Derivation Tree :-

$E \to E+E \mid E*E \mid id$.

$E \to E+E$

$E \to E*E$

$E \to id$

⇒ It is Top down approach. $S \to w$

$w \to S$ it is bottom up approach.

⇒ If we get a string from start of the grammar then it is derivation.

⇒ If we represent this ~~Bo~~ Derivation in hierarchical structure it is derivation Tree.

$G = [\{E\}, \underbrace{\{+, *, id\}}_{\text{Terminal}}, \underset{P}{P}, \underset{\text{Start}}{E}]$

Ques :- $E \to E+E \mid E*E \mid id$

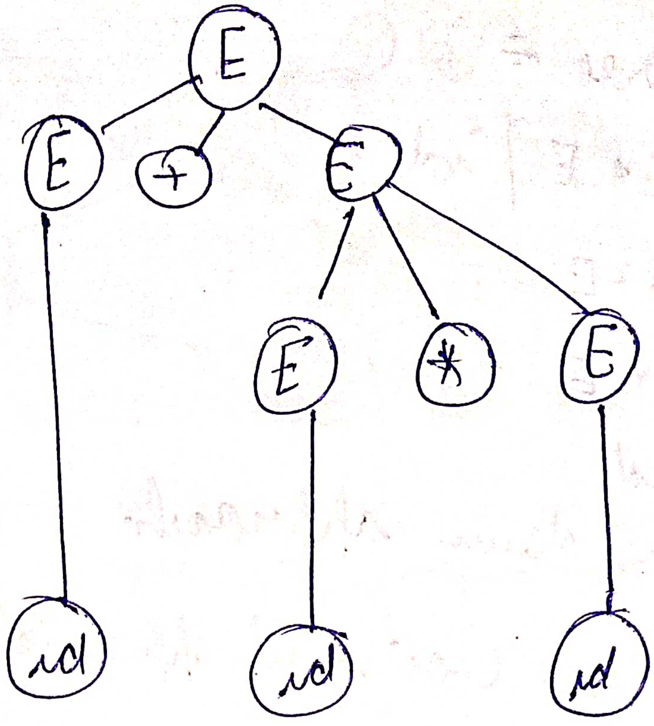
$w = id + id * E$

Sol^n :- $E \to E+E$

$E \to E+E*E$

$E \to id + id * id$

$S \to w$



⇒ Our process to expand the terminal is wrong.

⇒ Derivation should be right most or leftmost

⇒ By derivation tree we cannot tell leftmost or rightmost.

Left Most Derivation :- If we derive left most ~~derivation~~ variable first then go left to right. (LMD)

Right Most Derivation :- If we derive Right most variable first then go right to left. (RMD)

⇒ LMD

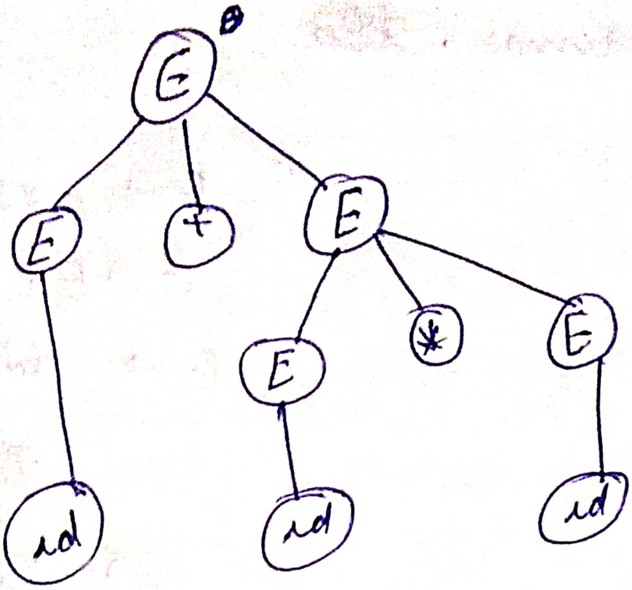
$E \rightarrow E + E \quad \{E \rightarrow E + E\}$

$E \rightarrow id + E \quad \{E \rightarrow id\}$

$E \rightarrow id + E * E \quad \{E \rightarrow E * E\}$

$E \rightarrow id + id * E$ $\{E \rightarrow id\}$

$E \rightarrow id + id * id$ $\{E \rightarrow id\}$.



