Chapter 1

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Number Sense and Operations

Numbers are everywhere in your daily life. From the time you wake up until you go to bed, you will encounter numbers in a variety of forms. You will use numbers and math to help understand situations, solve problems, and make decisions. During your morning commute you may have to use a toll booth or take a bus or subway. It is important to make sure you have the correct change. At work, your boss may ask you to calculate the number of sales for the month. On your way home, you might stop to pick up food for dinner and use coupons to get the best deal. These scenarios all involve numbers, likely written as fractions and decimals. They will show up everywhere in your day, and it is important to understand what the numbers mean and how to calculate and use them.

Lesson 1.1
Order Rational Numbers

When you are handed a memo at work, you may see numbers written as fractions or decimals. How do you compare the numbers and understand what the memo is trying to communicate? Learn how to identify and compare different types of numbers using a number line.

Lesson 1.2
Apply Number Properties

Numerical expressions can represent situations you encounter in your daily life, such as calculating a tip on a restaurant bill. You can use properties of numbers to quickly and accurately evaluate expressions. Learn how to apply the order of operations and such properties as the Distributive Property.

Lesson 1.3
Compute with Exponents

How can you find the area of the floor you need to tile or the volume of a container? Exponents are useful to calculate volume and area, as well as to solve other real-world situations. Learn how to apply the rules of exponents to rewrite and calculate exponent expressions.

Lesson 1.4
Compute with Roots

If addition is the inverse operation of subtraction, what is the inverse operation of exponents? Roots are operations that can "undo" the process of applying exponents. Learn how to calculate with roots and use roots to work backwards and solve problems involving exponents.

Goal Setting

Think about the last time you were in the grocery store. How did you decide what to buy? Do you always buy the bulk size or do you choose the cheapest option? If you have coupons, how do you figure out the reduced price of the item? How are prices labeled at your store? Where do you see fractions and decimals used in labels and packaging?

How could the lessons in this chapter help you make decisions while shopping? How could understanding how to compare rational numbers help compare prices and options?

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LESSON 1.1 Order Rational Numbers

LESSON OBJECTIVES

- Identify rational numbers
- Order fractions and decimals on a number line
- Calculate absolute value

CORE SKILLS & PRACTICES

- Use Math Tools Appropriately
- Apply Number Sense

Key Terms

absolute value
the distance a number is from zero

integers
the set of whole numbers and their opposites

rational number
the set of numbers that can be expressed as the ratio of two integers

Vocabulary

denominator
the bottom number of a fraction that represents the total number of parts contained in the whole of a fraction

numerator
the top number in a fraction that represents the part of the whole the fraction is describing

order
to place in the proper sequence

Key Concept

Rational numbers include whole numbers, fractions, decimals, and their opposites. A number line is a useful math tool for comparing and ordering rational numbers.

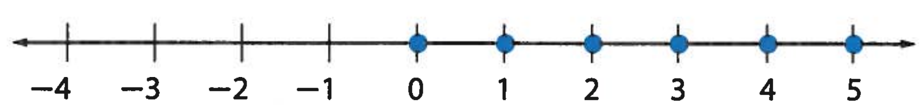
Rational Numbers

Rational numbers are part of the set of real numbers. A real number is any number you would find on a number line, and there are many different types. The numbers you use every day are examples of rational numbers. A number identifies the subway line that you need. Other numbers tell you the cost of the fare, the time your train arrives at the station, and how many stops you will pass before reaching your destination.

Types of Numbers

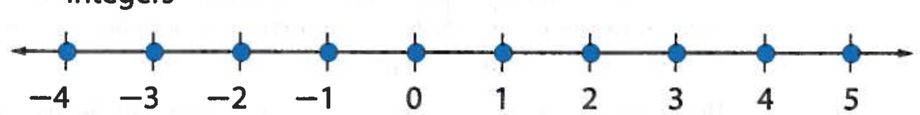
When we count, we use the numbers 1, 2, 3, 4, 5… These are called natural numbers. If there are no objects to count, the number 0 is included. The set of natural numbers and 0 are the whole numbers.

Whole Numbers



In some instances, we need more than whole numbers to describe or measure a quantity. Think about temperature. Negative numbers describe temperatures below zero. **Integers** are the whole numbers and their opposites.

Integers



We often use numbers that fall between integers, like a 26.2 mile-long race or a −22.5°F temperature. Most of the numbers you encounter can be expressed as fractions or terminating decimals (decimals that have a finite number of figures). These form a larger set of numbers, called the **rational numbers,** or all numbers that can be expressed as the ratio of two integers, $\frac{a}{b}$, where $b \neq 0$. The rational numbers include the natural numbers, whole numbers, integers, fractions, and terminating or repeating (continuing a pattern forever) decimals. When writing repeating decimals, a bar is written over the number or numbers that repeat. For example, the number 0.416666..., where 6 repeats forever, would be written as $0.41\overline{6}$.

Example 1: Examples of Numbers

Natural Numbers	1 2 30 127
Whole Numbers	0 7 64 591
Integers	−27 −4 0 28
Fractions	$\frac{1}{2}$ $\frac{4}{9}$ $7\frac{3}{8}$ $\frac{12}{7}$
Terminating Decimals	−0.5 3.2 27.704
Repeating Decimals	$-2.\overline{3}$ $0.\overline{12}$ $7.4\overline{63}$ $12.71\overline{4}$

Unlike rational numbers, **irrational numbers** cannot be expressed as the ratio of two integers. They are non–terminating decimals that do not repeat. They include the square roots of many whole numbers, such as $\sqrt{2} = 1.41421\ldots$. Another example is the number pi, the ratio of the circumference of a circle by its diameter. Pi is represented by the symbol π, which is 3.14159…. When calculating using pi, most people use the estimation 3.14.

Fractions and Decimals

Whole numbers are not always as common as rational numbers in daily life. A kitchen is an example of a place where whole–number measurements are rare. For example, a recipe may call for $\frac{3}{4}$ cup of sugar.

Fractions

Fractions represent equal parts of a whole. The top number, or **numerator,** identifies the number of parts of the whole you are describing. The bottom number, or **denominator,** identifies the total number of parts contained in the whole. Together, whole numbers and fractions form mixed numbers like $3\frac{1}{5}$.

$$\frac{5}{8} \begin{array}{l} \longleftarrow \text{numerator—parts of the whole you have} \\ \longleftarrow \text{denominator—total number of parts in the whole} \end{array}$$

WORKPLACE SKILL

Check, Examine, and Record

Calculations using rational numbers are done in the workplace each day. For example, many jobs involve handling money. You may be asked to purchase items for your work or verify the cost of a customer's purchase.

It is important to know how to calculate correctly in situations that involve money.

As a bank teller, Alisha counts and records the total dollar amount of the coins in her drawer at the end of each day. Today she counted 110 quarters. What dollar amount will Alisha record?

CALCULATOR SKILL

Many calculators are able to convert numbers between fractions and decimals. To convert a fraction to a decimal using the TI-30XS MultiView™, press the 2nd key to access the second function, then the table key, whose second function allows you to "toggle" the number shown on the display back and forth from a fraction to a decimal.

Decimals

Terminating and repeating decimals are types of rational numbers. You rely on ten digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) to write every number in our number system. Each digit in a number has a specific place value, or value based on its position in the number.

In a place–value chart like the one shown, a decimal point separates whole numbers from parts of a whole, or decimals. Whole numbers are to the left of the decimal point, and decimals are to the right. As you move to the right, each place value is one–tenth the value. The opposite is true as you move to the left.

$\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$ $\times\frac{1}{10}$

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones	and	tenths	hundredths	thousandths	ten thousandths	hundred thousandths	millionths
				3	2	6	.	7	5				

×10 ×10 ×10 ×10

Since decimals represent fractional values, we read them as tenths, hundredths, thousandths, and so on. When reading decimals aloud, the word *and* represents the decimal point and only the last decimal place is named.

For example, read 326.75 as "three hundred, twenty–six and seventy–five hundredths." As a fraction, this would be written $326\frac{75}{100}$ or $326\frac{3}{4}$.

Think about Math

Directions: Answer the following questions.

1. Which number has a 3 in the tens place?
   A. 317.426
   B. 623.109
   C. 8,234.67
   D. 1,970.32

2. Which of the following categories apply to the number 7? Select all that apply.
   A. Whole number
   B. Integer
   C. Rational number
   D. Irrational number

Working With Fractions and Decimals

Fractions and decimals are common rational numbers you see and use each day. At a post office, for example, guidelines show the dimensions of letters, postcards, and boxes in fractions. Price charts show the cost of stamps and delivery options in decimals.

Compare and Order Fractions

To compare two fractions, you first want to take note if they have the same denominators or numerators.

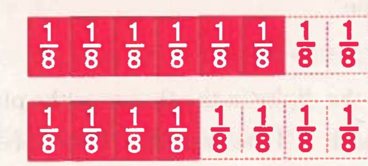
Example 1: Same Denominators

Compare $\frac{6}{8}$ and $\frac{4}{8}$.

Step 1 Observe that both fractions have the same denominator. To compare them, read the numerators.

Step 2 The fraction with the greater numerator is the greater fraction. You can see this by comparing two fraction bars. Each bar is split into the same number of sections, but one has more filled in, and is therefore the greater fraction.

$\frac{6}{8} > \frac{4}{8}$



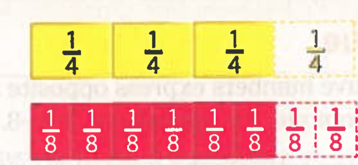
When fractions have different denominators and numerators, you can rewrite or rename one or more fractions in a set before you compare them. Once all of the fractions you are comparing share the same denominator, you can compare numerators.

Example 2: Different Numerators and Denominators

Compare $\frac{3}{4}$ and $\frac{5}{8}$. The fractions do not share the same denominator.

Step 1 Rewrite one or both fractions so that they have the same denominator by multiplying each fraction's numerator and denominator by the same number. Since $8 = 2 \times 4$, multiply $\frac{3}{4}$ by $\frac{2}{2}$ to get a fraction in eighths. As you can see in the fraction bars, it does not change the value of the fraction.

$\frac{3}{4} = \frac{2 \times 3}{2 \times 4} = \frac{6}{8}$



Step 2 Compare the fractions by comparing their numerators.

$\frac{6}{8} > \frac{5}{8}$, so $\frac{3}{4} > \frac{5}{8}$

CORE SKILL

Apply Number Sense

When you are given a set of numbers to compare, it is usually easiest to make sure they are of the same kind, either fractions or decimals. Since not all numbers can be written as fractions, decimals are an easier way to compare numbers.

For example, suppose a factory manager wants to know which of three products uses the most feet of plastic wrapping. Three employees report the average length of wrapping they use. The manager records the information in a data table.

Product A	Product B	Product C
5.25 feet	4.8 feet	$5\frac{3}{8}$ feet

Two of the values in the data table are decimals, and one is a fraction. To convert a fraction into a decimal, divide the numerator by the denominator.

$$5\frac{3}{8} = \frac{43}{8}$$

$$43 \div 8 = 5.375$$

$$5\frac{3}{8} = 5.375$$

Order the three numbers by comparing each digit from left to right. In this case, $4 < 5$ and so 4.8 is the smallest number. Since $0.2 < 0.3$, the next smallest number is 5.25. Therefore, the largest number is 5.375.

