LESSON 7.1

Ratios and Rates

Lesson Objectives

You will be able to

- Understand and write ratios
- Understand and find unit rates and unit prices

Skills

- **Core Skill:** Understand Ratio Concepts
- **Core Skill:** Use Ratio and Rate Reasoning

Vocabulary

paraphrase
rate
ratio
unit price
unit rate

KEY CONCEPT: Understand how to write ratios to understand the meaning of a unit rate.

Write each fraction in lowest terms.

1. $\frac{15}{25}$
2. $\frac{14}{28}$
3. $\frac{8}{12}$
4. $\frac{9}{15}$
5. Draw a picture that shows that 3 out of 4 equal parts are shaded.

Understand Ratios

A **ratio** is a comparison of two numbers. Use a ratio to compare the numbers 2 and 7. There are three different ways to write the ratio:

Use the word *to*.	Use a colon (:).	Write a fraction.
2 to 7	2:7	$\frac{2}{7}$

Think of a ratio as a fraction. Just as the fractions $^2/_7$ and $^7/_2$ are not the same, the ratios "2 to 7" and "7 to 2" are not the same. So the order of the two numbers in a ratio is important. A ratio is usually reduced to lowest terms.

Example 1 Write a Ratio

In Robertos' family, there are 2 boys and 4 girls. What is the ratio of boys to girls in Roberto's family?

Step 1 Write the ratio in one of these three ways. Follow the order in the phrase *ratio of boys to girls* for the order of numbers in the ratio. *Boys* is first, so the number representing the number of boys goes to the left of the word *to* or colon or in the numerator of the fraction. The number representing the number of *girls* will be on the right of the word *to* or colon or in the denominator of the fraction.

Use the word *to*.	Use a colon.	Write a fraction.
2 boys to 4 girls	2 boys:4 girls	$\frac{2 \text{ boys}}{4 \text{ girls}}$
2 to 4	2:4	$\frac{2}{4}$

Step 2 Reduce to lowest terms.

Use the word *to*.	Use a colon.	Write a fraction.
1 boy to 2 girls	1 boy:2 girls	$\frac{1 \text{ boy}}{2 \text{ girls}}$
1 to 2	1:2	$\frac{1}{2}$

Step 3 Make a statement about the ratio.
The ratio of boys to girls in Roberto's family is 1 to 2 or 1: 2 or $\frac{1}{2}$.

RESTATE OR PARAPHRASE INFORMATION

When you read, you may come across a long or complicated sentence or paragraph. To make sure you have understood what you just read, go back and read it again slowly. **Paraphrase,** or use your own words, to restate the information.

Look at each key word or important idea and imagine that you are explaining it to someone unfamiliar with the information. Then use clear, simple language to restate the ideas in an understandable way.

Read the following paragraph. Then read the two paraphrases below, and choose the best one.

> Ratios and rates are two ways of comparing quantities. A ratio is used when two items have different measures of the same thing. For example, a scale drawing might use a scale of 1 inch: 5 feet. Because feet and inches are both units of length, this is a ratio. A rate compares two items that are of a different nature. Miles per hour is a rate, because miles are a unit of length and hours are a unit of time. There are many rates that people use every day: hourly wages ($15 per hour) and fuel efficiency (20 miles per gallon) are both examples of rates. There are three ways to express ratios or rates: 3: 5, |, or 3 to 5.

1. You can compare with ratios and rates. You use ratios if you are comparing two things that are of the same type, as in a scale drawing. You use rates if you are comparing two things that are different, such as miles per hour, miles per gallon, or dollars per hour. You can write a ratio or rate as 3: 5, |, or 3 to 5.
2. Both ratios and rates compare things. Rates are common in everyday life. Ratios and rates can be written in different ways.

Paraphrase 1 restates the information in the original paragraph in different words. Paraphrase 2 leaves out several details, such as the difference between a ratio and a rate and the different ways to write them. Paraphrase 1 is the better choice.

Core Skill
Understand Ratio Concepts

Often, you can demonstrate your understanding of a problem and then solve it by restating the information in a way that breaks it down into simpler terms.

Some ratio problems are a good place to practice this skill. Look at Example 1 on the previous page. You are told the number of boys and girls in Roberto's family and asked to find the ratio of boys to girls. What if, instead, you were asked to find the ratio of boys to all of the children? You can restate the problem as two simpler problems: "First, I need to find the total number of children by adding the number of boys and girls. Then I can find the ratio of boys to total children. "

Select a partner with whom you can practice calculating ratios. Each of you write several situations that would require finding a ratio. Swap the situations that each of you wrote. Then restate each problem before solving.

Some ratio problems require calculation before you write the ratio.

Example 2 Solve a Multistep Ratio Problem

In a class of 30 students, there are 14 boys. What is the ratio of boys to girls in the class?

Step 1 Calculate the number of girls in the class because this number is not given.
30 students - 14 boys = 16 girls

Step 2 Write the ratio of boys to girls.
$\frac{14}{16}$

Step 3 Reduce the ratio to lowest terms.
$\frac{14}{16} = \frac{7}{8}$

Step 4 Make a statement about the ratio.
The ratio of boys to girls in the class is 7 to 8.

To write a ratio comparing two numbers, do the following:

- Use the word *to,* use a colon, or write a fraction.
- Remember that the order of the numbers in a ratio is important.
- Always reduce the ratio to lowest terms.

THINK ABOUT MATH

Directions: Write each ratio in two other ways.

1. 6 to 7
2. 1: 50
3. $\frac{10}{19}$

Directions: Write a ratio for each situation described.

4. To make orange juice, combine 1 can of concentrate with 3 cans of water. Write the ratio of concentrate to water.
5. A ski resort advertised that it had 23 snow days during January. Write a ratio of snow days to days without snow (there are 31 days in January).

21st Century Skill
Critical Thinking and Problem Solving

Math offers ways to exercise your creative skills. For instance, in Example 2, you are asked to find the ratio of boys to girls in a class of 30 that has 14 boys. However, what if the ratio of boys to total students was $^{7}/_{15}$?

This fraction of boys tells you two things: 1) the fraction of boys, which was given, and 2) the fraction of girls. Since $\frac{7}{15}$ is boys, the rest is girls. The question asks for the ratio of boys to girls, not girls to total students, which is what $^{8}/_{15}$ represents. But 7 parts of the whole are boys, and 8 parts of the whole are girls. So the ratio is 7: 8. Without even knowing the number of students you can determine the appropriate ratio.

In a notebook, write down the ratio of boys to total students if the ratio of boys to girls is 3: 7.

MATH LINK

A ratio is a comparison of two different quantities. A ratio can be written in fraction form, but a ratio cannot be written as a decimal.

Understand Unit Rates

A **rate** shows a relationship between two quantities measured in different units. Rates are a commonly used type of ratio. A **unit rate** is the rate for one unit of a quantity. To find a unit rate, divide the two numbers in the ratio.

Example 3 Find a Unit Rate

Huy and Kim used 16 gallons of gasoline to drive 500 miles. Find the unit rate of gas mileage (miles per gallon).

Step 1 Write the ratio of miles to gallons: $\frac{500 \text{ miles}}{16 \text{ gallon}}$.
Note that this is a rate, but not a unit rate.

Step 2 Divide so that you have a unit rate.
500 miles ÷ 16 gallons = 31.25 miles to 1 gallon =
31.25 miles per gallon

A **unit price** is the price per unit of an item. Divide to find a unit price.

Example 4 Find a Unit Price

Soap is priced at $1.19 for 4 bars. Find the unit price.

Step 1 Write the ratio of price to number of bars: $\frac{\$1.19}{4 \text{ bars}}$.

Step 2 Divide so that you have a unit price:
$1.19 ÷ 4 bars = $0.2975 for 1 bar.
The unit price is about $0.30 per bar.

To find a unit rate or a unit price, do the following:

- Write a ratio of the two quantities.
- Divide the numerator by the denominator to find the unit rate or unit price.

THINK ABOUT MATH

Directions: Write the unit rate for each situation described.

1. Julietta and Won drove 135 miles and used 6 gallons of gasoline. Find the gas mileage rate of their car (miles per gallon).
2. This week, Ravi worked 20 hours and earned $130. What is his earning rate (dollars per hour)?

Directions: Write each unit price. Round your answer to the nearest cent.