⇒ RMD

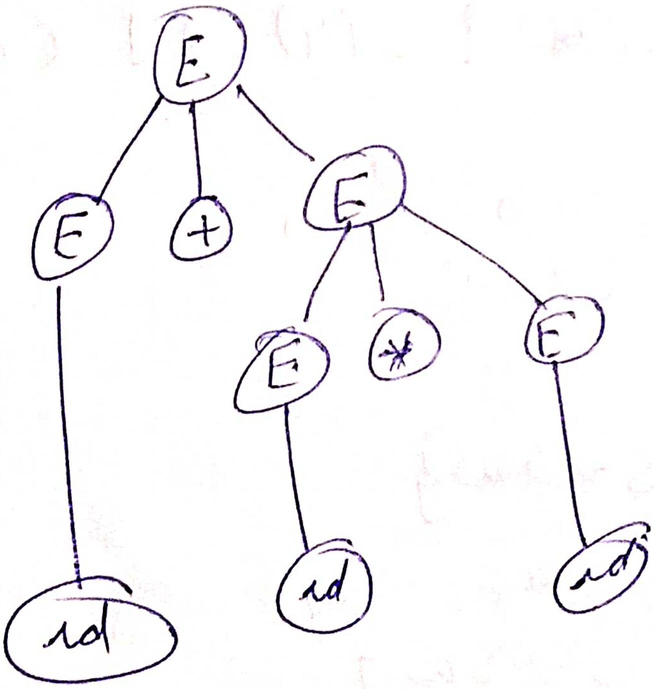
$E \rightarrow E + E$ $(E \rightarrow E * E)$

$E \rightarrow E + E * E$ $(E \rightarrow E * E)$

$E \rightarrow E + E * id$ $(E \rightarrow id)$

$E \rightarrow E + id * id$ $[E \rightarrow id]$

$E \rightarrow id + id * id$ $(E \rightarrow id)$



Ambiguous Grammar ÷ If we are having more than 1 LMD & RMD for any grammar.

$E \to E+E$

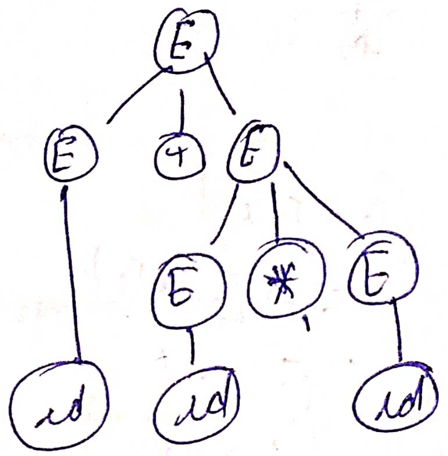
$E \to E+E*E$

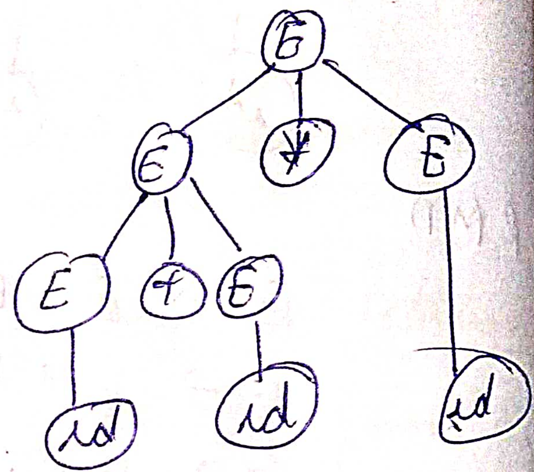
$E \to id+id*id$

$E \to E*E$

$E \to E+E*E$

$E \to id+id*id$





Note ÷ [Some that prove this grammar ambiguous] but you not have 2 different LMD or RMD then you can choose 1 LMD & 1 RMD