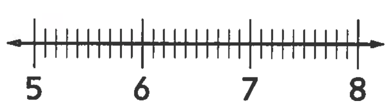
CORE PRACTICE

Use Math Tools Appropriately

To use math tools appropriately, first think about the problem you are trying to solve and which math tools can aid you in finding the answer. One such tool is a number line. A number line shows numbers from least to greatest. By plotting all the numbers on the number line, you can determine the order of the numbers from least to greatest by reading the numbers from left to right.

To compare and order decimals, it is helpful to use a number line marked off by tenths. This divides the space between each integer into 10 equal sections. To plot a number like 6.25, find the marks for 6.2 and 6.3 and plot the number halfway between.

Use the number line below to plot the decimals 6.75, 6.25, 6.4, and 7.1, and order them from least to greatest.



Compare and Order Decimals

To compare and **order** two decimals, you need to make sure you compare digits with the same place value. Suppose you want to compare 1.21 and 1.213.

Example 3: Compare Decimals

Step 1 To give both decimals the same number of digits before you compare them, add a zero to the end of 1.21. Adding a zero to the end of a decimal does not change its value.

1.210 1.213

Step 2 Compare the digits in the ones place, or whole number position.

1 = 1. The ones digits have the same value.

$\underline{1}.210$ $\underline{1}.213$

Step 3 Compare the digits in the tenths place.

2 tenths = 2 tenths. The tenths digits have the same value.

$1.\underline{2}10$ $1.\underline{2}13$

Step 4 Compare the digits in the hundredths place.

1 hundredth = 1 hundredth. The hundredths digits have the same value.

$1.2\underline{1}0$ $1.2\underline{1}3$

Step 5 Compare the digits in the thousandths place.

0 thousandth < 3 thousandths. Therefore 1.210 is less than 1.213.

$1.21\underline{0}$ $1.21\underline{3}$

1.21 < 1.213

Think about Math

Directions: Compare each number to 4.65. Check the line that each number corresponds to.

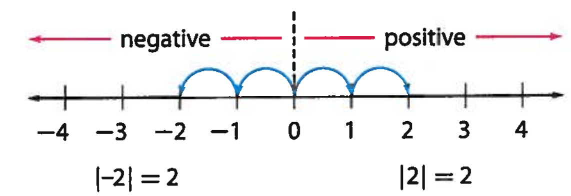
1. $4\frac{3}{4}$ ___ Less than 4.65 ___ Greater than 4.65
2. 4.37 ___ Less than 4.65 ___ Greater than 4.65
3. 4.72 ___ Less than 4.65 ___ Greater than 4.65
4. $4\frac{1}{5}$ ___ Less than 4.65 ___ Greater than 4.65

Absolute Value

Positive and negative numbers express opposite amounts. Every integer has an opposite. For example, the opposite of 3 is –3.

On a number line, opposite numbers are always the same distance from zero. The distance from zero is called the absolute value of the number. The symbol for **absolute value** is | |. Because absolute value is the distance to 0, it is always a positive amount or 0.

The absolute value of 2 (written |2|) is 2, and the absolute value of –2 (written |–2|) is also 2 because both numbers are a distance of 2 units from 0. Because 0 is zero distance from itself, the absolute value of 0 is 0.



Adding and Subtracting Integers Using Absolute Value

When adding two integers, look at the signs of the integers. If the integers have like signs, find the sum of the integers' absolute values. Then give the sum the same sign as both integers. For example, –6 + –12 = –18. If the integers have unlike signs, subtract the integers' absolute value as shown.

Example 4: Unlike Signs

Add –8 + 6.

Step 1 Subtract the integers' absolute values.

$|-8| - |6| = 8 - 6 = 2$

Step 2 Give the difference the sign of the integer with the greater absolute value.

$|-8| > |6|$, so make the difference negative. $-8 + 6 = -2$

Subtracting an integer is the same as adding the opposite of that integer. Change the number that is being subtracted to its opposite. Then add the integers. Once you know how to subtract integers, you can find the distance between two points.

Example 5: Finding Distance on a Number Line

The distance between two integers on a number line is the absolute value of their difference. Find the distance between –4 and –9.

Step 1 Find the difference of the two numbers. It does not matter in which order you subtract them because you will be taking the absolute value of the difference.

$|-4 - (-9)| = |-4 + 9| = |5|$

Step 2 Take the absolute value of the difference. $|5| = 5$

Think about Math

Directions: Choose the best answer to each question.

1. What is the distance between the numbers –1 and 5?
   A. –4
   B. –6
   C. 6
   D. 4

2. What is the sum of –7 + 3?
   A. –10
   B. 4
   C. –4
   D. 10

21ST CENTURY SKILL

Environmental Literacy

In chemistry, a pH level indicates whether a solution is acidic, basic, or neutral. On a pH scale from 0 to 14, pure water has a pH of 7. Chlorine is added to swimming pools to destroy harmful organisms that may be in the water. For chlorine to be effective, a water pH of 7.3 is ideal. However, a pH level that is more than 0.3 away from ideal is considered unacceptable.

You can use absolute value to identify which pools in the table below have acceptable or unacceptable pH levels. For example, if a pool had a pH value of 7.8, you can find the distance from the ideal using absolute value and compare to 0.3.

$|7.3 - 7.8| = |-0.5| = 0.5$

$0.5 > 0.3$

The pH level is more than 0.3 away from the ideal. Therefore, the pool is unacceptable. Using the values in the table, determine which pools have acceptable pH levels and which do not.

pH Level	
Pool A	7.4
Pool B	7.7
Pool C	7.9
Pool D	7.1

Vocabulary Review

Directions: Write the missing term in the blank.

absolute value	**denominator**	**integers**
order	**numerator**	**rational number**

1. Rational numbers can be placed in ________________ from least to greatest.
2. In the fraction $\frac{3}{4}$, the number 3 is the ________________.
3. A(n) ________________ is any number that can be expressed as a ratio of two numbers.
4. The ________________ is the total number of parts in a whole.
5. The ________________ of –4 is 4.
6. The set of natural numbers, their opposites, and the number zero form the set of ________________.

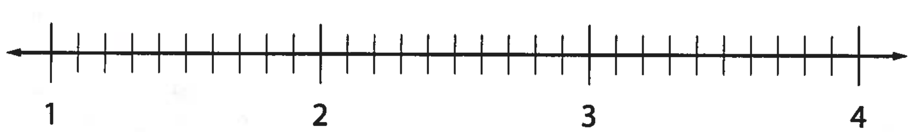
Skill Review

Directions: Read each problem and complete the task.

1. A lab technician measured the temperature of four different substances and recorded the temperatures in a data table. Now she wants to compare them.

Substance	X	Y	Z	W
Temperature	3.3°	3.15°	3.9°	3.55°

Order the temperatures on the number line. Then choose the appropriate ordering from least to greatest from the choices below.



A. 3.3°, 3.15°, 3.9°, 3.55°
B. 3.15°, 3.55°, 3.3°, 3.9°
C. 3.3°, 3.55°, 3.9°, 3.15°
D. 3.15°, 3.3°, 3.55°, 3.9°

2. The factory manager asks employees to use a new kind of transparent wrapping for their three top–selling items. The lengths recorded in the data table indicate how much wrapping each item requires. Compare the values and order them from least to greatest.

Item A	Item B	Item C
3.65 feet	4.1 feet	$3\frac{11}{16}$ feet

3. Explain the difference between rational and irrational numbers. Give examples of both in your explanation.

4. Determine which number has the greatest distance from the number 3.

A. –6
B. –2
C. 7
D. 11

5. A foot contains 12 inches. 5 inches is what fraction of a foot?

A. $\frac{5}{12}$
B. $\frac{1}{5}$
C. $\frac{1}{12}$
D. $\frac{5}{1}$

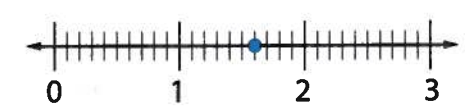
6. A wooden crate weighing $2\frac{5}{16}$ pounds contains grapefruit weighing $24\frac{1}{2}$ pounds. What is the combined weight of the crate and the grapefruit?

A. $26\frac{6}{18}$ pounds
B. 27 pounds
C. $26\frac{13}{16}$ pounds
D. $26\frac{3}{8}$ pounds

Skill Practice

Directions: Read each problem and complete the task.

1. Determine which rational number is represented on the number line shown.



A. 0.5
B. 0.6
C. 1.5
D. 1.6

2. Which rational numbers are within 1 unit of the rational number represented on the number line shown above?

A. 1.5 and –1.5
B. 2.5 and 0.5
C. 2.6 and 0.6
D. 1.6 and –1.6

3. Use a number line to compare the fractions $\frac{9}{5}$ and $\frac{5}{2}$.

4. Which of the following numbers have a distance of 3 from the number 8? Select all that apply.

A. –5
B. 2
C. 5
D. 11

5 Explain why absolute value is always positive or zero.

6. Find the sum of $3\frac{1}{3} + 2\frac{3}{4} + 5\frac{5}{6}$.

A. $10\frac{9}{13}$
B. $11\frac{11}{12}$
C. $11\frac{8}{13}$
D. $10\frac{11}{12}$

LESSON 1.2 Apply Number Properties

LESSON OBJECTIVES

- Determine LCM and GCF of two positive numbers (not necessarily different)
- Apply number properties (Distributive, Commutative, and Associative Properties) to rewrite numerical expressions
- Determine when a numerical expression is undefined

CORE SKILLS & PRACTICES

- Apply Number Sense Concepts
- Perform Operations

Key Terms

greatest common factor (GCF)
the greatest factor that is shared between the numbers

least common multiple (LCM)
the least multiple that is shared between the numbers

order of operations
the rules for the order that calculations should be done when evaluating an expression

Vocabulary

addend
a number that is added to another number

factor
a number that is multiplied by another number

undefined
an expression that cannot be evaluated

Key Concept

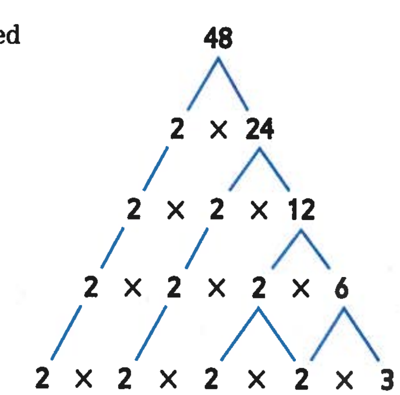
The least common multiple and greatest common factor of a pair of numbers can be used to solve problems. Awareness of number properties can be helpful in evaluating numerical expressions, although some expressions are undefined.

Factors and Multiples

Suppose you and a friend start jogging around a track at different speeds. You may meet up with each other at different points around the track. You can use the mathematical concepts of factors and multiples to find out how long it will take to meet up again at the starting point.

