LESSON

8.1

Exponents

Lesson Objectives

You will be able to

- Evaluate exponents
- Evaluate arithmetic expressions with exponents

Skills

- **Core Skill:** Evaluate Expressions
- **Core Skill:** Calculate Area and Volume

Vocabulary

base
exponent
power

KEY CONCEPT: Extend understanding of numbers to exponents and arithmetic expressions that contain exponents.

Evaluate each expression when x = *3 and* y = -2.

1. $x + y$	4. 6x - 5y
2. $2x - 3$	**5.** 4(x + 2y)
3. 3y + 4	6. -2(3x - y)

Evaluate Exponents

The expression 2^4 is called a **power:** 2 is the **base,** and 4 is the **exponent.** The expression 2^4 is read *two to the fourth power.* To find the value, use the base as a factor and the exponent as the number of times it is multiplied. The expression 2^4 has the same product as 2 x 2 x 2 x 2. The product of both expressions is 16.

Example 1 Find the Value of an Exponential Expression

Find the value of 3^5.

Step 1 Identify the base and exponent.
Base: 3
Exponent: 5

Step 2 Use the base as a factor as many times as the exponent indicates.
3 x 3 x 3 x 3 x 3

Step 3 Multiply.
3 x 3 x 3 x 3 x 3 = 243
The value of 3^5 is 243.

Example 2 Find the Value of an Expression with Zero as the Exponent

Find the value of 5^0.

Step 1 Identify the base and exponent.
Base: 5
Exponent: 0

Step 2 The exponent is 0. 5 is used as a factor 0 times, and the value of the expression is 1.
The value of 5^0 is 1.

MATH LINK

When the exponent of a non-zero base is zero, the value of the expression is always **1**.

MATH LINK

You can use a mnemonic device (a memory aid) such as PEMDAS to help recall the order of operations for arithmetic expressions. PEMDAS stands for Parentheses, Exponents, Multiplication, Division, Addition, Subtraction.

Example 3 Use a Calculator to Evaluate an Exponential Expression

Find the value of 9^6.

Press 2nd 9 $x^{\square}$ 6 enter

The display should read

9^6 Math ▲
531441

.

The value of 9^6 is 531,441.

THINK ABOUT MATH

Directions: Find the value of each expression.

1. 4^3
2. 2^5
3. 5^2
4. 3^3

Directions: Use a calculator to find the value of each expression.

5. 8^7
6. 24^5
7. 43^4
8. 12^6

Evaluate Arithmetic Expressions with Exponents

Recall that an arithmetic expression contains numbers and one or more operations. It is evaluated by using the order of operations. When you don't follow the order of operations, you will get the wrong value for an arithmetic expression.

An arithmetic expression can also contain powers with exponents or square roots. Square roots will be introduced in Lesson 8. 2. The order of operations that you learned about in Chapter 1 is expanded to include powers with exponents and square roots. Follow this sequence of steps when evaluating expressions:

1) Operations within parentheses
2) Exponents and roots
3) Multiplication and division from left to right
4) Addition and subtraction from left to right

Core Skill
Evaluate Expressions

In mathematics, it is important to learn the sequence of steps you follow to solve expressions and equations, use your calculator, and perform operations. As you gain experience in mathematics, you will begin to recognize one of the singular features of the subject: its regularity. You work your way to the solution of similar problems by following the same sequence of steps over and over again.

When you are evaluating an expression that contains exponents, for example, you must understand the meaning of each part of the expression. You want to identify the base, the number that will be the factor that is multiplied by itself. You also want to identify the exponent, the number that tells you how many times to multiply the base by itself. Then you perform the multiplication to get the product. The steps you take to arrive at a solution never vary.

After reading the section that lists the sequence in the order of operations, describe in a notebook why it is important to use the order of operations to evaluate $12 - (5 + 3)^2 / 2^3$. Include in your description the value of the expression if you use the order of operations. Use a possible value if you do not use the order of operations.

Core Skill
Calculate Area and Volume

Two of the things that exponents are used for are finding area and volume. The most basic shapes that have their area and volume calculated are the square and cube. In fact, the units to describe area and volume are square units and cubic units, respectively, and are named after the square and cube.