3. 5 pounds of potatoes for $1. 49
4. 12 eggs for $1. 09

Core Skill
Use Ratio and Rate Reasoning

Finding unit rates are extremely useful, especially when it involves money. For instance, suppose you go to the store hoping to buy a shirt for less than $35. However, the store lists the price as "4 shirts for $128. " Do you have enough money? Instead of looking at the rate $\frac{128}{4}$ you would need to see the unit price. Therefore, by multiplying both the numerator and denominator by the scale factor of $\frac{1}{4}$ you would compute to find the unit price per shirt. Therefore, each shirt costs $32 and you would have enough money to buy one shirt.

Consider the following problem. You are taking a trip across the country, and you have borrowed your friend's new fuel-efficient car. You friend claims that the car easily gets 30 miles to the gallon. After driving 420 miles, you fill up the tank with 15 gallons. In a notebook, determine if your friend is correct in his assumption.

Vocabulary Review

Directions: Complete each sentence with the correct word.

rate ratio unit price unit rate

1. A__________ shows a relationship between two quantities measured in different units.

2. To find a________________, divide the price by the number of units.

3. To find the_______________ of gas mileage, you find the number of miles driven per gallon.

4. A_______________can be written with the word *to*, with a colon (:), or as a fraction.

Skill Review

Directions: Match each description of a ratio on the left with its unit rate on the right.

1. _______A store is selling 3 pairs of pants for $72.
2. _______Camille walked 2. 5 miles in a half-hour.
3. _______Francis made $21 in 1. 5 hours.
4. _______ The school club received $36 in dues for every 4 students that joined.
5. _______Sasha baked 147 cupcakes in 7 hours.

A. 9: 1

B. 24: 1

C. 21:1

D. 5:1

E. 14:1

Skill Practice

Directions: Choose the best answer to each question.

1. What is the unit price of shampoo if a 15-ounce bottle costs $2. 79?

   A. about $0. 19 per ounce
   B. about $0. 29 per ounce
   C. about $1. 90 per ounce
   D. about $5. 38 per ounce

2. Which statement has the same meaning as "the ratio of males to females is 1 to 2"?

   A. Half are male, and half are female.
   B. There are twice as many males as females.
   C. Two out of three are female.
   D. One out of three is female.

3. In a local election, Marshall received 582 votes, and Emilio received 366 votes. Which shows a ratio for the number of votes for Emilio to the number of votes for Marshall?

   A. 366 to 948
   B. 97: 61
   C. $^{61}/_{97}$
   D. 588 to 948

4. Briana ran 6 miles in 50 minutes. How many miles per minute did she run?

   A. 0. 083
   B. 0. 12
   C. 1. 2
   D. 8. 3

LESSON 7.2

Unit Rates and Proportional Relationships

Lesson Objectives

You will be able to

- Use unit rates to solve mathematics problems
- Interpret representations of proportional relationships

Skills

- **Core Skill:** Compute Unit Rates Associated with Ratios of Fractions
- **Core Skill:** Evaluate Reasoning

Vocabulary

constant of proportionality
proportional relationship

KEY CONCEPT: A unit rate is a special example of a ratio. When it is expressed in fractional form, the denominator equals one. When expressed verbally, a ratio is an example of a unit rate if the second value being compared is one.

Recall that a ratio is comparison of two values. For example, in a college mathematics class, 20 students may be math majors and 8 students may be majors of a different kind. You can say that the ratio of math majors to nonmath majors is "20 to 8." You can also represent this ratio as a fraction, $\frac{20}{8}$. When you rewrite the fraction in simplest form, the ratio becomes $\frac{5}{2}$. This ratio means that for every five students who are math majors, there are two students who are majoring in a different field.

$20 \text{ to } 8 = \frac{20}{8} = \frac{5}{2}$

What Is a Unit Rate?

Recall that a unit rate is a ratio that is used to compare two different types of quantities. You encounter unit rates every day. You may see a road sign indicating that the speed limit is 30 miles per hour $\left(\frac{30 \text{ miles}}{\text{hour}}\right)$. You may receive a flyer from your local grocery store advertising tomatoes on sale for 69 cents per pound $\left(\frac{69¢}{\text{lb}}\right)$. You may read a financial news report that reports the stock price of a software company is 15 dollars per share.

Not all ratios you hear or read about represent unit rates. What are some examples of ratios that are *not* unit rates? Perhaps you hear a commercial that declares that 8 out of every 10 dentists surveyed recommends a particular brand of toothpaste. Maybe a digital music website announces a special deal encouraging you to buy four songs for $3.00. Or a newspaper cites a statistic that 89 out of every 100 people nationwide passed a newly developed test for getting a driver's license.

Let's express all of these examples in fractional form, as shown in the figure below. Notice that unit rates are always ratios with a denominator of 1. In other words, ratios that are unit rates have 1 unit, such as 1 hour, 1 pound, or 1 share, in their denominators.

Unit Rates	Not Unit Rates
$\frac{\text{30 miles}}{\text{1 hour}}$	$\frac{\text{8 dentists recommend}}{\text{10 dentists surveyed}}$
$=1 \leftarrow \frac{\text{69 cents}}{\text{1 pound}}$	$\neq 1 \leftarrow \frac{\text{3 dollars}}{\text{4 songs}}$
$\frac{\text{15 dollars}}{\text{1 share}}$	$\frac{\text{89 passed test}}{\text{100 took test}}$

Converting Ratios to Unit Rates

Recall the special digital music deal you read about. For $3. 00, you can buy four songs. What if you wanted to convert this ratio into a unit rate? In other words, if you took advantage of this deal, how much would you pay for each song?

To convert this ratio into a unit rate, the denominator must be 1. So, divide both the numerator and denominator in $\frac{3}{4}$ by the denominator, 4.

$$\frac{\text{3 dollars}}{\text{4 songs}} = \frac{\frac{3}{4}\text{ dollars}}{\frac{4}{4}\text{ songs}} = \frac{\text{0.75 dollars}}{\text{1 song}} = \frac{\text{0.75 dollars}}{\text{song}}$$

The unit rate is $0. 75, or 75 cents per song. Notice that when you divide the numerator and denominator by the denominator, you use only the number, not the units (songs) associated with the denominator.

THINK ABOUT MATH

Directions: Let's look at a more complicated ratio, where there is a fraction in the denominator. Your friend tells you that he can run three miles in a half hour. You want to calculate his running speed in miles per hour. The ratio is:

$$\frac{\text{3 miles}}{\frac{1}{2}\text{ hour}}$$

Convert this ratio into a unit rate.

____

____

Core Skill
Compute Unit Rates Associated with Ratios of Fractions

If you know a unit rate, you can use it to generate an infinite number of value pairs. Consider high-definition television (HDTV) screens, for example. These televisions have different shapes from earlier television models. A new HDTV has an "aspect ratio" of 16: 9, or $\frac{16}{9}$. This means that in terms of unit rates, the ratio between the width and height of the screen is $\frac{\text{16 inches}}{\text{9 inches}} = \frac{\text{1. 78 inches}}{\text{1 inch}}$

You can use this information to find the height of common HDTV screens. Complete the chart below. To find each screen height, divide the width by 1. 78. Round your answers to the nearest whole number.

Screen Width (inches)	Calculation	Screen Height (inches)
16	16 ÷ 1. 78	9
41	41 ÷ 1. 78	23
44. 5		
46. 2		

Core Skills
Evaluate Reasoning

When you want to write a ratio that is expressed verbally, be sure that you put the numbers in the correct places. Let's look at an example.

You and a friend are working on a landscaping project. Working as a team, you know that in 3 hours, you can fill 28 wheelbarrows full of soil and move them to another area. To plan for a future landscaping project, you want to determine the unit rate for this ratio, or wheelbarrows per hour.

You perform the following calculations:

$$\frac{3}{28} = \frac{\frac{3}{28}}{\frac{28}{28}} =$$

$$\frac{0.11}{1} = \frac{0.11 \text{ wheelbarrows}}{\text{hour}}$$

This number does not seem right to you. You know that you can fill more than $^{1}/_{10}$ of a wheelbarrow in one hour! How can you determine where you went wrong? Always include units in your calculations, unlike the calculation above. If you had indicated units, you would have seen that you had set up the ratio incorrectly.

$$\frac{3 \text{ hours}}{28 \text{ wheelbarrows}}$$

Using units will help you determine if your calculations make sense. Recalculate the unit rate in wheelbarrows per hour.

