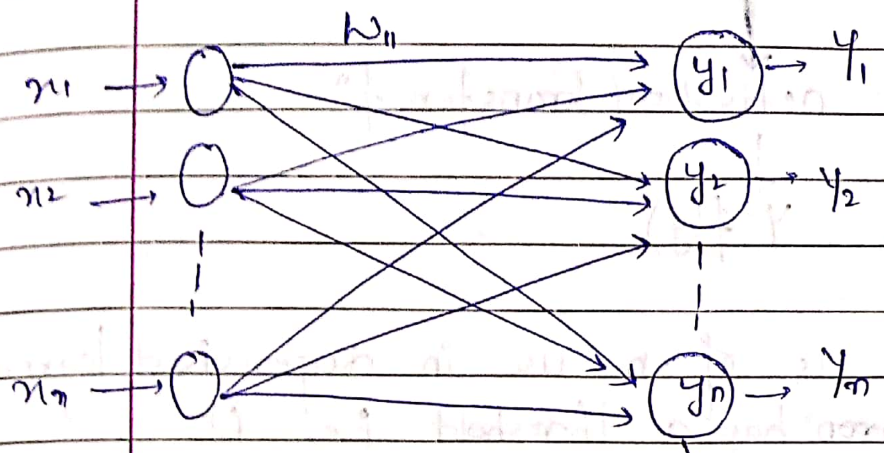
• ARTIFICIAL NEURAL NETWORK (ANN)



(i/p layer) (o/p layer)

net i/p and further processing is done to get o/p. y

Single layer feed forward artificial neural network

Single layer → one stage of transfer is there only

feed forward → ∴ feeding data in forward dirn only

✓ All connections are weighted.

W_{11} represent connection from x_1 to y_1 with some wt.

$$x \to \bigcirc \xrightarrow{w} (y) \to y$$

$\boxed{y = wx}$ → net i/p

wt

+ve → Excitatory wt. (∵ inc o/p)

−ve → inhibitory wt. (∵ dec o/p)

$\boxed{y = xw}$

↓

Activation/transfer f^n

↓

$Y = f(y)$

net i/p is of no use in supervised learning

- Every neuron has a threshold i.e. θ

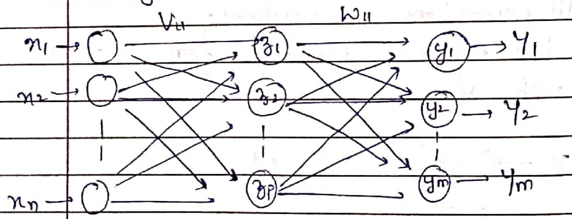
→ When neuron exceed its threshold then only it comes in firing state and generate o/p.

→ Rest, it remains in non-firing state

→ After acheiving firing state once, each neurons needs come rest before again acheiving firing state.

- While retraining, we change wt. of connections.

Multilayer Architecture



$\boxed{z_1 = xv}$

↓

Activation f^n

$\boxed{f(z_1) = Z_1}$

↓

$\boxed{y_1 = Z_1 w}$

↓

activation f^n

$Y_1 = f(y_1)$

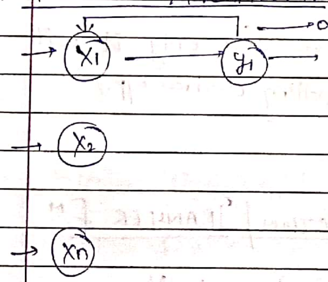
✓ Any layer added b/w first and last layer is called hidden layer.

✓ There is no restriction on no. of hidden layers as well as on no. of neurons in each layer.

(individual multiplication as well as matrix multiplication both depend on coding style)

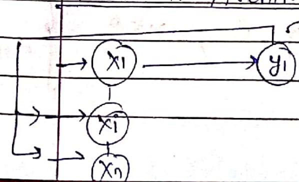
Feedback Architecture

→ o/p is fed to the same i/p



Recurrent Architecture (Eg. Hop field Network)

o/p is fed to other neurons except the same one.



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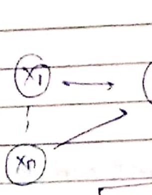
Bias Architecture

extra wt. applied to neurons and has a unit activation i.e. const. (+1)*



b/wo
x_1

$\boxed{y = xw + b} \equiv \boxed{y = mx + c}$

st. line eqn

Here, 'b' decides shape of curve.

Importance of bias wt.

→ It helps in better conversion
→ Conversion closer to the desired o/p

Why b is +ve

→ b if +ve then only it will help in getting desired o/p.

Activation Function / Transfer fn

(1) Identity fn : $f(x) = x; \ \forall x$

(2). Binary fn : $f(x) = \begin{cases} 1 ; & x \geq 0 \\ 0 ; & x < 0 \end{cases}$

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(3). Bipolar stepfn : $f(x) = \begin{cases} 1 ; & x \geq 0 \\ -1 ; & x < 0 \end{cases}$

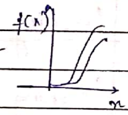
discrete function

→ equality sign can be shifted for desired o/p's.

(4). Binary sigmoidal fn :

$$f(x) = \frac{1}{1 + e^{-\lambda x}}$$

λ → stepness fn



derivative of f(x)

$$\boxed{f'(x) = \lambda f(x) [1 - f(x)]}$$

* λ → scalar
generally (0.5 to 1)
if not given, assume (λ=1)

(5). Bipolar sigmoidal :

$$f(x) = \frac{2}{1 + e^{-\lambda x}} - 1$$

continuous activation fn (mainly used in back propogation fn.)

derivative

$$\boxed{f'(x) = \frac{\lambda}{2} \left(1 + f(x)\right)\left(1 - f(x)\right)}$$

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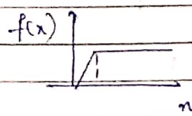
6. Ramp f^n :

$$f(x) = \begin{cases} 1 & ; x > 0 \\ x & ; x = 0 \\ 0 & ; x < 0 \end{cases}$$

$$\begin{cases} 1 & ; x > 0 \\ 0 & ; x = 0 \\ -1 & ; x < 0 \end{cases}$$

↳ for perceptual neuron



Learning rate α'

Learning machine to get desired o/p which we already know.

if, actual o/p ≠ desired o/p

- retraining is done by changing wt.
- Δw is calculated using α' to get desired o/p
- α need to smaller to provide fast learning

α → not given assume (α=1) → for ease of computation.

