LESSON
1.1 Place Value

Lesson Objectives

You will be able to

- Use place value to read and write whole numbers
- Compare and order whole numbers

Skills

- **Core Skill:** Apply Number Sense Concepts
- **Core Practice:** Model with Mathematics

Vocabulary

approximate
chart
digit
number line
period
value
whole number

KEY CONCEPT: Represent, compare, and order whole numbers to better understand the meaning and value of whole numbers.

1. What number is 1 more than 8?

2. What number is 1 less than 73?

3- What number is 10 more than 60?

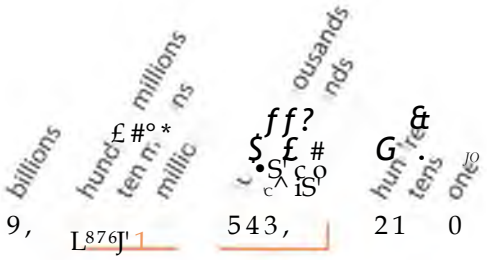
4. What number is 10 less than 45?

Place Value

Digits are the ten number symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. A number is an arrangement of digits in a particular order. The numbers beginning with 0, 1, 2, 3, and so on are the set of **whole numbers.** The position of a digit in a number determines its **value,** or how much it represents.

Starting from the ones place, commas are inserted every third number to separate a number into groups of three, called **periods.**

PLACE VALUE CHART



Example 1 Use a Place-Value Chart

In the number 137,258,406, which digit is in the ten millions place?

Step 1 Locate the ten millions place in the place-value chart. A chart is an arrangement of numbers or other information.

Step 2 Find the digit in 137,258,406 that is in that position. The 3 is in the ten millions place.

Example 2 Determine the Value of the Digits

What is the value of each digit in the number 105?

Step 1 1 is in the hundreds place. Its value is 1 hundred or 100.

Step 2 0 is in the tens place. No tens are in the number 105.

Step 3 The 5 is in the ones place. Its value is 5 ones, or 5.

IDENTIFY THE MAIN IDEA

Most of the material you read both at home and in school contains a **main idea.** The main idea tells what the paragraph, article, or lesson is about. The other sentences support the main idea.

The main idea is not always found in the first sentence or even in the first paragraph of a passage. It might be found almost anywhere within a passage. Sometimes the main idea is not even stated directly.

To identify the main idea ask: ***What is this passage about?***

Read the following paragraph and identify the main idea.

> (1) For some problems, an exact answer is not needed. (2) An estimate (an **approximate,** or "about," answer) will be sufficient. (3) It is also good to estimate an answer, then solve the problem, and finally check the solution by comparing the estimate to the exact answer.

Sentence 1 is a general suggestion that exact answers are not always needed. Sentence 2 explains what an estimate is and states that it may be all that is needed to answer a math problem. Sentence 3 states the usefulness of making an estimate first, finding an exact solution, and then comparing the two. Sentences 1 and 3 support Sentence 2. Sentence 2 states the main idea.

Core Practice
Model with Mathematics

Charts, which are diagrams that show information, can help you understand place value. In the place-value chart on page 12, for example, the place value is shown for each digit in the number, and periods are indicated. In a notebook, write a sentence explaining how the value of each digit in the number 5, 555 changes as the digits move from the right of the number to the left.

MATH LINK

Remember that zeros hold a position and should not be ignored. When writing numbers, write a zero for each place that is not expressed in words.

THINK ABOUT MATH

Directions: Identify the value of the underlined digit.

1. 3, 4̲78 _____
2. 15, 7̲89, 200 _____
3. 702, 432, 5̲16 _____
4. 7, 3̲00, 561, 892 _____
5. 8, 5̲70, 213, 000 _____

Read and Write Whole Numbers

In general, we read whole number in words and use the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 to write them.

Example 3 Read Whole Numbers

Read the number 28, 304.

Step 1 Begin at the left of the number. Read the number in each period, and replace the comma with the name of the period. So, the number 28, 304 means 28 thousands, 3 hundreds, 0 tens, 4 ones.

Step 2 Read the number 28, 304 as "twenty-eight thousand, three hundred four."

When reading whole numbers, remember to concentrate on each period and the positions of all the digits.

Example 4 Write Whole Numbers

Write the number *six million, two hundred ninety-one thousand, fifty* as a whole number.

Step 1 Six million becomes 6, 000, 000.
Two hundred ninety-one thousand becomes 291, 000.
Fifty becomes 50.

Step 2 Combine the whole number parts.
6, 000, 000 + 291, 000 + 50 = 6, 291, 050

When you write whole numbers, think about the place-value chart. Remember to insert zeros as needed.

THINK ABOUT MATH

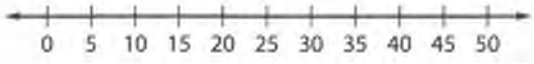
Directions: Match the number to its name in words.

_____ 1. fourteen thousand, two hundred sixty
_____ 2. five thousand, eighty-five
_____ 3. seventy-eight
_____ 4. twenty-six million
_____ 5. one hundred eleven thousand

A. 78
B. 111, 000
C. 26, 000, 000
D. 5, 085
E. 14, 260

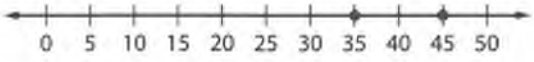
Compare and Order Whole Numbers

Compare numbers by using a **number line.** A number line is a list of numbers arranged in order from left to right on a line. The numbers are larger the farther right they are on the number line.



Example 5 Use a Number Line to Compare Numbers

Which is greater, 35 or 45?



Step 1 Locate each number on the number line.

Step 2 45 is to the right of 35, so 45 is greater than 35. Write this as 45 > 35 (45 is greater than 35) or 35 < 45 (35 is less than 45).

Example 6 Use Place Value to Compare Numbers

Compare. Write < or > in the blank to make a true statement.

12,358 _____ 12, 421

Step 1 To compare, align the numbers by place value.

12,358
12, 421

Step 2 Start at the left. Compare the digits until they differ.

12,358
12,421

The digits in the hundreds place are different. 3 is less than 4, so 12, 358 < 12, 421.

Example 7 Order Whole Numbers

Write the set of numbers in order from greatest to least.

4, 134, 805 5, 883, 081 4, 147, 001

Step 1 Align the numbers by place value. Start at the left and compare digits.

4,134,805
5,883,081
4,147,001

Step 2 5 > 4, so 5, 883, 081 is the greatest number. Continue comparing the other numbers until the digits differ.

4,134,805
4,147,001

4, 147, 001 > 4, 134, 805
So, 5, 883, 081 > 4, 147, 001 > 4, 134, 805.

21st Century Skill
Access Information

You obviously learn math from your textbook and teacher. But in this Information Age, your math education can be enriched if you explore other ways to master the subject. You can join an online Math Club, for example, where like-minded students share what they have learned in their own exploration of the mysteries of math. Your teacher can also provide you with books that would further your math education.

MATH LINK

When using the inequality symbols < and >, remember that the pointed end of the symbol always points to the smaller number.

THINK ABOUT MATH

Directions: Answer the following.

1. How would you use place value to compare 203, 478 and 204, 210?

_____

_____

2. Place the numbers below in order from least to greatest.
   701, 286 698, 321 698, 432

_____

Vocabulary Review

Directions: Complete the sentences below using one of the following words.

approximate digits number line periods value whole numbers

1. The position of a digit in a number determines its_____
2. The set of numbers beginning with 0, 1, 2, 3, and so on is the set of_____
3. Starting from the ones place, insert commas every third place to separate a number into groups of three called_____
4. The number system used today is based on the_____ 0 through 9 arranged in a particular pattern.
5. Antonia did not need an exact answer, so she found an_____ answer.
6. Numbers can be compared by using a_____

Skill Review

Directions: Write the value of each digit based on the place-value chart.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
2	4	7	3	0	1	5

1. 2_____ 4. 3_____ 6. 1 _____

2. 4_____ 5. 0_____ 7. 5_____

3. 7_____

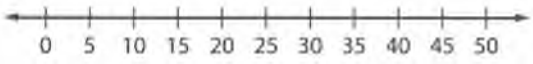
Skill Review (continued)

Directions: Create a place value chart for each number below.

8. 6, 729

9. one million, thirty-five

Directions: Locate each point on a number line. Then compare the numbers.



10. 16; 40

11. 20; 13

12. 5; 27

13. 50; 2

Directions: Identify the main idea of the passage.

14. One way to compare numbers is to line the digits up by place value, then compare the first place where the digits are different. A number line can also be used. You should arrive at the same conclusion with either method. There are several ways to compare numbers correctly.

