

LESSON 4.1

Introduction to Integers and Absolute Value

Lesson Objectives

You will be able to

- Identify integers
- Compare and order integers
- Find the absolute value of an integer

Skills

- **Core Skill:** Apply Number Sense Concepts
- **Core Skill:** Represent Real-World Problems

Vocabulary

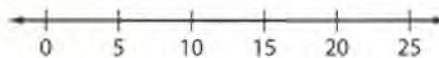
absolute value
infinite
integer
opposite

MATH LINK

The symbol "... " is an **ellipsis**. When it appears at either end of a list of numbers, it indicates that the list goes on forever. So all integers are included in the list..., -2, -7, 0, 1, 2.... When an ellipsis appears in the middle of a list (0, 1, 2, ..., 7), it shows that there are more numbers in the list that have been omitted for space.

KEY CONCEPT: Identify, compare, and order integers, as well as find their absolute value, in order to better understand the meaning and value of integers.

Locate each number on the number line below.



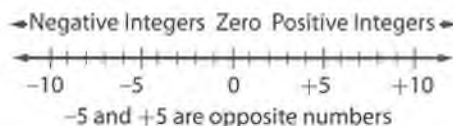
1. 1 2. 23 3. 7 4. 4

Use a less than (<), greater than (>), or equal to (=) symbol to compare each pair of numbers.

5. $5 \square 9$ 6. $31 \square 0$ 7. $62 \square 14$ 8. $18 \square 18$

Understand Integers

The positive and negative whole numbers, -3, -2, -1, 0, 1, 2, 3, ... make up the set of **integers** and can be represented on a number line. The set of integers is **infinite**, that is, it extends without end to the left and to the right of 0 on the number line. An integer has both a distance from 0 and a direction (positive or negative). For example, the integer +5 has a positive direction and a distance of 5 from 0. The integer -5 has a negative direction and a distance of 5 from 0. Because +5 and -5 have the same distance from 0 but have different directions, they are called **opposite** numbers.



Example 1 Find Opposites of Integers

Use the number line on the previous page to find the opposite of -8.

Step 1 Count how far the given number is from 0.

-8 is 8 units to the left of 0.

Step 2 Count the same distance from 0 in the opposite direction. The number located at this distance is the opposite of the given number. The number 8 units to the right of 0 is +8.

The opposite of -8 is +8.

Example 2 Use a Number Line to Compare Integers

Compare -13 and +7.

Step 1 Locate both integers on a number line.

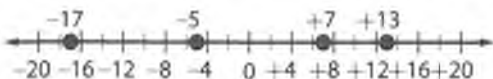


Step 2 The numbers increase in value as you move from left to right on the number line. Since -13 is to the left of +7, -13 is less than ($<$) +7. Since +7 is to the right of -13, +7 is greater than ($>$) -13. Both statements are accurate comparisons of -13 and +7. $-13 < +7$ or $+7 > -13$

Example 3 Use a Number Line to Order Integers

Order the integers +13, -5, +7, and -17 from least to greatest.

Step 1 Locate each integer on a number line.



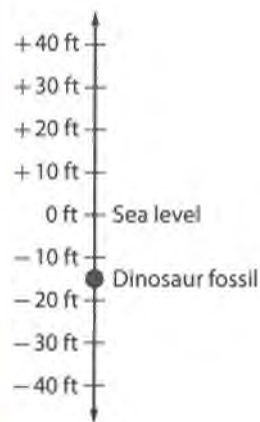
Step 2 Write the numbers from left to right as they appear on the number line. -17, -5, +7, +13

Core Skill

Apply Number Sense Concepts

You already have a solid foundation in number concepts as a result of your study of whole numbers, positive numbers, and negative numbers. You can apply this knowledge of numbers in general as you begin your study of absolute value. You can also rely on the number lines in this lesson when learning about absolute value. They are a great visual tool that can help you grasp and explain this new concept.

After you have read the lesson, write in your notebook and explain the concept of absolute value in your own words.



MATH LINK



An integer with no + or - sign is understood to be a positive integer.

Core Skill Represent Real-World Problems

It may seem that negative numbers are not used in the real world, because it's not possible to drive -100 miles in a car. The same may seem true for absolute values, but this is not the case for either topic. Negative numbers are used in everyday life, but you may not know it. This is because absolute values have taken their place!

Consider the following example. Suppose you open a credit card with a local bank. Since you have yet to purchase anything with the card, your balance is \$0.00. After making some purchases, you receive a monthly statement saying your balance is \$125.00. Does this mean that you have \$125.00 in the bank that you can withdraw? It doesn't. Because you made purchases totaling \$125.00, it means that your balance is -\$125.00, since you owe the bank that money. The number given to you as your balance is the total amount of money from owing nothing (away from zero). This is just absolute value! In a notebook, write down what it would mean if your credit card balance was negative.

THINK ABOUT MATH



Directions: Identify whether each number is an integer. If it is an integer, give its opposite.

1. +4 2. -19 3. 4.5 4. 3 5. $\frac{2}{3}$

Directions: Compare. Write > or < for each \square .

6. +1 \square -5 7. -1 \square +8 8. 23 \square -9 9. 0 \square +12

Absolute Value

Because opposite integers have the same distance from zero, they have the same **absolute value**. Absolute value is the distance between a number and 0. Distance is never negative, so absolute value is always a positive number. Vertical bars are used to show absolute value. For example, the absolute value of +2 is written $|+2|$. Since +2 is 2 units from 0, $|+2| = 2$.

Example 4 Absolute Value

Find the values of $|+4|$ and $|-4|$.

Step 1 The integer +4 is 4 units from 0 on a number line. $|+4| = 4$.

Step 2 The integer -4 is 4 units from 0 on a number line. $|-4| = 4$.

Notice that $|+4| = |-4|$.

THINK ABOUT MATH

Directions: Find each absolute value.

1. $|+9|$ 2. $|-12|$ 3. $|+13|$ 4. $|-25|$

Vocabulary Review

Directions: Complete each sentence with the correct word.

absolute value infinite integer opposite

- The _____ of a positive integer is a negative integer.
- Zero is neither a positive nor a negative _____.
- When a set of numbers is _____, the numbers continue without an end.
- The _____ of a number is always a positive value.

Skill Review

Directions: Use the number line below to help you determine which number is greater. Indicate your answer by writing *greater than* or *less than* in the blank.



1. -4,351 is _____ +10.
2. +380 is _____ -200.
3. -4,999 is _____ -5,000.
4. 0 is _____ +800.

Directions: Solve each problem. Use a number line to justify your answer.

5. The average low temperatures this week were 20°F , 15°F , -2°F , 0°F , 18°F , -6°F , and 4°F . Order the temperatures from coldest to warmest.