Prime Factorization

Whole numbers greater than 1 are considered either prime or composite. A prime number has only itself and the number 1 as its factors. A **factor** is any whole number that can be multiplied by another number. The result is the product. A composite number has itself, 1, and at least one other whole number as its factors. The number 1 is neither prime nor composite.



$$48 = \underbrace{2 \times 2 \times 2 \times 2}_{} \times 3$$

$$48 = 2^4 \times 3$$

To list the factors of a composite number, identify whole numbers that divide evenly into the number. The number 48 can be divided evenly by 2, so both 2 and 24 are factors of 48.

The prime factorization of a composite number shows the number written as the unique product of its prime factors. Tree diagrams like the one shown are often used to break apart the number into its factors.

You can use powers to simplify a number's prime factorization by using repeated factors as the base and the number of times the factor appears as the exponent.

Don Paulson Photography/Purestock/SuperStock

Greatest Common Factor

A pair of whole numbers can have many factors in common. To find common factors of a pair of numbers, list the factors of each number and identify the shared factors. There are four common factors for 24 and 30.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
common factors

The **greatest common factor (GCF)** is the greatest factor that is shared between two composite numbers. You can find the GCF by finding the largest common factor from the list of the factors for each number. The GCF for 24 and 30 is 6.

$$189 = 3^3 \times 7$$
$$440 = 2^3 \times 5 \times 11$$
$$\text{GCF}(189, 440) = 1$$

189 and 440 are relatively prime.

Two numbers for which the GCF is 1 are said to be relatively prime. The numbers 189 and 440 are relatively prime because they have no common factors other than 1.

Least Common Multiple

A multiple of a number is the product of the number and any natural number. Just as with factors, pairs of numbers can have common multiples. Common multiples of 4 and 5 are 20, 40, 60, etc.

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...
Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, ...
common multiples

The **least common multiple (LCM)** is the least multiple that is shared between the numbers. To find the LCM, you can write the first several multiples of each number and identify the least number in both lists. The LCM for 4 and 5 is 20.

You can also find the LCM of a pair of numbers by examining their prime factorizations. The least common multiple is the product of the highest power of each factor. Notice that the factors do not need to be shared by the numbers to be included in the LCM calculation.

$$120 = 2^3 \times 3 \times 5$$
$$252 = 2^2 \times 3^2 \times 7$$
$$\text{LCM}(120, 252) = 2^3 \times 3^2 \times 5 \times 7 = 2{,}520$$

Think about Math

Directions: Answer the following questions.

1. Which is the GCF of 36 and 90?
   A. 2
   B. 3
   C. 6
   D. 18

2. Which statement is true?
   A. The GCF of 5 and 15 is 15.
   B. The LCM of 7 and 21 is 21.
   C. The GCF of 60 and 126 is 36.
   D. The LCM of 24 and 27 is 648.

CORE SKILL

Apply Number Sense Concepts

The greatest common factor of two numbers is useful when reducing fractions, and the least common factor is used when adding and subtracting fractions. For instance, to reduce $\frac{8}{12}$, you find the GCF of 8 and 12, which is 4. Therefore, you can divide both 8 and 12 by 4 to get the reduced fraction $\frac{2}{3}$. When adding or subtracting fractions, you find the least common factor of the denominators (also known as least common denominator) instead of finding the GCF. Knowing whether the GCF or LCM is being asked in a problem is an important problem-solving skill.

For example, suppose a jeweler has 60 lengths of wire and 48 charms to use to make bracelets. He will use the same number of lengths of wire and the same number of charms on each bracelet, and he wants to make as many bracelets as possible, using all of his materials. How many bracelets will he be able to make?

Properties of Numbers

Construction workers rely on using the right tools in order to perform each job properly. Often times, different tools might be used for the same job, but using a specific tool makes the job less challenging. In mathematics, there are certain properties of numbers that you can use as tools to help make your calculations easier.

Commutative Property

The Commutative Properties deal with the order of numbers. The Commutative Property of Addition states that you can add two numbers in either order without affecting the sum. Think about a person training for a race. If they run 4 miles then 3 miles, or 3 miles then 4 miles, they still have run 7 miles total.

Similarly, the Commutative Property of Multiplication allows you to switch the order of two factors without changing the product. Both of these properties hold true for whole numbers, integers, and rational numbers, including fractions and decimals.

Commutative Property of Addition
$a + b = b + a$

Example:
$4 + 5 = 5 + 4$
$0.5 + (-1) = (-1) + 0.5$
$-\frac{1}{3} + \frac{2}{3} = \frac{2}{3} + \left(-\frac{1}{3}\right)$

Commutative Property of Multiplication
$a \times b = b \times a$

Example:
$2 \times 3 = 3 \times 2$
$-0.25 \times (-8) = -8 \times (-0.25)$
$\frac{4}{5} \times \frac{1}{4} = \frac{1}{4} \times \frac{4}{5}$

The Commutative Property does not hold for the operations of subtraction or division. For these two operations, the order in which the numbers are written have an effect on the difference and quotient.

Associative Property

The Associative Property of Addition states that you can group **addends**, or numbers added to get another number, in different ways without affecting the sum.

Similarly, the Associative Property of Multiplication allows you to change the grouping of factors without changing the product.

Associative Property of Addition
$(a + b) + c = a + (b + c)$

Example:
$(1 + 2) + 3 = 1 + (2 + 3)$
$3 + 3 = 1 + 5$
$6 = 6$

Associative Property of Multiplication
$(a \times b) \times c = a \times (b \times c)$

Example:
$(2 \times 4) \times 6 = 2 \times (4 \times 6)$
$8 \times 6 = 2 \times 24$
$48 = 48$

Using the Associative and Commutative Properties together, you can reorder and change the grouping of addends or factors to make computing with them easier. This can be helpful in evaluating complicated expressions using mental math.

Examples:

$\left(-\frac{1}{4} + 3.9\right) + \frac{5}{4}$ and $\left(-0.25 \times \frac{7}{9}\right) \times (-4)$

Step 1

$\left(-\frac{1}{4} + 3.9\right) + \frac{5}{4}$ $\quad$ $\left(-0.25 \times \frac{7}{9}\right) \times (-4)$

$= \left(3.9 + \left(-\frac{1}{4}\right)\right) + \frac{5}{4}$ $\quad$ $= \left(\frac{7}{9} \times (-0.25)\right) \times (-4)$ ← Commutative Property

Step 2

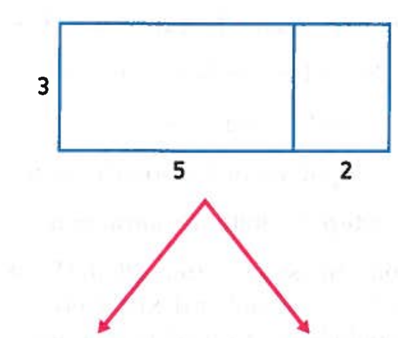
$= 3.9 + \left(-\frac{1}{4} + \frac{5}{4}\right)$ $\quad$ $= \frac{7}{9} \times (-0.25 \times (-4))$ ← Associative Property

Step 3

$= 3.9 + 1 = 4.9$ $\quad$ $= \frac{7}{9} \times 1 = \frac{7}{9}$ ← Simplify

Distributive Property

The Distributive Property can be illustrated with the area of adjoining rectangles like the ones shown. On the left, the area of the entire rectangle is found by first finding the sum of the partial side lengths and then multiplying by the common side. On the right, the area is found by adding the areas of each of the smaller rectangles together. The result is the same.



$3(5 + 2)$ $\quad$ $(3 \times 5) + (3 \times 2)$
$= 3(7)$ $\quad$ $= 15 + 6$
$= 21$ $\quad$ $= 21$

Stated mathematically, you can see how the Distributive Property gets its name. The factor outside the parentheses is distributed to each of the addends inside the parentheses.

Distributive Property:
$a(b + c) = ab + ac$

Think about Math

Directions: Use the property listed to simplify the expression.

1. $25(4 \times 7)$; Associative
2. $9(12 - 3)$; Distributive
3. $\left(5 \times \frac{1}{4}\right) \times 20$; Commutative and Associative

21ST CENTURY SKILL

Business Literacy

In business, a company's profit is calculated by subtracting the operating costs from the revenue. Potential investors will look at predictions of yearly revenue and cost to determine if a company is worth investing time and money in.

A small business's average daily revenue over the past few months was \$875.22. Its daily operating cost during that time was \$750.79. Write and solve an expression that determines the total profit for the business during the whole year.

CORE PRACTICE

Perform Operations

Understanding how to simplify expressions using the order of operations is an important skill to have. However, sometimes parentheses can be forgotten, which may or may not change the value of an expression.

For example, the expression $18 + 4 \times 7 + 10 - 3^2 \times 2$ simplifies to 38. If a set of parentheses are added to create the expression $18 + 4 \times 7 + (10 - 3^2) \times 2$, the expression now simplifies to 48.

Rewrite the expression $3 + 4^2 - 5 \times 7 + 11$ adding in one set of parentheses so that the expression equals 25.

Order of Operations

There is a certain order in which everyday tasks must be performed. For example, you cannot put on your shoes before you put on your socks. The same is true with operating with numbers in mathematics. The order in which different operations are performed has a direct impact on the final answer, so certain rules and conventions must be followed.

Understanding the Order of Operations

There may be several operations involved in evaluating an expression. You might reach a different answer depending on the order in which you do those operations.

For the expression $3^2 - 2 \times 3$, the correct value is 3. To make sure there is only one correct answer for any given problem, mathematicians have agreed on an order in which to perform the operations when evaluating expressions with multiple operations. This order is called the **order of operations**.

- First simplify inside parentheses or other grouping symbols.
- Second, evaluate any exponents.
- Next, work in order from left to right to multiply or divide.
- Finally, work in order from left to right to add or subtract.

So, to evaluate the expression $3^2 - 2 \times 3$, we use the following steps:

Step 1 Parentheses (none)	$3^2 - 2 \times 3$
Step 2 Exponents	$= 9 - 2 \times 3$
Step 3 Multiplication/Division	$= 9 - 6$
Step 4 Addition/Subtraction	$= 3$

You can use the letters *PEMDAS* or the phrase *Please Excuse My Dear Aunt Sally* to remember the first letters of the operations in the order that they should be performed: parentheses, exponents, multiplication and division, and addition and subtraction.

Using the Order of Operations

To evaluate an expression containing multiple operations, be sure to follow the order of operations.

	$30 - (6 - 3)^2 + 8 \div 2$
Step 1 Simplify inside the parentheses.	$= 30 - (3)^2 + 8 \div 2$
Step 2 Evaluate exponents.	$= 30 - 9 + 8 \div 2$
Step 3 Multiply and divide in order from left to right.	$= 30 - 9 + 4$
Step 4 Add and subtract in order from left to right.	$= 21 + 4 = 25$

The value of this expression is 25.

Undefined Expressions

Not all numerical expressions can be evaluated to obtain a numerical result. Such expressions are said to be **undefined**.

The most common example of an undefined expression involves division by 0, which is itself undefined.

To understand why, think about the equal-groups representation of division. The expression $8 \div 4$ can be interpreted as separating 8 objects into 4 groups of 2, or into 2 groups of 4. The expression $8 \div 0$ would then mean separating 8 objects into 0 groups, or into groups of 0, which is not possible. Therefore, division by zero is undefined.