15. Digits represent numbers. 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0 are the digits we use. The digits can be combined and re-ordered to create infinitely many numbers. Digits have different meanings based on which position they are in.

Skills Practice

Directions: Choose the best answer to each question.

1. Which of the following has the numbers arranged from greatest to least?

   A. 45, 378; 55, 210; 56, 345
   B. 5, 010, 000; 5, 100, 321; 5, 002, 146
   C. 78; 75; 72
   D. 379; 389; 398

2. Laquita wrote a check for $241 to pay her heating bill. How did she write this number in words on the check?

   A. two hundred forty-one
   B. two hundred forty
   C. two hundred fourteen
   D. twenty-one

3. Ashley priced four cars she was interested in buying. Which car was the most expensive?

   A. Car A: $23, 456
   B. Car B: $22, 201
   C. Car C: $22, 345
   D. Car D: $23, 712

4. For the number 601, 295, what is the place value of 0?

   A. hundred thousands
   B. ten thousands
   C. thousands
   D. hundreds

LESSON

1. 2

Add and Subtract Whole Numbers

Lesson Objectives

You will be able to

- Add whole numbers
- Subtract whole numbers

Skills

- **Core Skill:** Perform Operations
- **Core Practice:** Attend to Precision

Vocabulary

addition
calculate
context clue
difference
operations
subtraction
sum

MATH LINK

Numbers in an addition problem can be added in any order.

5 + 6 = 11 6 + 5 = 11

KEY CONCEPT: Addition and subtraction are basic operations in mathematics.

Write each number in words.

1. 37_____ 2. 1, 008_____ 3. 152_____ 4. 32, 000_____

Use a less than (<), greater than (>), or equal to (=) symbol to compare each pair of numbers.

5. 15 □ 39 6. 301 □ 108 7. 222 □ 44 8. 1, 234 □ 1, 324

Add Whole Numbers

The most basic of all **operations,** or processes, in mathematics is addition. **Addition** is the combining of two or more numbers. Suppose you have two sets of pencils: 4 in one set and 3 in the other. Find the total number of pencils by combining the two sets or adding 4 + 3.

4 + 3 = 7

The answer to an addition problem is called the **sum,** or total. So the sum of 4 and 3 is 7.

Example 1 Add Two-Digit Numbers

Find the sum of 32 and 47.

Step 1 To **calculate** means to find the answer using a mathematical process. To calculate the sum of an addition problem, line up the digits with ones under ones, tens under tens, and so on.

$$\begin{array}{r} 32 \\ +47 \\ \hline 79 \end{array}$$

Step 2 Add the ones column.

Step 3 Add the tens column.

Example 2 Column Addition

Add: 248 + 36 + 1, 987

Step 1 Line up the digits by place value.

Step 2 Add the digits in the ones column and write the sum at the bottom. If the sum has more than one digit, carry the left digit to the next column.

$$\begin{array}{r} {\scriptstyle 1\ 1\ 2\ \ } \\ 248 \\ 36 \\ +1987 \\ \hline 2,271 \end{array}$$

Step 3 Repeat until all columns have been added.

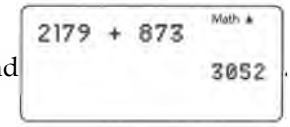
Example 3 Add Whole Numbers on a Calculator

Use a calculator to find the sum of 2, 179 + 873.

Press on

Press 2 1 7 9 + 8 7 3 enter

The display should read



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THINK ABOUT MATH

Directions: Solve each problem.

1. 48 + 31
2. 57 + 28
3. 14 + 165 + 374

Subtract Whole Numbers

Subtraction is deducting, or taking away, an amount from another amount. To find how many objects remain in a set of objects after some of them are removed, use subtraction. Suppose you have 8 pencils in a set and take away 5 of them. Find the number of pencils by performing the operation 8 - 5 .

8 – 5 = 3

The answer to a subtraction problem is called the **difference.** The difference between 5 and 8 is 3. Subtraction is also used to compare one amount to another: for example, the question, "How many more people registered to vote this year than last year? "

Core Practice
Attend to Precison

In mathematics, the words *accuracy* and *precision* are related to measurement. Accuracy refers to how close an answer is to a true, or real, value. For example, consider the problem 18 + 15. Say that three people find three sums: 32, 33, 34. The accurate sum is the one that comes closest to the true value of 18 + 15, which is 33. The answer 33 is accurate, and it takes only one process or calculation to find the value.

Now consider precision. Precision usually refers to the extent that more measurements or calculations give the same results. In other words, if you use a process to add 18 + 15 and then use a calculator to check your answer, the result is the same. If you ask a friend to solve the problem, the answer is also the same. It is a precise answer, or one that remains the same in multiple trials.

Work with a partner, and take turns posing addition problems. Solve the problems by hand and again by using a calculator to apply both accuracy *and* precision.

MATH LINK

The order of the numbers in subtraction cannot be changed. For example, 5 - 2 is not the same as **2 – 5**.

Example 4 Subtract Numbers Without Regrouping

Subtract 254 from 497.

Step 1 The sentence translates to **497 - 254.** In order to calculate the difference, write the digits in the ones under ones, tens under tens, and so on. Start with subtracting the ones, 7 - 4 .

$$\begin{array}{r} 497 \\ -\ 254 \\ \hline 243 \end{array}$$

Step 2 Subtract the digits in the tens and then the hundreds columns.

Example 5 Subtract Numbers with Regrouping

Find 2, 754 - 657.

Step 1 Begin in the ones column. Since you cannot subtract 7 from 4, regroup a ten as 10 ones. Subtract.

Step 2 Move to the tens column. Since you cannot subtract 5 from 4, regroup a hundred as 10 tens. Subtract.

Step 3 Move to the hundreds column. Subtract.

Step 4 Bring down the 2 in the thousands place.

$$\begin{array}{r} {\scriptstyle 14} \\ {\scriptstyle 6\ \not{7}\ \not{5}\ {}^{1}\not{4}} \\ 2,\not{7}\not{5}\not{4} \\ -\ 6\ 5\ 7 \\ \hline 2,\ 0\ 9\ 7 \end{array}$$

MATH LINK

Addition and subtraction are opposite operations. Subtraction can be used to check a sum.

To check 7 + 3 = 10, use 10 - 7 = 3.

Addition can be used to check a difference.

To check 15 – 9 = 6, use 6 + 9 = 15.

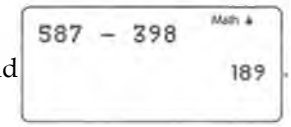
Example 6 Subtract Whole Numbers on a Calculator

Use a calculator to find the difference of 587 - 398.

Press on

Press 5 8 7 – 3 9 8 enter.

The display should read



THINK ABOUT MATH

Directions: Solve each problem.

1. 73 - 48
2. 2, 387 - 455
3. **800 - 171**

Vocabulary Review

Directions: Complete each sentence with the correct word.

calculate difference operations sum

1. The result in subtraction is called the_____
2. To_____is to find an answer using a mathematical process.
3. The answer to an addition problem is its _____
4. Addition and subtraction are basic_____ in mathematics.

Skill Review

Directions: Identify the context clue for each word problem. Solve the problem.

1. Last night, the attendance at the Ten Screen Movie Theater was 3, 183. Tonight, the attendance increased by 459 people. How many people were at the movie theater tonight?

2. The football stadium store sold 2, 498 T-shirts on one Saturday and 3, 565 on the next Saturday. How many T-shirts were sold on those two Saturdays combined?

3. Ahmed's gross pay for the week is $525. If his deductions are $138, what is his net pay?

4. Janine bought a car for $8, 750. After two years, the car had lost value. It had depreciated $1, 900. How much is her car worth now?

5. Jermaine has $576 in his checking account. He uses his ATM card and withdraws $110. How much money is left in his checking account?

6. The bookstore ordered 317 books on Friday. On Monday, 489 books were ordered. How many total books were ordered?

7. The James family's earnings for last year were $63, 789. This year, the earnings were $69, 123. How much greater were the earnings this year than last year?

Skill Practice

Directions: Choose the best answer to each question.

1. During the week, Bernardo drove these distances: 456 miles, 482 miles, 449 miles, 479 miles, and 468 miles. How many miles did he drive during the week?

   A. 2, 000
   B. 2, 034
   C. 2, 304
   D. 2, 334

2. Last year, a food company sold 937, 642 packages of carrots. This year, 1, 000, 000 packages were sold. How many more packages were sold this year?

   A. 62, 358
   B. 124, 598
   C. 999, 098
   D. 1, 937, 642

3. Warren County collected $23, 470 in taxes last year and $31, 067 this year. How much more money was collected this year?