6. A river near Maria's home is 194 feet above sea level. A river near Sheila's home is 600 feet above sea level. Compare the heights of the two rivers using integers and $>$ or $<$ symbols.

Skill Practice

Directions: Choose the best answer to each question.

1. When the furnace broke last winter, the temperature in the auditorium dropped to -3°C . What is the opposite of -3?
A. -6
B. -3
C. +3
D. +6
2. Which shows -32, +24, -10, 0, and +316 in order from least to greatest?
A. 0, -10, +24, -32, +316
B. +316, +24, 0, -10, -32
C. 0, -32, -10, +24, +316
D. -32, -10, 0, +24, +316
3. While scuba diving, Ariana dove to 89 feet below sea level. Which integer expresses her position?
A. 89
B. -89
C. +89
D. |89|
4. Which explains one way to find the absolute value of any integer?
A. Write the opposite of the integer that is inside the vertical bars.
B. Compare the integer to its opposite.
C. Count how far the integer is from 0 on a number line.
D. Count how far the integer is from its opposite on a number line.

Add Integers

Lesson Objectives

You will be able to

- Use a number line to find the sum of two integers
- Use integer addition rules to find the sum of two integers

Skills

- **Core Practice:** Reason Abstractly
- **Core Skill:** Perform Operations

Vocabulary

addend
illustrate
negative
positive
sequence
sign

KEY CONCEPT: Two ways to find the sum of two integers include using a number line and using a sequence of rules.

Find each sum or difference.

1. $4 + 5$

2. $8 + 2$

3. $9 - 7$

4. $10 - 4$

Use a less than ($<$), greater than ($>$), or equal to ($=$) symbol to compare each pair of numbers.

5. $|+5| \square |-9|$

6. $|-3| \square |0|$

7. $|+8| \square |+4|$

8. $|-81| \square |+14|$

Use a Number Line to Add Integers

You can use a number line to add two integers. Use the **sign** (whether the integer is positive $[+]$ or negative $[-]$) of the **addends**, or numbers that are added together, to tell which way to move on the number line.

Example 1 Add a Negative Integer

Add $+4 + (-6)$.

Step 1 Locate the first addend, $+4$, on the number line.

Step 2 Move 6 units to the left, since you are adding a **negative** ($-$), less than zero, number whose absolute value is 6.



Step 3 The sum is the number of the point where you end. In this case, it is -2 .

$$+4 + (-6) = -2$$

Example 2 Adding a Positive Integer to a Negative Integer

Michael accidentally overdraws his bank account by \$15. To avoid being charged a fee, he immediately deposits \$100 into his account. How much money does he now have in his bank account?

Step 1 Determine the first addend. Because he overdrawed his account by \$15, Michael has $-\$15$ in his account.

Step 2 Write down the addition problem. Michael added \$100 to his account, so the sum is $-15 + 100$.

Step 3 Using a number line, start at -15 and then move 100 units to the right.

Step 4 The sum is the number of the point where you end. In this case, it is 85. Therefore, Michael has \$85 in his bank account.

FOLLOW A SEQUENCE OF STEPS

Some passages describe actions that happen in a certain order, called a **sequence**. Words, such as *first*, *then*, *next*, *after*, and *finally*, signal the order in which the actions happened.

Sequences explain the order of an individual's activity. *Sabina went to the store, then she walked to the post office and bought stamps. After that, she mailed her letters. Finally, she went home.*

Actions might not be written in the order in which they happened. *Tahir finished reading the last sixty pages of a novel. Before that, he had done the dishes. When he was done with the novel, he made coffee and poured it into the mug he had washed that morning.* Four actions are described, but they occurred in a different order than they are written. Tahir washed a mug, read sixty pages to finish a novel, made coffee, and poured coffee into the clean mug. The phrases *before that*, *when he was done*, and *had washed that morning* provide clues about the sequence.

Sometimes, a sequence of events can be given as a set of instructions. *To follow the order of operations, first do operations in parentheses, then perform multiplication and division from left to right. Finally, do addition and subtraction from left to right.*

Read the passage below. Put the sentences in the order in which the actions occurred. Write the clue words that helped you order them.

(1) Nayo measured the temperature in the evening as 25°F using a thermometer. (2) She earlier found the afternoon temperature of 57°F using the same method. (3) Nayo had wanted to find the difference in temperature from midday to evening. (4) She then subtracted 25°F from 57°F and got 32°F. (5) She finally knew the temperature dropped 32°F from midday to evening.

3 (had wanted), 2 (earlier), 1 (in the evening), 4 (then), 5 (finally)

Core Practice Reason Abstractly

You may sometimes hear people talking about multitasking, which refers to doing more than one job at the same time. Numbers also multitask. They do the "work" of positive numbers and the work of negative numbers. Understanding what a negative number represents allows you to work with such numbers when performing simple operations, such as addition and subtraction. In these kinds of simple calculations, you are reasoning abstractly—that is, you are working directly with numbers and not with things you can touch and count.

For example, suppose you want to know how far you walk every day in a two-week period. Reason abstractly to find the answer. In other words, write the problem out mathematically, leaving out any words.

MATH LINK

Addends are the numbers being added.

Core Skill Perform Operations

Number lines are a great tool for adding a positive and negative integer, because they allow you to **illustrate**, or use graphic detail, to explain operations with integers. Using number lines, you can actually demonstrate the results of each operation you perform. To add a positive integer, you move right on the number line. To add a negative integer, you move left. When you add two positive integers, you always get a positive answer. Two negative numbers that are added together result in a negative answer. When you add a positive and negative number, first you move in one direction, and then you move in the opposite direction. Your answer will be positive if the absolute value of the negative number was the smaller number. Otherwise, your answer will be zero or negative.

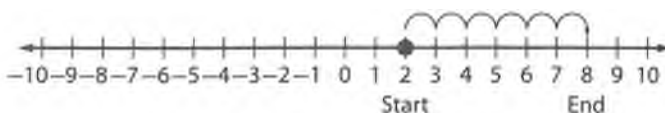
In your notebook, explain why you might get a zero for your answer when adding a positive and negative number. You can use a number line to demonstrate cases in which the two numbers that you picked add up to zero.

Example 3 Add a Positive Integer

Add $+2 + (+6)$.

Step 1 Locate the first addend, $+2$, on the number line.

Step 2 Move 6 units to the right, since you are adding a **positive (+)** (greater than zero) number whose absolute value is 6.



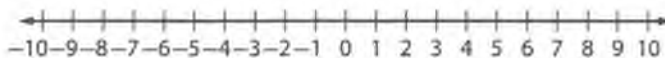
$$+2 + (+6) = +8$$

Step 3 The sum is the number at the point where you end. In this case, it is $+8$.