Proportional Relationships

Recall that a ratio is a comparison of two values. You can extend this comparison to sets of value pairs that have the same ratio. These sets of data represent a **proportional relationship.**

Let's look at an example. Suppose you are working as a construction planner and need to order cement mix, which is priced by the pound. You receive the pricing chart on the right from a cement company.

Weight (lbs.)	Cost ($)
1, 000	350
2, 000	700
3, 000	1, 050
4, 000	1, 040
5, 000	1, 750

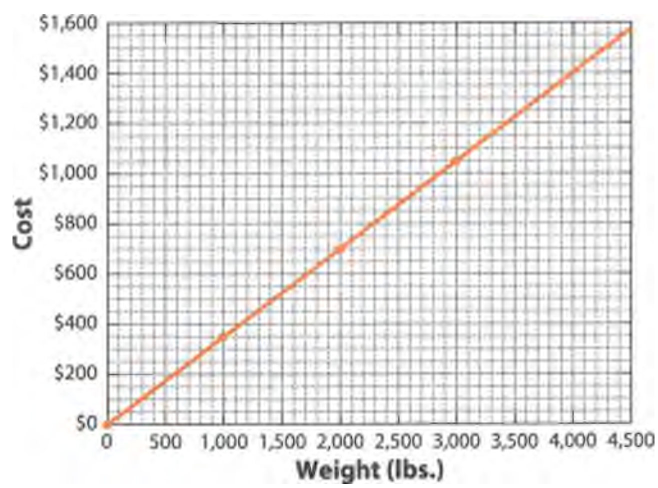
You notice that each time the weight increases by 1, 000 pounds, the price increases proportionally by $350. So, this means that ratio of cost to weight is:

$$\frac{\$350}{1{,}000 \text{ lbs}}$$

... and the unit rate is:

$350/1, 000 lbs. = $0. 35/lb

So, the cement costs 35 cents per pound ($0. 35/lb). Now look at a graph of cost and weight data.



Now, you can determine the equation of this line. The y-intercept on the y-axis is zero, since zero pounds of cement would cost $0. 00. To calculate the slope you can use the points (2, 000, 700) and (3, 000, 1, 050). Recall the formula for slope:

slope = rise/run

So, $\frac{\$1{,}050 - \$700}{3{,}000 \text{ lbs.} - 1{,}000 \text{ lbs.}} = \frac{\$350}{1{,}000 \text{ lbs.}} = \$0.35/\text{lb.}$

Notice that the slope of the line equals the unit rate that you calculated previously. Therefore, the equation of the line is:

$$y = 0.35x$$

This is an example of a proportional relationship. It is a linear equation that has the form:

$$y = kx$$

The y-intercept is zero, and the slope is represented by k, which is called the **constant of proportionality.**

Apply Proportional Relationships

You can use the equation for a proportional relationship, $y = kx$, to solve different types of problems using unit rates. Sometimes, it is helpful to rearrange this equation into the form $y/_x = k$. Remember that k is the unit rate, so this will help you keep track of the units for y and x.

For example, if a unit rate is expressed in kilometers per hour, you can use the equation $y/_x = k$ to tell you the units for y and x.

$$\frac{y}{x} = k$$

$$\frac{y}{x} = \frac{\text{kilometers}}{\text{hours}}$$

Then you know that the units for x are hours and the units for y are kilometers.

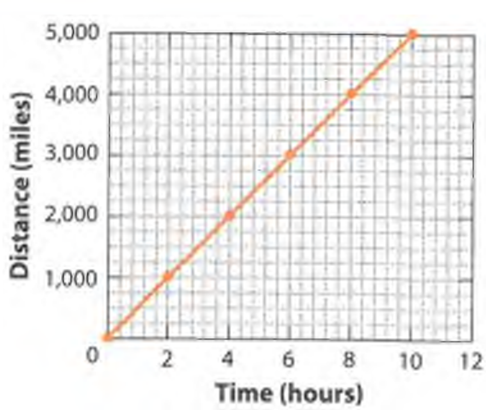
Say your family purchased a pool for the summer. The pool holds 12, 000 gallons of water when it is full. As you fill up the pool, you record the time and the number of gallons marked on the water meter outside your house.

Time (hours)	Gallons
0. 5	600
1. 5	1, 800
2. 5	3, 000
3. 5	4, 200
4. 5	5, 400

You can write and solve a linear equation for a proportional relationship to determine the rate of the water flow, or unit rate, in gallons per hour. If you let x represent the time and y represent the number of gallons in the pool, the linear equation for the proportional relationship is: $y = 1, 200x$

THINK ABOUT MATH

Directions: Use the equation $y = 1, 200x$ to determine how long it will take the pool to fill up to its 12, 000 gallon capacity.



MATH LINK

It may help you find unit rates if you remember that the constant of proportionality in a proportional relationship equals the slope of the line that represents the proportional relationship. So, the constant of proportionality equals the unit rate.

Look at this example. An airline uses two different models of aircraft to make the long flight between New York City and Istanbul, Turkey. The time and distance data for the first aircraft is recorded in a table below. The time and distance data for the second aircraft is recorded in the graph on the bottom of this page.

Time (hours)	Distance (miles)
1	450
3	1, 350
5	2, 250
7	3, 150
9	4, 050
11	4, 950

The speed, or unit rate, of the first aircraft is $\frac{450 \text{ miles}}{\text{hour}}$.

The speed, or unit rate, of the second aircraft is $\frac{500 \text{ miles}}{\text{hr.}}$

The second aircraft travels to Istanbul faster than the first.

Vocabulary Review

Directions: Use the following terms to complete each sentence. Some words may be used more than once.

constant of proportionality proportional relationship

1. A__________ exists between two variables if the ratio between them is always the same.

2. The________________is the value of the ratio between two variables that are proportionally related.

Skill Review

Directions: Choose the best answer to each question.

1. Which of the following is an example of a unit rate?
   - A. A car travels 100 miles in two hours.
   - B. You spent $1. 98 for two pounds of potatoes.
   - C. An Olympic sprinter can run 100 meters in 10. 5 seconds.
   - D. A taxi driver charges $1. 75 per mile.

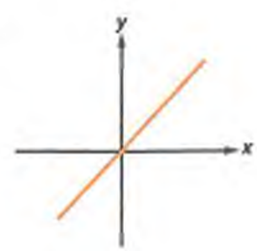
2. You went to a farmer's market where they were offering 3 pints of strawberries for $8. 25. What is the unit rate in dollars per pint?
   - A. 2. 75
   - B. 0. 36
   - C. 3. 00
   - D. 8. 25

3. If a pound of sugar costs $0. 65 and you have $3. 00, how many pounds of sugar can you buy?
   - A. 1. 95
   - B. 3. 65
   - C. 4. 62
   - D. 0. 21

4. Which graph represents a proportional relationship?

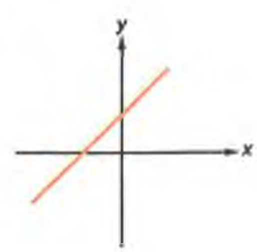
A.

B.



C.

D.



Skill Practice

Directions: Read the problem. Then answer the questions.

1. Imagine that you are looking at a map of the United States, but it doesn't have a scale that indicates the number of miles per inch on the map. You use a ruler to determine the distance between New York City and Washington, DC, on the map. The distance is $2\,{}^{1}/_{2}$ inches. You check a website and find out that the actual distance between New York City and Washington, DC, is 200 miles.

   A. What is the scale of the map in miles per inch (unit rate)?

   ____

   ____

   B. Write the equation for the proportional relationship whose constant proportionality is the scale of the map. Let *y* represent the distance in miles and x represent this distance on the map in inches.

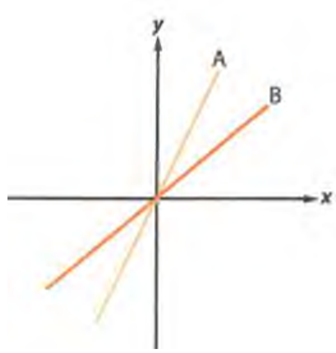
   ____

   C. Using a ruler, you determine that the distance between New York City and Atlanta on the map is 8. 75 inches. What is the actual distance between New York City and Atlanta in miles?

