(15, 15)$

Skill Practice

Directions: Read each problem and complete the task.

1. Hillary packs her lunch with three small dishes: one pasta, one fruit, and one vegetable. She can choose from 2 different pasta dishes, 2 different fruit dishes, and 2 different vegetable dishes. Complete this tree diagram, then find how many lunch combinations Hillary can make.



2. There are 12 people who are trying to get 3 spots on a school council. Andrew calculates that there are 1,320 different outcomes. Is Andrew correct? Explain.

3. Anthony is packing clothes for a trip. He packs 4 shirts, 3 pairs of pants, and 2 pairs of shoes. He can wear any shirt with any pair of pants, and with any pair of shoes. How many different combinations of outfits has Anthony packed? Explain your answer.

4. Keisha has 14 songs in a playlist. Keisha wants to know how many different ways she can play the songs in her playlist. What are two different ways to show this?

5. There is another formula for combinations. Explain how it is like the formula in the lesson.

$$C(n, k) = \frac{n!}{k!(n-k)!}$$

LESSON 2.4 Determine Probability

LESSON OBJECTIVES

- Determine the probability of simple events
- Determine the probability of compound events

CORE SKILLS & PRACTICES

- Determine Probabilities

Key Terms

compound event
an event formed by two or more simple events

probability
the study of how likely it is for an event to occur

tree diagram
a branching diagram that shows possible outcomes of an experiment

Vocabulary

complement
an event that shows all the ways that an event cannot happen

dependent event
a second event whose probability depends upon a first event

independent event
a second event whose probability does not depend upon a first event

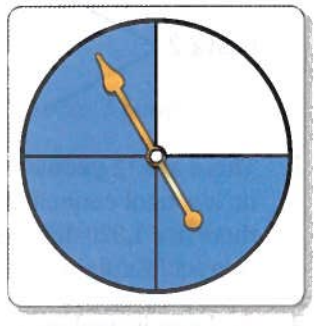
Key Concept

The probability of a chance event uses a number between 0 and 1 to describe the likeliness that the event will occur. You can use the number of total and favorable outcomes of an event to determine the probabilities of simple or compound events.

Probability of Simple Events

Genetic traits, like eye color, are inherited from your parents; theirs from their parents, and so on. You can analyze possible combinations in order to find the probability that your children will have certain colored eyes or other genetic traits.

The Basics of Probability



Probability is the study of how likely it is for an event to occur. The different results that can occur are called outcomes. This spinner shows two different outcomes: blue and white.

The probability of an event is often expressed with words such as *likely* or *unlikely*. If the outcome is sure to happen, we say it is certain; if there is no chance for the outcome to occur, we say it is impossible.

- Spinning blue is likely.
- Spinning white is unlikely.
- Spinning blue or white is certain.
- Spinning red is impossible.

Probability can also be described as a ratio. The ratio is found by comparing the number of ways that are favorable for an event to occur with all of the outcomes that are possible.

$$\text{probability} = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

For this spinner, write the probability of spinning blue as the ratio of the number of blue sections in the spinner (favorable outcomes) to the total number of sections in the spinner (total outcomes). The probability of spinning blue is 3 out of 4.

Probability as a Number

The number that represents a probability can be expressed using words, as a fraction, as a decimal, or as a percent.

Example 1: Find the Probability

Art, Etan, José, Ryan, and Dwight each write their names on slips of paper and put the papers in a box. One name will be randomly drawn to win a prize. What is the probability Ryan will win?

Step 1 Find the total number drawn.
- There are five names that can be of outcomes.

Step 2 Find the number of favorable outcomes.
- Only 1 name is Ryan's.

Step 3 Write the probability as a fraction, decimal, or percent.

$$\text{probability of Ryan winning} = \frac{\text{favorable outcomes}}{\text{possible outcomes}} = \frac{1}{5} = 0.2 = 20\%$$

The probability that Ryan wins is 20%.

You can also write the probability of the **complement** of an event. The complement is an event that shows all the ways that an event cannot happen. To write the probability that Ryan does not win, the number of possible outcomes is still 5, but the number of favorable outcomes—the other names in the box—is 4.

