Chapter 6

The Time Value of Money

LEARNING OBJECTIVES

After studying this chapter you should be able to:
- Calculate the future value and present value of a single amount.
- Calculate the future value and present value of an annuity.
- Set up a loan amortisation table.
- Explain how compounding frequency impacts on the effective rate of interest.

Money has time value. A rupee today is more valuable than a rupee a year hence. Why? There are several reasons:
- Individuals, in general, prefer current consumption to future consumption.
- Capital can be employed productively to generate positive returns. An investment of one rupee today would grow to $(1 + r)$ a year hence (r is the rate of return earned on the investment).
- In an inflationary period, a rupee today represents a greater real purchasing power than a rupee a year hence.

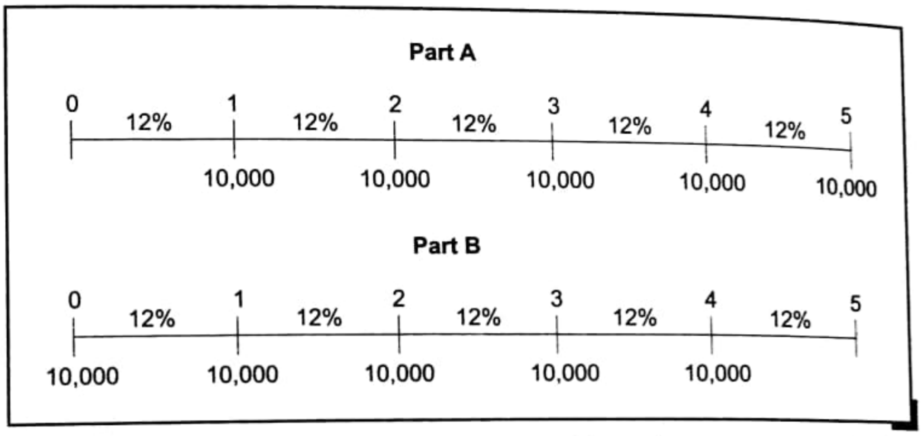
Most financial problems involve cash flows occurring at different points of time. These cash flows have to be brought to the same point of time for purposes of comparison and aggregation. Hence you should understand the tools of compounding and discounting which underlie most of what we do in finance - from valuing securities to analysing projects, from determining lease rentals to choosing the right financing instruments, from setting up the loan amortisation schedules to valuing companies, so on and so forth.

6.1 TIME LINES AND NOTATION

When cash flows occur at different points in time, it is easier to deal with them using a **time line**. A time line shows the timing and the amount of each cash flow in a cash flow stream. Thus, a cash flow stream of Rs.10,000 at the end of each of the next five years can be depicted on a time line like the one shown in Part A of Exhibit 6.1.

In Exhibit 6.1, 0 refers to the present time. A cash flow that occurs at time 0 is already in present value terms and hence does not require any adjustment for time value of money. You must distinguish between a *period of time* and a *point of time*. Period 1 which is the first year is the portion of time line between point 0 and point 1. The cash flow occurring at point 1 is the cash flow that occurs at the end of period 1. Finally, the discount rate, which is 12 percent in our example, is specified for each period on the time line and it may differ from period to period. If the cash flow occurs at the beginning, rather than the end, of each year, the time line would be as shown in Part B of Exhibit 6.1.

Exhibit 6.1 **Time Line**



Note that a cash flow occurring at the end of year 1 is equivalent to a cash flow occurring at the beginning of year 2.

Cash flows can be positive or negative. A positive cash flow is called a *cash inflow*; a negative cash flow, a *cash outflow*.

The following notation will be used in our discussion:

PV	:	Present value
FV_n	:	Future value n years hence
C_t	:	Cash flow occuring at the end of year t
A	:	A stream of constant periodic cash flows over a given time
r	:	Interest rate or discount rate
g	:	Expected growth rate in cash flows
n	:	Number of periods over which the cash flows occur.

6.2 FUTURE VALUE OF A SINGLE AMOUNT

Suppose you invest Rs.1,000 for three years in a savings account that pays 10 percent interest per year. If you let your interest income be reinvested, your investment will grow as follows:

First year	:	Principal at the beginning	1,000
		Interest for the year	100
		(Rs.1,000 × 0.10)	1,100
		Principal at the end	

Second year	:	Principal at the beginning	1,100
		Interest for the year	110
		(Rs.1,100 × 0.10)	
		Principal at the end	1,210
Third year	:	Principal at the beginning	1,210
		Interest for the year	121
		(Rs.1,210 × 0.10)	
		Principal at the end	1,331

Formula

The process of investing money as well as reinvesting the interest earned thereon is called compounding. The future value or compounded value of an investment after n years when the interest rate is r percent is :

$$\boxed{FV_n = PV(1 + r)^n} \tag{6.1}$$

In this equation $(1 + r)^n$ is called the future value interest factor or simply the future value factor.

To solve future value problems you have to find the future value factors. You can do it in different ways. In the example given above, you can multiply 1.10 by itself three times or more generally $(1 + r)$ by itself n times. This becomes tedious when the period of investment is long.

Fortunately, you have an easy way to get the future value factor. Most calculators have a key labelled "y^x". So all that you have to do is to enter 1.10, press the key labelled y^x, enter 3, and press the "=" key to obtain the answer.

Alternatively, you can consult a future value interest factor (FVIF) table. Exhibit 6.2 presents one such table showing the future value factors for certain combinations of periods and interest rates. A more comprehensive table is given in Appendix A at the end of the book.

Suppose you deposit Rs.1,000 today in a bank which pays 10 percent interest compounded annually. How much will the deposit grow to after 8 years and 12 years ?

The future value 8 years hence will be:

$$\text{Rs.}1{,}000\,(1.10)^8 = \text{Rs.}1{,}000\,(2.144)$$
$$= \text{Rs.}2{,}144$$

The future value 12 years hence will be :

$$\text{Rs.}1{,}000\,(1.10)^{12} = \text{Rs.}1{,}000\,(3.138)$$
$$= \text{Rs.}3{,}138$$

Exhibit 6.2 **Value of $FVIF_{r,n}$ for Various Combinations of r and n**

n/r	6%	8%	10%	12%	14%
2	1.124	1.166	1.210	1.254	1.300
4	1.262	1.360	1.464	1.574	1.689
6	1.419	1.587	1.772	1.974	2.195
8	1.594	1.851	2.144	2.476	1.853
10	1.791	2.159	2.594	3.106	3.707
12	2.012	2.518	3.138	3.896	4.817

While tables are easy to use they have a limitation as they contain values only for a small number of interest rates. So often you may have to use a calculator or a spreadsheet – the use of spreadsheet is illustrated later.

Compound and Simple Interest

So far we assumed that money is invested at compound interest which means that each interest payment is reinvested to earn further interest in future periods. By contrast, if no interest is earned on interest the investment earns only simple interest. In such a case the investment grows as follows:

Future value = Present value [1 + Number of years x Interest rate]

For example, an investment of Rs.1,000, if invested at 12 percent simple interest rate, will in 5 years time become :

$1{,}000\ [1 + 5 \times 0.12] = \text{Rs.}1{,}600$

Exhibit 6.3 shows how an investment of Rs.1,000 grows over time under simple interest as well as compound interest when the interest rate is 12 percent. From this exhibit you can feel the power of compound interest. As Albert Einstein once remarked: "I don't know what the seven wonders of the world are, but I know the eighth -compound interest". You may be wondering why your ancestors did not display foresight. Hopefully, you will show concern for the posterity.

Exhibit 6.3 **Value of Rs.1000 Invested at 10 percent Simple and Compound Interest**

	Simple Interest					Compound Interest				
Year	Starting Balance	+	Interest	=	Ending Balance	Starting Balance	+	Interest	=	Ending Balance
1	1000	+	100	=	1100	1000	+	100	=	1100
5	1400	+	100	=	1500	1464	+	146	=	1610
10	1900	+	100	=	2000	2358	+	236	=	2594
20	2900	+	100	=	3000	6116	+	612	=	6728
50	5900	+	100	=	6000	106,718	+	10672	=	117,390
100	10,900	+	100	=	11,000	12,527,829	+	1,252,783	=	13,780,612

Exhibit 6.4 shows graphically how money grows under simple interest and compound interest. Note that under simple interest the growth is linear and under compound interest the growth is exponential.

Power of Compounding

The power of compounding is often illustrated with the sale of Manhattan Island in 1626. It was sold by Red Indians to Peter Minuit for \$24. Looking at the New York real estate prices today, it appears that Peter Minuit got a real bargain. But consider the future value of \$24 in 2010 if Red Indians had invested for 384 years (2010 minus 1626) at an interest rate of 8 percent per year:

$$\$\,24 \times (1.08)^{384} = \$\,164{,}033{,}800{,}000{,}000$$
$$= \$\,164 \text{ trillion}$$

The total value of land on Manhattan in 2010 may perhaps have not more than \$ 500 billion. So the deal was not really a bargain.

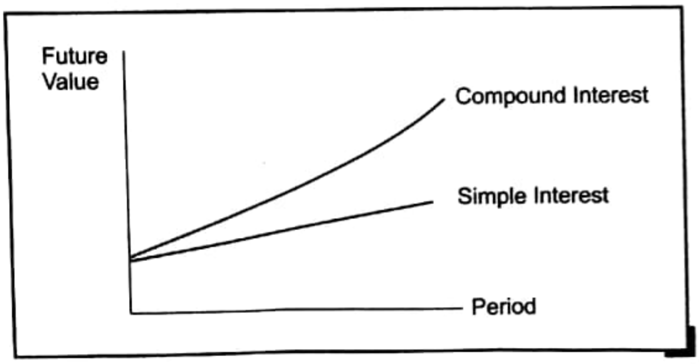
Although interesting, this comparison is misleading. First, judged by historical standards, the 8 percent rate is much higher. If we use a 3.5 percent rate, which is more consistent with historical experience, the future value of \$ 24 would be just $\$24 \times (1.035)^{384} = \$\,13{,}10\,1{,}034$. Second, we ignored the rental income to Peter Minuit and his successors' over the last 384 years.

Considering everything, Peter Minuit got a real good deal.