When you are evaluating a numerical expression according to the order of operations, be on the lookout for steps that result in division by 0. Such expressions are undefined and cannot be evaluated further.

$$17 + 2^3 \div (16 - 2 \times 8) + 9$$
$$17 + 2^3 \div (16 - 16) + 9$$
$$17 + 2^3 \div 0 + 9$$

undefined

Think about Math

Directions: Find the value of each expression.

1. $12.5 + 6\,(15 - 12)^2 - 6.5$
2. $8 \times 5 \div 10 + 50 \div 5 \times (3 - 1)^2$
3. $25 - 30 \div (15 - 3 \times 5)^2$

CALCULATOR SKILL

Recognizing when an expression is undefined can help as a check when you are solving problems. One of the most recognizable undefined expressions is any expression that requires division by zero. This is because division by zero doesn't make sense in a realistic sense. When you enter an expression into your calculator that requires division by zero, the TI-30XS MultiView™ will show the error DIVIDE BY 0.

Enter expression $\frac{2}{((-1)^2 - 1)}$ into your calculator and see what the calculator shows. Can you think of a different mathematical expression that is undefined and will give an error on your calculator?

Vocabulary Review

Directions: Write the missing term in the blank.

addend	**factor**	**greatest common factor**
least common multiple	**order of operations**	**undefined**

1. The ________________ of two numbers is the smallest number for which both numbers are factors of that number.
2. When finding the sum of two numbers, each number is called a(n) ________________.
3. The number 6 is the ________________ of 24 and 42.
4. A(n) ________________ expression is one that has no answer.
5. When using the ________________, evaluate parentheses, exponents, multiplication/division, and addition/subtraction in that order, from left to right.
6. The number 12 has six ________________: 1, 2, 3, 4, 6, and 12.

Skill Review

Directions: Read each problem and complete the task.

1. Which expression completes the equation?

$5\,(12 + 23) = 5 \times 12 +$ ____

A. $5 + 23$
B. 5×23
C. $5 + 12$
D. 5×12

2. Which is the value of the expression?

$\frac{1}{8} \times (24 - 22)^3 - 3^2$

A. -8
B. 9
C. 55
D. undefined

3. Which is the greatest common factor of 25 and 45?

A. 25
B. 15
C. 5
D. 1

4. What is the prime factorization of 90?

5. Marquita owns a small business producing wooden toys. She has received an order for 120 toy trains and 95 toy soldiers. Because of the large order, she wants to break it up into equal shipments. How many boxes will she need if each box must contain an equal number of toy trains and an equal number of toy soldiers?

6. Which is the least common multiple of 15 and 20?

A. 300
B. 60
C. 35
D. 1

Skill Practice

Directions: Read each problem and complete the task.

1. Jay is making a painting based on a 10-inch by 15-inch picture. He divides the picture into grid squares. What are the greatest size grid squares he can make?

2. What numbers would make this expression undefined?

$18 \div (25 - x^2)$

3. To evaluate expressions with several sets of parentheses, find the inner set of parentheses and apply the order of operations within the parentheses before evaluating the entire expression. Which is the value of this expression?

$10 \times (12 - (3 \times 2)^2 + 6)$

A. -180
B. -8
C. 60
D. undefined

4. Find the GCF and LCM of $2^4 \times 3^2 \times 7^3 \times 13$ and $2^2 \times 3^3 \times 5 \times 11^2$.

A. $GCF = 2 \times 3 \times 5 \times 7 \times 11 \times 13$;
$LCM = 2^6 \times 3^5 \times 5 \times 7^3 \times 11^2 \times 13$
B. $GCF = 2^2 \times 3^2$;
$LCM = 2^4 \times 3^3 \times 5 \times 7^3 \times 11^2 \times 13$
C. $GCF = 2^4 \times 3^2 \times 7^3 \times 13$;
$LCM = 2^2 \times 3^3 \times 5 \times 11^2$
D. $GCF = 2 \times 3$
$LCM = 2 \times 3 \times 5 \times 7 \times 11 \times 13$

5. Which property can be applied to this expression?

$85 \times (100 - 5)$

6. A store receives shipments each day. Every 4 days it receives a shipment of milk, and every 10 days it receives a shipment with cookies. If they receive a shipment that includes both milk and cookies on April 2, when is the next date that they will receive a shipment that includes both?

7. Ethan is working on his monthly bills. He currently has $1,000 in his savings account. After receiving two paychecks of $1,100 each, he now needs to pay rent and other bills. His rent is $800, cell phone $90, groceries $200, utilities that cost $120 and are paid twice a month, and other expenses which total $350. Which expression shows how much Ethan will have in his savings account after paying bills?

A. $1{,}000 + 1{,}100 - 800 + 90 + 200 + 120 \times 2 + 350$
B. $1{,}000 + 1{,}100 - (800 + 90 + 200 + 120 \times 2 + 350)$
C. $1{,}000 + 2 \times 1{,}100 - (800 + 90 + 200 + 120 \times 2 + 350)$
D. $1{,}000 + 2 \times 1{,}100 - 800 + 90 + 200 + 120 \times 2 - 350$

8. Evaluate this expression, showing each step. Write the property you used or an explanation for each step.

13×24

$(10 + 3) \times (20 + 4)$
rewrite the problem for easier multiplication

$(10 + 3) \times 20 + (10 + 3) \times 4$

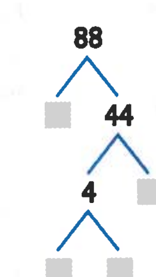
uses the ________________

$10 \times 20 + 3 \times 20 + 10 \times 4 + 3 \times 4$

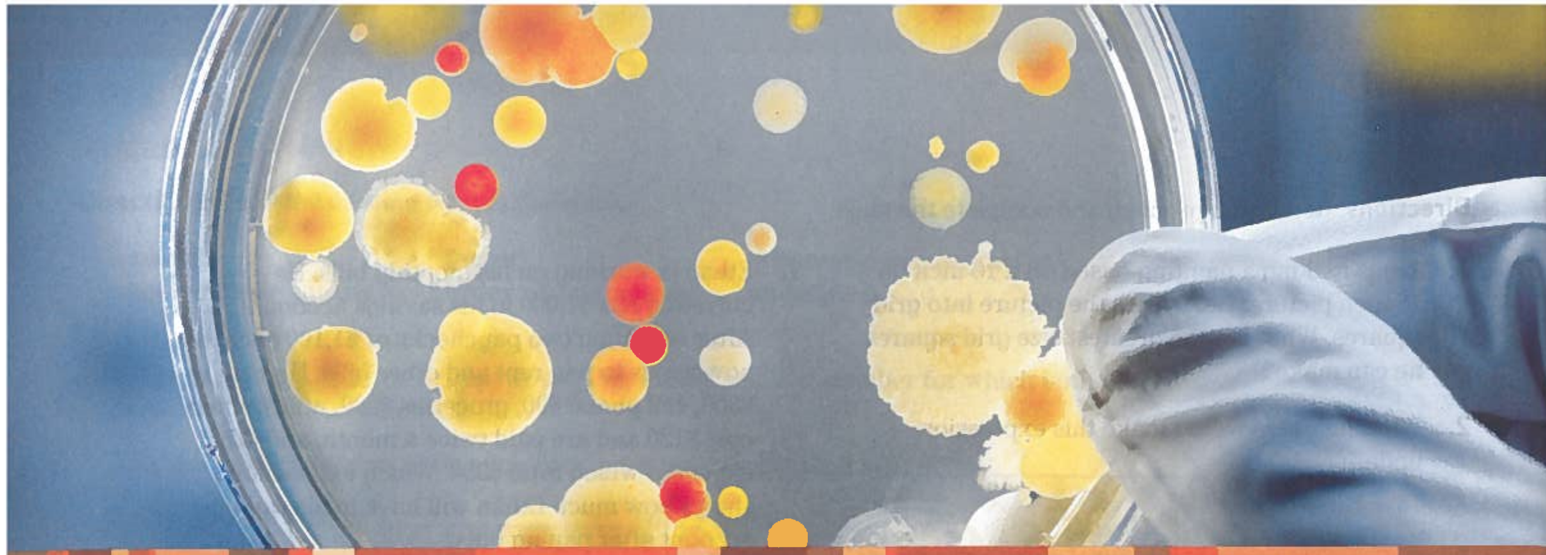
uses the ________________

$200 + 60 + 40 + 12 = 312$ simplified

9. Complete the prime factorization. Then write the prime factorization in exponent form.



88 = ________________



Andreas Reh

LESSON 1.3 **Compute with Exponents**

LESSON OBJECTIVES

- Apply rules of exponents to expressions
- Perform operations on numbers written in scientific notation
- Solve real-world problems involving squares and cubes

CORE SKILLS & PRACTICES

- Represent Real-World Problems
- Make Use of Structure

Key Terms

cube
a number raised to the third power

scientific notation
a system of writing a number as the product of a decimal and a power of 10

square
a number raised to the second power

Vocabulary

order of operations
the rules for the order that calculation should be done when evaluating an expression

reciprocals
two numbers or expressions whose product is 1

standard notation
the way in which a number is typically written, using place value

Key Concept

Exponents can be used to represent and solve problems, such as those involving squares and cubes or scientific notation. You can rewrite and simplify expressions involving exponents.

Exponential Notation

If you open a bank account with compound interest, you can use a formula involving an exponent to calculate the amount of money in your account after a certain amount of time.

Defining Powers

Repeated multiplications, like 4 × 4, can be expressed using powers. In a power, the number that is repeatedly multiplied is called the base. The small raised number is called an exponent. It tells you how many times to use the base as a factor. You can read the power shown here as "4 to the second power."

4^2 ← exponent; ← base; power

To evaluate a power, simply perform the repeated multiplication. The expressions 4 × 4 and 4^2 have the same value, 16.

You can evaluate powers involving exponents of 1, 0, or negative numbers. You can also raise decimal and rational numbers to a given power.

$a^1 = a$	A number to the 1st power uses the base number as a factor only 1 time. It is usually written without the exponent.
$a^0 = 1$	Any number to the zero power is equal to 1.
$a^{-n} = \frac{1}{a^n}, a \neq 0$	A nonzero number raised to a negative power is equal to the reciprocal of the number raised to a positive power. The **reciprocal** is any one of two numbers whose product is 1. Find the reciprocal by inverting the number written as a fraction.

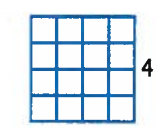
Example 1: Examples of Numbers in Exponential Notation

Write each number using exponential notation.

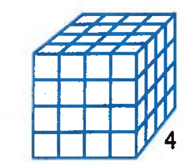
$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$ $\frac{1}{27} = \frac{1}{3 \times 3 \times 3} = \frac{1}{3^3} = 3^{-3}$

Squares and Cubes

The product of a number to the second power is usually called the **square** of the number. This is because if you show the multiplication visually with rows and columns, a square is formed. Similarly, a number raised to the third power is called the **cube** of the number. If you show the multiplication visually, a cube is formed. The expressions shown here can be read as "4 squared" and "4 cubed."



$4^2 = 4 \times 4 = 16$



$4^3 = 4 \times 4 \times 4 = 64$

Cubes and squares have special names because they frequently appear in real-world problems.

Example 2: Solving Real-World Problems

If an object is dropped, the distance it has fallen after t seconds is given by the expression $16t^2$. Find the number of feet that a dropped object has fallen after 2 and 3 seconds.