In summary, to add integers using a number line, first find the first addend. Then move right to add a positive number or move left to add a negative number.

THINK ABOUT MATH

Directions: Match each problem with its sum. Use a number line to help you.



- | | |
|-----------------------|---------|
| _____ 1. $+10 + (-3)$ | A. -1 |
| _____ 2. $-2 + (+1)$ | B. -7 |
| _____ 3. $+3 + (+2)$ | C. -5 |
| _____ 4. $-5 + (-2)$ | D. $+7$ |
| _____ 5. $-10 + (+5)$ | E. $+5$ |
| ----- 6. $-2 + (+3)$ | F. $+1$ |

Use Rules to Add Integers

Adding integers with the same sign is similar to adding whole numbers. The sign of the sum depends on the sign of the integers.

Example 4 Add Two Positive Integers

Find $+5 + (+7)$.

Step 1 Find the absolute values.

$$\begin{array}{l} +5 + (+7) \\ |+5| = 5 \quad |+7| = 7 \end{array}$$

Step 2 Add the absolute values. Then use the sign of the integers to determine the sign of the answer. If the addends are positive, the answer will be positive.

$$\begin{array}{l} 5 + 7 = 12 \\ +5 + (+7) = +12 \end{array}$$

MATH LINK

The number 0 is neither positive nor negative. Numbers less than 0 are negative. Numbers greater than 0 are positive.

Example 5 Add Two Negative Integers

Find $-5 + (-7)$.

Step 1 Find the absolute values.

$$\begin{array}{l} +5 + (+7) \\ |+5| = 5 \quad |-7| = 7 \end{array}$$

Step 2 Add the absolute values. Then use the sign of the integers to determine the sign of the answer. If the addends are negative, the answer will be negative.

$$\begin{array}{l} 5 + 7 = 12 \\ +5 + (-7) = -12 \end{array}$$

Adding integers with different signs is similar to subtracting whole numbers. The sign of the sum depends on the sign of the integer with the greater absolute value.

Example 6 Add Integers with Opposite Signs

Find $+5 + (-7)$.

Step 1 Find and subtract the absolute values.

$$\begin{array}{l} +5 + (-7) \\ = 5 \quad |-7| = 7 \quad 7 > 5 \end{array}$$

Step 2 Use the sign of the integer with the greater absolute value. Because the -7 has a greater absolute value, use the negative sign to get -2 .

$$\begin{array}{l} 7 - 5 = 2 \\ +5 + (-7) = -2 \end{array}$$

To summarize, to use rules to add integers with the same sign, follow the same sequence of steps each time. First, add the absolute values of the integers, and then use the sign of the integers.

To add integers with different signs, likewise follow the same sequence of steps each time. First, subtract the absolute values of the integers, and then use the sign of the integer with the greater absolute value.

THINK ABOUT MATH

Directions: Match each problem with its sum.

- | | |
|-----------------------|----------|
| _____ 1. $+10 + (+2)$ | A. -14 |
| _____ 2. $-2 + (-12)$ | B. -10 |
| _____ 3. $+8 + (+8)$ | C. -16 |
| _____ 4. $-7 + (-9)$ | D. 0 |
| _____ 5. $-6 + (+6)$ | E. $+12$ |
| _____ 6. $-12 + (+2)$ | F. $+16$ |

Vocabulary Review

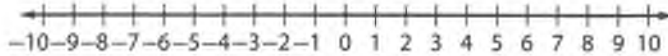
Directions: Complete the sentences below using one of the following words:

addends negative positive sign

1. Numbers to the left of 0 on a number line are _____ numbers.
2. Numbers to the right of 0 on a number line are _____ numbers.
3. _____ are the numbers to be added in an addition expression.
4. The _____ of a number tells whether it is positive or negative.

Skill Review

Directions: Copy the number line and illustrate how to find the sum for each expression below.



- | | | |
|----------------|-----------------|----------------|
| 1. $+8 + (+1)$ | 3. $-10 + (+6)$ | 5. $-9 + (-1)$ |
| 2. $+7 + (-3)$ | 4. $+3 + (-5)$ | 6. $-3 + (+9)$ |

Directions: Write the sequence of steps that must be followed to find the sum of the following expressions.

- | | | |
|-----------------|-----------------|-----------------|
| 7. $+8 + (+12)$ | 8. $+14 + (-9)$ | 9. $-15 + (+7)$ |
|-----------------|-----------------|-----------------|

Directions: Read each passage, then put the sentences in the order in which the action occurred. Write any clue words that helped you order the sentences.

10. (1) First, Samuel drew a number line. (2) Then he plotted the location of -3 on the number line. (3) After that, he counted to the right 5 units because he was adding a positive 5 to negative -3. (4) Samuel finally knew that the sum of -3 and +5 is +2.
11. (1) Julio knew the sum would have a positive sign. (2) He had earlier found the absolute value of +6 to be greater than the absolute value of -1. (3) Julio had wanted to find the sum of +6 + (-1). (4) After all his calculations, he found the sum to be +5. (5) He had subtracted 6 - 1 and gotten 5.

Skill Practice

Directions: Choose the best answer to each question.

1. Which of the following explains why the sum of -9 and $+16$ is a positive number?
 - A. $+16$ is to the right of 0 on a number line, and -9 is to the left of 0 .
 - B. A positive number plus a negative number is always a positive number.
 - C. -9 has a greater absolute value than $+16$, and its sign is not positive.
 - D. $+16$ has a greater absolute value than -9 , and its sign is positive.
2. The average temperature today was -5°F . Yesterday the average temperature was 6°F warmer than today. What was the average temperature yesterday?
 - A. -11°F
 - B. -1°F
 - C. 1°F
 - D. 11°F
3. The solution to the problem Alba was working on is a negative integer. Which of the following problems could Alba be trying to solve?
 - A. $+8 + (-4)$
 - B. $-8 + (+4)$
 - C. $-6 + (+6)$
 - D. $+7 + (-6)$
4. A diver first dove to 35 feet below sea level to meet another diver. Then the diver came up 10 feet to untangle a line. At what depth did the diver stop to untangle a line?
 - A. -45 feet
 - B. -25 feet
 - C. 25 feet
 - D. 45 feet

Subtract Integers

Lesson Objectives

You will be able to

- Use a number line to subtract integers
- Use addition of opposite integers to find the difference between two integers

Skills

- **Core Skill:** Perform Operations
- **Core Skill:** Represent Real-World Problems

Vocabulary

point
solve
tic mark

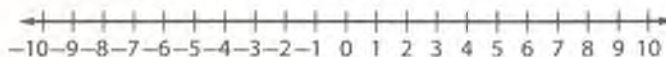
MATH
LINK

Addition and subtraction are opposite operations. To subtract integers on a number line, do the opposite of what you did to add integers.

