LESSON

3.1 Introduction to Fractions

Lesson Objectives

You will be able to

- Understand fractions
- Name equivalent fractions
- Compare and order fractions

Skills

- **Core Skill:** Interpret Data Displays
- **Core Skill:** Perform Operations

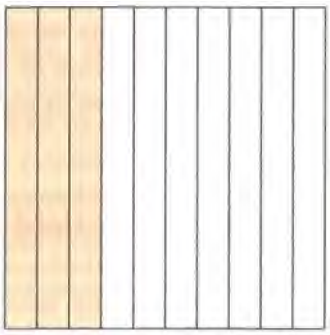
Vocabulary

common multiple
denominator
diagram
equivalent fractions
fraction
lowest terms
numerator
represent

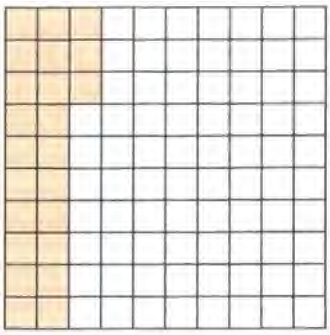
KEY CONCEPT: Represent, compare, and order fractions to understand and develop the meaning and value of fractions.

Write the decimal shown in each diagram.

1.



2.

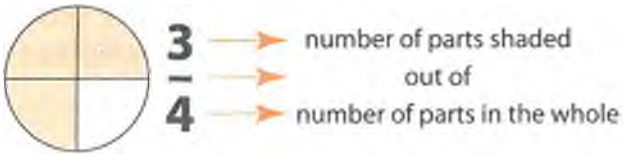


Compare each pair of decimals, using <, >, or =.

3. 0. 2_____ 0. 8 4. 0. 67_____ 0. 55 5. 0. 7_____ 0. 70

Understand Fractions

Decimals involve any number of parts out of a whole made up of 10, 100, 1, 000, and so on. **Fractions** are another way of showing parts of a whole. A **fraction** is made up of two numbers: the numerator and the denominator. For example, the fraction 3/4 has a numerator of 3 and a denominator of 4. The **numerator** indicates the number of parts. The **denominator** refers to the number of parts in the whole.



Example 1 Write the Fraction Shown in a Diagram

What fraction is shown in the diagram?

Step 1 Count the total number of shaded parts in the diagram. This is the numerator of the fraction. There are 7 shaded parts, so the numerator is 7.

Step 2 Count the total number of parts in the whole. This is the denominator of the fraction. There are 10 parts in the whole, so the denominator is 10.

Step 3 Write the fraction with the numerator on the top and the denominator on the bottom: $\frac{7}{10}$.

The numerator of a fraction is the top number. It names the number of parts. The denominator of a fraction is the bottom number. It names the number of parts in the whole.

RECOGNIZE DETAILS

In addition to main ideas, passages usually contain details. These are usually specific pieces of information that relate to the main idea. Details can describe, quantify, or support the main idea.

Recognizing details is a skill that will help you to understand what you read.

Read the passage below for details.

> (1) A **fraction** is a way to represent parts of a whole. (2) The **denominator** is the number on the bottom. (3) It shows how many equal parts the whole has been broken into. (4) The **numerator** is the number on top. (5) It tells how many of the equal parts are being counted.

What are the details in the passage above?

_____

_____

_____

The main idea of the passage is stated in sentence 1: A fraction represents parts of a whole. The details that support and expand on this idea are in sentences 2, 3, 4, and 5. Sentences 2 and 4 describe what the different numbers in a fraction represent. Sentences 3 and 5 explain how the numbers relate to parts of a whole.

Core Skill

Interpret Data Displays

A **diagram** is an illustration or picture that shows mathematical or other types of information. Learning how to interpret the information in diagrams is thus an important skill. The diagrams that appear in this lesson are tools that can be used to represent fractions visually. Each whole diagram is divided into parts, and some of the parts are shaded. The shaded parts of the diagram **represent,** or stand for, the numerator. The total number of parts in the whole represents the denominator.

In a notebook, draw a diagram to represent the fraction 2/5. Explain how the diagram shows this fraction.

MATH LINK

When you draw a diagram to represent a fraction, it does not matter what kind of shape you draw as long as it can be divided easily into the correct number of equal pieces.

21st Century Skill:
Collaborate with Others

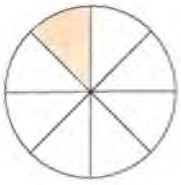
Because students bring different strengths to the classroom, it is often helpful to work together. Each individual can contribute something that will lead to the successful completion of an assignment or project. Math problems may sometimes seem really overwhelming when you try to solve them on your own. But when you partner with a classmate or friend or when you join a group, you can accomplish more together than you could if you were to work alone.

THINK ABOUT MATH

Directions: Write the fraction shown in the diagram.

1. 

_____

2. 

_____

3. 

_____

4. 

_____

Name Equivalent Fractions

Equivalent fractions are fractions that have the same value. Any fraction multiplied by some form of 1 will yield an equivalent fraction. Forms of 1 are any fraction in which the numerator is the same as the denominator, such as 1/1, 6/6 or 102/102. Study the diagram below.

$$\frac{3}{4} = \frac{6}{8} = \frac{12}{16}$$

$$\frac{3}{4} \times \boxed{1} = \frac{3}{4} \qquad \frac{3}{4} \times \boxed{\frac{2}{2}} = \frac{6}{8} \qquad \frac{3}{4} \times \boxed{\frac{4}{4}} = \frac{12}{16}$$

Same as 1 Same as 1

A fraction is in **lowest terms** if the numerator and denominator cannot be divided evenly (remainder of zero) by any whole number other than 1. Since fractions can be written as many equivalent fractions, writing a fraction in lowest terms makes comparing values easier.

Example 2 Name Equivalent Fractions

Name three fractions equivalent to $\frac{4}{5}$.

Step 1 Multiply $\frac{4}{5}$ by three different forms of 1.

$$\frac{4}{5} \times \frac{4}{4} = \frac{16}{20}; \qquad \frac{4}{5} \times \frac{20}{20} = \frac{80}{100}; \qquad \frac{4}{5} \times \frac{50}{50} = \frac{200}{250}$$

Step 2 Write the three equivalent fractions: $\frac{16}{20}, \frac{80}{100}, \frac{200}{250}$.

Example 3 Find a Specific Equivalent Fraction

Write a fraction with a denominator of 12 that is equivalent to $\frac{21}{36}$.

Step 1 Set up the problem: $\frac{21}{36} = \frac{?}{12}$.

