Chapter 6

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Functions

Factories produce diverse items from cars to loaves of bread. A production line on a factory floor begins with raw materials and ends with a fully packaged product, ready for shipping to the customer. In a similar way, a function has an input, and certain operations are performed on this input value to yield an output value. In this chapter you will understand how to work with functions, graphing, and identifying key features.

Lesson 6.1

Identify a Function

Every square is a rectangle, but not every rectangle is a square. Similarly every function is an equation, but not every equation defines a function. Learn how to identify a function using the vertical line test.

Lesson 6.2

Identify Linear and Quadratic Functions

You learned methods for factoring quadratic equations to find solutions. Those solutions correspond to x-intercepts on the graph of quadratic functions. Learn how to identify linear and quadratic functions by analyzing consecutive differences.

Lesson 6.3

Identify Key Features of a Graph

What are some important features of a function? How can you identify those features in a graph? What do those features translate to in the equation of the function? Learn to identify key features of a function in tables, graphs, and equations.

Lesson 6.4

Compare Functions

When solving a real-world problem, what is your first step? Do you make a table, graph, or equation? How do you decide which type will be most appropriate? Learn techniques for using multiple representations to solve real-world problems.

Goal Setting

Think about solving a linear equation. What steps do you follow? Did you graph the equation or solve the equation algebraically? What information is easy to tell from an equation that is not immediately found on a graph? What information is easy to find from a graph?

How do you solve a quadratic equation algebraically? How might graphing the quadratic equation help you solve the problem?

LESSON 6.1 Identify a Function

LESSON OBJECTIVES

- Recognize a function as a table of values, a graph, an equation, and in the context of a scenario
- Evaluate linear and quadratic functions
- Plot points in a coordinate plane

CORE SKILLS & PRACTICES

- Use Math Tools Appropriately
- Solve Real-World Problems

Key Terms

function
a rule that assigns exactly one output to each input

linear function
a function that can be written in the form $f(x) = mx + b$, where m and b are constants, whose graph is a non-vertical line

quadratic function
a polynomial that has 2 as its highest power of x

Vocabulary

domain
the set of inputs of a function

one-to-one function
a function for which every value in the range has exactly one element assigned to it from the domain

range
the set of outputs of a function

Key Concept

A function assigns exactly one output for each input. The inputs of a function are a given set, and the outputs for this function create another set. The outputs are what the function did to the set of inputs. A good way to identify a function is to use the Vertical Line Test.

Functions

Functions are used in physics, environmental science, biology, economics, business, and finance. They are used in every field of study. Interpreting the output of functions tells us a lot about our world!

Function and Its Purpose

A **function** is a rule that assigns exactly one output to each input. The inputs and the outputs can be represented by sets. An input is an element from a set called the **domain**. An output is an element from a set called the **range**. We can represent a function using the symbol $f(x)$.

$$f(x) = x^2$$

function name — input — output

Functions allow people to see unique relationships both numerically and graphically.

Tables of Values

We can use tables of values to represent functions. A table represents a function if there is exactly one value, $f(x)$, in the range for each value, x, in the domain.

The table shows a function in which every domain value is multiplied by 2. The results are the range.

Domain x	Range $f(x)$
1	2
2	4
3	6
4	8

$f(x) = 2x$

Example 1: Identifying a Function from a Table

Tell whether each table represents a function.

a.

Domain	Range
−1	4
0	0
1	4
2	8

b.

Domain	Range
0	0
1	1
4	−2
4	2

Check whether each domain value has exactly one range value.

a. Each domain value has exactly one range value. Even though 4 appears twice in the range, the domain value −1 has only one range value, and 1 has only one range value.

b. The domain value 4 has two range values, −2 and 2. This table does not represent a function.

The table in Example 1a represents a function, but it does not represent a **one-to-one function**. A one-to-one function is a function in which every domain value has a different range value.

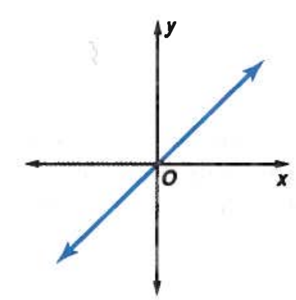
Graphs

We can represent a function as a graph on a coordinate plane. The elements of the domain are represented by x-coordinates and the elements of the range are represented by y-coordinates. A graph represents a function if no two points have the same x-coordinate and different y-coordinates.

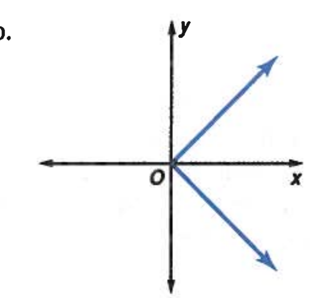
Example 2: Identifying a Function from a Graph

Tell whether each graph represents a function.

a.



b.



a. This graph represents a function because no two points have the same x-coordinate and different y-coordinates.

b. This graph does not represent a function because there are many points with the same x-coordinate and different y-coordinates.

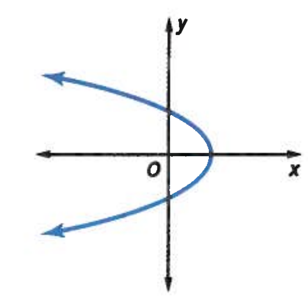
CORE PRACTICE

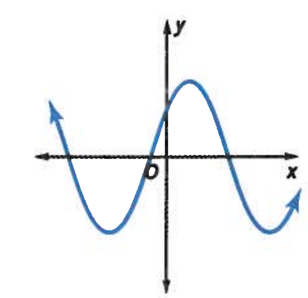
Use Math Tools Appropriately

You can use a tool called the Vertical Line Test to determine whether a graph represents a function.

For a graph to represent a function, no two points can have the same x-coordinate and different y-coordinates. If two points did have the same x-coordinate but different y-coordinates, these two points would lie on a vertical line. For example, the points (4, 2) and (4, −3) lie on the vertical line $x = 4$. Therefore, a graph represents a function if there is no vertical line that intersects the graph at more than one point.

Use the Vertical Line Test to determine whether each graph represents a function.





Equations

We can use equations to represent functions. Equations describe the rules for inputs to outputs mathematically. An example of an equation that represents a function is $f(x) = 2x + 3$.

Example 3: Writing an Equation for a Function

Suppose every item in a store is priced at $5.00. Write a function to represent the cost of x items. What is the cost of 4 items? If someone spent $30.00, how many items did this person buy?

Step 1 Because the cost of each item is $5.00, the cost of x items is $5x$. The function is $f(x) = 5x$.

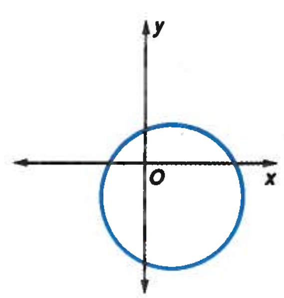
Step 2 To find the cost of 4 items, evaluate $f(x)$ when $x = 4$: $f(4) = 5(4) = 20$. The cost of 4 items is $20.00.

Step 3 Someone spent $30.00 and bought x items. Substitute 30 for $f(x)$ and solve for x: $30 = 5x$, so $x = 6$. This person bought 6 items.

Think about Math

Directions: Tell whether the table and the graph represent functions. Explain your answers.

1.



2.

Domain	Range
1	1
2	1
3	1

Linear and Quadratic Functions

No one can predict the future, but with functions you can make predictions based on certain rules. Businesses use functions to measure costs, price, and revenue. This can help determine how many of an item must be sold at a certain price to make a profit.

Evaluate Linear Functions

A **linear function** expresses a linear equation using function notation. A linear function can be written in the form $f(x) = mx + b$. The graph of a linear function is a non-vertical line. Vertical lines do not represent functions because every point on a vertical line has the same x-coordinate.

Example 4: Evaluating a Linear Function

Find the value of $f(x) = \frac{1}{2}x + 3$ when $x = 4$.

Step 1 Substitute 4 for x. $\quad f(4) = \frac{1}{2}(4) + 3$

Step 2 Simplify by multiplying and then adding. $\quad = 2 + 3$

$= 5$

When $x = 4$, $f(x) = 5$.

Evaluate Quadratic Functions

A **quadratic function** expresses a quadratic equation using function notation. A quadratic function can be written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$. Evaluating a quadratic function is similar to evaluating a linear function. In a quadratic function, however, there will be an exponent that must be evaluated before any other operations are performed.

Example 5: Evaluating a Quadratic Function

Find the value of $f(x) = x^2 + 4x - 3$ when $x = -2$.

Step 1 Substitute -2 for x. $\quad f(-2) = (-2)^2 + 4(-2) - 3$

Step 2 Simplify. First evaluate the exponent. Then multiply. Finally, add and subtract from left to right. $\quad = 4 - 8 - 3$

$= -4 - 3$

$= -7$

When $x = -2$, $f(x) = -7$.

Think about Math

1. Find the value of the function $f(x) = -7x + 4$ when $x = -2$, $x = 1$, and $x = 4$.
2. Find the value of the function $f(x) = 2x^2 + 2$ when $x = -2$, $x = 0$, and $x = \frac{1}{2}$.

21ST CENTURY SKILL

Business Literacy

A business uses the function $C(x) = x^2 - 6x + 8$ to determine the cost to produce x tennis rackets. How much does it cost to produce 20 tennis rackets?

CORE SKILL

Solve Real-World Problems

The function $H(t) = -16t^2 + 256$ is used in physics to describe the height H in feet of an object t seconds after it has been dropped from a height of 256 feet.

Evaluate this function when $t = 0$, $t = 1$, $t = 2$, $t = 3$, and $t = 4$. What do your answers represent? Graph the corresponding points in the coordinate plane and connect them to form the graph of the function.

Functions in the Coordinate Plane

We graph functions in the coordinate plane to learn more about them. We can observe a graph's characteristics to better understand the behavior of the function. We can look at rates of change and tell how much a racecar driver is accelerating or decelerating through a curve.

Plot Points on the Coordinate Plane

When function notation was introduced, we replaced y with $f(x)$.

$$y = x + 1$$
$$f(x) = x + 1$$
$$\text{So, } y = f(x).$$

On a coordinate plane, we plot points (x, y). For a function, each point will be $(x, f(x))$. Again, y is replaced with $f(x)$.

We can evaluate a function for any value of x in the domain and graph the corresponding point on the coordinate plane.

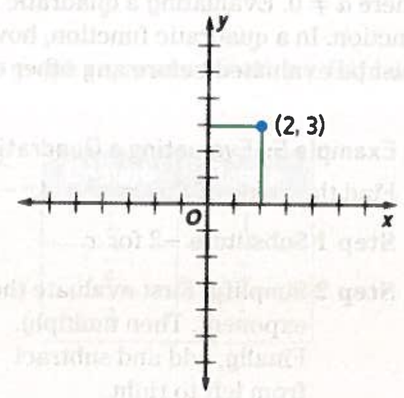
Example 6: Using Functions to Graph Points

a. For the function $f(x) = x + 1$, graph the point whose x-coordinate is 2.