Doubling Period

Investors commonly ask the question: How long would it take to double the amount at a given rate of interest? To answer this question we may look at the future value interest factor table. Looking at Exhibit 6.2 we find that when the interest rate is 12 percent it takes about 6 years to double the amount, when the interest is 6 percent it takes about 12 years to double the amount, so on and so forth. Is there a rule of thumb which dispenses with the use of the future value interest factor table? Yes, there is one and it is called the rule of 72. According to this rule of thumb, the doubling period is obtained by dividing 72 by the interest rate. For example, if the interest rate is 8 percent, the doubling period is about 9 years (72/8). Likewise, if the interest rate is 4 percent the doubling period is about 18 years (72/4). Though somewhat crude, it is a handy and useful rule of thumb.

Exhibit 6.4 Graphic View of Simple and Compound Interest



If you are inclined to do a slightly more involved calculation, a more accurate rule of thumb is the rule of 69. According to this rule of thumb, the doubling period is equal to:

$$0.35 + \frac{69}{\text{Interest Rate}}$$

As an illustration of this rule of thumb, the doubling period is calculated for two interest rates, 10 percent and 15 percent.

Interest Rate	*Doubling Period*
10 percent	$0.35 + \frac{69}{10} = 7.25$ years
15 percent	$0.35 + \frac{69}{15} = 4.95$ years

Finding the Growth Rate

The formula we used to calculate future value is quite general and it can be applied to answer other types of questions related to growth. Suppose your company currently has 5,000 employees and this number is expected to grow by 5 percent per year. How many employees will your company have in 10 years? The number of employees 10 years hence will be:

$$5{,}000 \times (1.05)^{10} = 5000 \times 1.629 = 8{,}145$$

Consider another example. Phoenix Limited had revenues of Rs.100 million in 2000 which increased to Rs.1000 million in 2010. What was the compound growth rate in revenues? The compound growth rate may be calculated as follows:

$$100\,(1 + g)^{10} = 1{,}000$$

$$(1 + g)^{10} = \frac{1000}{100} = 10$$

$$(1 + g) = 10^{1/10}$$

$$g = 10^{1/10} - 1$$

$$= 1.26 - 1 = 0.26 \text{ or } 26 \text{ percent}$$

6.3 PRESENT VALUE OF A SINGLE AMOUNT

Suppose someone promises to give you Rs.1,000 three years hence. What is the present value of this amount if the interest rate is 10 percent? The present value can be calculated by discounting Rs.1,000, to the present point of time, as follows :

Value three years hence	= Rs.1,000
Value two years hence	$= \text{Rs.}1{,}000\left[\frac{1}{1.10}\right]$
Value one year hence	$= \text{Rs.}1{,}000\left[\frac{1}{1.10}\right]\left[\frac{1}{1.10}\right]$
Value now	$= \text{Rs.}1{,}000\left[\frac{1}{1.10}\right]\left[\frac{1}{1.10}\right]\left[\frac{1}{1.10}\right]$

Formula

The process of discounting, used for calculating the present value, is simply the inverse of compounding. The present value formula can be readily obtained by manipulating the compounding formula:

$$FV_n = PV\,(1 + r)^n \tag{6.2}$$

Dividing both the sides of Eq. (6.2) by $(1 + r)^n$, we get :

$$\boxed{PV = FV_n\,[1 / (1 + r)^n]} \tag{6.3}$$

The factor $1/(1 + r)^n$ in Eq. (6.3) is called the discounting factor or the present value interest factor ($PVIF_{r,n}$). Exhibit 6.5 gives the value of $PVIF_{r,n}$ for several combinations of r and n. A more detailed table of $PVIF_{r,n}$ is given in Appendix A at the end of this book.

What is the present value of Rs.1,000 receivable 6 years hence if the rate of discount is 10 percent ?

The present value is:

$$\text{Rs.}1{,}000 \times PVIF_{10\%,6} = \text{Rs.}1{,}000(0.565) = \text{Rs.}565$$

What is the present value of Rs.1,000 receivable 20 years hence if the discount rate is 8 percent? Since Exhibit 6.5 does not have the value of $PVIF_{8\%,20}$ we obtain the answer as follows:

$$\text{Rs.}1{,}000\left[\frac{1}{1.08}\right]^{20} = \text{Rs.}1{,}000\left[\frac{1}{1.08}\right]^{10}\left[\frac{1}{1.08}\right]^{10}$$

$$= \text{Rs.}1{,}000\,(PVIF_{8\%,10})(PVIF_{8\%,10})$$

$$= \text{Rs.}1{,}000\,(0.463)(0.463) = \text{Rs.}214$$

Exhibit 6.5 **Value of $PVIF_{r,n}$ for Various Combinations of r and n**

prepare

n/r	6%	8%	10%	12%	14%
2	0.890	0.857	0.826	0.797	0.770
4	0.792	0.735	0.683	0.636	0.592
6	0.705	0.630	0.565	0.507	0.456
8	0.626	0.540	0.467	0.404	0.351
10	0.558	0.463	0.386	0.322	0.270
12	0.497	0.397	0.319	0.257	0.208

Present Value of an Uneven Series

In financial analysis we often come across uneven cash flow streams. For example, the cash flow stream associated with a capital investment project is typically uneven. Likewise, the dividend stream associated with an equity share is usually uneven and perhaps growing.

The present value of a cash flow stream - uneven or even - may be calculated with the help of the following formula:

$$PV_n = \frac{A_1}{(1+r)} + \frac{A_2}{(1+r)^2} + \cdots + \frac{A_n}{(1+r)^n} = \sum_{t=1}^{n} \frac{A_t}{(1+r)^t} \tag{6.4}$$

where, PV_n is the present value of a cash flow stream, A_t is the cash flow occurring at the end of year t, r is the discount rate, and n is duration of the cash flow stream.

Exhibit 6.6 shows the calculation of the present value of an uneven cash flow stream, using a discount rate of 12 percent.

Exhibit 6.6 **Present Value of an Uneven Cash Flow Stream**

Year	Cash Flow Rs.	$PVIF_{12\%,n}$	Present Value of Individual Cash Flow
1	1,000	0.893	893
2	2,000	0.797	1,594
3	2,000	0.712	1,424
4	3,000	0.636	1,908
5	3,000	0.567	1,701
6	4,000	0.507	2,028
7	4,000	0.452	1,808
8	5,000	0.404	2,020
Present Value of the Cash Flow Stream			13,376

PYQ

Spreadsheet Analysis To calculate the present value of the cash flow stream given in Exhibit 6.6, you can use the Excel spreadsheet as shown below:

	A	B	C	D	E	F	G	H	I
1	Year	1	2	3	4	5	6	7	8
2	Cash flow	1,000	2,000	2,000	3,000	3,000	4,000	4,000	5,000
3	Discount rate	12%	NPV		=NPV(B3, B2:12)			→	13,375

Type the cash flows for years 1 through 8 in the cells B2 to I2 and the discount rate in the cell B3. If you want the present value in say cell I3, select I3 and type = NPV(B3,B2:I2) and press enter and the value will appear therein. Here it should be noted that in Excel the term NPV is used to denote the net result of adding the present values of a stream of future cash flows unlike our usual practice of using the term NPV, net present value, to denote the excess of the total present value of the future receipts (payments) over the initial investment (cash inflow).

6.4 FUTURE VALUE OF AN ANNUITY

An annuity is a stream of constant cash flows (payments or receipts) occurring at regular intervals of time. The premium payments of a life insurance policy, for example, are an annuity. When the cash flows occur at the end of each period, the annuity is called an **ordinary annuity** or a **deferred annuity**. When the cash flows occur at the beginning of each period, the annuity is called an **annuity due**. Our discussion here will focus on a deferred annuity.

The formula for an annuity due is simply $(1 + r)$ times the formula for the corresponding ordinary annuity.

Suppose you deposit Rs.1,000 annually in a bank for 5 years and your deposits earn a compound interest rate of 10 percent. What will be the value of this series of deposits (an annuity) at the end of 5 years? Assuming that each deposit occurs at the end of the year, the future value of this annuity will be:

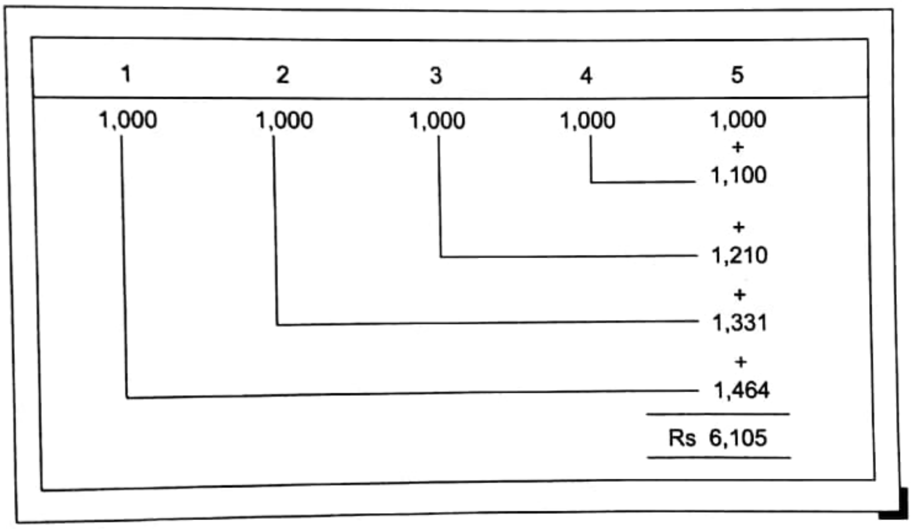
$$\text{Rs.}1{,}000(1.10)^4 + \text{Rs.}1{,}000(1.10)^3 + \text{Rs.}1{,}000(1.10)^2 + \text{Rs.}1{,}000(1.10) + \text{Rs.}1{,}000$$

$$= \text{Rs.}1{,}000(1.464) + \text{Rs.}1{,}000(1.331) + \text{Rs.}1{,}000(1.21) + \text{Rs.}1{,}000(1.10) + \text{Rs.}1{,}000$$

$$= \text{Rs.}6{,}105$$

The time line for this annuity is shown in Exhibit 6.7

Exhibit 6.7 **Time Line for an Annuity**



Formula

In general terms the future value of an annuity is given by the following formula:

$$\text{FVA}_n = A\,(1 + r)^{n-1} + A\,(1 + r)^{n-2} + \cdots + A$$

$$\boxed{\text{FVA}_n = A\,[(1 + r)^n - 1] \,/\, r} \qquad (6.5)^1$$

[1] The formula for the future value of an annuity is derived as follows:

The future value of an annuity is:

$$\text{FVA}_n = A\,(1 + r)^{n-1} + A\,(1 + r)^{n-2} + \cdots + A\,(1 + r) + A \qquad (1)$$

Multiplying both the sides of (1) by $(1+ r)$ gives:

$$\text{FVA}_n(1 + r) = A\,(1 + r)^n + A\,(1 + r)^{n-1} + \cdots + A\,(1 + r)^2 + A\,(1 + r) \qquad (2)$$

Subtracting (1) from (2) yields:

$$\text{FVA}_n\, r = A\,[(1 + r)^n - 1] \qquad (3)$$

Dividing both the sides of (3) by r gives:

$$\text{FVA}_n = A\left[\frac{(1+r)^n - 1}{r}\right] \qquad (4)$$

where FVA_n is the future value of an annuity which has a duration of n periods, A is the constant periodic flow, r is the interest rate per period, and n is the duration of the annuity.

