4th Jan 2019

Modeling and Simulation

* Components of a System

1. Entity → An object of interest in the system.
2. Attribute → A property of an entity.
3. Activity → A time of specified length.
4. State → The collection of variable necessary to describe the system at any time, relative to the object of the study.
5. Event → An instantaneous occurance that may change the state of the system.
6. Endogenous → to describe activities and events occuring with in the system.
7. Exogenous → to describe activities and event in an environment that affect the system.

Example

System	Attribute	Activities	State variables
Bank	Checking	Arrival	No. of busy tellers.
	Deposit	Departure	No. of customer waiting.

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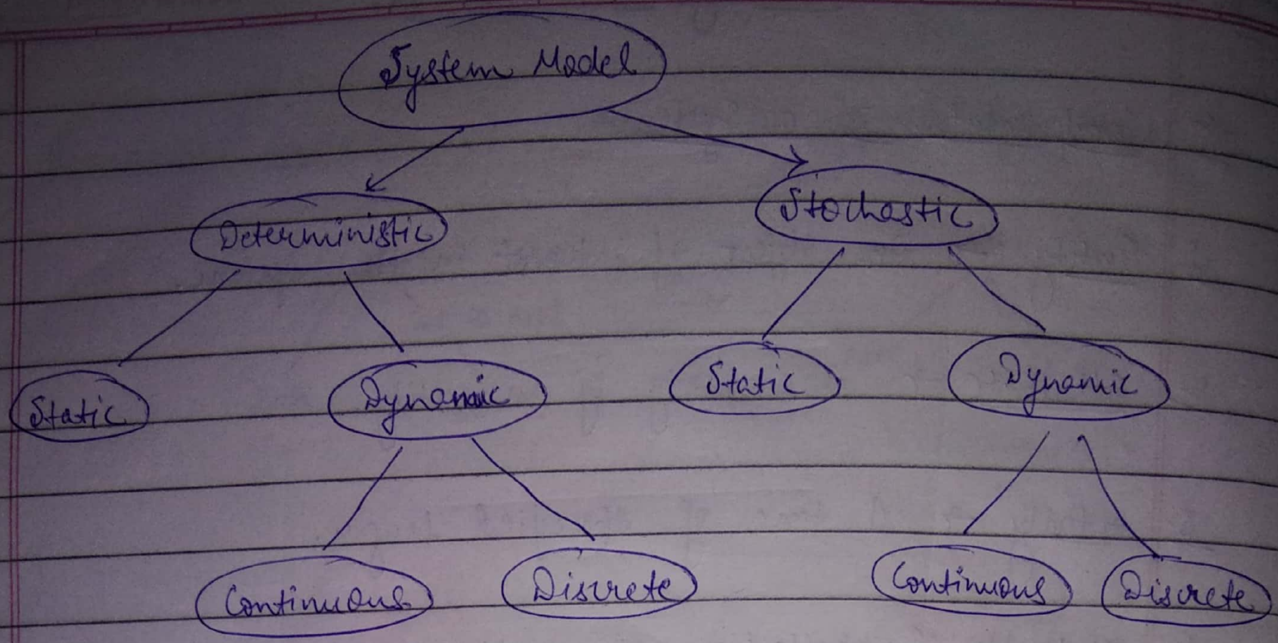