Step 1 Find the value of the function when $x = 2$.

$f(2) = 2 + 1 = 3$

Step 2 When $x = 2$, $f(x) = 3$. Graph the point (2, 3).

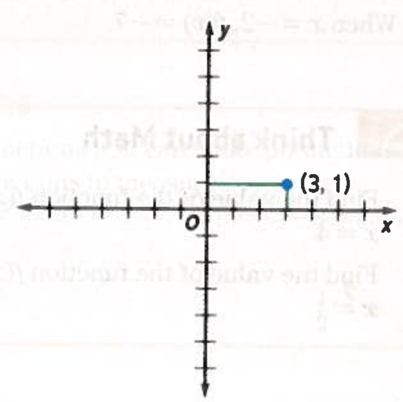


b. For the function $f(x) = 2x - 5$, graph the point whose x-coordinate is 3.

Step 1 Find the value of the function when $x = 3$.

$f(3) = 2(3) - 5 = 6 - 5 = 1$

Step 2 When $x = 3$, $f(x) = 1$. Graph the point (3, 1).



Piecewise Functions

A piecewise function consists of two or more parts. It applies two or more algebraic rules to different parts of the domain. Even though it consists of more than one rule, it is treated as one function. This creates graphs with unique characteristics. We still have to make sure all the combined parts assign exactly one range value for each domain value.

Example 7: Evaluating and Graphing a Piecewise Function

Evaluate the piecewise function when $x = -2$, $x = -1$, $x = 1$, $x = 2$, and $x = 3$. Then graph the function.

$$f(x) = \begin{cases} x \text{ when } x < 1 \\ 3 \text{ when } x = 1 \\ -x \text{ when } x > 1 \end{cases}$$

Step 1 Evaluate the function when $x = -2$.
$-2 < 1$, so use the first part of the function, $f(x) = x$. $\quad f(-2) = -2$

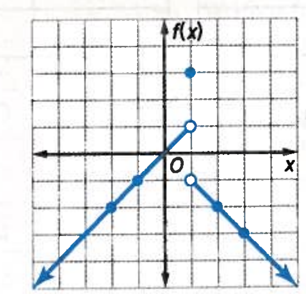
Step 2 Evaluate the function when $x = -1$.
$-1 < 1$, so use the first part of the function, $f(x) = x$. $\quad f(-1) = -1$

Step 3 Evaluate the function when $x = 1$.
Use the second part of the function, $f(x) = 3$. $\quad f(1) = 3$

Step 4 Evaluate the function when $x = 2$.
$2 > 1$, so use the third part of the function, $f(x) = -x$. $\quad f(2) = -2$

Step 5 Evaluate the function when $x = 3$.
$3 > 1$, so use the third part of the function, $f(x) = -x$. $\quad f(3) = -3$

Step 6 Graph the points $(-2, -2)$, $(-1, -1)$, $(1, 3)$, $(2, -2)$, and $(3, -3)$. Connect the points to form the graph.



Think about Math

Evaluate the piecewise function when $x = -2$, $x = -1$, $x = 1$, $x = 2$, and $x = 3$. Then graph the function.

$$f(x) = \begin{cases} -2x \text{ when } x < 0 \\ 2x \text{ when } x \geq 0 \end{cases}$$

Vocabulary Review

Directions: Write the missing term in the blank.

domain	**quadratic function**	**function**
linear function	**one-to-one function**	**range**

1. The set of outputs of a function is the ________________.
2. In a ________________, each input has exactly one output.
3. A ________________ can be written in the form $f(x) = ax^2 + bx + c$, where $a \neq 0$.
4. The set of inputs of a function is the ________________.
5. A ________________ can be written in the form $f(x) = mx + b$.
6. In a ________________, each input has a different output.

Skill Review

Directions: Read each problem and complete the task.

1. Which table or tables represent functions? Which function or functions are one-to one?

Table A

Domain	Range
−3	3
−2	2
−1	1
0	0

Table B

Domain	Range
3	0
6	1
9	0
12	1

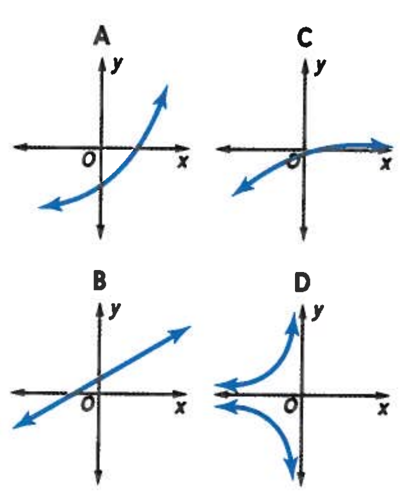
Table C

Domain	Range
−1	1
0	2
−1	3
−2	4

Table D

Domain	Range
0.2	0.04
0.5	0.25
0.8	0.64
1.1	1.21

2. Which graph does not represent a function?



3. The one-to-one function $f(x) = 6.5x$ represents the cost to download x books from a Web site.
   a. What is the cost per book?
   b. Find the cost of downloading 4, 5, 6, and 7 books.

4. What is the value of the function $f(x) = x^2 - 2x + 7$ when $x = -3$?
   A. 22
   B. 10
   C. 7
   D. 4

5. Evaluate the function $f(x) = -x^2 - x + 2$ when $x = -2$, $x = -1$, $x = 0$, $x = 1$, and $x = 2$. Then graph the corresponding points.

6. Graph the piecewise function.

$$f(x) = \begin{cases} x - 1 \text{ when } x < -1 \\ 0 \text{ when } x = -1 \\ -x + 1 \text{ when } x > -1 \end{cases}$$

Skill Practice

Directions: Read each problem and complete the task.

1. Identify the table that does not represent a function. Use the definition of a function to explain why this table does not represent a function.

Table A

Domain	−1	0	1	2
Range	−1	0	1	2

Table B

Domain	−1	0	1	2
Range	0	0	0	0

Table C

Domain	−1	0	−1	2
Range	−1	0	1	2

Table D

Domain	−1	0	1	2
Range	−1	0	−1	−2

2. Explain why the Vertical Line Test can be used to determine whether a graph represents a function.

3. Which situation could be represented by the function $f(x) = 8x$?
   A. Lindsey bought 8 pieces of candy for \$1.00.
   B. Pat earns \$8.00 per hour.
   C. Ken worked for 8 hours.
   D. Julian sold 8 more raffle tickets than his cousin sold.

4. Lou ran at a speed of 8 miles per hour.
   a. Write a function to represent the number of miles that Lou would run in x hours if he maintained this speed.
   b. If Lou could run at this speed for 2 hours, what distance would he run?
   c. Is the function a one-to-one function? Explain.
   d. At this speed, how many minutes will it take Lou to run one mile?
   e. Is the function linear or quadratic? Explain.

5. The function $H(t) = -16t^2 + 400$ gives the height H in feet of a ball t seconds after it has been dropped from a height of 400 feet. What is the value of this function when $t = 5$? What is the meaning of this value in the context of the problem?

6. Ricardo said that the piecewise relationship below is a function. Explain to Ricardo why he is incorrect. How could you modify the relationship so that it is a piecewise function?

$$f(x) = \begin{cases} x + 1 \text{ when } x \geq 1 \\ x - 1 \text{ when } x \leq 1 \end{cases}$$

LESSON 6.2 Identify Linear and Quadratic Functions

LESSON OBJECTIVES

- Evaluate linear and quadratic functions in the form of a table or graph
- Recognize linear and quadratic functions in the form of a table or graph

CORE SKILLS & PRACTICES

- Critique the Reasoning of Others

Key Terms

common difference
the amount that is the same between consecutive differences

consecutive difference
the subtraction between the next and current terms in a table

Vocabulary

linear function
a function that represents a line

quadratic function
a polynomial function that has 2 as its highest power of x

coordinate
the pairs (x, y) graphed on a plane

Key Concept

Linear and quadratic functions express a relationship between two variables—one independent and the other dependent. As the independent variable changes, the dependent variable of linear functions changes at a constant rate while the dependent variable of quadratic functions does not change at a constant rate.

Evaluating Linear and Quadratic Functions

Linear and quadratic equations help model interest rates and gravitational forces, respectively. Plugging values into these equations helps determine the amount of interest and the amount of gravity.

Linear Functions

A **linear function** represents a line. Algebraically, the equation of a line is written as $f(x) = mx + b$.

The table shows values of the linear function $f(x) = 2x - 3$ for several values of x.

x	$f(x) = 2x - 3$	Consecutive Difference
−3	$2(-3) - 3 = -9$	$(-7) - (-9) = 2$
−2	$2(-2) - 3 = -7$	$(-5) - (-7) = 2$
−1	$2(-1) - 3 = -5$	$(-3) - (-5) = 2$
0	$2(0) - 3 = -3$	$(-1) - (-3) = 2$
1	$2(1) - 3 = -1$	$1 - (-1) = 2$
2	$2(2) - 3 = 1$	$3 - 1 = 2$
3	$2(3) - 3 = 3$	

Notice in the table that the x-values differ by 1. In a table with this quality, if you take any two y-values next to each other and subtract them, you find their **consecutive differences**. Notice in the table above that the consecutive differences are the same. For this reason, the difference is called a **common difference**.

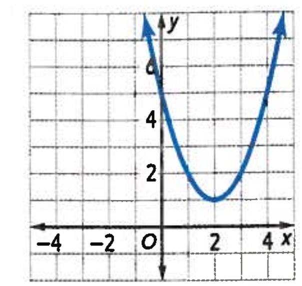
Think about Math

Directions: Answer the following questions.

1. What is the common consecutive difference for the function $f(x) = -3x + 2$?
2. What is the common consecutive difference for the function $f(x) = x - 4$?

Quadratic Functions

A **quadratic function** is a polynomial function that has 2 as its highest power of x. The graph of a quadratic function is a parabola.



Just like with linear functions, you can evaluate quadratic functions and make a table of values.