The term $[(1 + r)^n - 1] / r$ is referred to as the future value interest factor for an annuity ($FVIFA_{r,n}$). The value of this factor for several combinations of r and n is given in Exhibit 6.8. A more detailed table is given in Appendix A at the end of this book.

Exhibit 6.8 **Value of $FVIFA_{r,n}$ for Various Combinations of r and n**

n/r	6%	8%	10%	12%	14%
2	2.060	2.080	2.100	2.120	2.140
4	4.375	4.507	4.641	4.779	4.921
6	6.975	7. 336	7.716	8.115	8.536
8	9.897	10.636	11.436	12.299	13.232
10	13.181	14.487	15.937	17.548	19.337
12	16.869	18.977	21.384	24.133	27.270

Applications

The future value annuity formula can be applied in a variety of contexts. Its important applications are illustrated below.

Knowing What Lies in Store for You Suppose you have decided to deposit Rs.30,000 per year in your Public Provident Fund Account for 30 years. What will be the accumulated amount in your Public Provident Fund Account at the end of 30 years if the interest rate is 8 percent?

The accumulated sum will be:

$$\text{Rs.}30{,}000\ (FVIFA_{8\%,30yrs})$$

$$= \text{Rs.}30{,}000\left[\frac{(1.08)^{30} - 1}{.08}\right]$$

$$= \text{Rs.}30{,}000\ [\ 113.283]$$

$$= \text{Rs.}3{,}398{,}490$$

Use of Excel Spreadsheet Time value calculations can be easily done using a spreadsheet. In Excel, there are customised notations and functions for the various time value parameters as shown below:

Parameter	*Notation/Symbol*	*Built in Formula in Excel*
Present value	PV	=PV(rate,nper,pmt,[fv],[type])
Future value	FV	=FV(rate,nper,pmt,[pv],[type])
No. of continuous successive periods	NPER	=NPER(rate,pmt,pv,[fv],[type])
Payment per period	PMT	=PMT(rate,nper,pv,[fv],[type])
Interest rate	RATE	=RATE(nper,pmt,pv,[fv],[type])

The following printout of an Excel worksheet, to calculate the accumulated sum in the above illustration, may be used to understand how Excel is used.

	A		*B*
1	Amount of deposit per period(PMT)	Rs.	30,000
2	No. of periods(NPER)	Years	30
3	Interest rate (RATE)	p.a	8%
4	Accumulated amonut(FV)	Rs.	3,398,496
5		Formula used	=FV(B3, B2, –B1)

Open the worksheet and input the respective given values for the various parameters inside the cells A1 to A3, respectively inside cells B1 to B3 as shown above. To get the future value in cell B4, select B4 and type =fv(and even as you type this, the formula template, viz. FV(rate,nper,pmt,[pv].[type]) will become visible nearby(by way of a tip) to guide you further. What you have to do thereafter is just give the cell reference numbers of the respective parameter values inside the parenthesis in the order cited, duly separated by commas. Thus in the place for rate type B3(just point the cursor onto cell B3 and that cell no. will get typed automatically), in the place for nper type B2, in the place for pmt type –B1. Note that if there is a payment/outflow, a – sign should precede the cell reference.

Where a notation is inside a square bracket, it indicates that if you skip that place, Excel will take that the value is 0. In our case as there is no PV figure in the given data just skip that place. The notation 'type' can take only one of the two values viz, 0 if the outflow/inflow takes place at the end of each period or 1 if that takes place in the beginning of each period. Again, if nothing is typed in the space marked [type] , the value would be 0.

To sum up, type =FV(B3,B2,–B1) inside B4 and press enter and the future value will automatically appear in that cell.

How Much Should You Save Annually You want to buy a house after 5 years when it is expected to cost Rs.2 million. How much should you save annually if your savings earn a compound return of 12 percent?

The future value interest factor for a 5 year annuity, given an interest rate of 12 percent, is:

$$\text{FVIFA}_{n=5,\ r=12\%} = \frac{(1+0.12)^5 - 1}{0.12} = 6.353$$

$$FVA = A\left[\frac{(1+r)^n - 1}{r}\right]$$

The annual savings should be :

$$\frac{\text{Rs. } 2000{,}000}{6.353} = \text{Rs.}314{,}812$$

$$2000000 = A\left[\frac{(1+.12)^5 - 1}{.12}\right]$$

$$= A\,(6.353$$

Annual Deposit in a Sinking Fund Futura Limited has an obligation to redeem Rs.500 million bonds 6 years hence. How much should the company deposit annually in a sinking fund account wherein it earns 14 percent interest, to cumulate Rs.500 million in 6 years time?

The future value interest factor for a 6 year annuity, given an interest rate of 14 percent is:

$$FVIFA_{n=6,\, r=14\%} = \frac{(1+0.14)^6 - 1}{0.14} = 8.536$$

The annual sinking fund deposit should be:

$$\frac{\text{Rs. 500 million}}{8.536} = \text{Rs.58.575 million}$$

Finding the Interest Rate A finance company advertises that it will pay a lump sum of Rs.8,000 at the end of 6 years to investors who deposit annually Rs.1,000 for 6 years. What interest rate is implicit in this offer?

The interest rate may be calculated in two steps :

1. Find the $FVIFA_{r,6}$ for this contract as follows :

$$\text{Rs.8,000} = \text{Rs.1,000} \times FVIFA_{r,6}$$

$$FVIFA_{r,6} = \frac{\text{Rs. 8,000}}{\text{Rs. 1,000}} = 8.000$$

$FVA_n = A\left[\frac{(1+r)^n - 1}{r}\right]$

$8000 = 1000\left[\frac{(1+r)^n - 1}{r}\right]$

2. Look at the $FVIFA_{r,n}$ table and read the row corresponding to 6 years until you find a value close to 8.000. Doing so, we find that

$FVIFA_{12\%,6}$ is 8.115

So, we conclude that the interest rate is slightly below 12 percent.

An Excel worksheet of the above is as under:

	A	B	C	D	E
1	Future value (Fv)	8,000			
2	Periods in years (Nper)	6	Rate	→	11.13%
3	Periodic payment (Pmt)	1000	= (B2, –B3, 0B1)		

In the rate formula, viz. =RATE (nper,pmt,pv,[fv],[type]), the value for pv here is nil. In typing out the rate formula you may either type a 0 in the place for pv or just put two commas between B3 and B1.

How Long Should You Wait You want to take up a trip to the moon which costs Rs.1,000,000 – the cost is expected to remain unchanged in nominal terms. You can save annually Rs.50,000 to fulfill your desire. How long will you have to wait if your savings earn an interest of 12 percent?

The future value of an annuity of Rs.50,000 that earns 12 percent is equated to Rs.1,000,000.

$$50{,}000 \times FVIFA_{n=?,12\%} = 1{,}000{,}000$$

$$50{,}000 \times \left[\frac{1.12^n - 1}{0.12}\right] = 1{,}000{,}000$$

$$1.12^n - 1 = \frac{1{,}000{,}000}{50{,}000} \times 0.12 = 2.4$$

$$1.12^n = 2.4 + 1 = 3.4$$
$$n \log 1.12 = \log 3.4$$
$$n \times 0.0492 = 0.5315$$
$$n = \frac{0.5315}{0.0492} = 10.8 \text{ years}$$

You will have to wait for about 11 years.

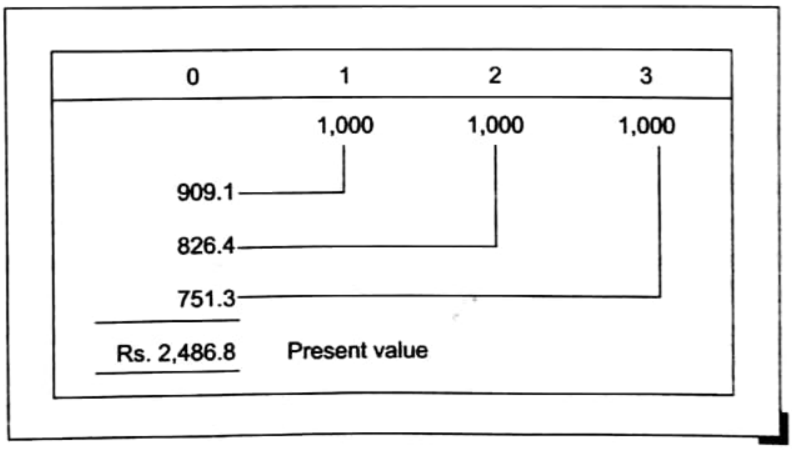
6.5 PRESENT VALUE OF AN ANNUITY

Suppose you expect to receive Rs.1,000 annually for 3 years, each receipt occurring at the end of the year. What is the present value of this stream of benefits if the discount rate is 10 percent? The present value of this annuity is simply the sum of the present values of all the inflows of this annuity:

$$\text{Rs. } 1{,}000\left[\frac{1}{1.10}\right] + \text{Rs. } 1{,}000\left[\frac{1}{1.10}\right]^2 + \text{Rs. } 1{,}000\left[\frac{1}{1.10}\right]^3$$
$$= \text{Rs.}1{,}000 \times 0.9091 + \text{Rs.}1{,}000 \times 0.8264 + \text{Rs.}1{,}000 \times 0.7513$$
$$= \text{Rs.}2{,}486.8$$

The time line for this problem is shown in Exhibit 6.9

Exhibit 6.9 **Time Line**



Formula

In general terms the present value of an annuity may be expressed as follows:

$$\text{PVA}_n = \frac{A}{(1+r)} + \frac{A}{(1+r)^2} + \cdots + \frac{A}{(1+r)^{n-1}} + \frac{A}{(1+r)^n}$$

$$A\left(\frac{1-\left(\frac{1}{1+r}\right)^n}{r}\right)$$

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$$= A\left[\frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \ldots + \frac{1}{(1+r)^{n-1}} + \frac{1}{(1+r)^n}\right]$$

$$= A\,[\{1 - (1/1+r)^n\}/\,r] \qquad (6.6)^2$$

$$= A\left[\frac{\{1-\frac{1}{(1+r)^n}\}}{r}\right]$$

where PVA_n is the present value of an annuity which has a duration of n periods, A is the constant periodic flow, and r is the discount rate.