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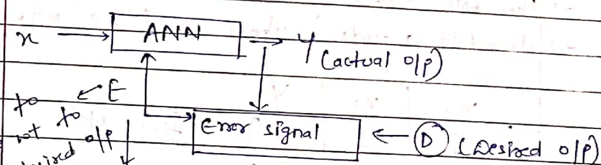
Types of Learning :-

1. Supervised Learning   Eg. Classification



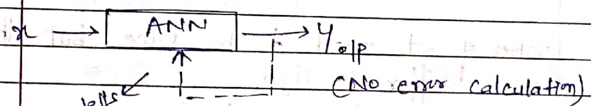
used to update wt to get desired o/p

least mean square error.

$(MS(E) = \frac{1}{2}(D-Y)^2$.

↳ random factor to cancel out 2 at time of derivative.

2. Unsupervised   Eg. Clustering   (Have learning rule)



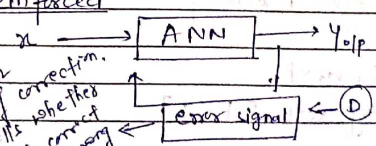
(No error calculation)

neither tells right or wrong

3. Reinforced

(b/w supervised & unsupervised)

↳ may not give correct result



to compare with only

not used in processing only of correction.

tells whether o/p is correct or wrong

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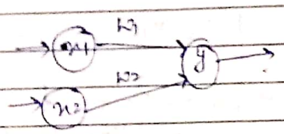
Mc Culloch Pitts Model (MCP)

- the i/p & o/p value will be 0 or 1.
- discrete binary step fn. (activation)
- Uses both exhibitory & inhibitory model
- threshold depends upon -ve & +ve wt. applied

	x_1	x_2	y	
1.	1	1	1	→ firing state
2.	1	0	0	non-firing state.
3.	0	1	0	
4.	0	0	0	

training pairs.



(single layer feed forward model)

x, y or $s:t$

string target

values of wt need to be same but they can differ in sign.

Eg. if 2 then -2 not -1

y_{in} → denotes net input

$$\boxed{y_{in} = x_1 w_1 + x_2 w_2}$$

→ $w_1 = 1$, $w_2 = 1$

1st training sets [1 1]

$y_{in_1} = 2$

→ [1 0]

$y_{in_2} = 1$

→ [0 1]

$y_{in_3} = 1$

→ [0 0]

$y_{in_4} = 0$

Find threshold value.

$$y = \begin{cases} 1 ; & y_{in} \geq \theta \\ 0 ; & y_{in} < \theta \end{cases}$$

from computation, [1 1] results in 2.

$\Rightarrow (\theta = 2)$ ∵ [1 1] is the only firing state

$$y = \begin{cases} 1 ; & y_{in} \geq 2 \\ 0 ; & y_{in} < 2 \end{cases}$$

Assumption model for realization

Decision Boundary

if the pts. can be demarked on two sides of a line then it is linearly separable.

Q. Realise OR gate using MCP model.

x_1	x_2	y	
1	0	1	firing states
0	1	1	
1	1	1	
0	0	0	non-firing "

$w_1 = w_2 = 1$ $\quad$ $x_1 w_1 + x_2 w_2 = y_{in}$

[1 1]

$y_{in} = 1 + 1 = 2$

[1 0]

$y_{in} = 1 + 0 = 1$ $\quad$ $\theta = 1$ based on computation.

[0 1]

$y_{in} = 1$

[0 0]

$y_{in} = 0$

$$y = \begin{cases} 1 ; & y_{in} \geq 1 \\ 0 ; & y_{in} < 1 \end{cases}$$

AND-NOT gate. $(x_1 \wedge x_2')$

and first variable with not of second variable.

x_1	x_2	x_2'	y	
1	1	0	0	
1	0	1	1	→ firing state.
0	1	0	0	
0	0	1	0	

[1 1] $w_1 = 1$ & $w_2 = 1$ ✗

$y_{in} = x_1 w_1 + x_2 w_2$
$= 1 + 1 = 2.$

[1 0]

$y_{in} = 1 + 0 = 1$

[0 1]

$y_{in} = 1$

[0 0]

$y_{in} = 0$

✓ $w_1 = 1 \quad w_2 = -1$

$1(1) + 1(-1)$
$= 1 - 1 = 0$

$1(1) + 0(-1)$
$1 + 0 = 1$ ✓

$0(1) + 1(-1)$
$= -1$

$0(1) + 0(-1)$
$= 0$

$\therefore \theta = 1$

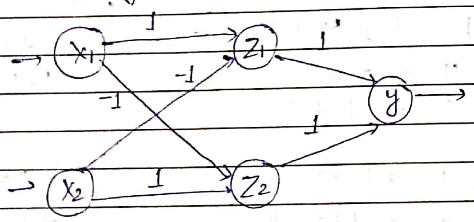
$$y = \begin{cases} 1 ; & y_{in} \geq 1 \\ 0 ; & y_{in} < 1 \end{cases}$$

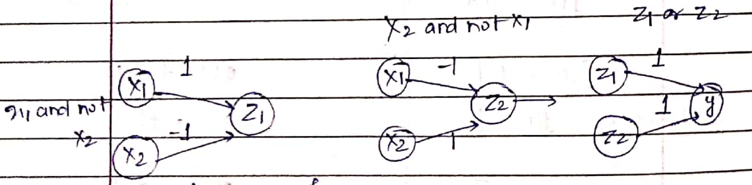
XOR Gates

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

On computing, we unable to get a linear separable architecture.