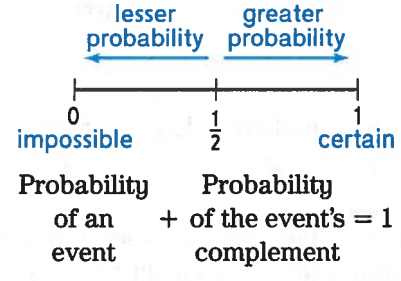
$$\text{probability of Ryan not winning} = \frac{\text{favorable outcomes}}{\text{possible outcomes}} = \frac{4}{5} = 0.8 = 80\%$$

So, the probability that Ryan will not win is 80%.

Probability Facts

Here are some other important facts to know about probabilities expressed as numbers.

- If there is no chance that an event can happen, the probability is 0.
- If an event is certain to happen, its probability is 1, or 100%.
- The greater the probability that an event will occur, the closer the number will be to 1 or 100%; the less likely the probability, the closer the number will be to 0.
- The sum of the probability of an event and its complement is 1.
- If two events are equally likely, the probability of each is 50%.



Probability of an event + Probability of the event's complement = 1

Find Favorable Outcomes

For many probability problems, making a list of possible outcomes is the best first step. Use your list to count total outcomes and favorable outcomes.

Example 2: Find Favorable Outcomes

Jenni, David, and Lauren have tickets to a play and got three seats together in the same row. If each person randomly chooses a ticket, what is the probability that Lauren will sit next to David?

Step 1 Make a list of all possible outcomes.

Jenni	David	Lauren
Jenni	Lauren	David
David	Jenni	Lauren
David	Lauren	Jenni
Lauren	Jenni	David
Lauren	David	Jenni

Step 2 Count the number of possible outcomes.

- There are 6 possible seating arrangements.

Step 3 Count the number of favorable outcomes.

- There are 4 arrangements with Lauren next to David.

Step 4 Write the probability as a fraction.

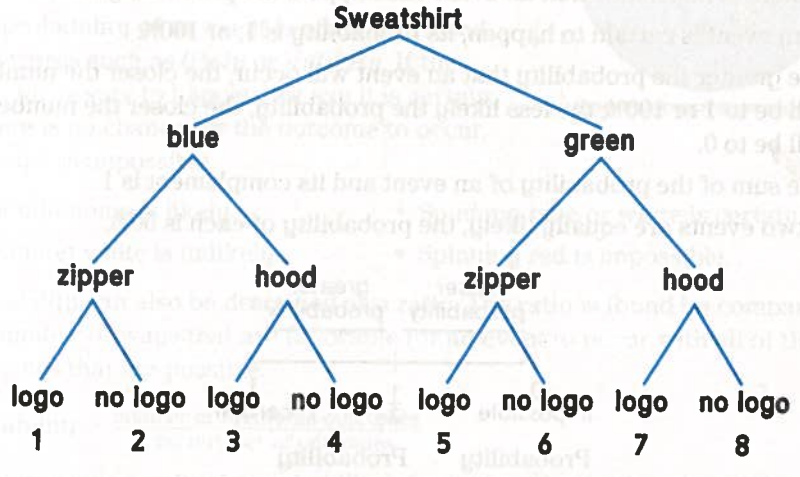
$$\text{probability of Lauren sitting next to David} = \frac{4}{6} = \frac{2}{3}$$

Tree Diagrams

Another way to count outcomes is to draw a **tree diagram,** which resembles a branching tree. Each path identifies a possible outcome of an experiment.

Example 3: Tree Diagrams

The Clothes Factory has XL sweatshirts on sale. The sweatshirts come only in blue or green. Each sweatshirt comes with either a zipper or a hood. And each may or may not have a store logo printed on it. How many combinations of sweatshirts are available?



This tree diagram shows 8 possible outcomes. If you wanted to find the probability of a randomly chosen sweatshirt on sale that has a hood, you can use the tree diagram to find there are 4 favorable equally likely outcomes and write the probability $\frac{4}{8}$, or $\frac{1}{2}$.

Predict with Probability

When you flip a coin, roll a number cube, or spin a spinner, you are basing probability on the laws of chance. This is called theoretical probability. Theoretical probability can be used to make predictions.

Example 4: Theoretical Probability

If you roll a number cube 50 times, how many times are you likely to roll the number 4? You can't predict with certainty, but you can say how many 4s are *most likely* to occur.

Step 1 Write the probability of rolling a 4 on one roll.

$$\text{probability for 4 on one roll} = \frac{\text{favorable outcomes}}{\text{possible outcomes}} = \frac{1}{6}$$

Step 2 Multiply by the number of rolls.

Out of 50 rolls, you are likely to roll a 4 about 8 times.

You can also base probability on past performance or on data. This type of probability is called experimental probability.