Que ÷ $L = [\{S\}, \{a,b\}, P, S]$

$w = $ 'aba baba

$S \to SbS \mid a$

LMD

$S \to SbS \quad \{S \to SbS\}$

$S \to abS \quad \{S \to a\}$

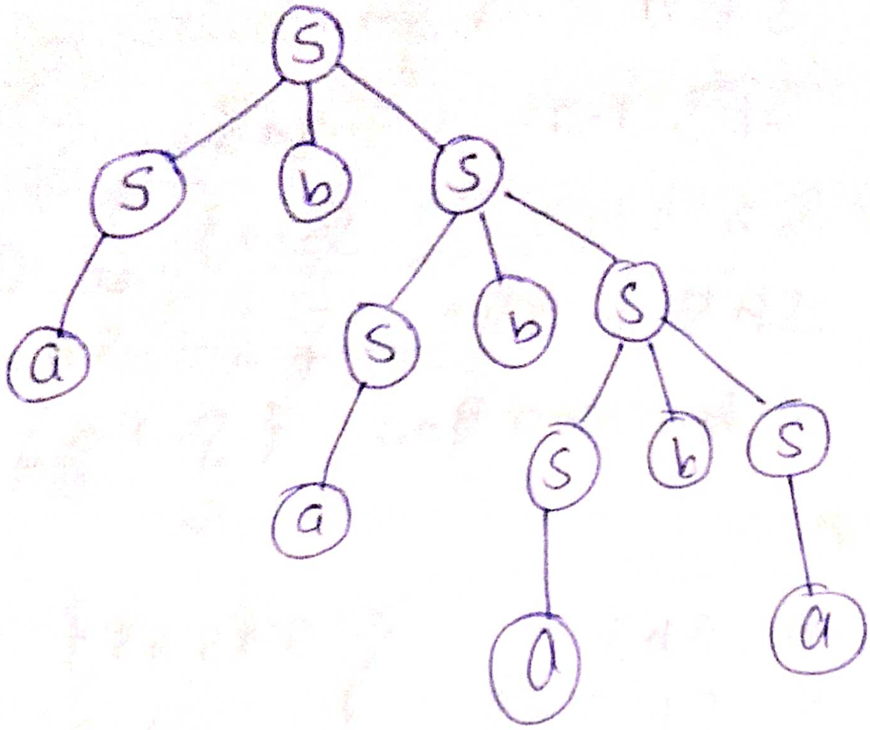
$S \to abSbS \quad \{S \to SbS\}$

$S \to ababS \quad \{S \to a\}$

$S \to ababSbS \quad \{S \to SbS\}$

$S \to ababab S$ $\{S \to a\}$

$S \to abababa$ $\{S \to a\}$.



