LESSON

5.1

Expressions

Lesson Objectives

You will be able to

- Translate between verbal and symbolic representations of expressions
- Simplify expressions
- Evaluate expressions

Skills

- **Core Skill:** Evaluate Expressions
- **Core Practice:** Make Sense of Problems

Vocabulary

algebraic expression
coefficient
constant term
mathematical expression
symbolic expression
variable
verbal expression

KEY CONCEPT: Mathematical and real-world situations can be represented by expressions that can be simplified and evaluated.

Find each sum or difference.

1. $1.24 + 8.3$ 2. $-12 - (-3)$ 3. $-18 + 3$

Find each product or quotient.

4. 1.2×3.5 5. $-56 \div (-4)$ 6. $-3 \times (-6)$

Verbal and Symbolic Representations of Expressions

A numeric or **mathematical expression** is any combination of symbols, numbers, and operations. Mathematical expressions can be used to represent many different things. Also, multiple expressions can be used to describe the same object. For example, 2×5 and $6 + 4$ both represent the number 10. However, the expression 2×5 may represent the total number of players on the court at the same time during a basketball game, whereas the expression $6 + 4$ may represent the total number of fruit bought at a store (6 apples and 4 oranges). An expression can be thought of as a whole or it can thought of as a collection of parts.

Example 1 Mathematical Expressions

-3 $5 \times (-8)$ $78 - 23(2 + 1)$

A numeric expression may be simplified by using the order of operations.

Example 2 Simplify a Mathematical Expression

Simplify $21 - 5(3)$.

Use the order of operations and multiply 5 times 3. $5 \times 3 = 15$
Then subtract that result from 21. $21 - 15 = 6$

A variable or **algebraic expression** is a combination of numbers, one or more variables, and operations. A **variable** is a symbol that stands for an unknown number or value and is written in italics, such as x. A number that multiplies the variable is called the **coefficient**. A number that is added or subtracted in a variable expression is called a **constant term**.

Algebraic expressions can also be used to represent real-world scenarios. For example, the expression $2l+2w$ can represent the perimeter of a rectangle because both lengths need to be added to both widths. Also, using the Distributive Property, the expression can be rewritten as $2(l+w)$, saying that the sum of the length and width need to be doubled to find the perimeter.

Example 3 Algebraic or Variable Expressions

Identify the variable, coefficient, and constant term for each algebraic or variable expression.

expression	variable	coefficient	constant term
$2 + x$	x	1	2
m	m	1	0
$3n - 8$	n	3	-8

A **symbolic expression** is an expression that uses variables, numbers, and symbols for operations.

MATH LINK

Multiplication in expressions can be shown in several ways. Each of the following represents multiplication.

$8n$ $\quad 8 \cdot n$ $\quad 8(n)$

IDENTIFY KEY WORDS

Some sentences can be confusing or complicated. Focusing on the most important words in the sentence, or key words, can help the reader to better understand the meaning of that sentence. To identify key words, look for the subject (*who* or *what*) and the action (what the subject did). Consider the following sentence: *Marion, by adding, subtracting, multiplying, and simplifying several numbers, found the value of the expression.*

Who? Marion
Did what? found the value of the expression

Marion found the value of the expression. Everything else is just details.

Read the passage below and identify the key words in each sentence.

> (1) Variables are a part of math expressions that stand for an unknown value. (2) They can be a letter or other symbol as long as the same variable is used consistently in a problem. (3) Variables can be positive or negative numbers, decimals, fractions, or other type of real numbers. (4) There are many rules for learning to work with variables.

1. variables, unknown value
2. letter, other symbol
3. real number
4. rules

MATH LINK

When a variable appears to stand alone, the coefficient is always understood to be 1.

Core Skill
Evaluate Expressions

Often, as you have learned, solving a math problem requires you to follow a series of steps in a logical order. The step-by-step approach to solving problems in this book provides a **framework,** or structure, for explaining how you arrived at a particular solution. Use the same step-by-step approach to explain a solution. For example, to explain the solution for Example 4, you might say, "First, I assigned the variable n to represent the unknown number. Then I identified the correct operation in the verbal phrase, 5 *less than*. This phrase told me that I should subtract..."

After you complete question 1 in Skill Practice on p. 139, write in your notebook the steps you took to solve the problem.

Example 4 Write a Variable Expression That Represents a Verbal Phrase

Write the variable expression for 5 *less than twice a number*.

Step 1 Assign a variable to the unknown number.
Let n represent the unknown number.

Step 2 Identify the operation or operations in the verbal phrase.
The phrase 5 *less than* means that 5 is subtracted from a number, so the operation is subtraction.
The phrase *twice a number* means 2 times a number, so the other operation is multiplication.

Step 3 Write the variable expression: $2n - 5$.

Example 5 Write an Expression That Represents a Real-World Situation

Tomas has a cell phone plan that charges a flat rate of $35.00 per month plus $0.25 for each text message. Write an algebraic expression that shows the monthly fee for Tomas's cell phone plan.

Step 1 Assign a variable to the unknown value.
The unknown value is the number of text messages.
Let t represent the number of text messages.

Step 2 Identify the operation or operations in the situation.
The word *plus* indicates addition. The phrase *$0.25 for each text message* indicates multiplication.

Step 3 Write the algebraic expression.
$35.00 is the monthly charge, and 0.25t$ is the charge for text messages.
$35.00 + 0.25t$

A **verbal expression** is written in words and numbers. The verbal representation might be mathematical or it might refer to a real-world situation.

Example 6 Translate an Expression Into a Verbal Phrase

$3m + 4$

Step 1 Identify the variable.
The variable is m.

Step 2 Identify the operation or operations.
$3m$ means 3 times m + 4 means add 4.

Step 3 Write a verbal phrase.
One way to write a verbal phrase for $3m + 4$ is *three times a number m plus four.*
Another way is to write a real-world situation for $3m + 4$:
Martin paid a one time fee of $4 to join the coffee club at work and paid $3 each of the following weeks.

THINK ABOUT MATH

Directions: Write each verbal phrase as a variable expression.

1. a number increased by 12 ____________
2. $250 less than twice Rahman's salary ____________

Directions: Write each variable expression as a verbal phrase.

3. $4t \div 2$ ____________ 4. $c - 9$ ____________

Evaluate Expressions

When you **evaluate**, or find the value of, an expression, you substitute a given value for the variable and then perform the operation.

Example 7 Evaluate an Expression

Evaluate $x + 7$ when $x = -2$.

Step 1 Substitute the given value for the variable.
$x + 7 = -2 + 7$

Step 2 Perform the operation.
$-2 + 7 = 5$
The value of $x + 7$ when $x = -2$ is 5.

When there is more than one operation, use the order of operations:

1. Do operations within parentheses.
2. Do multiplication and division from left to right.
3. Do addition and subtraction from left to right.

Core Practice
Make Sense of Problems

Because real-world world problems are set within situations like the ones we experience in everyday life, they provide a way to add, subtract, multiply, and divide real objects. You are dealing with actual quantities. Suppose you wanted to write such a problem of your own. Somebody reading your problem would have to add in order to solve it. What would you say? Fortunately, there are key words that tell readers what operation they should use to solve this kind of problem. See the chart below, which lists key words for the four major operations.