Example 5: Experimental Probability

The table shows the last 50 sales at Motor City. What is the best prediction for the number of Japanese cars sold out of the next 200 car sales?

Type	Number
Domestic	24
European	6
Japanese	20

Step 1 Use the given data to write the probability that the next car sold is a Japanese car.

$$\frac{\text{favorable outcomes}}{\text{possible outcomes}} = \frac{20}{50} = \frac{2}{5}$$

Step 2 Multiply by the number of cars that will be sold.

$$\frac{2}{5} \times 200 = \frac{400}{5} = 80$$

Out of the next 200 car sales, 80 of the sales are likely to be Japanese cars.

Think about Math

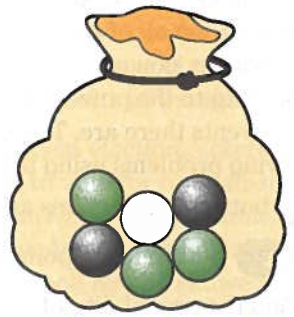
Directions: Answer the following questions.

1. If you flip a pair of coins 48 times, how many times are you most likely to get 2 heads?
2. A basketball player makes 15 free throws out of his last 45 attempts. How many free throws is he most likely to make out of his next 90 attempts?

CORE SKILL

Determine Probabilities

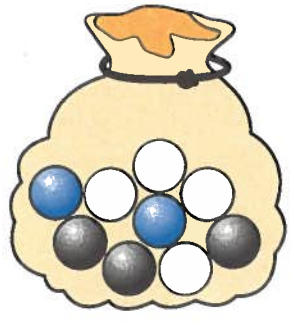
Regina is playing a game in which she draws marbles from a bag. The bag contains 1 white marble, 2 gray marbles, and 3 green marbles. What is the probability that Regina will draw a gray marble on her first try?



Each marble in the bag represents a possible outcome. For this problem, drawing a gray marble is a favorable outcome. Two of the six marbles in the bag are gray, so there are 2 favorable outcomes out of 6 possible outcomes.

$$\text{probability of gray marble} = \frac{\text{favorable outcomes}}{\text{possible outcomes}} = \frac{2 \text{ gray marbles}}{6 \text{ total marbles}} = \frac{1}{3}$$

Now find each of these probabilities using the bag of marbles shown below: blue, white, gray, not blue, not white, and not gray.



CALCULATOR SKILL

Calculating Probabilities of Independent Events

Remember that when calculating the probability of independent events, the probability of each independent event is multiplied. If the independent events have equal probabilities, then instead of multiplying all of the probabilities, you can raise that probability to the power of how many events there are. Try the following problems using the $\frac{a}{d}$ button for fractions and the x^2 button for exponents.

1. Find the probability of flipping a coin 20 times and getting tails every time.
2. Find the probability of a basketball player who has a 70% probability of making a free throw making his next seven free throws.

Probability of Compound Events

Probability also has applications in weather forecasting. An "80% chance of rain" may seem like a simple statement, but there is a lot of data and probability of many different events that are at work quantifying the probability with a single number.

Independent Events

So far, you have been working with simple events. Sometimes, you might want to know the probability of an event formed by two or more simple events, like rolling a number cube and spinning a spinner. This is called a **compound event** and is found by multiplying the probability of the first event by the probability of the second.

Example 6: Independent Events

When flipping a penny twice, what is the probability of getting two heads in a row?

Probability of two *heads* in a row $= \frac{1}{2} \times \frac{1}{2}$

probability of a head on first flip; probability of a head on second flip

Step 1 The probability of getting a head each time you flip the coin is $\frac{1}{2}$.

Step 2 Multiply the probabilities of getting a head on each flip.

The probability of two heads in a row is $\frac{1}{4}$. Events like this double coin flip are **independent events**, meaning the probability of a second event does not depend on the first event.

For each draw, there is a probability of $\frac{6}{9} = \frac{2}{3}$ drawing a white marble.

$$\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$$

first draw; second draw

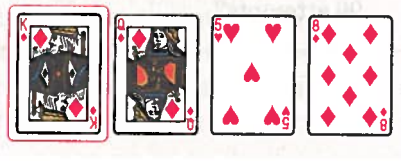
Independent events are often found in probability problems involving replacement. An example is when you draw a marble from a bag and then replace it before drawing another. The events are independent. Each marble has the same probability of being drawn on both draws.