Suppose that the length of the side of a square is s inches long. What would the area be? The formula for the area of a rectangle is A = *l* x *w*. A square is a rectangle with both sides equal, so the area of the square is $A = s^2$. What about a cube with a side length s? The volume of the cube is $V = s^3$.

Finding area and volume will be covered later in the book. But in both examples, exponents are used. When speaking of these two formulas, the squares area would be said "s to the second power, or s squared", while the volume would be "s to the third power, or s cubed". These shortcuts for the two powers (2nd and 3rd), are used because they describe the shape whose area or volume is being calculated.

In a notebook, determine what would happen to a square's area if all of the sides doubled. Make sure to use the properties of exponents correctly.

Example 4 Use the Order of Operations to Find the Value of an Expression

Find the value of $45 - 3 \times 2^2 + (8 \times 5)$.

Step 1 Do operations within parentheses. $(8 \times 5) = 40$

Step 2 Do exponents and roots. $2^2 = 4$

Step 3 Do multiplication and division. Work from left to right. $3 \times 4 = 12$

Step 4 Do addition and subtraction.
$45 - 12 + 40 = 33 + 40 = 73$
The value of the expression is 73.

THINK ABOUT MATH

Directions: Find the value of each expression.

1. $(1 + 2 + 3)^2$
2. $3^2 + 6^2 / 3$
3. $(2^3 + 3^3) / 7$
4. $9 \times 8^0 + (6 - 1)$
5. $24 / (1^5 + 5)$
6. $3 \times (10 - 4) / 9 + 4^2$

Vocabulary Review

Directions: Match each word to one of the phrases below.

1. ________base
2. ________exponent
3. ________power

A. contains a base and an exponent
B. the number 3 in the expression 3^4
C. the number that indicates how many times a number is multiplied by itself

Skill Review

Directions: Answer the following question.

1. How does understanding sequence help you find the value of an expression that contains two or more operations?

____

Directions: Describe the sequence you would use to find the value of each of the following expressions. Then find the value of the expression.

2. $4^2 + 3^3 / 9$

____

3. $2 \times (14 - 7^{\circ}) + 28 + 2^2$

____

Skill Practice

Directions: Choose the best answer to each question.

1. Which of the following has the same value as 4^3?

A. 3^4
B. 8^2
C. 43'
D. 64°

2. Which operation in the following expression should be performed first?

$(3 + 6)^2 - 2^3 / 4 \times 3$

A. Evaluate 2^3.
B. Evaluate 6^2.
C. Multiply 4×3.
D. Add $3 + 6$.

3. By what factor would the volume of a cube change if all of the sides doubled?

A. 1 (the volume would stay the same)
B. 2 (the volume would double)
C. 4 (the volume would quadruple)
D. 8 (the volume would octuple)

4. Tabina sold 2^5 air conditioners last week and 3^3 air conditioners this week. What is the difference in the number of air conditioners she sold?

A. 1
B. 2
C. 5
D. 8

LESSON
8.2 Roots

Lesson Objectives

You will be able to

- Find square roots
- Find cube roots

Skills

- **Core Skill:** Evaluate Reasoning
- **Core Skill:** Interpret Data Displays

Vocabulary

cell
cube root
perfect cube
perfect square
radical sign
square root
squared

MATH LINK

Many square roots are memorized as part of the multiplication facts. Look at columns **1** and **2** in the table on the next page. You probably already know the square roots of the perfect squares to **10**. You might also know that **11** x **11** is **121** and **12** x **12** is 144. Both 121 and 144 are perfect squares.

KEY CONCEPT: Develop and extend understanding of numbers to include the concepts of square roots and cube roots.

Find the value of each expression.

1. 7^2
2. 2^5
3. 3^4
4. 6^3

Use a calculator to find the value of each expression.

5. 5^6
6. 8^8
7. 12^4
8. 41^5

Find Square Roots

The expression 7^2 is sometimes called "7 **squared**" or "the square of 7." The exponent 2 indicates that the base is squared. Recall that 7^2 is the same as 7×7, so the value of 7^2 is 49.

The expression $\sqrt{49}$ is read, "the **square root** of 49." The symbol $\sqrt{}$ is called a **radical sign**. Finding the square root of a number is the opposite of finding the square of a number. A number's square root is the number that, multiplied by itself, will yield the original number.