Step 1 Substitute $t = 2$ and $t = 3$ into the expression.

After 2 seconds: $16t^2 = 16(2)^2$

After 3 seconds: $16t^2 = 16(3)^2$

Step 2 Rewrite the powers using repeated multiplication.

After 2 seconds: $16(2)^2 = 16 \times 2 \times 2$

After 3 seconds: $16(3)^2 = 16 \times 3 \times 3$

Step 3 Evaluate.

After 2 seconds: $16 \times 2 \times 2 = 64$ ft

After 3 seconds: $16 \times 3 \times 3 = 144$ ft

Think about Math

Directions: Write and evaluate an exponential expression to solve the problem.

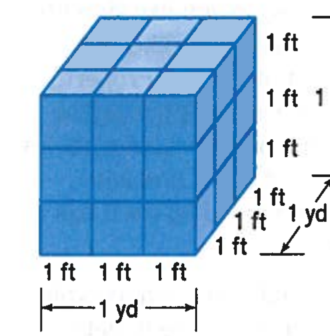
1. To the nearest dollar, what is the total cost of installing new carpet in a room that is 15.5 ft by 15.5 ft, if the carpet costs $3.75 per square foot and there is a $50 installation fee?

Expression: ______________ Cost: ________

CORE SKILL

Represent Real-World Problems

When you are solving a real-world problem, take note of any repeated multiplication that can be represented using exponential shorthand. For example, one cubic foot of granite weighs about 170 pounds. To find the weight in pounds of a cubic yard of granite, use the following diagram to understand how many cubic feet are in one cubic yard.



The exponential expression 3^3 gives the number of cubic feet in one cubic yard, so the weight of one cubic yard of granite can be calculated by evaluating the exponential expression 170×3^3.

If one cubic inch of gold weighs about 0.7 pound, what exponential expression represents the weight of one cubic foot of gold?

CORE PRACTICE

Make Use of Structure

You can use properties of exponents to understand why any number raised to a power of 0 is 1.

Consider the expressions shown in the table, and examine them in terms of their structure.

Expression	$\frac{4^7}{4^4}$	$\frac{4^7}{4^5}$	$\frac{4^7}{4^6}$	$\frac{4^7}{4^7}$
Power	4^3	4^2	4^1	4^0

The expressions on the top have like bases, so you can simplify each expression using the Quotient of Powers Property to get the expressions on the bottom. The last column shows that $4^0 = 4^7 \div 4^7$, and any number divided by itself has a value of 1.

Use a similar method to explain why it makes sense to define negative exponents using the reciprocal of the positive exponent.

Rules of Exponents

When you go to a science museum you will often see the skeletons of animals from thousands of years ago. When scientists discover these bones they can use a method called "carbon dating" to figure out the age. The mathematics behind carbon dating involves negative exponents, which you can understand by looking at some of the properties of exponents.

Products and Quotients of Powers

You can simplify expressions involving the product of two powers with like bases by rewriting the powers using repeated multiplication. Count the number of times the base is used as a factor and use that as the new exponent. A shortcut for multiplying powers with like bases is to keep the same base and add the exponents of each power for the new exponent. The answers are the same.

$$\begin{aligned}3^4 \times 3^2 &= (3 \times 3 \times 3 \times 3) \times (3 \times 3)\\ &= 3 \times 3 \times 3 \times 3 \times 3 \times 3\\ &= 3^6\end{aligned}$$

Shortcut:

$$\begin{aligned}3^4 \times 3^2 &= 3^{4+2}\\ &= 3^6\end{aligned}$$

You can also simplify the quotient of two powers with like bases by rewriting the power using repeated multiplication. Simplify by dividing out pairs of factors from the numerator and denominator. Notice that a shortcut for dividing powers with like bases is to keep the same base and subtract the exponents of each power for the new exponent.

$$\begin{aligned}4^7 \div 4^5 &= \frac{4^7}{4^5}\\ &= \frac{\cancel{4} \times \cancel{4} \times \cancel{4} \times \cancel{4} \times \cancel{4} \times 4 \times 4}{\cancel{4} \times \cancel{4} \times \cancel{4} \times \cancel{4} \times \cancel{4}}\\ &= 4^2\end{aligned}$$

Shortcut:

$$\begin{aligned}4^7 \div 4^5 &= 4^{7-5}\\ &= 4^2\end{aligned}$$

These two shortcuts hold for all numbers (except when $a = 0$ for a quotient of powers).

Product of Powers Property
$a^m \times a^n = a^{m+n}$

Quotient of Powers Property
$\frac{a^m}{a^n} = a^{m-n}$ (for $a \neq 0$)

Power of Powers

You can also simplify expressions involving the power of a power. Consider the power inside the parentheses as the base and rewrite using repeated multiplication. Then expand out each of those factors and count to determine the new exponent. A shortcut for finding the power of a power is to keep the base and multiply the exponents for the new exponent.

$$\begin{aligned}(5^2)^3 &= 5^2 \times 5^2 \times 5^2\\ &= (5 \times 5) \times (5 \times 5) \times (5 \times 5)\\ &= 5 \times 5 \times 5 \times 5 \times 5 \times 5\\ &= 5^6\end{aligned}$$

Shortcut:

$$\begin{aligned}(5^2)^3 &= 5^{2\times 3}\\ &= 5^6\end{aligned}$$

This shortcut also holds true for all numbers.

Power of a Power Property
$(a^m)^n = a^{mn}$

You can use the Power of a Power Property along with the Product and Quotient of Powers Properties to simplify more complicated expressions involving powers with like bases. Remember to follow the **order of operations**, or the order that the calculation should be done, any time you are evaluating an expression with multiple operations.

Example 3: Using Properties of Exponents

Simplify $\frac{(4^2 \times 4^6)^2}{(4^3)^5}$.

Step 1 Use the Product of Powers Property to simplify inside the parentheses of the numerator.

$$\frac{(4^2 \times 4^6)^2}{(4^3)^5} = \frac{(4^{2+6})^2}{(4^3)^5} = \frac{(4^8)^2}{(4^3)^5}$$

Step 2 Use the Power of a Power Property to simplify in the numerator and denominator.

$$\frac{(4^8)^2}{(4^3)^5} = \frac{4^{8\times 2}}{4^{3\times 5}} = \frac{4^{16}}{4^{15}}$$

Step 3 Use the Quotient of Powers Property to write the expression as a single power.

$$\frac{4^{16}}{4^{15}} = 4^{16-15} = 4^1 = 4$$

Powers of Products and Quotients

You can also simplify expressions that involve the power of a product (or quotient). Use repeated multiplication to expand the expression using the product (or quotient) as the base. Then rewrite the expression using the Commutative and Associative Properties to group like factors, and rewrite again to show each base raised to the same exponent.

$$\begin{aligned}(5 \times 2)^3 &= (5 \times 2) \times (5 \times 2) \times (5 \times 2)\\ &= (5 \times 5 \times 5) \times (2 \times 2 \times 2)\\ &= 5^3 \times 2^3\end{aligned}$$

$$\begin{aligned}(2 \div 5)^3 &= \left(\frac{2}{5}\right)^3\\ &= \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}\\ &= \frac{2^3}{5^3}\end{aligned}$$

You can also use shortcuts to simplify the above expressions. Unlike the previous properties, these properties involve different bases, but the same exponent.

Power of a Product Property
$a^n b^n = (ab)^n$

Power of a Quotient Property
$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$ (for $b \neq 0$)

Example 4: Using Properties of Exponents

Simplify $\frac{(4^2)^3 \times 5^6}{2^6}$.

Step 1 Use the Power of a Power Property to simplify the parentheses.

$$\frac{(4^2)^3 \times 5^6}{2^6} = \frac{4^{2\times 3} \times 5^6}{2^6} = \frac{4^6 \times 5^6}{2^6}$$

Step 2 Use the Power of a Product Property to simplify in the numerator.

$$\frac{4^6 \times 5^6}{2^6} = \frac{(4 \times 5)^6}{2^6} = \frac{20^6}{2^6}$$

Step 3 Use the Power of a Quotient Property to write the expression as a single power.

$$\frac{20^6}{2^6} = \left(\frac{20}{2}\right)^6 = 10^6 = 1{,}000{,}000$$

CALCULATOR SKILL

The TI-30XS MultiView™ calculator has an x^2 key for calculating squares. For other exponents, you have to use the ^ key. To input exponents on the calculator, enter the base number, press ^, enter the exponent number, press enter.

You can use your calculator to input negative exponents by using parentheses around the exponent. For example, to input 4^{-2} press

4 ^ (−
2) enter.

Scientific Notation

When you look through a telescope some stars will appear larger than others based on their distance from Earth. These distances are very large numbers of miles and can be written in a shorthand form called scientific notation.

Use Scientific Notation

Scientific notation is a system for writing very large or very small numbers, using exponents. Numbers in scientific notation are written as a product of two factors. The first factor is a decimal number greater than or equal to 1 and less than 10. The second factor is a power of 10.

a decimal number A, where $1 \le A < 10$ { 7.5×10^5 } a power of 10

The power of 10 factor tells how many places to move the decimal point when changing from scientific notation to **standard notation**, the way numbers are usually written. The sign of the exponent indicates whether the number is greater than 10 or less than 1.

Positive Exponents	**Negative Exponents**
$7.5 \times 10^5 = 750{,}000$	$4.2 \times 10^{-3} = 0.0042$
move 5 places to the right	move 3 places to the left

To write a number in scientific notation, move the decimal point until the number is greater than or equal to 1 but less than 10, counting the number of place values moved. Use this decimal number as the first factor. Use the number of place values moved as the exponent in the power of 10.

Example 5: Writing Numbers in Scientific Notation

Write each number in scientific notation.

$60{,}000{,}000 \rightarrow 6 \times 10^7$ $\quad 0.000075 \rightarrow 7.5 \times 10^{-5}$

Add and Subtract in Scientific Notation

To add or subtract two numbers in scientific notation, the powers of 10 must be the same. If they are not the same, use properties of exponents to rewrite the expression until the exponents are the same. Be sure to write the final answer in scientific notation, where the first factor is greater than or equal to 1 but less than 10.

Example 6: Adding Numbers in Scientific Notation

Add: $(3.4 \times 10^7) + (8.9 \times 10^5)$

Step 1 Rewrite the numbers so the powers of 10 are the same.

$(3.4 \times 10^7) + (8.9 \times 10^5) = (340 \times 10^5) + (8.9 \times 10^5)$

Step 2 Use the Distributive Property to factor the power of 10.

$(340 \times 10^5) + (8.9 \times 10^5) = (340 + 8.9) \times 10^5$

Step 3 Combine inside the parentheses.

$(340 + 8.9) \times 10^5 = 348.9 \times 10^5$

Step 4 Write in scientific notation. 348.9×10^5 is not in scientific notation because the decimal, 348.9, is greater than 10.

$348.9 \times 10^5 = 3.489 \times 10^2 \times 10^5$

$= 3.489 \times 10^{2+5} = 3.489 \times 10^7$ Use the Product of Powers Property.

Follow similar steps when adding or subtracting numbers in scientific notation less than 1. The exponents should always be the same.