The table below shows that, unlike linear functions, quadratic functions do not have common consecutive differences.

x	$f(x) = x^2 - 4x + 5$	Consecutive Difference
−3	$(-3)^2 - 4(-3) + 5 = 26$	$17 - 26 = -9$
−2	$(-2)^2 - 4(-2) + 5 = 17$	$10 - 17 = -7$
−1	$(-1)^2 - 4(-1) + 5 = 10$	$5 - 10 = -5$
0	$(0)^2 - 4(0) + 5 = 5$	$2 - 5 = -3$
1	$(1)^2 - 4(1) + 5 = 2$	$1 - 2 = -1$
2	$(2)^2 - 4(2) + 5 = 1$	$2 - 1 = 1$
3	$(3)^2 - 4(3) + 5 = 2$	

However, the consecutive differences of the quadratic function are *exactly* the y-values found in the previous linear example. So quadratic functions have common *second* consecutive differences.

x	$f(x) = x^2 - 4x + 5$	Consecutive Difference	2nd Consecutive Difference
−3	$(-3)^2 - 4(-3) + 5 = 26$	$17 - 26 = -9$	$(-7) - (-9) = 2$
−2	$(-2)^2 - 4(-2) + 5 = 17$	$10 - 17 = -7$	$(-5) - (-7) = 2$
−1	$(-1)^2 - 4(-1) + 5 = 10$	$5 - 10 = -5$	$(-3) - (-5) = 2$
0	$(0)^2 - 4(0) + 5 = 5$	$2 - 5 = -3$	$(-1) - (-3) = 2$
1	$(1)^2 - 4(1) + 5 = 2$	$1 - 2 = -1$	$1 - (-1) = 2$
2	$(2)^2 - 4(2) + 5 = 1$	$2 - 1 = 1$	
3	$(3)^2 - 4(3) + 5 = 2$		

TEST-TAKING SKILL

Eliminate Unnecessary Information

Sometimes on a test you are given extra information not needed to solve the problem. Read each test question carefully and determine what information is needed to solve the problem.

The table below shows the first, second, and third differences for a function $f(x)$.

x	$f(x)$	Consecutive Differences 1st	2nd	3rd
8	2	1	1	0
9	3	2	1	0
10	5	3	1	0
11	8	4	1	
12	12	5		
13	17			

Is $f(x)$ a quadratic function? What information in the table is not needed to answer this question?

Examples of Non-Linear/Quadratic Functions

All polynomial functions eventually turn out to have common consecutive differences. For example, the function $f(x) = x^3$ has common third consecutive differences.

x	$f(x) = x^3$	Consecutive Difference	2nd Consecutive Difference	3rd Consecutive Difference
−3	$(-3)^3 = -27$	$(-8) - (-27) = 19$	$7 - 19 = -12$	$(-6) - (-12) = 6$
−2	$(-2)^3 = -8$	$(-1) - (-8) = 7$	$1 - 7 = -6$	$0 - (-6) = 6$
−1	$(-1)^3 = -1$	$0 - (-1) = 1$	$1 - 1 = 0$	$6 - 0 = 6$
0	$(0)^3 = 0$	$1 - 0 = 1$	$7 - 1 = 6$	$12 - 6 = 6$
1	$(1)^3 = 1$	$8 - 1 = 7$	$19 - 7 = 12$	
2	$(2)^3 = 8$	$27 - 8 = 19$		
3	$(3)^3 = 27$			

Some functions never turn out to have common consecutive differences. For example, the function $f(x) = 2^x$ has no common consecutive differences. Notice how the second consecutive differences are the same as the first consecutive differences.

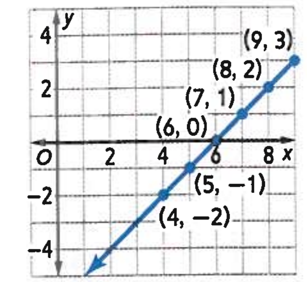
x	$f(x) = 2^x$	Consecutive Difference	2nd Consecutive Difference
0	$2^0 = 1$	$2 - 1 = 1$	$2 - 1 = 1$
1	$2^1 = 2$	$4 - 2 = 2$	$4 - 2 = 2$
2	$2^2 = 4$	$8 - 4 = 4$	$8 - 4 = 4$
3	$2^3 = 8$	$16 - 8 = 8$	$16 - 8 = 8$
4	$2^4 = 16$	$32 - 16 = 16$	$32 - 16 = 16$
5	$2^5 = 32$	$64 - 32 = 32$	
6	$2^6 = 64$		

Recognizing Linear and Quadratic Functions

Some computer scientists make and break computer codes. These codes are needed to secure national secrets, as well as your banking information. One of the earliest codes in history was the Caesar Cipher, which uses a linear function to move each letter to another letter.

Linear Functions

The function below appears to be linear. To be sure, check by finding consecutive differences.



Example 1: Identifying Linear Functions

Step 1 Record the **coordinates,** or the pairs (x, y) graphed on a plane, of several points from the graph in a table. Make sure that the x-values change by 1.

Step 2 Find the consecutive differences of the y-values.

x	$f(x)$	Consecutive Differences
4	−2	1
5	−1	1
6	0	1
7	1	1
8	2	1
9	3	

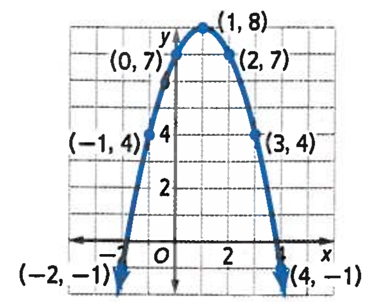
Because the function has common first consecutive differences, it is linear.

CALCULATOR SKILL

Many 4-function calculators do not have the ability to enter in tables. The TI-30XS MultiView™ calculator can not only enter a table of values, it can generate the table for you. Press the table key, enter the function you want to find a table of values for, choose a start value for x and a step value (the increment between each x value). A reasonable start value for x may be -3 and a reasonable step value for x may be 1. Using the down arrow, you can scroll throughout the table to see the table of values.

Quadratic Functions

The function below appears to be quadratic. To be sure, check by finding second consecutive differences.



Example 2: Identifying Quadratic Functions

Step 1 Record the coordinates of several points from the graph in a table. Make sure the x-values change by 1.

Step 2 Find the first consecutive differences.

Step 3 Find the second consecutive differences.

x	$f(x)$	Consecutive Differences	2nd Consecutive Differences
−2	−1	5	−2
−1	4	3	−2
0	7	1	−2
1	8	−1	−2
2	7	−3	−2
3	4	−5	
4	−1		

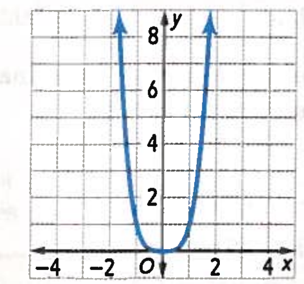
Because the function has common second consecutive differences, it is quadratic.

Examples of Non-Linear/Quadratic Functions

Sometimes the graph of a function may appear to be linear or quadratic, but in fact it is not.

Example 3: Showing that a Function is Not Quadratic

Is the function in the graph quadratic?



The function appears to be quadratic. To be sure, check by finding second consecutive differences.

Step 1 Record the coordinates of several points from the graph in a table. Make sure the x-values change by 1.

Step 2 Find the first consecutive differences.

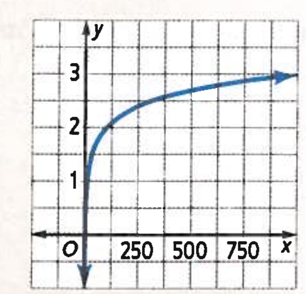
Step 3 Find the second consecutive differences.

x	$f(x)$	Consecutive Differences	2nd Consecutive Differences
−3	81	−65	50
−2	16	−15	14
−1	1	−1	2
0	0	1	14
1	1	15	50
2	16	65	
3	81		

The second consecutive differences show that the function is not quadratic.

Example 4: Showing that a Function is Not Linear

Is the function in the graph linear?



The function appears to be linear. To be sure, check by finding consecutive differences.

Step 1 Record the coordinates of several points from the graph in a table.

Step 2 Find the first consecutive differences. It appears that this function has common consecutive differences and is therefore linear. However, this is not the case. The function is not linear. What went wrong? Look at the table in this example. The x-values do not differ by 1 and so the differences between the values of $f(x)$ are not consecutive differences.

x	$f(x)$	Consecutive Differences?
1	0	1
10	1	1
100	2	1
1000	3	1
10000	4	

Making sure the x-values differ by 1 is very important when trying to determine whether a graph is linear or quadratic.

CORE PRACTICE

Critique the Reasoning of Others

Mario makes the table of values below and calculates the first differences as shown. He claims that the function $f(x)$ is linear because there are common first consecutive differences. Do you agree with Mario? Why or why not?

x	$f(x)$	Consecutive Differences
3	1	6
4	7	6
5	1	6
6	7	6
7	1	6
8	7	

Vocabulary Review

Directions: Write the missing term in the blank.

common difference **linear function** **consecutive difference** **quadratic function** **coordinate**

1. A function that represents a line is a ________________.
2. When differences are the same, they are called ________________.
3. A polynomial function that has 2 as its highest power of x is a ________________.
4. A point graphed on a plane is also called a ________________.
5. The difference between terms that follow each other in a table is a ________________.

Skill Review

Directions: Read each problem and complete the task.

1. Which table of values corresponds to the function $f(x) = -5x - 3$?

A.

x	$f(x)$
−3	−18
−2	−13
−1	−8
0	−3
1	2
2	7
3	12

B.

x	$f(x)$
−3	12
−2	7
−1	2
0	−3
1	−8
2	−13
3	−18

C.

x	$f(x)$
−3	15
−2	10
−1	5
0	0
1	−5
2	−10
3	−15

D.

x	$f(x)$
−3	−18
−2	−13
−1	−8
0	−3
1	−8
2	−13
3	−18

2. The table of values below corresponds to which function?

x	−3	−2	−1	0	1	2	3
$f(x)$	−3	2	5	6	5	2	−3

A. $f(x) = x^2 - 6$
B. $f(x) = -x^2 + 6$
C. $f(x) = -x - 6$
D. $f(x) = x - 6$

3. Use second differences to show that the function $f(x) = 3x^2 - 1$ is quadratic.

Skill Practice

Directions: Read each problem and complete the task.

1. Heidi says that the function $f(x) = -4x + 7$ is linear because it is in the form $f(x) = mx + b$. Neal made the table of values below for the function and concluded that the function is not linear because the first consecutive differences are not common. Which student is correct? Describe the other student's error.

x	$f(x)$	Consecutive Differences
−2	15	−4
−1	11	−8
1	3	−8
3	−5	−4
4	−9	

2. You have seen that linear functions have common first consecutive differences and that quadratic functions have common second consecutive differences. You have also seen that $f(x) = x^3$ has common third consecutive differences. Make a conjecture about $f(x) = x^4$ and common consecutive differences. Then test your conjecture by making a table of values for $f(x) = x^4$.

3. Enter values into the tables so that Table A represents a linear function and Table B represents a quadratic function. Show that your answers are correct.