Operation	Key word
Addition	add, sum, total, altogether, increased, in all
Subtraction	subtract, difference, more than, less than, farther than
Multiplication	multiply, total, product, times, twice
Division	divide, quotient, each, average, split

Milena walked 3 miles on Saturday and 2 miles on Sunday. How much farther did she walk on Saturday? The key words *how much farther* and *walk* indicate that you should subtract the amounts that Milena walked.

In a notebook, make a list of key words that appear in the problems in this lesson. Write whether they successfully achieved their goal—namely, to lead a reader to add, subtract, multiply, or divide in order to solve the problem.

Example 8 Evaluate an Expression Using the Order of Operations

Evaluate $a + 4b$ when $a = -10$ and $b = 1.5$.

Step 1 Substitute the given values for the variables.
$a + 4b = -10 + 4 \times 1.5$

Step 2 Use the order of operations.
First perform the multiplication: $-10 + 4 \times 1.5 = -10 + 6$
Then perform the addition: $-10 + 6 = -4$
The value of $a + 4b$ when $a = -10$ and $b = 1.5$ is -4.

THINK ABOUT MATH

Directions: Evaluate each expression. Use the order of operations if necessary.

1. $13.6 - a$ when $a = 2.9$ ____________
2. $-3m + n$ when $m = 4$ and $n = 6$ ____________
3. $c + d$ when $c = -5$ and $d = 2$ ____________
4. $-5x \div 2y$ when $x = 8$ and $y = -5$ ____________

Vocabulary Review

Directions: Match each word to the letter that describes the word.

1. ------------algebraic expression
2. ______ coefficient
3. ______ constant term
4. ______ mathematical expression
5. --------------symbolic expression
6. ______ variable
7. ______ verbal expression

A. a number that is added or subtracted in a variable expression

B. a symbol that represents an unknown number or value

C. an expression written using a combination of words and numbers

D. an expression that uses variables, numbers, and symbols for operations

E. the numerical factor of a variable

F. another name for a variable expression

G. another name for a numeric expression

Skill Review

Directions: Identify the key words in each passage.

1. (1) To write an algebraic expression from a verbal expression, identify the key words. (2) The unknown quantity in the expression is what the variable represents. (3) Key words also indicate what numbers and operations should be used.

____

2. (1) When evaluating the expression $g + 2h$ when $g = 4$ and $h = -3$, first substitute the given values for the variables. (2) To avoid errors, make sure you substituted the correct values for each variable, especially if there is more than one variable. (3) The second step is to perform the operation or use the order of operations, if there is more than one operation. In this case, multiply first and then add.

____

Directions: Answer the following question using what you have learned about evaluating expressions.

3. What ideas would you apply to evaluate the expression $2x + y$ when $x = -3$ and $y = 4$? Evaluate the expression.

____

Skill Practice

Directions: Choose the best answer to each question.

1. What is the value of $s + 5t$ when $s = 3$ and $t = -2$?
   - A. −16
   - B. −7
   - C. 9
   - D. 13

2. Which phrase best describes the expression $3x - 8$?
   - A. the number x less eight times three
   - B. eight minus three times a number x
   - C. three times a number x decreased by eight
   - D. three times the number x and negative 8

3. Jamila bought a ladder and rented a rototiller to do some work in her backyard. The ladder cost $48.75, and the cost to rent the rototiller is $18 per day. Which expression can Jamila use to find the cost of doing the work in her yard?
   - A. $48.75 + 18$
   - B. $(48.75 + 18)d$
   - C. $48.75 - 18d$
   - D. $48.75 + 18d$

4. Which expression represents −17 less than the product of −12 and some number?
   - A. $-17 - (-12)x$
   - B. $-12 - 17x$
   - C. $-12x - (-17)$
   - D. $-12x + (-17)$

LESSON
5. 2

Solve One-Step Equations

Lesson Objectives

You will be able to

- Understand and write equations
- Solve one-step equations

Skills

- **Core Practice:** Make Sense of Problems
- **Core Skill:** Represent Real-World Arithmetic Problems

Vocabulary

equal sign
equation
equivalent equation
inverse operations
solution

MATH LINK

The Addition, Subtraction, Multiplication, and Division Properties of Equality state that when you add, subtract, multiply, or divide each side of an equation by the same value, you produce an equivalent equation. An **equivalent equation** is one that has the same solution as the original equation.

KEY CONCEPT: Use equations to represent situations, and use inverse operations to solve one-step equations.

Write an expression for each situation. Use n as the variable.

1. the sum of a number and four ______________
2. three times a number decreased by one ______________
3. a number split into eight equal parts ______________

Evaluate each expression when $x = -1$ *and* $y = 3$.

4. $x - y$ ________ 5. $2y + 1$ ________ 6. $3(x + y)$ ________

Understand and Write Equations

An **equation** is a mathematical statement that two expressions are equal. In algebra, an equation has at least one variable, which represents an unknown value. It is usually a letter. An **equal sign** (=) is placed between the two expressions to show a mathematical statement of equivalence.

Example 1 Write an Equation for a Situation

Bao works 4 hours and earns $72. Write an equation that can be used to find how much he earns per hour.

Step 1 Identify the variable.
Let d represent the dollar amount Bao earns per hour.

Step 2 Identify the operation.
Multiply the number of hours by dollars per hour to get total earnings. $4d$ represents the operation.

Step 3 Write the equation using the two quantities.
$4d$ is equal to $72. The equation is $4d = 72$.

A **solution** of an equation is the value of the variable that makes the equation a true statement.

Example 2 Check for Solutions

Is 13 a solution of the equation $x + 12 = 25$?

Step 1 Substitute the number, or solution, for the variable.
$13 + 12 = 25$

Step 2 Determine whether the solution results in a true statement.
$25 = 25$, so 13 is a solution of $x + 12 = 25$.

THINK ABOUT MATH

Directions: Write an equation for each situation.

1. the sum of a number and two is three ____________
2. Anya's age decreased by five is twelve ____________

Directions: Explain whether the value given for the variable is a solution of the equation.

3. $7c = 42$; $c = 8$ ____________
4. $y + 7 = 3$; $y = -4$ ____________

Core Practice
Make Sense of Problems

Math questions can sometimes be tricky, because they may contain information that you do not need to know in order to solve a problem. For example, consider this problem: "Mariska's school ordered 100 box lunches for a student field trip. Twenty-five were vegetarian lunches, and 75 of the box lunches contained meat sandwiches. The cost of each lunch was $4. 50. How much did the school spend to buy the lunches? " Before considering the information you were given, you should focus on the actual question, which appears in the last sentence: "How much did the school spend to buy the lunches? " Then look back at the question to see what information you need to know in order to solve the problem. Do you need to know the number of vegetarian or meat lunches? No, you can ignore those details, because you do not need them to solve the problem.