   A. 7, 597
   B. 8, 567
   C. 7, 957
   D. 8, 957

4. At an insurance company, there were 380 cubicles on the first floor, 407 cubicles on the second floor, 298 cubicles on the third floor, and 321 cubicles on the fourth floor. How many cubicles are there in total at the insurance company?

   A. 986
   B. 1, 406
   C. 2, 314
   D. 1, 043

LESSON
1.3

Multiply and Divide Whole Numbers

Lesson Objectives

You will be able to

- Multiply whole numbers
- Divide whole numbers

Skills

- **Core Skill:** Find Reverse Operations
- **Reading Skill:** Draw Evidence from Text

Vocabulary

context
dividend
division
divisor
factor
multiplication
product
quotient

KEY CONCEPT: Multiplication is the operation of adding a certain quantity a set number of times. Division is the operation that is used to separate a quantity into parts.

Add.

1. 4 + 8
2. 57 + 13
3. 142 + 89
4. 909 + 111

Subtract.

5. 86 - 53
6. 718 - 81
7. 100 - 54
8. 21 - 9

Multiply Whole Numbers

The answer to a **multiplication** problem is called the **product.** The numbers that are multiplied are the **factors.**

factor x factor = product

An x or a dot (•) can be used to show multiplication. Here are two ways to write 3 times 9 equals 27.

3x9 = 27 3 • 9 = 27

Example 1 Multiply Two Numbers

Multiply 736 x 45.

Step 1 Line up the digits you want to multiply with ones under ones, tens under tens, and so on. Put the number with more digits on top. Multiply each digit in 736 by 5 in 45 to find the first partial product.

```
  736
×  45
3,680
```

Step 2 Multiply each digit in 736 by the 4 in 45. Start the second partial product under the 8.

Step 3 Add the partial products.

```
  736
×  45
 3680
29440
33,120
```

MATH LINK

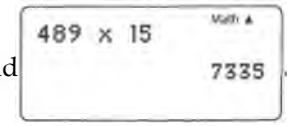
In multiplication, the product of two single-digit numbers can be a double-digit number. In such cases, place the ones digit of the number in the partial product, and carry the tens digit over to the next multiplication. In the first step of Example 1 (5 x 6 = 30), for example, the 0 is placed in the ones place. Then add (or carry) 3 to the next product, 5 x 3 = 15, to get 18. Now place 8 in the tens place, and carry and add 1 to the next product, 5 x 7 = 35, to get 36. The final product is 3, 680. In most multiplication problems, some carrying to the next place value is needed.

Example 2 Multiply Whole Numbers on a Calculator

Use a calculator to find the product of 489 × 15.

Press [on]

Press [4] [8] [9] [×] [1] [5] [enter]

The display should read 

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THINK ABOUT MATH

Directions: Find the product.

1. 17x4
2. 46 x 9
3. 390 x 4
4. 63 x 311
5. 394 x 29
6. 38 • 18
7. 96 • 37
8. 48 ·· 207
9. 100 x 482
10. 1, 467 x 35

Divide Whole Numbers

The answer to a **division** problem is called the **quotient.** The number that is divided is the **dividend,** and the number that is dividing it is the **divisor.** There are several ways to show division.

dividend ÷ divisor = quotient			divisor$\overline{)\text{dividend}}$ quotient
24 ÷ 8	$8\overline{)24}$	24/8	$\frac{24}{8}$

Example 3 Divide Two Numbers

Divide: 372 6

Step 1 Find the largest number that you can multiply the divisor by to get a product that is less than or equal to the dividend. Since you cannot divide 6 into 3, start this problem by dividing 6 into 37.

$$6\overline{)372}$$

$$\begin{array}{r} 62 \\ 6\overline{)372} \\ -36 \\ \hline 12 \end{array}$$

Step 2 Multiply 6x6 = 36, and subtract 37 - 36 = 1. Continue to multiply, subtract, and bring down the next number. Divide 6 into 12.

$$\begin{array}{r} 62 \\ 6\overline{)372} \\ -36 \\ \hline 12 \end{array}$$

Step 3 Multiply and subtract.

$$\begin{array}{r} 62 \\ 6\overline{)372} \\ -36 \\ \hline 12 \\ -12 \\ \hline 0 \end{array}$$

Core Skill
Find Reverse Operations

You learned that addition and subtraction and also multiplication and division are reverse operations. This is not simply a "fun fact. " A reverse operation allows you to check an answer that you came up with when solving a problem. Consider this division problem: Divide 25 by 5. You come up with the answer 20. You can check the answer by using the reverse operation. You turn the original divisor and the quotient in your solution into factors, and after multiplying the two numbers, you check the product against the original dividend. As soon as you do so, you see that your original answer was wrong, because 5 x 20 ≠ 25. You didn't divide; you subtracted when solving the problem.

After you complete the Skill Practice questions on page 25, check your answers by using the reverse operation in each case.

Reading Skill
Draw Evidence from Text

Context is the setting, events, or ideas surrounding something. In reading, context is the surrounding words and sentences that help explain the meaning of a certain word. In math word problems, context clues can help the reader determine which operation to use in a word problem. Phrases such as *product* and *times* indicate multiplication. Phrases such as *quotient*, *split*, and *divided* mean division.

Sometimes a little more detective work is needed to determine the operation(s) that will solve a problem. Read the following word problems.

Jorge puts $20 in his savings every week. What is the total he will have saved at the end of 17 weeks?

Keisha has a case of 100 granola bars. She eats five bars every week. How many weeks will the case of granola bars last?

Both problems contain the words *every week*. The first problem gives the number of weeks and is asking for a *total*, so multiplication is indicated. The second problem tells the *total* number of granola bars in a case and is asking for *how many weeks*, so division is the best choice.

In a notebook, make a list of phrases that are clues to using multiplication and another list of clues to using division.

Example 4 Divide

Divide: $4\overline{)2,374}$

Step 1 Divide 4 into 23.

Step 2 Multiply, subtract, and bring down the next number.

Step 3 Divide 4 into 37.

Step Multiply, subtract, and bring down the next number.

Step 5 Divide 4 into 14.

Step 6 Multiply and subtract. There are no more numbers to bring down. The number 2 is the remainder.

```
  593 R2
4)2,374
 -20
  37
 -36
  14
 -12
   2
```

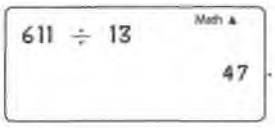
Example 5 Divide Whole Numbers on a Calculator

Use a calculator to find the quotient of 611 / 13.

Press on

Press 6 1 1 ÷ 1 3 enter.

The display should read



THINK ABOUT MATH

Directions: Find the quotient.

1. 41 ÷ 3
2. 186 ÷ 6
3. $15\overline{)480}$
4. $16\overline{)3,246}$
5. 1,200/6
6. 409 ÷ 12
7. 200 ÷ 10
8. $16\overline{)248}$
9. $9\overline{)984}$
10. 625/39

Vocabulary Review

Directions: Write each word next to its meaning.

dividend division divisor factor multiplication product quotient

_____ 1. the number by which the dividend is being divided

_____ 2. one of the numbers in a multiplication problem

_____ 3. the answer to a multiplication problem

_____ 4. the number being divided

Vocabulary Review (continued)

_____5. repeating a quantity a set number of times

_____6. the answer to a division problem

_____7. separating a quantity into parts

Skill Review

Directions: Write the words or phrases that give a context clue to the correct operation. Solve the problem.

1. During the year, Juanita spent $480 for electricity. She paid the same amount each month. How much did she pay monthly for electricity? (1 year = 12 months)
2. Bob owns a hardware store. He sells hammers for $15. On Saturday, he sold 36 hammers. How much money did he collect for hammers that day?
3. Harold had 4, 866 stamps in his stamp collection that were equally divided into six stamp books. How many stamps are in each of his six stamp books?
4. If Rafael pays $75 monthly for health insurance, how much will he have paid after two years?
5. The ticket office sold 64, 750 tickets to the playoff game. If the cost is $27 per ticket, how much money did the ticket sales generate?
6. A city swim team generated $432 in ticket sales. If the tickets are $3 each, how many tickets were purchased?
7. Sun is decorating tables for an event. She needs to place 30 rose petals on each table. There are 17 tables. How many petals does she need?

Skill Practice

Directions: Choose the best answer to each question.