Multiply and Divide in Scientific Notation

When you multiply or divide numbers in scientific notation, the powers of 10 do not need to be the same. To multiply two numbers in scientific notation, use the Commutative and Associative Properties of Multiplication to group the decimal factors together and the powers of 10 together. Multiply to combine the decimal factors, and use the Product of Powers Property to combine the powers of 10.

Example 7: Multiplying Numbers in Scientific Notation

Multiply: $(2.1 \times 10^3) \times (6.5 \times 10^5)$

Step 1 Use the Commutative and Associative Properties.

$(2.1 \times 10^3) \times (6.5 \times 10^5) = (2.1 \times 6.5) \times (10^3 \times 10^5)$

Step 2 Multiply the coefficients, and use the Product of Powers to multiply the powers of 10.

$(2.1 \times 6.5) \times (10^3 \times 10^5) = 13.65 \times 10^{3+5} = 13.65 \times 10^8$

Step 3 Write in scientific notation. 13.65×10^8 is not in scientific notation because the decimal, 13.65, is greater than 10.

$13.65 \times 10^8 = 1.365 \times 10^1 \times 10^8$

$13.65 \times 10^{1+8} = 1.365 \times 10^9$ Use the Product of Powers Property

Follow similar steps to divide two numbers written in scientific notation. Divide the decimal factors, and divide the powers of 10. Use the Quotient of Powers Property, and rewrite until the quotient is properly written in scientific notation as needed.

Example 8: Dividing Numbers in Scientific Notation

Divide: $(8.4 \times 10^6) \div (2.2 \times 10^9)$

Step 1 Write the quotient using a fraction bar.

$(8.4 \times 10^6) \div (2.2 \times 10^9) = \frac{8.4 \times 10^6}{2.2 \times 10^9} = \frac{8.4}{2.2} \times \frac{10^6}{10^9}$

Step 2 Divide the coefficients, and use the Quotient of Powers Property to divide the powers of 10.

$\frac{8.4}{2.2} \times \frac{10^6}{10^9} = 3.82 \times 10^{6-9} = 3.82 \times 10^{-3}$

Think about Math

Directions: Perform each indicated operation and express the answer in scientific notation.

1. $(2.6 \times 10^3) \times (4.3 \times 10^4)$
2. $(5.1 \times 10^7) + (4.8 \times 10^6)$
3. $(4.4 \times 10^{-3}) - (6.9 \times 10^{-2})$
4. $(8.7 \times 10^8) \div (2.4 \times 10^5)$

21ST CENTURY SKILL

Health Literacy

Red blood cells are responsible for transporting oxygen throughout the body and removing waste. They are the most common of the blood cells, and make up between 40–45% of human blood. They are naturally very small, averaging a length of 7×10^{-6} meters. There are about 2.5×10^{13} red blood cells in the adult human body.

If you laid all of your red blood cells end to end, how long would they be?

Vocabulary Review

Directions: Match each term to its definition.

1. ____ cube
2. ____ order of operations
3. ____ reciprocal
4. ____ scientific notation
5. ____ square
6. ____ standard notation

a. the way in which a number is typically written, using place value
b. a number raised to the third power
c. a system of writing a number as the product of a decimal and a power of 10
d. the rules for the order that calculation should be done when evaluating an expression
e. two numbers or expressions whose product is 1
f. a number raised to the second power

Skill Review

Directions: Read each problem and complete the task.

1. A gardener plants one aster seedling in each square foot of a 12-ft by 12-ft garden. Which expression shows the cost to plant all the asters if each seedling costs $7?
   A. 7×12^2
   B. 7×12^3
   C. 12×7^2
   D. 12×7^3

2. Which property can be applied to this expression?
   $5^4 \div 2^4$

3. The table shows the distances between several planets and the sun.

Planet	Distance from the Sun (mi)
Venus	6.71×10^7
Earth	9.3×10^7
Mars	1.4×10^8
Jupiter	4.84×10^8

What is the distance between Earth and Mars when they are on opposite sides of the sun?
   A. 1.07×10^7
   B. 1.07×10^8
   C. 2.33×10^7
   D. 2.33×10^8

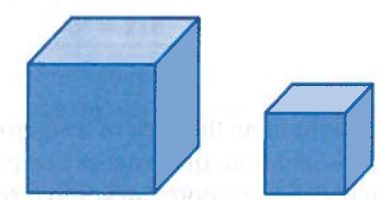
4. Rodrigo invested $2,000 in a fund that returns 8% interest compounded yearly (at the end of the year) and makes no additional deposits or withdrawals. The total value of the fund, including accrued interest, at the end of n years is given by the equation $V = 2{,}000(1.08)^n$. What is the total value of the fund at the end of the third year?

5. What is the value of the expression shown?
   $\frac{(5^2)^3 \times 5^4}{5^{12}}$
   A. 0.04
   B. 0.2
   C. 5
   D. 25

Skill Practice

Directions: Read each problem and complete the task.

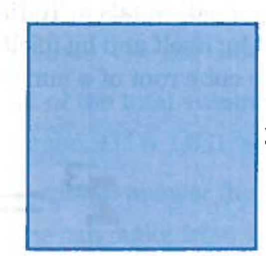
1. A party planner will use colored sand to fill 10 large and 5 small cubic vases like the ones shown to use for centerpieces and other decorations.



Which expression shows the number of cubic inches of colored sand she will need to completely fill all the vases?
   A. $10 \times 8^2 \times 5^3$
   B. $10 \times 8^4 \times 5^3$
   C. $10 \times 8^2 \times 5^4$
   D. $10 \times 8^3 \times 5^4$

2. Simplify the expression shown, justifying your work using properties of exponents.
   $\frac{5^2 \times 2^6 \times 5^6}{10^6}$

3. Write an expression using an exponent to show the total cost for carpeting the square room shown if the carpet costs $3.42 per square foot and there is a $100 installation fee.



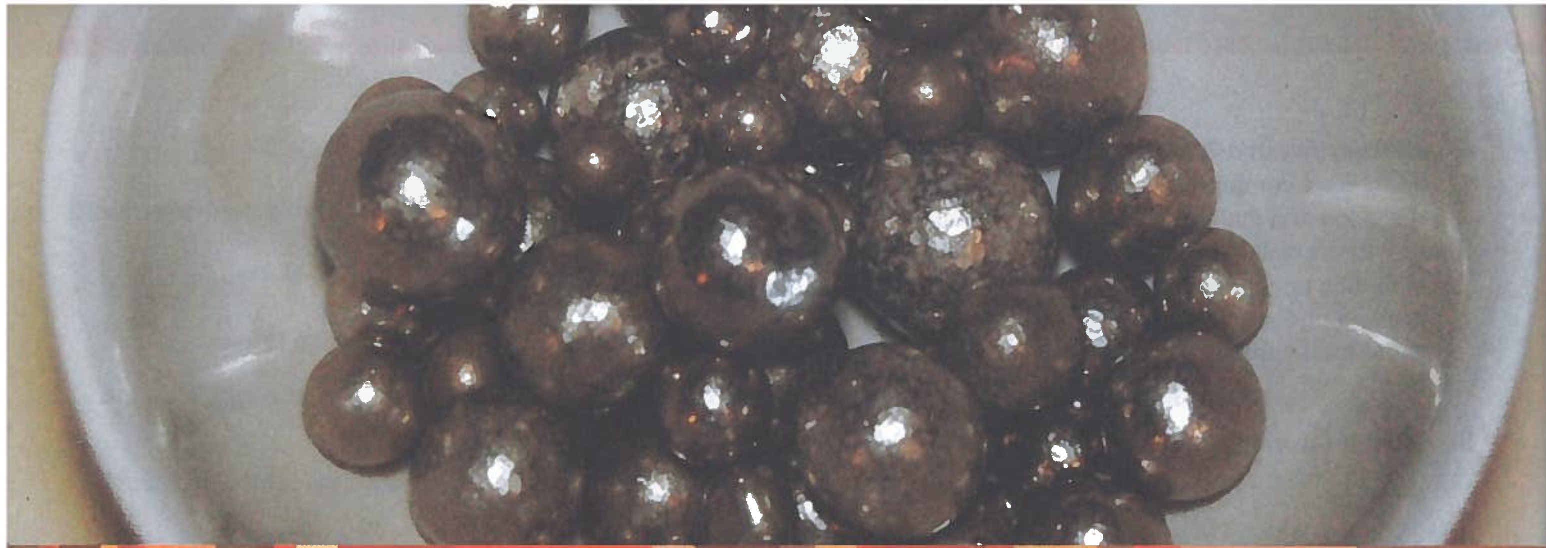
4. The table shows the lengths of two bacteria.

Bacterium	Length (cm)
A	2.0×10^{-5}
B	1.2×10^{-4}

How many of the smaller bacteria do you need to place end-to-end to equal the length of the larger bacterium?
   A. 4
   B. 6
   C. 8
   D. 10

5. Find the product of the numbers shown by expressing them both in scientific notation.
   $250{,}000 \times (7.6 \times 10^{-4})$

6. What value for n makes the expression shown below have a value of 4^3?
   $\frac{4^3 \times (4^2)^n}{4^{10}}$
   A. 1
   B. 3
   C. 5
   D. 7



Mark Dierker/McGraw-Hill Education

LESSON 1.4 Compute with Roots

LESSON OBJECTIVES

- Perform computations with square and cube roots
- Solve real-world problems involving square and cube roots
- Simplify expressions involving roots using the properties of rational exponents

CORE SKILLS & PRACTICES

- Represent Real-World Arithmetic Problems
- Attend to Precision

Key Terms

cube root
a number that, when cubed, equals a given number

rational exponent
an exponent that is a rational number

square root
a number that, when squared, equals a given number

Vocabulary

index
the small number next to a radical sign that indicates the degree of the root

irrational numbers
the set of numbers that cannot be expressed as the ratio of two integers

prime factorization
a number written as the product of its prime factors

Key Concept

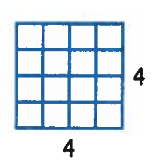
Numerical expressions involving roots (often called radicals) can be written using rational exponents and then simplified using the rules of exponents.

Square Roots and Cube Roots

Roots, including square roots and cube roots, often appear in real-world problems.

Defining Roots

The square of a number n can be thought of as the area of a square with side lengths n. Just like subtraction undoes addition, the process of squaring has an inverse operation, called finding the square root. The **square root** of a positive number n is a number which, when squared, equals n. To find the square root of 4, ask, "What number multiplied by itself equals 16?"

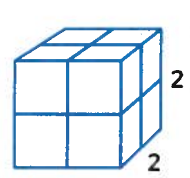


$\underbrace{4}_{\text{square root}}^2 = \underbrace{16}_{\text{square}}$

This question actually has two answers; one positive and one negative. For real-world problems, you usually only need to consider the positive square root.

$4^2 = 16 \rightarrow 4$ is a square root of 16. $(-4)^2 = 16 \rightarrow -4$ is also a square root of 16.

Cubing a number also has an inverse. The **cube root** of a given number n is the number which, when cubed, equals n. To find the cube root of 8, ask, "What number multiplied by itself and by itself again equals 8?" Unlike square roots, there is only one cube root of a number.