Dependent Events

Think about how you might find the probability of randomly choosing a pair of face cards from the four cards shown. For this compound event, the probability of the second event depends on the first event. These events are said to be **dependent events**.

Example 7: Dependent Events

Suppose you randomly take two cards from these four cards: king of hearts, queen of diamonds, five of hearts, eight of diamonds. What is the probability that both cards will be face cards?

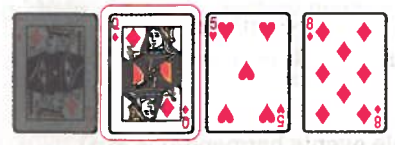


Step 1 The probability that the first card is a face card is $\frac{2}{4}$.

Step 2 For your second card, you need to remove your first card from the possible outcomes. There are now 3 cards remaining, one face card and two number cards. Choosing a face card for your second card occurs with a probability of $\frac{1}{3}$.

Step 3 Multiply the first probability by the second probability.

$\frac{2}{4} \times \frac{1}{3} = \frac{2}{12} = \frac{1}{6}$



Compound probabilities like this one are said to be without replacement, because once the first card is chosen it cannot be chosen again. When you are asked to find probabilities without replacement, you know that the number of possible outcomes for the event decreases, and so the events are dependent.

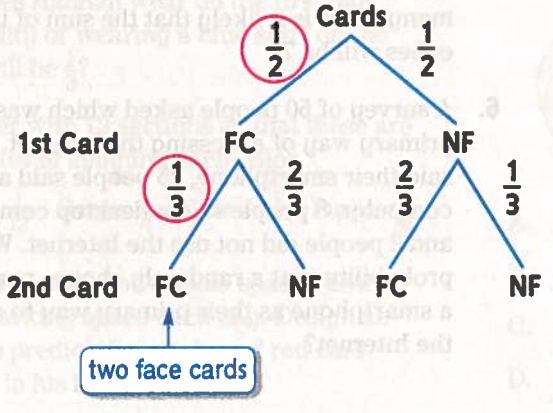
More with Tree Diagrams

You can also use tree diagrams to help you find probabilities of compound events. For choosing two cards of the four cards in Example 7, let the first level of the diagram show outcomes for the first card, and the second the outcomes for the second card.

Example 8: Tree Diagrams

Step 1 Label each branch of the diagram with its corresponding probability.

FC = face card
NF = not a face card



$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Step 2 Multiply the probabilities along the branches of the favorable outcomes. The result is the same as what you found by analyzing possible outcomes.

So the probability of choosing two face cards is $\frac{1}{6}$.

21ST CENTURY SKILL

Civics Literacy

News media and other organizations often conduct exit polls after people have cast their vote. Voters are asked, for example, which candidates they voted for and why, as well as whether they voted for or against any ballot initiatives. Exit polls can then be used to make predictions about the overall results of the election.

At a recent election, voters were asked to vote for or against two unrelated referendums, Referendum A and Referendum B. Exit poll results showed that 2 out of 3 people voted for Referendum A, and 3 out of 5 people voted for Referendum B. Based on the results of the exit poll, what is the probability of at least one of these Referendums passing?

Vocabulary Review

Directions: Write the missing term in the bank.

compound event	**probability**	**complement**
independent event	**tree diagram**	**dependent event**

1. The probability of two or more simple events happening is a(n) _____.
2. _____ is the study of how likely it is for an event to occur.
3. A(n) _____ is a way to show possible outcomes that resembles a branching tree.
4. You flip a coin fifteen times. The probability that on the sixteenth flip it will be heads is a(n) _____.
5. The probability that today is not Tuesday is the _____ of the probability that today is Tuesday.
6. An event that depends on the result of a prior event is a(n) _____.

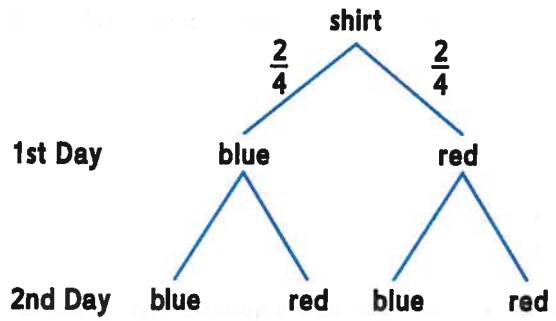
Skill Review

Directions: Read each problem and complete the task.