Before you answer question 4 in Skill Practice on page 143, make a two-column chart in your notebook: "Information I Need" and "Information I Can Ignore. " Then fill out the two columns in your list. Finally, solve the problem.

Solve Equations

Use **inverse operations** to solve an equation. Inverse operations are operations that are the opposite of each other and undo each other's results.

- The inverse of addition is subtraction.
- The inverse of subtraction is addition.
- The inverse of multiplication is division.
- The inverse of division is multiplication.

Example 3 Solve an Addition Equation

Solve the equation $y + 4 = 21$.

Step 1 Identify the operation used in the equation.
There is an addition sign, so the operation is addition.

Step 2 Identify the inverse operation.
The inverse of addition is subtraction.

Step 3 Use the inverse operation to solve the equation. In this case, subtract 4 from each side of the equal sign.

$$\begin{array}{r} y + 4 = 21 \\ \underline{-4} \quad \underline{-4} \\ y = 17 \end{array}$$

Step 4 Check the solution.
$17 + 4 = 21$ is a true statement, so the solution is 17.
$y = 17$.

Core Skill
Represent Real-World Arithmetic Problems

Learning how to perform operations in the classroom or when doing your homework prepares you for solving the kinds of real-world arithmetic problems you will encounter in everyday life. You can use addition, for example, to calculate the cost of two pairs of jeans when the price tag in the clothing store says that each pair costs \$23. 95. First, translate the problem into an equation: \$23. 95 + \$23. 95 = x. On the other hand, since the price of each pair was the same, you might realize that you can use multiplication: \$23. 95 x 2 = x. Both equations will produce the same answer: \$47. 90.

Consider the following problem. The local library charges \$0. 10 a copy to make photocopies. You have designed a flyer announcing an upcoming Carnival Night that will be held at the school, and you need to make 150 copies. How much will you spend to make the copies? In a notebook, first write an equation for the problem.

Example 4 Solve a Multiplication Equation

Solve the equation $3a = 24$.

Step 1 Identify the operation used in the equation.
$3a$ means $3 \times a$, so the operation is multiplication.

Step 2 Identify the inverse operation.
The inverse of multiplication is division.

Step 3 Use the inverse operation to solve the equation. In this case, divide each side of the equal sign by 3.
$3a = 24 \qquad \frac{3a}{3} = \frac{24}{3} \qquad 1a = 8 \qquad a = 8$

Step 4 Check the solution.
$3 \times 8 = 24$ is a true statement, so the solution is 8. $a = 8$

Example 5 Solve a Subtraction Equation

Solve the equation $x - 8 = 18$.

Step 1 Identify the operation used in the equation.
There is a subtraction sign, so the operation is subtraction.

Step 2 Identify the inverse operation.
The inverse of subtraction is addition.

Step 3 Use the inverse operation to solve the equation. In this case, add 8 to each side of the equal sign.

$$\begin{array}{r} x - 8 = 18 \\ \underline{+8} \quad \underline{+8} \\ x = 26 \end{array}$$

Step 4 Check the solution.
$26 - 8 = 18$ is a true statement, so the solution is 26.
$x = 26$.

Example 6 Solve a Division Equation

Solve the equation $\frac{z}{6} = 5$.

Step 1 Identify the operation used in the equation.
$\frac{z}{6}$ means $z \div 6$, so the operation is division.

Step 2 Identify the inverse operation.
The inverse of division is multiplication.

Step 3 Use the inverse operation to solve the equation. In this case, multiply each side of the equal sign by 6.

$$\frac{z}{6} \times 6 = 5 \times 6$$
$$z = 30$$

Step 4 Check the solution.
$\frac{30}{6} = 5$ is a true statement, so the solution is 30. $z = 30$.

THINK ABOUT MATH

Directions: Solve each equation.

1. $5y = 35$ ________
2. $n - 1 = 10$ ________
3. $\frac{z}{3} = 2$ ________
4. $y + 20 = 21$ ________

Vocabulary Review

Directions: Match each word to one of the statements below.

_______ 1. equal sign

_______ 2. equation

_______ 3. equivalent equation

_______ 4. inverse operations

_______ 5. solution

A. has the same solution as another equation
B. the value of a variable that makes an equation a true statement
C. a statement that two expressions are equal
D. used between the two expressions in an equation
E. operations that are opposites and undo each other

Skill Review

Directions: Use what you learned about determining important information to answer the following questions.

1. What part of a problem do you focus on to determine what information you need to know in order to solve the problem?

____

2. What tool allows you to undo the result of another operation?

____

Skill Practice

Directions: Choose the best answer to each question.

1. Maemi ordered a lunch for each attendee at her meeting. Each lunch cost \$16. The total cost was \$80. Which of the following equations can be used to find how many lunches Maemi ordered?

A. $16t = 80$
B. $16 + t = 80$
C. $80 - t = 16$
D. $\frac{t}{16} = 80$

2. Which of the following is a solution of $2 + b = 14$?

A. $b = 28$
B. $b = 16$
C. $b = 12$
D. $b = 7$

3. Nizhoni wants to check the solution of $\frac{n}{6} = 6$. What should she do to check the solution?

A. Multiply the solution by 6.
B. Add 6 to the solution.
C. Subtract 6 from the solution.
D. Substitute the solution for n in the equation.

4. Tamika is a computer solutions associate who works in a call center. She usually works a 7-hour day and handles 84 calls in an average shift. She uses the equation $7x = 84$ to find the number of calls she answers in an hour. How many calls can Tamika answer in an hour?

A. 12
B. 18
C. 77
D. 588

LESSON
5.3

Solve Two-Step Equations

Lesson Objectives

You will be able to

- Translate verbal sentences into two-step equations
- Solve two-step equations

Skills

- **Core Practice:** Make Sense of Problems
- **Core Skill:** Evaluate Expressions

Vocabulary

affect
isolate
two-step equation

KEY CONCEPT: Use two inverse operations to solve two-step equations.

Match each equation to its solution.

_____ 1. $x + 3 = 12$

_____ 2. $x - 3 = -4$

_____ 3. $4x = 24$

_____ 4. $\frac{x}{3} = 4$

A. $x = 6$ B. $x = 9$ C. $x = 12$ D. $x = -1$

Translate Verbal Sentences into Two-Step Equations

A **two-step equation** is an equation that contains two different operations, such as multiplication and subtraction in the equation $3t - 7 = 14$. To translate a verbal sentence into an equation, represent the unknown with a variable, identify the two operations, and then write the equation.

Example 1 Translate a Sentence into an Equation

Four times a number plus four is twelve.

Step 1 Choose a variable to represent the unknown value. Let n represent the unknown number.

Step 2 Identify the expressions on each side of the equal sign. The word *is* stands for the equal sign, so *four times a number plus four* is on one side, and 12 is on the other side.

Step 3 Identify the two operations. The phrase *four times a number* indicates multiplication, or $4n$. The phrase *plus four* indicates addition, or $+ 4$.

Step 4 Write the equation. The equation is $4n + 4 = 12$.