Step 2 Divide by a form of 1.
Since $36 \div 3 = 12$, divide the numerator and denominator by $3\left(\frac{3}{3} = 1\right)$.
$\frac{21}{36} \div \frac{3}{3} = \frac{7}{12}$

Step 3 Check your answer by multiplying $\frac{7}{12}$ by $\frac{3}{3}$.
$\frac{7}{12} \times \frac{3}{3} = \frac{21}{36}$
The answer is correct. $\frac{7}{12}$ is equivalent to $\frac{21}{36}$.

Example 4 Rewrite a Fraction in Lowest Terms

Rewrite $\frac{20}{24}$ as a fraction in lowest terms.

Step 1 Divide the numerator and denominator by a number or numbers until both numerator and denominator can no longer be divided.
$\frac{20}{24} = \frac{20}{24} \div \frac{2}{2} = \frac{10}{12} \div \frac{2}{2} = \frac{5}{6}$ or
$\frac{20}{24} = \frac{20}{24} \div \frac{4}{4} = \frac{5}{6}$

Step 2 Write $\frac{20}{24}$ in lowest terms: $\frac{5}{6}$.

Example 5 Use a Calculator to Reduce a Fraction to Lowest Terms

Use a calculator to reduce $\frac{28}{32}$ to lowest terms.

Press [on].

Press [▫/▫] [2] [8] [▼] [3] [2]

Press [enter]

The display should read

The fraction $\frac{28}{32}$ reduced to lowest terms is $\frac{7}{8}$.

In summary, to find equivalent fractions, multiply the fraction by forms of 1, such as $\frac{2}{2}$, $\frac{10}{10}$, or $\frac{50}{50}$. To find specific equivalent fractions, multiply or divide by forms of 1.

To rewrite fractions in lowest terms, divide the numerator and denominator by the same number or numbers until the numerator and denominator can no longer be divided, or use a calculator.

MATH LINK

When rewriting a fraction in lowest terms, try to identify the greatest number by which both the numerator and denominator are evenly divisible.

MATH LINK

When using the inequality symbols < and >, remember that the open end of the symbol opens to the greater number.

THINK ABOUT MATH

Directions: Match the letter of each fraction or number to the appropriate fraction or whole number.

_____ 1.	an equivalent fraction for $\frac{2}{3}$	A. 3
_____ 2.	the missing numerator in the equation $\frac{15}{25} = \frac{?}{5}$	B. $\frac{1}{3}$
_____ 3.	the missing numerator in the equation $\frac{7}{9} = \frac{?}{27}$	C. $\frac{10}{15}$
_____ 4.	$\frac{16}{18}$ written in lowest terms	D. $\frac{8}{9}$
_____ 5.	$\frac{14}{42}$ written in lowest terms	E. 21

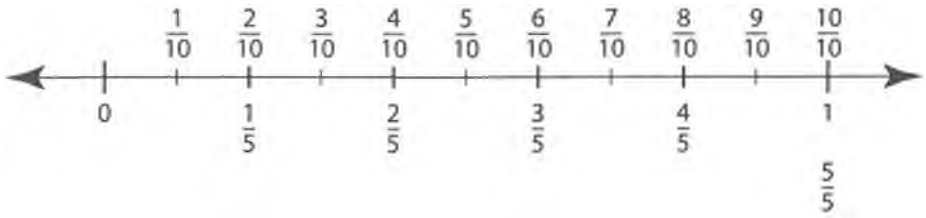
Compare and Order Fractions

There is often a need to compare fractions. When there are more than two fractions involved, the fractions are usually listed in order from least to greatest or greatest to least. Using a number line is one way to compare fractions. Another way is to find a common denominator for both fractions and then compare the numerators.

Example 6 Use a Number Line to Compare Fractions

Which is greater, $\frac{7}{10}$ or $\frac{3}{5}$?

Step 1 Draw a number line with both fifths and tenths.



Step 2 Locate both fractions on the number line.

Since 7/10 is to the right of 3/5, 7/10 is greater than 3/5. Write 7/10 > 3/5.

Example 7 Find a Common Denominator to Compare Fractions

Compare $\frac{5}{6}$ and $\frac{7}{8}$.

Step 1 Identify a common denominator for the two fractions. List the multiples of 6 and 8 until a **common multiple**, or a number that is multiple of both denominators, is identified.
Multiples of 6: **6** (6×1), **12** (6×2), **18** (6×3), **24** (6×4)
Multiples of 8: **8** (8×1), **16** (8×2), **24** (8×3)

Step 2 Rewrite $\frac{5}{6}$ and $\frac{7}{8}$ as fractions having denominators of 24 (the common multiple).

$\frac{5}{6} = \frac{?}{24} \longrightarrow \frac{5 \times 4}{6 \times 4} = \frac{\mathbf{20}}{\mathbf{24}}$ $\quad$ $\frac{7}{8} = \frac{?}{24} \longrightarrow \frac{7 \times 3}{8 \times 3} = \frac{\mathbf{21}}{\mathbf{24}}$

Step 3 Compare the numerators.

Since $20 < 21$, $\frac{20}{24} < \frac{21}{24}$ and $\frac{5}{6} < \frac{7}{8}$.

Example 8 Order Fractions

Write the set of fractions in order from least to greatest.

$\frac{3}{4}, \frac{2}{3}, \frac{5}{6}$

Step 1 Rewrite the fractions using a common denominator.
Since 12 is a common multiple of 3, 4, and 6, rewrite the fractions using 12 as the common denominator.

$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{\mathbf{9}}{\mathbf{12}}$ $\quad$ $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{\mathbf{8}}{\mathbf{12}}$ $\quad$ $\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{\mathbf{10}}{\mathbf{12}}$

Step 2 Compare the numerators.

Since $8 < 9 < 10$, $\frac{8}{12} < \frac{9}{12} < \frac{10}{12}$ and $\frac{2}{3} < \frac{3}{4} < \frac{5}{6}$.

THINK ABOUT MATH

Directions: Answer the following.

1. How would you use a number line to compare the fractions $\frac{3}{4}$ and $\frac{7}{8}$?

_____

2. How would you use a common denominator to compare $\frac{1}{4}$ and $\frac{1}{6}$?

_____

3. Place the numbers below in order from greatest to least.
$\frac{7}{9}, \frac{2}{3}, \frac{5}{6}$

_____

Core Skill
Perform Operations

Equivalent fractions are fractions that represent the same number, such as 1/2 and 2/4 or 3 and 3/1. Using equivalent fractions can help solve problems found in everyday life. You can use equivalent fractions to determine the number of 51 x 44 = 204 or 20 quarters.