   ____

   ____

2. Which of the lines on the graph represents a proportional relationship with a greater constant of proportionality? Explain your answer.



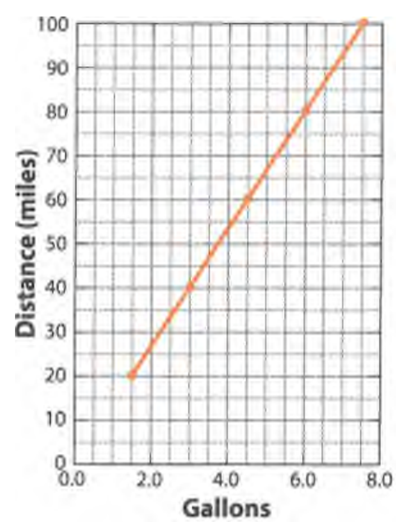
____

____

3. A car company conducts mileage testing for two new models of cars. For model A, the data for distance traveled and the number of gallons of gasoline consumed is shown in the following table.

Distance (miles)	Gallons
30	0. 7
60	1. 4
90	2. 1
120	2. 8
150	3. 5

For model B, the distance traveled and the number of gallons of gasoline consumed is given in the following graph.



What is the gas mileage (unit rate) of each model car in miles per gallon? Which car gets better gas mileage?

Model A: ______________________________

____

Model B: ______________________________

____

LESSON 7.3

Solve Proportions

Lesson Objectives

You will be able to

- Understand and write proportions
- Solve proportions

Skills

- **Core Skill:** Represent Real-World Problems
- **Core Practice:** Build Solution Pathways

Vocabulary

cross-multiplication
equivalent
proportion
value

MATH LINK

Recall that equivalent fractions are fractions that represent the same number, such as $^1/_2$ and $^3/_6$.

KEY CONCEPT: Understand how to use proportions to solve problems.

Write each ratio in two other ways.

1. $\frac{3}{5}$
2. 9:4
3. 5 to 9

Simplify each fraction.

4. $\frac{6}{10}$
5. $\frac{24}{60}$
6. $\frac{6}{26}$

Understand Proportions

A **proportion** is an equation made up of two **equivalent,** or equal, ratios. If you think of ratios as fractions, then the two ratios in a proportion are equivalent fractions.

Example 1 Write a Proportion

Write a proportion using 3 to 4 as one of the ratios.

Step 1 Write the given ratio as a fraction: $\frac{3}{4}$.

Step 2 Write an equivalent fraction.

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

Step 3 Write the proportion.

$$\frac{3}{4} = \frac{6}{8}$$

Sometimes, you can use the idea of equivalent fractions to decide if two ratios form a proportion or not. You are posing the same question if you ask either, "Do the two ratios form a proportion? " or "Are the two fractions equivalent? "

Example 2 Use Equivalent Fractions to Identify a Proportion

Do the two ratios $\frac{4}{10}$ and $\frac{5}{12}$ form a proportion?

Step 1 Find a common denominator: 60.

Step 2 Write each as a fraction with the denominator 60.

$$\frac{4}{10} = \frac{4 \times 6}{10 \times 6} = \frac{24}{60}$$
$$\frac{5}{12} = \frac{5 \times 5}{12 \times 5} = \frac{25}{60}$$

Step 3 Are the two fractions equivalent? No, since $\frac{24}{60} \neq \frac{25}{60}$.
Do the two ratios form a proportion? No.

Another way to see if two ratios form a proportion or not is to compare their two products using **cross-multiplication**. Each product is the numerator of one ratio multiplied by the denominator of the other ratio. If the products of cross-multiplication are equal, the ratios form a proportion. If they are not equal, the ratios do not form a proportion.

Example 3 Use Cross-Multiplication to Identify a Proportion

Do the two ratios $\frac{2}{15}$ and $\frac{3}{16}$ form a proportion?

Step 1 Perform cross-multiplication:
$2 \times 16 = 32$, and
$3 \times 15 = 45$.

Step 2 Are the products equal? No, because $32 \neq 45$.
Do the two ratios form a proportion? No.

In summary:

- A proportion is made up of two equal ratios.
- A product of cross-multiplication is the numerator of one ratio multiplied by the denominator of the other ratio.
- To see if two ratios form a proportion, check to see if the products of cross-multiplication are equal.

MATH LINK

Two ratios that form a proportion must be equivalent. So the two ratios must be equivalent fractions or equivalent rates. Also, their products of cross-multiplication must be equal.

Core Skill
Represent Real-World Problems

The American Flag has a set of specifications that must be met in order to be considered an official flag and can be flown at a governmental building. These specifications govern not only the length and width of the flag but also the distance between the stars and the size of the stars. The hoist fly (width:length) ratio is 10: 19.

Consider the following problem. Vanessa is visiting her statehouse and wants to buy an official flag for her house. The gift shop has official flags, and Vanessa can tell that the flag is 12 inches wide. How long must the flag be in order for it to be official? In a notebook, write down the correct proportion, and then solve it.

THINK ABOUT MATH

Directions: Decide if the two ratios form a proportion. If so, write the proportion.

1. 50 to 20, 10 to 4
2. 8:3, 24:9
3. $\frac{1}{4}, \frac{2}{9}$

Directions: Form a proportion using the ratio given. There is more than one correct answer.

4. 12 to 13
5. 6:3
6. $\frac{25}{20}$

Solve Proportions

When solving a proportion, much like when solving an equation, you find the missing **value,** or amount, of the variable in the proportion. You can use what you have already learned about proportions to solve them.

Example 4 Use Proportions to Solve a Rate Problem

On Monday, Chetan drove 100 miles in 2 hours. On Tuesday, he drove 125 miles in 2 $^1/_2$ hours. Did he drive the same rate on Tuesday as on Monday? If so, write a proportion showing that the two ratios (rates) are equal.

Step 1 Divide to find each rate.

100 miles ÷ 2 hours = 50 miles per hour

125 miles ÷ $2\frac{1}{2}$ hours = 50 miles per hour

Step 2 Answer the question.
Yes, Chetan drove the same rate on Tuesday as on Monday (50 miles per hour).

Step 3 Write a proportion showing that the rates are equal.

$\frac{100}{2} = \frac{125}{2.5}$

Step 4 Use cross-multiplication to check your answer.

$100 \times 2.5 = 250$; $125 \times 2 = 250$

The products are equal, so the rates are equal.

For many problems, you can write and solve a proportion.

Example 5 Solve a Proportion

Solve the proportion for x. $\frac{5}{8} = \frac{x}{10}$

Step 1 Write the cross-products. Since this is a proportion, they are equivalent.

$5(10) = 8 \times x$

$50 = 8x$

Step 2 To solve, divide by 8 on both sides.

$\frac{50}{8} = \frac{8x}{8}$

$6.25 = x$

Step 3 Write the value for x into the proportion, and check using cross-multiplication.

$\frac{5}{8} = \frac{6.25}{10}$; $5 \times 10 = 50$; $6.25 \times 8 = 50$

The products of cross-multiplication are equal, so $x = 6.25$.

Example 6 Write a Proportion to Solve a Problem

A college advertises a 4:5 ratio of male students to female students. If there are about 1,200 male students, how many female students are there?

Step 1 Write a proportion. Make sure the order of the values is the same for each of the two ratios. Let f represent the number of female students.

$\frac{\text{male}}{\text{female}} = \frac{4}{5} = \frac{1{,}200}{f}$

Step 2 Solve the proportion.

$(5 \times 1{,}200) = 4f$

$\frac{6{,}000}{4} = \frac{4f}{4}$

$1{,}500 = f$

Step 3 Check using cross-multiplication in the proportion $\frac{4}{5} = \frac{1{,}200}{1{,}500}$.

$4 \times 1{,}500 = 6{,}000$; $1{,}200 \times 5 = 6{,}000$

Step 4 State your answer.

There are about 1,500 female students.

THINK ABOUT MATH

Directions: Solve the proportion (find the missing value).

1. $\frac{1}{3} = 10a$ 2. $\frac{9}{2} = \frac{b}{12}$ 3. $\frac{c}{42} = \frac{6}{7}$ 4. $\frac{1}{2} = \frac{d}{42}$

Core Practice
Build Solution Pathways

Math often presents more than one way to solve problems. When the problems are complex, it pays to find shortcuts that take you to the correct answer every time. Can you find a pattern in the solutions that quickly lead to the answers? Use that solution pathway when solving similar problems.

