LESSON 6.1

Linear Equations

Lesson Objectives

You will be able to

- Plot a line that represents the linear relationship between two sets of numbers
- Graphically determine the value of the dependent variable
- Determine whether an independent and a dependent variable are linearly related
- Write the equation of a line from a verbal description of the relationship between two sets of numbers

Skills

- **Core Skill:** Solve Real-World Arithmetic Problems
- **Core Skill:** Solve Linear Equations

Vocabulary

dependent variable
independent variable
linear equation
linear relationship
rise
run
slope
y-intercept

KEY CONCEPT: A variable is something you are trying to measure. There are two kinds of variables, independent and dependent. An **independent variable** has a value that remains the same. That is, it is not affected by a **dependent variable.** A dependent variable is a value that depends on other factors.

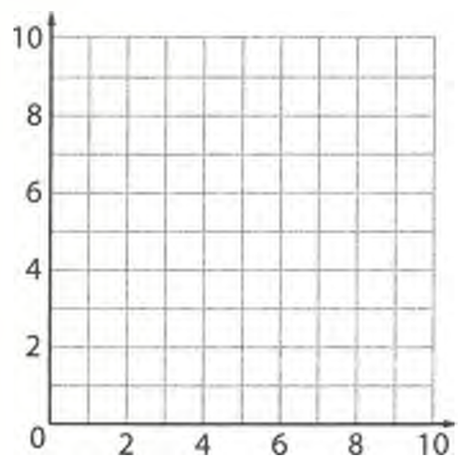
An independent variable, such as how much time you spend preparing for a math test, is something you control. It affects the dependent variable, which is your test score. Or in another example, the kind of exercise you choose to do is an independent variable. It affects your heart rote, which is the dependent variable.

*Two variables hove a **linear relationship** if their corresponding points lie on the same line in the coordinate plane. This means that as the independent variable increases (or decreases), the dependent variable increases (or decreases) proportionally.*

A coordinate plane is a two-dimensional surface on which you can plot points. Each point is located by a pair of x- and y-coordinates.

Plot the following coordinate pairs on the coordinate plane below.

(1, 2), (3, 3), (6, 0), (0, 7), (5, 8), (9, 10)



Linear Relationships

Linear relationships are a very important concept in mathematics and science. They also appear in many applications that you come across in everyday life—sometimes without even noticing them!

Imagine that you have saved enough money to buy a new cell phone. As part of your new cell phone plan, you are allowed to send up to 200 text messages per month for a flat fee of $5. 00. For each text message over the 200-message limit, you pay an additional 20 cents, or $0. 20.

In January, you sent 210 text messages, which means that you sent 10 extra messages. You can calculate the text-message charge.

$$\text{Total text-message charge} = \underset{\text{flat fee}}{\$5.00} + \underbrace{(\$0.20 \times 10)}_{\text{charge for extra messages}} = \$5.00 + \$2.00 = \$7.00$$

So, for 10 extra messages, your monthly text-message charge is $7.00.

In February, you sent 215 text messages, which means that you sent 15 extra messages. You can calculate the total text-message charge.

$$\text{Total text-message charge} = \$5.00 + (\$0.20 \times 15) = \$5.00 + \$3.00 = \$8.00$$

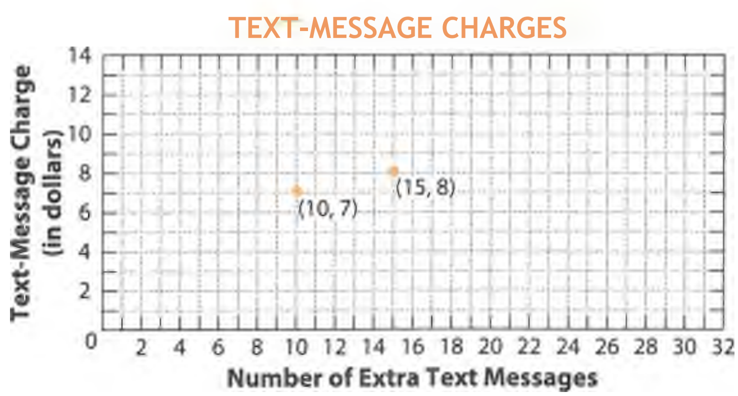
The following table displays the total text-messaging costs for two months.

Number of Extra Text Messages	Total Text-Message Charge
10	7
15	8

The same information can be expressed as two coordinate pairs, where the first number is the number of extra text messages, and the second number is the text-message charge.

(10, 7)   (15, 8)

You can plot these points on the coordinate plane, where the horizontal axis represents the number of extra text messages and the vertical axis represents the text-message charge in dollars.



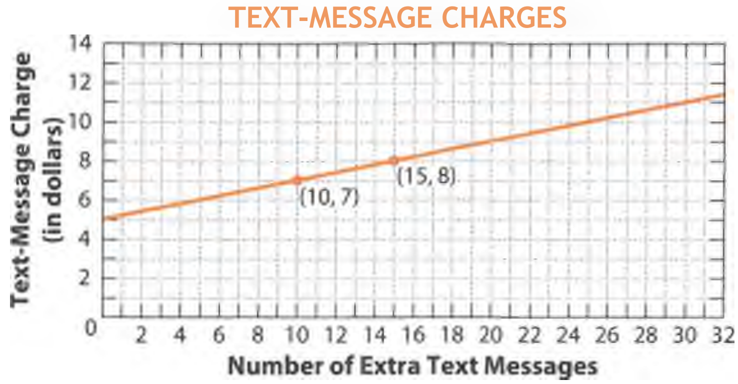
Core Skill
Solve Real-World Arithmetic Problems

Mathematics is used to solve a variety of everyday problems. Say, for example, that a credit counselor suggests that her clients spend no more than 25 percent of their monthly income on rent. She can use mathematics to model costs. This allows her to help her clients limit their apartment searches to affordable apartments. How much would a person need to earn each month to afford a monthly rent of $600.00?

Monthly Salary (in dollars)	Maximum Rent (25% of monthly salary in dollars)
1, 000	250. 00
1, 250	312. 50
1, 500	375. 00
1, 750	437. 50
2, 000	500. 00
2, 250	562. 50
2, 500	625. 00

Remember from geometry that if you have two points on a plane, you can draw a line that goes through both of them.

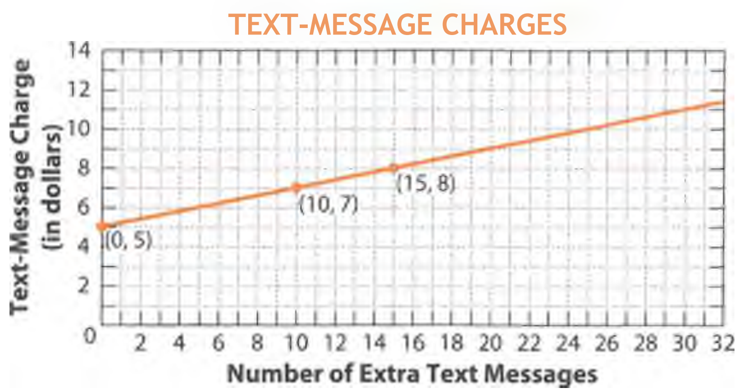
You can use this line to determine the text-message charge for any number of extra text messages *without* having to do the calculations.



How do you know that the line you drew will give you the correct text-message charge for any number of extra text messages? You can check it against another piece of information that you already know. Remember that your cell-phone plan says that you pay $5. 00 if you send 200 text messages or less, which means that there are zero extra text messages.

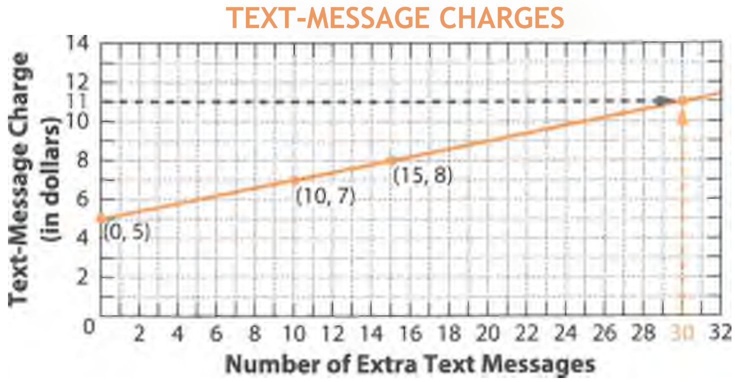
(0, 5)

You can express this information as a coordinate pair and plot it on the coordinate plane.



The point (0, 5) is indeed on the line, so you can feel confident that you can use this line to determine the text-message charge for any given number of extra text messages.

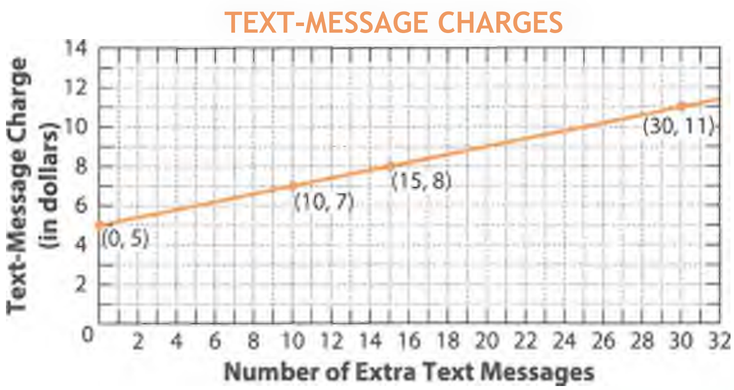
Let's say that in March, you sent 230 text messages. That means that you sent 30 extra messages. Locate the number 30 on the horizontal axis. Then, draw a vertical dashed line straight up to the line, as shown in orange in the figure on the next page. Next, draw a horizontal dashed line back to the vertical axis, as shown in the figure at the top of page 169.



The graph tells you that if you sent 30 extra text messages in March, your text-message charge is $11. 00. This corresponds to the point (30, 11) on the coordinate plane.

Complete a Data Table

Use the plot to complete the data table.



The points, or coordinate pairs, are on the same line. This means that there is a linear relationship between the number of extra text messages and the text-message charge.

In mathematics, special terms are used to describe two values that have a linear relationship. The number of extra text messages is the independent variable. The text-message charge is the dependent variable. The text-message charge is the dependent variable because its value *depends* on the number of extra text messages. In other words, you need to know the number of extra text messages before you can determine the total text-message charge.

You can also identify whether there is a linear relationship between the number of extra text messages and the text-message charge. Look at the data table that you completed. A pattern in the data exists. Each time the number of extra text messages increases by 5, the text-message charge increases by one dollar. This indicates that there is a linear relationship between these two variables.

Number of Extra Text Messages	Text-Message Charge (in dollars)	Coordinate Pair
0	5	(0, 5)
5		
10	7	(10, 7)
15	8	(15, 8)
20		
25		
30	11	(30, 11)

21st Century Skills
Critical Thinking and Problem Solving

Applications of linear equations are widely encountered in business, social sciences, economics, science, and engineering fields. In business, for example, linear equations model total costs after sales tax is added to purchases. In engineering, linear equations show the relationship between speed and time in calculations of distance. In science, linear equations show relationships between animal behavior and the environment, such as how often crickets chirp in different outdoor temperatures. Linear equations model relationships between variables in social data, too, such as the frequency of cell-phone use and times of day. People use linear equations to predict events and understand and solve difficult problems more easily.

Choose an electronic device that you use often throughout the day. Record the times you use the device and how long each use occurs over a period of several days. Then graph the results. Describe any patterns that emerge from the data.