$[\{1-(1/1+r)^n\}/r]$ is referred to as the present value interest factor for an annuity ($PVIFA_{r,n}$). It is, as can be seen clearly, simply equal to the product of the future value interest factor for an annuity ($FVIFA_{r,n}$) and the present value interest factor ($PVIF_{r,n}$). Exhibit 6.10 shows the value of $PVIFA_{r,n}$ for several combinations of r and n. A more detailed table of $PVIFA_{r,n}$ values is found in Appendix A at the end of this book.

Exhibit 6.10 **Value of $PVIFA_{r,n}$ for Different Combinations of r and n**

n/r	6%	8%	10%	12%	14%
2	1.833	1.783	1.737	1.690	1.647
4	3.465	3.312	3.170	3.037	2.914
6	4.917	4.623	4.355	4.111	3.889
8	6.210	5.747	5.335	4.968	4.639
10	7.360	6.710	6.145	5.650	5.216
12	8.384	7.536	6.814	6.194	5.660

Applications

The present value annuity formula can be applied in a variety of contexts. Its important applications are discussed below.

How Much Can You Borrow for a Car After reviewing your budget, you have determined that you can afford to pay Rs.12,000 per month for 3 years toward a new car. You call a finance company and learn that the going rate of interest on car finance is 1.5 percent per month for 36 months. How much can you borrow?

To determine how much you can borrow, we have to calculate the present value of Rs.12,000 per month for 36 months at 1.5 percent per month.

Since the loan payments are an ordinary annuity, the present value interest factor of annuity is:

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[2] The formula for the present value of an annuity is derived as follows:

$$PVA_n = A\,(1+r)^{-1} + A\,(1+r)^{-2} + \cdots + A\,(1+r)^{-n} \qquad (1)$$

Multiplying both the sides of (1) by $(1 + r)$ gives :

$$PVA_n\,(1+r) = A + A\,(1+r)^{-1} + \cdots + A\,(1+r)^{1-n} \qquad (2)$$

Subtracting (1) from (2) yields :

$$PVA_n\,r = A\,[1 - (1+r)^{-n}] = A\,\{[(1+r)^n - 1]\,/\,(1+r)^n\} \qquad (3)$$

Dividing both the sides of (3) by r results in :

$$PVA_n = A\,\{[(1+r)^n - 1]\,/\,r\,(1+r)^n\} = A\,[\{1 - (1/1+r)^n\}/\,r] \qquad (4)$$

$$\text{PVIFA}_{r,n} = \frac{1-\dfrac{1}{(1+r)^n}}{r} = \frac{1-\dfrac{1}{(1.015)^{36}}}{0.015} = 27.66$$

$A\left[\frac{\left\{1-\left(\frac{1}{1+r}\right)^n\right\}}{r}\right]$

Hence the present value of 36 payments of Rs.12,000 each is:
Present value = Rs.12,000 × 27.66 = Rs.331,920
You can, therefore, borrow Rs.331,920 to buy the car.
The above can be worked out in a spreadsheet as shown below:

	A	B	C	D	E
1	Monthly payment (Pmt) Rs.	12,000			
2	Period in months (Nper)	36	Present value	→	331,928
3	Rate of interest per month (Rate)	1.50%	= PV (B3, B2, –B1)		

Period of Loan Amortisation You want to borrow Rs.1,080,000 to buy a flat. You approach a housing finance company which charges 12.5 percent interest. You can pay Rs.180,000 per year toward loan amortisation. What should be the maturity period of the loan?

The present value of annuity of Rs.180,000 is set equal to Rs.1,080,000.

$$180{,}000 \times \text{PVIFA}_{n,r} = 1{,}080{,}000$$

$$180{,}000 \times \text{PVIFA}_{n=?,\, r=12.5\%} = 1{,}080{,}000$$

$$180{,}000\left[\frac{1-\dfrac{1}{(1.125)^n}}{0.125}\right] = 1.080{,}000$$

Given this equality the value of n is calculated as follows:

$$\frac{1-\dfrac{1}{(1.125)^n}}{0.125} = \frac{1{,}080{,}000}{180{,}000} = 6$$

$$\frac{1}{(1.125)^n} = 0.25$$

$$1.125^n = 4$$

$$n \log 1.125 = \log 4$$

$$n \times 0.0512 = 0.6021$$

$$n = \frac{0.6021}{0.0512} = 11.76 \text{ years}$$

You can perhaps request for a maturity of 12 years.

Determining the Loan Amortisation Schedule Most loans are repaid in equal periodic instalments (monthly, quarterly, or annually), which cover interest as well as principal repayment. Such loans are referred to as **amortised loans.**

For an amortised loan we would like to know (a) the periodic instalment payment and (b) the loan amortisation schedule showing the break up of the periodic instalment payments between the interest component and the principal repayment component. To illustrate how these are calculated, let us look an example.

Suppose a firm borrows Rs.1,000,000 at an interest rate of 15 percent and the loan is to be repaid in 5 equal instalments payable at the end of each of the next 5 years. The annual instalment payment A is obtained by solving the following equation.

$$\text{Loan amount} = A \times PVIFA_{n=5,\, r=15\%}$$

$$1{,}000{,}000 = A \times 3.3522$$

$$\text{Hence } A = 298{,}312$$

The amortisation schedule is shown in Exhibit 6.11. The interest component is the largest for year 1 and progressively declines as the outstanding loan amount decreases.

Exhibit 6.11 **Loan Amortisation Schedule**

Year	Beginning Amount (1)	Annual Instalment (2)	Interest (3)	Principal Repayment (2)-(3) = (4)	Remaining Balance (1)-(4) = (5)
1	1,000,000	298,312	150,000[a]	148,312[b]	851,688
2	851,688	298,312	127,753	170,559	681,129
3	681,129	298,312	102,169	196,143	484,986
4	484,986	298,312	72,748	225,564	259,422
5	259,422	298,312	38,913	259,399	23*

a. Interest is calculated by multiplying the beginning loan balance by the interest rate.
b. Principal repayment is equal to annual instalment minus interest.
* Due to rounding off error a small balance is shown.

The above schedule can be set up using a spreadsheet as below:

	A	B	C	D	E	F
1		Present value	Interest rate	No. of instalments (in years)	Annual instalment amount	
2		1,000,000	15%	5	298,316	
3	Year	Beginning amount	Annual instalment	Interest	Principal repayment	Remaining balance
4	1	1,000,000	298,316	150000	148,316	851,684
5	2	851,684	298,316	127,753	170,563	681,121
6	3	681,121	298,316	102,168	196,148	484,973
7	4	484,973	298,316	72,746	225,570	259,403
8	5	259,403	298,316	38,910	259,406	-3

To create the above spreadsheet, proceed as follows: In B4, type =B2 to get the beginning amount. To get the instalment amount in C4, type =E2 and press F4. A $ sign will appear before E and 2 (E2). This will make the value in this cell absolute, that is, constant throughout. Use the formula =B4*C2 to get interest amount in D4 (note that C2 here is made absolute by pressing F4). Fill in the principal repayment amount in E4 using the formula =C4-D4 and the remaining balance in F4 using the formula =B4-E4. Copy this value to B5 by typing =F4. Next, click on C4. Observe that there is a tiny black box at the lower right corner of the cell. This is called a fill handle. Point the cursor to the fill handle (it will turn into a black cross) and drag it down upto C8.This will autofil the value in C4 (whether an absolute value or a formula) upto C8. Use the fill handle to autofil all the remaining cells by dragging down the values in the respective cells above them.

Determining the Periodic Withdrawal Your father deposits Rs.300,000 on retirement in a bank which pays 10 percent annual interest. How much can be withdrawn annually for a period of 10 years?

$$A = \text{Rs.}300{,}000 \times \frac{1}{PVIFA_{10\%,10}}$$

$$= \text{Rs.}300{,}000 \times \frac{1}{6.145}$$

$$= \text{Rs.}48{,}820$$

A spreadsheet calculation of the above is as under.

	A	B	C	D	E
1	Initial deposit Rs.	30,000			
2	Interest rate	10%	Annual withdrawal	→	48,824
3	Period years	10	= PMT (B2, B3, –B1)		

Finding the Interest Rate Suppose someone offers you the following financial contract: If you deposit Rs.10,000 with him he promises to pay Rs.2,500 annually for 6 years. What interest rate do you earn on this deposit? The interest rate may be calculated in two steps:

Step 1 Find the $PVIFA_{r,6}$ for this contract by dividing Rs.10,000 by Rs.2,500

$$PVIFA_{r,6} = \frac{\text{Rs. }10{,}000}{\text{Rs. }2{,}500} = 4.000$$

Step 2 Look at the PVIFA table and read the row corresponding to 6 years until you find a value close to 4.000. Doing so, you find that

$$PVIFA_{12\%,6} \text{ is } 4.111 \text{ and } PVIFA_{14\%,6} \text{ is } 3.889$$

Since 4,000 lies in the middle of these values the interest rate lies (approximately) in the middle. So, the interest rate is 13 percent.

Valuing an Infrequent Annuity Raghavan will receive an annuity of Rs. 50,000, payable once every two years. The payments will stretch out over 30 years. The first payment will be received at the end of two years. If the annual interest rate is 8 percent, what is the present value of the annuity?