Consider the following problem. Mrs. Lenhart made 3 apple pies for her 5th grade class. She cut each pie into 8 slices for her students to share. How many slices of pie were shared among Mrs. Lenhart's students? In a notebook, write the appropriate equivalent fraction by multiplying the correct form of 1.

Before Cut

After Cut

Vocabulary Review

Directions: Complete the sentences below using one of the following words.

common multiple denominator equivalent fractions
fraction lowest terms numerator

1. A_____ is a made up of two parts. It is a way of showing parts of a whole.
2. The bottom number of a fraction is the_____ It tells the number of parts in the whole.
3. A fraction is in_____ when the numerator and denominator can no longer be divided by a whole number other than 1.
4. The_____of a fraction is the top number. It tells the number of parts.
5. A common denominator of two or more fractions can be found by finding a_____ ____ of the fractions.
6. Fractions that have the same value are_____

Skill Review

Directions: Underline and number the details in each passage. Then explain how the details support the main idea.

1. Equivalent fractions have the same value. One way to find equivalent fractions is to multiply each fraction by a form of 1. A form of 1 is any fraction in which the numerator and denominator are the same, such as 5/5. Another way to find an equivalent fraction is to divide by a form of 1.

_____

_____

_____

2. There are several ways you can compare two fractions. One way is by finding a common denominator. List the multiples of each denominator. The first common multiple is the least common denominator of the two fractions. Rewrite the fractions with the common denominator. Then compare the numerators of the fractions to determine the lesser or greater fraction.

_____

_____

Directions: Use a diagram to represent each fraction below. Explain how the diagram represents the fraction.

3. $\frac{6}{7}$ 4. $\frac{5}{6}$ 5. $\frac{3}{8}$ 6. $\frac{4}{9}$

Directions: Use diagrams to show the equivalent fractions below.

7. the number of twelfths equivalent to $\frac{3}{4}$ 8. the number of fifths equivalent to $\frac{8}{10}$

Skill Practice

Directions: You may use a calculator for these questions. Choose the best answer to each question.

1. Which of the following has the fractions arranged from least to greatest?

   A. $\frac{2}{5}, \frac{1}{2}, \frac{5}{8}$
   B. $\frac{2}{3}, \frac{5}{6}, \frac{5}{8}$
   C. $\frac{3}{4}, \frac{4}{5}, \frac{7}{10}$
   D. $\frac{1}{2}, \frac{3}{5}, \frac{5}{9}$

2. Which fraction represents the shaded part of the diagram?

   A. $\frac{1}{8}$
   B. $\frac{1}{6}$
   C. $\frac{5}{6}$
   D. $\frac{7}{8}$

3. A community center is sponsoring mixed softball teams this summer. Out of the people who signed up for the teams, $\frac{4}{7}$ are female. If 42 people signed up, how many are female?

   A. 28
   B. 24
   C. 20
   D. 16

4. Which fraction is written in lowest terms?

   A. $\frac{7}{35}$
   B. $\frac{4}{18}$
   C. $\frac{18}{21}$
   D. $\frac{14}{25}$

LESSON
3. 2

Add and Subtract Fractions

Lesson Objectives

You will be able to

- Add and subtract fractions with like denominators
- Add and subtract fractions with unlike denominators

Skills

- **Core Skill:** Perform Operations
- **Core Skill:** Apply Number Sense

Vocabulary

common denominator
like denominators
simplify
unlike denominators

MATH LINK

Another way to look at Step 2 in Example 1 is to think: "How many twelfths result from combining the parts? " *12 twelfths*

KEY CONCEPT: Understand and apply strategies for finding the sums and differences of fractions that have like or unlike denominators.

Match each fraction with an equivalent fraction.

_____ 1. $\frac{6}{10}$ _____ 2. $\frac{5}{14}$ _____ 3. $\frac{9}{15}$

A. $\frac{27}{45}$ B. $\frac{25}{70}$ C. $\frac{12}{20}$

Write each fraction in lowest terms.

4. $\frac{12}{15}$ _____ 5. $\frac{18}{42}$ _____ 6. $\frac{26}{50}$ _____

Add and Subtract Fractions with Like Denominators

When adding or subtracting fractions, look to see if the denominators are **like denominators** or **unlike denominators**. In other words, do the fractions have denominators that are the same or different?

Example 1 Add Fractions with Like Denominators

Add $\frac{5}{12}$ and $\frac{7}{12}$.

Step 1 Look at the denominators. If they are the same (12), the sum will have this denominator (12).

$$\frac{5}{12} + \frac{7}{12} = \frac{\square}{12}$$

Step 2 Add the numerators.

$$\frac{5}{12} + \frac{7}{12} = \frac{12}{12}$$

Step 3 **Simplify** the answer, or reduce it to lowest terms.

$\frac{12}{12} = 1$, so $\frac{5}{12} + \frac{7}{12} = \frac{12}{12} = 1$

Example 2 Subtract Fractions with Like Denominators

Subtract $\frac{1}{10}$ from $\frac{7}{10}$.

Step 1 Look at the denominators. They are the same (10), so the difference will have the same denominator (10).

$$\frac{7}{\mathbf{10}} - \frac{1}{\mathbf{10}} = \frac{\square}{\mathbf{10}}$$

Step 2 Subtract the numerators.

$$\frac{\mathbf{7}}{10} - \frac{\mathbf{1}}{10} = \frac{\mathbf{6}}{10}$$

Step 3 Simplify the answer.

$$\frac{6}{10} \div \frac{2}{2} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}, \text{ so } \frac{7}{10} - \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$$

When reviewing the previous two examples for adding or subtracting fractions with like denominators, remember the sequence of steps. First, add or subtract the numerators. The denominators will be the same as the original ones. Then, if necessary, write the sum or difference in lowest terms.

MATH LINK

When adding or subtracting fractions with like denominators, make sure the denominators remain the same. However, if the answer can be simplified (reduced to lowest terms), the final denominator will change.

THINK ABOUT MATH

Directions: Find each sum or difference. Reduce the answer to lowest terms.

1. $\frac{5}{12} + \frac{3}{12}$ _____
2. $\frac{31}{36} - \frac{5}{36}$ _____
3. $\frac{7}{18} - \frac{1}{18}$ _____
4. $\frac{11}{20} + \frac{7}{20}$ _____

MATH LINK

One way to find a common denominator when adding or subtracting fractions with unlike denominators is to multiply the original denominators by each other. Remember to simplify or reduce the answer to lowest terms.