Linear Equations

In the previous section, you graphed a line to represent a linear relationship. There may be cases where it is preferable to have an equation that represents the linear relationship between two values. These equations are called **linear equations.**

Recall your text-message plan. You can write an equation that will help you calculate the text-message charge when you have extra text messages. You know that the text-message charge is equal to the flat fee ($5. 00) plus the number of extra text messages multiplied by $0. 20 (20 cents.)

$$\text{text-message charge} = \underset{\text{flat fee}}{\$5.00} + \underbrace{\$0.20 \times \text{number of extra messages}}_{\text{charge for extra messages}}$$

Before continuing, remove the dollar sign ($) and assign letter symbols to the values, or variables, that you are going to work with. This will make it easier to write the equation.

Use the letter y to identify the dependent variable.

$$y = \text{text-message charge}$$

Use the letter *x* to identify the independent variable.

$$x = \text{number of extra text messages}$$

After making substitutions, the equation becomes:

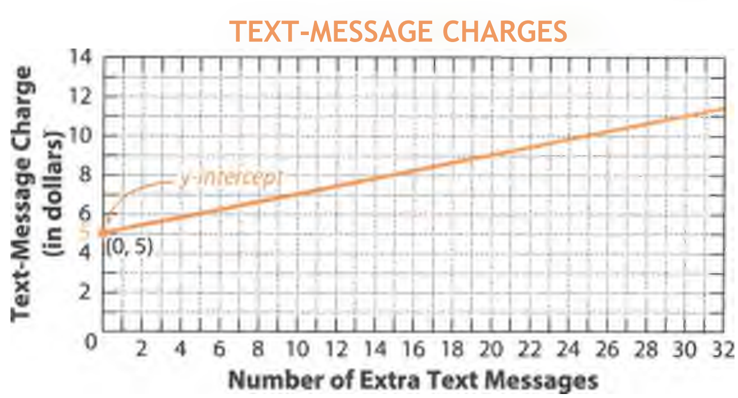
$$y = 5 + 0.\ 2x$$

Rearrange the terms on the right side of the equation.

$$y = 0.\ 2x + 5$$

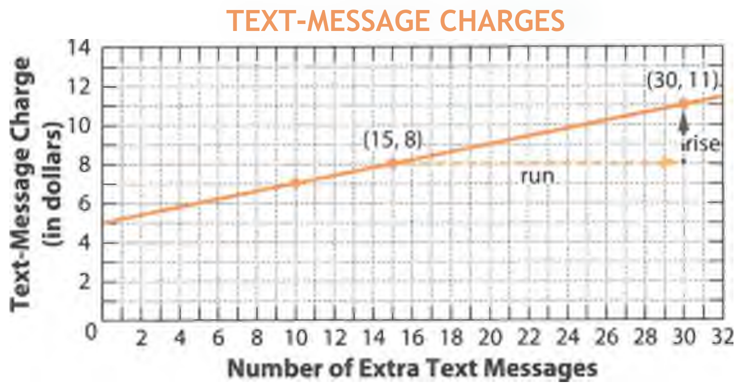
This equation represents the line that you plotted in the previous section. Look at the how the equation and the line are related.

The y-axis corresponds to the values of text-message charge. Each charge is represented by the letter *y*. The number 5 is called the **y-intercept** of the line. It represents the point where the line crosses the y-axis (vertical axis).



The number 0. 2 is called the **slope** of the line. The slope of a line represents how steep it is. A horizontal line has a slope of zero. A line that slants upward to the right has a positive slope. A line that slants downward to the right has a negative slope.

You can calculate the slope of a line by comparing any two points on the line. For example, use the points (15, 8) and (30, 11), as shown on the graph below.



The slope of a line is defined as the ratio between the rise and the run, or

$$slope = \frac{rise}{run}$$

The **rise** is the vertical distance between two coordinates. It is calculated by finding the difference between the vertical, or y-coordinates.

$$rise = 11 - 8 = 3$$

In this case, the rise is positive.

The **run** is the horizontal distance between two coordinates. It is calculated by finding the difference between the horizontal, or x-coordinates.

$$run = 30 - 15 = 15$$

So, the slope of this line is:

$$slope = \frac{3}{15} = 0.2$$

In general, the equation of a line, or linear equation, is written as

$$y = mx + b,$$

where m is the slope and b is the y-intercept. So, for this example

$$m = 0.2$$
$$b = 5$$

THINK ABOUT MATH

Directions: Use the graph to determine two coordinate points. Use the coordinate points to calculate the slope.

What is m, or the slope of the line?

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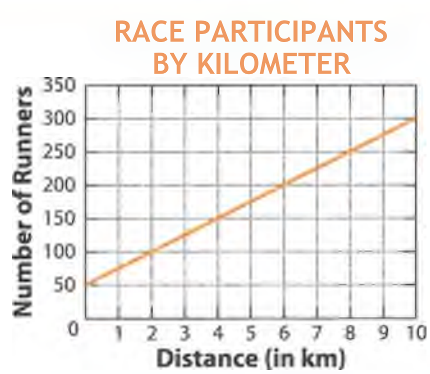
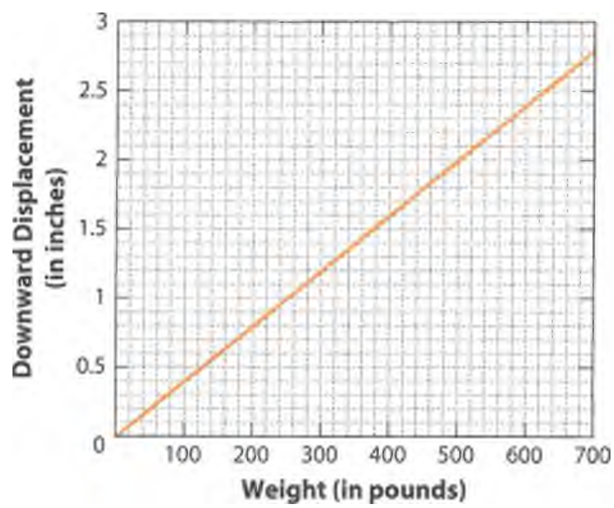


Have you ever put something heavy in the back of a car or pickup truck and noticed that back of the car moved downward? This happens because the weight causes springs in the suspension system to compress. Imagine that you are an automotive engineer and you're testing the suspension of a pickup truck by placing different weights in the back of the truck and measuring how much the back of the truck moves downward. Engineers call the ratio of the downward movement to the force of the weight the **compliance** of the suspension springs. The smaller the compliance, the harder it is to compress a spring. The compliance of the suspension springs can be shown as the slope of the graph below. See the sidebar on the left titled Solve Linear Equations for steps in calculating the slope.



Core Skill
Solve Linear Equations

The data for calculating the compliance of the suspension springs can be plotted as the slope of the graph. Follow these steps to calculate the slope.

1. Pick two points on the line. It will be easier to perform the calculations if you pick points whose values are easy to work with, such as (300, 1. 2) and (500, 2).

2. The slope is the rise divided by the run. First, determine the rise by calculating the vertical distance between the points, which is equal to the difference between the y-coordinates of each point. So, the rise = 2 - 1. 2 = 0. 8.

3. Next, determine the run by calculating the horizontal distance between the points, which is equal to the difference between the x-coordinates of the each point. So, the run = 500 - 300 = 200.

4. Now, calculate the slope = $\frac{rise}{run}$ = $\frac{0.8}{200}$ = 0. 004. The suspension springs have a low compliance, which means it takes a lot of weight to compress them.

Vocabulary Review

Directions: Match each term to its example.

1. ___slope
2. _______y-intercept
3. ______ rise
4. _______run
5. ______ linear relationship
6. _______dependent variable
7. ______ independent variable

A. the horizontal distance between two points on a line

B. the value where a line crosses the vertical axis

C. a variable that remains the same, or unaffected by other variables

D. the vertical distance between two points on a line

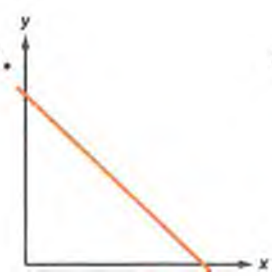
E. the steepness of a line

F. a variable whose value depends on other factors

G. a relationship between two variables on a graph that can be shown by drawing a straight line between them

Skill Review

Directions: Indicate whether the lines on the plots below are linear or nonlinear. Explain your answers.

1.


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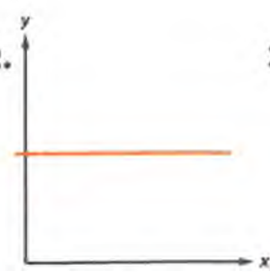

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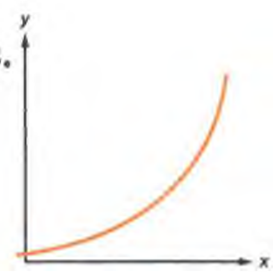


1. ____

____

2. ____

____

3. ____

____

MATH LINK

Earlier in this lesson, we used the graph of a linear equation to determine the value of the dependent variable if we were given the value of the independent variable. We can also determine the value of these variables from the linear equation itself.

In the example where we examined the relationship between the weight applied to the back of a pickup truck and the downward movement of the suspension springs, we can write the linear equation as

$y = 0.004x$

where y is the downward movement of the suspension in inches and x is the weight in pounds. If we want to calculate the downward movement, y, for a given weight, x, we plug in the value of x and solve for y. For example, if we put an object that weighs 200 pounds in the back of the pickup, the downward movement will be

$y = 0.004 \times 200 = 0.8$ inches

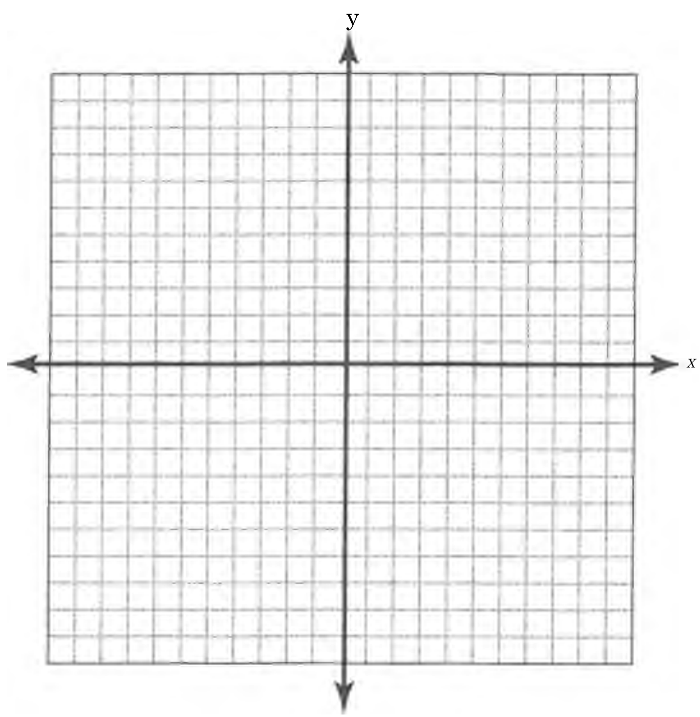
Skill Review (continued)

Directions: Read the problem below. Then answer items 5-8 that follow.

You are planning a trip to Canada, where the temperature is measured in degrees Celsius instead of degrees Fahrenheit. You find out that the linear equation that determines the temperature in degrees Fahrenheit from the temperature in degrees Celsius is

$$F = 1.8C + 32,$$

where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius. Before you go, you create a linear graph that you can use to convert degrees Celsius to degrees Fahrenheit quickly. This will help you know how to dress appropriately for outside temperatures.



5. What is the independent variable?

____

6. What is the dependent variable?

____

7. What will the slope of the line be?

____

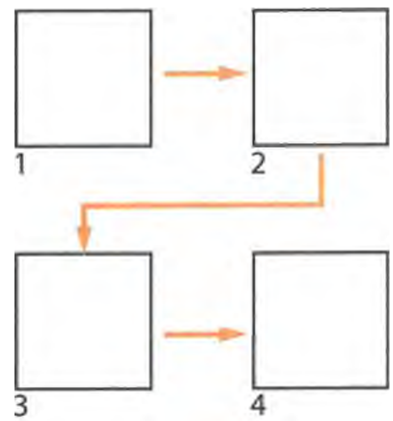
8. What will the y-intercept be?