- Move left on the number line to subtract a positive integer.
- Move right on the number line to subtract a negative integer.

KEY CONCEPT: Subtract two integers by adding the opposite of the integer that is being subtracted.

Use the number line below to find each sum.



1. $+1 + (-7)$ 2. $-6 + (+4)$ 3. $-10 + (+2)$ 4. $+4 + (-10)$

Find each sum.

5. $+10 + (-15)$ 6. $-7 + (+13)$ 7. $-6 + (+8)$ 8. $+5 + (-12)$

Use a Number Line to Subtract Integers

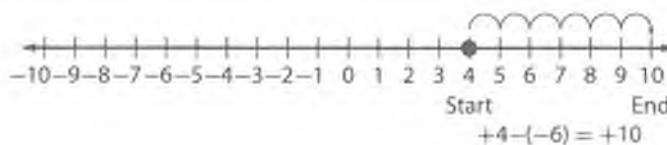
You can use a number line to **solve**, or find the answer to, a subtraction problem involving integers. On the number line below, each **tic mark**, or division, indicates an integer.

Example 1 Subtract a Negative Integer

Find $+4 - (-6)$.

Step 1 Locate the first number, $+4$, on the number line.

Step 2 Move 6 units to the right, since you are subtracting a negative number whose absolute value is 6. If you were adding -6 , you would move 6 units to the left. Because you are performing the opposite of addition by doing subtraction, move 6 units to the right.



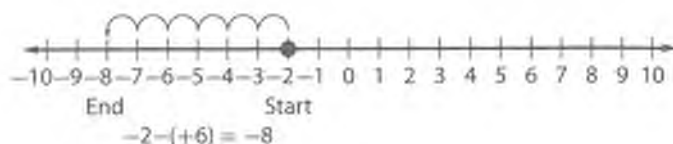
Step 3 The number of the **point**, or specified place, where you end is the difference. In this case it is $+10$.

Example 2 Subtract a Positive Integer

Find $-2 - (+6)$.

Step 1 Locate the first number, -2 , on the number line.

Step 2 Move 6 units to the left, since you are subtracting a positive number whose absolute value is 6.



Step 3 The number of the point where you end is the difference. In this case, it is -8 .

Use Addition to Subtract Integers

Subtracting integers is similar to adding integers. In fact, you can rewrite a subtraction problem as an addition problem and then add.

Example 3 Subtract a Positive Integer

Find $+5 - (+7)$.

Step 1 Rewrite the problem so that you are adding the opposite of the integer being subtracted.

Step 2 Follow the rules for adding integers.

$$\begin{aligned} -7 \text{ is the opposite of } +7 \\ +5 - (+7) &\longrightarrow +5 + (-7) \\ | +5 | = 5 & \quad | -7 | = 7 \\ 7 - 5 = 2 & \quad 7 > 5, \text{ so} \end{aligned}$$

$$\begin{aligned} +5 + (-7) &= -2 \\ \text{and} \\ +5 - (+7) &= -2 \end{aligned}$$

Example 4 Subtract a Negative Integer

Find $+5 - (-9)$.

Step 1 Rewrite the problem so that you are adding the opposite integer.

Step 2 Follow the rules for adding integers.

$$\begin{aligned} +9 \text{ is the opposite of } -9 \\ +5 - (-9) &\longrightarrow +5 + (+9) \\ 5 + 9 &= 14, \text{ so} \end{aligned}$$

$$\begin{aligned} +5 + (+9) &= +14 \\ \text{and} \\ +5 - (-9) &= +14 \end{aligned}$$

Core Skill

Perform Operations

You have already used number lines to compare fractions. You can also use number lines to show operations such as addition and subtraction.

When using number lines, you generally move left when subtracting one number from another—but not always. Recall that subtracting a number is the same thing as adding the opposite of the number. When you are subtracting a negative number, you are adding the opposite of the negative number—that is, a positive number.

In a notebook, draw a number line, and use it to show how to solve these two problems: $-5 + (-1)$ and $-5 - (-1)$. What does this problem show you about the relationship between subtracting a number and adding its opposite?

MATH LINK

Subtracting an integer is the same as adding its opposite.

Core Skill
Represent Real-World
Problems

Consider the following problem involving operations of integers. Ginger ran 11 miles North to her friend's house for lunch. Afterward, she ran back South 6 miles to the park, where she teaches dog obedience classes. How many miles is Ginger from her house? In a notebook, create a number line and use it to determine the correct answer to the problem.

THINK ABOUT MATH

Directions: Match each problem with its difference.

- | | |
|-----------------------|--------|
| _____ 1. $+10 - (+2)$ | A. -12 |
| _____ 2. $-2 - (-12)$ | B. -10 |
| _____ 3. $+8 - (+8)$ | C. 0 |
| _____ 4. $-3 - (-9)$ | D. +6 |
| _____ 5. $-6 - (+6)$ | E. +8 |
| _____ 6. $-8 - (+2)$ | F. +10 |

Vocabulary Review

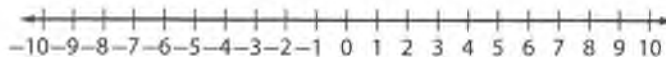
Directions: Complete each sentence with the correct word from the list below.

point(s) solve tic mark(s)

1. Number lines have evenly divided _____ to indicate certain numbers.
2. When using a number line to subtract, the _____ where you end is the difference.
3. To _____ an addition problem means to find its sum.

Skill Review

Directions: For each pair of expressions, copy the number line. Then use the number line to perform the operations and solve the expressions. If they are equal, add an equal sign to make an equation.



1. $+8 + (+2)$ and $+8 - (-2)$

2. $+4 + (-5)$ and $+4 - (+5)$

Directions: Describe how to find each sum and difference without using a number line.

3. $+12 + (+11)$ and $+12 - (-11)$

4. $+7 + (-3)$ and $+7 - (+3)$

Directions: Explain the following.

5. Use what you have learned about the addition and subtraction of integers to explain how you know that $+10 - (-3)$ equals a positive number without solving the problem.

Skill Practice

Directions: Choose the best answer to each question.