The interest rate over a two-year period is:

$$(1.08) \times (1.08) - 1 = 16.64 \text{ percent}$$

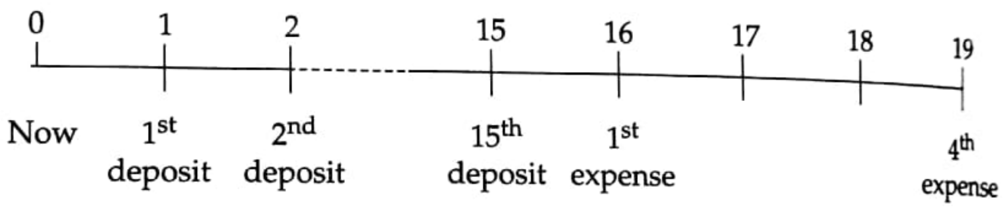
This means that Rs. 100 invested over two years will yield Rs. 116.64.

We have to calculate the present value of a Rs.50,000 annuity over 15 periods, with an interest rate of 16.64 percent per period. This works out to:

$$\text{Rs. } 50{,}000\ [\{ 1 - (1/1.1664)^{15} \}] \,/\, 0.1664 = \text{Rs. } 270{,}620$$

Equating Present Value of Two Annuities Ravi wants to save for the college education of his son, Deepak. Ravi estimates that the college education expenses will be rupees one million per year for four years when his son reaches college in 16 years – the expenses will be payable at the beginning of the years. He expects the annual interest rate of 8 percent over the next two decades. How much money should he deposit in the bank each year for the next 15 years (assume that the deposit is made at the end of the year) to take care of his son's college education expenses?

The time line for this problem is as follows:



The present value of college education expenses when his son becomes 15 years old is:

$$\text{Rs. } 1{,}000{,}000 \times \text{PVIFA (4 years, 8\%)}$$

$$= \text{Rs. } 1{,}000{,}000 \times 3.312 = \text{Rs. } 3{,}312{,}000$$

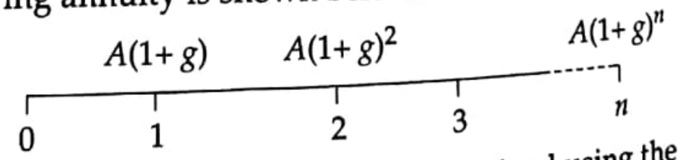
The annual deposit to be made so that the future value of the deposits at the end of 15 years is Rs.3,312,000 is:

$$A = \frac{\text{Rs. } 3{,}312{,}000}{\text{FVIFA(15 years, 8\%)}} = \frac{\text{Rs. } 3{,}312{,}000}{27.152}$$

$$= \text{Rs. } 121{,}980$$

Present Value of a Growing Annuity

A cash flow that grows at a constant rate for a specified period of time is a growing annuity. The time line of a growing annuity is shown below:



The present value of a growing annuity can be determined using the following formula:

$$\text{PV of a Growing Annuity} = A\,(1+g)\left[\frac{1-\dfrac{(1+g)^n}{(1+r)^n}}{r-g}\right] \qquad (6.7)^{[3]}$$

The above formula can be used when the growth rate is less than the discount rate ($g < r$) as well as when the growth rate is more than the discount rate ($g > r$). However, it does not work when the growth rate is equal to the discount rate ($g = r$) - in this case, the present value is simply equal to $n\,A$.

For example, suppose you have the right to harvest a teak plantation for the next 20 years over which you expect to get 100,000 cubic feet of teak per year. The current price per cubic feet of teak is Rs.500, but it is expected to increase at a rate of 8 percent per year. The discount rate is 15 percent. The present value of the teak that you can harvest from the teak forest can be determined as follows:

$$\text{PV of teak} = \text{Rs.}500 \times 100{,}000\,(1.08)\left[\frac{1-\dfrac{1.08^{20}}{1.15^{20}}}{0.15-0.08}\right]$$

$$= \text{Rs.}551{,}736{,}683$$

A Note on Annuities Due

So far we discussed ordinary annuities in which cash flows occur at the end of each period. There is a variation, which is fairly common, in which cash flows occur at the beginning of each period. Such an annuity is called an **annuity due**. For example, when you enter into a lease for an apartment, the lease payments are due at the beginning of the month. The first lease payment is made at the beginning; the second lease payment is due at the beginning of

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[3] The formula for the present value of a growing annuity (PVGA) is derived as follows :

$$\text{PVGA} = \frac{A(1+g)}{(1+r)} + \frac{A(1+g)^2}{(1+r)^2} + \cdots + \frac{A(1+g)^n}{(1+r)^n} \qquad (1)$$

Multiplying both the sides of (1) by $(1+g)/(1+r)$ gives:

$$\text{PVGA} \times = \frac{(1+g)}{(1+r)} = \frac{A(1+g)^2}{(1+r)^2} + \frac{A(1+g)^3}{(1+r)^3} + \cdots + \frac{A(1+g)^{n+1}}{(1+r)^{n+1}} \qquad (2)$$

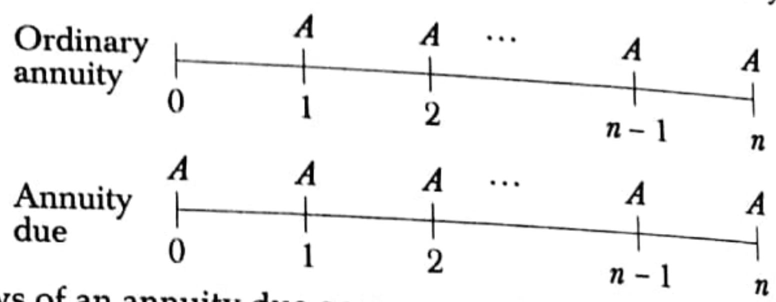
Subtracting (2) from (1) yields:

$$\text{PVGA } 1 - = \frac{(1+g)}{(1+r)} = \frac{A(1+g)}{(1+r)} = \frac{A(1+g)^{n+1}}{(1+r)^{n+1}} \qquad (3)$$

This simplifies to:

$$\text{PVGA} = A\,(1+g)\left[\frac{1-\dfrac{(1+g)^n}{(1+r)^n}}{r-g}\right] \qquad (4)$$

the second month, so on and so forth. The time lines for ordinary annuity and annuity due are shown below:



Since the cash flows of an annuity due occur one period earlier in comparison to the cash flows on an ordinary annuity, the following relationship holds:

$$\text{Annuity due value} = \text{Ordinary annuity value} \times (1 + r)$$

This applies for both present and future values. So, two steps are involved in calculating the value of an annuity due. First, calculate the present or future value as though it were an ordinary annuity. Second, multiply your answer by $(1 + r)$.

6.6 PRESENT VALUE OF A PERPETUITY

A perpetuity is an annuity of infinite duration. For example, the British government has issued bonds called consols which pay yearly interest forever.

Present Value of a Perpetuity

The present value of a perpetuity may be expressed as follows:

$$P_\infty = A \times \text{PVIFA}_{r,\infty} \tag{6.8}$$

where P_∞ is the present value of a perpetuity and A is the constant annual payment.

What is the value of $\text{PVIFA}_{r,\infty}$? It is equal to:

$$\sum_{t=1}^{\infty} \frac{1}{(1+r)^t} = \frac{1}{r} \tag{6.9}$$ [4]

Put in words, it means that the present value interest factor of a perpetuity is simply 1 divided by the interest rate expressed in decimal form. Hence, the present value of a perpetuity is simply equal to the constant annual payment divided by the interest rate. For example,

[4] The formula for $\text{PVIFA}_{r,\infty}$ is derived as follows:

$$\text{PVIFA}_{r,\infty} = 1(1 + r)^{-1} + 1(1 + r)^{-2} + \cdots + 1(1 + r)^{-\infty} \tag{1}$$

Multiplying both the sides of (1) by $(1 + r)$ gives:

$$\text{PVIFA}_{r,\infty}\,(1 + r) = 1 + 1(1 + r)^{-1} + \cdots + 1(1 + r)^{-(\infty-1)} \tag{2}$$

Subtracting (1) from (2) yields:

$$\text{PVIFA}_{r,\infty} \times r = 1 - 1(1 + r)^{-\infty} \tag{3}$$

Since the second term on the right hand side of (3) vanishes, we get:

$$\text{PVIFA}_{r,\infty} \times r = 1 \tag{4}$$

This results in:

$$\text{PVIFA}_{r,\infty} = \frac{1}{r} \tag{5}$$

the present value of a perpetuity of Rs.10,000 if the interest rate is 10 percent is equal to: Rs.10,000/0.10 = Rs.100,000. Intuitively, this is quite convincing because an initial sum of Rs.100,000 would, if invested at a rate of interest of 10 percent, provide a constant annual income of Rs 10,000 for ever, without any impairment of the capital value.

Growing Perpetuity

An office complex is expected to generate a net rental of Rs. 3 million next year, which is expected to increase by 5 percent every year. If we assume that the increase will continue indefinitely, the rental stream is a growing perpetuity. If the discount rate is 10 percent, the present value of the rental stream is:

$$PV = \frac{3{,}000{,}000}{(1.10)} + \frac{3{,}000{,}000(1.05)^1}{(1.10)^2} + \cdots + \frac{3{,}000{,}000(1.05)^{n-1}}{(1.10)^n} + \cdots \quad (6.10)$$

Algebraically, it may be expressed as follows:

$$PV = \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \cdots + \frac{C(1+g)^{n-1}}{(1+r)^n} + \cdots$$

where C is the rental to be received a year hence, g is the rate of growth per year, and r is the discount rate.

While Eq. (6.10) looks daunting, it reduces itself to the following simplification.

$$PV = \frac{C}{r-g} \quad (6.11)$$ [5]

6.7 INTRA-YEAR COMPOUNDING AND DISCOUNTING

So far we assumed that compounding is done annually. Now we consider the case where compounding is done more frequently. Suppose you deposit Rs 1,000 with a finance company which advertises that it pays 12 percent interest semi-annually–this means that the interest is paid every six months. Your deposit (if interest is not withdrawn) grows as follows:

First six months	:	Principal at the beginning	= Rs. 1,000.0
		Interest for 6 months	= Rs. 60.0
		Rs 1,000 × $\frac{\cdot 12}{2}$	
		Principal at the end	= Rs. 1,060.0
Second six months	:	Principal at the beginning	= Rs. 1,060.0
		Interest for 6 months	= Rs. 63.6
		Rs.1,060 × $\frac{\cdot 12}{2}$	
		Principal at the end	= Rs. 1,123.6

Note that if compounding is done annually, the principal at the end of one year would be Rs.1,000 (1.12) = Rs.1,120. The difference of Rs.3.6 (between Rs.1,123.6 under semiannual compounding and Rs.1,120 under annual compounding) represents interest on interest for the second half year.