1. The computer repair shop pays each of five employees $589 each week. How much is this total each week?

   A. $2, 505
   B. $2, 914
   C. $2, 945
   D. $2, 954

2. Chantou pays $525 for rent every month. How much rent does she pay in one year?

   A. $40 R5
   B. $43 R9
   C. $1, 575
   D. $6, 300

3. There are 320 people expected for an awards dinner. Each table can seat 16 people. How many tables will be needed?

   A. 10
   B. 20
   C. 24
   D. 32

4. A rock band earns $1, 315 from ticket sales at a concert. For each ticket sold, the band gets $5. How many tickets were sold?

   A. 263
   B. 343
   C. 1, 163
   D. 6, 575

LESSON
1.4

Factoring

Lesson Objectives

You will be able to

- Determine the set of factors of a number
- Determine the greatest common factor of two numbers
- identify and apply patterns

Skills

- **Core Skill:** Apply Number Sense Concepts
- **Core Skill:** Build Solutions Pathways

Vocabulary

Commutative Property of Multiplication
Distributive Property of Multiplication
equation
evaluate
expression
factor
greatest common factor
operation

KEY CONCEPT: A whole number is the product of two or more factors, and the greatest common factor is the greatest factor shared by those whole numbers.

Begin at 0 and count forward: 0, 1, 2, 3, 4, and so on. The number 0 and the numbers that follow are whole numbers. A place-value chart is a valuable tool for reading the value of each digit in a whole number. For example, the value of the 2 in the whole number 721, 465 is 20, 000.

Billions			Millions			Thousands			Ones		
H	T	0	H	T	O	H	T	0	H	T	0
						7	2	1	4	6	5

What Is a Factor?

We can use the following equation, or mathematical statement, to answer the question "What is a factor? "

$$6 \times 4 = 24$$

A factor is a number that is multiplied by another number to give a product, or total. Start with the number 6 in the equation. The example shows that you can multiply 6 by another number to get the product 24. So, the number 6 is a factor of 24.

Look at the equation again. It shows that you can multiply 4 by another number to get the product 24. So, the number 4 is also a factor of 24.

Apply Number Sense Concepts: Finding the Factors of a Number

You know that the numbers 6 and 4 are factors of 24. There are more factors of 24, too. Look at the different factors you can multiply to get 24, starting with 1 and working your way up to 24.

$1 \times 24 = 24$
$2 \times 12 = 24$
$3 \times 8 = 24$
$4 \times 6 = 24$
$6 \times 4 = 24$
$8 \times 3 = 24$
$12 \times 2 = 24$
$24 \times 1 = 24$

Do you notice any similarities among these equations? Think about the **Commutative Property of Multiplication.** This property states that if you switch the order in which two numbers are multiplied, the product is the same. For example:

$$2 \times 12 = 24$$
$$12 \times 2 = 24$$

Because 12 x 2 = 24 and 2 x 12 = 24, both of these equations tell us the same thing—that the numbers 2 and 12 are factors of 24. There is no need to write the same factors more than once, so you can cross out one of these equations from the list.

Look for other examples of the Commutative Property of Multiplication in the list. Cross out one equation in each pair.

1 x 24 = 24
2 x 12 = 24
3 x 8 = 24
4 x 6 = 24
~~6 x 4 = 24~~
~~8 x 3 = 24~~
~~12 x 2 = 24~~
~~24 x 1 = 24~~

Now that you have crossed out equations that use identical factors, you can write each factor in the list of equations in numerical order. Starting from the top, the factors are 1, 2, 3, 4, 6, 8, 12, and 24.

You can shorten the set of factors even further. Notice the first multiplication equation in the list:

1 x 24 = 24

The number 1 is a factor of 24, but 1 is a factor of every number! For this reason, we do not include 1 in any set of factors.

The number 24 is a factor of 24, but every number is a factor of itself! For this reason, we do not include a number in its own set of its factors.

So, the factors of 24 are 2, 3, 4, 6, 8, and 12. You can follow the same procedure to find the set of factors of any number.

Core Skill
Apply Number Sense Concepts

You can apply properties of numbers to solve mathematical problems. The Associative Property, for example, applies to grouping. In addition, the rule is $a + (b + c) = (a + b) + c$, so the order of the **addends,** or numbers being added, doesn't matter. The sum remains the same. The property also applies to multiplication. The order of the factors, or numbers being multiplied $a(b \times c) = (a \times b)c$, doesn't matter. In either case, the product, or answer, remains the same.

The Commutative Property also applies to addition and multiplication. In addition, the sum remains the same even when the order of two addends changes: $a + b = b + a$

In multiplication, the product remains the same even when the order of two factors changes: $ab = ba$

Core Skill
Build Solution Pathways

You can shorten the amount of time you spend factoring a number by looking for patterns among the equations you write when factoring. For example, factor the number 28.

$2 \times 14 = 28$
$4 \times 7 = 28$
$\mathbf{7 \times 4 = 28}$ ←
$14 \times 2 = 28$

Notice that the third equation is the first time you can apply the Commutative Property of Multiplication. It tells you that these factors have been multiplied before: $7 \times 4 = 4 \times 7$. When you move to the fourth equation in the list, you see the same thing: $14 \times 2 = 2 \times 14$. You have found a pattern. You can use this pattern to determine the factors of a number more quickly.

As long as you begin factoring with the lowest factor, you can stop writing equations the first time that you identify an equivalent equation using the Commutative Property of Multiplication. By this time, you will have identified all of the factors of a number. In this example, the only equations you have to look at are

$2 \times 14 = 28$
$4 \times 7 = 28$

So, the factors of 28 are 2, 4, 7, and 14.

Comparing Sets of Factors

Often in mathematics, it is necessary to compare sets of factors for two different numbers. For example, you can compare the sets of factors for 24 and 100. Recall the factors of 24. They are 2, 3, 4, 6, 8, and 12.

Now you can use the procedure for finding factors to determine the factors of 100.

- List the equations.
- Apply the Commutative Property of Multiplication. Cross out equivalent equations.
- Delete equations containing the factors 1 and the number itself.
- Order the remaining factors.

~~$1 \times 100 = 100$~~
$2 \times 50 = 100$
$4 \times 25 = 100$
$5 \times 20 = 100$
$10 \times 10 = 100$
~~$20 \times 5 = 100$~~ ← This is the first time the Commutative Property of Multiplication applies.
~~$25 \times 4 = 100$~~
~~$50 \times 2 = 100$~~
~~$100 \times 1 = 100$~~

The factors of 100 are 2, 4, 5, 10, 20, 25, and 50.

Now that you know both sets of factors, you can find common factors, or factors the numbers share.

Factors of 24: 2, 3, 4, 6, 8, 12
Factors of 100: 2, 4, 5, 10, 20, 25, 50

The **greatest common factor** is the largest common factor shared by two numbers. In this example, there are only two shared factors, and the greater factor is 4. So, the greatest common factor of 24 and 100 is 4.

Factoring Mathematical Expressions

In mathematics, an **expression** is a phrase that shows terms separated by **operation** symbols, such as symbols for addition, subtraction, multiplication, and division. You can use the greatest common factor of two numbers to rewrite mathematical expressions to make them easier to **evaluate,** or find the value of. For example, consider the addition expression 36 + 8.

First, find the greatest common factor of 36 and 8. Begin by factoring each number and identifying common factors.

Factors of 36: 2, 3, 4, 6, 8
Factors of 8: 2, 4
The common factors are 2 and 4.
The greatest common factor is 4.

Next, use the factors to simplify the expression. Each number in the expression is an addend, or a number that is added to another number. Express the addends in 36 + 8 as a product of two numbers, one of which is the greatest common factor, 4.

$$36 + 8 = (4 \times 9) + (4 \times 2)$$

Now you can apply the **Distributive Property of Multiplication** to rewrite this expression as:

$$(4 \times 9) + (4 \times 2) = 4 \times (9 + 2)$$
$$36 + 8 = 4 \times (9 + 2)$$

THINK ABOUT MATH

Directions: Think about this example of a subtraction expression: 63 - 28.

The factors of 63 are; _____

The factors of 28 are; _____

The only common factor is_____

So, the greatest common factor is_____

Now, apply the Distributive Property of Multiplication to rewrite the subtraction expression.

63 − 28 = _____ − _____

_____ − _____ = 7 × (_____ − _____)

21st Century Skills

Critical Thinking and Problem Solving

Few people in the workplace will ever need to factor numbers in their daily work. So why is it important to learn factoring in school? The reason is that learning the process of factoring is an exercise in developing methods of thought that help you solve problems. For example, you can use a standard procedure, or **algorithm,** to evaluate the expression 63 - 28. When you factor the numbers and apply the Distributive Property, you actually simplify the problem. Learning to factor, then, helps you recognize opportunities to simplify complex problems, saving time and effort.