____

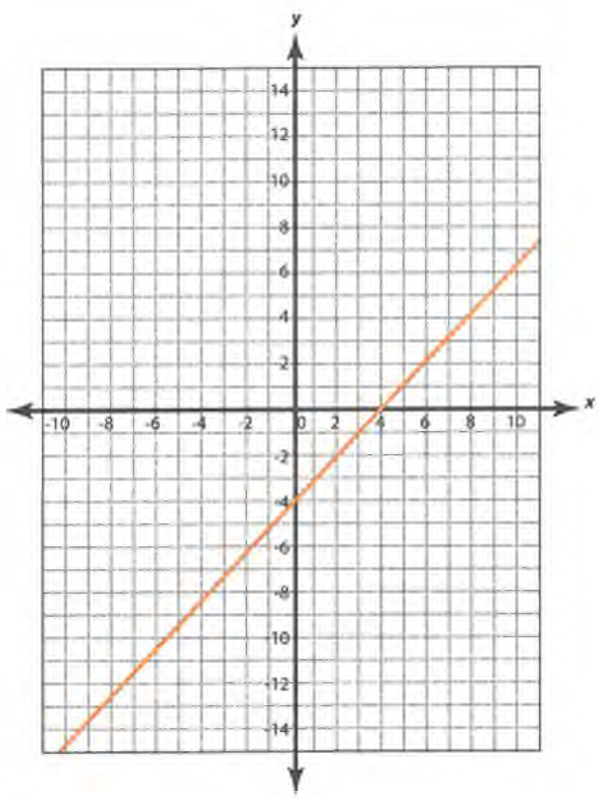
Skill Practice

Directions: Use the flow chart below to write your answers.

1. Show the steps for determining the slope of a line that is drawn on a coordinate plane. Number each stage.



Directions: Use the graph below to choose the best answer for each question.



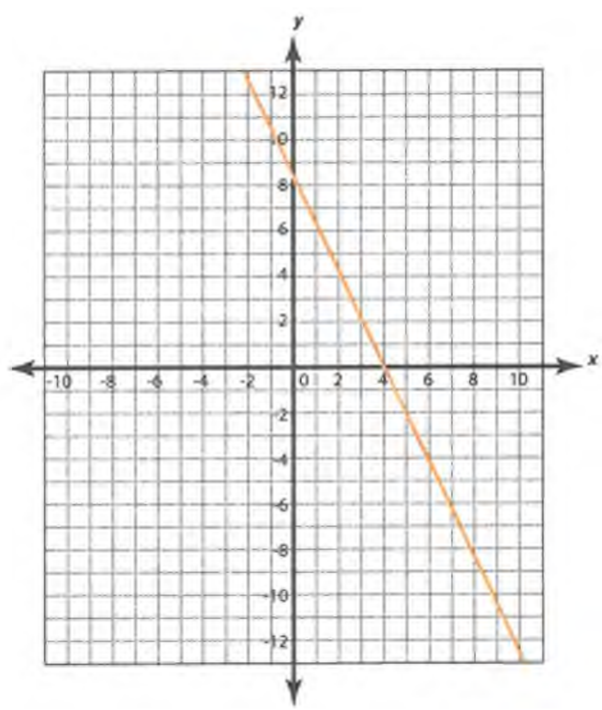
2. What is the slope of the line?

A. 4 **C.** 1
B. -4 **D.** -1

3. What is the y-intercept?

A. 4 **C.** 0
B. -4 **D.** 1

Directions: Use the graph below to choose the best answer.



4. What is the equation for this line?

A. $y = 2x + 8$
B. $y = 2x - 8$
C. $y = -2x - 8$
D. $y = -2x + 8$

LESSON
6.2

Graphing Linear Equations

Lesson Objectives

You will be able to

- Use the point-slope form to graph the equation of a line
- Use the slope-intercept form to graph the equation of a line
- Use the two-point form to graph the equation of a line

Skills

- **Core Skill:** Perform Operations
- **Core Skill:** Interpret Graphs and Functions

Vocabulary

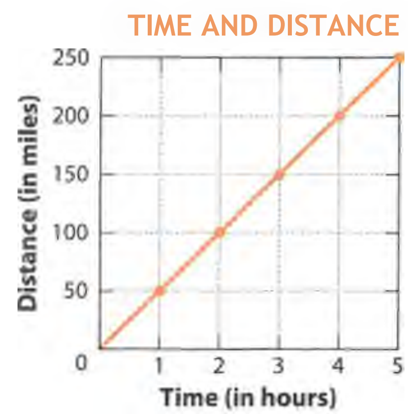
intersect
point-slope form
slope-intercept form
subscript
two-point form

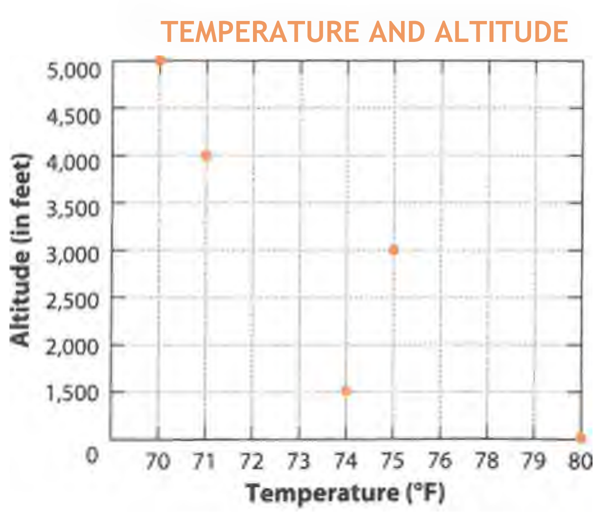
KEY CONCEPT: There are two ways to graph a linear equation. (1) If two coordinate pairs that lie on the line are known, then the graph of the line can be constructed, or (2) if one coordinate pair that lies on the line and the slope of the line are known, then the graph of the line can be constructed.

If an independent variable increases (or decreases), and the dependent variable increases (or decreases) proportionally, the two variables are linearly related. If the increase and decrease are disproportional, there is no linear relationship.

The data from the following tobies were graphed. Examine the line of the equation in the first graph. The straight line shows a proportional relationship between the variables of Time and Distance.

Now examine the second graph. You con see that it is impossible to connect the points on a straight line, meaning there is no proportional, or linear, relationship between altitude and temperature in this particular example.



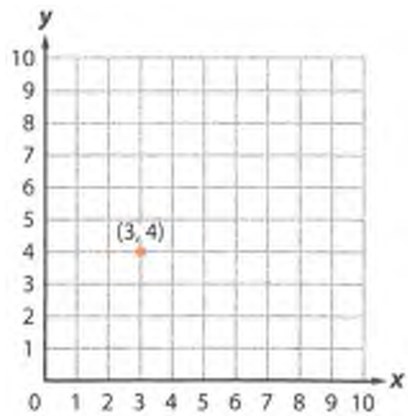


Graphing Linear Equations

To plot the graph of a linear equation, you need to know the line's slope and one point on the line. The slope, which is also called the **gradient,** is how steep a line on a plane is.

Say, for example, that the slope of a linear relationship between two variables is 5. Say also that the point (3, 4) lies on the line. How can you graph the line that represents the linear equation?

First, plot the point (3, 4) on a coordinate plane. Recall that a coordinate plane is a two-dimensional surface on which you plot points located by their x- and y-coordinates.



The slope is said to represent the "rise over run." Recall from the previous lesson that the rise is the change in *y*, which goes up or down. The run is the change in *x*, which goes left or right. The slope is the rise divided by run.

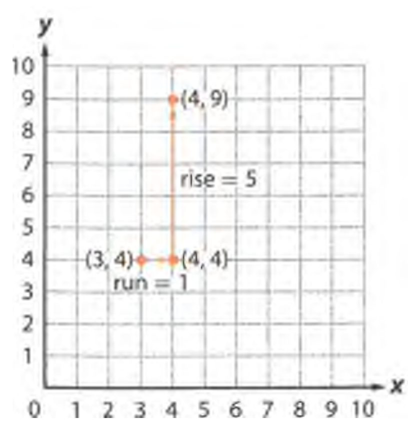
$$slope = \frac{rise}{run} = 5 = \frac{5}{1}$$

$$rise = 5$$

$$run = 1$$

In the figure to the right, start from the point (3, 4) and go to the point (4, 4). Notice that the run, or horizontal distance from point (3, 4) to point (4, 4), equals 1.

Then, go to the point (4, 9). Notice that the rise, or vertical distance from the point (4, 4), equals 5.



Core Skill
Perform Operations

When you calculate the slope of a line, you perform a combination of operations. To calculate the rise, you determine the vertical distance between two points on the line by subtracting the y-coordinate of one of the points from the other. Similarly, to calculate the run, you determine the horizontal distance between two points on the line by subtracting the x-coordinates of one of the points from the other. Then, to calculate the slope, you divide the rise by the run.

If the points on the line are represented by (x_1, y_1) and $[x_2, y_2)$ the equation for calculating the slope, m, is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Summarize what you learned above as a three-step process.

Core Skill
Interpret Graphs and Functions

You decided to save money so that you can buy a new smartphone. You start with 20 dollars in your bank account, and each month, you add an additional 30 dollars. Suppose you wanted to graph the line that represents the total amount in your bank account each month.

You know one point on the line. It is (0, 20), where the first number is the number of months—you start at zero months, and the second number is the total amount in your bank account—you start at 20 dollars.

You can also determine the slope of the line. You save 30 dollars each month, which means that the total amount in your bank account *increases* by 30 dollars for every month that goes by. Think of these values as the rise and run.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{30}{1}\ \text{month}$$

Use the point-slope formula to graph this line: $y - y_1 = m(x - x_1)$

Use the variable x to represent the number of months, and the variable y to represent the total amount in your bank account. Let the horizontal axis be the number of months and the vertical axis be the total amount in your bank account.

Use the graph of the line to predict how much money you will have in seven months.

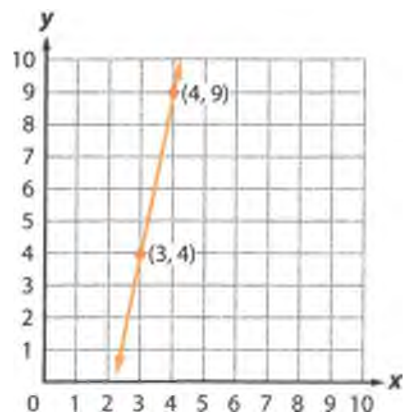
THINK ABOUT MATH

Directions: Two sets of data have a linear relationship. A graph of this relationship includes the points (5, 35) and (10, 45). What is the slope of the line that joins these points?

____

Point-Slope Form

Recall that you must have two points to define a line. Now that you have plotted (3, 4) and (4, 9), you can draw the line that connects them. This line is the graph of the linear equation.