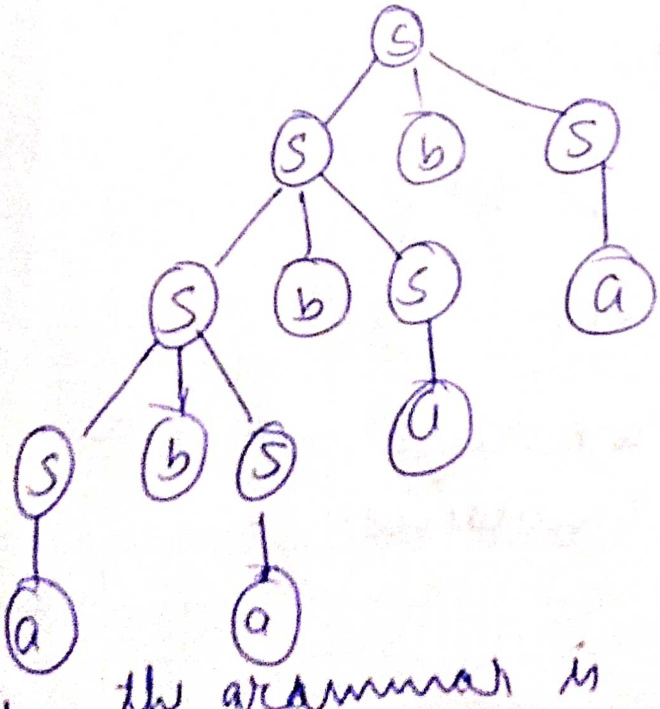
2nd LMD,

$S \to SbS$ $\{S \to SbS\}$

$S \to SbSbS$ $\{S \to SbS\}$

$S \to SbSbSbS$ $\{S \to SbS\}$

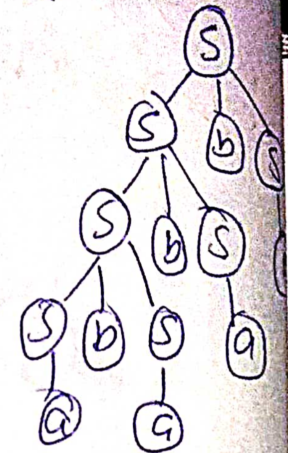
$S \to abababa$ $\{S \to a\}$.



$w = abababa$

Hence, the grammar is Ambagious.

I^{st} RMD : $w = abababa$



$S \to SbS$

$S \to Sba \quad \{S \to a\}$

$S \to SbSba \quad \{S \to SbS\}$

$S \to Sbaba \quad \{S \to a\}$

$S \to SbSbaba \quad \{S \to SbS\}$

$S \to ababa\cancel{b}ba \quad \{S \to a\}.$

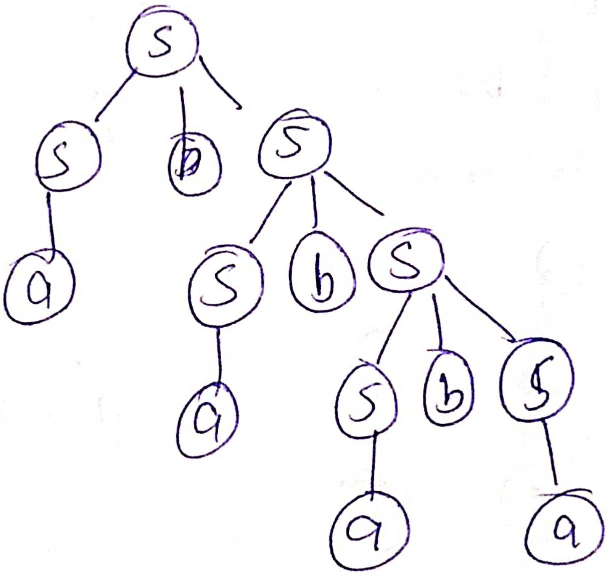
II^{nd} RMD :

$S \to SbS \quad \{S \to SbS\}$

$S \to SbSbS \quad \{S \to SbS\}$

$S \to SbSbSbS \quad \{S \to SbS\}$

$S \to abababa \quad \{S \to a\}$



As, I^{st} & II^{nd} RMD are different then the grammar is Ambiguous.

Simplification of the CFG =

(i). Reduction of useless variable.
(ii) Elemination of null production.
(iii) Elimination of unit production.