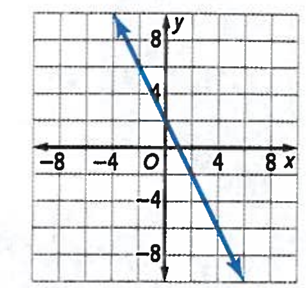
Table A

x	$f(x)$

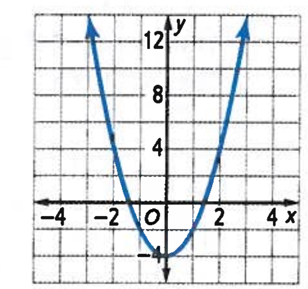
Table B

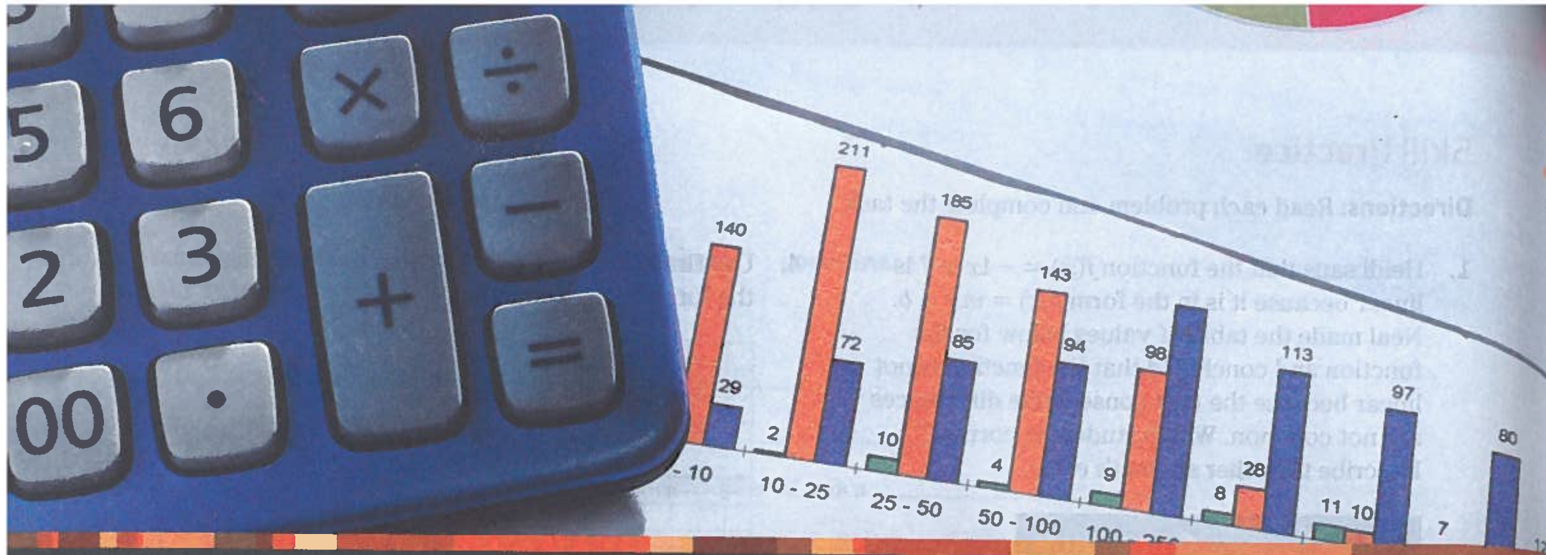
x	$f(x)$

4. Use first consecutive differences to show that the function is linear.



5. Use second consecutive differences to show that the function is quadratic.





John Lamb/Getty Images

LESSON 6.3 Identify Key Features of a Graph

LESSON OBJECTIVES

- Identify key features of a graph
- Draw a graph when given its key features
- Graph a real-world relationship by identifying key features

CORE SKILLS & PRACTICES

- Make Use of Structure
- Gather Information

Key Terms

end behavior
describes the appearance of a graph as it extends in both directions away from zero

relative maximum/minimum
the y-coordinate of any point that is the highest/lowest point for some section of the graph

Vocabulary

line symmetry
a figure displays this when there is a line that divides the figure into two halves that are the mirror images of each other

rotational symmetry
a figure displays this when it can be rotated less than 360° around a point to coincide with itself

x-intercept
the x-coordinate of a point where a graph crosses the x-axis

y-intercept
the y-coordinate of a point where a graph crosses the y-axis

Key Concept

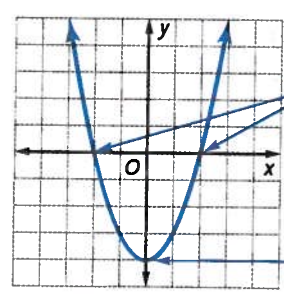
You can sketch graphs if you know or can determine their key features.

Key Features

Businesses often use information from graphs to make important decisions. Making a good decision requires knowing how to read a graph correctly and how to identify and interpret its most important features.

Intercepts and Intervals

An ***x*-intercept** is the x-coordinate of a point where a graph crosses the x-axis. A ***y*-intercept** is the y-coordinate of a point where a graph crosses the y-axis. Intercepts can sometimes be determined by examining a graph.

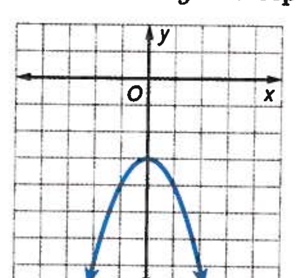


The graph crosses the x-axis at $(-2, 0)$ and $(2, 0)$. The x-intercepts are -2 and 2.

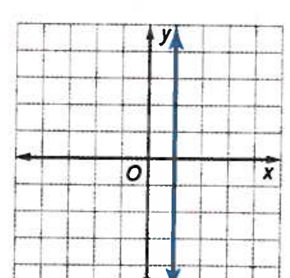
The graph crosses the y-axis at $(0, -4)$. The y-intercept is -4.

This graph crosses the x-axis at $(-2, 0)$ and $(2, 0)$, so the x-intercepts are -2 and 2. The graph crosses the y-axis at $(0, -4)$, so the y-intercept is -4.

Not all graphs have x- and y-intercepts.

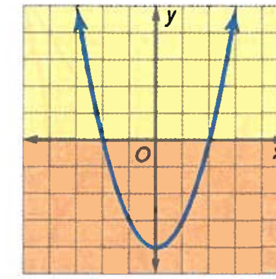


No x-intercepts



No y-intercepts

A *positive interval* describes the values of x for which the graph is above the x-axis. This graph is above the x-axis when $x < -2$ and $x > 2$.



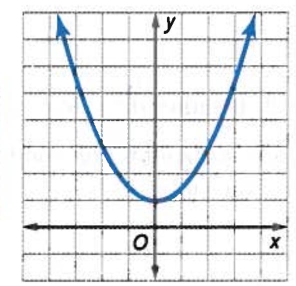
Positive interval: The graph is above the x-axis when $x < -2$ and $x > 2$.

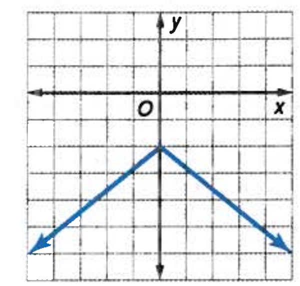
Negative interval: The graph is below the x-axis when $-2 < x < 2$.

A *negative interval* describes the values of x for which the graph is below the x-axis. This graph is below the x-axis when $-2 < x < 2$.

Some graphs are always above the x-axis, and some graphs are always below the x-axis.

Always above the x-axis



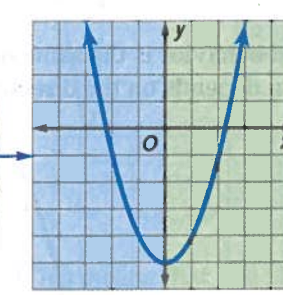


Always below the x-axis

Increasing and Decreasing

A *decreasing interval* describes the values of x for which the graph falls from left to right. On a decreasing interval, the values of y decrease as the values of x increase. This graph falls from left to right when $x < 0$.

Decreasing interval: The graph falls from left to right when $x < 0$.

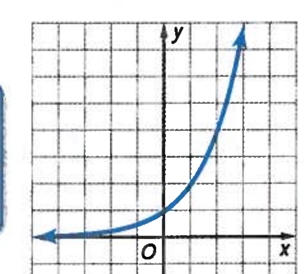


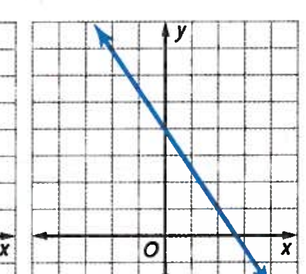
Increasing interval: The graph rises from left to right when $x > 0$.

An *increasing interval* describes the values of x for which the graph rises from left to right. On an increasing interval, the values of y increase as the values of x increase. This graph rises from left to right when $x > 0$.

Some graphs are always rising or always falling for all values of x.

always rising from left to right



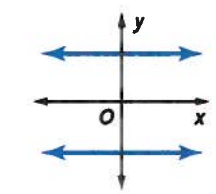


always falling from left to right

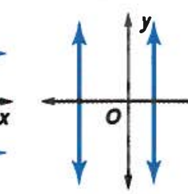
CORE PRACTICE

Make Use of Structure

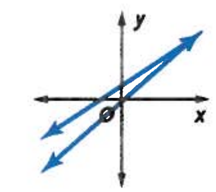
You can use the structure of a linear graph to determine general information about key features of linear graphs. For example, a linear graph could be vertical, horizontal, or neither. If the graph is not vertical or horizontal, it will rise from left to right or it will fall from left to right.



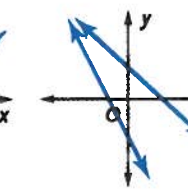
Horizontal



Vertical



Rises from left to right



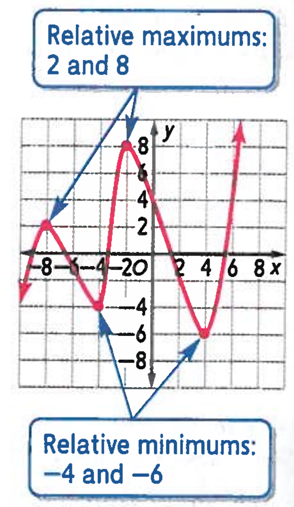
Falls from left to right

From these possibilities, you can see that all linear graphs have one or two intercepts.

Now you try. Think about the structure of a quadratic graph. Visualize or sketch the different possibilities. What are the possible numbers of intercepts for a quadratic graph?

Relative Minimums and Maximums

A **relative maximum/minimum** is a y-coordinate of any point that is the highest/lowest point for some section of the graph. Relative maximums/minimums occur at "hills"/"valleys" in the graph.