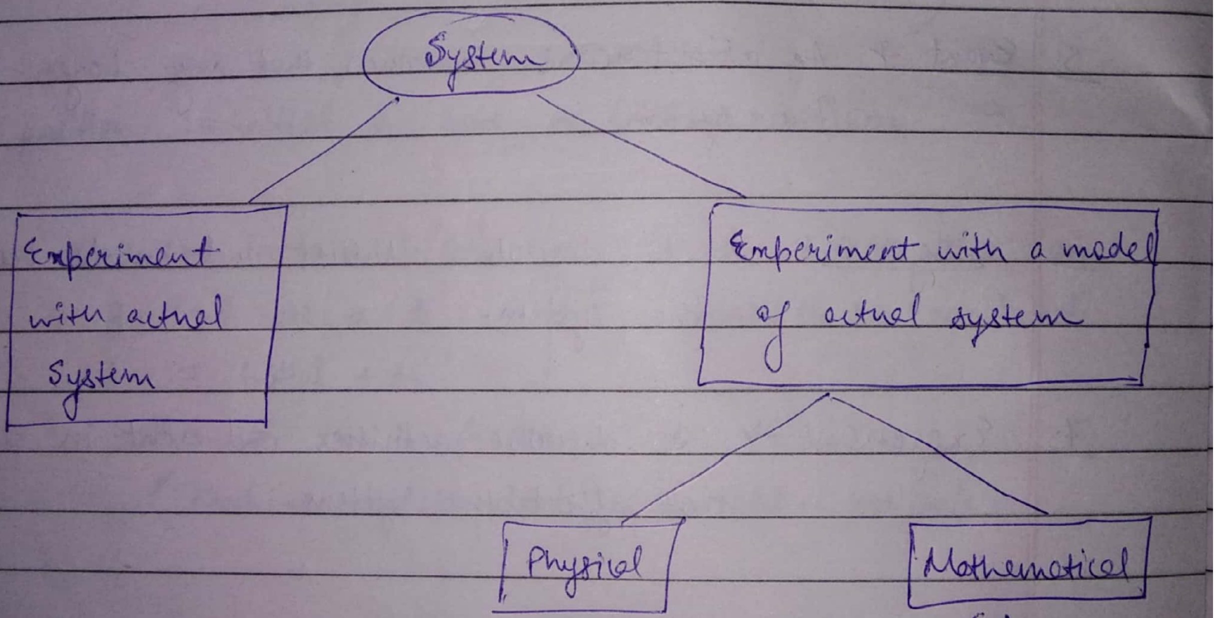

How to develop a model

1. Build a conceptual model
2. Convert it to specification model.
3. Convert it to computational model
4. verify.
5. Validate.