Example 2 Translate a Real-World Situation into an Equation

Mai-Ling is saving \$2,000 for a down payment on a car. She has saved \$500 so far and plans to save the rest in 6 months. Write an equation that shows how much she needs to save per month to have a \$2,000 down payment.

Step 1 Choose a variable for the unknown value. Mai-Ling wants to know how much to save each month for the down payment. This is the unknown. Let d represent the dollar amount she needs to save each month.

Step 2 Identify the expressions on each side of the equal sign. \$2,000 is the total Mai-Ling needs to save, so it goes on one side of the equal sign. On the other side of the equal sign is the unknown dollars she needs to save each month for the next 6 months *and* the \$500 she has already saved.

Step 3 Identify the two operations on one side of the equation. 6 months $\times$ d indicates multiplication, or $6d$. The phrase *and \$500* indicates addition, or $+ 500$

Step 4 Write the equation. $6d + 500$ represents one side of the equation. 2,000 represents the other side. It shows that the money Mai-Ling saves in 6 months plus the money she already has saved is equal to \$2,000. The equation is $6d + 500 = 2{,}000$.

MATH LINK

Remember that the variable in an equation can be any letter. In general, the letter n or x is used to represent an unknown number, but any letter can be used. In real-world situations, the variable is often the first letter of the unknown, such as c for *cost*. Even though this is often the case, it is not always the case. What is important to remember is that the variable always stands for the unknown value or number.

Solve Two-Step Equations

The goal in solving any equation is to **isolate** the variable. To isolate the variable means to get the variable by itself on one side of the equation. In a two-step equation, perform two inverse operations to isolate the variable.

Solving two-step equations involves a sequence of steps that must be performed in a certain order. When solving two-step equations, therefore, perform the operations in the reverse order to the order of operations.

Core Practice
Make Sense of Problems

What do the values in a problem represent? Answering this question is the first step in solving a problem, so it is important to make sense of a problem before you start. Look at the labels. Does a number represent distance? Time? Pay attention to the placement of decimal points because their position in a number **affects,** or has an effect on, the place value of each digit in the number. Are you multiplying tenths? Hundredths? Also notice whether the equation contains a mix of positive and negative numbers. You will get the wrong answer if you treat a negative number as a positive number in your calculations.

After you have finished reading Example 4, redo the problem, but this time, solve it as if the first expression was a positive value—that is, 2x. Then compare your answer with the original answer. Notice how the solution would be affected if you had misread the value in the original problem and had treated the expression -2x as a positive value. Compare the two answers. -2≠2

Example 3 Solve an Equation That Involves Addition and Multiplication

Solve the equation $3x + 6 = 24$.

Step 1 Identify each operation and its inverse in the equation. $3x$ means $3 \times x$, so the first operation is multiplication. Its inverse is division. There is a plus sign, so the second operation is addition. Its inverse is subtraction.

Step 2 Identify the order in which the inverse operations should be performed. First, use subtraction to undo addition. Then use division to undo multiplication.

Step 3 Perform the first inverse operation on each side of the equation. Subtract 6 from each side of the equation.

$$\begin{array}{rcr} 3x + 6 & = & 24 \\ -\ 6 & = & -6 \\ \hline 3x & & 18 \end{array}$$

Step 4 Perform the second inverse operation on each side of the equation. Divide each side of the equation by 3.
$3x = 18$ $\quad \frac{3x}{3} = \frac{18}{3}$ $\quad x = 6$

Step 5 Check the solution by substituting the solution for the variable in the original equation.
$3(6) + 6 = 24$ $\quad 18 + 6 = 24$ $\quad$ The statement is true, so $x = 6$.

Example 4 Solve an Equation That Involves Subtraction and Multiplication

Solve the equation $-2x - 4 = -8$.

Step 1 Identify each operation and its inverse in the equation. The first operation is multiplication. Its inverse is division. The second operation is subtraction. Its inverse is addition.

Step 2 Identify the order in which the inverse operations should be performed. First, use addition to undo subtraction. Then use division to undo multiplication.

Step 3 Perform the first inverse operation on each side of the equation. Add 4 to each side of the equation.

$$\begin{array}{rcr} -2x - 4 & = & -8 \\ +4 & = & +4 \\ \hline -2x & & -4 \end{array}$$

Step 4 Perform the second inverse operation on each side of the equation. Divide each side of the equation by -2.
$-2x = -4$ $\quad \frac{-2x}{-2} = \frac{-4}{-2}$ $\quad x = 2$

Step 5 Check the solution by substituting the solution for the variable in the original equation.
$-2(2) - 4 = -8$ $\quad -4 + (-4) = -8$
The statement is true, so $x = 2$.

Notice that in Example 4, you added 4 to both sides of the equal sign to solve the problem. However, this is the same as subtracting its negative. That way, in both examples, subtraction took place first. This is because addition and subtraction are essentially the same operation, since subtracting a negative number is the same as adding a positive number.

Example 5 Solve an Equation That Involves Addition and Division

Solve the equation $\frac{x}{-8} + 2 = 5$.

Step 1 Identify each operation and its inverse in the equation.
The first operation is division. Its inverse is multiplication.
The second operation is addition. Its inverse is subtraction.

Step 2 Identify the order in which the inverse operations should be performed. First, use subtraction to undo addition. Then use multiplication to undo division.

Step 3 Perform the first inverse operation on each side of the equation. Subtract 2 from each side of the equation.
$\frac{x}{-8} + 2 = 5 \qquad \frac{x}{-8} + 2 - 2 = 5 - 2 \qquad \frac{x}{-8} = 3$

Step 4 Perform the second inverse operation on each side of the ≠ equation. Multiply each side of the equation by −8.
$\frac{x}{-8} = 3 \qquad \frac{x}{-8} \times (-8) = 3(-8) \qquad x = -24$

Step 5 Check the solution by substituting the variable into the original equation.
$\frac{-24}{-8} + 2 = 5 \qquad 3 + 2 = 5$ The statement is true, so $x = -24$.

Example 6 Solve an Equation That Involves Subtraction and Division

Solve the equation $\frac{x}{4} - 7 = 3$.

Step 1 Identify each operation and its inverse in the equation.
$\frac{x}{4}$ means $x \div 4$, so the first operation is division. Its inverse is multiplication. There is a minus sign, so the second operation is subtraction. Its inverse is addition.

Step 2 Identify the order in which the inverse operations should be performed. First, use addition to undo subtraction. Then use multiplication to undo division.

Step 3 Perform the first inverse operation on each side of the equation. Add 7 to each side of the equation.
$\frac{x}{4} - 7 = 3 \qquad \frac{x}{4} - 7 + 7 = 3 + 7 \qquad \frac{x}{4} = 10$

Step 4 Perform the second inverse operation on each side of the equation. Multiply each side of the equation by 4.
$\frac{x}{4} = 10 \qquad \frac{x}{4} \times 4 = 10 \times 4 \qquad x = 40$

Step 5 Check the solution by substituting the solution for the variable in the original equation.
$\frac{40}{4} - 7 = 3 \qquad 10 - 7 = 3$ The statement is true, so $x = 40$.