Now, we'll go for multilayer architecture by introducing a hidden layer.





find θ for each three gates.

x_1	x_2	z_1	
0	0	0	1
0	1	0	0
1	0	1	1
1	1	0	0

$0\times1 + 0(-1) = 0$

$0\times1 + 1(-1) = -1$

$1\times1 + 0\times(-1) = 1$

$1\times1 + 1\times(-1) = 0$

$\theta = 1$

22nd — Test

x_2 and not x_1

x_1	x_2	z_2	
0	0	0	1
0	1	1	1
1	0	0	0
1	1	0	0

$0\times(-1) + 0(1) = 0$

$0\times(-1) + 1(1) = 1$

$1\times(-1) + 0(1) = -1$

$1\times(-1) + 1(1) = 0$

$\theta = 1$

z_1 or z_2

x_1	x_2	z_1	z_2	w
0	0	0	0	0
0	1	0	1	1
1	0	1	0	1
1	1	0	0	0

$0(1) + 0(1) = 0$

$0(1) + 1(1) = 1$

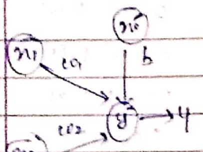
$1(1) + 0(1) = 1$

$0(1) + 0(1) = 0$

$\theta = 1$

$$\begin{cases} 1 ; & y_{in} \geq 1 \\ 0 ; & y_{in} < 1 \end{cases}$$

Linear Seprability



$y_{in} = x_1 w_1 + x_2 w_2 + b$

if pt. lies on line.

$\Rightarrow y_{in} = 0$

$\Rightarrow x_1 w_1 + x_2 w_2 + b = 0$

$\Rightarrow \boxed{x_2 = -x_1 \frac{w_1}{w_2} - \frac{b}{w_2}}$

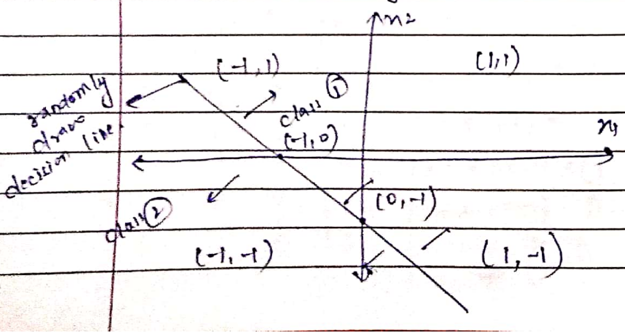
eqn for decision line.

compare to $\boxed{y = mx + c}$

Eg. (1).

OR Gate

x_1	x_2	y	
1	1	1	same o/p ⇒ belongs to same class.
1	-1	1	
-1	1	1	
-1	-1	-1	— Class 2



Outliers → can never be classified under any class.

→ They do not listen to problem.

→ Every problem consist of such pts.

∴ Ignore them as they will always dec. the performance.

Now,

$(x_1, y_1) = (-1, 0)$

$(x_2, y_2) = (0, -1)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = -1$

$y_1 = m x_1 + c$

$0 = (-1)(-1) + c$

$\Rightarrow c = -1$

$\Rightarrow \boxed{y = -x_1 - 1}$

Now compare it with standard eqn.

$\boxed{x_2 = -x_1 \frac{w_1}{w_2} - \frac{b}{w_2}}$

$\Rightarrow \frac{w_1}{w_2} = 1, \quad \frac{b}{w_2} = 1$

$\Rightarrow \boxed{w_1 = w_2 = b}$

Now, for ease of computation take them 1.

$\Rightarrow \boxed{w_1 = w_2 = b = 1}$

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const. activation of (+1).

x_1	x_2	x_0	y_{in}	y
1	1	1	3	1
1	-1	1	1	1
-1	1	1	1	1
-1	-1	1	-1	-1

firing state in original one

$\Rightarrow \theta = 1$

table based on this eqn

$$\boxed{y_{in} = x_1 w_1 + x_2 w_2 + x_0 b}$$

Compare with y to see 'θ'

$$y = \begin{cases} 1 & y_{in} \geq 1 \\ -1 & y_{in} < 1 \end{cases}$$

$\because$ θ can be figured out

$\Rightarrow$ OR gate is linearly separable. (if coming y is same as expected or desired y.)

linearly separable.

Q. 2. Check whether AND NOT gate is linearly separable or not.

x_1	x_2	x_2'	y
1	1	-1	-1 ✓
1	-1	1	1 — 2nd class
-1	1	-1	-1 ✓
-1	-1	1	-1 ✓

1 class

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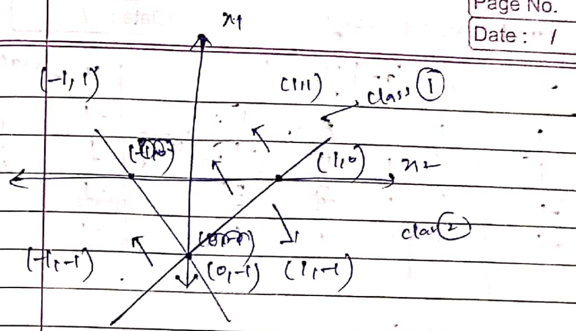