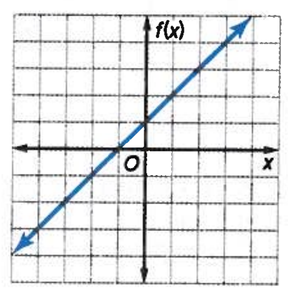
A relative maximum or minimum may or may not occur at the highest or lowest point on the entire graph. Relative maximums and minimums can exist even if the graph overall does not have a highest or lowest point, as long as there are one or more sections of the graph that have a highest or lowest point.

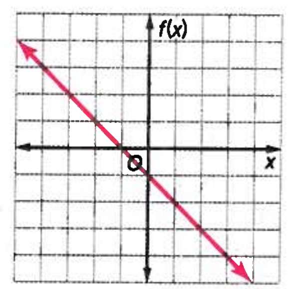
Graphs that are always rising or always falling do not have relative minimums or maximums. There are no "hills" or "valleys."

End Behavior

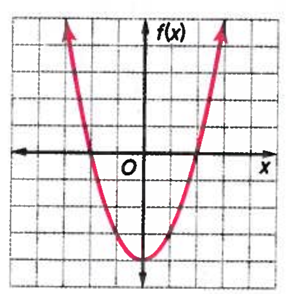
End behavior describes a graph as it extends in either direction away from 0.

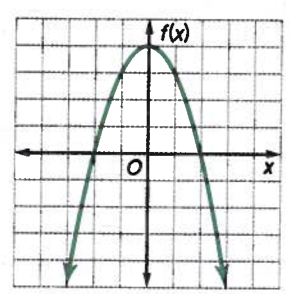
For linear graphs that are neither vertical nor horizontal, the end behavior on the left is different from the end behavior on the right.



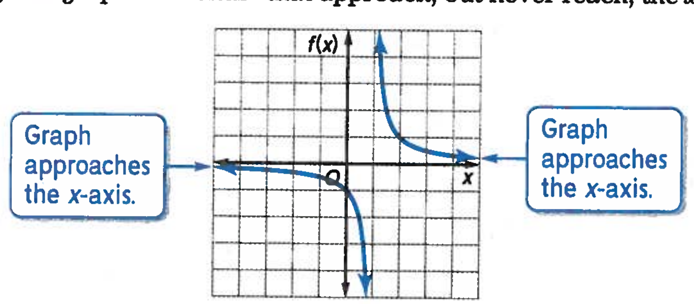


For quadratic graphs, the end behavior is the same on both sides. Whether the graph extends up or down depends on the direction the graph opens.



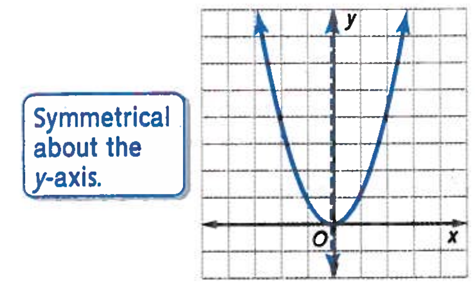


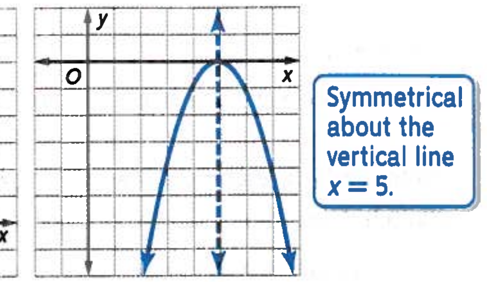
Some graphs have "ends" that do not extend up or down indefinitely. For example, this graph has "ends" that approach, but never reach, the x-axis.



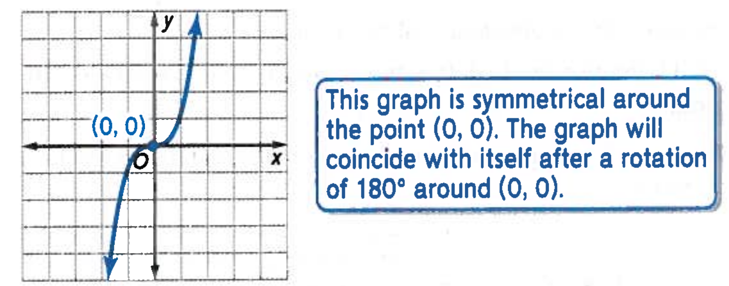
Symmetry

A graph has **line symmetry** if there is a line that divides the figure into two halves that are mirror images of each other. For example, a quadratic graph is symmetrical about a vertical line.

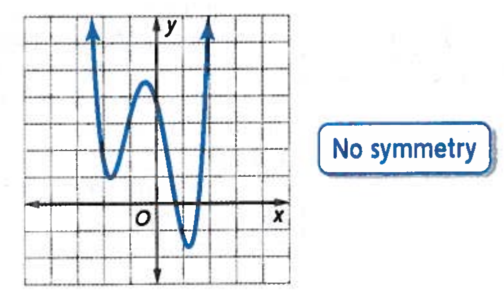




A graph has **rotational symmetry** if it can be rotated less than 360° around a point to coincide with itself.

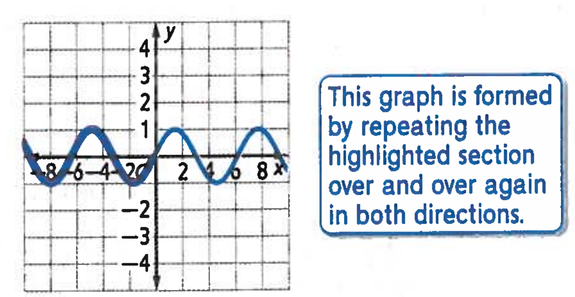


Some graphs have no symmetry.



Periodic Graphs

Some graphs are periodic. This means that one piece of the graph repeats over equal intervals.



CORE SKILL

Gather Information

When you are asked to identify the graph of a function, one method is to substitute values for x and generate ordered pairs. However, it may require less work to use the function rule to find information about the key features of the graph. Then you can match the key features to the correct graph.

Suppose you were given several graphs and asked to identify the graph of $f(x) = x^2 + 5x + 6$.

- Find the y-intercept by substituting 0 for x in the function rule.

$$f(0) = 0^2 + 5(0) + 6 = 0 + 0 + 6 = 6$$

- Find the x-intercepts by substituting 0 for $f(x)$ and factoring to solve the quadratic equation.

$$0 = x^2 + 5x + 6 = (x + 2)(x + 3)$$
$$x = -2 \qquad x = -3$$

Think about a graph that shows a y-intercept of 6 and x-intercepts of -2 and -3. What key features might you use to identify the graph of $4x + 2y = 12$?

Use Key Features to Draw a Graph

Forensic artists make sketches of people based on witnesses' descriptions of physical features—hair and eye color, jaw line, eyebrow thickness and shape, and so on. Similarly, when you are given a description of the key features of a graph, you can make a sketch of the graph.

Sketch a Graph

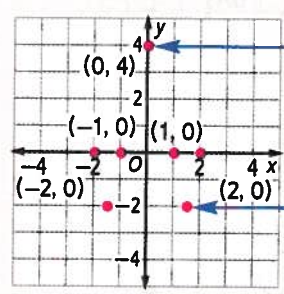
You can sketch a graph when you know some of its key features.

Example 1: Sketching a Graph Using Key Features

Sketch a graph with the following features.

- The y-intercept is 4.
- The x-intercepts are -2, -1, 1, and 2.
- There is one relative maximum, 4. It occurs at one point.
- There is one relative minimum, -2. It occurs at two points.
- The graph is symmetrical about the y-axis.
- The end behavior on the left is the same as the end behavior on the right.

Step 1 Graph the intercepts, the relative maximum, and the relative minimum.



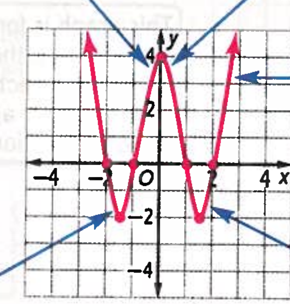
The y-intercept and the relative maximum are the same.

The relative minimum, -2, occurs at two points. We don't know the x-coordinates of these points, but they must be somewhere between the x-intercepts as shown. The graph is symmetric about the y-axis, so these two points are the same distance from the y-axis.

Step 2 Sketch the graph through the points. Make sure that the graph has all of the key features listed above.

2. The graph rises from the relative minimum to the relative maximum.

3. The graph falls from the relative maximum until it reaches the relative minimum.



5. Because there is only one relative maximum, the graph must continue to rise indefinitely. The end behavior is the same on both sides, so the left side of the graph must also rise indefinitely.

1. The graph falls until it reaches the relative minimum.

4. The graph rises from the relative minimum.

Real-World Graphs

You can make a graph to describe a real-world situation. Think about the key features of a graph and how they translate to the situation.

Example 2: Graphing a Real-World Situation

Chloe leaves home driving at a speed of 30 miles per hour. She drives at this speed for 8 minutes, stops at a red light for 2 minutes, and then drives at 30 miles per hour for 10 more minutes until she arrives at work. Make a graph to represent the situation.

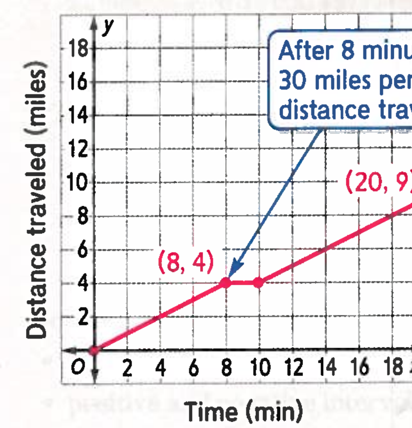
Step 1 Assign a variable to each quantity. Let x = time and y = distance traveled.

Step 2 Identify the intercepts. When x (time) is 0, y (distance traveled) is also 0. Both the x- and y-intercepts are 0.

Step 3 Identify increasing and decreasing intervals. Distance traveled increases when Chloe is driving – during the first 8 minutes and the last 10 minutes. Chloe drove for a total of $8 + 2 + 10 = 20$ minutes, so the graph will rise for $0 < x < 8$ and $10 < x \leq 20$. Distance traveled does not decrease, so there are no decreasing intervals. Distance does not change during the 2 minutes that Chloe is stopped at the red light, so the graph will neither rise nor fall when x is between 8 and 10.

Step 4 Use key features and the given information to sketch the graph. Because Chloe was driving at a constant rate (30 miles per hour), the beginning and end of her trip will be linear.

The part of her trip stopped at the red light will be constant.



After 8 minutes of driving 30 miles per hour, the distance traveled is 4 miles.