Add and Subtract Fractions with Unlike Denominators

When adding or subtracting fractions with denominators that are not the same, first find equivalent fractions that have a **common denominator**—a common multiple of the denominators of the two fractions. Then add or subtract the fractions.

Core Skill
Perform Operations

Real-world word problems involving fractions will sometimes talk about things like one half of a pizza pie. It is easy to add or subtract fractions when you can visualize individual slices of a pizza that represent part of a whole pie. A pizza has 8 slices, and you and your sister eat 4 of them. How much of the pizza remains? It is easy to see that half of the pie is left. Once you become familiar with fractions, of course, you no longer need to visualize actual parts of a whole in order to add and subtract. You will be able to perform operations involving fractions on paper or with a calculator.

Example 3 Add Fractions with Unlike Denominators

Add $\frac{7}{12}$ and $\frac{1}{3}$.

Step 1 Look at the denominators. They are different (12 and 3). Find a common denominator of these two numbers: 12.

Step 2 Find an equivalent fraction for $\frac{1}{3}$ using a common denominator of 12.

$\frac{1}{3} = \frac{\square}{12}$ Think: $3 \times ? = 12$

$\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$ Multiply by $\frac{4}{4}$ or 1.

Step 3 Add using twelfths.

$$\begin{array}{r} \frac{7}{12} = \frac{7}{12} \\ + \frac{1}{3} = \frac{4}{12} \\ \hline \frac{11}{12} \end{array}$$

Add the new numerators.

The new denominator is 12.

The answer is already in lowest terms.

Example 4 Subtract Fractions with Unlike Denominators

Subtract $\frac{1}{5}$ from $\frac{3}{4}$.

Step 1 Look at the denominators. They are different (5 and 4). One common denominator is 20, since $5 \times 4 = 20$.

$$\begin{array}{r} \frac{3}{4} = \frac{\square}{20} \\ - \frac{1}{5} = \frac{\square}{20} \\ \hline \end{array}$$

Step 2 Find equivalent fractions for $\frac{1}{5}$ and $\frac{3}{4}$ with 20 as the denominators.

$\frac{3}{4} = \frac{\square}{20}$ Think: $4 \times ? = 20$

$\frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$ Multiply by $\frac{5}{5}$ or 1.

$\frac{3}{4} = \frac{15}{20}$

$\frac{1}{5} = \frac{\square}{20}$ Think: $5 \times ? = 20$

$\frac{1}{5} \times \frac{4}{4} = \frac{4}{20}$ Multiply by $\frac{4}{4}$ or 1

$\frac{1}{5} = \frac{4}{20}$

Step 3 Subtract using twentieths.

$$\begin{array}{r} \frac{3}{4} = \frac{15}{20} \\ - \frac{1}{5} = \frac{4}{20} \\ \hline \frac{11}{20} \end{array}$$

Subtract the new numerators.

The new denominator is 20.

The answer is already in lowest terms.

Example 5 Use a Calculator to Add or Subtract Fractions.

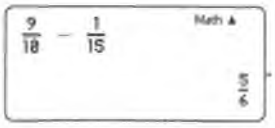
Solve $\frac{9}{10} - \frac{1}{15}$.

Press on.

Press $\frac{n}{d}$ 9 ▼ 1 0 ◀▶ − $\frac{n}{d}$ 1 ▼ 1 5.

Press enter.

The display should read



$\frac{9}{10} - \frac{1}{15} = \frac{5}{6}$

Example 6 Solve a Problem with Unlike Denominators

Ray spent $\frac{1}{12}$ hour stretching before he began his morning run. He spent $\frac{2}{3}$ hour running. What part of an hour did Ray spend stretching and running?

Step 1 Set up the problem.

$$\begin{array}{r} \frac{1}{12} = \frac{\square}{12} \\ +\ \frac{1}{3} = \frac{\square}{12} \\ \hline \end{array}$$

Step 2 Add.

$$\begin{array}{r} \frac{1}{12} = \frac{1}{12} \\ +\ \frac{2}{3} = \frac{8}{12} \\ \hline \frac{9}{12} \end{array}$$

Step 3 Simplify. $\frac{9}{12} = \frac{3}{4}$

The sequence for adding and subtracting fractions with unlike denominators is to first find a common denominator. Second, write the equivalent fractions with the new denominator. Third, add or subtract the numerators. Fourth, if necessary, write the answer in lowest terms.

THINK ABOUT MATH

Directions: Fill in the blanks.

1. To find $\frac{3}{8} + \frac{1}{3}$, you need to find a common denominator for _____ and _____. One common denominator is _____. The sum in lowest terms is _____
2. To find $\frac{5}{6} - \frac{1}{3}$, you need to find a common denominator for _____ and _____. One common denominator is _____. The difference in lowest terms is _____

Core Skill

Apply Number Sense

The ability to look at the solution of a problem and determine its inaccuracy is a skill that can strengthen your mathematical ability. For instance, Michael claims that since he received a 70% on his first exam, and an 80% on his second exam, he now has a 150% in the course. This is obviously not the case, since doing better than 100% is rare, depending on the course.

Consider the following problem. Gina works 3/5 of an hour on her homework on Monday, 7/6 of an hour on Tuesday, and 4/3 of an hour on Thursday. What fraction of an hour did Gina work on her homework? Without finding the answer, could Gina have spent more than 6 hours on her homework? In a notebook, solve the equation needed to determine how much time Gina spent.

Vocabulary Review

Directions: Match each word to one of the statements below.

_____ 1. common denominator

_____ 2. like denominators

_____ 3. simplify

_____ 4. unlike denominators

A. It is what you do to reduce a fraction, such as $\frac{2}{10}$, to lowest terms.

B. It describes fractions such as $\frac{11}{12}$ and $\frac{2}{15}$, because the denominators of the fractions are different.

C. An example would be 12 because it is a multiple of the denominators of $\frac{2}{3}$ and $\frac{3}{4}$.

D. The fractions $\frac{3}{10}$ and $\frac{7}{10}$ are examples, since the denominators are the same.

Skill Review

Directions: Write the sequence of steps you would use to find the sum'or difference of the following fractions. Then find the sum or difference.

1. $\frac{3}{10} + \frac{2}{5}$

2. $\frac{13}{15} - \frac{4}{15}$

Directions: Describe the steps you would take to find the difference of $\frac{7}{8}$ and $\frac{3}{10}$ on your calculator. Then find the difference.

3. _____

_____

_____

_____

_____

_____

Skill Practice

Directions: Choose the best answer to each question.