1. There are an apple, orange, and pear in a fruit basket. Marcus randomly selects one piece of fruit to eat each day. What is the probability that Marcus will choose an apple before choosing a pear?
2. There is a bag of 4 red marbles, 5 blue marbles, and 1 white marble. Wendy draws a marble, and then places it back in the bag. Then Peter draws a marble. What is the probability that both Wendy and Peter drew a blue marble?
3. Daniel, Tiana, Elise, Mary, and Robert put their names on slips of paper and put the papers in a box. What is the probability that Tiana's name will be picked?
4. There are five rides at a theme park, two of which are roller coasters. Maria wants to ride each ride once. She chooses a roller coaster for her first ride, then randomly chooses the second ride. What is the probability that the second ride is a roller coaster?
5. If you roll two number cubes 30 times, how many times is it likely that the sum of the two cubes will be 7?
6. A survey of 50 people asked which was their primary way of accessing the Internet. 15 people said their smartphone, 25 people said a laptop computer, 8 people said a desktop computer, and 2 people did not use the Internet. What is the probability that a randomly chosen person uses a smartphone as their primary way to access the Internet?
7. Dave packs 8 pairs of pants for a business trip. Three of the pants are khaki, and 5 of the pants are black. Draw a tree diagram to find the probability of Dave randomly choosing khaki pants for his first two days.
8. There is a bag of marbles. Five times a marble has been drawn and placed back in the bag. Of the five draws, 2 were green marbles, 1 was a yellow marble, and 2 were blue marbles. How likely is it that the next drawing will be a blue marble?

Skill Practice

Directions: Read each problem and complete the task. Use the tree diagram to answer questions 1–3.



1. Hannah drew this tree diagram to help her find probabilities of wearing a red or blue shirt over 4 days. She wears a shirt for one day and then chooses from the remaining shirts the next day. Is this probability situation independent or dependent?
2. Complete the tree diagram. Extend the branches to show each day Hannah wears the shirts.
3. What shirt does Hannah wear on the first day if the probability of wearing a blue shirt on the second day will be $\frac{1}{3}$?
4. Draw a spinner with 12 sections so that these are the probabilities of spinning each color.

   Red: $\frac{1}{3}$ Blue: $\frac{1}{12}$ Green: $\frac{1}{4}$ Yellow: $\frac{1}{6}$ Purple: $\frac{1}{6}$
5. Walter owns a parking lot with 25 spaces. Only 1 car uses a parking space each day. Design an experiment to predict the number of red cars that will park in his lot on Friday.
6. A red and a blue number cube are used for one turn of a game. If the outcome of rolling the red number cube is 4, you then get to roll the blue number cube. If 24 people are playing the game, how many people are likely to roll an even number on the blue number cube?
7. The table shows the transportation method used by the employees in an office. What is the probability that the next two employees that join the office staff will take the bus?

Transportation	Number
car	18
bus	6
walk	6

   A. 0.04
   B. 0.12
   C. 0.33
   D. 0.36
8. Julian randomly chose a penny from the three shown. Then Matt chose one from the two that remained. What is the probability that Matt chose tails?

   A. $\frac{1}{6}$
   B. $\frac{1}{3}$
   C. $\frac{1}{2}$
   D. $\frac{2}{3}$

Directions: Choose the best answer to each question

1. Amber bought 10 pounds of potatoes for $4.50. What is the price per pound of the potatoes?

A. $45.00 per pound
B. $14.50 per pound
C. $4.50 per pound
D. $0.45 per pound

2. In a survey of 300 shoppers, 40% reported bringing a bottle of water with them. How many of the surveyed shoppers does this represent?

A. 75
B. 120
C. 260
D. 340

3. The probability that it will rain tomorrow and the probability that it will not rain tomorrow is 1, or _____.

4. A spinner has four equal parts. One part is red, one part is blue, one part is yellow, and one part is green. If you spin the spinner 64 times, you are likely to spin a red part _____ times.

5. Anna and her friend had a dinner that cost $45 and they would like to leave an 18% tip. Which equation can you use to find the amount of the tip?

A. $\frac{18}{45} = \frac{x}{100}$
B. $18 \times 100 = 45x$
C. $\frac{18}{100} = \frac{x}{45}$
D. $\frac{100}{45} = \frac{x}{18}$

6. Ellis spends $\frac{17}{20}$ of his allowance on toys. Marie spends 85% of her allowance on toys. $\frac{17}{20}$ and 85% are _____ because both numbers can be written as the same ratio.