Just as addition and subtraction are very similar (as discussed on the previous page), multiplication and division operations are just as similar. Notice that in both division examples, the division could have been written as a product of the variable and a fraction. For instance, $x \div 4$ could have be rewritten as $\frac{1}{4} \times x$. Therefore you could have divided by $\frac{1}{4}$, which is the same as multiplying by 4. This is because multiplication and division are just as similar as addition and subtraction.

MATH LINK

Follow the rules for integer operations when solving equations with integers. Recall that integers are the set of whole numbers and their negatives.

To add same signs: Add the absolute values. Use the sign.

To add different signs: Subtract the lesser absolute value from the greater absolute value. Use the sign of the greater absolute value.

To subtract an integer: Add its opposite.

To multiply and divide: If the signs are the same, the product or quotient is positive. If the signs are different, the product or quotient is negative.

Core Skill
Evaluate Expressions

Arithmetic and algebraic expressions are similar in some ways. Both contain numbers, symbols, and operations. Of course, there are some important differences. Arithmetic expressions such as $5 \times (-8)$ or $78 - 23(2 + 1)$ are pretty easy to solve. You merely add, subtract, multiply, or divide the numbers to get a solution. But algebraic expressions involve unknowns that are represented by a mix of both numbers and variables. What must you do to solve the algebraic equation $5x + 4 = 24$? In a notebook write the equation and the solution.

THINK ABOUT MATH

Directions: Solve each equation.

1. $5x + 1 = 41$ ________
2. $\frac{x}{6} - 12 = -10$ ________
3. $\frac{x}{-8} + 6 = 14$ ________
4. $-2x + 3 = 1$ ________

Vocabulary Review

Directions: Fill in the blanks with one of the words or phrases below.

affect isolate two-step equation

1. A(n)_______________ requires that you do two operations to solve it.
2. When you_______________ a variable, you get the variable by itself on one side of the equation.
3. If you undo operations in the wrong order, it will___________________ your answer.

Skill Review

Directions: Answer the questions below.

1. Explain how the skill of understanding sequence helped you solve two-step equations.

____

____

____

2. Describe the sequence you would use to solve the equation **$6x + 3 = -9$.**

____

____

____

3. Describe a sequence that you could use to solve any two-step equation.

____

____

____

Skill Practice

Directions: Choose the best answer to each question.

1. Which equation best describes this sentence: A *number divided by eight plus three is fifty-one?*

   A. $n + 3/8 = 51$

   B. $8/n + 3 = 51$

   C. $n/8 + 3 = 51$

   D. $8 + 3/n = 51$

2. Dan drives 49 miles to work, which is 7 miles less than twice the number of miles that Eduardo drives. Which equation shows the number of miles *m* that Eduardo drives to work?

   A. $2m - 7 = 49$

   B. $2(m - 7) = 49$

   C. $2m + 7 = 49$

   D. $2(m + 7) = 49$

Directions: Solve the problems.

3. What is the solution for the equation $n/7 + 12 = 58$?

   ____

4. Ibrahim has saved \$2, 000. He is planning on buying a new computer and accessories that cost \$3, 650. Ibrahim earns \$150 in commission for each elliptical trainer that he sells. He writes this equation to figure out how many trainers he will need to sell this month in order to be able to buy his computer.

   $150t + 2,000 = 3,650$

   How many elliptical trainers does Ibrahim need to sell?

   ____

LESSON
5.4 Solve One- and Two-Step Inequalities

Lesson Objectives

You will be able to

- Translate verbal statements into inequalities
- Solve one-step inequalities
- Solve two-step inequalities

Skills

- **Core Skill:** Solve Inequalities
- **Core Practice:** Evaluate Reasoning

Vocabulary

inequality
infinite
reverse

MATH LINK

Use the following symbols when translating verbal statements to inequalities.

$<$ less than fewer than
$\leq$ less than or equal to; at most no more than
$>$ greater than more than
$\geq$ greater than or equal to; at least no less than

KEY CONCEPT: Use inverse operations to solve one- and two-step inequalities.

Compare each pair of numbers, using $<$, $>$, or $=$.

1. 0.10 ______ 0.1
2. 14 ______ 23
3. 186 ______ 183
4. −2 ______ −5
5. −54 ______ −50
6. 8.57 ______ 8.5

Translate Verbal Statements into Inequalities

An **inequality** is a statement in which an inequality symbol is placed between two expressions, such as $t + 6 \geq 2$. The inequality symbols are $<$ (less than), $\leq$ (less than or equal to), $>$ (greater than), and $\geq$ (greater than or equal to).

Inequalities and equations are similar in some ways. For example, both contain expressions that combine numbers and variables. In other words, there is a connection, or relationship, between inequalities and equations. There are, however, differences. The inequality symbols tell you the expressions that are being compared may not be equal.

Because there is this connection between inequalities and equations, translating a verbal statement into an inequality is similar to translating a statement into an equation. First, choose a variable for the unknown. Next, identify the operations in the statement, and choose the inequality symbol that represents the statement. Then write the inequality.

Example 1 Translate a Verbal Statement into a One-Step Inequality

A number increased by 5 is no more than 9.

Step 1 Choose a variable to represent the unknown number.
Let n represent the unknown number.

Step 2 Identify the operation and write the expression.
The phrase *increased by* 5 indicates addition.
The expression is $n + 5$.

Step 3 Identify the inequality symbol in the verbal statement.
The phrase *no more than* means less than or equal to, so the symbol is $\leq$.

Step 4 Write the inequality. $n + 5 \leq 9$

Example 2 Translate a Verbal Statement into a Two-Step Inequality

Antonio will pay no less than \$115 total for two tickets to the football game plus \$35 for snacks.

Step 1 Choose a variable to represent the unknown.
The unknown is the cost of a ticket.
Let t represent the cost of one ticket.

Step 2 Identify the operation and write the expression.
The phrase *two tickets* indicates multiplication, since $2 \times t$ will be the cost of two tickets. The expression *plus \$35* indicates addition.
The expression is $2t + 35$.

Step 3 Identify the inequality symbol in the verbal statement.
The phrase *no less than* means greater than or equal to, so the symbol is $\geq$.

Step 4 Write the inequality.
$2t + 35 \geq 115$

THINK ABOUT MATH

Directions: Translate each verbal statement into a one-step or two-step inequality.

1. A number minus 8 is more than 12.
2. The cost of lunch plus a \$3 tip is at least \$10.
3. Four more than twice a number is less than 25.
4. A representative at Ace Plumbing told a customer that three hours of labor and a service fee of \$25 is no less than \$310.

MATH LINK

The Addition, Subtraction, Multiplication, and Division Properties of Inequality state that if you add, subtract, multiply, or divide each side of the inequality by the same number, then the inequality remains true. The exception is multiplying or dividing by a negative number. In these cases, you must reverse the direction of the inequality symbol for the inequality to remain true.

Solve One-Step Inequalities

Solve an inequality in the same way that you solve equations. Use inverse operations to isolate the variable.