Consider Example 4. Are the two ratios equivalent? Once you organize the two ratios, you could go through the process of finding the common denominator for 2 and 2. 5. You still wouldn't be done, because now you'd have to convert both ratios to fractions with the common denominator you've chosen.

There is a simpler way to solve the problem. Cross-multiplication is a one-step method for testing if two ratios are equivalent. In Example 4, you could have jumped directly to Step 3 and then cross-multiplied. Both products, 250, are the same, so you would know immediately the ratios are equivalent.

In a notebook, determine if 12: 50 and 35: 165 are equivalent ratios, or form a proportion.

Vocabulary Review

Directions: Complete each sentence with the correct word,

cross-multiplication equivalent proportion value

1. If the products of________________ are equal, the proportion is true.
2. Two fractions that name the same value are called___________________ fractions.
3. The missing________________that needs to be found in a proportion is indicated by a variable.
4. A(n)_______________is a relationship of equivalency between two ratios.

Skill Review

Directions: Show four ways a proportion can be written for the data given in each of the following:

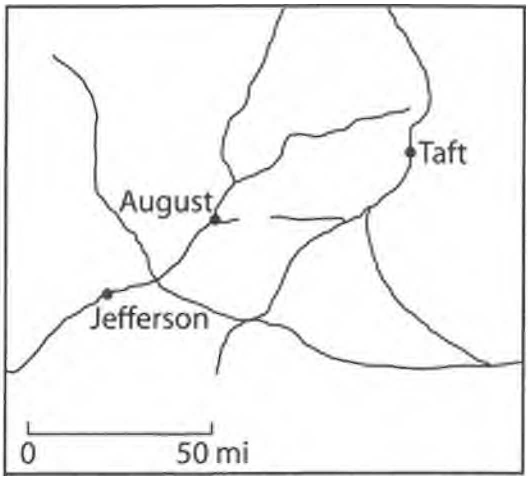
1. 5 circles for every 8 triangles and 10 circles for every 16 triangles
2. $2 for 7 miles and $6 for 21 miles
3. 80 seeds every 15 feet and 32 seeds every 6 feet
4. 7 cups of flour for every 5 tablespoons of sugar and 10. 5 cups of flour for every 7. 5 tablespoons of sugar
5. 72 chairs for 8 tables and 126 chairs for 14 tables

Directions: Solve each problem.

6. Photographs can measure either 3 inches by 5 inches or 4 inches by 6 inches. Do the ratios of $\frac{\text{width}}{\text{length}}$ form a proportion? Show your work.
7. Guadelupe mixed three parts of blue paint with one part of yellow paint to make green paint. Asura mixed six parts of blue paint with two parts of yellow paint to make green paint. Will the two shades of green paint be the same? Why or why not?
8. Sanaye typed 650 words in 10 minutes. Rashid typed 780 words in 12 minutes. Do the two rates form a proportion? Are their typing speeds the same? Explain.

Skill Practice

Directions: Choose the best answer to each question.

1. 

The scale on a highway map is 1 inch: 50 mites. What is the distance in miles between Jefferson and Taft, which are 2 $^1/_2$ inches apart on the map?

A. 20

B. 52 $^1/_2$

C. 100 $^1/_2$

D. 125

2. Jakob earned $100 mowing 8 lawns. Which proportion can be solved to determine how much he will earn mowing 10 lawns at this rate?

A. $\frac{100}{10} = \frac{d}{8}$

B. $\frac{10}{d} = \frac{100}{8}$

C. $\frac{8}{10} = \frac{d}{100}$

D. $\frac{100}{8} = \frac{d}{10}$

3. Which statement is true of the proportion $^2/_3$ = $^{10}/_{15}$

A. 2: 3 and 15: 10 are equivalent ratios.

B. $^2/_3$ and $\frac{10}{15}$ are equivalent fractions.

C. 2x10 and 3 x 15 are equal.

D. It is not a true proportion.

4. The ratio of rock songs to pop songs on Jason's mp3 player is 3: 8. If the number of pop songs he has on his player is 400, how many rock songs does he have?

A. 50

B. 100

C. 150

D. 300

LESSON
7.4

Introduction to Percents

Lesson Objectives

You will be able to

- Understand and write percents
- Change fractions to decimals and decimals to fractions
- Change fractions to percents and percents to fractions
- Change decimals to percents and percents to decimals

Skills

- **Core Skill:** Interpret Data Displays
- **Core Practice:** Construct Viable Arguments

Vocabulary

percent
repeating decimal
similarity

KEY CONCEPT: Percents, like decimals and fractions, represent part of a whole.

Divide.

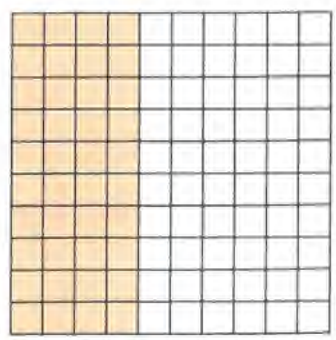
1. $230 \div 5$
2. $23 \div 5$
3. $2.3 \div 5$
4. $0.23 \div 5$

Understand Percents

A **percent** is another means of expressing a number as part of a whole. The word *percent* means "for each 100." For example, you can write the ratio 3 to 100 as 3% and read it as 3 percent.

Example 1 Use Percents

The 10-by-10 grid below has 100 squares. What percent of the grid is shaded?



Step 1 Count the number of shaded squares: 40.

Step 2 Write a percent.
40 of the 100 squares are shaded: 40%.

Step 3 Summarize: 40% of the grid is shaded.

COMPARE AND CONTRAST

Writers use a **comparison** when they want to examine a **similarity,** or the way two or more people, things, or ideas are alike. Writers use **contrast** to look at the differences between people, things, or ideas. In comparing items that are alike, writers use terms such as *similar, both*, or *like*. When contrasting unlike things, terms such as *different, but, in contrast,* and *instead of* are often used.

Read the following paragraphs. As you read, look for the similarities and differences between percents and fractions.

> Fractions are a way of comparing a part to a whole. The whole can be divided into any number of equal parts. This number is used as the denominator. A certain number of the parts are compared to the whole. This is the numerator. Operations such as adding, subtracting, multiplying, and dividing can be performed on fractions. Fractions are written with a slash (1/2) or with a fraction bar $^1/_2$
>
> Percents are another way to compare a part to a whole. *Percent* means “per hundred. ” Unlike fractions, percents always have the same number of equal parts: 100. The number out of 100 that is being specified is the percent. Percents can also have operations, such as addition, subtraction, multiplication, and division, performed on them. One hundred percent is a whole. Percents are written with a percent sign (%) or the word *percent.*

Similarities	Differences
both compare parts to a whole operations performed in both both can be greater or less than one	fractions: any number of equal parts percents: always 100 equal parts fractions: written with a slash or fraction bar percents: written with a percent sign (%) or the word *percent*

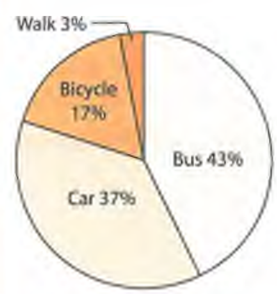
The writer has given the same type of information for both percents and fractions—what they are, whether operations can be performed on them, and how they are written. You must decide when the information about the items is the same or is different.

Core Skill
Interpret Data Displays

You have learned that data can be displayed in many different ways: in tables, in number lines, and in diagrams. Percents also can be displayed visually. A circle graph shows parts of a whole. The whole looks like a pizza, and the parts look like the slices. The size of each slice represents the percent part of the whole that is assigned to that slice.

Monica takes a poll of her class to find out how the students get to school every day. Here are the results of her poll: 43 percent take the school bus, 37 percent are driven to school, 17 percent ride a bicycle, and 3 percent walk. Monica represents this information in the following circle graph. If the labels in the circle graph didn't contain percentages, could you tell just by looking at it whether the majority of the polled students took a bus to school? Explain your answer.

TRANSPORTATION TO SCHOOL



Percents as Fractions

Because a percent is a form of a ratio and because you can write any ratio as a fraction, you can write a percent as a fraction.