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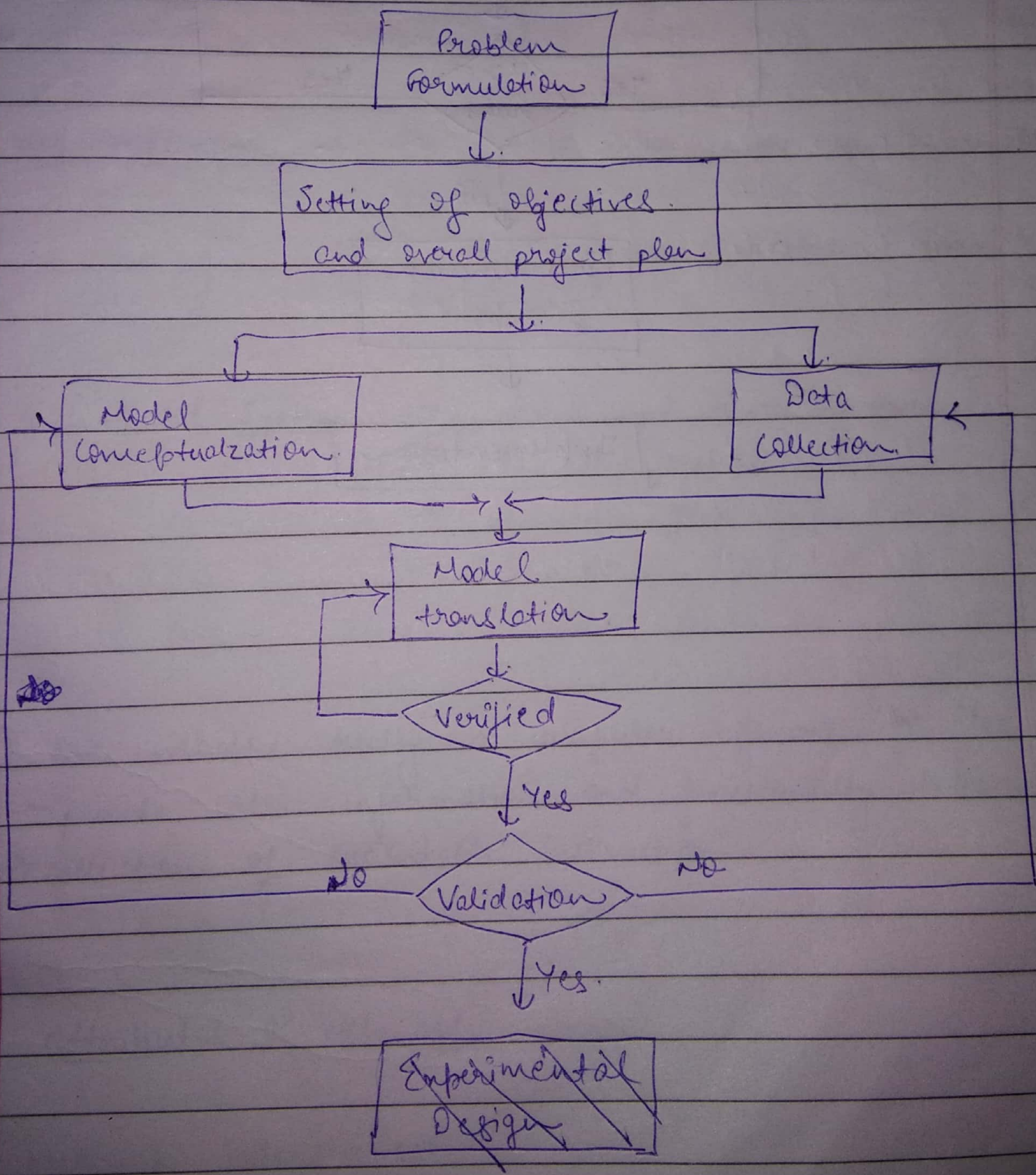
* 3- Model level.

1. Conceptual
2. Specification → Pseudo code.
3. Computational → Coding part (implementation).

* Flow chart of simulation



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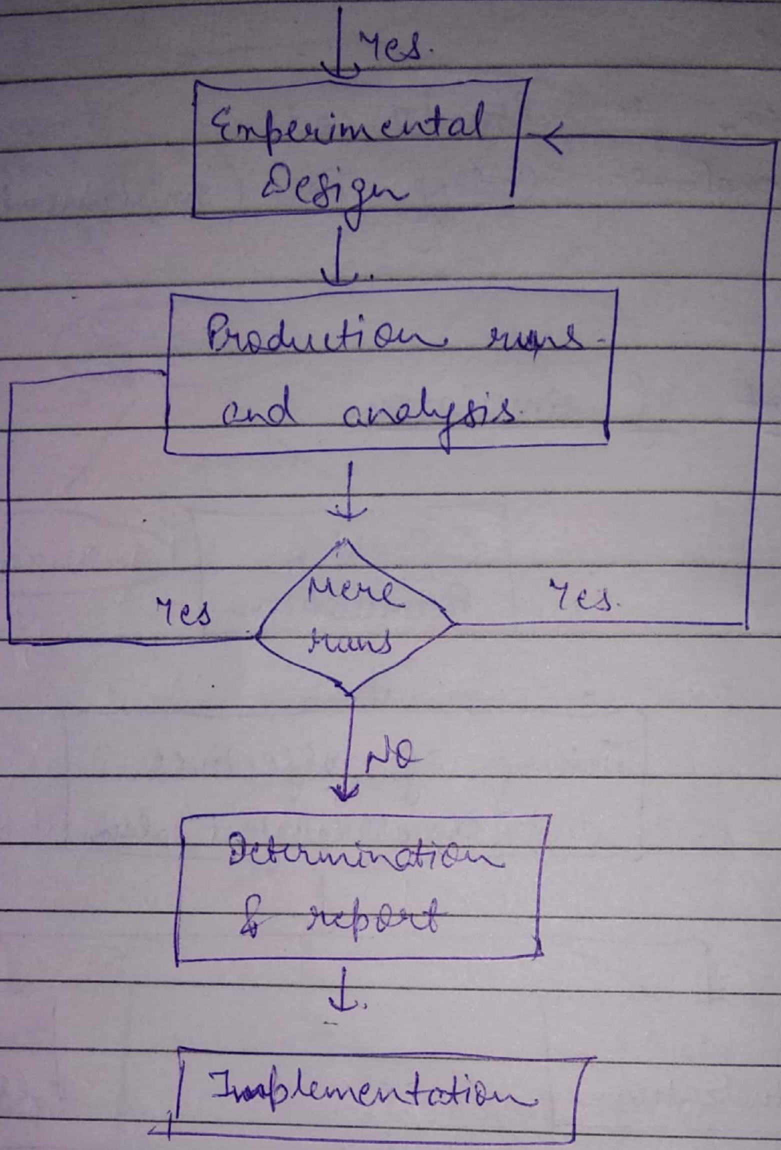