- The temperature outside at 3:00 p. m. was 3°F . By 9 p. m., the temperature had dropped 8°F . What was the temperature at 9 p. m.
A. $+11^{\circ}\text{F}$ C. -5°F
B. $+5^{\circ}\text{F}$ D. -11°F
- When solving the equation $+7 - (-5)$ on a number line, which of the following steps would you take?
A. Locate $+7$ on a number line, then move left 5 units.
B. Locate $+7$ on a number line, then move right 5 units.
C. Locate -5 on a number line, then move left 7 units.
D. Locate -5 on a number line, then move right 7 units.
- The basement of an office building is 40 feet below the surface street. The exterior wall of the foundation extends down an additional 8 feet. Where in relation to the surface street is the bottom of the exterior wall of the basement?
A. +48 feet
B. +32 feet
C. -32 feet
D. -48 feet
- The balance in May's checking account was \$245. She wrote a check for \$302. If there is no fee charged to the account, what is the balance now?
A. -\$57
B. -\$55
C. \$57
D. \$547
- The melting temperature for tin is about 231°C , and the melting temperature for oxygen is about -218°C . What is the difference in the melting temperatures for tin and oxygen?
A. 453°C C. -249°C
B. 449°C D. -449°C
- What is the difference between -92 and 114?
A. -206
B. -22
C. 22
D. 206
- Which of the following will have a negative answer?
A. $-23 + 24$
B. $-23 - (-24)$
C. $24 + (-23)$
D. $-23 - 24$
- Which of the following explains why -9 minus $+16$ is a negative number?
A. $+16$ is to the right of 0 on a number line, and -9 is to the left of 0.
B. -9 is the first number in the problem, and it is negative.
C. A negative number minus a positive number is the same as adding two negative numbers.
D. A negative number minus a positive number is a negative number whenever the second number is greater than the first number.

Multiply and Divide Integers

Lesson Objectives

You will be able to

- Multiply two integers
- Divide two integers

Skills

- **Core Skill:** Interpret Data Displays
- **Core Practice:** Use Appropriate Tools Strategically

Vocabulary

column
inverse
repeated
row
table
title

KEY CONCEPT: Use rules to find products and quotients of integers.

Find each sum or difference.

1. $-4 + (-4)$

3. $+9 + (+9)$

5. $-12 - (-6)$

7. $+14 - (+7)$

2. $-8 + (-8)$

4. $+4 + (+4)$

6. $-10 - (-5)$

8. $+20 - (+10)$

Multiply Integers

Recall that multiplication is repeated addition. A **repeated** action happens again and again. You can use your knowledge of adding integers with the same signs to help you when multiplying integers.

Example 1 Positive x Positive = Positive

Find $+4 \times (+3)$.

Step 1 Rewrite the problem as repeated addition.

Step 2 Add $+3$ four times. Recall $4 \times (+3) =$

that a positive number

$$(+3) + (+3) + (+3) + (+3) = +12$$

plus a positive number

is a positive number.

$$+4 \times (+3) = +12$$

Example 2 Positive x Negative = Negative

Find $+4 \times (-3)$.

Step 1 Rewrite the problem as repeated addition.

Step 2 Add -3 four times. Recall $4 \times (-3) =$

that a negative number

$$(-3) + (-3) + (-3) + (-3) = -12$$

plus a negative number

is a negative number.

$$+4 \times (-3) = -12$$

Recall that every integer has an opposite. Use this idea to help you see that a negative number times a positive number is a negative number, and that a negative number times a negative number is a positive number.

READ A TABLE

A **table** is a way to organize information in **rows** and **columns**. A table has cells of information arranged horizontally (rows) and vertically (columns) to make information easier to understand. A table has a title and labels. The **title** tells what the table is meant to show. The labels identify the information in the separate rows and columns.

Study the table below and then answer the questions.

ATIAN'S CELL PHONE MINUTES USED

	Wednesday	Thursday	Friday	Saturday
Daytime	23	46	10	0
Night & Weekend	113	38	24	176

The title tells that the table will show the cell phone minutes used by Atian over four days. You can find a specific piece of data. You can also find more than one piece of information and add, subtract, or compare the numbers. To find specific information, first identify which row and column the information is in. Then find the cell where the row and column intersect. To find the daytime minutes used on Thursday, find *Daytime*. Then follow the row across to the *Thursday* column.

1. How many total daytime minutes did Atian use for the 4 days?
To find the total, add all the minutes from the *Daytime* row together: $23 + 46 + 10 + 0 = 79$ minutes, or 1 hour, 19 minutes.

2. How many more *Night & Weekend* minutes than *Daytime* minutes did he use on Wednesday?
Find the number of minutes used on Wednesday: 113 night, 23 daytime. Because the question asks *how many more*, subtract the values: $113 - 23 = 90$ minutes, or 1 hour 30 minutes.

3. Did Atian use more *Daytime* minutes or *Night & Weekend* minutes over the four days?
This question asks about total minute usage for each type. Add all of the *Daytime* minutes together: $23 + 46 + 10 + 0 = 79$.
Then add all the *Night & Weekend* minutes together: $113 + 38 + 24 + 176 = 351$. Compare the two numbers. $351 > 79$, so Atian used more *Night & Weekend* minutes.

Core Skill Interpret Data Displays

Tables are a handy way of organizing and displaying information and data. On the other hand, tables can sometimes present an overwhelming amount of information. Think back to the time when you learned how to read. Eventually, the black marks that stared up at you when you opened a book made sense. You unlocked their mystery. So too you will learn how to read tables.

Notice the arrangement of data in rows and columns. Also pay attention to a table's title and labels; they both provide clues that explain what specific information is being presented.

Read the table below. Then, in a notebook, write a paragraph describing the information presented in the table.

Park Attendance

	Boys	Girls
June	77	80
July	110	157
August	191	142

What rows or columns might you add to the table above to provide more information?

Core Practice

Use Appropriate Tools Strategically

It is easy to be distracted by all of the information in a table. Always spend a few moments to analyze, or study, the information there so that you understand what the table is telling you. Look at the title and labels. For example, a table titled "Temperature ($^{\circ}\text{F}$)" might be reporting the "temperature in degrees Fahrenheit" recorded during a week.

Look at the table below and record your answers to the questions.

Temperatures ($^{\circ}\text{F}$)

	High	Low
Monday	13 $^{\circ}$	2 $^{\circ}$
Tuesday	20 $^{\circ}$	3 $^{\circ}$
Wednesday	6 $^{\circ}$	-10 $^{\circ}$

1. Which temperature scale is being used?
2. What was the range of temperatures for Tuesday?
3. Which day had the greatest difference in high and low temperatures?

Example 3 Negative x Positive = Negative

Find $-4 \times (+3)$.

Recall that $+4$ is the opposite of -4 . Think of $-4 \times (+3)$ as the *opposite of* $+4 \times (+3)$.

$+4 \times (+3) = +12$ (See Example 1.)

The *opposite of* $+4 \times (+3) = \text{opposite of } +12$ (or -12), so $-4 \times (+3) = -12$. =

Example 4 Negative x Negative = Positive

Find $-4 \times (-3)$.

Think of $-4 \times (-3)$ as the *opposite of* $+4 \times (-3)$.