After 10 minutes of driving 30 miles per hour, the distance traveled is 5 miles. The total distance traveled in 20 minutes is $4 + 5 = 9$ miles.

Think about Math

Directions: Sketch a graph with the following key features.

- There are three x-intercepts.
- There is a relative maximum.
- The end behavior on the left is different from the end behavior on the right.
- The graph has no symmetry.

CALCULATOR SKILL

When given a function rule, it is usually fairly straightforward to find y-intercept(s)—simply substitute 0 for x. However, finding x-intercept(s) can be more involved. If the function is quadratic, you may need to factor or use the quadratic formula. When using a calculator and the quadratic formula to find x-intercepts, it may be useful to first calculate $b^2 - 4ac$ and write the value on paper. Then enter **(−b + "value")/2a** and **(−b − "value")/2a**. The parentheses are important because your calculator uses the order of operations. If you do not include the parentheses, the calculator will compute the division before the addition/subtraction.

Use a calculator and the quadratic formula to find the x-intercepts of $f(x) = 6x^2 - 7x - 3$.

Vocabulary Review

Directions: Fill in each blank with a word from the list below.

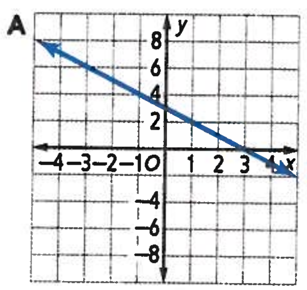
end behavior	**line symmetry**	**relative maximum/minimum**
rotational symmetry	**x-intercept**	**y-intercept**

1. A(n) ________________ is the y-coordinate of a point where a graph crosses the y-axis.
2. A(n) ________________ is the y-coordinate of any point that is the highest/lowest point for some section of a graph.
3. A(n) ________________ is the x-coordinate of a point where a graph crosses the x-axis.
4. A figure has ________________ if there is a line that divides the figure into two halves that are mirror images of each other.
5. ________________ describes the appearance of a graph as it extends in both directions away from 0.
6. A figure has ________________ if it can be rotated less than 360° around a point to coincide with itself.

Skill Review

Directions: Read each problem and complete the task.

1. Which is the graph of $2x + 2y = 6$? Explain how you can use key features to identify the correct graph.

A


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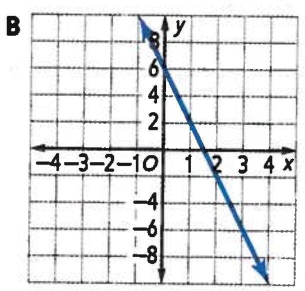

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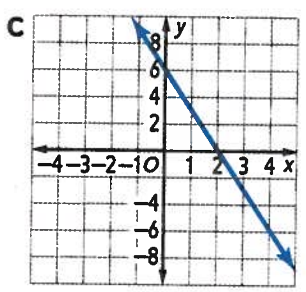

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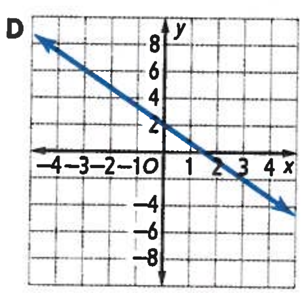
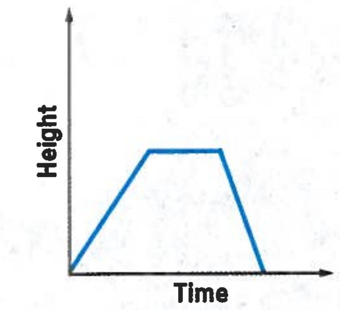


2. Sketch a graph with the following key features.
   - The x-intercepts are -3, -1, 1, and 3.
   - The y-intercept is -9.
   - There is one relative minimum, -9. It occurs at one point.
   - There is one relative maximum, 16. It occurs at two points.
   - The graph extends down indefinitely in both directions.
   - The graph is symmetrical about the y-axis.

3. Which situation is best represented by the graph?

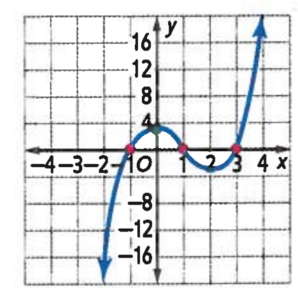


   A. A rubber ball is dropped from a height of 5 feet. It bounces several times before rolling to a stop on the ground.
   B. An elevator begins at the ground floor of an office building. It goes up three floors and remains on the third floor for several minutes. Then it goes down to the second floor, where a passenger gets in and goes up to the fifth floor.
   C. Leanne hikes up to a mountain peak at a speed of 2.5 miles per hour. When she reaches the top, she rests for a while, and then hikes back down at a speed of 4 miles per hour.
   D. A diver is on a board one meter above the ground. He dives into a pool that is 12 feet deep and then swims up to the water's surface.

Skill Practice

Directions: Read each problem and complete the task.

1. Describe the key features of the graph.



   Identify the:
   - x- and y-intercepts
   - positive and negative intervals
   - increasing and decreasing intervals
   - relative minimum(s) and maximum(s)

2. Sketch a quadratic graph that matches each description. If a graph is not possible, explain why.
   a. The graph has no relative minimum.
   b. The graph has no relative minimums or maximums.
   c. The graph has no symmetry.
3. Write a real-world situation about a vehicle or an object whose speed changes over time. Make a graph to represent your situation. Identify the key features of your graph and describe their meanings in the context of your situation.
4. Use key features to sketch the graph of $f(x) = x^2 - 9$. Describe the key features you used.
5. Drake claims that the only key features needed to sketch a linear graph are the intercepts. Do you agree with Drake? Explain why or why not.

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LESSON 6.4 Compare Functions

LESSON OBJECTIVES

- Compare proportional relationships represented in different ways
- Compare linear functions represented in different ways
- Compare quadratic functions represented in different ways

CORE SKILLS & PRACTICES

- Use Ratio and Rate Reasoning
- Make Sense of Problems

Key Terms

proportional relationship
a relationship between two quantities x and y such that the ratio of y to x is always equal to a nonzero constant k

Vocabulary

slope
the ratio of rise to run

y-intercept
the y-coordinate of a point where a graph crosses the y-axis

quadratic function
a function that can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$

Key Concept

Functions can be represented in many ways—graphs, tables, equations, verbal descriptions, and so on. To compare two or more functions represented in different ways, you will have to use the information given in each representation to determine key features that can be compared.

Compare Proportional Relationships

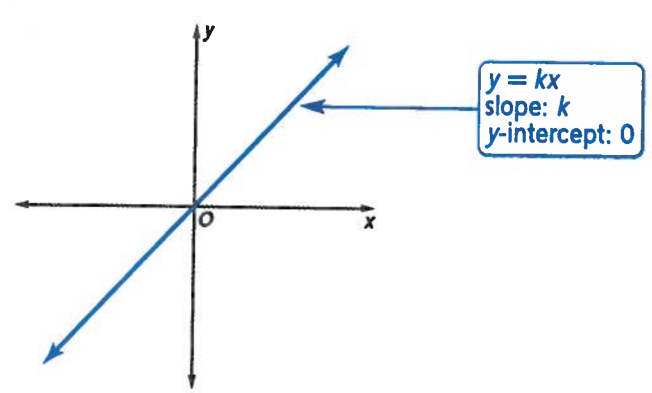
Many real-world relationships are proportional. For example, an employee's pay is proportional to the number of hours the employee works. Being able to compare proportional relationships will allow you to evaluate situations and make decisions.

Distance-Rate-Time

Two variable quantities x and y are in a **proportional relationship** if the ratio of y to x is always equal to a nonzero constant k.

$$\frac{y}{k} = x$$

Solving the equation above for y gives $y = kx$. The linear equation $y = kx$ is a line with slope k that passes through (0, 0). Remember that the **slope** of a line is the ratio of rise to run.

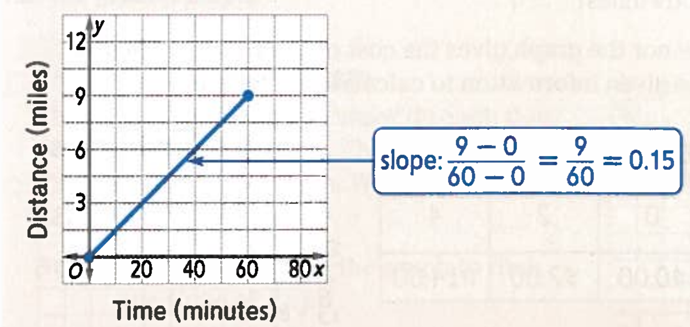


When a person or an object is traveling at a constant speed (r), the relationship between distance (d) and time (t) is a proportional relationship described by the equation $d = rt$. The speed (r) is the rate of change or slope.

Example 1: Comparing Distance Relationships

Alison and Lia competed in a race. The graph and equation describe their race performances. Who ran faster, Alison or Lia?

Alison's Race



Lia's Race

$d = 0.1t$, where d represents distance in miles and t represents time in minutes

Step 1 Find Alison's speed. Alison's speed is equal to the slope of the graph, 0.15. This means that Alison's speed was 0.15 miles per minute.

Step 2 Find Lia's speed. Lia's speed is represented by the coefficient of t, 0.1. Lia's speed was 0.1 miles per minute.

Step 3 Compare the speeds. $0.15 > 0.1$, so Alison ran faster.

Hourly Pay Rates

When a person is paid a fixed amount per hour, the relationship between time worked and total pay is a proportional relationship. The amount earned per hour is the rate of change.

Example 2: Comparing Hourly Pay Rates

The table and the equation describe Kyle's and Reese's pay at their jobs. Who earns more per hour, Kyle or Reese?

Kyle's Pay

Hours Worked	0	1	2	3	4	5
Total Pay	0	20	40	60	80	100

Reese's Pay

$p = 10h$, where p represents total pay and h represents hours worked

Step 1 Find the amount that Kyle earns per hour. You can see from the table that when the number of hours worked increases by 1, the total pay increases by \$20. Kyle earns \$20 per hour.

Step 2 Find the amount that Reese earns per hour. This amount is represented by the coefficient of h, 10. Reese earns \$10 per hour.

Step 3 Compare. $20 > 10$, so Kyle earns more money per hour.

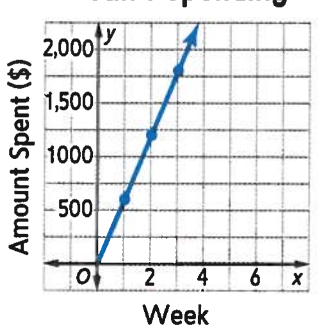
CORE SKILL

Use Ratio and Rate Reasoning

Sometimes when comparing proportional relationships, you will have to convert units.