The **solution of an inequality** is the set of all of the numbers that make the inequality true. These sets contain an **infinite,** or endless, number of numbers.

MATH

LINK

The solution r > 12 means that all numbers greater than 12 are a solution for the inequality. That means there are infinitely many correct solutions to this inequality.

Example 3 Use Addition to Solve an Inequality

Solve the inequality $r - 2 > 10$.

Step 1 Identify the operation in the inequality and its inverse. The operation is subtraction, and its inverse is addition.

Step 2 Use the inverse operation to solve the inequality. In this case, add 2 to each side of the inequality.

$$\begin{array}{rcl} r - 2 & > & 10 \\ \underline{+2} & & \underline{+2} \\ r & > & 12 \end{array}$$

Step 3 Check the solution. Since r is greater than 12, substitute a number greater than 12, such as 12.5, for the variable in the original inequality.

$12.5 - 2 > 10$ $\quad$ $10.5 > 10$ $\quad$ This is true, so $r > 12$.

Core Skill

Solve Inequalities

You have some experience already with solving one- and two-step equations with one variable. Much of what you have learned about solving such equations applies to the process of solving inequalities. For example, you want to isolate the variable. You must also follow the order of operations. There is one important difference between solving equations and solving inequalities. When multiplying or dividing both sides of an inequality by a negative number, you must reverse the inequality symbol.

In a notebook solve the equations 14y > -2 and then solve -14y > 2.

Example 4 Use Subtraction to Solve an Inequality

Solve the inequality $h + 6 \leq 3$.

Step 1 Identify the operation in the inequality and its inverse. The operation is addition, and its inverse is subtraction.

Step 2 Use the inverse operation to solve the inequality.

$$\begin{array}{rcl} h + 6 & \leq & 3 \\ \underline{-6} & & \underline{-6} \\ h & \leq & -3 \end{array}$$

Step 3 Check the solution. Since h is less than or equal to -3, substitute -3 for the variable in the original inequality.

$-3 + 6 \leq 3$ $\quad$ $3 \leq 3$ $\quad$ This is true, so $h \leq -3$.

As you might have noticed in the two previous examples, both involved subtracting the number that was being added (or subtracted). In Example 3, you subtracted a -2 from both sides to solve the inequality (subtracting a negative is the same as adding). Similarly, in Example 4, you subtracted 6 from both side to solve the inequality. There are also similarities between solving inequalities involving multiplication and division.

One major difference in solving equations and solving inequalities is that if you divide or multiply by a negative number to solve an inequality, you must **reverse**, or change the direction, of the inequality symbol. If a symbol is $\leq$, it becomes $\geq$ when you reverse its direction. If you multiply or divide by a positive number, the direction of the symbol is not reversed.

Example 5 Use Division to Solve an Inequality

Solve the inequality $-4b \geq 20$.

Step 1 Identify the operation in the inequality and its inverse. The operation is multiplication, and its inverse is division.

Step 2 Use the inverse operation to solve the inequality. In this case, the inverse operation is division by a negative integer. You will need to reverse the direction of the inequality symbol from $\geq$ to $\leq$.

$\frac{-4b}{-4} \geq \frac{20}{-4}$ $\qquad$ $b \leq -5$

Step 3 Check the solution. Since b is less than or equal to -5, substitute both -5 and a number less than -5, such as -6, for the variable in the original inequality.

$-4(-5) \geq 20$ $\quad$ $20 \geq 20$

$-4(-6) \geq 20$ $\quad$ $24 \geq 20$ $\quad$ This is true, so $b \leq -5$.

THINK ABOUT MATH

Directions: Solve each inequality.

1. $-3t > 15$
2. $x - 14 \leq -12$
3. $\frac{a}{6} \leq -1$
4. $c + 2 < 9$

Example 6 Use Multiplication to Solve an Inequality

Solve the inequality $\frac{y}{3} < -12$.

Step 1 Identify the operation in the inequality and its inverse. The operation is division, and its inverse is multiplication.

Step 2 Use the inverse operation to solve the inequality. The inverse operation is multiplication by a positive number, so you will *not* need to reverse the direction of the inequality symbol.

$\frac{y}{3} < -12$ $\quad$ $\frac{y}{3} \times 3 < -12 \times 3$ $\quad$ $y < -36$

Step 3 Check the solution. Since y is less than -36, substitute a number less than -36 in the original inequality, such as -39.

$\frac{-39}{3} < -12$ $\quad$ $-13 < -12$ $\quad$ This is true, so $y < -36$.

In Examples 3 and 4, you noticed that in both inequalities, subtraction was used to solve the inequality. The same is true of Examples 5 and 6. In both examples, you can divide by the number being multiplied to the variable. Example 5 has the variable being multiplied by -4 and in Example 6, the variable is multiplied by $\frac{1}{3}$. To solve those examples, you divided by -4 and 3 (the reciprocal of $\frac{1}{3}$), respectively.

Solve Two-Step Inequalities

You use two inverse operations to solve a two-step inequality. In general, perform addition or subtraction before multiplication or division. As with one-step inequalities, reverse the direction of the inequality symbol if you multiply or divide by a negative number.

Core Practice
Evaluate Reasoning

Precision is all-important when solving two-step inequalities. First, you must do the two steps in the correct order: addition and subtraction before multiplication and division. You must also be mindful of those cases in which your calculations lead to a reversal of the direction of the inequality symbol.

Enesta solves the following problem: $-5x - 25 \geq 50$. First, she divides each side by -5, but she doesn't reverse the direction of the inequality symbol when dividing by a negative number. As a result, she gets $x + 5 \geq -10$. Then she subtracts 5 from both sides to get $x \geq -15$. She checks her solution by replacing x with -14. $-5(-14) - 25 \geq 50$; $70 - 25 \geq 50$; $45 \not\geq 50$.

What did she do wrong? In her first step, she divided both sides of the inequality by -5 correctly, but she should have remembered to reverse the inequality sign. She divided first, then added, which was a valid path to finding the solution. Her error occurred during the division step.

Do Enesta's problem again, but this time, add first and then divide. Check your solution.

Example 7 Use Addition and Multiplication to Solve a Two-Step Inequality

Solve the inequality $\frac{k}{-8} - 1 < 2$.

Step 1 Identify each operation in the inequality and its inverse.
The first operation is division, and its inverse is multiplication.
The second operation is subtraction, and its inverse is addition.

Step 2 Identify the order in which the inverse operations should be performed.
Use addition to undo subtraction, and then use multiplication to undo division.

Step 3 Perform the first inverse operation on each side of the inequality.
Add 1 to each side of the inequality.
$\frac{k}{-8} - 1 < 2$ $\frac{k}{-8} - 1 + 1 < 2 + 1$ $\frac{k}{-8} < 3$

Step 4 Perform the second inverse operation on each side of the inequality.
Multiply each side by –8. Since you are multiplying by a negative integer, you need to reverse the direction of the inequality symbol from < to >.
$\frac{k}{-8} < 3$ $\frac{k}{-8} \times (-8) > 3 \times (-8)$ $k > -24$

Step 5 Check the solution. Substitute a number greater than –24 into the original inequality. Choose a number that can be easily divided by –8, such as –16.
$\frac{-16}{-8} - 1 < 2$ $2 - 1 < 2$ $1 < 2$ This is true, so $k > -24$.