7. Jen borrows $4,000 from her bank. She has a simple interest loan at a 5% interest rate and will pay it back over 4 years. How much interest will she pay on this loan?

A. $8,000
B. $4,009
C. $800
D. $50

8. Lisa is a video editor. She has 4 videos that she needs to edit. How many different orders are there for her to edit the videos?

A. 48
B. 24
C. 10
D. 6

9. Liam, Mia, Aiden, Lily, Avery, and Logan each write their name on a slip of paper and place the slips of paper in a box. One name will be randomly drawn to win a prize. What is the probability that Aiden will win?

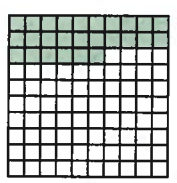
A. 6%
B. $\frac{1}{5}$
C. $\frac{1}{6}$
D. 60%

10. When you see 8! you can read it as *eight factorial* and it means to multiply _____.

11. A bag has 3 blue marbles and 5 red marbles. After a marble is drawn, it is placed back in the bag. What is the probability of drawing a blue marble twice?

A. $\frac{9}{64}$
B. $\frac{9}{25}$
C. $\frac{3}{8}$
D. $\frac{3}{5}$

12. This model shows the fraction $\frac{26}{100}$, the percentage _____, and the decimal 0.26.



13. Mrs. Albert buys 10 boxes of crayons for $30. How many boxes of crayons can she buy for $75, if the ratio stays the same?

A. 20 boxes
B. 25 boxes
C. 60 boxes
D. 75 boxes

14. Mr. Thomas is sketching a scale model of a park. The length of the park is 500 yards and the width of the park is 200 yards. He drew the length as 25 inches and the width as _____ inches.

15. A bag has 5 yellow marbles and 7 green marbles. After a marble is drawn, it is set aside. What is the probability of drawing a green marble twice?

A. $\frac{35}{144}$
B. $\frac{7}{22}$
C. $\frac{49}{144}$
D. $\frac{7}{12}$

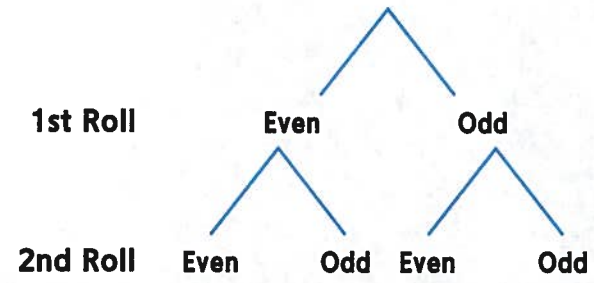
16. A gym charges $40 a month for membership in 2013. In 2014 the price increases to $45 a month. What was the percent change in the price?

A. −12.5%
B. 11.1%
C. 12.5%
D. 88.9%

17. A club is electing a president, vice president, and treasurer. There are 7 people running for positions and no one can hold more than one position. How many ways can the positions be filled?

A. 1.75
B. 210
C. 840
D. 5,040

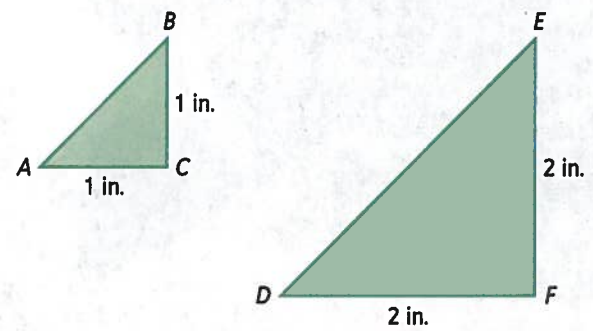
18. This tree diagram shows all possible outcomes of rolling a _____.



19. A company has 4 summer intern positions. There are 12 applicants. How many ways can the positions be filled?

A. 19,958,400
B. 11,880
C. 495
D. 384

20. The two triangles are _____ because the measures of the sides are proportional.



Check Your Understanding

On the following chart, circle the items you missed. The last column shows pages you can review to study the content covered in the question. Review those lessons in which you missed half or more of the questions.

Lesson	Item Number(s)			Review Page(s)
	Procedural	Conceptual	Problem Solving	
2.1 Apply Ratios and Proportions	5	6, 12, 20	1, 13	48–53
2.2 Calculate Real-World Percentages	14		2, 7, 16	54–61
2.3 Use Counting Techniques	19	10	8, 17	62–69
2.4 Determine Probability	4, 15, 18	3	9, 11	70–77