Dean earns $75 per day and he works 7 days per week. The graph below describes Dean's spending.

Dean's Spending



To determine whether Dean's pay supports his spending, you will have to compare the amount he earns to the amount he spends. Notice that Dean's spending is described weekly while his pay is described daily. You will have to use reasoning to compare weekly amounts to daily amounts.

Use the graph to determine how much Dean spends per week.

How can you determine how much Dean earns per week?

Does Dean's pay support his spending? Explain.

Cost

When you buy more than one of the same item, the relationship between the number of items purchased and total cost is a proportional relationship. The cost per item is the rate of change.

Example 3: Comparing Cost

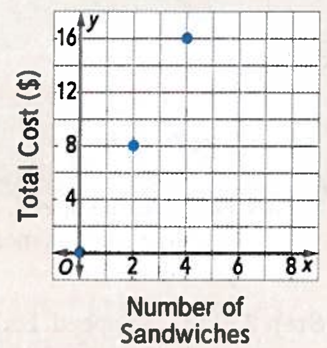
The table and the graph give information about the cost of turkey sandwiches at two different delis. At which deli does it cost more to buy three turkey sandwiches?

Neither the table nor the graph gives the cost of three sandwiches, so you must use the given information to calculate the answer.

Dawn's Deli

Number of Sandwiches	0	2	4
Total Cost	$0.00	$7.00	$14.00

Sam's Sandwiches



Step 1 Find the cost of a turkey sandwich at Dawn's Deli.
Two sandwiches cost $7.00.
Therefore, one sandwich costs $7.00 ÷ 2 = $3.50.

Step 2 Find the cost of a turkey sandwich at Sam's Sandwiches.
Two sandwiches cost $8.00.
Therefore, one sandwich costs $8.00 ÷ 2 = $4.00.

Step 3 Compare the costs. $3.50 < $4.00, so a turkey sandwich costs more at Sam's Sandwiches.
Therefore, as long as the price stays constant, three turkey sandwiches cost more at Sam's Sandwiches.

Think about Math

Three proportional relationships are represented by the table, the equation, and the graph. Order the relationships from greatest rate of change to least rate of change.

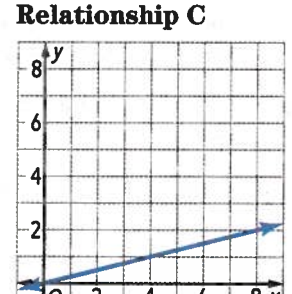
Relationship A

x	y
0	0
5	2
10	4

Relationship B

$y = 1.5x$

Relationship C



Compare Linear Functions

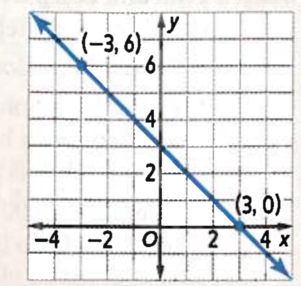
When choosing a monthly plan for text messages, comparing the monthly charge and rate per text message for each plan can help you determine which plan is best for you. Plans such as these can often be modeled by linear functions.

Compare Slopes

When given information about two linear functions, you can determine which has the greater slope.

Example 4: Compare Slopes

The linear function $g(x)$ passes through the points (1, 12) and (−2, 0). The linear function $f(x)$ is shown in the graph. Which function has the greater slope?



Step 1 Use two points on the graph to find the slope of $f(x)$:

$$m = \frac{6-0}{-3-3} = \frac{6}{-6} = -1$$

Step 2 Use the given points to find the slope of $g(x)$: $m = \frac{0-12}{-2-1} = \frac{-12}{-3} = 4$

Step 3 Compare. $-1 < 4$, so $g(x)$ has the greater slope.

Compare *y*-intercepts

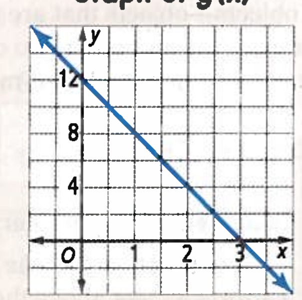
A ***y*-intercept** is the y-coordinate of a point where a graph crosses the y-axis. When given information about two linear functions, you can determine which has a greater y-intercept.

Example 5: Compare *y*-intercepts

Two linear functions are described in the table and graph. Which function has the greater y-intercept?

x	−3	−2	−1	0	1	2	3
f(x)	15	12	9	6	3	0	−3

Graph of $g(x)$



Step 1 Identify the y-intercept of $f(x)$. The y-intercept is the y-coordinate of the point whose x-coordinate is 0. You can see from the fourth row of the table that the y-intercept is 6.

Step 2 Identify the y-intercept of $g(x)$. The graph of $g(x)$ crosses the y-axis at (0, 12), so the y-intercept is 12.

Step 3 Compare. $6 < 12$, so $g(x)$ has the greater y-intercept.

Linear functions can model cost situations in which there is an initial cost as well as a cost per item. In these situations, the initial cost is represented by the y-intercept and the cost per item is represented by the slope.

21ST CENTURY SKILL

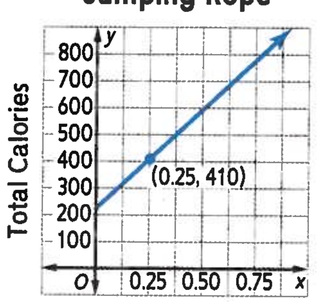
Health Literacy

Yesterday, Max spent one hour bowling and burned about 224 calories. Today Max will either swim or jump rope. The total number of calories burned during exercise for yesterday and today will depend on the amount of time Max spends swimming or jumping rope, as shown in the table and the graph.

Swimming

Hours Spent Swimming Today	Total Calories Burned Yesterday and Today
0	224
0.5	447
1	670

Jumping Rope

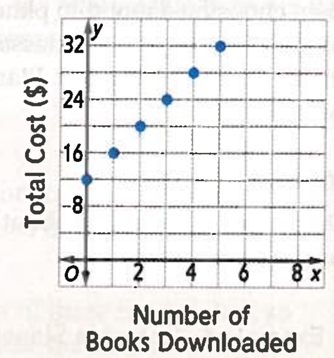


Max wants to burn a total of at least 400 calories for yesterday and today. If Max decides to swim today, how long must he swim to meet his goal? If Max decides to jump rope today, how long must he jump rope to meet his goal?

Example 6: Compare Costs

Luisa wants to join an online book club that allows her to download books to her tablet. For Book Club A, there is a one-time membership fee of \$15.99, and each book downloaded costs \$3.00. The costs for Book Club B are described in the graph. Which book club costs more per download? Which book club has the greater membership fee?

Book Club B



Step 1 Find and compare the cost per download for each club.

- The cost per download for Club A is \$3.00.
- From the graph, you can see that the total cost increases by \$4.00 for each book downloaded. The cost per download for Book Club B is \$4.00.
- Compare. $\$3.00 < \$4.00$, so the cost per download is greater for Book Club B.

Step 2 Find and compare the membership fee for each club.

- The membership fee for Club A is \$15.99.
- The membership fee is the cost when 0 books are downloaded. From the graph, you can see that the membership fee for Book Club B is \$12.00.
- Compare. $\$12.00 < \$15.99$, so the membership fee is greater for Book Club A.

Compare Quadratic Functions

Quadratic functions, functions that can be written in the form $y = ax^2 + bx + c$ where $a \neq 0$, are commonly used to model the motion of objects—objects that are dropped, thrown, kicked, and so on. You can compare the motion of two different objects by comparing the quadratic functions that model their motion.

Compare Zeros

Example 7: Compare Zeros

A red golf ball and a blue golf ball were hit at the same time from a platform 48 feet above the ground. The equation and the table describe the motion of each golf ball. Which golf ball reached the ground first?

Red Golf Ball

$y = -16x^2 + 52x + 48$, where y represents the height above the ground in feet and x represents the time in seconds after the ball is hit.

Blue Golf Ball

Time After the Ball is Hit (seconds)	0	1	2	3
Height Above the Ground (feet)	48	64	48	0

Step 1 Determine when the red golf ball reached the ground. When the ball reaches the ground, the height $y = 0$. Substitute 0 for y and solve the quadratic equation.

$$0 = -16x^2 + 52x + 48$$
$$= (-2x + 8)(8x + 6)$$
$$-2x + 8 = 8 \quad \text{or} \quad 8x + 6 = 0$$
$$x = 4 \quad \text{or} \quad x = -\frac{3}{4}$$

Factor and use the Zero Product Property.

Step 2 Determine when the blue golf ball reached the ground. According to the table, the blue golf ball reached the ground (height = 0) after 3 seconds.

Step 3 Compare. $3 < 4$, so the blue golf ball reached the ground first.

Because x represents time, the negative solution does not make sense. The red golf ball reached the ground after 4 seconds.

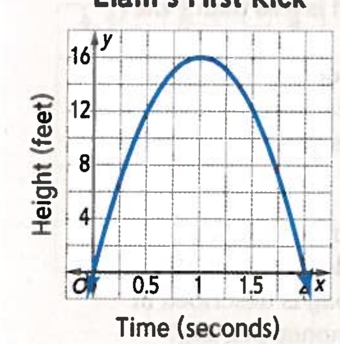
Compare Maximums

You can use quadratic functions to determine the maximum height reached by an object. You can compare quadratic functions to determine which of two objects reached a greater height.

Example 8: Compare Maximums

The graph and the table describe two of Liam's kicks in yesterday's soccer game. For which kick did the soccer ball reach a greater height?

Liam's First Kick



Liam's Second Kick

Time After the Ball is Kicked (seconds)	0	0.5	1	1.5	2
Height Above the Ground (feet)	0	13	18	15	4

Step 1 Examine the graph. The greatest height reached by the soccer ball is represented by the maximum—about 16 feet.

Step 2 Examine the table. You can see that the soccer ball reached a height of 18 feet after 1 second.

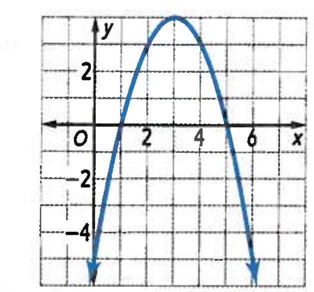
Step 3 Compare. We cannot be sure that the ball's maximum height in the second kick was 18 feet, but 18 is greater than the maximum height in the first kick. Therefore, the soccer ball reached a greater height in Liam's second kick.

CORE PRACTICE

Make Sense of Problems

To make sense of problems, look at all of the given information. Identify what you are asked to find and develop a plan to determine the answer.

How would you determine if a function had the same x-intercepts as the function shown?