Note ÷ If any CFG contains all 3 abnormalities hence sequence to simplify is
null → unit → Useless

Reduction of useless variable ÷

Ex ÷ $S \rightarrow AB$, $A \rightarrow a$, $B \rightarrow b$
$B \rightarrow C$, $E \rightarrow C$ · useless

$G = [V_n, \Sigma, P, S] \Rightarrow G' = \{V_n', \Sigma, P', S\}$

a & b are terminal
A B are Variable

Step 1 ÷ Construction of V_n'
$W_i = \{A \in V_n : A \rightarrow w$ where $w \in \Sigma^*\}$

So, V_n'
$W_1 = \{A, B, E\}$

$W_{i+1} = W_i \cup \{A \in V_n : A \rightarrow \alpha$ where $\alpha \in (W_i \cup \Sigma)^*\}$

* W_i mai wo variables include honge jo directly terminals produce kr rhe hai

* W_{i+1} W_i ke variable + wo variable jo terminals and W_i se milke banne walli string agar right hand side mai hain then uska left hand side ka variable include hoga.

now $\Sigma = \{a, b, c\}$ $W_1 = \{A, B, E\}$

$W_2 = W_1 \cup \{S\}$

$W_3 = \{S, A, B, E\} \cup \phi$

$\boxed{V_N' = \{S, A, B, E\}}$

Step 2:-

$\Rightarrow$ Construction of P':-

$A \to \alpha$ where $\alpha \in (V_N' \cup \Sigma)^*$

1. $S \to AB$
2. $A \to a$
3. $B \to b$
5. $E \to c$

4. we are not including $B \to C$ as C is not present in $(V_N' \cup \Sigma)^*$

~~prammeliation~~

Theorem 2 ÷ (V_N'', Σ, P'', S)

① Construction of W_i for $i \geq 1$

$W_1 = \{S\}$

$W_{i+1} = W_i \cup \{x \in V_N \cup \Sigma :$

$A \to \alpha$ in P' where

α contains $x\}$

$W_1 = \{S.\}$

$W_2 = \{S\} \cup \{A, B\}$

$W_3 = \{S, A, B\} \cup \{A, B, a, b\}$

$W_4 = \{S, A, B, a, b\} \cup \phi.$

② Construction of (V_N'', Σ', P'', S)

$V_N'' = V_N' \cap W_K$

$= \{S, A, B, E\} \cap \{S, A, B, a, b\}$

$\boxed{V_N'' \Rightarrow \{S, A, B\}}$

$\Sigma' = \Sigma \cap W_K = \{a, b, c\} \cap \{S, A, B, a, b\}$

$\Rightarrow \{a, b\}$

$P' = \{A \to \alpha$ where $A \in W_k\}$

$$\boxed{\begin{array}{c} S \to AB \\ A \to a \\ B \to b \end{array}}$$

Ans.

$E \to c$ is not a Part of W_k hence it is not included in P'.

Que 2 :- $S \to AB \mid CA$ $\quad B \to BC \mid AB$

$A \to a \quad C \to aB \mid b$

Sol^n :-

$W_1 = \{A, C\}$

$W_2 = \{A, C\} \cup \{S\}$

$W_2 = \{A, C, S\}$

$W_3 = \{A, C, S\} \cup \phi$

$$\boxed{V_N' = \{S, A, C\}}$$

$$\boxed{\begin{array}{ll} P' :- & S \to CA \\ & A \to a \\ & C \to b \end{array}}$$

Ans.

Que 3 : $S \to aAa$ $A \to Sb \mid bCC \mid DaA$

$C \to abb \mid DD$ $E \to aC$

$D \to aDA$.

Soln :

$G = [V_N, \Sigma, P, S] \Rightarrow G' = [V_N', \Sigma, P', S]$

① Construction of V_N',

$W_1 = \{C\}$

$W_2 = \{C\} \cup \{A, E\}$

$W_3 = \{C, A, E\} \cup \{S\}$

$W_4 = \{C, A, E, S\}$

$\boxed{V_N' = \{C, A, E, S\}}$

② Construction of P',

P' : $S \to aAa$

$A \to Sb \mid bCC$

$C \to abb$

$E \to aC$

Theorem 2 (Lemma 2) :