$\underbrace{2}_{\text{cube root}}^3 = \underbrace{8}_{\text{cube}}$

You can use similar logic to define the nth root of a number. Roots are shown using a radical sign. The small number is called the **index** of the root. If the index is odd, there is only one possible value for the root of a number. If the index is even, you should consider both positive and negative values for the root.

If n is odd:
$a^n = b \rightarrow \sqrt[n]{b} = a$

If n is even:
$a^n = b \rightarrow \sqrt[n]{b} = a$
$(-a)^n = b \rightarrow \sqrt[n]{b} = -a$

Roots of Perfect Squares and Cubes

Numbers that have whole number square roots are called perfect squares. Perfect squares are easily found by squaring whole numbers.

Example 1: Perfect Squares

$1^2 = 1$	$4^2 = 16$	$7^2 = 49$	$10^2 = 100$	$13^2 = 169$
$2^2 = 4$	$5^2 = 25$	$8^2 = 64$	$11^2 = 121$	$14^2 = 196$
$3^2 = 9$	$6^2 = 36$	$9^2 = 81$	$12^2 = 144$	$15^2 = 225$

Similarly, numbers that have whole-number cube roots are called perfect cubes. You can find perfect cubes by cubing whole numbers.

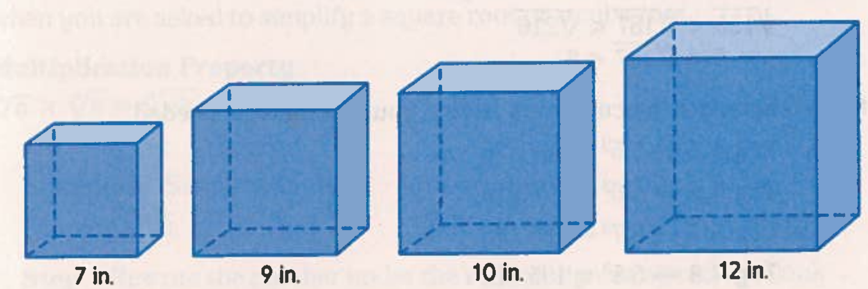
Example 2: Perfect Cubes

$1^3 = 1$	$4^3 = 64$	$7^3 = 343$	$10^3 = 1{,}000$	$13^3 = 2{,}197$
$2^3 = 8$	$5^3 = 125$	$8^3 = 512$	$11^3 = 1{,}331$	$14^3 = 2{,}744$
$3^3 = 27$	$6^3 = 216$	$9^3 = 729$	$12^3 = 1{,}728$	$15^3 = 3{,}375$

It is good to remember the first several perfect squares and perfect cubes so that you can easily use them to solve real-world problems.

Example 3: Solving Real-World Problems

A sculptor has 1,331 cubic inches of clay to make a single cube to use as part of a large sculpture. Which of the cubes shown could he make?



Step 1 Determine the index of the root needed to solve the problem.

The volume of a cube is measured in cubic units, and *side length* = $\sqrt[3]{volume}$. Use the cube root.

Step 2 Find the cube root of the total volume.

1,331 is a perfect cube. $11^3 = 1{,}331$, so $\sqrt[3]{1{,}331} = 11$.

Step 3 Interpret the cube root to answer the question.

The largest cube he can make from 1,331 cubic inches is 11 inches on each side. He could make the 7 in., 9 in., or 10 in. cube.

CORE SKILL

Represent Real-World Arithmetic Problems

When you are solving a real-world problem involving roots, you need to be able to determine the *type* of root needed to solve the problem. For problems involving measurements, you can often tell the index of the root that is needed by considering the units of measurement. Cubic units often indicate the need to find a cube root, while square units often imply the need for a square root.

A sculptor uses a thin layer of colored clay to cover a large square tile he is using for a large sculpture. He has enough clay to cover 576 square inches. What are the dimensions of the largest square he could cover with the colored clay?

CALCULATOR SKILL

Your TI-30XS MultiView™ calculator can be used to calculate square roots and cube roots. There is a key specifically for calculating square root.

You must press 2nd x^2 to access the square root button. There is not a specific button for calculating cube root, but you can use the ^ feature, it is located above the ^ key. In order to calculate a cube root, such as $\sqrt[3]{125}$, type 3 2nd ^ 1 2 5 enter. You should get the answer 5. Use your calculator to find the cube roots of 1 through 10, and verify the approximations in the table.

Approximating Square and Cube Roots

Roots of nonperfect squares and cubes are often **irrational numbers**, the set of numbers that cannot be expressed as the ratio of two integers. The decimal expansion of irrational numbers does not terminate or repeat, but you can approximate them with terminating decimals or whole numbers. You can use what you know about perfect squares and cubes to approximate roots of nonperfect squares and cubes.

Example 4: Approximate a Square Root of a Number

To the nearest whole number, what is $\sqrt{61}$?

Step 1 Identify the perfect squares that the number is between.

$49 < \mathbf{61} < 64$

Step 2 Find the square roots of the perfect squares.

$$\sqrt{49} < \sqrt{61} < \sqrt{64}$$
$$7 < \sqrt{61} < 8$$

Step 3 Estimate the square root. Compare to determine the integer to which the root of the number is closer.

Try 7.5 $\rightarrow 7.5^2 = 56.25$

$56.25 < 61 < 64$, so $\sqrt{61}$ is closer to 8 than to 7.

You can use a similar method to approximate cube roots. To obtain more accurate approximations, you can check your decimal estimate by cubing it, and then refine your estimate as needed.

Example 5: Approximate a Cube Root of a Number

To the nearest tenth, what is $\sqrt[3]{187}$?

Step 1 Identify the perfect cubes that the number is between.

$125 < \mathbf{187} < 216$

Step 2 Find the cube roots of the perfect cubes.

$$\sqrt[3]{125} < \sqrt[3]{187} < \sqrt[3]{216}$$
$$5 < \sqrt[3]{187} < 6$$

Step 3 Estimate the cube root. Refine your estimate as needed.

Try 5.5 $\rightarrow 5.5^3 = 166.375$

Try 5.6 $\rightarrow 5.6^3 = 175.616$

Try 5.7 $\rightarrow 5.7^3 = 185.193$

Try 5.8 $\rightarrow 5.8^3 = 195.112$

So, $\sqrt[3]{187} \approx 5.7$.

It can be helpful to memorize some of the decimal approximations of common square and cube roots.

Square Roots	
$\sqrt{3} \approx 1.73$	$\sqrt{6} \approx 2.45$
$\sqrt{4} = 2$	$\sqrt{7} \approx 2.65$
$\sqrt{5} \approx 2.24$	$\sqrt{8} \approx 2.83$

Cube Roots	
$\sqrt[3]{3} \approx 1.44$	$\sqrt[3]{6} \approx 1.82$
$\sqrt[3]{4} \approx 1.59$	$\sqrt[3]{7} \approx 1.91$
$\sqrt[3]{5} \approx 1.71$	$\sqrt[3]{8} = 2$

Think about Math

Directions: Choose the best answer to the question.

1. The maximum walking speed in inches per second of an animal with leg length in inches can be approximated by the formula shown.

$$\frac{\textit{maximum walking}}{\textit{speed (in. per sec)}} = 19.6\sqrt{\textit{leg length (in.)}}$$

Use the formula to approximate the maximum walking speed of a giraffe with a leg length of 72 inches. Approximate any square roots to the nearest whole number.

A. 117.6 inches per second
B. 166.3 inches per second
C. 176.4 inches per second
D. 235.2 inches per second

Radicals and Rational Exponents

As you might expect, the length of time it takes a planet to orbit the sun is related to the distance that planet is from the sun. Johannes Kepler, a seventeenth-century German mathematician and astronomer, derived an equation that indeed relates these two quantities, and it includes what is known as a rational exponent. This relationship is given by the equation $d = t^{\frac{2}{3}}$.

Multiplying Like Radicals

Expressions involving roots are called radical expressions, or sometimes just radicals. There are several properties you can use when operating with radicals that have the same index.

$$\underbrace{\sqrt[n]{a} \text{ and } \sqrt[n]{b}}_{\text{same index}}$$

A multiplication property of radicals allows you to write the product of roots (with the same index) as the root of a product. This property comes in handy when you are asked to simplify a square root or a cube root.

Multiplication Property

$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

Example 6: Simplify Roots

Simplify $\sqrt{90}$.

Step 1 Rewrite the number under the radical sign as a product. Look for factors of the number that are perfect squares.

$$\sqrt{90} = \sqrt{9 \times 10}$$

Step 2 Use the property of multiplying radicals to rewrite the root of the product as the product of roots.

$$\sqrt{9 \times 10} = \sqrt{9} \times \sqrt{10}$$

Step 3 Simplify the known root.

$$\sqrt{9} \times \sqrt{10} = 3 \times \sqrt{10}$$

CORE PRACTICE

Attend to Precision

To attend to precision when computing with radicals, you need to ensure that you use the correct property. You also attend to precision when you identify which of given numbers to use to solve a problem, and when you approximate to a given place value.

An object traveling at a rate of r miles per hour for t hours travels a total of d miles as given by the formula $d = rt$. The table shows the rates and times covered by different runners expressed using square roots.

Runner	r	t
A	$\sqrt{54}$	$\sqrt{5}$
B	$\sqrt{50}$	$\sqrt{6}$
C	$\sqrt{45}$	$\sqrt{8}$

To the nearest tenth of a mile, how far did Runner B travel?

Dividing Like Radicals

You can also divide radicals with the same index using a property of radicals. The quotient of two roots (with the same index) is equal to the root of the quotient.

Division Property

$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \quad (b \neq 0)$

You can use the division property along with the multiplication property to help you simplify and approximate radical expressions. It is often helpful to use the **prime factorization** of the number under a radical sign in order to identify perfect squares and cubes that can be removed from underneath the radical. (Remember, the prime factorization shows a number written as a product of its prime factors.)

Example 7: Simplify Radical Expressions

Simplify and approximate $\frac{\sqrt[3]{2{,}880}}{\sqrt[3]{60}}$.

Step 1 Use the property of dividing radicals to rewrite the quotient of roots as the root of a quotient. Evaluate the quotient.

$\frac{\sqrt[3]{2{,}880}}{\sqrt[3]{60}} = \sqrt[3]{\frac{2{,}880}{60}} = \sqrt[3]{48}$

Step 2 Write the prime factorization of the number under the radical.

$\sqrt[3]{48} = \sqrt[3]{2^4 \times 3}$

Step 3 Use the properties of exponents and the property of multiplying radicals to rewrite the root of the product as the product of roots.

$\sqrt[3]{2^3} \times \sqrt[3]{6} \approx 2 \times 1.82 \approx 3.64$

Step 4 Simplify the root of the perfect cube and approximate the root of the nonperfect cube.

Defining Rational Exponents

What if the index of the radicals are not the same? You can still perform operations on these radicals by first writing them in an equivalent form using a rational exponent. A **rational exponent** is an exponent that is a rational number.

$\sqrt[n]{b} = b^{\frac{1}{n}}$

Think about why this notation makes sense:

- On the left, the nth root of a number raised to the nth power is the number.
  $(\sqrt[n]{b})^n = b$
- On the right, raising the expression to the nth power using the rules for exponents results in the number raised to the first power, or just the number.
  $(b^{\frac{1}{n}})^n = b^{\frac{1}{n} \times n} = b^{\frac{n}{n}} = b^1 = b$

You can also use non-unit fractions in lowest terms as rational exponents.