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8th Jan 2019 — Modeling and Simulation

Category	Time Req	No of Patients	Prob	C.P.D	R.N. Interval
Filling	45 min	40	0.40	0.40	00 – 39
Crown	60 min	15	0.15	0.55	40 – 54
Cleaning	15 min	15	0.15	0.70	55 – 69
Extracting	45 min	10	0.10	0.80	70 – 79
Checkup	15 min	20	0.20	1.00	80 – 99
		100			

Ques. Strong is a Dentist whose schedule of all patients for 30 mins appointment. Some of the patients take more or less depending on the type of dental work to be done. The following summary shows the various categories of work, their responsibilities, their probabilities and the time actually needed to compute the work.

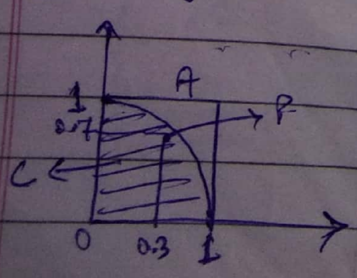
* Monte Carlo Simulation

A method of estimating the value of an unknown quantity using the principles of "inferential statistics".

→ (same property as the parent). (minimum variance)

→ if variance is high then it fails.

* Value of π



$\sqrt{r_1^2 + r_2^2} < 1$

$C = 78$

$A = 100$

Let, P = points inside 'C' out of 10,000,
$A = 10,000$

$$\text{estimated area of } C = \boxed{\frac{P}{10,000} \times 1}$$ → estimated area

$$\frac{\pi}{4} = \frac{P}{10,000} \Rightarrow \boxed{\pi = \frac{4P}{10,000}}$$

* Steps of Monte Carlo

→ Establishing Prob. distribution → {Given}
→ Cummulative Prob. distribution.
→ Setting Random Number Intervals.
→ Generating RN (generally given).

Ques Simulate the dentist clinic for 4 hrs and find average waiting time for patient as ~~well~~ well as idleness of doctor.
Assume that all patients arrive at their scheduled ~~ar~~ A.T. starting at 8. A.M.
Use the following ~~R.N~~ R.N. for handling prob.
40, 82, 11, 34, 25, 66, 17, 79.

Patient	Sch. Arrival	R.NO.	Category	Service Time
1	8:00	40	crown	60 min
2	8:30	82	check up	15
3	9:00	11	filling	45
4	9:30	34	filling	45
5	10:00	25	filling	45
6	10:30	66	cleaning	15
7	11:00	17	filling	45
8	11:30	78	extracting	45

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Patient	Sch. Arr.	Service Start	Duration	Ser. End	Waiting	Idle time
1	8:00	8:00	60 min	9:00	0	0
2	8:30	9:00	15	9:15	30	0
3	9:00	9:15	45	10:00	15	0
4	9:30	10:00	45	10:45	30	0
5	10:00	10:45	45	11:30	45	0
6	10:30	11:30	15	11:45	60	0
7	11:00	11:45	45	12:30	45	0
8	11:30	12:30	45	1:15	60	0
					285	

Avg. waiting time $= \frac{285}{8} = \boxed{35.62 \text{ min}}$

15th Jan 2016 Modeling & Simulation

Ques.