How can you determine the equation of this line? You can use the **point-slope form** of the equation of a line:

$$y - y_1 = m(x - x_1)$$

In the point-slope form of the equation of a line, x_1 is the x-coordinate of the known point of the line, whose coordinates you were initially given. You were also given y_1 the y-coordinate of that point, and m is the slope of the line. So, for this example, you have:

$$x_1 = 3$$

$$y_1 = 4$$

$$m = 5$$

Putting these numbers into the point-slope form, you get:

$$y - 4 = 5(x - 3)$$

$$y - 4 = 5x - 15$$

$$y = 5x - 11$$

Slope-Intercept Form

There is a special case of point-slope form for the equation of a line. If the x-coordinate of the point provided is zero, then that point is the y-intercept of the line. Recall that the y-intercept is where the line **intersects,** or crosses, the y-axis. In this case, the form of the equation is called the **slope-intercept form,** and it is represented by the equation

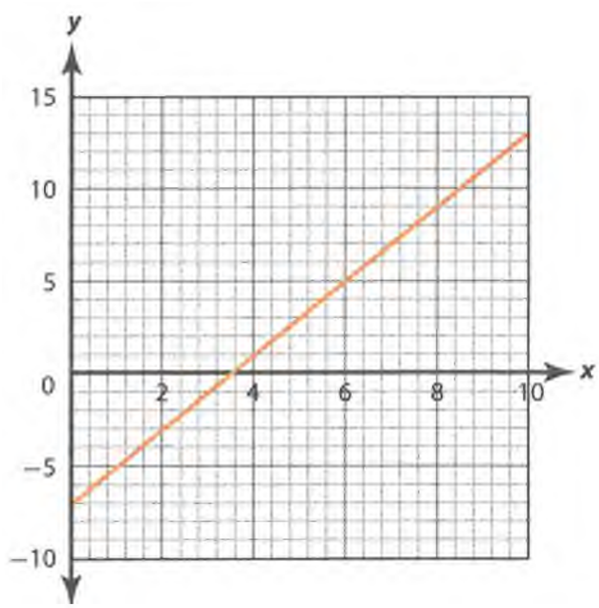
$$y = mx + b,$$

where m is the slope of the line and b is the y-intercept. The coordinate pair representing the y-intercept is $(0, b)$.

This equation is very useful when you are given the equation of a line and want to plot it. For example, say you are given the following equation:

$$y = 2x - 7$$

You can use the same procedure for graphing the line as you learned for the point-slope form. The slope, $m = 2$, and the y-intercept, $b = -7$, which means that the graph of the line crosses the y-axis at the point (0, -7), as shown below.



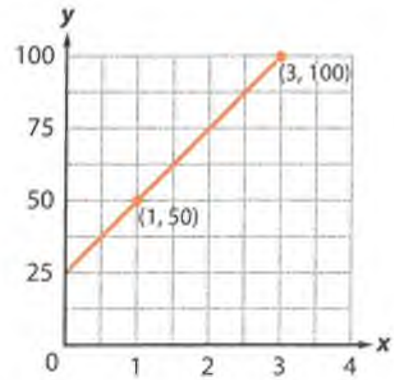
21st-Century Skills
Critical Thinking and Problem Solving

Graphs of linear equations are very useful in predicting trends that follow a linear pattern. For example, an economist determines that the increase in the cost of living in a particular city has followed a linear pattern for the past thirty years. She may use this trend to reasonably predict the cost of living in that city over the next several years. City managers can use this data to plan the city's budgets in preparation for rising costs.

Think about a regular cost you have each month. Perhaps it is purchases you make at an online music or bookstore. Or perhaps you pay a cell phone bill each month. Record the payments you have made over the last six months. Graph the data to determine if your costs are increasing, decreasing, or staying about the same each month. How can the results help you plan for future months?

Two-Point Form

You have already learned that if you know two points on a line, you can plot the graph of the linear equation. Let's say that you are given two points (1, 50) and (3, 100), and you draw the line that goes through both of them, as shown below.



How can you determine the equation of this line? You can use the **two-point form** of the equation of a line:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

In the two-point form of the equation, x_1 and y_1 are the x-coordinate and y-coordinate of the first point, and x_2 and y_2 are the x-coordinate and y-coordinate of the second point. So, for this example, you have:

$$x_1 = 1$$

$$y_1 = 50$$

$$x_2 = 3$$

$$y_2 = 100$$

Putting these numbers into the two-point form, you get

$$y - 50 = \frac{100 - 50}{3 - 1}(x - 1)$$

$$y - 50 = \frac{50}{2}(x - 1)$$

$$y - 50 = 25(x - 1)$$

$$y - 50 = 25x - 25$$

$$y = 25x + 25$$

If you compare this two-point form of the equation to the slope-intercept form, you see that the slope is 25 and the y-intercept is 25. Check the graph of the line in the figure above to see if the slope and y-intercept are correct.

Vocabulary Review

Directions: Use one of the words below to complete each sentence.

intersects   point-slope form   slope-intercept form
subscript   two-point form

1. When you know one point on a line and the line's slope, you can use the ______ to find the equation of the line.

2. The y-intercept of a line is the point where the line's graph ______ the y-axis.

3. When you use the form $y = mx + b$ to find the equation of a line, you use the__________________

4. When you have two points on a coordinate plane, you can use the ______ to find the equation of a line.

5. A letter, figure, or number is a__________________ if it is set below the line.

Skill Review

Directions: Match each form for the equation of a line with when to use it.

1. ______slope-intercept form
2. ______ point-slope form
3. ______ two-point form

A. Use this to determine the equation of a line when you know its y-intercept and slope.

B. Use this to determine the equation of a line when you know two points on a line.

C. Use this to determine the equation of a line when you know the slope of a line and a point on the line.

MATH LINK

Recall that in point-slope form, you can use a point (x_1, y_1) and a given slope m to find the equation of a straight line using the formula $y - y_1 = m(x - x_1)$. It is important not to be distracted by the subscript, the number written below and to the right of each coordinate in the coordinate pair. Remember that the subscript only indicates the coordinate pair you are given that you will use to find the equation.

For example, say you want to find the equation of the straight line that has a slope (m) of 3 and passes through the point (2, 4). You would use the point-slope form $y - y_1 = m(x - x_1)$ to find the equation:

$y - y_1 = m(x - x_1)$
$y - 4 = 3(x - 2)$
$y - 4 = 3x - 4$
$y = 3x - 2$

Skill Review (continued)

4. You're on your first trip to New York City and you take a taxi from the airport to your hotel. At the taxi stand, you see a sign that provides information about taxi fares into the city.

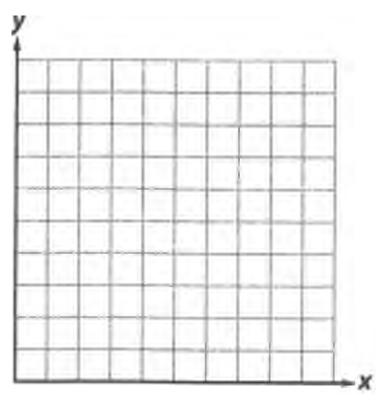
NYC TAXI SERVICE FARES

$2. 00 Airport charge
$5. 00 Starting fare

PLUS

$2. 00 per mile

Plot the line that represents the total fare, where the total fare values are on the vertical axis and the number of miles is on the horizontal axis. Use the variable y to represent the total fare and use the variable x to represent the number of miles. Plot the graph for x = 0 to 10 miles. What is the equation for this line? Express the equation using slope-intercept form.



Skill Practice

Directions: Answer the following.

1. A classmate tells you that it is possible to graph a line given only its slope. Explain whether this statement is correct.

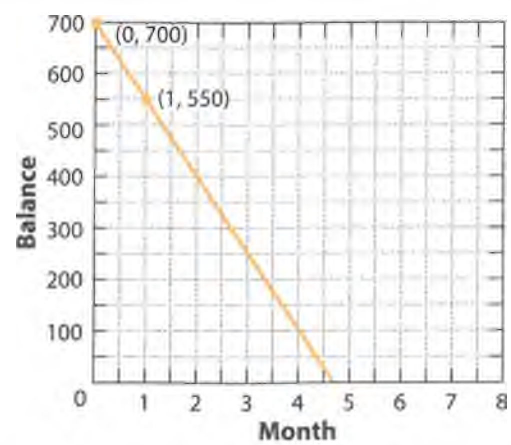
____

____

____

Directions: Use the graph below to answer the questions.

You recently took advantage of a zero-interest credit card offer to purchase a new mountain bike that costs $700. Each month you pay either $150 or the balance, whichever amount is lower. The graph of the line is plotted, where the vertical axis is the balance owed and the horizontal axis is the number of months.



2. How many months it will take to pay off the credit card? Refer to the graph to explain your answer.

____

____

____

3 Calculate the slope of the graph. Explain what the slope represents. Use the terms *rise* and *run* in your explanation.

____

____

____

4 What is the y-intercept of the graph? What does it represent?

____

LESSON 6. 3

Pairs of Linear Equations

Lesson Objectives

You will be able to

- Solve systems of two linear equations
- Interpret graphs of two linear equations
- Use linear equations to solve problems

Skills

- **Core Skill:** Solve Pairs of Linear Equations
- **Core Skill:** Solve Simple Equations by Inspection

Vocabulary

addition method
eliminate
equilibrium point
simultaneous
system of simultaneous linear equations
substitution method

KEY CONCEPT: A pair of linear equations forms a system of two simultaneous linear equations. The solution to a system of two linear equations in two variables corresponds to a point of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

In previous lessons, you graphed lines in separate graphs to represent specific linear relationships and represented the linear relationship between two variables as a linear equation.

Two variables have a linear relationship if their corresponding points lie on the same line in the coordinate plane, the two-dimensional surface on which you can plot points by a pair of x- and y-coordinates. Every pair of (x, y) coordinate points on the line is a solution to the linear equation that represents the line. These values make the relationship between the left and right sides of the equation sign true. These points satisfy the equation.

Simultaneous Linear Equations

Two equations that are satisfied by the same set of variables form a system of two simultaneous linear equations. The word *simultaneous* means "occurring at the same time." A **system of simultaneous linear equations** is a collection of linear equations. To solve a system, look for the values of the variables that make all of the equations true simultaneously. If you graph each of the linear equations in the set, the point where they intersect is a single common solution, meaning that the point lies on each of the lines. Simultaneous linear equations can be mathematical objects or models of the real world.

A Real-World Model

You may find yourself making a decision that requires you to compare two financial alternatives. Often, these alternatives can be represented by a linear relationship. For example, imagine you are purchasing a text-messaging plan. You have two options:

Option 1: Pay a flat fee of \$5. 00 for the first 200 text messages, and \$0. 20 for each message thereafter.

Option 2: Pay a flat fee of \$20. 00 for the first 200 text messages, and \$0. 05 cents for each message thereafter.

Which option is better for you? How can you tell? You can use the descriptions of each option to write the equation of a line.

Option 1: $y = 0.20x + 5$
Option 2: $y = 0.05x + 20$

To tell which option is better, you calculate costs for a given number of messages. You select the number 210 to represent the number of text messages sent in a given month, meaning you exceeded the text-message allowance by 10 messages.

Option 1: $y = 0.\ 20x + 5$
Total text-message charge = \$5. 00 + (\$0. 20 x 10) = \$5. 00 + \$2. 00 = \$7. 00

Option 2: $y = 0.\ 05x + 20$
Total text-message charge = \$20. 00 + (\$0. 05 x 10) = \$20. 00 + \$0. 50 = \$20. 50

Option 1 appears to be a better plan if you send 210 messages. However, you decide to test the plans using a different number of text messages. This time, you decide to send a total of 400 text messages. That's 200 more messages than your plan includes without an extra cost. You can write the number 200 in the place of x in the equation for each option and solve each equation to determine the cost.

Option 1: $y = 0.\ 20x + 5$
Total text-message charge = \$5. 00 + (\$0. 20 x 200) = \$5. 00 + \$40. 00 = \$45. 00

Option 2: $y = 0.\ 05x + 20$
Total text-message charge = \$20. 00 + (\$0. 05 x 200) = \$10. 00 + \$20. 00 = \$30. 00

If you send lots of text messages per month, Option 1 will cost \$15. 00 more than Option 2. It seems that the number of text messages you anticipate sending will determine which option is more economical for you.

You can look at these alternative options as a system of related equations.

Now suppose that you decide that a graph of the alternative cost plans will help you compare them more closely. You can use lines on the graph to determine the text-message charge for any number of text messages *without* doing calculations. You know that a point on a line satisfies its linear equation. So, a common point of intersection will tell you how many sent messages will result in equal costs.