Example 2 Write Percents as Fractions

Write 14% as a fraction.

Step 1 Write the percent as a ratio that compares the number to 100 (a fraction with denominator 100).

$$14\% = 14 \text{ of } 100 = \frac{14}{100}$$

Step 2 Write the fraction in lowest terms.

$$\frac{14}{100} = \frac{14 \div 2}{100 \div 2} = \frac{7}{50}$$

Step 3 Summarize: $14\% = \frac{7}{50}$

Percents as Decimals

Notice from Step 1 of Example 2 that 14% is equal to 14 of 100, or 14 hundredths. Recall that 0.14 is read *14 hundredths*. This is helpful information to remember when you want to write a percent as a decimal.

Example 3 Write Percents as Decimals

Write 35% as a decimal.

Step 1 Write the percent as a ratio that compares the number to 100 (a fraction with denominator 100).

$$35\% = 35 \text{ of } 100 = \frac{35}{100}$$

Step 2 Write the fraction as a decimal.

$$\frac{35}{100} = \text{thirty-five hundredths} = 0.35$$

Step 3 Summarize: $35\% = 0.35$

THINK ABOUT MATH

1. Write 67% as a decimal. ______

2. Convert 346% to a fraction. ______

3. Write 3% as a decimal. ______

4. Write 40% as a fraction. ______

Core Practice
Construct Viable Arguments

One of the most understood uses of percentages is in tipping at restaurants. Because waiters and waitresses are classified differently than kitchen workers at restaurants, they rely on tipping as part of their wages. Depending on the type of restaurant, you might have an automatic tip added to your bill if you have eight or more people eating at your table.

Consider the following scene. Ginger and her friend Marlo go to a restaurant and afterward get a bill of \$30. Mario wants to tip 15%, and she places \$15 on the table for the tip. However, Ginger says that \$15 is too much because $15/30 = 1/2 = 50\%$. Ginger then corrects Mario by saying that 15% of \$30 can easily be computed by using the facts that 15 = 10 + 5 and that 5 is half of 10.

In a notebook, use the facts Ginger provided to determine the correct amount of tip Marlo should place on the table instead of \$15.

Decimals as Percents and Fractions

Just as percents can be written as decimals, decimals can be written as percents. To change a decimal to a percent, simply move the decimal point two places to the right.

Example 4 Write Decimals as Percents

Write 0. 6 as a percent.

Step 1 To write 0. 6 as a percent, multiply it by $\frac{100}{100}$.

$$0.6 \times \frac{100}{100} = \frac{0.6 \times 100}{100} = \frac{60}{100} = 60\%$$

Step 2 Summarize: 0. 6 = 60%

Decimals can also be written as fractions or mixed numbers.

Example 5 Write Decimals as Fractions

Write 0. 6 as a fraction.

Step 1 To change 0. 6 to a fraction, write the decimal as a fraction with a denominator that is a power of 10. (10, 100, 1, 000,...)

You can also write the decimal as it is read out loud.

0. 6------> *6 tenths*--------> 6/10

Step 2 Then reduce to lowest terms.

$$0.6 = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

Example 6 Write Decimals as Mixed Numbers

Write 1. 76 as a fraction.

$$1.76 = 1\frac{76}{100} = 1\frac{76 \div 4}{100 \div 4} = 1\frac{19}{25}$$

Fractions as Decimals and Percents

Just as a percent can be written as a fraction and a decimal, a fraction can be written as a percent. To do this, you must first write the fraction as a decimal.

Example 7 Write Fractions as Decimals

Write 5/8 as a decimal.

To change | to a decimal, divide the numerator by the denominator until you get a remainder of 0. Add a decimal point and zeros as needed.

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -4\,8 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

$^5/_8$ = 0. 625

MATH LINK

Remember $\frac{100}{100}$ is equal to 1. So in Example 4, 0. 6 is actually being multiplied by 1 in the form of $^{100}/_{100}$ The short a decimal to a percent is to move the decimal point two places to the right (multiply by 100) and to write the percent sign (divide by 100). So, for example, 0. 13 = 13%.

MATH LINK

The three dots seen at the end of the decimal 0. 7777... are the symbol for *continues in this pattern*. The notation 0. $\overline{7}$ can also be used. A bar over digits means those digits repeat infinitely.

If two or more digits repeat in the same pattern, place the bar over all of the digits that repeat: 11 = 0. $\overline{09}$, 12 = 0. $\overline{83}$, and 7 = 0. $\overline{142857}$. When dividing on a calculator, the last digit in the display may be rounded up. $^2/_3$ will be displayed as 0. 6666667 because the 6 rounds to a 7. This does not mean that the decimal stops repeating, only that the space available for displaying the answer is limited.

MATH LINK

Do not be confused when you see a percent that already contains a decimal. Remember to move the decimal point two places to the left to change a percent to a decimal.

0. 1% is not the same as 0. 1.

0. 1% = 0. 001

10% = 0. 1

Example 8 Write Fractions as Percents

Write $^5/_8$ as a percent.

Step 1 First write | as a decimal (see Example 7).

$^5/_8 = 0.625$

Step 2 Move the decimal point two places to the right by multiplying by 100. Add a percent sign.

| = 0. 625 = 62. 5%

Sometimes when changing a fraction to a decimal, no matter how many zeros you add in the dividend, the answer either does not end or repeats one or more digits over and over. A **repeating decimal** is when digits are repeated over and over.

Example 9 Fractions as Repeating Decimals and Percents

Write | as a decimal and as a percent.

Step 1 Divide the numerator by the denominator.

7d9 = 0. 77f or 0. 777...

Step 2 To write as a percent, move the decimal two places to the right and write a percent sign:

77f %

0.777
9)7.000
−6 3
70
−63
70
−63
7

THINK ABOUT MATH

Directions: Write each number as a percent, decimal, and fraction.

1. 3/12	3. 0. 068	5. 37%
2. $^2/_3$	4. 2. 4	6. 0. 2%

Vocabulary Review

Directions: Complete each sentence with the correct word.

percent repeating decimal similarity

1. __________is a word that means "of 100. "

2. A--------------------between two items is the way in which the two items are the same.

3. A--------------------has one or more digits that repeat without end.

Skill Review

Directions: The table below shows how a number in each row is written as a percent, decimal, and fraction. Complete each column for the number given in each row. Identify which representations are similar and are easy to change between the two forms.

	Percent	Decimal	Fraction
1.	45%		
2.		0. 8	
3.			$\frac{7}{20}$
4.		2. 06	
5.	24. 1%		
6.			$1\,{}^{3}/_{8}$

Directions: Write a sentence that compares and contrasts each of the following pairs of numbers.

7. 0. 75 and ${}^{3}/_{4}$

8. 7| and $7.8\overline{3}$

9. ${}^{2}/_{3}$ and 66. 667%

10. ${}^{1}/_{9}$ and O1...

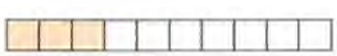
Skill Practice

Directions: Choose the best answer to each question.

1. A band hopes that 2% of the people attending its concert will purchase a CD. Which fraction is the same as 2%?

   A. $\frac{1}{5}$
   B. $\frac{1}{50}$
   C. $\frac{100}{2}$
   D. $\frac{20}{100}$

2. Juan is tiling a hallway in his home. The diagram below shows how much of the floor he has completed. Which percent represents how much of the hallway Juan has completed?



   A. 3%
   B. 7%
   C. 30%
   D. 70%

3. Lishan said that she completed only ${}^{4}/_{5}$ of her math test. What percent of her math test did she complete?

   A. 45%
   B. 80%
   C. 125%
   D. Not enough information is given.

4. Tomas ran 12 minutes out of the 30 minutes he had hoped to run. What percent of the minutes he had hoped to run did Tomas actually run?

   A. 2. 5%
   B. 12%
   C. 18%
   D. 40%

LESSON
7.5 Solve Percent Problems

Lesson Objectives

You will be able to

- Write percents as either decimals or fractions to solve problems
- Use proportions to solve percent problems

Skills

- **Core Skill:** Evaluate Reasoning
- **Core Skill:** Use Percents

Vocabulary

extremes
means
portion

MATH LINK

Remember, when you convert a percent to a decimal, move the decimal point two places to the right.

KEY CONCEPT: Decimals, fractions, and proportions can be used to solve percent problems.

Multiply.