Daily Demand	Prob.
0	0.01
15	0.15
25	0.20
35	0.50
45	0.12
50	0.02

R.No. → 48, 78, 09, 51, 56, 77, 15, 14, 68, 09

Check simulation for next 10 days.

cakes made = 35/day

Avg. daily demand.

Ans.

Daily Dmd	Prob	C.F	R.No
0	0.01	0.01	0-0
15	0.15	0.16	1-15
25	0.20	0.36	16-35
35	0.50	0.86	36-85
45	0.12	0.98	86-97
40	0.02	1.00	98-99

35 + 35 + 15 + 35 + 35 + 35 + 15 + 15 + 35 + 15 = 270

Day	R.No.	Dmd	35/day	stock
1	48	35	35	0
2	78	35	35	0
3	09	15	35	20
4	51	35	'	0
5	56	35	'	0
6	77	35	'	0
7	15	15		20
8	14	15	'	20
9	68	35	'	0
10	09	15	35	20

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Ques The automobile company manufactures around 150 scooters, the daily production varies from 146 - 154 the finished scooter are transported in container accumudated 150 scooter. Use the following R.No to find the answer
80, 81, 76, 75, 64, 43, 18, 26, 10, 12, 65, 58, 69, 61, 57.
1. Simulate avg. no. of scooter waiting in a factory.
2. Avg. empty space in a container

Prod/day	Prob.	C.P.	Rang.
146	0.04	0.04	0 - 3
147	0.09	0.13	4 - 12
148	0.12	0.25	13 - 24
149	0.14	0.39	25 - 38
150	0.11	0.50	39 - 49
151	0.10	0.60	50 - 59
152	0.20	0.80	60 - 79
153	0.12	0.92	80 - 91
154	0.08	1.00	92 - 99

		waiting	
80	153	3	0
81	153	3	0
76	152	2	0
75	152	2	0
64	152	2	0
43	150	0	0
18	148	0	2
26	149	0	1
10	147	0	3
12	147	0	3
65	152	2	0
58	151	1	0
69	152	2	0
61	152	2	0
57	151	1	0

Ans-1 $\boxed{\frac{20}{15}}$

Ans-2 $\boxed{\frac{9}{15}}$

1 2

$\boxed{\frac{20}{15}, \frac{9}{15}}$

$\frac{9}{15}$

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23rd Jan 2019 — **Modeling & Simulation**

★ **Random Variable** — Discrete (B/w 1-6) / Continuous

★ **Discrete R/V.**

R_x of a Random variable X; (Range)
If it is discrete finite no's or countable infinite