① V_N'' : $W_1 = \{S\}$

$W_1 = \{S, a, A\}$

$W_2 = \{S, a, A, b, C\}$

$W_4 = \{A, S, C, a, b\} = W_K$

$V_N'' = V_N' \cap W_K$

$\Rightarrow \{S, A, C, E\} \cap \{S, A, C, a,$

$\Rightarrow \{S, A, C\}$

$\Sigma' = \Sigma \cap W_K = \{a, b\} \cap \{S, A, C, a,$

$\Rightarrow \{a, b\}$

$P'' = S \to aAa, \quad A \to Sb \mid bCC$

$C \to abb.$

Elimination of Null Production :-

$G = [V_N, \Sigma, P, S] \Rightarrow G' = [V_N, \Sigma, P', S$

Step 1 :- Construction of set of nullable variables,

$W_i = \{A : A \in V_N \text{ \& } A \to \Lambda\}$

$W_{i+1} = W_i \cup \{X : X \in V_N \text{ \& } X \to \alpha$

$\therefore \alpha \to W_i^* \}$

Que :- $S \to aS \mid AB \quad A \to \Lambda \quad B \to \Lambda \quad D \to b$

$W_1 = \{A, B\}$

$W_2 = \{A, B\} \cup \{S\}$

$\Rightarrow \{A, B, S\} =$ final

Step: Construction of P' :-

(i). Add those production whose RHS don't contain any nullable variable.

① P' : $D \to b$.

(ii). If $A \to x_1, x_2 \ldots x_k$ is in P. then add $A \to \alpha_1, \alpha_2 \ldots \alpha_k$ in P' where $\alpha_1 = x_1$ or $\alpha_1 = \Lambda$ if $x_i \in W_k$:

② $S \to \underset{x_1}{a}\ \underset{x_2}{S}$

③ $S \to a\ \Lambda$

$S \to a$.

④ $S \to \underset{x_1}{A}\ \underset{x_2}{B}$

⑤ $S \to B$

~~$S \to AB$~~

⑥ $S \to A$

$P' =$		
$S \to AB$	$S \to aS$	$D \to b$
$S \to A$	$S \to a$	
$S \to B$		

Elimination of unit Production :-

Single Variable $\to$ Single Variable

Direct method

$A \to B$	$A \to a$
$B \to C$	$B \to a$
$C \to D$	$C \to a$
$D \to a$	$D \to a$

Step 1 ÷ Construction of set of variables W_k derivable from A [{A→B} : sin G]

$W_i(A) = \{A\}$

$W_{i+1}(A) = W_i(A) \cup \{B : A \to B$ is in G $\}$

Que ÷ $S \to AB$, $A \to a$ $B \to C / b$

$C \to D$, $D \to E$, $E \to a$

$W_0(S) = \{S\}$

$W_1(S) = \{S\} \cup \{\phi\} = \{S\}$

$W_0(A) = \{A\}$ $W_1(A) = \{A\}$

$W_0(B) = \{B\}$ $W_1(B) = \{B\} \cup \{C\}$

$W_2(B) = \{B\ C\ D\}$

$W_3(B) = \{B, C, D, E\}$

$W_4 = (B) = \{B, C, D, E\} \cup \phi$

$W(B) = \{B, C, D, E\}$

$W_0(C) = \{C\}$ $W_1(C) = \{C, D\}$

$W_2(C) = \{C, D, E\}$

$W_0(D) = \{D\}$ $W_1(D) = \{D, E\}$

$W(D) = \{D, E\}$

$W_0(E) = \{E\}$

Step 2 ÷ P' ÷ (i) All non unit Production add.

(ii) $A \to \alpha$: if $B \to \alpha$ is in G with $B \in W(A)$ and $\alpha \notin V_N$

(i). G' ⇒ P' ÷ ① $S \to AB$

② $A \to a$

③ $B \to b$

④ $E \to a$

(ii). $B \to a$, $C \to a$, $D \to a$

So/
$S \to AB$
$A \to a$
$B \to a | b$
$C \to a$
$D \to a$
$E \to a$

Final ÷
$S \to AB$
$A \to a$
$B \to b$

(6.1). $a^*b + a^*b$

(6.5) $S \to aB \mid bA$
$A \to aS \mid bAA \mid a$
$B \to bS \mid aBB \mid b$

$w = aaabbabbba$.