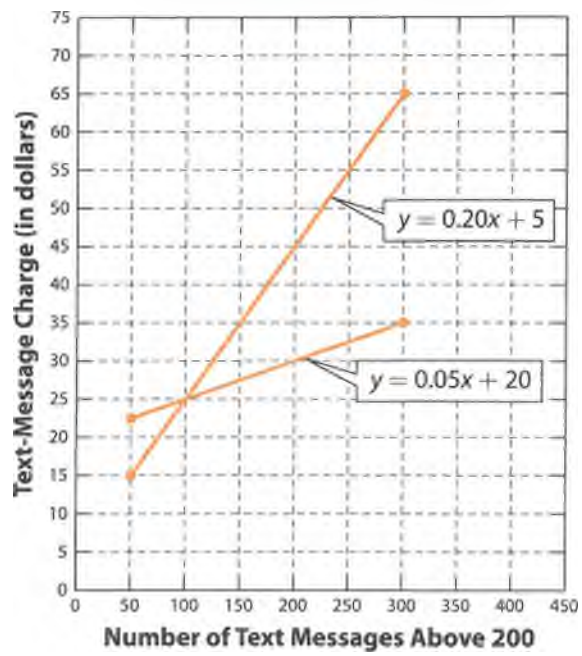
The following table displays the information for the total text-messaging costs for the two plans you are considering.

Number of Messages Above 200	Option 1 Cost (\$)	Option 2 Cost (\$)
0	5. 00	20. 00
100	25. 00	25. 00
150	35. 00	27. 50
200	45. 00	30. 00

To draw their lines, plot two points for each plan on the coordinate plane. Plot the points where the horizontal axis represents the number of text messages exceeding each plan's 200-message plan, and the vertical axis represents the cost in dollars. Note the point of intersection of the lines.

You can use the lines to determine the text-message charge for any number of text messages greater than 200 *without* having to do the calculations. The lines intersect at the point (100, 25). This point is the solution of both lines, and it is called the **equilibrium point.** It shows where both plans cost the same amount, or \$25. 00. If you follow the lines upward from this point, the difference in costs becomes more obvious.

TEXT-MESSAGE CHARGES



THINK ABOUT MATH

Directions: Say that you planned to share your phone and the phone cost with a family member. Each of you is likely to send 250 messages per month. If you pay half of the phone bill, what would you pay under each plan?

Option 1: ______________________________

Option 2: ______________________________

Core Skill
Solve Pairs of Linear Equations

The substitution method provides a direct way to solve a pair of linear equations without graphing. In the substitution method, you solve one of the equations for one of the variables and then substitute that solution into the other equation. For example, given $3a + 4\mathbf{b} = 5$ and $a - b = 3$, you can solve for a in terms of **b** in the second equation. Then, you can substitute the value of a into the first equation to solve for **b.** Finally, you can substitute the value found for **b** into an equation to solve for a.

1. $3a + 4b = 5$

2. $a - b = 3$
$a = 3 + b$

3. $3(3 + b) + 4b = 5$
$9 + 3b + 4b = 5$
$7b + 9 = 5$
$7b = -4$
$b = -\frac{4}{7}$

4. $a - (-\frac{4}{7}) = 3$
$a + \frac{4}{7} = 3$
$a = 3 - \frac{4}{7}$
$a = 2\frac{3}{7}$

Use the substitution method to find the solution to this linear pair:

1. $3a + 4\mathbf{b} = 5$
2. $a + \mathbf{b} = 3$

Combining Methods to Solve Pairs of Linear Equations

Earlier in the lesson, you graphed lines to represent a system of simultaneous linear relationships. There may be cases where it is preferable to solve the system without graphing.

The **addition method** is based on the principle that adding the same value to each side of an equation does not change the equality of that relationship. The addition method is also known as the elimination method because it uses a process to **eliminate,** or cancel out, variables to solve an equation. Consider this pair of equations:

$$2x + 3y = -12$$
$$2x - 3y = 4$$

If you add the equations, the terms with ***y*** cancel out, or add to 0:

$$\begin{aligned} 2x + 3y + 2x - 3y &= -12 + 4 \\ 4x &= -8 \\ x &= -2 \end{aligned}$$

To find ***y,*** use the substitution method you applied earlier in the lesson. Substitute ***x*** = -2 in the first equation:

$$\begin{aligned} 2(-2) + 3y &= -12 \\ -4 + 3y &= -12 \\ 3y &= -8 \\ y &= -\tfrac{8}{3} \end{aligned}$$

Now substitute $(-2, -\frac{8}{3})$ in the second equation to check your answer:

$$\begin{aligned} 2(-2) - 3(-\tfrac{8}{3}) &= 4 \\ -4 + 8 &= 4 \\ 4 &= 4 \end{aligned}$$

Core Skill
Solve Simple Equations by Inspection

Not all pairs of equations have a single solution. Consider the equations 3 P + 2*q* = 4 and 3p + 2 *q* = 5. There is no single solution because 3p + 2q cannot simultaneously equal both 4 and 5. If you graphed these equations, you would draw parallel lines. Their slopes would be the same, but their y-intercepts would differ.

Now consider the equations 3p + 2*q* = *4* and 6p + 4q = 8. Notice that you can divide both equations by the common factor 2. So, these equations are equivalent. Consequently, there are an infinite number of solutions. If you graphed the equations, you would see they are identical lines, meaning their slopes and intercepts are the same.

Examine the following pair of equations: x + y = 1 and 3x + 3y = 3. How would you describe the lines that would appear on a graph of these equations?

MATH LINK

To determine if using the addition method is an appropriate way to eliminate a variable, think first about whether you can manipulate one or both of the equations to eliminate one of the variables. Consider the following example:

$3a + 2b = 4$
$4a + 2b = 5$

To eliminate a variable, you can multiply one equation by -1.

$-(3a + 2b) = -4$
$-3a - 2b = -4$
$4a + 2b - 3a - 2b = 5 - 4$
$4a - 3a = 1$
$a = 1$

Vocabulary Review

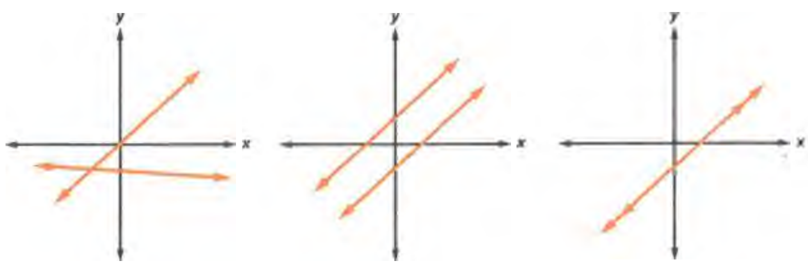
Directions: Match each term to its example.

1. __addition method
2. _______eliminate
3. _______equilibrium point
4. _______simultaneous
5. _______substitution method
6. _______system of simultaneous linear equations

A. cancel out
B. keeps the equality relationship between both sides of an equation
C. occurring at the same time
D. a set of equations that are satisfied by the same set of variables
E. the coordinate pair that represents a common solution to two simultaneous equations in two variables
F. a technique for solving simultaneous equations by first solving for one variable in terms of the other

Skill Review

Directions: Indicate whether the systems of two linear equations have 0, 1, or an infinite number of solutions. Explain your answers.



1 2 3

1. ____

2. ____

3\-

Directions: Read the problem. Then answer the questions that follow.

A portion of \$100, 000 (x) was invested with a return of 3 percent after one year. The remainder of the investment (y) was invested at a return of 1 percent. The total return on the investment was \$1, 800.

4. What is the equation that shows the investment yield?

____

Skill Review (continued)

5. What is the equation that shows the way the $100, 000 was split?

____

6. How much money was invested at a 3 percent rate of return?

____

7. How much money was invested at a 1 percent rate of return?

____

Skill Practice

Directions: Solve each pair of linear equations.

1. $x + y = 20$
   $x + y = 40$

   A. $x = 120; y = -100$
   B. $x = 30; y = -10$
   C. infinite solutions
   D. no solution

2. $5a - 4b = 12$
   $3a + 4b = 20$

   A. $a = 2; b = 4$
   B. $a = 4; b = 2$
   C. $a = 4; b = -2$
   D. $a = -2; b = 4$

3. $5a - 3b = 12$
   $3a - 5b = 14$

   A. $a = \frac{17}{8}, b = \frac{29}{8}$
   B. $a = \frac{29}{8}, b = \frac{17}{8}$
   C. $a = \frac{-9}{8}, b = \frac{17}{8}$
   D. $a = \frac{9}{8}, b = -\frac{17}{8}$

Directions: Read the problem. Find the solution.

4. Part of a $20, 000 investment was invested at 6 percent return rate. The remainder of the investment was invested at 4 percent return rate. The total return was $1, 000. Write a pair of equations that can be used to answer how much was invested at each rate.

____

____

LESSON

6.4

Scatter Plots

Lesson Objectives

You will be able to

- Describe the information that a trend line provides about two correlated variables
- Describe various aspects of the correlation between two variables

Skills

- **Core Skill:** Represent Real-World Problems
- **Core Skill:** Interpret Data Displays

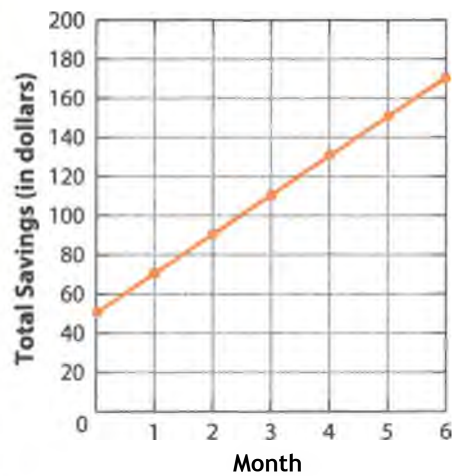
Vocabulary

cluster
correlation
outlier
scatter plot
trend line

KEY CONCEPT: We can use the concept of correlation to describe the relationship between two variables that generally follow a linear pattern but cannot be described by a linear equation. Plotting data on a scatter plot and constructing a trend line can determine the strength and direction of the correlation between such variables.

Up to now, we have used equations to define linear relationships between two variables. For example, say you begin saving money to buy a new set of noise-canceling headphones. You start with $50 *and then save $20 per month. You can plot the data as shown below.*

Month	Total Savings (in dollars)
0	50
1	70
2	90
3	no
4	130
5	150
6	170



You can write the equation for this line:

$$y = 20x + 50$$

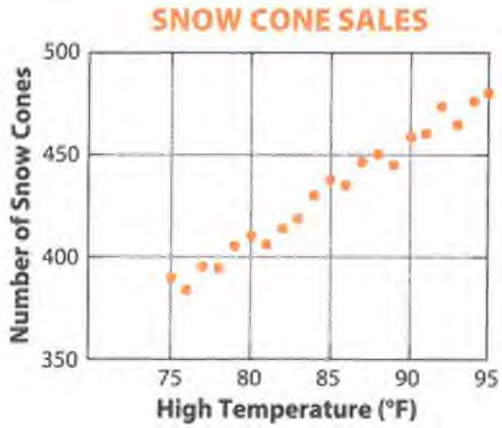
where y *is the total savings and* x *is the month. If you look carefully at the graph, you will notice that alt of the points lie exactly on a line that connects them.*

Scatter Plots

Sometimes you will encounter two sets of variables that result in a graph whose data points do not lie *exactly* on a line but may be close to a line. Let's look at an example.

The owner of a snow cone stand records the average number of snow cones that she sells, based on the forecasted high temperature of the day. The data are shown on page 191 in a table and a plot of each coordinate pair.

The plot makes it easy to see that the data points do not all lie on the same line. Instead, they appear to be scattered. Therefore, graphs such as these are called **scatter plots.**

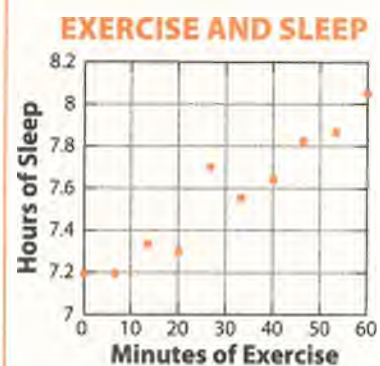


In this scatter plot, all of the data points are so close you can draw a line right through them. Such a line is called a **trend line,** and it graphically shows the relationship between daily high temperatures and the number of snow cones sold.