1. 4.3 x 19 2. 0.35 x 8 3. 28 x 1/4 4. 600 x $^1/_5$

Solve for the missing part of each proportion.

5. $\frac{3}{4} = \frac{\square}{28}$ 6. $\frac{\square}{10} = \frac{5}{25}$ 7. $\frac{6}{4} = \frac{9}{\square}$ 8. $\frac{60}{\square} = \frac{25}{100}$

Percent of a Number

A percent is a **portion** (part) of a number. Often, it is necessary to figure out what that portion is. For example, if you decide to leave a 20% tip at a restaurant, you need to calculate the amount of money that is 20% of the total bill.

Example 1 Use a Percent as a Decimal

What is 20% of $25?

Step 1 Write the percent as a decimal.
20% = 0.20

Step 2 Multiply.
0.20 x $25 = $5.00

Step 3 Summarize.
20% of $25 is $5.00.

To calculate with percents, you first need to change the percent to either a decimal or a fraction. Example 1 uses a decimal, and Example 2 uses a fraction. You may use whichever form of the number is easier for you to work with, because both forms will give you the same answer.

Example 2 Use a Percent as a Fraction

What is 25% of $400?

Step 1 Write the percent as a fraction.

$$25\% = \frac{25}{100} = \frac{1}{4}$$

Step 2 Multiply.

$\frac{1}{4}$ x $400 = $100

Step 3 Summarize.

25% of $400 is $100.

Some problems require you to find the percent of a number. Be sure to read the problem carefully. With percent problems, look for the word *of* since most percent problems ask you to find the percent *of* a number. When the answer you need is the percent of a number, multiply to find the answer.

Example 3 Percent of a Number

A state official predicts that 70% of the registered voters in a state will vote on election day. If there are 514, 000 registered voters in that state, how many are predicted to vote?

Step 1 Understand the question.

You must find how many of the 514, 000 registered voters are predicted to vote on election day.

Step 2 Decide what information is needed.

Use the percent of voters predicted to vote: 70%.

Also, use the number of registered voters: 514, 000.

Step 3 Choose the most appropriate operation.

This problem asks, "What is 70% of 514, 000? "

The operation to use is multiplication.

Step 4 Solve the problem.

70% of 514, 000 = 0. 70 x 514, 000 = 359, 800 voters

Step 5 Check your answer.

To check, see if the ratio 359,800/514,000 equals 70%.

359, 800 / 514, 000 = 0. 7 = 70%

So 359, 800 voters are predicted to vote.

THINK ABOUT MATH

Directions: Calculate each of the following.

1. 10% of 870
2. 47% of 1, 000
3. 75% of 16
4. 50% of $50

Core Skill
Evaluate Reasoning

Percents are one of the most important topics to understand because they are used everyday. From shopping sales at a store to tipping at restaurants, percents show up in all places. Being able to calculate percentages is a valuable tool you can have to make sure you are getting the correct price or tipping the correct amount.

Consider the following scenario. Silvia and Sandra are shopping at a local department store. Silvia sees a sign that says 40% off a pair of $60 jeans. She sets up the following proportion and solves it.

$\frac{x}{60} = \frac{40}{100}$

$x = 60 \times {}^{40}/\frac{40}{100}$

$x = 24$

She decides that the jeans now cost $24 and decides to buy them. In a notebook, determine if Silvia is correct. If not, explain what she did wrong and what the sale cost would have been.

Use Proportions to Solve Percent Problems

One way to solve percent problems is to use proportions (two equal ratios). Solve a proportion with the percent written as a ratio comparing a number to 100.

$$\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$$

By solving the proportion, you find the missing value: the part, the whole, or the percent. 100 is always in the proportion. Be careful to write the proportion correctly before you solve it. After solving the proportion, multiply to check your result.

Example 5 Find the Part

What is 30% of 90?

Step 1 Identify the part, the whole, and the percent.
Part: (missing) Whole: 90 Percent: 30

Step 2 Write the proportion $\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$.
$\frac{\square}{90} = \frac{30}{100}$

Step 3 Solve the proportion.
$(90 \times 30) \div 100 = 27$

Step 4 Summarize.
30% of 90 is 27.

Step 5 Use multiplication to check your answer.
30% of 90 $= 0.3 \times 90 = 27$

You are familiar with finding an arithmetic mean, or average. Say, for example, that you take two tests, and your scores are 84 and 92.

(Score 1 + Score 2) $\div$ 2 = the arithmetic mean
$(84 + 92) \div 2 = 176 \div 2 = 88$
Your average test score is 88.

There is another kind of mean, too, but it is not arithmetic. It is geometric, or a **geometric mean**. You can multiply any number of positive values (n) and find the *nth* root of the product to find the geometric mean.

For example, imagine multiplying two positive values, x and y. Find the geometric mean of x and y, when $x = 9$ and $y = 16$.

$9 \times 16 = 144$
$\sqrt{144} = 12$

When $x = 4$ and $y = 36$, the same geometric mean is also found because $4 \times 36 = 144$.

Now, apply what you know to write a proportion between the two pairs of numbers.

$\frac{4}{9} = \frac{16}{36}$

In the proportion, the numbers 9 and 16 represent the **means** because they are closest to the geometric mean (12). The numbers 4 and 36 represent the **extremes**, or numbers farthest from the mean.

Scientists often calculate geometric means to predict exponential growth. Exponential growth occurs more and more quickly over time, as a population of living things increases. For example, scientists can use geometric means to predict global human population growth over decades.

Example 6 Find the Percent

What percent of 90 is 18?

Step 1 Identify the part, the whole, and the percent.
Part: 18   Whole: 90   Percent: (missing)

Step 2 Write the proportion $\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$.
$\frac{18}{90} = \frac{\square}{100}$

Step 3 Solve the proportion.
$(18 \times 100) \div 90 = 20$

Step 4 Summarize.
20% of 90 is 18.

Step 5 Use multiplication to check your answer.
$20\% \text{ of } 90 = 0.2 \times 90 = 18$

Example 7 Find the Whole

80% of what number is 20?

Step 1 Identify the part, the whole, and the percent.
Part: 20   Whole: (missing)   Percent: 80

Step 2 Write the proportion $\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$.
$\frac{20}{\square} = \frac{80}{100}$

Step 3 Solve the proportion.
$(20 \times 100) \div 80 = 25$

Step 4 Summarize.
80% of 25 is 20.

Step 5 Use multiplication to check your answer.
$80\% \text{ of } 25 = 0.8 \times 25 = 20$

To solve a percent problem, set up a ratio, $\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$, comparing a number to 100. Insert each value that you are given into the ratio, and then solve for the missing value.

THINK ABOUT MATH

Directions: Answer each question.

1. What is 20% of 45?
2. What percent of 40 is 25?
3. 42% of what number is 14. 7?

Core Skill
Use Percents

You use percents everywhere—even when you're not aware of them. The tax you pay on things you buy, for example, is a percentage of the purchase price. The tip you give the server in a restaurant is a percentage of the cost of a meal. Baseball batting averages also are a percentage; you calculate a batting average by dividing the number of hits the player gets by the number of times a player comes to the plate.

The key to finding the percent in every problem is looking for the symbol % or the word *percent*. The whole is right after the word *of*. When attempting to solve such problems, therefore, you should begin by first identifying these three elements. In the problems below, the part is circled, the whole is underlined, and the percent is boxed.

$\boxed{50\%}$ of 62 is what number?

3 is $\boxed{75\%}$ of what number?

12 is $\boxed{\text{what percent}}$ of 24?

In a notebook, copy the questions below. Circle the part, underline the whole, and put a box around the percent. Then write the proportion $\frac{\text{Part}}{\text{whole}} = \frac{\%}{100}$ for each.

6 is 25% of what number?
17 is what percent of 51?
What is 100% of 49?

Vocabulary Review

Directions: Complete each sentence with the correct word.

extremes means portion

1. A percent is one way to represent a(n)______________ of a whole.

2. The two numbers in a proportion closest to the geometric mean are the_________________

3. The two numbers in a proportion furthest from the geometric mean are the ________________

Skill Review

Directions: Circle the *part,* underline the *whole,* and put a box around the *percent* in each problem below. Write the proportion $\frac{\text{part}}{\text{whole}} =$ ^ for each. Then answer the question.

1. What number is 25% of 80?

2. 10% of what number is 8?