MATH LINK

To *commute* means "to move or change. " The Commutative Property of Multiplication states that if you change the order of the numbers you're multiplying, the result will stay the same. In the box below, provide an example of the Commutative Property of Multiplication. Then use the example to explain how applying the Commutative Property of Multiplication shortens the time it takes to factor a number.

_____ x _____ = 20

_____ x _____ =20

_____ x _____ = 20

_____ x _____ = 20

Factors of 20: _____

Vocabulary Review

Directions: Match each term to its example.

1. _____greatest common factor
2. _____ equation
3. _____ Commutative Property of Multiplication
4. _____ Distributive Property of Multiplication
5. _____ evaluate
6. _____ operation
7. _____ expression
8. _____factor

A. 9 x 4 = 36

B. 6 x 54 = (6 x 50) + (6 x 4)

C. 12: 1, 2, 3, 4, 6, 12
16: 1, 2, 4, 8, 16

D. 9 + 3

E. 1 2 x 2 = 2 x 1 2

F. 3 cases = $45, so 1. 25 cases =?

G. division

H. the number 9 in 9 x 28

Skill Review

Directions: Use the factoring procedure to find all factors of the following numbers:

1. 12_____

2. 32_____

3. 45_____

4. 44_____

5. 88_____

Directions: Use the factoring procedure to find the greatest common factor of each of the following pairs of numbers:

6. 4 and 20_____

7. 30 and 42_____

8. 35 and 49_____

9. 66 and 88_____

10. 50 and 100_____

Directions: Use the Distributive Property to write the following expressions in the form a x (b + c) or *a* x (b - c), where *a* is the greatest common factor.

11. 24 + 36 ____

12. 45 - 27 ____

13. 20 + 64 _____

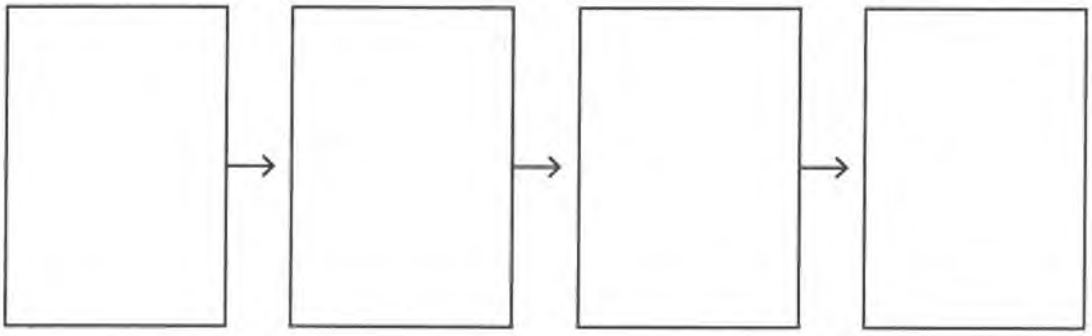
14. 48 + 72 _____

15. 66 - 44 _____

Skill Practice

Directions: Use the flow chart to show the stages of factoring. Number each stage.

1.



Directions: Choose the best answer.

2. Which of the following sets of numbers represents all of the factors of 52?

A. 2, 26
B. 2, 4, 13, 26
C. 1, 52
D. 2, 4, 6, 13, 26
E. 2, 6, 13, 26

3. Which of the following sets of numbers represents all of the factors of 90?

A. 2, 3, 5, 6, 9, 10
B. 2, 3, 5, 6, 9, 10, 15, 18, 30, 45
C. 2, 3, 5, 9, 10, 18, 30, 45
D. 2, 3, 30, 45
E. 2, 45

4. What is the greatest common factor of 36 and 60?

A. 2
B. 6
C. 12
D. 36
E. 60

5. What is the greatest common factor of 25 and 70?

A. 2
B. 5
C. 15
D. 25
E. 70

6. Which of the following expressions is equivalent to 60 + 84, where the first term is the greatest common factor of 60 + 84?

A. 2 x (30 + 42)
B. 3 x (15 + 21)
C. 6 x (10 + 14)
D. 12 x (5 + 7)
E. 24 x (5 + 7)

7. Which of the following expressions is equivalent to 96 - 72, where the first term is the greatest common factor of 96 - 72?

A. 2 x (48 - 36)
B. 4 x (24 - 18)
C. 6 x (16 - 12)
D. 12 x (8 - 6)
E. 24 x (4 - 3)

LESSON

1. 5 Rounding and Estimation

Lesson Objectives

You will be able to

- Identify situations in which rounding or estimating is appropriate
- Round numbers to the nearest specified place value
- Use estimation appropriately

Skills

- **Core Skill:** Paraphrase Data
- **Core Practice:** Use Appropriate Tools Strategically

Vocabulary

compatible numbers
estimate
front-end digits
rounding

KEY CONCEPT Rounding and estimation are useful when an answer does not need to be exact or when checking an exact answer.

Solve each problem.

1. 45 + 853
2. 23 x 100
3. 692 - 132
4. 112 / 16
5. 43 - 32
6. 18 x 912
7. 341 + 35
8. 1, 170 / 26

Rounding

For some problems, an exact answer is not necessary. An **estimate,** an approximate answer, will be sufficient. It is also good practice to estimate an answer first, solve the problem, and then check your solution by comparing the estimate to the exact answer.

One of the most common estimation strategies is **rounding.** Think of a number as being part of a hilly number line like the one below.

The numbers roll forward or backward to the closest valley. Numbers ending in 0, 1, 2, 3, and 4 roll back to the nearest 0. Numbers ending in 5, 6, 7, 8, or 9 roll ahead to the nearest 0.

Example 1 Round Numbers

Round 53 and 227 to the nearest ten or nearest hundred.

To the nearest ten, 53 rounds backward to 50.

To the nearest hundred, 53 rounds forward to 100.

To the nearest ten, 227 rounds forward to 230.

To the nearest hundred, 227 rounds backward to 200.

THINK ABOUT MATH

Directions: Round each number to the nearest ten.

1. 57
2. 92
3. 125
4. 1, 345

DRAW CONCLUSIONS

Drawing conclusions requires you to make decisions about information in the text. It is taking the facts presented by the author and determining their relationship to each other or their most logical outcome.

It is important to base conclusions on only the facts given and not to create meaning that is not presented in the text.

What conclusions can be drawn from the following paragraph?

> In the hot months of summer, some people use air conditioning to cool off. Air conditioners use electricity. Electric bills are based on the amount of electricity used by a household. They vary from month to month.

What conclusion can you draw about electric bills in the summer, based on the passage above?

_____ (A) They rise for all people.

_____ (B) They fall for all people.

_____ (C) They rise for people who use air conditioning.

_____ (D) They rise for people who do not use air conditioning.

The correct answer is C. The passage states that some people use air conditioning, which uses electricity. Using more electricity raises electric bills. The passage does not give any facts about people who do not use air conditioning, so no conclusions about their electric bills can be drawn.

Core Skill
Paraphrase Data

Paraphrasing is the ability to restate data in your own words. Paraphrasing is one way to determine if you have mastered the topic that was taught in the text. After reading the lesson, ask yourself, "What steps are involved in rounding? What steps are involved in estimating? " Breaking the process down to a series of steps makes it easier to lay out the process in your own words.

After you answer question 1 in Skill Practice, paraphrase the steps you took to round the numbers in the table.

21st Century Skill

Undestand Media Messages

You often encounter data in media stories. For example, you might see a table containing numbers in a news story that estimates television ratings. A sports story may present two numbers that represent a team's won-loss record. Recognizing what the numbers in media stories mean helps you to understand the story.

Look for a media story that contains numbers. Describe the story to a friend and explain what the numbers mean.

Core Practice

Use Appropriate Tools Strategically

Estimation strategies vary in their degree of precision. Front-end estimation is a convenient way to quickly estimate numbers, but it is not always the best choice. In Example 2, the actual answer (1, 042) is closer to 1, 000 than to 900. As numbers get larger, into the thousands and beyond, the estimates can become much further off from the actual number. This estimation strategy is best chosen when only a very general sense of the number is needed.

Estimation

Estimation can often save time when an exact answer is not needed or when you want to check whether an exact answer is reasonable.

Front-End Digits

We have already used the strategy of rounding. Another estimation strategy involves using the far-left digits or **front-end digits.**

Example 2 Estimate the Sum of 259, 673, and 110

Step 1 Rewrite each number using its front-end digit followed by zeros to replace the other numbers.

$259 \longrightarrow 200 \qquad 673 \longrightarrow 600 \qquad 110 \longrightarrow 100$

Step 2 Perform the appropriate operation, in this case addition.

$$\begin{array}{r} 259 \\ 673 \\ +\,110 \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 200 \\ 600 \\ +\,100 \\ \hline 900 \end{array}$$

The sum of 259, 673, and 110 is about 900.