$(x_1, y_1) = (+1, 0)$

$(x_2, y_2) = (0, -1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{+1 - 0}{0 - (-1)} = +1.$$

$$\boxed{y_1 = +x + c}$$

↓ compare

(1,0) $0 = x + c$

$0 = 1 + c$

$c = -1$

$$\boxed{y_1 = x - 1}$$

$$x_2 = -x_1 \frac{w_1}{w_2} - \frac{b}{w_2}$$

$\Rightarrow \frac{w_1}{w_2} = -1$ & $\frac{b}{w_2} = +1$

$\Rightarrow w_1 = -w_2 = b = 1$

$\Rightarrow \boxed{w_1 = -1, \; w_2 = +1, \; \& \; b = +1}$

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$w_1 = 1,\ w_2 = +1,\ b = 1$

x_1	x_2	x_0	y_{in}		y
1	1	1	1	1	-1
1	-1	1	-1	-1	1
-1	1	1	-3	3	-1
-1	-1	1	-1	1	-1

same
firing
linearly separable.

$y_{in} = x_1 w_1 + x_2 w_2 + x_0 b$
$= \cancel{x_1 - x_2 + 1}$ $\boxed{-x_1 + x_2 + b}$

$y_{in} = 1 - 1 + 1 = 1$ | $-(1) + (1) + (1) = 1$
$1 - (-1) + 1 = 1$ | $-(1) + (-1) + (1) = -1$
$-1 - 1 + 1 = -1$ | $-(-1) + (1) + (1) = 3$
$-1 - (-1) + 1 = 1$ | $-(-1) + (-1) + (1) = 1$

X

$\theta = 1$

$y = \begin{cases} -1 & ; y_{in} \geq 1 \\ 1 & ; y_{in} < 1 \end{cases}$

* IMP: coming wrong
$\Rightarrow$ we should take $\boxed{w_1 = 1,\ w_2 = -1 \text{ and } b = -1}$

x_1	x_2	x_0	y_{in}	y
1	1	1	-1	-1
1	-1	1	1	1
-1	1	1	-3	-1
-1	-1	1	-1	-1

1). $1(1) + 1(-1) + 1(-1) = 1 - 1 - 1 = -1$
2). $1(1) + (-1)(-1) + 1(-1) = 1$
3). $1(-1) + (-1)(1) + 1(-1) = -3$
4). $1(-1) + (-1)(-1) + (-1)(1) = -1$

$\boxed{\theta = 1}$

$y = \begin{cases} 1 & ; y_{in} \geq 1 \\ -1 & ; y_{in} < 1 \end{cases}$

LEARNING RULES

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$W_{new} = W_{old} + \Delta W$

$\Delta W = \alpha (LS)$ $\rightarrow$ product of learning rate or signal.

$b_{new} = b_{old} + \Delta b$

$\Delta b = \alpha (LS)$

* It is diffn for each learning rule.

(1). Hebb Learning Rule — eg of unsupervised learning.

✓ If neuron A is in firing mode & B is adjacent to it then, it will push neuron B towards acheiving firing state.

✓ Bipolar mode $\longrightarrow$ o/p is 1 (ON) or -1 (OFF).

✓ initial value of weights is always 0.
$w_1 = w_2 = \dots = w_n = 0$

* { We are finding new wts & training our system bcz. it assume's that old wt. might not be correct. }

$\Delta W = \alpha (x \cdot y)$

$\Delta b = \alpha (1)(y)$
$= \alpha y$

* Taking bias is a compulsion.

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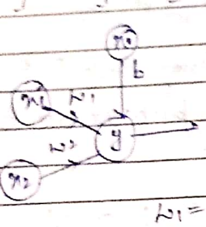
Q. Implement a Hebb Net for AND gate.

	x_1	x_2	x_0	y
1.	1	1	1	1
2.	1	-1	1	-1
3.	-1	1	1	-1
4.	-1	-1	1	-1

IMP. (target o/p y is based only on x_1 and x_2)

bias x_0 is only for implementation purpose.



$w_1 = w_2 = b = 0$

s : t
storage target

Training pair

1). $[1 \quad 1 \quad 1] : 1$ ← o/p

∵ unsupervised learning ⇒ no calculation for net i/p

⇒ θ i.e. threshold is not calculated.

⇒ m/c is learned with the pair we provide them.

But for case of computation, use, $\alpha = 1$

0.2 generally gives good result

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$\alpha = 1$

$\Delta w_1 = \alpha . x_1 . y$
$= (1)(1)(1) = 1$

$\Delta w_2 = \alpha . x_2 . y$
$= (1)(1)(1) = 1$

$(\Delta b = 1)$ $(\alpha . y)$

IMP: o/p depends on sequence of i/p taken.

$w_1 \text{ new} = 1 + 0 = 1$
$w_2 \text{ new} = 1 + 0 = 1$
$b \text{ new} = 1 + 0 = 1$

IMP: one epoch is equal to one iteration i.e. one learning/training cycle.

2). $[1 \quad -1 \quad 1] : -1$

$\Delta w_1 = \alpha . x_1 . y$
$= 1(1)(-1) = -1$

$\Delta w_2 = \alpha . x_2 . y$
$= 1(-1)(-1) = 1$

$\Delta b = \alpha . y = 1(-1) = -1$

IMP: When to stop.
When, $\Delta w = 0$
⇒ wt. will not change
⇒ ans. has been obtained.

$w_1 \text{ new} = 1 - 1 = 0$
$w_2 \text{ new} = 1 + 1 = 2$
$b \text{ new} = 1 - 1 = 0$

3). $[-1 \quad 1 \quad 1] : -1$

$\Delta w_1 = \alpha . x_1 . y$
$= 1(-1)(-1) = 1$

$\Delta w_2 = \alpha . x_2 . y = 1(1)(-1) = -1$

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$\Delta b = 1(-1) = -1$

$w_1 new = 0 + 1 = 1$

$w_2 new = 2 + (-1) = 1$

$b_{new} = 0 + (-1) = -1$

diff^n implies, $\boxed{w_{old} = w_{new}}$ → consecutive.