[5] Proof for this formula is given in Chapter 7.

The general formula for the future value of a single cash flow after n years when compounding is done m times a year is:

$$FV_n = PV\left[1+\frac{r}{m}\right]^{m\times n} \tag{6.12}$$

Suppose you deposit Rs.5,000 in a bank for 6 years. If the interest rate is 12 percent and the frequency of compounding is 4 times a year your deposit after 6 years will be:

$$\text{Rs.}5{,}000\left[1+\frac{0.12}{4}\right]^{4\times 6} = \text{Rs.}5{,}000\,(1.03)^{24}$$

$$= \text{Rs.}5{,}000 \times 2.0328 = \text{Rs.}10{,}164$$

Effective versus Stated Rate

We have seen above that Rs.1,000 grows to Rs.1,123.6 at the end of a year if the stated rate of interest is 12 percent and compounding is done semi-annually. This means that Rs.1,000 grows at the rate of 12.36 percent per annum. The figure of 12.36 percent is called the effective interest rate - the rate of interest under annual compounding which produces the same result as that produced by an interest rate of 12 percent under semi-annual compounding.

The general relationship between the effective interest rate and the stated annual interest rate is as follows:

$$\text{Effective interest rate} = \left[1+\frac{\text{Stated annual interest rate}}{m}\right]^{m} - 1$$

where m is the frequency of compounding per year.

Suppose a bank offers 12 percent stated annual interest rate. What will be the effective interest rate when compounding is done annually, semiannually, and quarterly?

$$\text{Effective interest rate with annual compounding} = \left[1+\frac{0.12}{1}\right]^{1} - 1 = 0.12$$

$$\text{Effective interest rate with semi-annual compounding} = \left[1+\frac{0.12}{2}\right]^{2} - 1 = 0.1236$$

$$\text{Effective interest rate with quarterly compounding} = \left[1+\frac{0.12}{4}\right]^{4} - 1 = 0.1255$$

When compounding becomes continuous, the effective interest rate is expressed as follows:

$$\text{Effective interest rate} = e^{r} - 1 \tag{6.13}$$

where e is the base of natural logarithm and r is the stated interest rate.

Exhibit 6.12 shows how compounding frequency impacts on the effective interest rate. From this exhibit is clear that the effect of increasing the frequency of compounding is not as dramatic as some would believe it to be - the additional gains dwindle as the frequency of compounding increases.

Exhibit 6.12 **Compounding Frequency and Effective Interest Rate**

Frequency	*Stated Interest Rate (%)*	*m*	*Formula*	*Effective Interest Rate (%)*
Annual	12	1	0.12	12.00
Semi-annual	12	2	$\left[1+\frac{0.12}{2}\right]^2-1$	12.36
Quarterly	12	4	$\left[1+\frac{0.12}{4}\right]^4-1$	12.55
Monthly	12	12	$\left[1+\frac{0.12}{12}\right]^{12}-1$	12.68
Weekly	12	52	$\left[1+\frac{0.12}{52}\right]^{52}-1$	12.73
Daily	12	365	$\left[1+\frac{0.12}{365}\right]^{365}-1$	12.75
Continuous	12		$e^{0.12}-1$	12.75

Shorter Discounting Periods

Sometimes cash flows have to be discounted more frequently than once a year - semi-annually, quarterly, monthly, or daily. As in the case of intra-year compounding, the shorter discounting period implies that (i) the number of periods in the analysis increases and (ii) the discount rate applicable per period decreases. The general formula for calculating the present value in the case of shorter discounting period is:

$$PV = FV_n \left[\frac{1}{1+r/m}\right]^{mn} \tag{6.14}$$

where PV is the present value, FV_n is the cash flow after n years, m is the number of times per year discounting is done, and r is the annual discount rate.

To illustrate, consider a cash flow of Rs.10,000 to be received at the end of four years. The present value of this cash flow when the discount rate is 12 percent (r = 12 percent) and discounting is done quarterly (m = 4) is determined as follows:

$$\begin{aligned} PV &= \text{Rs.}10{,}000 \times PVIF_{r/m,\, m \times n} \\ &= \text{Rs.}10{,}000 \times PVIF_{3\%,16} \\ &= \text{Rs.}10{,}000 \times 0.623 = \text{Rs.}6{,}230 \end{aligned}$$

SUMMARY

- Money has time value. A rupee today is more valuable than a rupee a year hence.
- When cash flows occur at different points in time, it is easier to deal with them using a **time line**. A time line shows the timing and the amount of each cash flow in a cash flow stream.
- The process of investing money as well as reinvesting the interest earned thereon is called **compounding**. The future or compounded value of an investment after n years when the interest rate is r percent is:

$$\text{Future value}_n = \text{Present value } (1 + r)^n$$

- If no interest is earned on interest the investment earns only simple interest. In such a case the investment grows as follows:

$$\text{Future value} = \text{Present value } [1 + nr]$$

- According to the **rule of 72**, the doubling period under compounding is obtained by dividing 72 by the interest rate.
- The process of **discounting**, used for calculating the present value, is simply the inverse of compounding. The present value formula is:

$$PV = FV_n \, [\, 1/(1 + r)^n \,]$$

- The present value of a cash flow is equal to:

$$PV_n = \sum_{t=1}^{n} \frac{A_t}{(1+r)^t}$$

- An **annuity** is a stream of constant cash flow (payment or receipt) occuring at regular intervals of time. When the cash flows occur at the end of each period the annuity is called an **ordinary annuity** or a deferred annuity. When the cash flows occur at the beginning of each period, the annuity is called an **annuity due**.
- The future value of an annuity is given by the formula:

$$FVA_n = A\,[(1 + r)^n - 1] \,/\, r$$

- The present value of an annuity is given by the formula:

$$PVA_n = A\,[\{\, 1 - (1/1 + r)^n\}/\, r\,]$$

- A cash flow that grows at a constant rate for a specified period of time is a growing annuity. The present value of a **growing annuity** is given by the following formula:

$$\text{PV of a Growing Annuity} = A(1 + g)\,\{[1 - (1 + g)^n \,/\, (1 + r)^n\,] \,/(r - g)\}$$

- Since the cash flows of an annuity due occur one period earlier in comparison to the cash flows of an ordinary annuity, the following relationship holds:

$$\text{Annuity due value} = \text{Ordinary annuity value} \times (1 + r)$$

- A **perpetuity** is an annuity of infinite duration. The present value of a perpetuity is:

$$\text{Present value of a perpetuity} = A/r$$

- The general formula for the future value of a single cash flow after n years when compounding is done m times a year is:

$$FV_n = PV\,[1 + r/m]^{mn}$$

- The relationship between **effective interest rate** and the **stated annual interest rate** is as follows:

$$\text{Effective interest rate} = \left[1 + \frac{\text{Stated annual interest rate}}{m}\right]^m - 1$$

- When compounding becomes continuous the effective interest rate is expressed as:

$$\text{Effective interest rate} = e^r - 1$$

- The formula for calculating the present value in the case of shorter discounting period is:

$$PV = FV_n \, [1/\,(1 + r/m)\,]^{mn}$$

QUESTIONS

1. Why does money have time value?
2. State the general formula for the future value of a single amount.
3. What is the difference between compound and simple interest?
4. Explain the rule of 72.
5. Explain the rule of 69. How does it compare with the rule of 72.
6. State the general formula for calculating the present value of a single amount.
7. What is an annuity? What is the difference between an ordinary annuity and an annuity due?
8. State the formula for the future value of an annuity.
9. State the formula for the present value of an annuity.
10. What is a growing annuity? What is the formula for finding the present value of a growing annuity?
11. What is the formula for the present value of a perpetuity?
12. State the formula for the future value of a single cash flow after n years when compounding is done m times a year.
13. What is the relationship between the effective interest rate and the stated interest rate?
14. State the formula for calculating the present value of a single cash flow when discounting is done m times a year.
15. A firm's earnings grew from Re.1 per share to Rs.3 per share over a period of 10 years. The total growth was 200 percent, but the annual compound growth rate was less than 20 percent. Why?

SOLVED PROBLEMS

6.1 If you invest Rs.5,000 today at a compound interest of 9 per cent, what will be its future value after 75 years ?

Solution The future value of Rs.5,000 after 75 years, when it earns a compound interest of 9 per cent, is

$$\text{Rs.}5{,}000\ (1.09)^{75}$$

Since the FVIF table given in Appendix A has a maximum period of 30, the future value expression may be stated as

$$\text{Rs.}5{,}000\ (1.09)^{30}\ (1.09)^{30}\ (1.09)^{15}$$

The above product is equal to

$$\text{Rs.}5{,}000\ (13.268)\ (13.268)\ (3.642) = \text{Rs.}32{,}05{,}685.1$$

6.2 If the interest rate is 12 per cent, what are the doubling periods as per the rule of 72 and the rule of 69 respectively ?

Solution As per the rule of 72 the doubling period will be

$$72\ /\ 12 = 6 \text{ years}$$

As per the rule of 69, the doubling period will be

$$0.35 + \frac{69}{12} = 6.1 \text{ years}$$

6.3 A borrower offers 16 per cent nominal rate of interest with quarterly compounding. What is the effective rate of interest ?

Solution The effective rate of interest is

$$\left[1+\frac{0.16}{4}\right]^4 - 1 = (1.04)^4 - 1$$

$$= 1.17 - 1$$

$$= 0.17 = 17 \text{ percent}$$

6.4 Fifteen annual payments of Rs.5,000 are made into a deposit account that pays 14 percent interest per year. What is the future value of this annuity at the end of 15 years?

Solution The future value of this annuity will be:

$$\text{Rs.}5{,}000\ (\text{FVIFA}_{14\%,15}) = \text{Rs.}5{,}000\ (43.842)$$

$$= \text{Rs.}2{,}19{,}210$$

6.5 A finance company advertises that it will pay a lumpsum of Rs.44,650 at the end of five years to investors who deposit annually Rs.6,000 for 5 years. What is the interest rate implicit in this offer?