Compatible Numbers

Sometimes, numbers that are close to the original numbers are used instead of the original numbers to make the solution easier or quicker to achieve. These numbers are called **compatible numbers.** They are also helpful in mental math.

Example 3 Estimate the Quotient of 1, 239 Divided by 37

Step 1 Change 1, 239 and 37 to numbers that are easier to divide.

$1{,}239 \longrightarrow 1{,}200 \qquad 37 \longrightarrow 40$

Step 2 Divide using the new numbers.

$$40\overline{)1{,}200} = 30 \text{ R } 00 \quad (120)$$

Answer: 1, 239 divided by 37 is about 30.

Step 3 Check the estimate.

Multiply 30 and 40 (mentally if possible). The product 1, 200 is close to 1, 239, so the estimate is reasonable.

THINK ABOUT MATH

Directions: Estimate each answer using front-end numbers or compatible numbers.

1. 563 + 215
2. 2, 610 / 485
3. 251 + 358 + 608
4. 1, 115 + 8

Vocabulary Review

Directions: Match each word to its definition.

A. compatible numbers B. estimate C. front-end digits D. rounding

_____1. a common estimation strategy in which the number goes up or down to the nearest 0

_____2. an approximate answer

_____3. numbers that are close to the original numbers and make the solutions easier or quicker to achieve

_____4. an estimation strategy involving the use of far-left digits

Skill Review

Directions: State the conclusion you draw about the result of the estimation method. Explain.

1. Jamie needs to estimate the number of buses he will need in order to transport people to the company picnic. Each bus can carry 56 people, and there are 1, 344 people going to the picnic. Jamie decides to use the compatible numbers 60 and 1, 200. Will he have enough buses for the picnic?

2. Mai has $485 in her checking account. She writes a check for repairs totaling $211 done to her car. She also needs to pay her telephone bill of $232 in the next week. After writing the check for the car repair, she estimates the amount in her checking account and the amount of the car repair check by using front-end estimation. Does she have enough in her checking account to pay her phone bill?

Skill Practice

Directions: Choose the best answer to each question.

The following chart gives the weights of three animals on a particular farm.

Animal	Weight
Clydesdale Horse	2, 067 lb
Welsh Pony	478 lb
Arabian Horse	952 lb

1. About how many pounds more does the Clydesdale weigh than the Welsh pony?

   A. 500 **C.** 1, 500
   B. 1, 000 **D.** 2, 500

2. Estimate the quotient of 5, 398 ÷ 87 using compatible numbers.

   A. 54 **C.** 60
   B. 55 **D.** 62

3. Mario is buying a pair of pants for $49, a shirt for $25, and socks for $6. Which method of estimation should he use to make sure he has enough money to buy them?

   A. rounding up
   B. rounding down
   C. front-end digits
   D. compatible numbers

4. Julietta is buying six new windows for her home. The windows she wants cost $365 each. How much should she budget to buy the windows?

   A. $60
   B. $400
   C. $1, 800
   D. $2, 400

LESSON

1.6 Arithmetic Expressions

Lesson Objectives

You will be able to

- Understand that operations must be performed in a specific order
- Solve problems involving the order of operations
- Use mental math to solve problems without paper and pencil

Skill

- **Core Practice:** Make Use of Structure
- **Core Skill:** Solve Real-World Arithmetic Problems

Vocabulary

compensation
mental math
order of operations
strategy

MATH LINK

A way to remember the order of operations is PEMDAS, or parentheses, exponents, multiplication, division, addition, subtraction.

Exponents are covered in Chapter 7.

KEY CONCEPT: Basic mathematical operations must be performed in a specified order to obtain the correct answer.

Solve.

1. 46 + 51
2. 34, 762 - 4, 875
3. 15 x 172
4. 102 x 72
5. 1, 145 - 132
6. 3, 413 / 17
7. 1, 092 / 26
8. 1, 892 + 412

Order of Operations

An **arithmetic expression** is an expression that has a number value. It often includes more than one operation. When finding the value of an arithmetic expression, you need to know the proper **order of operations,** the order in which the operations should be performed. For example, the expression 8 - 4 x 2 suggests two possible ways to solve the problem. Working from left to right, subtract first and get 8-4 = 4; 4x2 = 8, which is an incorrect answer. Multiplying first gives 4 **x** 2 = 8; 8-8 = 0. This is the correct answer.

Use the following set of rules whenever you want to find the value of an arithmetic expression. If an expression includes more than one operation of the same type, work from left to right.

1) Do operations within parentheses.
2) Do multiplication and division.
3) Do addition and subtraction.

Example 1 Use the Order of Operations

Find the value of the expression 5 **x** (6-2) / 2-2.

Step 1 Do operations within parentheses.
5 **x** 4 4 - 2 - 2

Step 2 Do multiplication and division. Work from left to right.
20 4- 2 - 2 ; 10-2

Step 3 Do addition and subtraction.
10 - 2 = 8; The value of the expression is 8.

If the expression does not include all the operations listed in the rules, skip the step with that rule and go on to the next step. If an expression includes a set of parentheses inside another set, work from the inside to the outside.

Example 2 Find the Value of an Expression

$100 - (2 \times (9 - 2)) \times 3$

Step 1 Do operations within parentheses.
$100 - (2 \times 7) \times 3$; $100 - 14 \times 3$

Step 2 Do multiplication and division.
$100 - 42$

Step 3 Do addition and subtraction.
$100 - 42 = 58$; The value of the expression is 58.

THINK ABOUT MATH

Directions: Use the order of operations to solve each problem.

1. $(12 + 4) / (1 + 7)$
2. $((30 - 8) / 2) + 5$
3. $13 + 6 \times 2 / 4 - 9$
4. $60 - (25 - (13 - 4))$

Mental Math Strategies

Sometimes, a solution can be determined without using paper, pencil, or a calculator. **Mental math** is done by applying certain strategies to find an answer without writing. A **strategy** is a plan. Knowing and practicing these strategies will save time in situations such as taking timed tests.

Zeros

One strategy involves shortcuts for adding or multiplying when zeros are involved. For example, to multiply 48 and 100, first write 48. Since 100 ends in two zeros, insert two zeros after the 48.

Think: "$48 \times 100 = 4,800$."

Another example would be adding 5, 000, 14, 000, and 6, 000 by thinking "5 + 14 + 6 = 25 and then attach three zeros."

Example 3 Mentally Multiply 60 and 200

Multiply 6 and 2 and insert 3 zeros (1 zero in 60 + **2** zeros in 200).

$$60 \times 200 = 12{,}000$$
$$\uparrow \quad \uparrow \quad \uparrow$$
$$6 \times 2 = 12$$

Core Skill
Solve Real-World Arithmetic Problems

When you use information you know in a different way or in a new situation, you apply that information.

Many math skills can be applied to everyday problems that are encountered in the context of everyday life. For example, mental math strategies are useful for adding amounts to find a total when shopping.

Some word problems may suggest the kind of situation that you might experience in your own life. For example, Doe *Ho wants to save $400 over the next 4 months. How much should he save every week?* You know there are 4 weeks in a month, so first you multiply 4 weeks times 4 months to get 16 weeks. Then you divide $400 by 4 months to get $25. He must save $25 a week.

In a notebook, write about a time when you made a purchase of some kind. What was the cost of the item? How much money did you hand over when making the purchase? How much change did you receive? Do the calculation.

MATH LINK

Often, in print, when extra sets of grouping symbols are needed, symbols such as brackets [ ] and braces {} are used. This book uses the double sets of parentheses you saw in Example 2 to make the use of a calculator easier.

MATH LINK

When using the compensation strategy, work from left to right. The order in which the compensated numbers are added can change the value of the solution.

For example, 43 - 25:
43 - 5 - 25 + 5 = 38 -
25 + 5 = 13 + 5 = 18

However, changing the order in which the addition and subtraction are performed will give an incorrect solution.

(43 - 5) - (25 + 5) =
38 - 30 = 8, an incorrect solution.

Break Apart Numbers

Another strategy is known as **breaking apart numbers.** This is thinking of numbers as the sum of two or more smaller numbers. Apply this strategy when it is easier to think of a number as the sum of two numbers. For example, to add 73 + 25, mentally think of 73 as 70 + 3 and 25 as 20 + 5. Then add 70 + 20 and 3 + 5 to get an answer of 98.

Example 4 Break Apart Numbers

Jolene bought three cans of fruit juice. If each can costs 58 cents, how much did she spend?