$+4 \times (-3) = -12$, so the *opposite of* $+4 \times (-3)$

The *opposite of* $+4 \times (-3) = \text{opposite of } -12$ (or $+12$), so $-4 \times (-3) = +12$.

When two integers have the same sign, their product will be positive, and when two integers have different signs, their product will be negative.

THINK ABOUT MATH

Directions: Match each problem with its product.

- | | |
|---------------------------|----------|
| _____ 1. $+5 \times (-7)$ | A. $+35$ |
| _____ 2. $-5 \times (-7)$ | B. -35 |
| _____ 3. $+4 \times (+8)$ | C. -32 |
| _____ 4. $-4 \times (+8)$ | D. $+32$ |

Divide Integers

Recall that division and multiplication are **inverse**, or opposite, operations. This means that you can use a related multiplication problem to help you solve a division problem.

Example 5 Positive / Positive = Positive

Find $+10 \div (+5)$.

Think: What number can I multiply by $+5$ to get a product of $+10$?

Will that number be positive or negative?

Since $+5 \times +2 = +10$, then $+10 \div (+5) = +2$.

Example 6 Positive / Negative = Negative

Find $+10 \div (-5)$.

Think: What number can I multiply by -5 to get a product of $+10$?

Since $-5 \times -2 = +10$, then $+10 \div (-5) = -2$.

Example 7 Negative \div Positive = Negative

Find $-10 \div (+5)$.

Think: What number can I multiply by $+5$ to get a product of -10 ?

Since $+5 \times -2 = -10$, then $-10 \div (+5) = -2$.

Example 8 Negative \div Negative = Positive

Find $-10 \div (-5)$.

Think: What number can I multiply by -5 to get a product of -10 ?

Since $-5 \times +2 = -10$, then $-10 \div (-5) = +2$.

When two integers have the same sign, their quotient will be positive, and when two integers have different signs, their quotient will be negative.

MATH LINK

The table below shows the rules for dividing integers.

Divide Integers		
Dividend	Divisor	Quotient
+	+	+
+	-	-
-	+	-
-	-	+

THINK ABOUT MATH

Directions: Match each problem with its product.

- | | |
|--------------------------|---------|
| _____ 1. $+24 \div (+8)$ | A. $+5$ |
| _____ 2. $-24 \div (+8)$ | B. -5 |
| _____ 3. $+45 \div (-9)$ | C. -3 |
| _____ 4. $-45 \div (-9)$ | D. $+3$ |

Vocabulary Review

Directions: Complete the sentences below using one of the following words.

columns inverse repeated rows table title

1. The _____ of a table is usually found at the top of the table.
2. The _____ of multiplying by -4 is dividing by -4 .
3. -6 is _____ three times when writing -6×3 as an addition problem.
4. _____ are the vertical divisions in a table.
5. _____ are the horizontal divisions in a table.
6. A _____ is a tool for organizing information.

Skill Review

Directions: The table below shows each of the three integers assigned to various students. Use the table to answer the following questions.

INTEGERS ASSIGNED TO EACH STUDENT

Student	First Integer	Second Integer	Third Integer
Mandisa	-1	+8	-5
Tuan	+6	-2	+ 12
Reyna	-14	-7	-9
Wil	+2	+9	+ 18

1. What is the product of Reyna's second and third integers?
2. What is the product of Mandisa's second integer and Wil's first integer?
3. What is the quotient of Wil's third integer divided by his first integer?
4. What is the quotient of Tuan's third integer divided by his first integer?
5. What is the quotient of Tuan's third integer divided by his second integer?

Directions: Use the table below to answer the following questions.

Population of Smithville (in hundreds)

Year	1990	2000	2010
Population	42	55	68

6. What does the second column tell you?
7. What does the second row tell you?
8. How much greater was the population of Smithville in 2000 than in 1990?
9. If the population of Smithville increases by the same amount every 10 years, what would you expect the population of Smithville to be in 2020?

Skill Practice

Directions: Choose the best answer to each question.

1. Which of the following explains why the product of -3 and $+2$ is a negative number?
 - A. $+2$ is to the right of 0 on a number line, and -3 is to the left of 0 .
 - B. A negative number times a number with the greater absolute value is negative.
 - C. The sum of $+2 + (+2)$ is a positive number.
 - D. The sum of $-3 + (-3)$ is a negative number.
2. When Marcie plugs in her new freezer, the freezer's temperature is 20°C . The temperature of the freezer descends by 2° every hour. Which of the expressions represents how many hours it takes for the freezer's temperature go down 36° ?
 - A. $20 + (-36) \div (-2)$
 - B. $-36 \div (-2)$
 - C. $36 \div (-2)$
 - D. $(20 - (-36)) \div (-2)$
3. What is $-12 \times (-7)$?
 - A. 84
 - B. 72
 - C. -72
 - D. -84
4. A diver dives 10 feet every 20 seconds. What is the location of the diver in feet with relation to the surface after 120 seconds?
 - A. -60
 - B. -16
 - C. 6
 - D. 60

The Coordinate Grid

Lesson Objective

You will be able to

- Plot and identify points on a coordinate grid

Skills

- Core Skill: Interpret Data Displays
- Core Practice: Make Sense of Problems

Vocabulary

coordinate plane
grid
ordered pair
origin
perpendicular
x-axis
x-coordinate
y-axis
y-coordinate

MATH LINK



When two lines are **perpendicular**, they intersect to form 4 corners, or 90° angles.

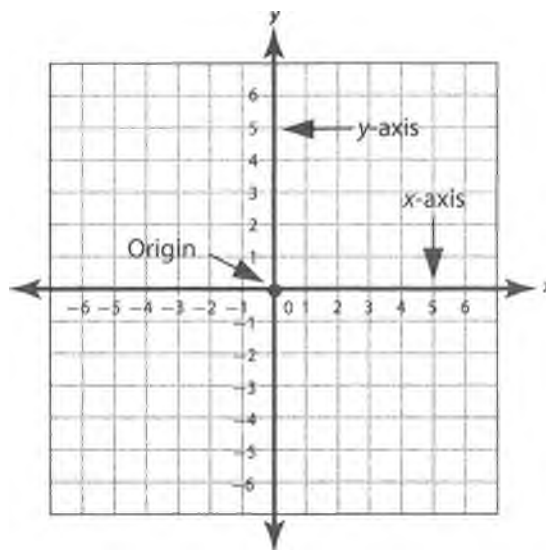
KEY CONCEPT: Coordinate grids are a method of locating points in the plane by means of directions and numbers.

Answer the following questions.

- Where are the positive numbers located on a horizontal number line?
- Where are the negative numbers located on a horizontal number line?