Daily High Temperature (°F)	Average Number of Snow Cones Sold
75	389
76	384
77	395
78	394
79	405
80	410
81	406
82	413
83	418
84	430
85	437
86	435
87	446
88	450
89	445
90	459
91	460
92	473
93	465
94	476
95	481

THINK ABOUT MATH

Directions: Draw an approximate trend line for the following scatter plot that shows the results of a study on the daily amount of exercise and the amount of sleep that a person gets.



If you look at all of the data points on the scatter plot, you see that some are above the trend line, some are below the trend line, and some lie almost exactly on the trend line.

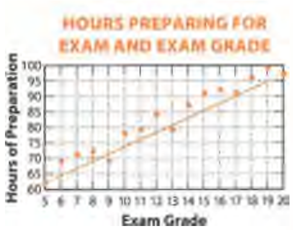
Now, imagine the values of the vertical distance between all of the points and the trend line. The trend line is the line for which the overall distance between the points and the trend line is minimized.

The data for the daily high temperature and the number of snow cones that are sold follow a close linear pattern, or **correlation.** Therefore, you can say that there is a **linear correlation** between the variables of daily high temperature and the number of snow cones sold.

Core Skill
Represent Real-World Problems

Scatter plots are one means of recording data, but the data have value only if you analyze them and look for relationships between variables. The positions of points on a scatter plot tell a story. It is a story of relationship. Drawing a trend line through the points is a quick and effective means of determining whether a relationship exists and how close it is.

Imagine that an education specialist wanted to know if there is a relationship between the number of hours a student studies and the student's exam grade. The specialist surveyed a group of math students to collect data and recorded the data in scatter plot.



Examine how the points fall along or near the trend line. These points and the trend line tell a story about the relationship between two variables. How would you describe that relationship?

Linear Correlations

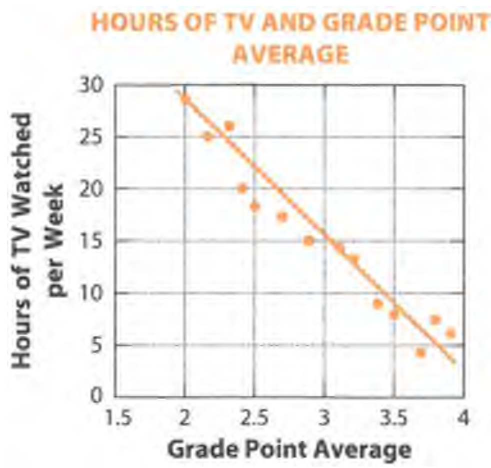
When relationships exist between two variables, those relationships can be described as positive or negative. Let's consider positive correlations first.

Positive and Negative Correlations

Recall the scatter plot that showed the relationship between the daily high temperature and the number of snow cones sold. As temperatures increased, so did sales. Since both variables increased together, the slope of the trend line is positive—therefore, you can say there is a **positive correlation** between the two variables.

It is possible for two variables to be negatively correlated, also. Consider the relationship between the variables of the number of hours a student watches television and the student's overall grade point average.

The following scatter plot shows the data collected from a student survey. Look closely at the points and the trend line.



A relationship between the two variables is plain to see. The points lie close to or on the line. However, the trend line for this set of data has a negative slope, meaning that as the number of hours of watching television increases, the grade point average decreases. The scatter plot shows a **negative correlation** between the variables.

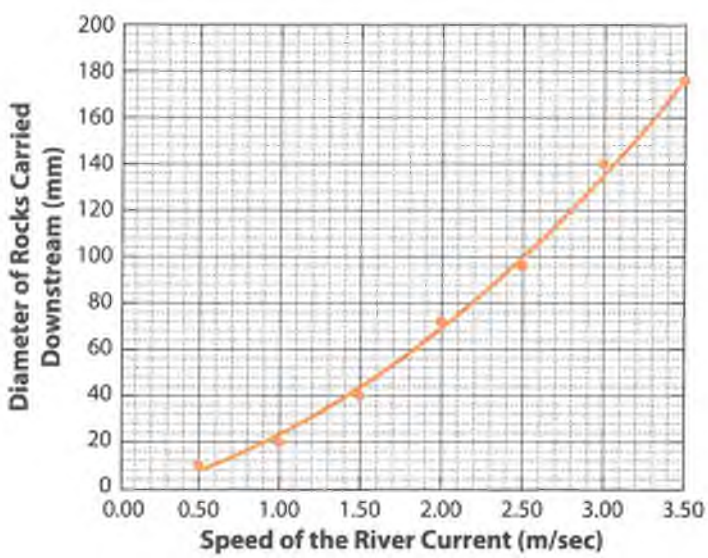
Nonlinear Correlations

You can investigate data that approximately follow a linear pattern, revealing a linear correlation between the variables. It is also possible to investigate and display data that approximately follow nonlinear patterns. Here are two examples.

Quadratic Model: Sizes of Rocks Moved by River Currents

A hydrologist works in a national park. She wants to know how the speed of a river's current determines the sizes of rocks that the river's current can carry downstream. She collects and graphs the data.

Speed of the River Current (m/sec)	Diameter of Rocks Carried Downstream (mm)
0. 5	10
1. 0	19
1. 5	40
2. 0	72
2. 5	96
3. 0	140
3. 5	175



Just as with data that closely follow a linear pattern, we can draw a trend line. Here, the trend line is a **quadratic curve,** which means that the size of the rocks that the river can carry downstream is proportional to the speed of the river raised to the second power. *Quadratic* is an algebraic term that refers to the square of an unknown quality. Thus, the size of the rock will increase much more quickly than the speed of the river.

Core Skills
Interpret Data Displays

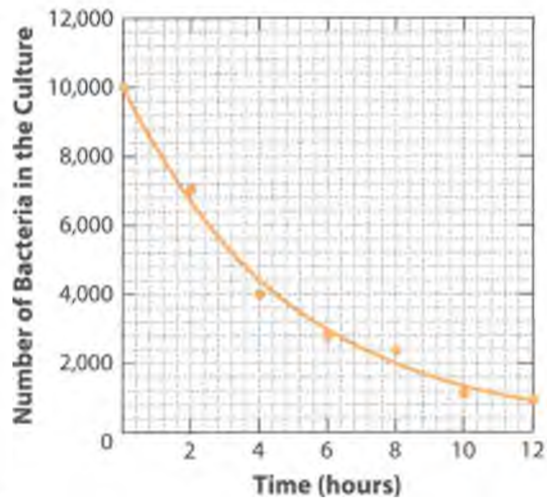
Imagine as an example that a web-based advertising company used two different styles of advertisements to promote the sales of their online products. The company wanted to know which advertisement style resulted in more sales. So, they collected data for each and plotted the data in a scatter plot.

You can draw an approximate trend line for each scatter plot. Explain what the shape of each trend line tells you about the relationship between the number of websites that uses each advertising style and sales.

Exponential or Power Model: Bacteria Response to an Antibiotic Treatment

A medical laboratory assistant tests how bacteria respond to a new type of antibiotic. He wants to know how quickly the antibiotic kills bacteria once it is added to a bacterial culture. He records and graphs the results.

Time (hours)	Number of Bacteria in the Culture
0	10, 000
2	7, 075
4	4, 018
6	2, 818
8	2, 333
10	1, 071
12	908



Here, the trend line is a "decaying" **exponential curve,** which means that the trend line decreases more rapidly at first and decreases less slowly as time increases. It is called an exponential curve because, when written as a formula, the variable is not in the base of the function (x2) but in the exponent (2x). The exponential curve can also be called a power curve.

Correlation Strength

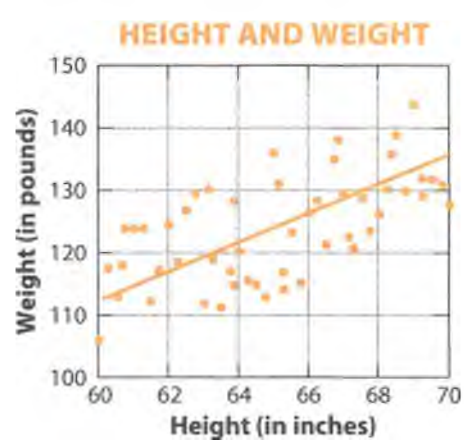
In addition to the direction of a correlation (positive or negative) and the form of a correlation (linear or nonlinear), you can also describe a correlation in terms of its strength.

Earlier in this lesson, you looked at a scatter plot that showed the relationship between the daily high temperature and the number of snow cones sold. The data points were very close to the trend line. Therefore, we say that a **strong correlation** exists between the two variables. Let's look at a different example to distinguish between a strong and a weak correlation.

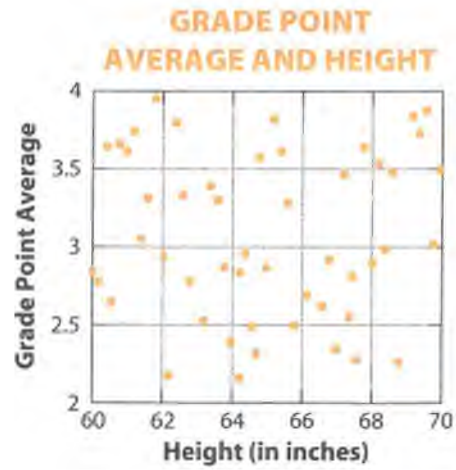
A physical education specialist surveys students to find the relationship between students' height and their weight.

The data points are not as close to the trend line as they were for the snow cone example above. Therefore, we say that a **weak correlation** exists between the two variables.

You may also encounter data that does not seem to follow any pattern. For example, you decide to do a survey to see if there is any relationship between students' grade point average and their height.



The data in the scatter plot show **no correlation.** Therefore, there is no trend line. There is no relationship between the variables of grade point average and height.



Outliers and Clusters

Sometimes the points in a scatter plot either do not appear where you expect them to be, or they do not appear uniformly along the trend line.

A business manager sells active wear for young adults. He studies purchasing records to determine if there is a relationship between the height and weight of his top customers. He recorded the data in a scatter plot like the one below.

The data are positively correlated, but there are several interesting features of this scatter plot. Notice the point that is clearly far away from the trend line. Such a point is called an **outlier.** Also, notice that the data points fall within two distinct groups, or **clusters.**

THINK ABOUT MATH

Directions: Look again at the scatter plot of the weight and height of the young adult customers. Although there are clusters and an outlier in the scatter plot, these hardly affect the trend line. Identify the pattern the data points follow and explain the pattern's meaning. In other words, explain how height and weight change together.

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MATH LINK

Sometimes outliers and clusters appear on a scatter plot as the result of an error in recording the data. However, this may not always be true, so it is important to investigate outliers and clusters carefully to be sure that they are the result of errors.

In the manager's scatter plot, the outlier is not an error. It represents a young man who is a freshman linebacker for a high school football team. The young man's weight is greater than a typical person of his height.

Also, the clusters in the scatter plot are not errors. After double-checking, the manager was surprised to learn that none of his customers has a height between 65. 5 and 68 inches.

Vocabulary Review

Directions: Choose a term to complete each sentence.

cluster correlation linear correlation negative correlation nonlinear correlation outlier positive correlation scatter plot trend line

1. If the value of one variable increases while the value of the second variable decreases, there is a________________ between the variables.

2. If the value of one variable increases as the value of the other variable increases, there is a_______________ between the variables.

3. A________________is a visual display of the relationship between two variables.

4. If two variables follow a clearly recognizable pattern, then there is a______________________ between the two variables.

5. If points in a scatter plot increase or decrease proportionally, then there is a between the variables that they represent.