Solution The interest rate may be calculated in two steps

(a) Find the FVIFA for this contract as follows:

Rs.6,000 (FVIFA) = Rs.44,650

So

$$\text{FVIFA} = \frac{\text{Rs. }44{,}650}{\text{Rs. }6{,}000} = 7.442$$

(b) Look at the FVIFA table and read the row corresponding to 5 years until 7.442 or a value close to it is reached. Doing so we find that

$\text{FVIFA}_{20\%,5yrs}$ is 7.442

So, we conclude that the interest rate is 20 percent.

6.6 What is the present value of Rs.1,000,000 receivable 60 years from now, if the discount rate is 10 percent ?

Solution The present value is

$$\text{Rs.}1{,}000{,}000\left[\frac{1}{1.10}\right]^{60}$$

This may be expressed as

$$\text{Rs.}1{,}000{,}000\left[\frac{1}{1.10}\right]^{30}\left[\frac{1}{1.10}\right]^{30}$$

$$= \text{Rs.}1{,}000{,}000\ (0.057)\ (0.057) = \text{Rs.}3249$$

6.7 A 12 – payment annuity of Rs.10,000 will begin 8 years hence. (The first payment occurs at the end of 8 years). What is the present value of this annuity if the discount rate is 14 percent?

Solution This problem may be solved in two steps.

Step 1 Determine the value of this annuity a year before the first payment begins, i.e., 7 years from now. This is equal to:

$$\text{Rs.}10{,}000\ (\text{PVIFA}_{14\%,12\text{ years}}) = \text{Rs.}10{,}000\ (5.660)$$

$$= \text{Rs.}56{,}600$$

Step 2 Compute the present value of the amount obtained in Step1:

$$0.35 + \frac{69}{12} = 6.1 \text{ years}$$

6.3 A borrower offers 16 per cent nominal rate of interest with quarterly compounding. What is the effective rate of interest ?

Solution The effective rate of interest is

$$\left[1+\frac{0.16}{4}\right]^4 - 1 = (1.04)^4 - 1$$

$$= 1.17 - 1$$

$$= 0.17 = 17 \text{ percent}$$

6.4 Fifteen annual payments of Rs.5,000 are made into a deposit account that pays 14 percent interest per year. What is the future value of this annuity at the end of 15 years?

Solution The future value of this annuity will be:

$$\text{Rs.5,000 } (FVIFA_{14\%,15}) = \text{Rs.5,000 } (43.842)$$

$$= \text{Rs.2,19,210}$$

6.5 A finance company advertises that it will pay a lumpsum of Rs.44,650 at the end of five years to investors who deposit annually Rs.6,000 for 5 years. What is the interest rate implicit in this offer?

Solution The interest rate may be calculated in two steps

(a) Find the FVIFA for this contract as follows:

Rs.6,000 (FVIFA) = Rs.44,650

So

$$FVIFA = \frac{\text{Rs. } 44{,}650}{\text{Rs. } 6{,}000} = 7.442$$

(b) Look at the FVIFA table and read the row corresponding to 5 years until 7.442 or a value close to it is reached. Doing so we find that

$FVIFA_{20\%,5yrs}$ is 7.442

So, we conclude that the interest rate is 20 percent.

6.6 What is the present value of Rs.1,000,000 receivable 60 years from now, if the discount rate is 10 percent ?

Solution The present value is

$$\text{Rs.1,000,000} \left[\frac{1}{1.10}\right]^{60}$$

This may be expressed as

$$\text{Rs.1,000,000} \left[\frac{1}{1.10}\right]^{30} \left[\frac{1}{1.10}\right]^{30}$$

$$= \text{Rs.1,000,000 } (0.057)\ (0.057) = \text{Rs.3249}$$

6.7 A 12 – payment annuity of Rs.10,000 will begin 8 years hence. (The first payment occurs at the end of 8 years). What is the present value of this annuity if the discount rate is 14 percent?

Solution This problem may be solved in two steps.

Step 1 Determine the value of this annuity a year before the first payment begins, i.e., 7 years from now. This is equal to:

$$\text{Rs.10,000 } (PVIFA_{14\%,12 \text{ years}}) = \text{Rs.10,000 } (5.660)$$

$$= \text{Rs.56,600}$$

Step 2 Compute the present value of the amount obtained in Step1:

$$\text{Rs.56,600 } (PVIF_{14\%,7 \text{ years}}) = \text{Rs.56,600 } (0.400)$$

$$= \text{Rs.22,640}$$

6.8 What is the present value of the following cash stream if the discount rate is 14 percent?

Year	0	1	2	3	4
Cash flow	5,000	6,000	8,000	9,000	8,000

Solution The present value of the above cash flow stream is:

Year	*Cash Flow*	*($PVIFA_{14\%, n}$)*	*Present Value*
0	Rs.5,000	1.000	Rs.5,000
1	6,000	0.877	5,262
2	8,000	0.769	6,152
3	9,000	0.675	6,075
4	8,000	0.592	4,736
			Rs.27,225

6.9 Mahesh deposits Rs.200,000 in a bank account which pays 10 per cent interest. How much can he withdraw annually for a period of 15 years ?

Solution The annual withdrawal is equal to:

$$\frac{\text{Rs. }200{,}000}{\text{PVIFA}_{10\%,\ 15\text{ yrs}}} = \frac{\text{Rs. }200{,}000}{7{,}606} = \text{Rs.}26{,}295$$

6.10 You want to take a world tour which costs Rs.1,000,000 – the cost is expected to remain unchanged in nominal terms. You are willing to save annually Rs.80,000 to fulfill your desire. How long will you have to wait if your savings earn a return of 14 percent per annum ?

Solution The future value of an annuity of Rs.80,000 that earns 14 percent is equated to Rs 1,000,000.

$$80{,}000 \times \text{FVIFA}_{n\,=\,?,14\%} = 1{,}000{,}000$$

$$80{,}000\left[\frac{1.14^n - 1}{0.14}\right] = 1{,}000{,}000$$

$$1.14^n - 1 = \frac{1{,}000{,}000}{80{,}000} \times 0.14 = 1.75$$

$$1.14^n = 1.75 + 1 = 2.75$$

$$n \log 1.14 = \log 2.75$$

$$n \times .0569 = 0.4393$$

$$n = 0.4393 \,/\, 0.0569 = 7.72 \text{ years}$$

You will have to wait for 7.72 years.

6.11 Shyam borrows Rs.80,000 for a musical system at a monthly interest of 1.25 per cent. The loan is to be repaid in 12 equal monthly instalments, payable at the end of each month. Prepare the loan amortisation schedule.

Solution

The monthly instalment A is obtained by solving the equation:

$$80{,}000 = A \times \text{PVIFA}_{n\,=12,\ r\,=\,1.25\%}$$

$$80{,}000 = A \times \frac{1 - \frac{1}{(1+r)^n}}{r}$$

$$80{,}000 = A \times \frac{1 - \frac{1}{(1.0125)^{12}}}{.0125}$$

$$= A \times 11.0786$$

Hence $A = 80{,}000 / 11.0786 =$ Rs.7221

The loan amortisation schedule is shown below:

Loan Amortisation Schedule

Month	Beginning Amount (1)	Monthly Instalment (2)	Interest Repayment (3)	Principal Balance (2)–(3) = (4)	Remaining (1)–(4) = (5)
1	80,000	7221	1000	6221	73779
2	73,779	7221	922.2	6298.8	67480.2
3	67,480.2	7221	843.5	6377.5	61102.7
4	61102.7	7221	763.8	6457.2	54645.5
5	54645.5	7221	683.1	6537.9	48107.6
6	48107.6	7221	601.3	6619.7	41487.9
7	41487.9	7221	518.6	6702.4	34785.5
8	34785.5	7221	434.8	6786.2	27999.3
9	27999.3	7221	350.0	6871.0	21128.3
10	21128.3	7221	264.1	6956.9	14171.4
11	14171.4	7221	177.1	7043.9	7127.1
12	7127.1	7221	89.1	7131.9	– 4.8@

@ Rounding off error

PROBLEMS

6.1 Calculate the value 5 years hence of a deposit of Rs.1,000 made today if the interest rate is (a) 8 percent, (b) 10 percent, (c) 12 percent, and (d) 15 percent.

6.2 If you deposit Rs.5,000 today at 12 percent rate of interest in how many years (roughly) will this amount grow to Rs.1,60,000 ? Work this problem using the *rule of 72*–do not use tables.

6.3 A finance company offers to give Rs.8,000 after 12 years in return for Rs.1,000 deposited today. Using the *rule of 72*, figure out the approximate interest offered.

6.4 You can save Rs.2,000 a year for 5 years, and Rs.3,000 a year for 10 years thereafter. What will these savings cumulate to at the end of 15 years, if the rate of interest is 10 percent?

6.5 Mr. Vinay plans to send his son for higher studies abroad after 10 years. He expects the cost of these studies to be Rs.1000,000. How much should he save annually to have a sum of Rs.1000,000 at the end of 10 years, if the interest rate is 12 percent ?

6.6 A finance company advertises that it will pay a lump sum of Rs.10,000 at the end of 6 years to investors who deposit annually Rs.1,000. What interest rate is implicit in this offer?

6.7 Someone promises to give you Rs.5,000 after 10 years in exchange for Rs.1,000 today. What interest rate is implicit in this offer?

6.8 Find the present value of Rs.10,000 receivable after 8 years if the rate of discount is (i) 10 percent, (ii) 12 percent, and (iii) 15 percent.

6.9 What is the present value of a 5-year annuity of Rs.2,000 at 10 percent ?

6.10 At the time of his retirement, Mr.Jingo is given a choice between two alternatives: (a) an annual pension of Rs.10,000 as long as he lives, and (b) a lump sum amount of Rs.50,000. If Mr.Jingo expects to live for 15 years and the interest rate is 15 percent, which option appears more attractive?

6.11 Mr.X deposits Rs.1,00,000 in a bank which pays 10 percent interest. How much can he withdraw annually for a period of 30 years. Assume that at the end of 30 years the amount deposited will whittle down to zero.

6.12 What is the present value of an income stream which provides Rs.1,000 at the end of year one, Rs.2,500 at the end of year two, and Rs.5,000 during each of the years 3 through 10, if the discount rate is 12 percent ?