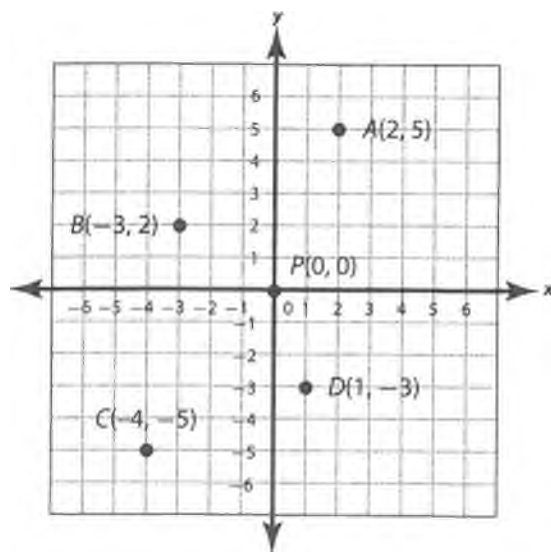
The Coordinate Plane

A **coordinate plane** is formed by two number lines perpendicular to each other. The horizontal number line is called the **x-axis**, and the vertical number line is called the **y-axis**. The point where the two lines meet, $(0, 0)$, is called the **origin**.

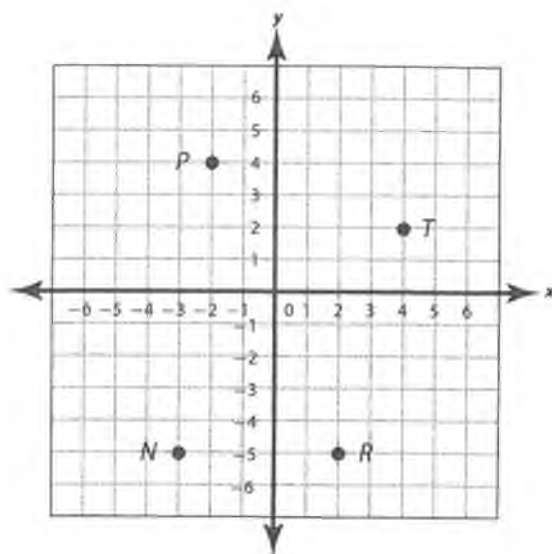


A point in the coordinate plane is named by two coordinates (numbers): an **x-coordinate** and a **y-coordinate**. Together the coordinates form an **ordered pair**. The pair is *ordered* because the *order* of the two coordinates matters. The x-coordinate is always first, and the y-coordinate is always second. They are written as (x, y) . The ordered pair tells exactly where the point lies in the coordinate plane.

In the coordinate plane below, point P is the origin: $(0, 0)$. The x -coordinate tells how far right or left a point is from the origin. The y -coordinate tells how far up or down the point is from the origin. Point A is at $(2, 5)$: 2 units right and 5 units up from the origin. Point B is at $(-3, 2)$: 3 units left and 2 units up from the origin.



Use the coordinate plane below for the examples that follow.



MATH LINK

A positive coordinate is right or up from the origin, while a negative coordinate is left or down from the origin.

Core Skill Interpret Data Displays

A grid is one of the many ways you can display data. The type of grid that appears throughout this lesson is called a coordinate grid. Knowing how to "read" the information in a coordinate grid is an important skill.

The scales in a coordinate grid are the same ones found on integer number lines. The two axes in a coordinate grid have names: the x -axis and the y -axis. You can use these two axes to help you locate points anywhere within the grid. Each point in the grid is identified by an ordered pair of numbers: first, the x -value, then the y -value.

Work with a partner to practice plotting points on a coordinate plane. Each partner take turns drawing a grid and plotting two or three coordinates. The other partner names the ordered pairs that lie on the coordinate plane.

Core Practice
Make Sense of
Problems

During a day of running errands, Javier stopped at three places. After leaving home, he drove 3 miles west to Zeeb's Auto Shop, 4 miles south to Nelson's Book Nook, and 6 miles north and 5 miles east to the D&L Market. Using the origin as his home, determine how many miles north, south, east, west, or any combination of those Javier is from home.

Using your notebook and a coordinate grid, draw the path Javier took to help find his current location.

Example 1 Plot a Point on a Coordinate Grid

What is the letter of the point at $(2, -5)$?

Step 1 Start at the origin. Count 2 units right and 5 units down.

Step 2 Identify the point for R . The point at $(2, -5)$ is R .

Example 2 Identify the Ordered Pair for a Point

What is the ordered pair for the point T ?

Step 1 Start at the origin. Count the number of units right until T is directly above: 4. Count the number of units up to T : 2.

Step 2 Write the ordered pair: $(4 \text{ right}, 2 \text{ up}) = (4, 2)$.
The ordered pair for point T is $(4, 2)$.

Example 3 Identify the Ordered Pair for a Point

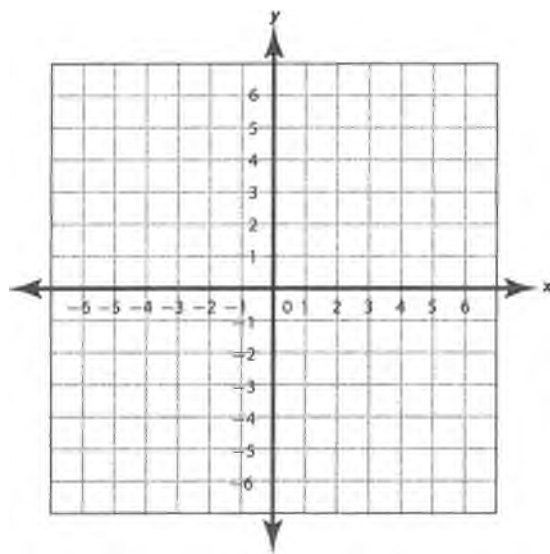
What is the ordered pair for point N ?

Step 1 Start at the origin. Count the number of units left until N is directly below: 3. Count the number of units down to N : 5.

Step 2 Write the ordered pair: $(3 \text{ left}, 5 \text{ down}) = (-3, -5)$.
The ordered pair for point N is $(-3, -5)$.

Example 4 Use a Coordinate Plane

Use the coordinate plane below to help you answer the question.



The coordinates of point W are $(-2, 5)$. What is the location of point M ? Mark your answer on the graph above.