Vocabulary Review

Directions: Fill in the blanks with one of the terms below. Terms may be used more than once.

slope **y-intercepts** **quadratic function** **proportional relationship**

1. A linear function is a function whose graph is a line. Features of linear functions that can be compared include ________________, which measures the steepness of a line, and ________________, which indicate where a line crosses the y-axis.

2. A function that can be written in the form $y = ax^2 + bx + c$ where $a \neq 0$ is a ________________. Features of this type of function that can be compared include minimums, maximums, and intercepts.

3. In a ________________, the ratio of y to x is equal to a nonzero constant k. The graph is a line whose ________________ is k.

Skill Review

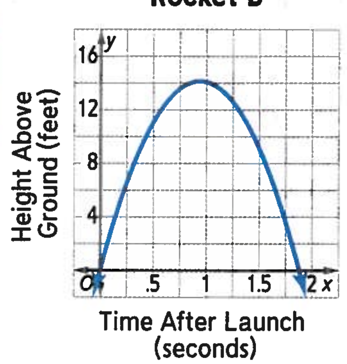
Directions: Read each problem and complete the task.

The heights of two model rockets that were launched at the same time are described in the table and graph below. Use this information for 1 and 2.

Rocket A

Time After Launch (seconds)	Height Above Ground (feet)
0	25
0.5	37
1	41
1.5	37
2	25
2.5	5

Rocket B

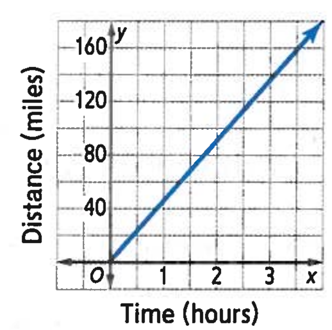


1. Jonas says that he cannot determine which rocket reached the ground first because the table does not contain any information about the time that Rocket A reached the ground. Is Jonas correct? Explain why or why not.

2. Which statement is correct?
   A. Rocket A was launched from 25 feet above the ground and Rocket B from the ground.
   B. Both rockets were launched from the ground.
   C. Rocket A was launched 5 feet above the ground and Rocket B from 25 feet above the ground.
   D. Both were launched from 25 feet above the ground.

3. A car is traveling at a rate of 60 miles per hour. A truck is traveling as shown in the graph. Which statement is correct?

Truck



   A. After 4 hours, the car will have traveled twice as far as the truck.
   B. The truck's rate of speed is less than $\frac{1}{2}$ the car's rate of speed.
   C. It will take the truck 4 hours to travel the same distance that the car travels in 3 hours.
   D. The car's rate of speed is $\frac{3}{4}$ the truck's rate of speed.

4. A work day is 8 hours. Sandra's total pay p when she works h hours is described by the equation $p = 21.5h$. Boris's pay is described in the table. Who earns more money per day? How much more?

Hours Worked	Total Pay
0	0
2	45
4	90

Skill Practice

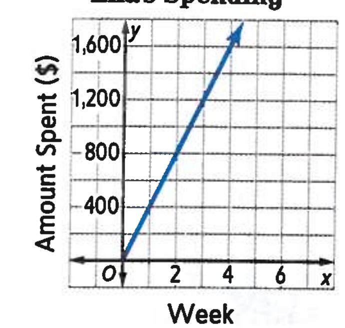
Directions: Read each problem and complete the task.

1. Lila works 5 days per week. Her weekly pay and spending are described in the table and graph below. Which statement or statements are correct?

Lila's Pay

Days Worked	0	1	2	3	4	5
Total Pay	\$0	\$100	\$200	\$300	\$400	\$500

Lila's Spending



   A. The amount that Lila spends each week is more than the amount she earns each week.
   B. Lila must work 4 days to earn as much as she spends in one week.
   C. After working 4 weeks, Lila could deposit \$500 in a savings account.
   D. If Lila reduces her weekly spending by \$50, she can save \$150 per week.

2. Order the functions from least y-intercept to greatest y-intercept.

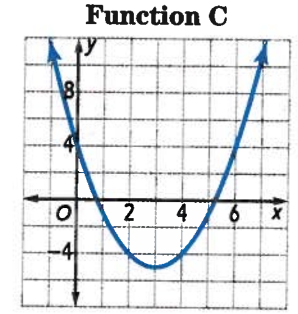
Function A

$f(x) = x^2 + 5x - 1$

Function B

x	$g(x)$
0	−7
1	1
4	1

Function C

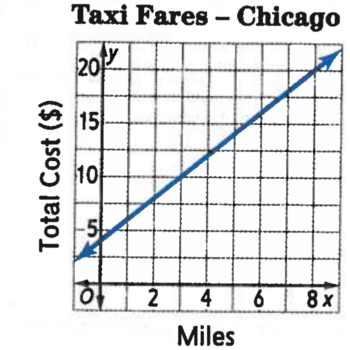


The total cost for a taxi ride usually consists of an initial charge plus a charge per mile. Taxi fares in four different cities are described below. Use this information for 3–5.

Taxi Fares – New York City

Miles	Total Cost
0	\$2.50
2	\$7.50
4	\$12.50
6	\$17.50

Taxi Fares – Chicago



The graph of Miami's taxi fares contains the points (1, 4.9) and (5, 14.5). The first coordinate in the ordered pairs represents distance in miles and the second coordinate represents cost.

In Dallas, the cost c to travel m miles in a taxi is $c = 1.8m + 2.25$.

3. Which city has the greatest initial charge? What is this initial charge?

4. Which city has the least charge per mile? What is this charge per mile?

5. Which shows the cities listed in order from least to greatest based on the total cost of a 15-mile taxi ride?
   A. Chicago, Dallas, New York City, Miami
   B. Dallas, Chicago, Miami, New York City
   C. New York City, Miami, Chicago, Dallas
   D. Miami, New York City, Dallas, Chicago

Directions: Choose the best answer to each question.

1. Which table of values corresponds to the function $f(x) = 2x^2 - 4x + 10$?

A.

x	f(x)
−2	26
−1	16
0	10
1	8
2	10

B.

x	f(x)
26	1
16	2
10	3
8	4
10	5

C.

x	f(x)
−2	10
−1	8
0	10
1	8
2	10

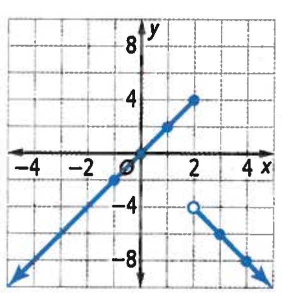
D.

x	f(x)
26	−2
16	−1
10	0
8	1
10	2

2. The function shown with the table is ________________ because the 2nd consecutive differences are the same value.

x	f(x)
−2	2
−1	1
0	2
1	5
2	10

3. Which function is represented by the graph?



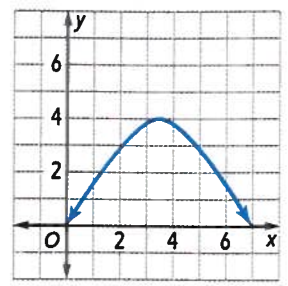
A. $f(x) = \begin{cases} -3x; & x \geq 2 \\ 3x; & x < 2 \end{cases}$

B. $f(x) = -3x$

C. $f(x) = 3x$

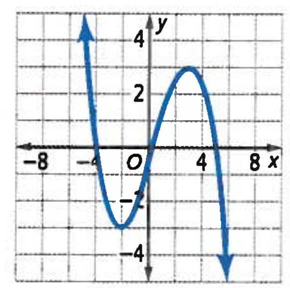
D. $f(x) = \begin{cases} -3x; & x > 2 \\ 3x; & x \leq 2 \end{cases}$

4. The graph below represents the situation of someone ________________.



5. A function is a rule that assigns exactly one output to each input. The set of inputs is called the ________________. The set of outputs is called the range.

Directions: Use the graph below to answer questions 6–8.



6. The y-intercept is −0.75 and the x-intercepts are ________________.

7. The increasing interval is ________________ and the decreasing intervals are $x < 2$ and $x > 3$.

8. The ________________ is 3 and the relative minimum is −3.

9. Ellen is paid \$18 an hour at her job. The equation $\frac{1}{15}p = h$ shows how Jake is paid, where p is the pay and h is the number of hours worked. So, ________________ is paid more per hour.

Directions: Choose the best answer to each question.

10. Which table of values corresponds to the function $f(x) = 12x - 20$?

A.

x	f(x)
−44	−2
−32	−1
−20	0
−8	1
4	2

B.

x	f(x)
−2	−44
−1	−32
0	−20
1	−8
2	4

C.

x	f(x)
0	20
2	4
3	16
4	28
5	80

D.

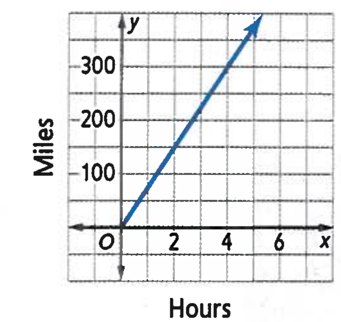
x	f(x)
−2	−4
−1	8
0	20
1	32
2	44

11. The input represents the number of products a company makes for a month. The output of a function represents the profit a company will make that month. How much profit will the company make if they make 100 products?

$f(x) = 2x^2 + 3x - 80$

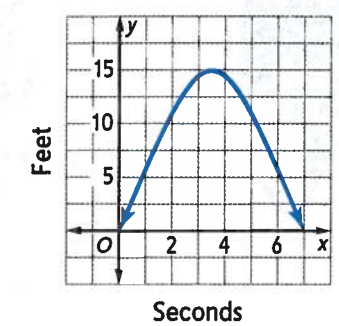
A. 420
B. 620
C. 20,220
D. 20,380

12. Which pair of points forms a line with a greater slope than the slope of the line shown?



A. (4, 153); (8, 313)
B. (2, 14); (4, 24)
C. (3, 25); (6, 55)
D. (2, 170); (5, 410)

13. Ava threw a ball in the air and graphed its heights. Bill also threw a ball in the air and recorded its heights in a table. Which is true?



time(s)	height(s)
2	10
3	15
4	18
5	15
6	10

A. Ava threw her ball higher than Bill threw his.
B. Bill threw his ball higher than Ava threw hers.
C. Ava and Bill threw their balls at the same height.
D. Bill didn't throw his ball as high as Ava threw hers.

Check Your Understanding

On the following chart, circle the items you missed. The last column shows pages you can review to study the content covered in the question. Review those lessons in which you missed half or more of the questions.

Lesson	Item Number(s)			Review Page(s)
	Procedural	Conceptual	Problem Solving	
6.1 Identify a Function	3	4, 5	11	184–191
6.2 Identify Linear and Quadratic Functions	2			192–199
6.3 Identify Key Features of a Graph		6, 7, 8	12	200–207
6.4 Compare Functions	1, 10		9, 13	208–215