$\sqrt[n]{b^m} = b^{\frac{m}{n}}$

The same rules of exponents that apply to powers also apply to rational exponents.

- **Product of Powers** To multiply powers with like bases, add the exponents.
  $a^m \times a^n = a^{m+n}$
- **Quotient of Powers** To divide powers with like bases, subtract the exponents.
  $\frac{a^m}{a^n} = a^{m-n} \quad (\text{for } a \neq 0)$
- **Power to a Power** To raise a power to a power, multiply the exponents.
  $(a^m)^n = a^{mn}$
- **Power of a Product** To multiply two powers with the same exponent, multiply the bases.
  $a^n b^n = (ab)^n$
- **Power of a Quotient** To divide two powers with the same exponent, divide the bases.
  $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \quad (\text{for } b \neq 0)$

Simplifying with Rational Exponents

Take a look at this example showing how to use rational exponents to simplify radical expressions. For this radical expression, the index of the radicals are not the same, meaning you cannot just write this quotient of roots as the root of a quotient.

Example 8: Simplify Rational Expressions

Simplify $\frac{\sqrt{64}}{\sqrt[4]{16}}$.

Step 1 Rewrite the radicals using rational exponents. Notice that you cannot yet use the Quotient of Powers Property, as the bases are not the same. However, you should recognize the bases as powers of 2.

$\frac{\sqrt{64}}{\sqrt[4]{16}} = \frac{64^{\frac{1}{2}}}{16^{\frac{1}{4}}}$

Step 2 Rewrite each base as a power of 2, and then use the Power to a Power Property.

$\frac{64^{\frac{1}{2}}}{16^{\frac{1}{4}}} = (2^6)^{\frac{1}{2}}/(2^4)^{\frac{1}{4}} = \frac{2^3}{2^1}$

Step 3 Use the Quotient of Powers Property and evaluate.

$\frac{2^3}{2^1} = 2^{3-1} = 2^2 = 4$

Think about Math

Directions: For each expression involving rational exponents, identify the equivalent radical expression.

1. $6^{\frac{1}{2}} \times 6^{\frac{1}{3}}$
   A. $\sqrt[6]{6}$
   B. $\sqrt[6]{6^5}$
   C. $2 \times \sqrt[3]{6}$
   D. $3 \times \sqrt{6}$

2. $\frac{(8^3)^{\frac{1}{2}}}{64^{\frac{1}{2}}}$
   A. $2\sqrt{2}$
   B. $2\sqrt{8}$
   C. $8\sqrt{2}$
   D. $3\sqrt{8}$

Vocabulary Review

Directions: Write the missing term in the blank.

cube root	**index**	**irrational number**
prime factorization	**rational exponent**	**square root**

1. In the radical expression $\sqrt[3]{8}$, the ________________ is 3.
2. The ________________ of a number shows the number written as the product of its prime factors.
3. The radical expression $\sqrt{4}$ can be rewritten using a(n) ________________ as $4^{\frac{1}{2}}$.
4. The ________________ of 64 is 4.
5. A number that cannot be written as the ratio of two integers is called a(n) ________________.
6. A(n) ________________ of 16 is −4.

Skill Review

Directions: Read each problem and complete the task.

1. Which property can be applied to this expression?
   $\sqrt{\frac{81}{144}}$
2. Four expressions are shown below.
   $16^{\frac{3}{2}}, 25^{\frac{3}{2}}, 27^{\frac{2}{3}}, 343^{\frac{2}{3}}$
   What is the difference between the expression with the greatest value and the expression with the least value?
   A. 219
   B. 116
   C. 61
   D. 40
3. To the nearest hundredth, what is the value of $\sqrt{191}$?
4. A farmer needs to build a fence to enclose a square plot of land with an area of 200 square yards. To the nearest tenth of a yard, how much fencing does the farmer need?

5. Kepler's Third Law of Planetary Motion relates the average distance d, in astronomical units (AU), from a planet to the sun to the time t, in years, it takes the planet to orbit the sun. This relationship is given by the equation $d = t^{\frac{2}{3}}$. Which is the best estimate of difference between the average distance of Uranus from the sun and the average distance of Jupiter from the sun? Approximate any roots to the nearest tenth.

Planet	Time to Orbit Sun (years)
Mars	2
Jupiter	12
Saturn	30
Uranus	84
Neptune	165

   A. 5 AU
   B. 14 AU
   C. 19 AU
   D. 20 AU

6. Which of the following shows the expression $\sqrt{12} \times \sqrt[4]{18}$ written with a single radical?
   A. $3\sqrt{2}$
   B. $3\sqrt[4]{2}$
   C. $6\sqrt{2}$
   D. $6\sqrt[4]{2}$

Skill Practice

Directions: Read each problem and complete the task.

1. To the nearest tenth, what is the side length of a cube with a volume of 439 cubic centimeters?
   A. 7.2 cm
   B. 7.4 cm
   C. 7.6 cm
   D. 7.8 cm
2. Evaluate the expression shown, justifying your work using properties of exponents and/or radicals.
   $\frac{\sqrt{9} \times \sqrt[3]{729}}{\sqrt{3} \times \sqrt{27}}$
3. Which property can be applied to this expression?
   $\sqrt{5} \times \sqrt{40}$
4. Which of the following expressions is equivalent to $\sqrt{90} \times \sqrt{450}$?
   A. $30\sqrt{2}$
   B. $90\sqrt{2}$
   C. $30\sqrt{5}$
   D. $90\sqrt{5}$
5. An artist is painting a large mural comprised of different size squares. She wants to paint one magenta square that is as large as possible, but she only has one can of magenta paint. If the can of paint can cover 220 square feet, what is the side length of the largest square that she can paint if she needs to apply 2 coats of paint? Round to the nearest whole number.
6. Which of the following expressions is not equivalent to $\frac{\sqrt{20} \times \sqrt[4]{4}}{\sqrt{2}}$?
   A. $\frac{4\sqrt{5} \times \sqrt[4]{4}}{\sqrt{2}}$
   B. $\frac{20^{\frac{1}{2}} \times 4^{\frac{1}{4}}}{2^{\frac{1}{2}}}$
   C. $10^{\frac{1}{2}} \times \sqrt[4]{4}$
   D. $\frac{2 \times 5^{\frac{1}{2}} \times 4^{\frac{1}{4}}}{2^{\frac{1}{2}}}$

Directions: Choose the best answer to each question.

1. Which is the value of the expression?
$15 + 10\,(2 + 5)^2 - 12$
A. 48
B. 62
C. 273
D. 493

2. Jen is ordering these expressions, so she would first like to find the expression with the greatest value. Which is the greatest?
A. $\sqrt{9}$
B. $\sqrt[3]{8}$
C. $\sqrt{8}$
D. $\sqrt[3]{10}$

3. Micah is ordering these expressions, so he would first like to find the expression with the least value. Which is the least?
A. $27^{\frac{1}{3}}$
B. $16^{\frac{1}{2}}$
C. $8^{\frac{2}{3}}$
D. $4^{\frac{2}{3}}$

4. Vanessa is making a painting based on a 12-inch by 24-inch picture. She wants to divide the picture into grid squares. What is the greatest size of grid squares she can make?
A. 36 inch by 36 inch
B. 12 inch by 12 inch
C. 6 inch by 6 inch
D. 2 inch by 4 inch

5. Allen works out at the gym every 6 days. Freddy works out at the gym every 4 days. Allen and Freddy see each other at the gym on a Monday. What day of the week will it be when they see each other again?
A. Monday
B. Wednesday
C. Friday
D. Saturday

6. Which is the absolute value of the expression $25 - 4^3$?
A. -39
B. -13
C. 13
D. 39

7. Which expression has the greatest value?
A. $\frac{8^5}{8^2}$
B. $8^5 \times 8^2$
C. $\frac{8^2}{8^5}$
D. $(8^5)^2$

8. Which shows this expression in simplest form?
$\sqrt{12}\ \sqrt{27}$
A. 18
B. $4\sqrt{3} \times 3\sqrt{3}$
C. $2\sqrt{3} \times 3\sqrt{3}$
D. 324

9. Which shows this expression in simplest form?
$\sqrt[3]{\frac{16}{27}}$
A. $\frac{4}{3}$
B. $\frac{\sqrt[3]{16}}{3}$
C. $\frac{2\sqrt[3]{2}}{3}$
D. $\frac{48}{81}$

Directions: Use the paragraph for Problems 10–11.

Four friends are on a basketball team. During a game, each friend kept track of how many shots they attempted and how many of those attempts they made.

Henry made 0.45 of his shots.

Allison made $\frac{4}{15}$ of her shots.

Arthur made $\frac{8}{20}$ of his shots.

Trevor missed 58% of his shots.

10. Which friend had the best record for the number of shots made?
A. Henry
B. Allison
C. Arthur
D. Trevor

11. Which friend had the worst record for the number of shots made?
A. Henry
B. Allison
C. Arthur
D. Trevor

12. This expression, $\frac{15 + 85}{4^3 - 8^2}$, is called ________________ because when simplifying, there is a 0 in the denominator.

13. The ________________ Property is represented by both $12 + 15 = 15 + 12$ as well as $4 \times 5 = 5 \times 4$.

14. Gianna has two savings accounts. One account has a rate of return of 3.75% while the other account has a rate of return of 0.375%. 3.75 is ________________ times greater than 0.375.

15. You can read the value of $\sqrt{16}$ as *the* ________________ *of sixteen.*

Directions: Use the paragraph for Problems 16–17.

Ellis is a botanist. He found the heights of three different Redwood trees and recorded the information in a table.

Heights of Redwood Trees	
Tree	**Height (in centimeters)**
1	1.0668×10^4
2	1.1430×10^4
3	9.114×10^3

16. What is the difference between the heights of Tree 1 and Tree 3 in Scientific Notation?
A. 1.554×10^3 cm
B. 15.54×10^4 cm
C. 15.54 cm
D. 1.554×10^4 cm

17. What is the total of the heights of Tree 2 and Tree 3 in Scientific Notation?
A. 2.0544×10^4 cm
B. 20.544×10^3 cm
C. 1.0257×10^4 cm
D. 10.257×10^3 cm

18. A ________________ number is one that can be written as a ratio of two integers.

19. Tile Company Pro charges $4.15 for each tile that is one square foot. To tile a room that is 12.5 feet wide and 12.5 feet long, the price would be ________________ for the tiles for the room.

20. You can read the number 8^3 as eight to the third power, or eight cubed, and can simplify it to ________________.

Check Your Understanding

On the following chart, circle the items you missed. The last column shows pages you can review to study the content covered in the question. Review those lessons in which you missed half or more of the questions.

Lesson	Item Number(s)			Review Page(s)
	Procedural	Conceptual	Problem Solving	
1.1 Order Rational Numbers	18	2, 3	10, 11	12–19
1.2 Apply Number Properties	1, 14	12, 13	4, 5, 16, 17	20–27
1.3 Compute with Exponents	6	7, 20	19	28–35
1.4 Compute with Roots	8, 9	15		36–43