Step 1 Choose an appropriate operation.
Since you are finding the price of several objects when one is given, choose multiplication.

Step 2 Apply the strategy of breaking apart numbers.
Think: 58 = 50 + 8 so 50 x 3 = 150 and 8x3 = 24.
150 + 24 = 174.

Step 3 Write or state the answer to the problem.
174 cents is more commonly referred to as $1. 74.
Jolene spent $1. 74 for three cans of juice.

The last, most common strategy is referred to as **compensation** (or **substitution).** To apply this strategy, change one number so it is easier to perform the necessary operation and then change the other number by its opposite. For example,
57 + 34 = (57 + 3) + (34 - 3) = 60 + 31 = 91.

Example 5 Use Compensation

Miguel drove 101 miles. Connie drove 28 miles. How much farther did Miguel drive than Connie?

Step 1 Choose an appropriate operation.
The key words *farther than* indicate subtraction.

Step 2 Apply the mental math strategy of compensation.
Think: 101 - 28 = 101 - 1 - 28 + 1 =
100 - 28 + 1 = 72 + 1 = 73

Step 3 Write or state the answer to the problem.
Miguel drove 73 miles farther than Connie drove.

Step 4 Check your answer. Since subtraction and addition are opposite, the answer plus 28 should equal 101.
Add 28 + 73 mentally.
Think: 20 + 8 + 70 + 3 = 90 + 11 = 101.
The answer checks out.

The mental math strategies should help save time by allowing shortcuts whenever possible. Remember that you still must know your basic addition and multiplication facts to achieve the correct solution.

THINK ABOUT MATH

Directions: Use a mental math strategy to solve each problem.

1. Meli worked 174 hours last month. She worked 162 hours this month. How many hours did she work in these two months?
2. Chantal has 45 boxes of envelopes. Envelopes are sold in boxes of 100. How many envelopes does she have?

Vocabulary Review

Directions: Fill in the blank with the correct word.

compensation mental math order of operations strategy

1. Use_____ to make numbers in a problem easier to work with mentally.
2. A_____is a plan for solving.
3. When there are multiple operations in a problem, the_____ tells what order to perform them.
4. _____is a way of solving problems without paper and pencil or a calculator.

Skill Review

Directions: Solve each problem. Write what information you applied from your own experience.

1. A coffee shop hopes to sell 145 cups of coffee every morning, with an extra 50 each on Saturday and Sunday. How many cups of coffee does it hope to sell in a week?
2. Coretta replaced all of the tires on her car. The tires cost $178 each. How much did she spend on the tires?
3. Sheila's car insurance costs $636 per year. How much does she pay every month?

Skill Practice

Directions: Choose the best answer to each question.

1. What is the value of (8 + 3) x 4 - 1?
   A. 17
   B. 19
   C. 33
   D. 43

2. What is the value of (44 - (2 x 14)) / (3 + 5) + 8?

   _____

3. Carter is ordering business cards for his coworkers. He needs 100 cards each for 36 people. How many cards should he order?
   A. 36
   B. 360
   C. 3, 600
   D. 36, 000

LESSON
1.7

Problem Solving

Lesson Objectives

You will be able to

- Use the five-step approach to solve word problems
- Use various strategies to solve word problems
- Judge the reasonableness of answers

Skills

- **Core Skill:** Build Lines of Reasoning
- **Core Practice:** Attend to Precision

Vocabulary

irrelevant
reasonable
solution

KEY CONCEPT: Problem solving Is an important part of the study of mathematics and an important part of everyday life.

Use a calculator to solve each problem.

1. 289 x 97
2. 850 x 381
3. 58, 590 / 62
4. 407 - 388
5. 2, 894 + 19, 073
6. 1, 411 x 3, 879
7. 96 / 4
8. 387 + 4, 296

The Five-Step Approach

The following five-step approach can be used to find a **solution,** or an answer, for all types of word problems and help organize thinking.

Step 1 Understand the question. After reading and rereading the problem carefully, decide what the problem asks you to find.

Step 2 Decide what information is needed to solve the problem. Then determine what information is irrelevant to the question.

Step 3 Choose the most appropriate operation or operations to solve the problem.

Step 4 Solve the problem. Make sure the solution answers the question asked.

Step 5 Check your answer by rereading the question to see if the answer is **reasonable,** that it makes sense.

Example 1 Use the Five-Step Approach

Paulo works from 4 p. m. to 7 p. m., Monday through Friday. How many hours does he work each week?

Step 1 Reread the problem for understanding. The problem asks for the number of hours Paulo works each week.

Step 2 Decide what information is needed.
Paulo works from 4 p. m. to 7 p. m. from Monday through Friday.

Step 3 Choose the most appropriate operation or operations.
Multiply 3 hours worked each day by 5 days per week.

Step 4 Solve the problem.
3x5 = 15. So, Paulo works 15 hours each week.

Step 5 Check your answer.
If Paulo works three hours for five days a week, he works 15 hours each week. The answer is reasonable.

IDENTIFY IRRELEVANT INFORMATION

Sometimes, a passage will include **irrelevant,** or unnecessary, information. There may be details or other information that can be ignored when reading a passage or problem. Learning to determine which information is irrelevant will help when reading problems.

Not all information given in a problem is necessary, or relevant, to finding the solution. In fact, in real-life situations, the most difficult part of using math is often looking at all the information available and deciding what is actually relevant to the problem. It is important to have a clear sense of what is being asked and which details will help to find the solution.

To find irrelevant information, ask: ***How does this detail relate to the question that is being asked?***

Read the following problem, and identify the irrelevant information.

> Sally is taking inventory of the glassware that her store sells. She has 15 cases of green glasses. They come 12 to a case and stand 8 inches tall. She also has 9 cases of blue glasses. They come 20 to a case. They are good for water and juice. If these are the only glasses Sally has in her store, how many glasses does she have in all?

The question asks how many total glasses Sally has. The size, color, and use for them are irrelevant information and can be ignored. The important information is 15 cases with 12 in a case and 9 cases with 20 in a case. Everything else in the problem is irrelevant information.

Core Practice
Attend to Precision

Precision is important in mathematics, engineering, and sciences. One of the purposes of making multiple measurements or repeating experimental trials is to find precise answers, or answers that repeat the same value.

You may have heard someone say, "Measure twice, cut once, " while undertaking a construction project. What is the purpose of measuring twice? And is twice enough?

When builders measure multiple times before they saw a length of wood, for example, they are attending to **precision.** If multiple measures result in the same value, builders can be confident that the value is precise.

In your notebook, describe a time you made multiple measurements or repeated the steps of an experiment to check mathematical values. Explain the value of precision in your work.

Choose an Operation

Many times, the hardest part of solving a problem is deciding whether to add, subtract, multiply, or divide.

One way to determine which operation will be used is to focus on key words. Noticing these words will often provide a clue to determining the appropriate operation. Some key words or phrases are listed below.

Addition	Subtraction	Multiplication	Division
sum	difference	product	quotient
total	more... than	times	split
altogether	less... than	twice (x 2)	divided by
increased by	minus	when finding several of a different amount	when given amount of many and finding one
when combining different amounts	decreased by	when given part and finding the whole	when sharing, cutting, or splitting
	farther than		
	when comparing one amount to another		

Compare the relevant numbers in the problem to the solution to determine if the correct operation was chosen.

Example 2 Choose an Operation to Solve a Problem

Lamar rode his bicycle for 17 miles on Saturday. On Sunday, he rode 25 more miles. How many miles did Lamar ride altogether?

Step 1 Reread the problem to understand the question.
The problem asks for the number of miles Lamar rode his bicycle on Saturday and Sunday combined.

Step 2 Decide what information is needed.
You need to find the total number of miles Lamar biked over the weekend.

Step 3 Choose the most appropriate operation.
The key word ***altogether*** gives the clue to add 17 and 25.

Step 4 Solve the problem.
17 + 25 = 42
Lamar rode a total of 42 miles over the weekend.

Step 5 Check your answer. To check an addition problem, subtract one of the numbers in the problem from the answer. In this case, subtract 25 from 42. The answer is 17, the other number in the problem. The answer checks out.

THINK ABOUT MATH

Directions: Solve the following word problem by using the five-step approach. Tell which operation you used.

Marcella bought shampoo for $3, hair rinse for $2, and a brush for $2. She gave the clerk a 10-dollar bill. How much did the items cost altogether?

Problem-Solving Strategies

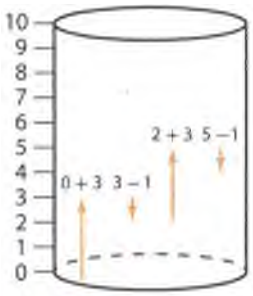
Draw a Picture

To solve a problem, you can sometimes draw a sketch or diagram to help you understand what is being asked.