~~this old no needs to be immediate.~~

~~It could have came in history~~

4). [-1 -1 1] : -1

$\Delta w_1 = \alpha . x_1 . y = 1(-1)(-1) = 1$

$\Delta w_2 = \alpha . x_2 . y = 1(-1)(-1) = 1$

$\Delta b = \alpha y = 1(-1) = -1$

$w_1 new = 1 + 1 = 2$

$w_2 new = 1 + 1 = 2$

$b_{new} = -1 - 1 = -2$

Q. Implement a Hebb Net for OR gate upto 2 epochs.

x_1	x_2	x_0	y
1	1	1	1
1	-1	1	1
-1	1	1	1
-1	-1	1	-1

** Neural network which uses Hebb Rule is called Hebb Net.

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$\alpha = 1$.

1. [1 1 1] : 1

$\Delta w_1 = \alpha . x_i y = 1(1)(1) = 1$

$\Delta w_2 = \alpha x_2 . y = 1(1)(1) = 1$

~~$\Delta b = \alpha . y = 1(1) = 1$~~

$w_{1 new} = 1 + 0 = 1$

$w_{2 new} = 1 + 0 = 1$

$b_{new} = 1$

2. [1 -1 1] : 1

~~$\Delta w_1 = 1(1)(1) = 1$~~

~~$\Delta w_2 = 1(-1)(1) = -1$~~

$\Delta b = 1(1) = 1$

$w_{1 new} = 1 + 1 = 2$

$w_{2 new} = 1 + (-1) = 0$

$b_{new} = 1 + 1 = 2$

3. [-1 1 1] : 1

$\Delta w_1 = 1(-1)(1) = -1$

$\Delta w_2 = 1(1)(1) = 1$

$\Delta b = 1(1) = 1$

~~$w_1 new = 2 - 1 = 1$~~

~~$w_2 new = 0 + 1 = 1$~~

$b_{new} = 2 + 1 = 3$

4. [-1 -1 +1] : -1

$\Delta w_1 = (-1)(-1)(1) = 1$

~~$\Delta w_2 = 1(-1)(-1) = 1$~~

$\Delta b = 1(-1) = -1$

$w_{1 new} = 1 + 1 = 2$

$w_2 new = 1 + 1 = 2$

$b_{new} = 3 - 1 = 2$

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2nd epoch

(2 , 2 , 2)

1. [1 (1) 1] : 1

~~$\Delta w_1 = 1(1)(1) = 1$~~
~~$\Delta w_2 = 1(1)(1) = 1$~~
~~$\Delta b = 1(1) = 1$~~

$w_{1N} = 3$
$w_{2N} = 3$
$b_N = 3$

2. [1 -1 1] : 1

$w_{1N} = 3 + 1 = 4$
$w_{2N} = 3 + (-1) = 2$
$b_N = 3 + 1 = 4$

3. [-1 1 1] : 1

$w_{1N} = 4 - 1 = 3$
$w_{2N} = 2 + 1 = 3$
$b_N = 4 + 1 = 5$

4. [-1 -1 1] : -1

$w_{1N} = 3 + 1 = 4$
$w_{2N} = 3 + 1 = 4$
$b_N = 5 - 1 = 4$

Tut – (12-1) – Tut-2
Thurs.

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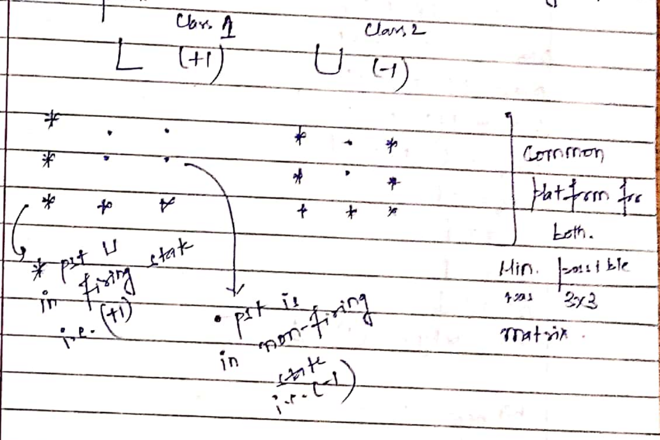
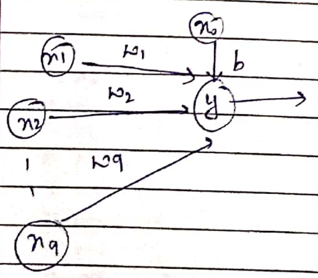
Q. Create a ~~simpl~~ suitable Heb Net to identify the patterns L and U where, L ∈ class 1 and U ∈ class 2. Find the wt.'s after learning for one epoch.