A **perfect square** is a whole number whose square root is a whole number. For example, 16 is a perfect square because 4^2 is 16.

Example 1 Find the Square Root of a Perfect Square

Find the value of $\sqrt{100}$.

Step 1 Think: What number multiplied by itself is 100?

$n \times n = 100$

$10 \times 10 = 100$

Step 2 Write the square root.

$\sqrt{100} = 10$

The value of $\sqrt{100}$ is 10.

Example 2 Use a Calculator to Find a Square Root

Find the value of $\sqrt{2{,}304}$.

Press [2nd] [x^2] [2] [3] [0] [4] [enter].

The display should read

$\sqrt{2{,}304}$	Math ▲
	48

The value of $\sqrt{2{,}304}$ is 48.

UNDERSTAND A TABLE

In addition to giving specific data, tables can help show information as a whole. Looking across a row or down a column can tell you certain things about that set of data. You may not have to analyze every cell in the table to understand the data. A **cell** is a place in a table or spread sheet where column and row intersect. Sometimes, trends are noticeable across rows or down columns. Other times, comparing two rows or columns can give information about the data.

Number (x)	Square (x^2)	Cube (x^3)
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1, 000

Study the table above. Without using the definition of squares and cubes, what can you tell about what happens to whole numbers when they are squared and cubed?

Notice that the numbers in the square column increase more rapidly than those in the number column as the list progresses. Likewise, the numbers in the cube column increase at a greater rate than the numbers in the square column. However, the numbers in the row for 1 are all the number 1. So you can infer that for whole numbers, with the exception of 1, cubed numbers will always be greater than squared numbers. Additionally, both squares and cubes increase more rapidly as the whole numbers increase.

Core Skill
Evaluate Reasoning

You probably have worked on group projects that your teacher assigned. You probably also have paired up with fellow classmates or friends and collaborated with them on a homework assignment. Working in pairs or small groups provides an opportunity to learn from each other. It also helps you solve problems together through reasoning, or making sense of what the problems are asking. Everybody benefits from collaborative efforts.

Kwan and Kolanda get together to do their math homework. Tonight's assignment: Complete some square root and cube root problems. Kwan says, "The cube root of 9 is 3, because you have to add 3 together three times to get 9. " "No, " Kolanda says. "In cube roots, you *multiply* a number three times. You don't add it. Three is the cube root of 27, and it is also the *square* root of 9. "

After you finish the lesson, find a partner to work with on one or two real-world problems in which you have to calculate square roots and cube roots. For example, would you need to find the square root or the cube root of a square garden if you knew the area and were asked to find the length of each side? Suppose you knew the volume of a cube and were asked to find the length of each side. Do you need to find the square root or cube root?

Core Skill
Interpret Data Displays

Tables will often present data or information in a way that makes it easy to use. First, study the information in the table so that you understand what it is telling you, and then use that information to solve problems. Some ways of using data to solve a problem include performing operations on the data, such as adding or subtracting, using the data to make a graph, or looking for a pattern.

Look again at the data in the table on the previous page. Think about ways in which you could use the data. Now look at Example **1**. You are asked to find the value ofvfOO. If you look at the first two columns in the table, you can see that the square of **10** is **100**. If the square of **10** is **100**, then the square root of **100** is **10**. So you can use column **2** in the table to find the square root in column **1**.

In a notebook, explain how you can use the data in the table to approximate square roots.

If a number is not a perfect square, you can **approximate,** or estimate, the square root by finding the two following, or **consecutive,** whole numbers, such as 8 and 9, between which the square root lies.

Example 3 Approximate a Square Root

Find the two consecutive whole numbers between which $\sqrt{150}$ lies.

Step 1 Think: Which two perfect squares are closest to 150.
Try the squares of 12 and 13.
$12 \times 12 = 144$
$13 \times 13 = 169$

Step 2 Write the perfect squares as square roots, and compare.
Write the square roots using $<$.
$\sqrt{144} < \sqrt{150} < \sqrt{169}$
$\sqrt{150}$ lies between $\sqrt{144}$ and $\sqrt{169}$.

Step 3 Find the square roots of the perfect squares.
$12 < \sqrt{150} < 13$
$\sqrt{150}$ is between 12 and 13.