Example 8 Use Subtraction and Division to Solve a Two-Step Inequality

Solve the inequality $6p + 21 \geq 39$.

Step 1 Identify each operation in the inequality and its inverse.
The first operation is multiplication, and its inverse is division.
The second operation is addition, and its inverse is subtraction.

Step 2 Identify the order in which the inverse operations should be performed.
Use subtraction to undo addition and then division to undo multiplication.

Step 3 Perform the first inverse operation on each side of the inequality.
Subtract 21 from each side of the inequality.
$6p + 21 \geq 39$ $6p + 21 - 21 \geq 39 - 21$ $6p \geq 18$

Step 4 Perform the second inverse operation on each side of the inequality.
Divide each side by 6. Since you are dividing by a positive number, you do *not* need to reverse the direction of the inequality symbol.
$6p \geq 18$ $\frac{6p}{6} \geq \frac{18}{6}$ $p \geq 3$

Step 5 Check the solution. Since p is greater than or equal to 3, choose a number greater than 3, such as 4, and substitute it into the original inequality.
$6(4) + 21 \geq 39$ $24 + 21 \geq 39$ $45 \geq 39$
This is true, so $p \geq 3$.

THINK ABOUT MATH

Directions: Solve each inequality.

1. $3x + 10 < 7$ ____________
2. $\frac{y}{5} - 7 \leq 2$ ____________
3. $\frac{m}{-2} + 6 \geq 20$ ____________
4. $-7b - 11 > 10$ ____________

Vocabulary Review

Directions: Match each word to a statement.

1. _______ inequality
2. _______ infinite
3. _____ reverse

A. extends indefinitely

B. type of mathematical sentence that contains <, >, ≤ or ≥ between the two expressions

C. what happens to an inequality symbol when multiplying or dividing by a negative number to solve the inequality

Skill Review

Directions: Answer the following questions.

1. Translate $x - 4 > 6$ into a verbal statement.

____

2. Describe how the connections you made between solving equations and solving inequalities helped you learn how to solve inequalities.

____

Skill Practice

Directions: Choose the best answer to each question.

1. Which statement best describes the inequality $x + 4 \leq 10$?

   A. A number increased by 4 is at least 10.
   B. A number plus 4 is at most 10.
   C. A number and 4 is less than 10.
   D. A number plus 4 is no less than 10.

2. Nina used the inequality $4s + 35 \geq 180$ to find the least amount that she will have to pay to learn how to use a database. What is the least she will have to pay per session s?

   A. $80. 00
   B. $53. 75
   C. $36. 25
   D. $10. 00

3. Which inequality is equivalent to the inequality $-5x > 5$?

   A. $x < -25$
   B. $x > -25$
   C. $x > -1$
   D. $x < -1$

4. A print shop manager estimated that it would cost no more than $65 to print 100 flyers, plus $15 for paper. Which inequality can be used to find the greatest cost per flyer?

   A. $100f + 15 \leq 65$
   B. $100f + 15 \geq 65$
   C. $100f - 15 \leq 65$
   D. $100f - 15 < 65$

LESSON
5.5

Identify Patterns

Lesson Objectives

You will be able to

- Write expressions to represent patterns
- Write equations to represent patterns

Skills

- **Core Skill:** Solve Real-World Arithmetic Problems
- **Core Skill:** Build Lines of Reasoning

Vocabulary

common difference
generalize
input variable
numerical pattern
output variable
sequence
term

MATH LINK

In an **arithmetic sequence,** consecutive terms increase or decrease by a **common difference.** In the sequence 1, 3, 5, 7,... consecutive terms increase by a common difference of +2. In the sequence 9, 6, 3, 0,... consecutive terms decrease by a common difference of -3.

KEY CONCEPT: Identify, represent, and generalize patterns using expressions and equations.

Translate each verbal phrase into an expression or equation.

1. Twice a number is ten.
2. three times a number minus five

Evaluate each expression when $x = 4$.

3. $12x$
4. $7x + 2$

Write Expressions to Represent Patterns

A **numerical pattern** is a set of numbers related by a rule. The **rule** is the operation or operations used to form the pattern. Once you know the rule, you can **generalize** it. That means that you can write an expression or equation that describes the pattern. The rule must apply to all of the numbers in the pattern. Sometimes, the pattern you discover is a **common difference**, which is when the difference between consecutive terms in a sequence of numbers is the same, or common.

A **sequence** is a set of numbers in a specific order. A **term** in the sequence is a number in the sequence. For example, 3 is the first term in the sequence 3, 6, 9, 12, 15, ...

Example 1 Write a Rule with One Operation

Write an expression to represent the pattern in the sequence 3, 6, 9, 12, 15, 18, ...

Step 1 Make a table that shows the position and number of each term in the sequence. Label the position of the term n.

Position of Term ***n***	1	2	3	4	5	6
Number in Sequence	3	6	9	12	15	18

Step 2 Find the common difference between consecutive numbers in each row.
In the first row, add 1 to the previous number to get the next number, so the common difference is 1.
In the second row, add 3 to the previous number to get the next number, so the common difference is 3.

Step 3 Use the common difference to find and write a rule for the pattern.
The common difference in the second row is 3 times the difference in the first row, so the rule could be 3 times n or $3n$.

Step 4 Try the rule on at least three numbers in the table. Substitute 1, 4, and 6 for n to see if you get 3, 12, and 18 in the sequence.
$3n = 3(1)$, or 3 $\quad$ $3n = 3(4)$, or 12 $\quad$ $3n = 3(6)$ or 18
These are true, so the rule is $3n$.

MAKE A TABLE

Tables usually have rows and columns that contain specific types of information. A passage may contain a lot of information, and it may not always be organized. Tables allow the reader to track information and review it quickly in an organized way.

Often, the most difficult part of making a table is deciding which information is to be included and how best to arrange it. The first step is to decide how many categories of information are being presented and how it would be best to organize them.

Read the following passage, and create a table of the information.

> Catalina found three apartments that she liked. The first had 1 bedroom and 1 bathroom. The rent was $600 per month, and heat and hot water were included. Parking was $50 per month in a covered garage. The second had 2 bedrooms and 1 bathroom. The rent was $750 per month. Heat and hot water were included, but parking was $75 per month. The last apartment had 1 bedroom, 1 bathroom, a deck, and a dishwasher. The rent was $700 per month, and parking was included. Heat and hot water were not included in the rent.

The passage describes three apartments, so the table should have three rows. Now, reread the passage and look for details about each apartment, such as the rent, number of bedrooms and bathrooms, and extra fees (for example, heat and parking). Two of the apartments also have extra information, so the table should also have a place for "other."