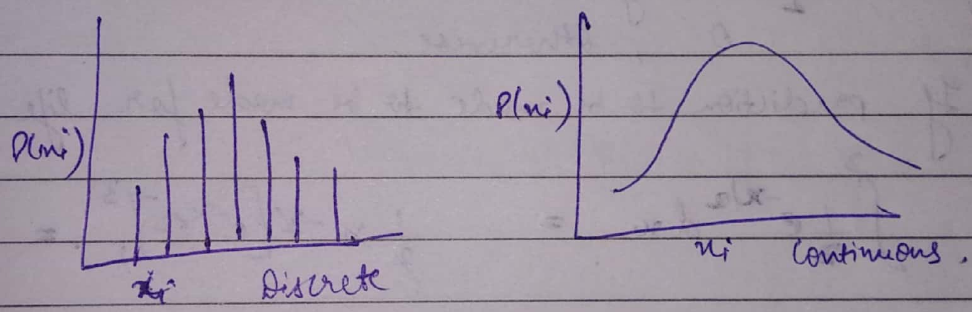
Properties

$P(x_i) \rightarrow$ Prob that X takes x_i

$P(x_i) \geq 0$, $\sum_{all} P(x_i) = 1$

$(x_i, P(x_i))$ is called prob dist of x

$P(x_i)$ is called prob mass function

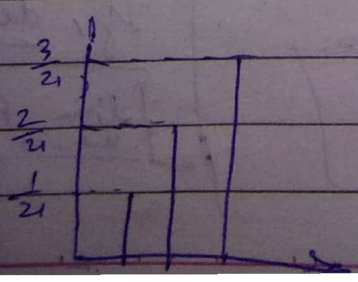


Ques (Die tossing exp. x = no. of spots on up. face.

$R_x = \{1, 2, 3, 4, 5, 6\}$

Assume prob of a given face getting landed is proportional to no. of spots shown.

x_i	1	2	3	4	5	6
$P(x_i)$	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{4}{21}$	$\frac{5}{21}$	$\frac{6}{21}$



DELTA

Continuous R.V

If R_x of a R.V X is an interval or collection of interval, X is cont. R.V.

Prob that X lie in interval (a,b) is

$$P(a \le X \le b) = \int_a^b f(x)\,dx$$

properties

$f(x) \ge 0$ for all x in R_x

$$\int_{R_x} f(x)\,dx = 1$$

$f(x) = 0$ If x is not in R_x

Ques $f(x) = \frac{1}{2} e^{-x/2}$ for $x \ge 0$

0 , otherwise

If prediction to be made to be made for life 2-3 years?

Ans $$\int_2^3 \frac{1}{2} e^{-x/2}\,dx = \frac{1}{\cancel{2}} \times -\cancel{2}\left[e^{-x/2}\right]_2^3 = e^{-\frac{2}{2}} - e^{-\frac{3}{2}}$$

Cummulitive distribution function

If X is discrete, $F(x) = \sum_{x_i < x} P(x_i)$

If cont. $F(x) = \int_{-\infty}^{x} f(t)\,dt$

F is ~~inc~~ non ~~decreasing~~ decaying function

then $\boxed{\lim_{x \to \infty} F(x) = 1}$

If, decaying

$\boxed{\lim_{x \to -\infty} F(x) = 0}$

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* <u>Expected value or Mean value</u>

$$E(x) = \sum_{\text{all } i} x_i \, P(x_i) \text{ for discrete } x$$

for <u>continuous</u>

$$E(x) = \int_{-\infty}^{\infty} x \, f(x) \, dx$$ ~~for conti x~~

for <u>continuous</u> x

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29th Jan 2019 Modelling

★ Variance

$$V = E[X - E(x)]^2.$$

$$V = E(x^2) - [E(x)]^2$$

★ Bernoulli Distribution

→ For experiments consisting of n-trials each can be a success or a failure

$x_j = 1$ If the jth experiment is success

and

$x_j = 0$ If ~~the~~ it is a failure

→ If n Bernoulli trails are independent It is called Bernoulli Process distribution.

$$P_i(x_j) = P(x_j) = \begin{cases} p, & x_j = 1, j = 1, 2 \ldots n \\ 1-p = q, & x_j = 0 \\ 0, & \text{otherwise} \end{cases}$$

Mean = $E(x_j) = 0 \times q + 1 \times p = p$

Variance = $V(x_j) = [0^2 \times q + 1^2 \times p] - p^2 = pq$

★ ~~Rand~~ Random Number

➡ every stochastic process should contain a Random number

→

DELTA Pg No. Date

☆ Considerations of R.No.
→ Generation should be fast
→ It should portable.
→ Should have a sufficiently long cycle
→ Should be replicable
→ Inden.
→ uniform

☆ Challenges in generating Pseudo R.No.
→ Generation should be uniformly distributed.
→ Mean of the generated no. may be too high or too low → X
→ Variance of generated no. may be too high or too low → X
→ Auto coreletion b/w no's

☆ Tech. generation of pseudo R.No.