(a) left most derivation :-

$S \to aB$ {$S \to aB$}
$S \to aaBB$ {$B \to aBB$}
$S \to aaaBB$ {$B \to aBB$}
$S \to aaabSB$ {$B \to bS$}
$S \to aaabbAB$ {$S \to bA$}
$S \to aaabbaSB$ {$A \to aS$}
$S \to aaabbabAB$ {$S \to bA$}
~~$S \to aaabbabbAAB$ {$A \to bAA$}~~
~~$S \to aaabbabb$~~

(a)

$S \to aB$ {$S \to aB$}
$S \to aaBB$ {$B \to aBB$}
$S \to aaaBBB$ {$B \to aBB$}
$S \to aaabBB$ {$B \to b$}

S→ aaa bb B {B→b}
S→ aaa bba BB {B→aBB}
S→ aaa bbabB {B→b}
S→ aaa bb a b bS {B→bS}
S→ a aa bbà bb bA {S→bA}
S→ aaa bba bb b a {A→a}.

(b) right most :-

S→ aB {S→aB}
S→ a a BB {B→aBB}
S→ a a.B bS {B→bS}
S→ a a B b bA {S→bA}
S→ a a B b b a {A→a}
S→ a a a BB bb a {B→aBB}
S→ a a a BB B b ba {B→b}
S→ a a a b S b b a {B→bS}
S→ a a a b b A b b a {S→bA}
S→ a a a b b a S b b a {A→aS}
S→ a a a b b a
S→ a a B b b a {A→a}
S→ a a a a BB b b a. {

$S \to aB$ $\{S \to aBB\}$

$S \to aaBB$ $\{S \to aBB\}$

$S \to aaBaBB$ $\{S \to aBB\}$

$S \to aaBaBbS$ $\{B \to bS\}$

$S \to aaBaBbbA$ $\{S \to bA\}$

$S \to aaBaBbba$ $\{A \to a\}$

$S \to aaBabbba$ $\{B \to b\}$

$S \to aaaBBabbba$ $\{B \to aBB\}$

$S \to aaabbabbba$ $\{B \to b\}$

Chomsky Normal Form (CNF):

$G = \{V_N, \Sigma, P, S\}$ $G' = \{V_N', \Sigma', P', S'\}$

① Elimination of Null & unit Production

CNF:- in production in CFG are like $A \to BC$
$A \to a$ Terminal

② Elimation of terminals on R.H.S

③ Restricting the number of variables on RHS of the Production

$S \to aAD$ $A \to aB \mid bAB$ ⑧ $B \to b$

⑨ $D \to d$

$G = [V_N, \Sigma, P, S] \to G'[V_N', \Sigma', P', S']$

Step 1 ÷ As there is no null and unit production. So proceed to step 2.

Step 2 ÷ Eliminations of terminals on RHS [if ~~the~~ there is mixing of variables and terminal on RHS then we should eliminate Terminal on RHS].

$S \rightarrow aAD$ $A \rightarrow aB$ $A \rightarrow bAB$

(5) $C_a \rightarrow a$
$S \rightarrow C_a AD$

(6) $A \rightarrow C_a B$

(7) $C_b = b$
$A \rightarrow C_b AB$

Step 3 ÷ Restricting no. of variables on RHS

$S \rightarrow C_a AD$ $A \rightarrow C_b AB$

(1) $C_1 \rightarrow AD$
(2) $S \rightarrow C_a C_1$

(3) $C_2 \rightarrow AB$
(4) $A \rightarrow C_b C_2$

(1) $S \rightarrow C_a C_1$
(2) $C_1 \rightarrow AD$
(3) $A \rightarrow C_b C_2$
(4) $C_2 \rightarrow AB$
(5) $C_a \rightarrow a$
(6) $A \rightarrow C_a B$
(7) $C_b = b$
(8) $D \rightarrow d$
(9) $D \rightarrow d$