	Number of beds/ bathroom	Rent ($)	Parking ($)	Heat/hot water included	Other
Apartment 1	1/1	600	50	Yes	Covered parking
Apartment 2	2/1	750	75	Yes	None
Apartment 3	1/1	700	0	No	Deck, dishwasher

Core Skill
Solve Real-World Arithmetic Problems

A passage in a math problem may contain a lot of information that is not organized. By organizing the information presented in a real-world problem, tables reveal patterns or relationships between numbers that may not have been obvious. Solving problems becomes easier when you've used a table to arrange or structure the important data.

Katiah recorded the high and low temperatures every day for a week. The high temperatures were Sunday 74°, Monday 76°, Tuesday 78°, Wednesday 72°, Thursday 71°, Friday 81°, and Saturday 80°. The low temperatures were Sunday 59°, Monday 61°, Tuesday 63°, Wednesday 57°, Thursday 56°, Friday 66°, and Saturday 65° What is the difference between the high and low for each day? Is there a common difference?

To answer the question, complete the table below.

Temperatures (°F)

Day	High Temperature	Low Temperature
Sun.	74	59
		62
Tues.	78	
Wed.		
Fri.		
Sat.		

Core Skill
Build Lines of Reasoning

At some point, step back and take the time to think about what you have learned. When solving problems, you don't want to simply repeat the series of steps from memory that you encountered in each one of the lesson examples you have studied. You want to be able to understand why each step is important as well as the connections among the various steps. If you truly understand the math, you can solve problems that may differ slightly from the ones you have seen before. Critical reflection provides a way to achieve this level of understanding.

Consider the following sequence that begins with 1, 4, 9, 16, 25,... Unlike other patterns you have seen, this pattern does not have a common difference (3, 5, 7, 9,...). Can you still find a pattern in this sequence? What rule is occurring? In a notebook, write out what is happening to each term to help find a pattern.

THINK ABOUT MATH

Directions: Make a table for each sequence. Then write a rule to represent the sequence.

1. 4, 8, 12, 16, 20, 24,...
2. 5, 8, 11, 14, 17, 20,...

A two-variable equation describes the pattern between two quantities and has two variables, such as x and y. The variable that you apply the rule to is the **input variable.** The result is the **output variable.** Write the output variable equal to the rule, such as $y = 2x$.

Example 2 Write a Rule with Two Operations

Write an equation to represent the pattern shown in the table.

x	1	2	3	4	5	6
y	9	14	19	24	29	34

Step 1 Find the common differences for the values of x and y.
The common difference for the x-values is 1.
The common difference for y-values is 5.

Step 2 Use the common difference to find and write a rule for the pattern. The common difference for the y-values is 5, so the rule could be $y = 5x$.

Step 3 Try the rule on at least three numbers in the table.
If $x = 1$, then $y = 9$, and $5(1) = 5$.
$5 \neq 9$. The rule is not correct.

Step 4 Try adding or subtracting a number from the rule to get the value for y. If this works, try the new rule on two more numbers. The difference between 9 and 5 is 4, so add 4 to the rule to give a new rule of $y = 5x$ 4- 4.
If $x = 1$, then $y = 9$, and $5(1) + 4 = 9$.
If $x = 4$, then $y = 24$, and $5(4) + 4 = 24$.
If $x = 6$, then $y = 34$, and $5(6) + 4 = 34$.
These are true, so the rule is $y = 5x + 4$.

Example 3 Find the Next Numbers in a Pattern

Find the next three numbers in the sequence 1, 3, 5, 7, 9, 11,....

Step Make a table that shows the position and the number of each term in the sequence. Label the position of the term *n*.

Position of Term, ***n***	1	2	3	4	5	6	7	8	9
Number in Sequence	1	3	5	7	9	11			

Step Write a rule for the sequence. Look at the first three numbers in each row. If you multiply 1 by 2 and subtract 1, you will get 1. Multiply 2 by 2 and subtract 1 to get 3. Multiply 3 by 2 and subtract 1 to get 5. So the rule is $2n - 1$.

Step To find the next three numbers in the sequence use the rule $2n - 1$:

$2 \times 7 - 1 = 13$ $2 \times 8 - 1 = 15$ $2 \times 9 - 1 = 17$

Position of Term, ***n***	1	2	3	4	5	6	7	8	9
Number in Sequence	1	3	5	7	9	11	13	15	17

Example 4 Find Any Number in a Pattern

What is the 20th term in the pattern 3, 7, 11, 15, 19, 23,...?

Step Make a table.

Position of Term, ***n***	1	2	3	4	5	6
Number in Sequence	3	7	11	15	19	23

Step Write a rule. There is a common difference of 1 for the first row and a common difference of 4 for the second row. If you multiply each term in the first row by 4 and then subtract 1, you will get the numbers in the pattern. The rule is $An - 1$.

Step 3 Apply the rule to the term you need to find. You need to find the 20th term. Substitute $n = 20$ in the rule, $An - 1$.
$4(20) - 1 = 79$; the 20th term of the pattern is 79.

Vocabulary Review

Directions: Fill in the blanks with the correct word or phrase below.

common difference **numerical pattern** **generalize** **input variable** **term** **output variable**

1. When you________________ a pattern, you write an expression or equation to describe the rule of the pattern.

2. A(n)_______________ is one of the numbers in a sequence.

3. When analyzing a pattern, the __________________ is the result of applying the rule to the _______________

4. The difference between consecutive terms in a sequence is called the________________________

5. A(n)_______________ is a set of numbers related by a rule.

Skill Review

Directions: Use what you learned about finding patterns to answer the following questions.

1. Explain the process you would use to find a pattern for the sequence 1, 4, 7, 10, 13,.... Then find the pattern and write the rule.

____

____

____

____

2. Explain why, after finding a pattern, it is useful to write the rule for the pattern using an expression or an equation.

____

____

____

____

Skill Practice

Directions: Choose the best answer to each question.

1. Dae-Jung is thinking about switching mail carriers for his business. The table below shows the costs for shipping and handling of different weights. Which equation shows the pattern between pounds p and cost c?

Number of pounds, *p*	1	2	3	4	5
Total Cost, c	8	10	12	14	16

A. $c = 8p$
B. $c = p + 7$
C. $c = 2p + 7$
D. $c = 2p + 6$

2. Elias says that the expression 4n describes the pattern of the sequence shown in the table.

Position of Term, *n*	1	2	3	4	5
Number in Sequence	5	9	13	17	21

Which of the following best describes Elias's statement?

A. He is correct.
B. He only used the common difference instead of the common difference plus 1, or $4n + 1$.
C. He should have added 4 to *n* for the rule $n + 4$.
D. The rule should be 5n because 1×5 is 5.

3. What is the next number in the pattern?

39, 77, 115, 153, 191

A. 209
B. 219
C. 229
D. 239

4. Zoey wrote the equation $n = 12m$ to describe the pattern in one of the tables below. Which table did she use?

A.

m	1	2	3	4	5
n	12	24	36	48	60

B.

m	1	2	3	4	5
n	11	22	33	44	55

C.

m	1	2	3	4	5
n	12	20	28	36	44

D.

m	1	2	3	4	5
n	12	22	32	42	52