Pattern	I/p's										O/P
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_0	
L	1	-1	-1	1	-1	-1	1	1	1	1	+1
U	1	-1	1	1	-1	1	1	1	1	1	-1

$w_1 = w_2 = w_3 = w_4 = w_5 = w_6 = w_7 = w_8 = w_9 = b = 0$

(initially, wt. = 0)



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($\alpha = 1$)

$[1 \ -1 \ -1 \ 1 \ -1 \ -1 \ 1 \ 1 \ 1 \ 1 \ 9] : +1$

$\Delta w_1 = \alpha x_1 \cdot y$

$= 1 \times 1 \times (1) = 1$

$\Delta w_2 = 1(-1)(1) = -1$

$\Delta w_3 = 1(-1)(1) = -1$

$\Delta w_4 = 1(1)(1) = 1$

$\Delta w_5 = 1(-1)(1) = -1$

$\Delta w_6 = -1$

$\Delta w_7 = 1$

$\Delta w_8 = 1$

$\Delta w_9 = 1$

$\Delta b = \alpha(y) = 1(1) = 1$

w_1 new

w_2 new

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Q. Use the heb net to learn the patterns, I and O. Where I ∈ Class I and O ∈ Class 2.
Find the final weights after 2 epochs.

```
*  *  *        *  *  *
.  *  .        *  .  *
*  *  *        *  *  *
```

I (+1)   O (-1)

Pattern	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_0	O/P
I	1	1	1	-1	1	-1	1	1	1	1	+1
O	1	1	1	1	-1	1	1	1	1	1	-1

(I/P spans x_1 to x_0)

($\alpha = 1$)

Set 1

Δw_1	$= 1$	w_1 new	$= 1$
Δw_2	$= 1$	w_2	$= 1$
Δw_3	$= 1$	w_3	$= 1$
Δw_4	$= -1$	w_4	$= -1$
Δw_5	$= 1$	w_5	$= 1$
Δw_6	$= -1$	w_6	$= -1$
Δw_7	$= 1$	w_7	$= 1$
Δw_8	$= 1$	w_8	$= 1$
Δw_9	$= 1$	w_9	$= 1$
Δb	$= 1$	b	$= 1$

Set 2

$1(-1) = -1$

$\Delta w_1 = -1$	$w_1 = 0$		$b = 0$
$\Delta w_2 = -1$	$w_2 = 0$		
$\Delta w_3 = -1$	$w_3 = 0$		
$\Delta w_4 = -1$	$w_4 = -2$		
$\Delta w_5 = 1$	$w_5 = 2$		
$\Delta w_6 = -1$	$w_6 = -2$		
$\Delta w_7 = -1$	$w_7 = 0$		
$\Delta w_8 = -1$	$w_8 = 0$		
$\Delta w_9 = -1$	$w_9 = 0$		
$\Delta b = -1$			

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epoch (2)

		set (2)
set (1)	w1 = 1	w1 = 0
	w2 = 1	w2 = 0
	w3 = 1	w3 = 0
	w4 = -3	w4 = -4
	w5 = 3	w5 = 4
	w6 = -3	w6 = -4
	w7 = 1	w7 = 0
	w8 = 1	w8 = 0
	w9 = 1	w9 = 0
	b = 1	b = 0

Supervised Learning

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1. Perceptron Algorithm

$$w_{new} = w_{old} + \Delta w$$

$$\Delta w = \alpha t x$$

calculate

$y \neq t \rightarrow$ target o/p

actual o/p we are getting

✓ This w_{new} can be calculated for more than one epoch

✓ Irrespective of I/P, we assume target o/p t in bi-polar form

✓ activation fn taken $\rightarrow$ step activation fn.

$$y = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } y_{in} = \theta \\ -1 & \text{if } y_{in} < \theta \end{cases}$$

✓ if, θ is not given assume it to be 0

Eg. AND Gate

x_1	x_2	x_0	t
1	1	1	1
1	-1	1	-1
-1	1	1	-1
-1	-1	1	-1

⟹ There is no condition on i/p weight

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if not given, we take them to be '0' initially

$\begin{cases} w_1 = w_2 = b = 0 \\ \alpha = 1 \\ \theta = 0 \end{cases}$

Now

$y = \begin{cases} 1 & ; y_{in} > 0 \\ 0 & ; y_{in} = 0 \\ -1 & ; y_{in} < 0 \end{cases}$

1. $[1 \quad 1 \quad 1] : 1$

$y_{in} = ~~\text{[illegible]}~~ 1(0) + 1(0) + 1(0) = 0$

$y = 0$

$\Rightarrow y \neq t$

$\Rightarrow$ training needs to be done.

$\Delta w_1 = \alpha \cdot t \cdot x_1 = 1(1)(1) = 1$	$w_{1N} = 0 + 1 = 1$	Correct set of for 1st.
$\Delta w_2 = 1(1)(1) = 1$	$w_{2N} = 0 + 1 = 1$	
$\Delta b = 1(1) = 1$	$b_N = 0 + 1 = 1$	

2. $[1 \quad -1 \quad 1] : -1$

$y_{in} = 1(1) + (-1)(1) + 1(1)$

$= 1$

$y = 1$

$y \neq t$

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$\downarrow$ train.

$\Delta w_1 = 1(-1)(1) = -1$	$w_{1N} = 1 - 1 = 0$
$\Delta w_2 = 1(-1)(-1) = 1$	$w_{2N} = 1 + 1 = 2$
$\Delta b = 1(-1)(1) = -1$	$b_N = 1 - 1 = 0$

3. $[-1 \quad 1 \quad 1] : -1$

$y_{in} = -1(0) + 1(2) + 1(0) = 2.$

$y = 1$

$y \neq t$

$\downarrow$ train

$\Delta w_1 = 1(-1)(-1) = 1$	$w_{1N} = 0 + 1 = 1$
$\Delta w_2 = 1(-1)(1) = -1$	$w_{2N} = 2 - 1 = 1$
$\Delta b = 1(-1)(1) = -1$	$b_N = 0 - 1 = -1$

4. $[-1 \quad -1 \quad 1] : -1$

$y_{in} = -1(1) + (-1)(1) + 1(-1) = -3$

$y = -1$

$y = t.$

$\Delta w_1 = 1(-1)(-1) = 1$
$\Delta w_2 = 1(-1)(-1) = 1$
$\Delta b = 1(-1)(1) = -1$

$\therefore$ No training is reqd.