6. A________________is located further away from the trend line than the other points in a scatter plot.

7. A________________is a grouping together of points on a scatter plot.

8. If the trend line on a scatter plot is exponential or quadratic, then there is a ________ ______ between the variables.

9. The line or curve around which the points in a scatter plot appear is called a _______________

Skill Review

Directions: Read the text. Then complete the activity.

1. How many outliers appear in the following scatter plot? Explain how you determined which points were outliers.

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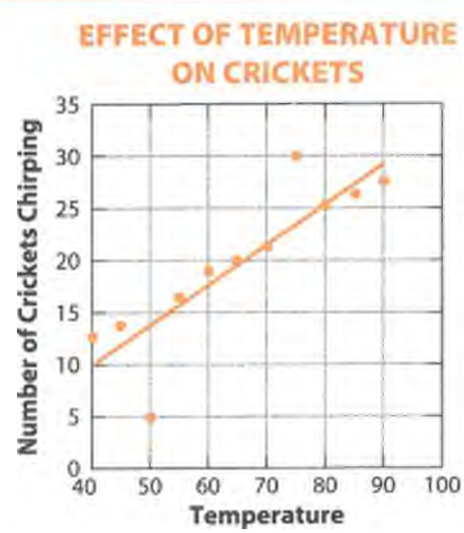
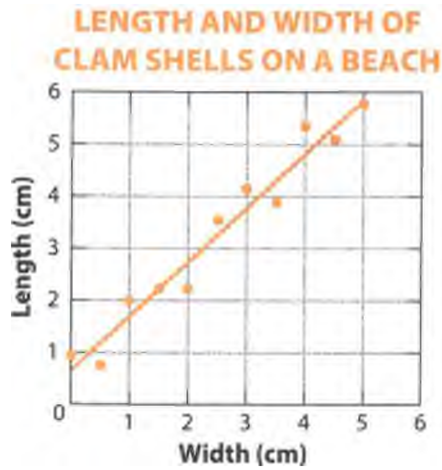


Skill Review (continued)

2. The following scatter plot shows the length and the width, measured in centimeters, of clamshells found along a beach.



Describe the correlation between the variables of length and width.

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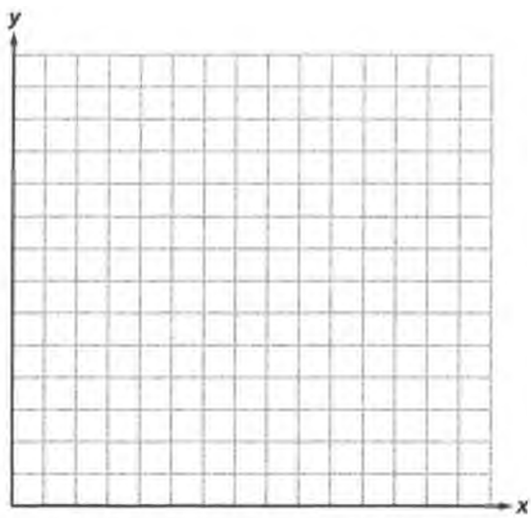
3. The data in the following table show the number of online posts to a music website and the number of recordings customers download daily. Create a scatter plot of these data and draw the trend line that you feel best fits the data.



Social Media Posts	Music Downloads
10, 000	900
20, 000	1, 300
25, 000	1, 800
30, 000	1, 500
35, 000	1, 600
40, 000	2, 100
50, 000	2, 200
55, 000	2, 700
60, 000	2, 500
65, 000	2, 600
70, 000	2, 800
75, 000	2, 900
80, 000	2, 800
90, 000	3, 000
95, 000	3, 100

Skill Practice

Directions: Read the text. Then complete the activity.

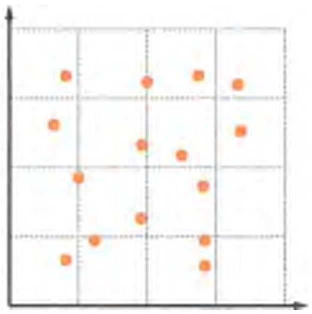
1. Match the descriptions of correlations with their corresponding scatter plots.

_______A. strong negative correlation

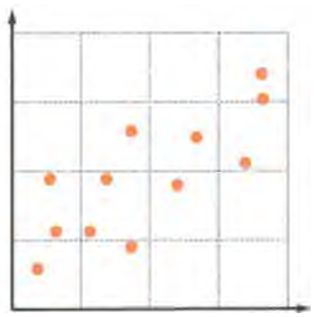
_______B. no correlation

_______C. weak positive correlation

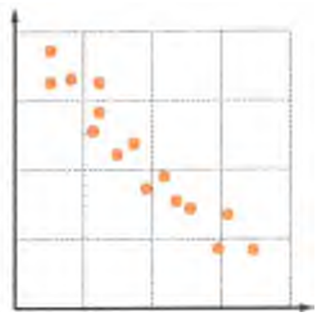
1.



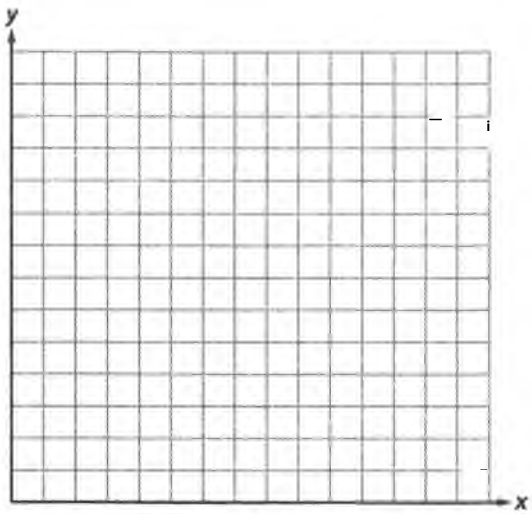
2.



3.



2. A health official collected data on the number of flu shots clinics gave in a month and the number of patients clinics treated for flu in the same month. The official put the data in the table on the right. Use the data to build a scatter plot. Draw a trend line through the data points.



Flu Shots Given	People with Flu
7, 846	6, 832
7, 994	6, 971
8, 216	6, 726
8, 364	6, 453
8, 631	6, 238
8, 772	5, 807
8, 901	5, 563
9, 114	5, 019

Use the trend line to identify the kind of relationship between the variables.

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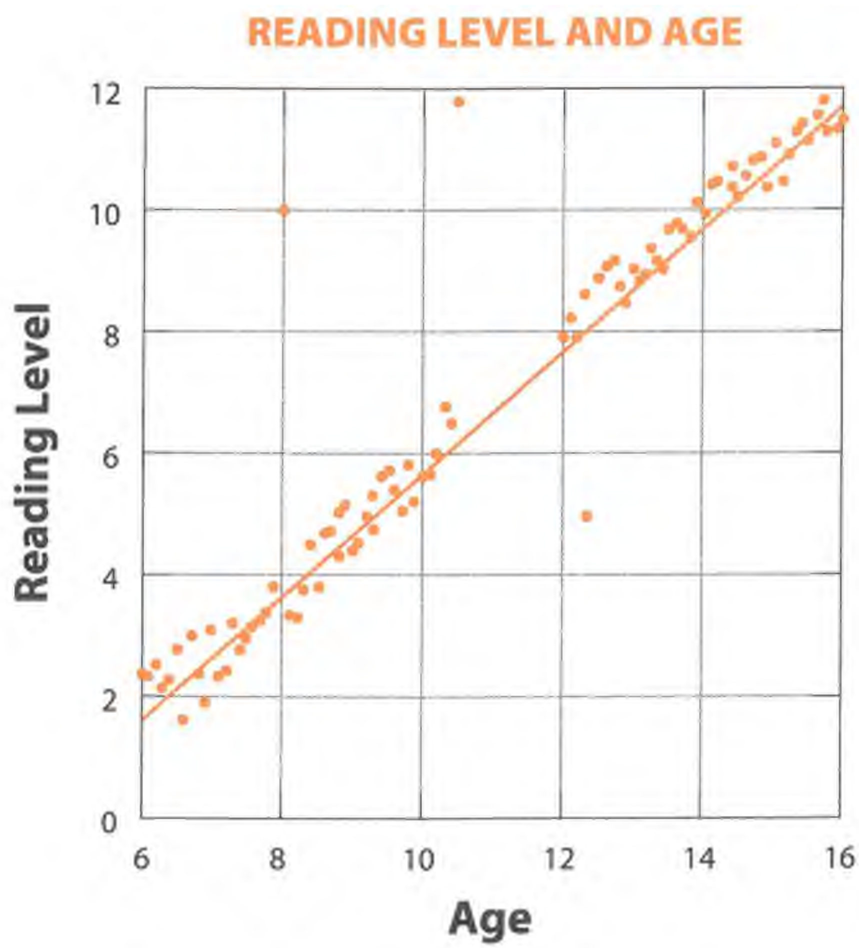
3. A Reading Specialist is doing research on the age and reading levels of children and adolescents. He collects the data shown in the scatter plot below. Identify any clusters in the data by circling them. Also, identify any outliers. Summarize the meaning of each outlier. Provide a possible explanation of what the outliers might indicate.



LESSON

6. 5

Functions

Lesson Objectives

You will be able to

- Identify a function
- Determine whether an equation represents a function

Skills

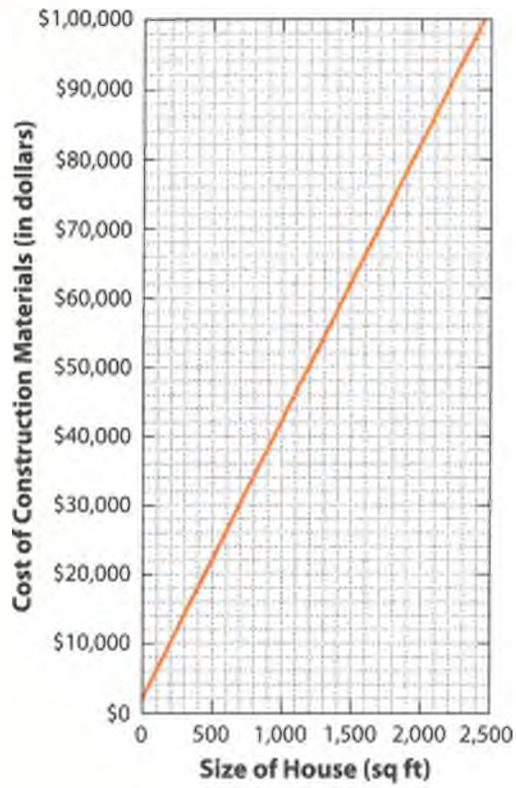
- **Core Skill:** Build Lines of Reasoning
- **Core Skill:** Interpret Graphs and Functions

Vocabulary

function
input
linear function
nonlinear function
output
vertical line test

KEY CONCEPT: You can look at a function as a set of instructions that tells you what to do with the input, or values you put in. The result of the instructions is called the output. Functions are equations that provide only one output for each input.

In a previous lesson, you learned about linear equations and some of the ways that they are used in different applications. For example, imagine you are a builder. You can use a linear equation to determine the cost of construction materials (in dollars) for any given size house (in square feet). The linear equation is



$$y = 40x + 2,000$$

where x *is the size of the house (in square feet) and* y *is the cost of the construction materials. Examine the graph of this linear equation.*

What Is a Function?

A **function** is a mathematical equation that has two variables, an **input** variable that goes into the equation and an **output** variable that results from the input. A rule to remember is that for each input, a function has only one output.

You can think of a function as a set of instructions that tells you how to take the input and use it to calculate the output. Mathematicians often describe a function as a "black box, " like the one in the drawing below. Think of it as a computer that takes in the input value, follows the instructions on what to do with the input value, and produces an output value. In the drawing, the input is labeled x and the output is labeled *y*.

Is It a Function?

Remember that the definition of a function states that for each input, there is only one output. This definition will help you determine whether or not an equation represents a function.