Example 3 Draw a Picture to Solve a Problem

An empty water container is 10 meters tall. As it is being filled, the water level goes up 3 meters during the day, but because of a leak, it goes down 1 meter each night. How many days will it be before the water reaches the top of the container?

Step 1 Understand the question. You want to figure out how many days it will take the container to fill.



Step 2 Find the necessary information. Just writing the three numbers that are in the problem (10, 3, and 1) and using division will result in the wrong answer. Drawing a picture should show that subtraction should be used as well.

Step 3 Choose an arithmetic operation or operations. Look at the picture and notice the lines go up 3 meters and then down 1 meter. This indicates that subtraction should be used to find the actual distance at which the water is going up each day and night. Then use division to find the number of days.

Step 4 Solve the problem. Make sure to answer the question asked. One way (but not the only way) is to subtract 1 from 3 and then divide 10 by the result. So, 10 / (3 - 1) = 5.

It will take five days to fill the water container.

Step 5 Check the answer. Divide 10 by 5 and get 2, which is the distance the water goes up each day and night. The answer is reasonable.

21st Century Skill
Technology Connection

The Internet is a wonderful learning resource that can be an effective tool in your education, but not all "hits" when doing online searches are equal. A good rule of thumb is to rely on hits that take you to "organizations. " You will be able to recognize organizations because they have ". org" in their domain names. Your math teacher can also recommend some good sites to go to.

As a class project, create a list of internet sites that could serve as reliable math-learning sources.

MATH LINK

You do not have to be an artist to practice the *"draw a picture"* strategy. If you know what you are drawing and why, this method can be very useful.

THINK ABOUT MATH

Directions: Explain which strategy you would use to solve the following word problems. Then solve the problem.

1. Susan has 32 onions she wants to sell. Each bag she has holds 7 onions. How many onions will Susan have left over after she sells the ones that have been placed in bags?
2. Five basketball players are in a circle. Each player throws the ball to every other player once. How many times is the ball thrown?

Guess and Check

Another popular problem-solving strategy is called guess and check. This is a method people often use in everyday situations.

Example 4 Guess and Check to Solve a Problem

Which two 2-digit numbers made from the digits 1, 2, 3, 4, and 5 give the largest sum when added together? No single digit may be used more than once.

Step 1 Understand the question.
It asks which two 2-digit numbers give the largest sum when added together.

Step 2 Find the necessary information.
Use only the digits 1, 2, 3, 4, and 5. No digit may be used more than once.

Step 3 Choose an arithmetic operation.
Because the question asks for a sum, add the two 2-digit numbers.

Step 4 Solve the problem.

First guess:	Second guess:
54 + 32 = 86	53 + 42 = 95

Answer: 53 + 42 = 95

Step 5 Check the answer.
Test the first guess. Ask: Is 86 a reasonable sum? No, adding 5 and 3 in the tens place gives 80. By changing 3 to 4 and then adding 5 and 4 in the tens place, you get 90. The number 90 is greater than 80.

Vocabulary Review

Directions: Fill in the blank with the correct word.

irrelevant reasonable solution

1. To find the---------- to a word problem is to find the answer.

2. When using addition to solve a word problem, check that you have found a_____ sum.

3. Knowing what a question is asking makes it easy to find_____ information.

Skill Review

Directions: Find the irrelevant information in the following problems. Solve the problem.

1. Manny spent $3. 15 for a 96-inch board of wood. How many 8-inch long pieces can he cut from the board?
2. Mrs. O'Rourke worked 28 hours this week. Mr. Martinez worked 32 hours, and Ms. Wong worked 41 hours. How many hours more did Ms. Wong work than Mrs. O'Rourke?
3. Carlos purchased 3 pounds of potatoes, 2 pounds of bananas, and 5 pounds of apples. He handed the clerk $12. 43. How many pounds total do his items weigh?
4. At a garage sale, Chaske sold 12 DVDs for $3 each. He also sold his DVD player for $20. How much money did he receive for his DVDs?

Skill Practice

Directions: Choose the best answer to each question.

1. Which two 2-digit numbers made with the digits 5, 6, 7, and 8 give the largest sum when added together? No digit may be used more than once.

   A. 87 + 65
   B. 86 + 75
   C. 88 + 77
   D. 85 + 67

2. In May, Jack's rent will increase $30. If he pays $415 now, how much will his new rent cost each month?

   _____

3. Which operation is best to use to solve the following problem: Last year, 64, 441, 087 passengers flew through a major airport. The year before, there were 51, 943, 567 passengers. How many more passengers flew through it last year than the year before?

   A. addition
   B. subtraction
   C. division
   D. multiplication

4. Garbage service in the town of Kankakee costs $156 for one year. What is the monthly charge for garbage collection?

   _____

CHAPTER 1

Review

Directions: Choose the best answer to each question.

1. What is the value of the underlined digit?

   6, 135, 012

   A. 3
   B. 30
   C. 30, 000
   D. 35, 012

2. What is the greatest common factor of 36, 48, and 60?

   A. 2
   B. 6
   C. 12
   D. 36

3. Which operation should be performed first in the expression
   7 + (12 - 4) x 15 / 3?

   A. add 7 and 12
   B. subtract 4 from 12
   C. multiply 4 times 15
   D. divide 15 by 3

4. There are 124 employees in Yilin's office. On a certain day, 14 people are out of the office for illness, and 12 people are out on vacation. How many people are working in the office on that day?

   A. 98
   B. 122
   C. 126
   D. 150

5. What is 4, 572, 013 rounded to the nearest thousand?

   A. 4, 600, 000
   B. 4, 570, 000
   C. 4, 572, 000
   D. 4, 573, 000

6. Sandrine went to the grocery store to buy more onions for the soup. Two onions cost $2. 79. She gave the grocer $5. 00 to pay for the onions. Use estimation to find out how much change Sandrine should receive.

   A. About $3. 00
   B. About $1. 75
   C. About $2. 50
   D. About $2. 00

7. What is the greatest common factor of 14, 21, and 42?

   A. 4
   B. 7
   C. 17
   D. 49

8. A stadium has 11, 260 seats. If the stadiums sells out for 5 home games in a row, approximately how many people attended those games?

   A. 2, 252
   B. 11, 000
   C. 55, 000
   D. 60, 000

9. Winona's mechanic charges $65 dollars per hour for labor plus the cost of parts. How much will she pay if her car needs a new part for $215 and 3 hours of labor?

   A. $280
   B. $410
   C. $710
   D. $840

10. Carl solved 7, 820 4- 68 = 115. How can he check his answer?

    A. add 115 plus 68
    B. subtract 115 from 7, 820
    C. multiply 115 times 68
    D. divide 115 by 68

Review

11. Three friends went bowling. The scores for their first game are shown in the chart.

Uppinder	James	Marietta
248	114	187

Which list shows the friends in order of greatest score to least score?

A. James, Marietta, Uppinder
B. Marietta, Uppinder, James
C. James, Uppinder, Marietta
D. Uppinder, Marietta, James

12. What is the value of 15 + (26 - 6) / (7 - (8 / 4))?

A. 19
B. 34
C. 114
D. 147

13. What is 842 / 27?

A. 11 R50
B. 21 R5
C. 30 R2
D. 31 R5

14. Mrs. Cortez is mailing postcards for a fund-raiser for the senior center to which she belongs. She has 2, 000 cards. She needs to send 582 cards to people who live near the center, 491 cards to local businesses, and 361 cards to other people on her mailing list. She also wants to have 500 cards to give out at a local shopping mall. How many postcards will she have left over?

A. 66
B. 557
C. 1, 934
D. She does not have enough.

15. Quietta has $12, 398 in a savings account. She withdraws $762 to pay her car insurance. How much money is left in her account?

A. $4, 778
B. $11, 636
C. $12, 436
D. $13, 160

Check Your Understanding

On the following chart, circle the number of any item you answered incorrectly. Under each lesson title, you will see the pages you can review to learn the content covered in the question. Pay particular attention to reviewing skill areas in which you missed half or more of the questions.

Chapter 1: Whole Numbers	Procedural	Conceptual	Application/ Modeling/ Problem Solving
Place Value pp. 12-17	1		11
Add and Subtract Whole Numbers pp. 18-21			4, 14
Multiply and Divide Whole Numbers pp. 22-25	13		8
Factoring pp. 26-31	2, 7		
Rounding and Estimation pp. 32-35	5		6
Arithmetic Expressions pp. 36-39	12	3	9
Problem Solving pp. 40-45		10	15