* After 1st epoch :-

$\boxed{w_1 = 1, \quad w_2 = 1, \quad b = -1}$

{ if not given, assume $\theta = 0$
{ make step activation

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Q. Use a perceptron network to find the final weights of the given data with $\alpha = 1$, initial wt. are 0 after 2 epochs.

x_1	x_2	x_3	x_4	b	t
1	1	1	1		1
-1	1	-1	-1		1
1	1	1	-1		-1
1	-1	-1	1		-1

(don't assume bias if not mentioned.)

IMP

IMP — only assume it in Gates by default

$$y = \begin{cases} 1 & ; y_{in} > 0.2 \\ 0 & ; -0.2 \le y_{in} \le 0.2 \\ -1 & ; y_{in} < -0.2 \end{cases}$$

1. $[1 \ 1 \ 1 \ 1] : 1$

$y_{in} = 1(0) + 1(0) + 1(0) + 1(0) = 0$

$y = 0$

$y \ne t$

train

$\Delta w_1 = 1(1)(1) = 1$	$w_1' = 1$
$\Delta w_2 = 1(1)(1) = 1$	$w_2' = 1$
$\Delta w_3 = 1(1)(1) = 1$	$w_3' = 1$
$\Delta w_4 = (1)(1)(1) = 1$	$w_4' = 1$

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2. $[-1 \ 1 \ -1 \ -1] : 1$

$y_{in} = -1(1) + 1(1) + -1(1) + (-1)(1)$

$= \cancel{-1} \cancel{+1} \cancel{-1} \cancel{-1} = 0$

$y = 0$

$y \ne t$

train

$\Delta w_1 = 1(1)(-1) = -1$	$w_1' = 0$
$\Delta w_2 = 1(1)(1) = 1$	$w_2' = 2$
$\Delta w_3 = 1(1)(-1) = -1$	$w_3' = 0$
$\Delta w_4 = 1(1)(-1) = -1$	$w_4' = 0$

3. $[1 \ 1 \ 1 \ -1] : -1$

$y_{in} = 1(0) + 1(2) + 1(0) + (-1)(0) = 2$

$y = 1$

$y \ne t$

train.

$\Delta w_1 = -1(1) = -1$	$w_1' = -1$
$\Delta w_2 = -1(1) = -1$	$w_2' = 1$
$\Delta w_3 = -1(1) = -1$	$w_3' = -1$
$\Delta w_4 = -1(-1) = 1$	$w_4' = 1$

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4 [1 -1 -1 1] : -1

$y_{in} = -1(1) + 1(-1) + (-1)(-1) + 1(1)$
$= -1 -1 +1 +1 = 0$

$y = 0$

$y \neq t$

↓ train

$\Delta w_1 = (-1)(1) = -1$	$w_1' = -2$
$\Delta w_2 = (-1)(-1) = 1$	$w_2' = 2$
$\Delta w_3 = (-1)(-1) = 1$	$w_3' = 0$
$\Delta w_4 = (-1)(1) = -1$	$w_4' = 0$

2nd epoch

1 [1 1 1 1] : 1

$y_{in} = -2(1) + 2(1) + 0 + 0 = 0$

$y = 0$

$y \neq t$

↓ train

$\Delta w_1 = 1(1) = 1$	$w_1' = -1$
$\Delta w_2 = 1(1) = 1$	$w_2' = 3$
$\Delta w_3 = 1(1) = 1$	$w_3' = 1$
$\Delta w_4 = 1(1) = 1$	$w_4' = 1$

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2 [-1 1 -1 -1] : 1

$y_{in} = -1(-1) + 3(1) + 1(-1) + 1(-1)$
$= 1 + 3 - 2 = 2$

$y = 1$

$y = t$

⇒ wt remains same.

$w_1' = -1$
$w_2' = 3$
$w_3' = 1$
$w_4' = 1$

3 [1 1 1 -1] : -1

$y_{in} = -1(1) + 3(1) + 1(1) + (-1)(1)$
$= -1 + 3 + 1 - 1 = 2$

$y = 1$

$y \neq t$

↓ train

$\Delta w_1 = -1$	$w_1' = -2$
$\Delta w_2 = -1$	$w_2' = 2$
$\Delta w_3 = -1$	$w_3' = 0$
$\Delta w_4 = 1$	$w_4' = 2$

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y [1 -1 -1 1] : -1

$y_{in} = 1(-2) + (-1)(2) + (-1)(0) + 1(2)$
$= -2 - 2 + 0 + 2 = -2$

$\boxed{y = -1}$

$y = t$

∴ final wt. are,

$w_1 = -2$
$w_2 = 2$
$w_3 = 0$
$w_4 = 2$

→ Perceptron is easily applicable to Linear Classification.

\# UCI Repository

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* Delta ~~Learning~~ Learning Rule

$w_{new} = w_{old} + \Delta w$
$\Delta w = \alpha \cdot (t - y_{in}) \cdot x$
Error, $E = (t - y_{in})^2$ → least mean square error.

* $y = \begin{cases} 1 & ; y_{in} \geq 0 \\ -1 & ; y_{in} < 0 \end{cases}$

This activation fn is used when we replace y_{in} in above formulas by y.

[ideally, bipolar in nature]

→ initially, weights should not be zero. They are low as possible but > 0.
→ This helps in better conversion as well as avoid over training.
→ multiple epoch are possible.
→ Error is calculated for each training pair.
→ epoch error is → sum error of each training pair & divide it by no. of training cases.

Que.