Example 4 Solve Problems Involving Square Roots

Find the side length of a square with an area of 324 square meters.

Step 1 The area of a square is the length of one side squared, so find the value of $\sqrt{324}$ to find the side length of the square. Try numbers that, when multiplied by themselves, are equal to 324.
Try 17. — Try 18.
$17 \times 17 = 289$ — $18 \times 18 = 324$

Step 2 Write the square root.
$\sqrt{324} = 18$
The side length of the square is 18 meters.

Example 5 Do Operations with Square Roots

Find the sum of $\sqrt{81}$ and $\sqrt{144}$.

Step 1 Find each square root.
$9 \times 9 = 81$, so $\sqrt{81} = 9$.
$12 \times 12 = 144$, so $\sqrt{144} = 12$.

Step 2 The operation is addition, so add the square roots.
$9 + 12 = 21$
The sum of $\sqrt{81}$ and $\sqrt{144}$ is 21.

Find Cube Roots

The expression 7^3 is sometimes called "7 cubed" or "the cube of 7." The exponent 3 indicates that the base is cubed. Since 7^3 is the same as $7 \times 7 \times 7$, the value of 7^3 is 343.

The expression $\sqrt[3]{343}$ is read, "the **cube root** of 343." The radical sign has a 3 in the corner to indicate that this is a cube root. The cube root of a number is the one number that multiplied three times will give the cube of the number. A **perfect cube** is a number whose cube root is an integer. For example, 8 is a perfect cube because $2^3 = 8$.

Example 6 Find the Cube Root of a Number

Find the value of $\sqrt[3]{125}$.

Step 1 Think: What number multiplied three times is 125?
$n \times n \times n = 125$
$5 \times 5 \times 5 = 125$

Step 2 Write the cube root.
$\sqrt[3]{125} = 5$; the value of $\sqrt[3]{125}$ is 5.

Example 7 Use a Calculator to Find a Cube Root

Find the value of $\sqrt[3]{1{,}728}$.

Press [2nd] [2nd] [x^3] [1] [7] [2] [8] [enter].

The display should read

$\sqrt[3]{1{,}728}$	Math ▲
	12

.

The value of $\sqrt[3]{1{,}728}$ is 12.

Example 8 Solve Problems Involving Cube Roots

Find the side length of a cube with a volume of 729 cubic centimeters.

Step 1 The volume of a cube is the length of one side cubed. Find the value of $\sqrt[3]{729}$ to find the side length of the cube.

Try 8.	Try 9.
$8 \times 8 \times 8 = 512$	$9 \times 9 \times 9 = 729$.

Step 2 Write the cube root.
$\sqrt[3]{729} = 9$; the side length of the cube is 9 centimeters.

THINK ABOUT MATH

Directions: Find the value of each of the following. Use a calculator if necessary.

1. $\sqrt[3]{8}$	3. $\sqrt[3]{27}$	5. $\sqrt[3]{216}$	7. $\sqrt[3]{64}$
2. $\sqrt[3]{1{,}000}$	4. $\sqrt[3]{3{,}375}$	6. $\sqrt[3]{15{,}625}$	8. $\sqrt[3]{5{,}832}$

MATH LINK

The radical sign is considered a **grouping symbol**. A grouping symbol is something that groups numbers and variables together. Some other examples are parentheses, brackets, and fraction bars. Make sure to perform all actions inside the grouping symbol first when using the order of operations.

For example, in the problem $\sqrt{41 - 16}$, first subtract 16 from 41 to get $\sqrt{25}$. Then find the square root of 25, which is 5. Performing these operations in the wrong order will lead to an incorrect answer.

MATH LINK

When using a calculator to find the square root of a number that is not a perfect square (or a cube root of a number that is not a perfect cube), you may find that the answer may be given on the screen in the form of numerals and radicals. For example $\sqrt{123}$ is displayed as $\sqrt{123}$, and the answer to $\sqrt{124}$ is displayed as $2\sqrt{31}$. To obtain a decimal result, simply press the [$\leftrightarrow 10^x$] button on the calculator.