6.13 What is the present value of an income stream which provides Rs.2,000 a year for the first five years and Rs.3,000 a year forever thereafter, if the discount rate is 10 percent ?
Hint: The present value for a perpetual annuity is derived by dividing the constant annual flow by the discount factor.

6.14 What amount must be deposited today in order to earn an annual income of Rs.5,000 beginning from the end of 15 years from now ? The deposit earns 10 percent per year.

6.15 Suppose someone offers you the following financial contract. If you deposit Rs.20,000 with him he promises to pay Rs.4,000 annually for 10 years. What interest rate would you earn on this deposit?

6.16 What is the present value of the following cash flow streams?

End of year	*Stream A*	*Stream B*	*Stream C*
1	100	1,000	500
2	200	900	500
3	300	800	500
4	400	700	500
5	500	600	500
6	600	500	500
7	700	400	500
8	800	300	500
9	900	200	500
10	1,000	100	500

The discount rate is 12 percent.

6.17 Suppose you deposit Rs.10,000 with an investment company which pays 16 percent interest with quarterly compounding. How much will this deposit grow to in 5 years?

6.18 How much would a deposit of Rs.5,000 at the end of 5 years be, if the interest rate is 12 percent and if the compounding is done quarterly ?

6.19 What is the difference between the effective rate of interest and stated rate of interest in the following cases:

Case A: Stated rate of interest is 12 percent and the frequency of compounding is six times a year.

Case B: Stated rate of interest is 24 percent and the frequency of compounding is four times a year.

Case C: Stated rate of interest is 24 percent and the frequency of compounding is twelve times a year.

6.20 If the interest rate is 12 percent how much investment is required now to yield an income of Rs.12,000 per year from the beginning of the 10^{th} year and continuing thereafter forever ?

6.21 You have a choice between Rs.5,000 now and Rs.20,000 after 10 years. Which would you choose? What does your preference indicate?

6.22 Mr.Raghu deposits Rs.10,000 in a bank now. The interest rate is 10 percent and compounding is done semi-annually. What will the deposit grow to after 10 years? If the inflation rate is 8 percent per year, what will be the value of the deposit after 10 years in terms of the current rupee?

6.23 How much should be deposited at the beginning of each year for 10 years in order to provide a sum of Rs.50,000 at the end of 10 years ?

6.24 A person requires Rs.20,000 at the beginning of each year from 2025 to 2029. How much should he deposit at the end of each year from 2015 to 2020? The interest rate is 12 percent.

6.25 What is the present value of Rs.2,000 receivable annually for 30 years? The first receipt occurs after 10 years and the discount rate is 10 percent.

6.26 After five years Mr.Ramesh will receive a pension of Rs.6000 per month for 15 years. How much can Mr.Ramesh borrow now at 12 percent interest so that the borrowed amount can be paid with 30 percent of the pension amount? The interest will be accumulated till the first pension amount becomes receivable.

6.27 Mr.Prakash buys a motorcycle with a bank loan of Rs.60,000. An instalment of Rs.3000 is payable to the bank for each of 24 months towards the repayment of loan with interest. What interest rate does the bank charge?

6.28 South India Corporation has to retire Rs.1000 million of debentures each at the end of 8, 9, and 10 years from now. How much should the firm deposit in a sinking fund account annually for 5 years, in order to meet the debenture retirement need? The net interest rate earned is 8 percent.

6.29 Mr.Longman receives a provident fund amount or Rs.1,000,000. He deposits it in a bank which pays 10 percent interest. If he withdraws annually Rs.200,000, how long can he do so ?

6.30 Phoenix Company borrows Rs.500,000 at an interest rate of 14 percent. The loan is to be repaid in 4 equal annual instalments payable at the end of each of the next 4 years. Prepare the loan amortisation schedule.

6.31 You want to borrow Rs.1,500,000 to buy a flat. You approach a housing company which charges 13 percent interest. You can pay Rs.200,000 per year toward loan amortisation. What should be the maturity period of the loan?

6.32 You are negotiating with the government the right to mine 100,000 tons of iron ore per year for 15 years. The price per ton of iron is expected to be Rs. 3,000 at the end of year 1 and increase thereafter at the rate of 6 percent per year. What is the present value of the iron ore that you can mine if the discount rate is 16 percent?

6.33 As a winner of a competition, you can choose one of the following prizes:

a. Rs. 500,000 now

b. Rs. 1,000,000 at the end of 6 years

c. Rs. 60,000 a year forever

d. Rs. 100,000 per year for 10 years

e. Rs. 35,000 next year and rising thereafter by 5 percent per year forever.

If the interest rate is 10 percent, which prize has the highest present value.

6.34 Pipe India owns an oil pipeline which will generate Rs. 12 crore of cash income in the coming year. It has a very long life with virtually negligible operating costs. The volume of oil shipped, however, will decline over time and, hence, cash flows will decrease by 3 percent per year. The discount rate is 12 percent.

a. If the pipeline is used forever, what is the present value of its cash flows?

b. If the pipeline is scrapped after 25 years, what is the present value of its cash flows?

6.35 An oil well presently produces 50,000 barrels per year. It will last for 15 years more, but the production will fall by 5 percent per year. Oil prices are expected to increase by 3 percent per year. Presently the price of oil is $50 per barrel. What is the present value of the well's production if the discount rate is 10 percent?

6.36 An oil well presently produces 80,000 barrels per year. It will last for 20 years more, but the production will fall by 6 percent per year. Oil prices are expected to increase by 4 percent per year. Currently the price of oil is $60 per barrel. What is the present value of the well's production if the discount rate is 12 percent?

6.37 You are considering whether your savings will be enough to meet your retirement needs. You saved Rs. 100,000 last year and you expect your annual savings to increase by 8 percent per year for the next 20 years. If your savings can be invested at 9 percent, how much would you have at the end of the twentieth year?

6.38 A bank offers an interest rate of 8 percent on deposits made with it. If the compounding is done on a weekly basis, what is the effective interest rate?

MINICASE I

As an investment advisor, you have been approached by a client called Ramesh, who wants some help in investment related matters.

Ramesh is currently 45 years old and has Rs 600,000 in the bank. He plans to work for 15 more years and retire at the age of 60. Ramesh's present salary is Rs 400,000 per year. He expects his salary to increase at the rate of 12 percent per year until his retirement.

Ramesh has decided to invest his bank balance and future savings in a portfolio in which stocks and bonds would be equally weighted. For the sake of simplicity, assume that these proportions will be maintained by him throughout. He also believes that bonds would provide a return of 7 percent and stocks a return of 13 percent. You concur with his assessment.

Once Ramesh retires at the age of 60 he would like to withdraw Rs 500,000 per year from his investments for the following 15 years as he expects to live upto the age of 75 years. He also wants to bequeath Rs 1,000,000 to his children at the end of his life. How much money would he need 15 years from now?

How much should Ramesh save each year for the next 15 years to be able to meet his investment objectives spelt out above? Assume that the savings will occur at the end of each year.

Suppose Ramesh wants to donate Rs 200,000 each year in the last three years of his life to a charitable cause. Each donation would he made at the beginning of the year. How much money would he need when he reaches the age of 60 to meet this specific need?

Ramesh recently attended a seminar on human capital where the speaker talked about a person's human capital as the present value of his life time earnings. Ramesh is curious to find out the present value of his lifetime salary. For the sake of simplicity assume that his present salary of Rs 400,000 will be paid exactly one year from now, and his salary will be paid in annual installments. What is the present value of his life time salary, if the discount rate is 8 percent? Remember that Ramesh expects his salary to increase at the rate of 12 percent per year until his retirement.

In answering the above questions, ignore the tax factor.

MINICASE II

Sardar Kartar Singh is a resident of Thailand for the past two decades and is the owner of a flourishing business there. He has a son, Satnam, 10 years old and a baby girl Jasleen who will be one year old this day. The family has come to India to celebrate her birthday in Punjab. Also, Kartar's wife has made some grand plans for the future financial security of the family and they intend to use their present visit for placing suitable deposits with their bank in New Delhi as per those plans.

According to the plan, Satnam would be doing his MBA after 10 years. It would be a two year course in a premier private business school in India. For that the all inclusive expenditure at present rates would be Rs.20 lakhs and Rs.25 lakhs in the beginning of the first and second year respectively. Jasleen would marry at the end of her 21st year and for that an amount of Rs.3 crores would then be needed. Kartar's wife is insistent that her presence would be essential in India in the best interests of both the children-to keep a watchful eye on Satnam during his stint at the business school and most importantly, to have ample time to renew the old network with family and friends for ensuring a very good match for the girl. Funds would have to be tied up for her and children's relocation to India at the end of ten years from now.

Kartar Singh always had great respect for his wife's commonsense and logic (though he was always shy of acknowledging it!). To arrange the funds, he has very recently sold one of his investments, a flat in a prime locality in Bangkok, for a hefty sum. For Satnam's MBA he has decided to open two recurring deposit accounts, maturing on the 10th and 11th years respectively. For Jasleen's marriage he wants to open a cumulative term deposit for 20 years. For family maintenance in India after 10 years, he wants to open another cumulative term deposit for 10 years with the maturity value of which he could immediately purchase an annuity due for the following 10 years. It is expected that after 10 years the family in India would need Rs.12 lakhs per year without taking inflation into consideration.

To make the calculations on the specific amounts needed etc. he has called you, an upcoming financial consultant. He asks you to make the calculations in such a way that he could easily understand the logic thereof. You understand from him that as all the deposits would be made out of his NRE account with the bank, it would not deduct any tax amount from the interest to be earned.

Specifically you are required to calculate the amounts that need to be deposited now in :

(i) the two recurring deposit accounts, in the beginning of each month.

(ii) a cumulative fixed deposit for meeting the cost of Jasleen's marriage.

(iii) a cumulative fixed deposit with the bank for purchasing the annuity due needed by the family in India after 10 years from an insurance company which is expected to give a return of 10 percent per year.

You set to work with the following data:

For both cumulative fixed deposit and Recurring deposit , nominal interest rate for periods of more than 5 years is 8 percent and compounding is done once in a quarter. Inflation in India after 10 years is expected to be 5 percent for the next ten years. The MBA course expenses are likely to grow at 5 percent per annum.

Show your detailed working.