At the beginning of this lesson, we looked at a linear equation that related the size of a house to the cost of the construction materials required to build it:

$$y = 40x + 2000$$

where x is the size of the house (in square feet) and y is the cost of the construction materials. If you select a value for the size of the house, you can calculate the cost of the materials. Say the value of x = 1, 500 square feet. Now you can calculate the value of y.

$$y = (40 \times 1500) + 2000 = 60,000 + 2,000 = 62,000$$

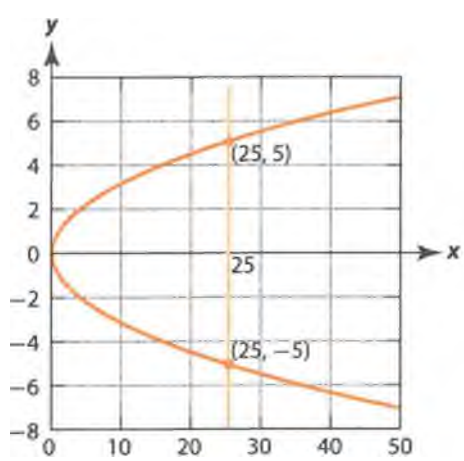
So, the cost of the construction materials to build a 1, 500 square foot house is $62, 000. The equation provides only one possible answer for each size of house. This is also evident from the graph of the linear equation. If you select any value of x, there is only one possible value of y. So, for each input, or house size, there is only one output, or cost of materials. Therefore, the linear equation y = 40x + 2, 000 is a function.

Now, consider the following equation

$$y = \sqrt{x}$$

If x = 25, then the value of y is 5, since 5 x 5 = 25. But the value of y could also be -5, since -5 X -5 = 25. So, for this equation, there are two possible answers. In other words, for any input x, there are two possible values for the output, y. This violates the definition of a function, which states that there is only one output for each input. Therefore, this equation is **not** a function.

Look at the graph of this equation, which shows the two values of y when x = 25. Note that because these points have the same x coordinate, they lie on the same vertical line.



Core Skill
Build Lines of Reasoning

You have already worked with functions, without realizing it. A linear equation is an example of a function. For example, in a previous lesson, you calculated the temperature in degrees Fahrenheit when given the temperature in degrees Celsius. The linear equation for this function is

$$F = 1.8C + 32$$

The variable C, the temperature in degrees Celsius, is the input. This is the value that we know. The variable F, the temperature in degrees Fahrenheit is the output. This is the value that we want to calculate.

Return to the idea of a function as a "black box" or computer. Imagine that when you type in the temperature in degrees Celsius, the computer displays the temperature in degrees Fahrenheit. Let's call this computer a "temperature converter. "

What "instructions" does the temperature converter follow? Use the linear equation $F = 1.8C + 32$ to write the instructions.

WORKPLACE CONNECTION

Computer Programming Input, Output Values

In addition to numerous applications in science and mathematics, the concept of a function is very important in computer programming. Computer scientists use the concept of a function when specifying input and output values for a computer program.

For example, a computer programmer writes a program for teachers. The program converts numerical grades (input values) into letter grades (output values). Without knowing the specific programming that makes the program work, a teacher is able to type in any numerical value and receive an equivalent letter grade. So, the computer program works like a "black box, " in that it accepts any input from the user and delivers an output.

You can do a **vertical line test** to determine if an equation is a function. Examine the line of the graph in the figure on the previous page. Then, observe the vertical line $x = 25$. Notice that it intersects the graph of the equation at two points. This indicates that there are two output values, 5 and -5, for one input value, 25. Because the vertical line crosses the graph at more than one point, the equation is not a function. It violates the definition of a function that there can be only one output for each input.

Function Categories

Functions can be classified into two broad categories—linear functions and nonlinear functions. You have been working with linear functions throughout this chapter. A **linear function** has the form

$$y = mx + b$$

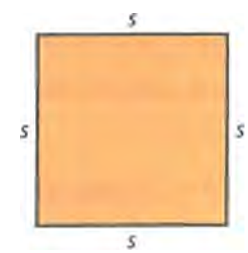
where m is the slope of the line and b is the y-intercept.

If a function does not have this form, then it is a **nonlinear function.** Let's look at two examples to illustrate the difference between linear and nonlinear functions.

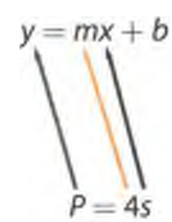
Perimeter of a Square

The perimeter of a square is the sum of the lengths of all its sides. The equation that represents the perimeter, P, is

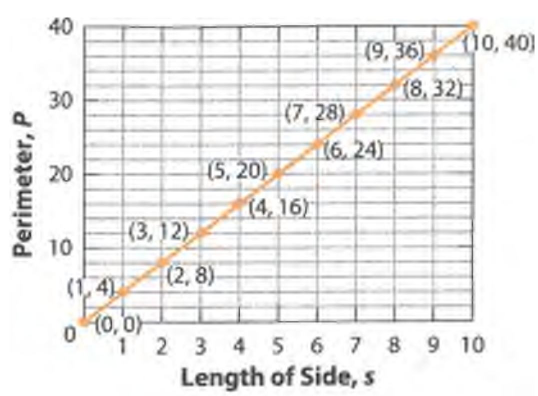
$$P = s + s + s + s$$
$$P = 4s$$



where s is the length of each side. Because it has the form $y = mx + b$, this equation represents a linear function.



The slope is 4 and the y-intercept is 0. Look at the graph of this function. Note that all of the points lie on a straight line, thus confirming that the formula for finding perimeter, P = 4s, is a linear function.

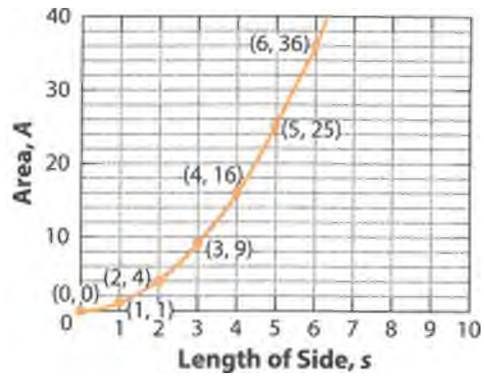


Area of a Square

The area of a square is the product of the length of two of its sides. The equation that represents the area, A, is

$$A = s \times s$$
$$A = s^2$$

You can graph this function, as you see. Note that the points do not lie on a straight line. Thus the equation $A = s^2$ is a nonlinear function. More specifically, it is a quadratic function.



Core Skill
Interpret Graphs and Functions

When you are asked to graph a function or interpret the graph of a function, it is always helpful to look at its equation, if it is available. You can look at the form of the function to determine whether it is linear or nonlinear. Remember, if the function has the form

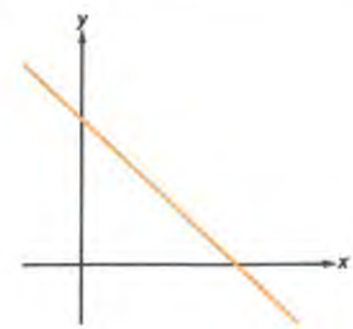
$$y = mx + b$$

then the function is linear—if it does not have that form, then the function is nonlinear.

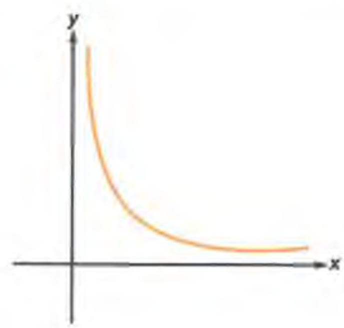
THINK ABOUT MATH

Directions: Label each graph as a Function or Not a Function.

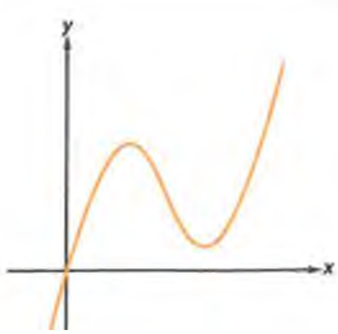
A. ____________________



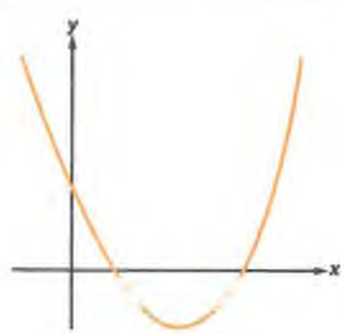
C. ____________________



B. ____________________



D. ____________________



MATH LINK

The circumference of a circle, C, is given by the equation $C = 2\pi r$ and the area of a circle, A, is given by the equation $A = 7\pi r^2$. Explain how you can determine whether each of these functions is linear or nonlinear, without performing any calculations or plotting their graphs.

Hint: You can compare each of these equations with the form of a linear function. Remember that the equation of a linear function is $y = mx + b$, where y is the dependent variable and x is the independent variable. If you can rewrite one or both of the equations in the form $y = mx + b$, it is a linear function. If you cannot rewrite one or either of the equations in the form $y = mx + b$, it is not a linear function.

Vocabulary Review

Directions: Use the following terms to complete each sentence. Note that some terms may be used in more than one sentence.

function input linear function nonlinear function output
vertical line test

1. An equation is a____________ if there is only one__________________ for each ________________

2. A________________has the form $y = mx + b$.

3. The points of a_________________ are not all on a straight line.

4. A________________can help determine whether the graph of an equation is a_________________or not.

Skill Review

Directions: Circle the best answer to each question.

1. You can think of a function as a
   - A. point on a line
   - B. set of instructions
   - C. set of geometric shapes
   - D. collection of mathematical questions

2. The vertical line test helps you determine whether
   - A. a function is linear or nonlinear
   - B. a graph represents a function
   - C. a function is a "black box"
   - D. an equation is graphed correctly

Directions: Read the problem. Then follow the directions.

3. Imagine you are programming a computer. Write the step-by-step "instructions" for the following nonlinear function. Include the words *input* and *output* in your instructions.

$$y = x^2 + 2x + 1$$

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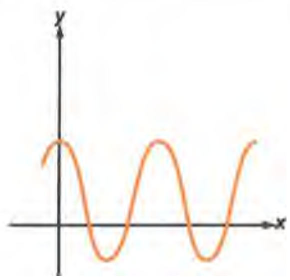
Skill Practice

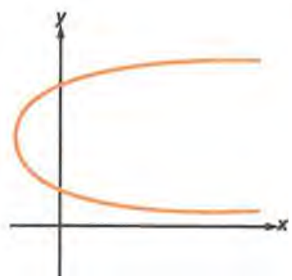
Directions: Read the problem. Then follow the directions.

1. Label each graph of an equation as a Linear Function or a Nonlinear Function.

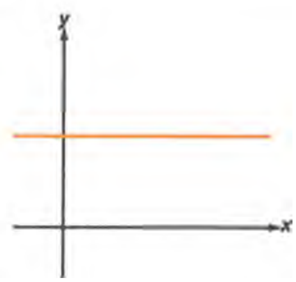
A. ____________________



B. ____________________



C. ____________________



2. The potential energy (PE) of an object is related to its vertical position. In other words, it is related to the object's height above the ground. The potential energy (in kilojoules) of an object is given by the equation

$$PE = \frac{mgh}{1000}$$

where m is the mass of the object, g is a constant related to gravity, or 9. 8, and h is the height above the ground.

Use the data in the table to calculate the PE of someone with a mass of 90 kg traveling upward in an elevator 10 meters at a time.

Mass (in kg)	g	Height (in m)	PE (in kj)
90	9. 8	0	
90	9. 8	10	
90	9. 8	20	
90	9. 8	30	
90	9. 8	40	
90	9. 8	50	

3. Draw a graph to plot the PE values you calculated in the previous problem.

4. Does the graph that you drew represent a function? If so, is it linear or nonlinear? Explain each of your answers.

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