Vocabulary Review

Directions: Fill in the blanks with one of the words or phrases below.

cube root perfect cube perfect square radical sign square root squared

1. A__________ is the symbol for the square root of a number.

2. The________________of 121 is 11.

3. If the square root of a number is a whole number, then it is a______________________

4. A number that is_________________ is multiplied by itself.

5. The________________of 64 is 4.

6. A________________has a whole number for the cube root.

Skill Review

Directions: Study the table below. Then answer the questions.

X	‾	X	‾
1	1	1, 331	11
8	2	1, 728	12
27	3	2, 197	13
64	4	2, 744	14
125	5	3, 375	15
216	6	4, 096	16
343	7	4, 913	17
512	8	5, 832	18
729	9	6, 859	19
1, 000	10	8, 000	20

1. Describe what the columns and rows in the table show.

2. Describe any patterns in the table.

3. Explain the ways in which you could use the data in the table.

4. Explain how you could use the data in the table to approximate the value of

Skill Practice

Directions: Choose the best answer to each question.

1. Between which two consecutive whole numbers does $\sqrt{33}$ lie?

   A. 3 and 4
   B. 4 and 5
   C. 5 and 6
   D. 6 and 7

2. What is the value of $\sqrt{289 - 225}$?

   A. 64
   B. 8
   C. 4
   D. 2

Directions: Answer the following questions.

3. A square parking lot has a total area of 6, 400 square meters. What is the length, in meters, of one side of the parking lot?

   ____

4. A photo shop makes custom photo cubes. Elliot needs to make a photo cube with a volume of 2, 744 cubic centimeters. What is the length, in centimeters, of each side of the cube?

   ____

LESSON
8. 3

Scientific Notation

Lesson Objectives

You will be able to

- Translate standard notation to scientific notation
- Translate scientific notation to standard notation

Skills

- **Core Practice:** Attend to Precision
- **Core Skill:** Perform Operations

Vocabulary

annex zeros
powers of ten
scientific notation
standard notation

MATH LINK

A number written in scientific notation will always have one non-zero digit to the left of the decimal point.

MATH LINK

If a number written in standard notation is greater than **1**, the exponent of the power of **10** will be positive.

KEY CONCEPT: Develop understanding of large numbers to include scientific notation and how to translate between numbers written in scientific notation and standard notation.

Evaluate each expression.

1. 12^4	3. 4^3	5. 2^6
2. 10^2	4. 10^1	6. 3^5

Translate Standard Notation to Scientific Notation

Scientific notation is a way to write very large numbers (or very small numbers) using multiplication and **powers of ten,** such as 10^3, 10^8, 10^{12}, and so on. Scientists and others who work with very large numbers, such as the distance from Earth to Saturn, use scientific notation, because the numbers are too great in standard notation. **Standard notation** is the way we generally represent numbers in everyday usage. In standard notation, the distance from Earth to Saturn is about 1, 320, 000, 000 kilometers. In scientific notation, the distance is written as 1.32×10^9 kilometers.

A number written in scientific notation includes a number greater than or equal to 1 and less than 10 multiplied by a power of 10. Some examples are 8×10^4, 2.1×10^{22}, and 5.273×10^5.

Example 1 Write a Number in Scientific Notation

Write 25, 500, 000 in scientific notation.

Step 1 Move the decimal point to the left so that the number to the left of the decimal point is between 1 and 10. Write the number as a decimal. Drop the zeros. 25,500,000 2. 55

Step 2 Count the number of places the decimal point moves to the left, and write the number of places as the exponent of a power of ten. 7 places = 10^7

Step Write the number times the power of ten. 25, 500, 000 written in scientific notation is 2.55×10^7. 2.55×10^7

THINK ABOUT MATH

Directions: Write each number in scientific notation.

1. 18, 400
2. 453, 260, 000
3. 20, 000, 000
4. 870, 000
5. 12, 650, 000, 000
6. 9, 348, 000

Translate Scientific Notation to Standard Notation

When translating standard notation to scientific notation, move the decimal point to the right and **annex zeros,** if necessary. Annex, or add, zeros so that the number of places after the decimal point in the original number is the same as the exponent in the power of ten.

Example 2 Write a Number in Standard Notation

Write 3.9×10^5 in standard notation.

Step 1 Use the power of ten to determine how many places to move the decimal point.
The exponent of 10^5 is 5, so move the decimal point 5 places.