1. The sum of $\frac{1}{6}$ and another fraction is $\frac{7}{12}$. What is the other fraction?

   A. $\frac{5}{12}$

   B. $\frac{6}{12}$

   C. $\frac{2}{3}$

   D. $\frac{3}{4}$

2. A chart shows the type of nail to use for different thicknesses of plywood. The thickness of the plywood ranges from $\frac{1}{4}$ inch to $\frac{3}{4}$ inch. In inches, what is the difference in the thicknesses of the plywood?

   A. $\frac{1}{4}$

   B. $\frac{1}{2}$

   C. $\frac{3}{4}$

   D. 1

Directions: Reduce fractions to their lowest terms.

3. Jorge spent $\frac{1}{2}$ hour pulling weeds in the backyard. He spent $\frac{3}{12}$ hour pulling weeds in the front yard. How much longer, in a fraction of an hour, did it take to pull weeds in the back?

   _____

4. Maya bought $\frac{3}{4}$ yard of fabric to cover a footstool. When she tried to cover the footstool, she was short $\frac{1}{8}$ yard. How many total yards of fabric did Maya need to cover the footstool?

   _____

LESSON
3. 3

Multiply and Divide Fractions

Lesson Objectives

You will be able to

- Multiply fractions
- Divide fractions

Skills

- **Core Skill:** Apply Number Sense
- **Core Skill:** Perform Operations

Vocabulary

invert
multiplicative inverse
reciprocal

MATH LINK

Whole numbers can be rewritten as fractions by placing the whole number in the numerator and writing the denominator as 1. $3 = \frac{3}{1}$, $100 = \frac{100}{1}$, and $67 = \frac{67}{1}$. Make sure not to confuse whole numbers with fractions that are equivalent to 1. $\frac{4}{4} = 1$, while $\frac{4}{1} = 4$.

KEY CONCEPT: Extend and develop ideas about multiplication and division to include multiplying and dividing fractions.

Write each fraction in lowest terms.

1. $\frac{16}{28}$ _____
2. $\frac{45}{80}$ _____
3. $\frac{88}{96}$ _____
4. $\frac{12}{16}$ _____

Multiply Fractions

Two fractions can be multiplied by multiplying the numerators and denominators and simplifying the product.

Example 1 Multiply a Fraction by a Fraction

Multiply $\frac{5}{8}$ and $\frac{3}{4}$.

Step 1 Multiply the numerators. Then multiply the denominators.

$$\frac{5}{8} \times \frac{3}{4} = \frac{5 \times 3}{8 \times 4} = \frac{15}{32}$$

Step 2 Rewrite the answer in lowest terms, if necessary. In this case, the answer is already in lowest terms.

Example 2 Use a Calculator to Multiply Fractions

Find $\frac{8}{15} \times \frac{5}{8}$.

Press ON

Press $\frac{\square}{\square}$ 8 ▼ 15 ▶ × $\frac{\square}{\square}$ 5 ▼ 8.

Press enter.

The display should read

$$\frac{8}{15} \times \frac{5}{8} = \frac{1}{3}$$

THINK ABOUT MATH

Directions: Match each multiplication problem to its product. All products are in lowest terms.

_____ 1. $\frac{1}{5} \times \frac{3}{8}$

_____ 2. $\frac{7}{12} \times \frac{2}{3}$

_____ 3. $\frac{9}{10} \times \frac{5}{6}$

_____ 4. $\frac{11}{25} \times \frac{9}{11}$

A. $\frac{3}{4}$ B. $\frac{9}{25}$ C. $\frac{3}{40}$ D. $\frac{7}{18}$

Divide Fractions

Multiplication and division are inverse operations. To divide by a fraction, you must multiply the dividend by the **reciprocal** (the number that when multiplied by the divisor, has a product of 1) of the divisor. To find the reciprocal of any number, **invert**, or turn over, the fraction or change the places of the numerator and denominator. For example, the inverted form of $\frac{3}{7}$ is $\frac{7}{3}$. In this case, $\frac{7}{3}$ is the reciprocal of $\frac{3}{7}$ because $\frac{7}{3} \times \frac{3}{7} = \frac{21}{21} = 1$.

After changing the division symbol to multiplication and the divisor to its reciprocal, multiply the fractions.

Example 3 Divide by a Fraction

Find $3 \div \frac{1}{16}$.

Step 1 Change the division symbol to multiplication and invert the divisor (multiply by the reciprocal of the divisor).

$3 \div \frac{1}{16} = \frac{3}{1} \times \frac{\mathbf{16}}{\mathbf{1}} \qquad (3 = \frac{3}{1})$

Step 2 The problem is now a multiplication problem. Multiply the numerators and multiply the denominators.

$\frac{3}{1} \times \frac{16}{1} = \frac{3 \times 16}{1 \times 1} = \frac{48}{1} = 48$

Step 3 Simplify, if necessary, by writing the answer in lowest terms. In this case, the answer is in lowest terms.

Example 4 Use a Calculator to Divide Fractions

Find $\frac{5}{9} \div \frac{2}{3}$.

Press ON.

Press $\frac{n}{d}$ 5 ▼ 9 ▶ ÷ $\frac{n}{d}$ 2 ▼ 3 ENTER.

The display should read

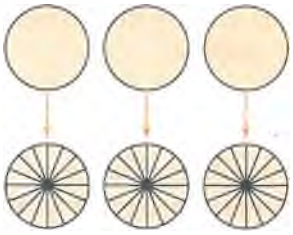
$\frac{5}{9} \div \frac{2}{3} = \frac{5}{6}$

Core Skill
Apply Number Sense

Juan owns a snow shoveling business. Of the money he makes, 1/2of it goes to employees (the other half for business costs). Juan has 5 employees (including himself) and pays everyone an equal amount. What fraction of the total money made does Juan pay to a single employee? Use the following fraction bar, if needed, to determine the correct fraction.

MATH LINK

Another way to look at Example 3 is to ask yourself: *How many sixteenths are in 3 wholes?*



1 whole is the same as 16 (1 × 16) sixteenths.

2 wholes is the same as 32 (2 × 16) sixteenths.

3 wholes is the same as 48 (3 × 16) sixteenths.

Therefore, $3 \div \frac{1}{16}$ is the same as 3 × 16.

MATH LINK

The reciprocal is also known as the multiplicative inverse.

Core Skill

Perform Operations

Math relies on regularity, or repeating patterns. You may have noticed that a step-by-step approach to problem solving is often used in this book. You may have also noticed that a specific sequence of steps is used regularly to solve related problems.

For example, when you divide a fraction by another fraction, you repeat the same steps in the same order. First, you change the division sign to a multiplication sign. Then you invert the divisor. Next, you multiply the numerators. Then you multiply the denominators. Finally, you simplify the answer, if possible.