(a) Congruential generator
→ linear C.G.
→ Mixed C.G.
→ Multiplicative C.G.
→ Combined linear C.G.

1 Linear C.G.

$$z_i = (a z_{i-1} + c) \bmod m.$$

$$R.No = \frac{z_i}{m} \{0 - 1\}$$

z_0 → seed value

Q/- Parameters, $M = 63$, $a = 22$, $c = 4$, $z_0 = 19$

$$z_i = [22 z_{i-1} + 4] \bmod 63$$

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	$22Z_{i-1}+4$	Z_i	R.No.
0		19	
1	422	44.	0.6984
2	972	27	0.4286
3	598	31	0.4921
⋮			
61	158	32	0.5079
63	334.	19	0.3016
64	422	44.	0.6384

⇒ For 'm' a power of 2, say $2^b = m$, and c is not equal to 0, longest possible period ($p = m = 2^b$) is obtained, If c is relatively prime to 'm' and $a = 1 + 4k$ (k is an integer)

⇒ For $m = 2^b$ & $c = 0$, longest possible period ($p = \frac{m}{4}$) is obtained If x_0 is odd and $a = 3 + 8k$ or $5 + 8k$ ($k = 0, 1, 2, \ldots$)

5th Feb 2019

Modelling & Simulation

i	x_i	x_{i+1}	x_i	x_{i+1}	x_i	x_{i+1}
0	1	13	2	26	4	52
1	13	41	26	18	52	36
2	41	21	18	42	36	20
3	21	17	42	34	20	4
4	17	28			4	52
					52	36

As the value of seeds increases in even no, the ~~value~~ random no will start to repeat earlier.

→ Speed and efficiency are aided by use of modulus M which is either a power of 2 or close to power of 2.

$m = 10^2, \quad a = 19, \quad c = 0, \quad x_0 = 63$

$X_1 = (1197) \bmod 10^2 = 97$

$(173) \bmod 2^6 = 45 \{1\times2^5 + 0\times2^4 + 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0\}$

$= 101101$

$173 \rightarrow 1\times2^7 +$

$= 10\underbrace{101101}_{}$ → same mod 2^6

Hypothesis for testing of uniformity

→ $H_0: R_i \sim U[0,1]$

→ $H_1: R_i \nsim U[0,1]$

→ The null hypothesis reads that the no. are distributed uniformly on [0,1]. Failure to reject the H_0 means that no evidence of uniformity has been detected. On the basis of this test this does not implies further

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testing of the generator for uniformity is unnecessary.

Hypothesis for testing of independency.

→ H_0 : $R_i \sim$ independency

→ H_1 : $R_i \nsim$ independency

⇒ For each test 'α' level of significance 'α' must be stated the 'α' is the ~~prog~~ probability of rejecting null hypothesis. Given that the null hypothesis is true.

$$\alpha = P\left(\frac{\text{reject } H_0}{H_0 \text{ true}}\right)$$

⇒ If several test are conducted on the same set of no. the prob. of rejecting the null hypothesis on at least one test by chance increases.

Uniformity Test

1. Kolmogorov ~~test~~ Smirnov test → (works on deviation)
2. Chi square test.

1. Kolmogorov Smirnov test

→ Based on the largest absolute deviation b/w $F(x)$ (uniform prob dist.) & $S_N(x)$.

$$S_N(x) = \frac{\text{No. of } R_1, R_2 \ldots R_N}{N} \text{ which are } \le x$$

$$D = \max |F(x) - S_N(x)|$$

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Steps

1. Ranking data from smallest to largest
2. Compute the parameters.

$$D^+ = \max_{1 \le i \le N} \left\{ \frac{i}{N} - R_i \right\}.$$

$$D^- = \max_{1 \le i \le N} \left\{ R_i - \frac{i-1}{N} \right\}$$

Compare $D = \max[D^+, D^-]$

$D\alpha$

Normalised value	$D\alpha$ 0.3	$D\alpha$ 0.5	$D\alpha$ 0.8
1	0.693		
2			
3			

Given Table (available online)

(i) If $D > D\alpha$ then does not follow uniformity.