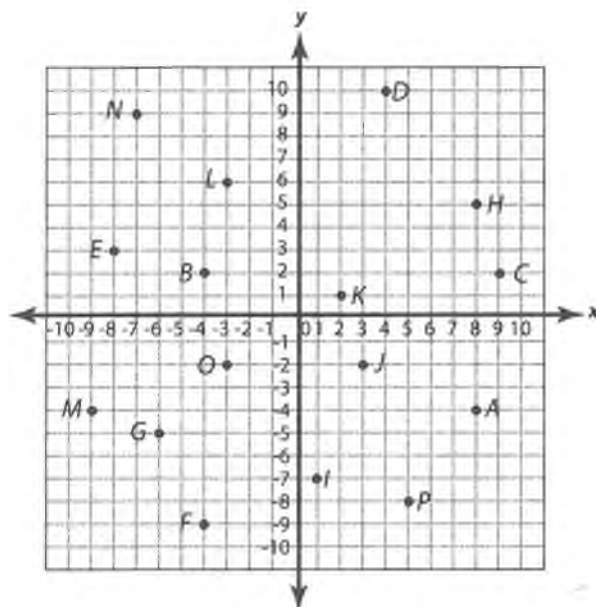
THINK ABOUT MATH

Directions: Write the letter of the point at each ordered pair on the coordinate grid.

1. $(3, -2)$
2. $(8, 5)$
3. $(-9, -4)$
4. $(-4, 2)$

Directions: Write the ordered pair for each point on the coordinate grid at the right.

5. $K(\quad , \quad)$
6. $E(\quad , \quad)$
7. $F(\quad , \quad)$
8. $P(\quad , \quad)$



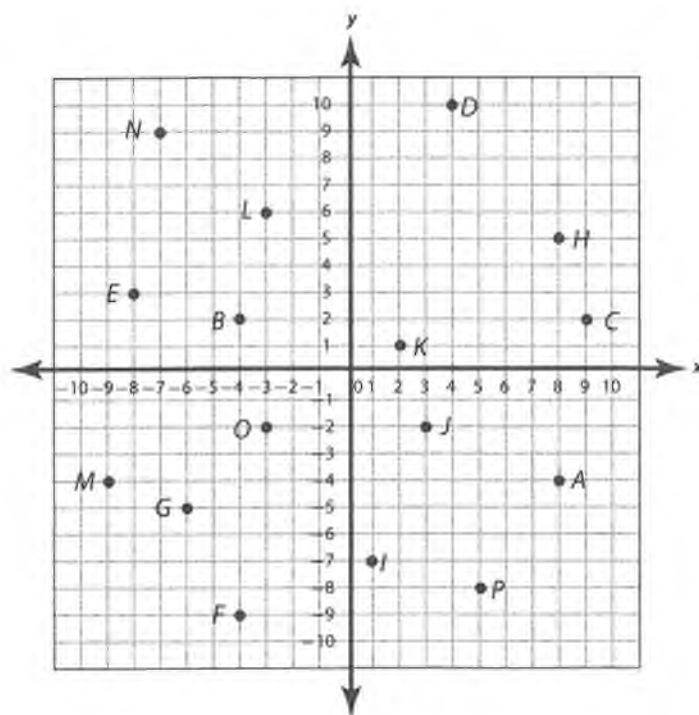
Vocabulary Review

Directions: Match the words in the column on the left with their description on the right.

- | | |
|---------------------------|--|
| 1. _____ coordinate plane | A. a grid made up of horizontal and vertical lines |
| 2. _____ ordered pair | B. the vertical number line on a grid |
| 3. _____ origin | C. the horizontal number line on a grid |
| 4. _____ perpendicular | D. the first number in an ordered pair |
| 5. _____ x-axis | E. the second number in an ordered pair |
| 6. _____ x-coordinate | F. the point where the x- and y-axes intersect |
| 7. _____ y-axis | G. two lines that cross to form a 90° angle |
| 8. _____ y-coordinate | H. a pair of numbers that corresponds to a point in the coordinate plane |

Skill Review

Directions: Use the coordinate grid below to answer each question.

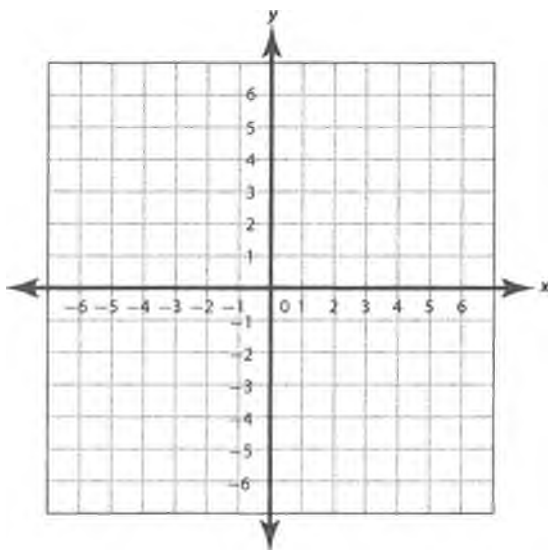


- For which point would you start at the origin, move right 8 units, and then move down 4 units?
- Explain how you would find the ordered pair that locates point C.
- Which points have a positive x-coordinate and a positive y-coordinate?
A. C, D, H, K B. F, G, M, O C. B, E, L, N D. A, J, L, P

Skill Practice

Directions: Choose the best answer to each question.

1. Which two points, when graphed on the coordinate grid and connected, form a line segment that is a vertical line?



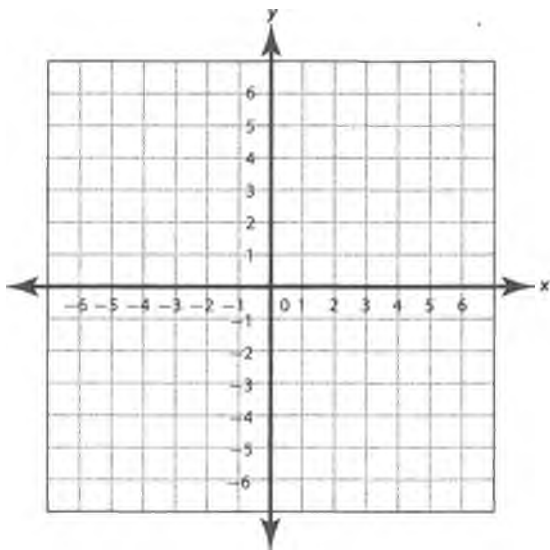
- A. $(2, 6), (8, 6)$
- B. $(2, -4), (0, 6)$
- C. $(-4, -4), (-6, -6)$
- D. $(2, 4), (2, -6)$

2. Which best describes how to plot the point at $(3, -7)$ on a coordinate grid?

- A. Start at 3. Move 7 units left.
- B. Start at the origin. Move 3 units to the left and 7 units up.
- C. Start at the origin. Move 3 units to the right and 7 units down.
- D. Start at -7 . Move 3 units down.

Directions: Mark each answer on the corresponding coordinate grid.

3. The coordinates of point A are $(-6, 5)$. What is the location of point A?



4. The coordinates of point B are $(4, -3)$. What is the location of point B?