In a notebook, describe the steps you take to multiply two fractions.

THINK ABOUT MATH

Directions: Solve the problems below.

1. Write a math problem that shows how to find the number of thirds in 9. Then find the number of thirds. Show your work.

2. A pica is a measure of $\frac{1}{6}$ inch. Picas are most commonly used to measure text. If a line of text is 3 inches long, how many picas is the line of text?

Vocabulary Review

Directions: Fill in the blanks with one of the words below.

invert **multiplicative inverse** **reciprocal**

1. The reciprocal is the same as the _____.
2. The _____ of 18 is $\frac{1}{8}$.
3. When you divide fractions, you need to _____ the divisor.

Skill Review

Directions: Describe the steps you would take to divide any two fractions using a calculator.

1. _____

Skill Practice (continued)

2. Fahrenheit and Celsius are temperature scales. You can convert temperatures from one scale to another. For example, to convert from 104 degrees Fahrenheit to degrees Celsius, subtract 32 from 104. Then multiply by $\frac{5}{9}$. Given what you know about inverse relationships, how would you convert 40 degrees Celsius to degrees Fahrenheit? Show calculations to support your reasoning.

Skill Practice

Directions: Choose the best answer to each question.

1. Which of the following is the quotient of $\frac{2}{3}$ divided by $\frac{4}{5}$?
   A. $\frac{8}{15}$
   B. $\frac{5}{6}$
   C. $\frac{6}{5}$
   D. $\frac{15}{8}$

2. How many $\frac{1}{2}$-inch columns can Anna fit in a table that is 10 inches long?
   A. 5
   B. 10
   C. 15
   D. 20

3. A crew is clearing a 15-mile trail after a storm. If the crew clears $\frac{3}{5}$ mile every day, how many days will it take to clear the trail?
   A. 25
   B. 15
   C. 9
   D. 3

4. Tyrone says the quotient of $\frac{1}{4} \div \frac{5}{8}$ is $\frac{5}{32}$. Which of the following best describes Tyrone's statement.
   A. He is correct.
   B. He forgot to invert the divisor before multiplying.
   C. He forgot to invert the dividend before multiplying.
   D. He forgot to invert both fractions before multiplying.

LESSON
3. 4 Mixed Numbers

KEY CONCEPT: Understand mixed numbers and perform the basic operations of addition, subtraction, multiplication, and division with mixed numbers.

Find each sum or difference. Simplify the answer, if necessary.

1. $\frac{3}{10} + \frac{3}{5}$ 2. $\frac{7}{8} - \frac{1}{3}$

Find each product or quotient. Simplify the answer, if necessary.

3. $\frac{5}{9} \times \frac{3}{7}$ 4. $\frac{2}{3} \div \frac{5}{6}$

Lesson Objectives

You will be able to

- Add and subtract mixed numbers
- Multiply and divide mixed numbers

Skills

- **Core Skill:** Represent Real-World Problems
- **Reading Skill:** Evaluate Arguments

Vocabulary

detail
improper fraction
mixed number
proper fraction
reduce
rename

MATH LINK

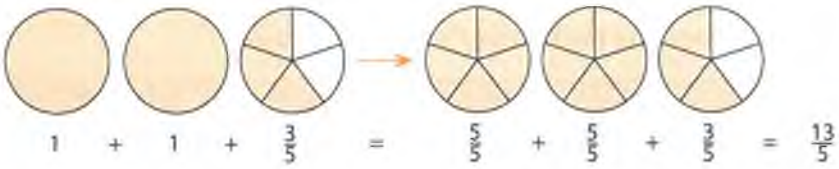
Follow these steps to change an improper fraction to a whole number or a mixed number:

1) Divide the numerator of the improper fraction by the denominator.

2) If the quotient has a remainder, the improper fraction is written as a mixed number with the remainder as the numerator and the same denominator as the original improper fraction.

Add and Subtract Mixed Numbers

The sum of a whole number and a fraction is a **mixed number**. For example, $2\frac{3}{5}$ is the same as $1 + 1 + \frac{3}{5}$, or $\frac{5}{5} + \frac{5}{5} + \frac{3}{5}$. The diagram shows that $2\frac{3}{5}$ is equal to $\frac{13}{5}$.



A fraction with a numerator greater than or equal to its denominator is an **improper fraction**. A fraction with a numerator that is less than the denominator is sometimes called a **proper fraction**. Remember that an improper fraction with a numerator equal to the denominator, such as $\frac{9}{9}$, $\frac{16}{16}$, or $\frac{100}{100}$, is equivalent to 1.

Example 1 Add Mixed Numbers

Add $2\frac{4}{5}$ and $1\frac{1}{3}$.

Step 1 Look at the denominators. They are different (5 and 3).

Step 2 **Rename** the fractions, or find equivalent fractions with a common denominator of 15.

$$2\tfrac{4}{5} = 2\tfrac{12}{15}$$
$$+1\tfrac{1}{3} = 1\tfrac{5}{15}$$

Step 3 Add the whole numbers.

Step 4 Add the renamed fractions.

$$3\tfrac{17}{15}$$

Step 5 **Reduce**, or simplify, the answer by writing it in lowest terms. Since $\frac{17}{15}$ is an improper fraction, think:

$$3\tfrac{17}{15} = 3 + \tfrac{15}{15} + \tfrac{2}{15} = 4\tfrac{2}{15}$$

SUMMARIZE SUPPORTING DETAILS

A passage will often contain many supporting **details**—information that supports the main idea. Sometimes, trying to remember every detail can be overwhelming. Summarizing details is a way to think about information in a general way. A summary does not include every piece of information from a selection. Only the most important details are presented, and they are presented in a clear manner.

When you summarize, details that are similar can be grouped together. For example, "Javier read 5 books in June, 6 books in July, and 4 books in August" can be summarized by the statement "Javier read 15 books over the summer."

Some details can be omitted from your summary. "Maggie's insurance plan covers health, vision, and dental. She has optional life insurance. Deductibles are low. Her insurance card has blue clouds on it." If the point of this passage is to describe Maggie's insurance, the detail about the clouds can be safely omitted.

Read the passage below. As you read, identify the most important details.

> Isra is shopping for a cable and internet package. The plan she likes is called the Silver Plan. It has 5 broadcast channels, 30 basic cable channels, and 40 specialized cable channels. It also includes 1 premium movie channel. She can add a digital video recording (DVR) device for a small fee. It comes with a remote that takes 2 AA batteries. Internet connection is also included. She thinks it is a good choice because she loves to watch movies. The other plans offered by the same company are not as appealing to her.