(ii) $(D < D\alpha)$ accepted uniformity

Example: 0.44, 0.81, 0.14, 0.05, 0.93 ; $D\alpha \rightarrow 0.565$ $(\alpha = 0.05)$

	i=1	i=2	i=3	i=4	i=5
R_i	0.05	0.14	0.44	0.81	0.93.
i/N	0.2	0.4	0.6	0.8	1.0
$i/N - R_i$	0.15	0.26	0.16	—	0.07
$R_i - \frac{i-1}{N}$	0.05	—	0.04	0.21	0.13.

$D^- = 0.21$, $D^+ = 0.26$

$D = 0.26$

$D < D_\alpha$ ~~∴~~ Accepted uniformity.

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★ Chi Square test

From uniform distribution no in each class.

$\left\{ \frac{N}{n} = E_i \right\}$

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

O_i = Observed no.
E_i = Expected no.

$\rightarrow \chi_\alpha^2 > \chi^2 \rightarrow$ uniform.
$\chi_\alpha^2 < \chi^2 \rightarrow$ non-uniform

Calculate expected freq 2x2 table

A/B	B_1	B_2	Total
A_1	F_{11}	F_{12}	N_1
A_2	F_{21}	F_{22}	N_2
Total	N_3	N_4	N.

M x N table

$\boxed{v = (m-1)(n-1)}$

degree of freedom

$fe_{11} = \frac{N_1 \times N_3}{N}$, $fe_{12} = \frac{N_1 \times N_4}{N}$, $fe_{21} = \frac{N_2 \times N_3}{N}$, $fe_{22} = \frac{N_2 \times N_4}{N}$

Ques. In a sample survey of public opinion answer to the question.

1) Do you drink
2) Are you in favour of local opinion sale liquor are tabulated below.

		Ques 1		
		Yes	No	Total
Ques 2	Yes	56	31	87
	No	18	6	24
	Total	74	37	111

Can you infer or not the local opinion on the sale of liquior is dependent on individual drink.

Given that ~~do~~ $\boxed{\chi_\alpha^2 = 3.841}$ corresponding to 5%

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2014

$$fe_{11} = \frac{87 \times 74}{111} = 58.$$

$$fe_{12} = \frac{87 \times 37}{111} = 29$$

$$fe_{21} = 16$$

$$fe_{22} = 8$$

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{(56-58)^2}{58} + \frac{2^2}{29} + \frac{2^2}{16} + \frac{2^2}{8}$$

$$= 0.068 + 0.137 + 0.25 + 0.5 = \boxed{0.955}$$

$$\chi_\alpha^2 = 3.841$$

$$\chi^2 = 0.955$$

$\chi_\alpha^2 > \chi^2 \Rightarrow$ uniform

Ans

★ Autocorrelation test :

→ Concerned with dependency b/w numbers with sequence

→ Computation of autocorrelation b/w ~~every~~ every 'm' numbers (m = lag). starting with i^{th} number

$\delta_{im} \rightarrow$ lag

Starting with i

highest integer

$R_i, R_{i+m}, R_{i+2m}, \ldots, R_{i+(M+1)m}$

M is largest integer such that $[i + (M+1)m \leq N]$.

→ A non-zero auto-correlation implies lack of dependence, so following detain test is appropriable.

$H_0\ S_{im} = 0 \Rightarrow$ dependent

$H_1\ S_{im} \neq 0 \Rightarrow$ independent / dependent

$$\left\{ Z_0 = \frac{\delta_{im}}{\sigma(\delta_{im})} \right\}$$ for non zero case

$$f = \frac{1}{M+1}\left[\sum_{k=0}^{M} R_i + k m R_i + (k+1)m\right] - 0.25$$

$$\sigma_{\delta_{im}} = \frac{\sqrt{13M+7}}{12(M+1)}$$

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