3. What percent of 44 is 11?

4. 9 is what percent of 100?

5. 16% of what number is 200?

6. 3% of 500 is what number?

7. 7 is 1% of what number?

8. What percent of 16 is 12?

9. Annabelle drank 340 cups of coffee in one year. She drank 73 cups during January alone. What percent of cups of coffee did Annabelle drink during January?

10. Lucio received 52% of the votes to win an election. There were 215, 400 voters. How many people voted for Lucio?

11. Panya bought some mittens on sale for $12. She paid only 80% of the original price. What was the original price of the mittens? How much money did she save by buying the mittens on sale?

Skill Practice

Directions: Choose the best answer to each question.

1. The sales tax rate in Marissa's home town is 6%. If she purchased a new car for $12, 500, how much will she pay in tax?

   A. $75
   B. $208. 33
   C. $750
   D. $2, 083. 33

2. Delfina is buying an $80 dress for 25% off. What calculation can help her to find how much less she will be paying for the dress?

   A. ^ x $80
   B. \ x $80
   C. \ x $80
   D. \ + $80

3. To pass an exam, Kamol must get at least 70% of the 30 problems correct. How many problems must he get correct to pass?

   A. 9
   B. 10
   C. 15
   D. 21

4. A newborn baby sleeps an average of 16 hours in a 24-hour period. What percent of its time does a newborn baby spend sleeping?

   A. about 15%
   B. about 38%
   C. about 67%
   D. about 75%

LESSON

7. 6

Use Percents in the Real World

Lesson Objectives

You will be able to

- Understand the interest formula
- Use a formula to find simple interest

Skills

- **Core Skill:** Make Sense of Problems
- **Core Skill:** Solve Real-World Arithmetic Problems

Vocabulary

convert
formula
interest
principal
rate
time

Core Skill

Make Sense of Problems

Because both the interest rate and time in the simple interest formula deal with time, both time units must be the same, whether it is in months, years, or centuries!

Suppose you deposit $100 into a 10% annual savings account, but pays interest monthly. In 12 months, you would have $10, but multiplying 100 x 0. 1 x 12 = $120! This happened because the time units were different.

In a notebook, figure out the monthly interest rate for the account.

KEY CONCEPT: Simple interest can be calculated using a formula and percents.

Write each ratio as a fraction.

1. 28 to 52   2. 55 to 365   3. 9 to 12

Multiply.

4. $500 x 0. 055   5. 0. 08 x \   6. 20 x 0. 3 x Y 12

Simple Interest Problems

Interest is money earned by an investment or paid when money is borrowed. Simple interest is the most basic type of interest. In simple interest, the amount of interest is calculated only on the principal. In other words, interest is not calculated on previous interest. The amount of simple interest depends upon three things: principal, rate, and time. Both the rate (%/time) and time have time units attached to them and must be the same in order for the formula to calculate interest correctly. You can use the formula...

Interest = Principal x Rate x Time

The **principal** is the amount of money invested or borrowed. The **rate** is the annual interest rate, usually given as a percent. The **time** is the length of time (in years) the money is invested or borrowed.

Example 1 Calculate Simple Interest with Time in Years

Laurel put $1, 000 into a savings account. The bank pays 3% simple interest annually on this account. How much interest will Laurel earn if she leaves the money in the account for 2 years?

Step 1 Identify the principal, the rate, and the time.
Principal: $1, 000 Rate: 3% = 0. 03 Time: 2 years

Step 2 Multiply.
Interest = Principal x Rate x Time
= $1, 000 x 0. 03 x 2
= $60

Step 3 Summarize.
Laurel will earn $60 in interest.

When the interest owed or earned is over a time that is not stated in years, you must **convert,** or change, it into years. Remember, there are 365 days (except for leap year), 12 months, or 52 weeks in a year.

Example 2 Convert Time to Years to Calculate Interest

A credit card company charges 12% annual interest on any balance due. If the balance due on Rafael's credit card is $150 and he waits 30 days to pay the bill, how much interest will he owe?

Step 1 Identify the principal, the rate, and the time.
Principal: $150 Rate: 12% = 0.12 Time: 30 days

Step 2 Write a ratio to convert the time to years.
There are 365 days in one year, so 30 out of 365 days is $\frac{30}{365}$ year.

Step 3 Multiply.
Interest = Principal × Rate × Time
$= \$150 \times 0.12 \times \frac{30}{365}$
$= \$1.479452055$ or $1.48

Step 4 Summarize.
Rafael will owe $1.48 in interest.

The amount of the interest depends upon three things: principal, rate, and time (in years). Use the formula, Interest = Principal × Rate × Time, to calculate interest. Any time not given in years must be converted to years. This assumes that the the rate given is per year. If the rate is a monthly rate, then time would need to be given in months. The same is true for any type of rate.

THINK ABOUT MATH

Directions: Calculate simple interest. If necessary, round to the nearest cent.

1. principal = $10, 000
   rate = 8%
   time = 5 years
2. principal = $2, 500
   rate = 4%
   time = 6 months
3. principal = $120, 000
   rate = 9%
   time = 30 years
4. principal = $600
   rate = 5. 5%
   time = 100 days

Core Skill
Solve Real-World Problems

When your parents or someone you know takes out a loan to buy a car, they pay interest on the money they borrowed. The bank pays you interest on money you put in a savings account. In each case, the interest that is paid is generally a percentage of the money that is borrowed or saved. This is another place where "percent" plays a role in everyday life.

When solving real-world interest problems, you are applying what you have already learned during your studies of ratios, percents, and multiplication with whole numbers, decimals, and fractions. Now, however, you are putting those skills to new use.

Look at the numbers being multiplied in Example 1: $1, 000 x 0. 03 x 2. In this example, which describes a real-world situation, a percent is changed to a decimal and then multiplied by whole numbers.

In a notebook, determine the simple interest rate on an account that paid $50 on a $1000 principal over 2 years.

MATH
LINK

Compound interest can be thought of as earning interest on interest. It is found by adding earned interest to the principal. It is the interest typically used by banks and investment institutions.

Vocabulary Review

Directions: Complete each sentence with the correct word,

convert formula interest principal rate time

1. The__________ is the amount of money invested or borrowed.
2. The length of_________________ is the number of years the money is being borrowed.
3. One________________ can be used to find the area of a square, while another is used to calculate interest owed.
4. You can________________ a percent to a decimal or a fraction.
5. The________________ of the interest owed is usually given as a percent.
6. ______________ can either be owed or earned.

Skill Review

Directions: Solve each problem. Apply information about percents, the interest formula, and problem solving that you learned in this lesson and previous lessons.

1. Find the interest earned in 1 year on a principal of $2, 000 that pays 10% annual interest.
2. Find the interest earned in 100 days on a principal of $5, 000 that pays 11% annual interest.
3. Jonah bought a car priced at $18, 500. He made a 10% down payment and borrowed the rest of the money at a 9% annual interest rate. He will pay the balance due in five years. How much interest will he pay?
4. On her eighteenth birthday, Enriqua put $1, 500 into a savings account. The annual interest rate on the account is 6%. How much money will she have in the account on her twenty-first birthday?

Skill Practice

Directions: Choose the best answer to each question.

1. Hye Su put $4, 000 into a savings account that pays 4% annual interest. How much interest will she earn in 3 years?

   A. $48
   B. $480
   C. $4, 800
   D. $48, 000

2. Jamil borrowed $75, 000 from a mortgage company to buy a house. He will repay the loan at 8% annual interest in 30 years. How much will he pay the mortgage company in interest?

   A. $180, 000
   B. $18, 000
   C. $1, 800
   D. $180

3. Tanya is comparing two loans. With loan A, she will pay 5% simple interest for 26 weeks. With loan B, she will pay 6. 5% simple interest for 18 weeks. She will borrow $12, 500. Which expresses the difference in what she will pay in interest on the two loans?

   A. Loan A costs $31. 25 more than loanB.
   B. Loan B costs $31. 25 more than loanA.
   C. Loan A costs $1, 625 more than loanB.
   D. Loan B costs $1, 625 more than loanA

4. A credit card company charges 24% annual interest on any balance due. If the balance due on Bette's credit card is $100, and she waits 60 days to pay the bill, about how much interest will she owe?

   A. $1. 20
   B. $3. 95
   C. $14. 40
   D. $39. 50