A good summary will include the features of the plan: 75 channels, 1 movie channel, option for DVR, internet included. Details such as what kind of batteries the remote uses can be left out of the summary.

Core Skill

Represent Real-World Problems

Because real-life scenarios don't always use whole numbers, it is more likely that mixed fractions, or improper fractions, will appear during calculations. Consider the following problem.

Claire is training for a marathon. She runs $8\frac{1}{3}$ miles on Monday, $9\frac{4}{5}$ miles on Tuesday, $11\frac{5}{6}$ miles on Wednesday, 12 miles on Thursday, and $14\frac{7}{8}$ miles on Friday. How many miles did Claire run during the week? If Claire wanted to run 100 miles during the entire week, including the weekend, how many more miles will she have to run on Saturday and Sunday to achieve her goal? In a notebook, write down the correct equation to determine how many miles Claire has run. Then answer the questions.

Reading Skill
Evaluate Arguments

You notice the following advertisement in the grocery store one day: "3 out of 4 dentists recommend Acme Teeth Foam for their patients. " This statement may be misleading. You might ask yourself, for example, "How many dentists gave an opinion? " "Do the dentists receive payment for favoring a certain toothpaste? "

When you read mathematical statements without questioning them, you risk making poor or even harmful decisions. Consider similar examples of math-based advertising you have encountered recently. List questions you should ask to determine if the advertiser is making unethical claims that are aimed at increasing sales.

MATH LINK

To convert between mixed numbers and improper fractions on your calculator, press 2nd [▶▶▶] ×10^x. This will change one kind of number to the other.

Example 2 Subtract Mixed Numbers

Subtract $1\frac{3}{4}$ from $3\frac{1}{6}$.

Step 1 Look at the denominators. They are different (4 and 6).

Step 2 Rename the fractions using 12 as the denominator.

$$3\tfrac{1}{6} = 3\tfrac{2}{12}$$
$$-\ 1\tfrac{3}{4} = 1\tfrac{9}{12}$$

Step 3 Subtract the fractional parts and whole numbers. If this is not possible, rename the first mixed number by removing 1 from the whole number, converting it to a fraction, and adding it to the fraction. So, $3\frac{2}{12} = 2 + 1 + \frac{2}{12} = 2 + \frac{12}{12} + \frac{2}{12} = 2\frac{14}{12}$.

Step 4 Subtract the renamed fractions and the new whole numbers.

Step 5 Simplify the answer if necessary. In this case, $1\frac{5}{12}$ is in lowest terms

$$3\tfrac{1}{6} = 3\tfrac{2}{12} = 2\tfrac{14}{12}$$
$$-\ 1\tfrac{3}{4} = 1\tfrac{9}{12} = 1\tfrac{9}{12}$$
$$1\tfrac{5}{12}$$

Multiply and Divide Mixed Numbers

When multiplying and dividing mixed numbers, you must first change the mixed numbers to improper fractions. Then follow the steps for multiplying and dividing fractions.

Example 3 Multiply Mixed Numbers

Multiply $1\frac{2}{3}$ and $2\frac{1}{5}$.

Step 1 Change the mixed numbers to improper fractions.

$$1\tfrac{2}{3} = \frac{(1 \times 3) + 2}{3} = \frac{5}{3};\ 2\tfrac{1}{5} = \frac{(2 \times 5) + 1}{5} = \frac{11}{5}$$

Step 2 Multiply the improper fractions by multiplying the numerators and multiplying the denominators.

$$1\tfrac{2}{3} \times 2\tfrac{1}{5} = \frac{5}{3} \times \frac{11}{5} = \frac{55}{15}$$

Step 3 Change the improper fraction to a mixed number, and simplify the fraction.

$$\frac{55}{15} = 3\frac{10}{15} = 3\frac{2}{3}$$

Example 4 Divide Mixed Numbers

Find $1\frac{3}{8} \div \frac{1}{2}$.

Step 1 Change any mixed numbers to improper fractions.

$$1\frac{3}{8} \div \frac{1}{2} = \frac{11}{8} \div \frac{1}{2}$$

Step 2 Change the division symbol to multiplication, and invert the divisor.

$$\frac{11}{8} \div \frac{1}{2} = \frac{11}{8} \times \frac{2}{1}$$

Step 3 The problem is now a multiplication problem. Complete the problem by multiplying the numerators and denominators and simplifying the result, if necessary.

$$\frac{11}{8} \times \frac{2}{1} = \frac{22}{8} = \frac{11}{4} = 2\frac{3}{4}$$

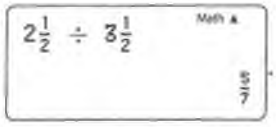
Example 5 Use a Calculator with Mixed Numbers

Solve $2\frac{1}{2} \div 3\frac{1}{2}$.

Press [on].

Press [2] [2nd] [$\frac{n}{d}$] [1] [▼] [2] [÷] [3] [2nd] [$\frac{n}{d}$] [1] [▼] [2] [enter].

The display should read



$$2\frac{1}{2} \div 3\frac{1}{2} = \frac{5}{7}$$

MATH LINK

Practice converting mixed numbers to improper fractions. You can change the mixed number $3\frac{2}{5}$ to the equivalent improper fraction: $3\frac{2}{5} = \frac{17}{5}$.

THINK ABOUT MATH

Directions: Indicate the operation you will use to solve the problem. Then solve the problem.

1. One horsepower is defined to be the power needed to lift 33,000 pounds a distance of one foot in one minute. If this is about $1\frac{1}{2}$ times the power an average horse can exert, how much weight would an average horse be expected to lift a distance of one foot in one minute?

_____

Vocabulary Review

Directions: Choose the correct term in parentheses to complete each sentence.

1. If a fraction is (proper, improper), then the numerator is equal to or greater than the denominator.

2. (Mixed numbers, Improper fractions) have a fractional part and a whole number part.

3. When adding or subtracting mixed numbers, you might need to (reduce, rename) the fractions using a common denominator.

4. The fractional part of a mixed number is a(n) (proper fraction, improper fraction).

5. You (reduce, rename) a fraction when you write it in lowest terms.

Skill Review

Directions: Summarize the details in each passage. Explain why any details are left out.

1. To divide fractions and mixed numbers, you need to change any mixed numbers to improper fractions. An improper fraction has a numerator that is equal to or greater than the denominator. For example, if the divisor of a division problem is 3^, you should change it to the improper fraction Next, you need to invert the divisor. The divisor is the number that you divide by. The dividend is the number that is divided. Then you multiply the numerators and the denominators. Sometimes, the result is not in lowest terms. If that is the case, simplify the result. If the simplified result is an improper fraction, rewrite the improper fraction as a mixed number or whole number.