Step 2 Move the decimal point to the right. Annex zeros, if needed. 3.90000

Step 3 Write the number in standard notation. 390,000
3.9×10^5 is 390, 000 in standard notation.

Example 3 Use a Calculator to Translate Scientific Notation to Standard Notation

Translate 5.9874×10^8 to standard notation.

Press 2nd 5 . 9 8 7 4 ×10ⁿ 8 enter.

The display should read

5.9874 × 10^8 Math ▲
598,740,000

5.9874×10^8 written in standard notation is 598,740,000.

Core Practice
Attend to Precision

Numbers that are expressed in scientific notation can be thought of as mathematical expressions. Fortunately, the expressions in scientific notation contain only one operation: multiplication. The calculation is simple—in theory, at least.

If you're not careful, however, when converting numbers in scientific notation to numbers in standard notation, you could come up with a number that is much too large or much too small. Make sure you convert to the correct number of decimal places.

Look at the pattern in the table below. Notice what happens to the standard form of the number as the power of ten increases by **1**.

Power of Ten	Standard Notation
10^0	1
10^1	10
10^2	100
10^3	1, 000
10^4	10, 000
10^5	100, 000

In a notebook, explain the pattern in the table. Then use the pattern to predict the number of zeros in 10^{14}. Finally, write the number in standard form.

Core Skill
Perform Operations

The Voyager Interstellar Mission is a pair of satellites, Voyager I and II, which NASA sent into space in September and August 1977, respectively. As stated on NASA's Voyager website, its mission is "to extend the NASA exploration of the solar system beyond the neighborhood of the outer planets to the outer limits of the Sun's sphere of influence, and possibly beyond".

Voyager I is approximately 18. 467 billion kilometers and Voyager II is approximately 15. 171 billion kilometers from Earth. In a notebook, write down both distances in scientific notation.

THINK ABOUT MATH

Directions: Write each number in standard notation.

1. 3. 1 x 10^5
2. 7 x 10^{12}
3. 4. 06 x 10^6
4. 2. 913 x 10^8

Directions: Use a calculator to find each number in standard notation.

5. 6. 641 x 10^8
6. **1. 002** x **10^7**
7. 5. 9 x 10^9
8. **8. 22** x **10^4**

Vocabulary Review

Directions: Write the word next to the statement it matches.

annex zeros powers of ten scientific notation standard notation

1. __________ a way of writing numbers in which a number between 1 and 10 is multiplied by a power of ten
2. ______________ expressions, such as 10^9, in which 10 is written with an exponent
3. ______________ a way of writing numbers, such as 4, 445
4. ______________ adding zeros to a number so that a number has the correct number of places

Skill Review

Directions: Apply what you have learned about converting numbers from standard notation to scientific notation or from scientific notation to standard notation to answer the questions below.

1. Translate the numbers 3, 786, 000, 000 and 92, 433 to scientific notation. [2]

____

2. Translate 4. 0 x **10^{12}** and 1. 92 36 x **10^7** to standard notation.

____

Skill Practice

Directions: Choose the best answer to each question.

1. What is 853, 491 written in scientific notation?

   A. 0. 853491 x 10^6
   B. 8. 53491 x 10^5
   C. 85. 3491 x 10^4
   D. 853. 491 x 10^3

2. What is 3. 4587 x 10^7 written in standard notation?

   A. 34, 587, 000
   B. 34, 587
   C. 3, 458, 700
   D. 345, 870

3. Which of the following is true of scientific notation?

   A. The number must be positive.
   B. The exponent of 10 must be zero.
   C. The number must be between 1 and 9.
   D. The exponent of 10 must be greater than zero.

4. What must be done to convert from scientific to standard notation?

   A. Annex the same amount of zeros as the exponent of 10 at the end of the number.
   B. Move the decimal point to the right past the last digit of the number.
   C. Rewrite the number without a decimal point.
   D. Move the decimal point to the right the same number as the exponent of ten and annex enough zeros (if needed).

5. The distance of Earth to the Sun is approximately 93, 000, 000 miles. What is that number using scientific notation?

   A. 93 x 10^6
   B. 9. 3 x 10^6
   C. 9. 3 x 10^7
   D. . 93 x 10^7