Directions: Use the connections you made in your notebook to describe or explain the following.

2. Describe the similarities either between adding fractions and adding mixed numbers or between subtracting fractions and subtracting mixed numbers.

3. Explain how the steps you used for multiplying or dividing fractions can be applied to multiplying or dividing mixed numbers.

Skill Practice

Directions: Choose the best answer to each question.

1. Kamil used $2\frac{2}{3}$ cups of flour to make one batch of lemon bars for the company picnic. If he wants to make $1\frac{1}{2}$ batches of lemon bars, how many cups of flour should he use?

   A. $1\frac{1}{6}$
   B. $1\frac{7}{9}$
   C. 4
   D. $4\frac{1}{6}$

2. Solve $10\frac{3}{4} - 6\frac{4}{5}$.

   A. $3\frac{19}{20}$
   B. $4\frac{1}{10}$
   C. $4\frac{9}{20}$
   D. $4\frac{19}{20}$

3. Which is the first step in solving the problem below?
   $4\frac{7}{12} \div 3\frac{1}{10}$

   A. Find a common denominator for $\frac{7}{12}$ and $\frac{1}{10}$.
   B. Invert the divisor and multiply.
   C. Multiply the numerators and denominators of the fractional parts.
   D. Change the mixed numbers to improper fractions.

4. A restaurant has $6\frac{1}{2}$ gallons of a secret sauce. A recipe for barbecue pork calls for $\frac{3}{4}$ gallon of the secret sauce. How many *full* recipes can be made with this sauce?

   A. 9
   B. 8
   C. 7
   D. 5

CHAPTER 3

Review

Directions: Choose the best answer to each question.

1. Which fraction is written in lowest terms?
   A. $\frac{4}{8}$
   B. $\frac{4}{14}$
   C. $\frac{4}{15}$
   D. $\frac{4}{16}$

2. What is the first step for finding the sum of $\frac{2}{4}$ and $\frac{1}{3}$?
   A. Add the numerators.
   B. Rename the fractions.
   C. Add the denominators.
   D. Find a common denominator.

3. Yamina worked $4\frac{2}{3}$ hours on Thursday morning and $2\frac{7}{8}$ hours on Thursday afternoon. How many hours did she work in total on Thursday?
   A. $7\frac{13}{24}$
   B. $7\frac{5}{24}$
   C. $6\frac{13}{24}$
   D. $6\frac{9}{24}$

4. A writer wrote $2\frac{3}{8}$ pages each hour for $7\frac{1}{2}$ hours. How many pages did he write in all?
   A. $5\frac{12}{17}$
   B. $14\frac{1}{8}$
   C. $16\frac{13}{17}$
   D. $17\frac{13}{16}$

5. What is the product of factors that are both positive proper fractions?
   A. a fraction whose value is less than either factor
   B. a fraction whose value is greater than either factor
   C. a mixed number
   D. a number whose value is equal to the greater factor

6. What is the next step to finding $4\frac{2}{7} \div 2\frac{1}{7}$?
   A. $\frac{7}{30} \times \frac{15}{7}$
   B. $\frac{30}{7} \div \frac{15}{7}$
   C. $2\frac{1}{7} \div 4\frac{2}{7}$
   D. $\frac{7}{30} \times \frac{15}{7}$

7. A recipe calls for 1 cup of shortening, $2\frac{2}{3}$ cups of flour, $\frac{1}{2}$ cup of sugar, and 1 cup of raisins. If Kalie doubles the recipe, how many cups of flour should she use?
   A. $6\frac{1}{3}$
   B. $5\frac{1}{3}$
   C. $4\frac{1}{3}$
   D. $3\frac{1}{6}$

8. What is the first step to finding $2\frac{1}{5} - 1\frac{3}{5}$?
   A. Find a common denominator for the fractions.
   B. Rename the first mixed number.
   C. Rename both mixed numbers.
   D. Subtract the numerators of the fractions.

Review

CHAPTER 3

9. Nichelle needs 6 boards that are each $3\frac{1}{3}$ feet long to make bookshelves. She wants to cut the shelves from one long board with no board left over. How many feet long does the original board need to be?

A. 15
B. 18
C. $19\frac{1}{2}$
D. 20

10. Jake had $7\frac{3}{5}$ gallons of wood stain. After staining a set of stairs, he had $5\frac{2}{3}$ gallons of wood stain left. How many gallons of wood stain did Jake use on the stairs?

A. $2\frac{14}{15}$
B. $1\frac{14}{15}$
C. $1\frac{1}{2}$
D. $1\frac{1}{15}$

11. Ayca subtracted $10\frac{5}{7} - 8\frac{2}{9}$ and got $2\frac{3}{63}$. What mistake did she make?

A. She did not find a common denominator.
B. She did not subtract the whole numbers correctly.
C. She did not rename the fractions correctly.
D. She added instead of subtracting.

12. What improper fraction in simplest form is equal to $4\frac{2}{7}$?

A. $\frac{15}{7}$
B. $\frac{30}{7}$
C. $\frac{13}{7}$
D. $\frac{28}{7}$

13. Which set of fractions is in order from greatest to least?

A. $\frac{4}{5}, \frac{7}{9}, \frac{3}{4}, \frac{4}{6}$
B. $\frac{3}{4}, \frac{4}{6}, \frac{7}{9}, \frac{4}{5}$
C. $\frac{4}{5}, \frac{4}{6}, \frac{7}{9}, \frac{3}{4}$
D. $\frac{3}{4}, \frac{4}{6}, \frac{4}{5}, \frac{7}{9}$

14. Which is the first step in a sequence of steps for dividing mixed numbers?

A. Change the division symbol to multiplication.
B. Find a common denominator for the fraction part of the mixed numbers.
C. Change any mixed numbers to improper fractions.
D. Multiply the improper fractions.

CHAPTER 3

Review

Check Your Understanding

On the following chart, circle the number of any item you answered incorrectly. Near each lesson title, you will see the pages you can review to learn the content covered in the question. Pay particular attention to reviewing skill areas in which you missed half or more of the questions.

Chapter 3: Fractions	Procedural	Conceptual	Application/ Modeling/ Problem Solving
Introduction to Fractions pp. 74-81	1	13	
Add and Subtract Fractions pp. 82-87		2	
Multiply and Divide Fractions pp. 88-91		5	
Mixed Numbers pp. 92-97	6, 8, 12	11, 14	3, 4, 7, 9, 10