OR Gate :-

x_1	x_2	x_0	t
1	1	1	1
1	-1	1	1
-1	1	1	1
-1	-1	1	-1

$\alpha = 1$

$w_1 = w_2 = b = 0.1$

1) [1 1 1] : 1

$y_{in} = 1(0.1) + 1(0.1) + 1(0.1) = 3 \times 0.1 = 0.3$

$\Delta w_1 = \alpha(t - y_{in}) \cdot x_1$

$= 1(1 - 0.3) \cdot 1$

$= 1(0.7) \cdot 1 = 0.7$

$\Delta w_2 = \alpha(t - y_{in}) \cdot x_2$

$= 0.7 \times 1 = 0.7$

$\Delta b = 0.7$

$Error = (t - y_{in})^2 = (1 - 0.3)^2$

$= (0.7)^2$

$= 0.49$

$w_1\ new = 0.1 + 0.7 = 0.8$

$w_2\ new = 0.1 + 0.7 = 0.8$

$b_{new} = 0.1 + 0.7 = 0.8$

2) [1 −1 1] : 1

$y_{in} = 1(0.8) - 1(0.8) + 1(0.8)$

$\cancel{= 1.7 - 1.7 + 1.7 = 1.7} = 0.8 - 0.8 + 0.8 = 0.8$

$\Delta w_1 = 1(1 - 0.8)(1) = 1(0.2)(1) = 0.2$

$\Delta w_2 = (+0.2)(-1) = -0.2$

$\Delta b = (+0.2)(1) = 0.2$

$E = (t - y_{in})^2 = \cancel{(-0.7)^2 = 0.49}\ (0.2)^2 = 0.04$

$w_{1N} = \cancel{1.7 - 0.7 = 1}$ | $0.8 + 0.2 = 1$

$w_{2N} = \cancel{1.7 + 0.7 = 2.4}$ | $0.8 - 0.2 = 0.6$

$b_N = \cancel{1.7 + 0.7 = 1}$ | $0.8 + 0.2 = 1$

3) [−1 1 1] : 1

$y_{in} = \cancel{(-1)(1) + 1(2.4) + 1(1)} = -1 + 0.6 + 1 = 0.6$

$= \cancel{2.4}\ 1 - 0.6$

$\Delta w_1 = 1(0.4)(-1) = -0.4$

$\Delta w_2 = 0.4(1)(1) = 0.4$

$\Delta b = 0.4(1)(1) = 0.4$

$E = (0.4)^2 = 0.16$

$w_{1N} = \cancel{1 + 1.4}\ 1 + (-0.4) = 0.6$

$w_{2N} = \cancel{2.4 - 1.4}\ 0.6 + 0.4 = 1$

$b_N = \cancel{1 - 1.4}\ 1 + 0.4 = 1.4$

$-1 + 3.8$

2.8

4) [−1 −1 1] : −1

$\cancel{y_{in} = -2.4 - 1 - 0.4 = -3.4 - 0.4 = -3.8}$

$\cancel{\Delta w_1 = 1(-1-(-3.8))(-1) = -2.8}$

$\cancel{\Delta w_2 = (2.8)(-1) =}$

$\cancel{\Delta b = (2.8)(1)}$

$y_{in} = -0.6 - 1 + 1.4 = \cancel{-0.6 + 0.4} = -0.2$

$\Delta w_1 = 1(-1 + 0.2)(-1) = 0.8$

$\Delta w_2 = 1(-0.8)(-1) = 0.8$

$\Delta b = (-0.8)(1) = -0.8$

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$E = (-0.8)^2 = 0.64$

$w_{1N} = 0.6 + 0.8 = 1.4$
$w_{2N} = 1 + 0.8 = 1.8$
$b_N = 1.4 - 0.8 = 0.6$

$$\text{epoch error} = \frac{0.49 + 0.04 + 0.16 + 0.64}{4}$$
$$= 0.33$$

Q2.

x_1	x_2	x_3	x_4	x_0	t
1	1	-1	1	1	1
1	1	1	1	1	1
1	-1	-1	-1	1	-1
-1	1	-1	-1	1	-1

$\alpha = 0.2$

$w_1 = w_2 = w_3 = w_4 = b = 0.1$

1). [1 1 -1 1 1] : 1

$y_{in} = 0.1 + \cancel{0.1} - \cancel{0.1} + 0.1 + 0.1$
$= 0.3$

0.2×0.7
$= 0.14$

$\Delta w_1 = 0.2(1-0.3)(1) = 0.14$
$\Delta w_2 = 0.14\ (1) = 0.14$
$\Delta w_3 = 0.14\ (-1) = -0.14$
$\Delta w_4 = 0.14\ (1) = 0.14$
$\Delta b = 0.14\ (1) = 0.14$

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$E = (0.7)^2 = 0.49$

$w_1 = 0.1 + 0.14 = 0.24$
$w_2 = 0.1 + 0.14 = 0.24$
$w_3 = 0.1 - 0.14 = -0.04$
$w_4 = 0.1 + 0.14 = 0.24$
$b = 0.1 + 0.14 = 0.24$

2). [1 1 1 1 1] : 1

$y_{in} = 0.24 + 0.24 - 0.04 + 0.24 + 0.24$
$= 0.96 - 0.04 = 0.92$

0.2×0.08
$= 0.016$

$\Delta w_1 = 0.2(1-0.92) = 0.016$
$\Delta w_2 = 0.2\ (\text{"}) = 0.016$
$\Delta w_3 = 0.016$
$\Delta w_4 = 0.016$
$\Delta b = 0.016$

$E = (0.08)^2 = 0.0064$

$w_1 = 0.24 + 0.016 = 0.256$
$w_2 = 0.24 + 0.016 = 0.256$
$w_3 = -0.04 + 0.016 = -0.024$
$w_4 = 0.24 + 0.016 = 0.256$
$b = 0.24 + 0.016 = 0.256$

3). [1 -1 -1 -1 1] : -1

$y_{in} = \cancel{0.256} - \cancel{0.256} + 0.024 - \cancel{0.256} + \cancel{0.256}$
$= 0.024$

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$0.2\times(-1.024)$
$=-2.048$

$\Delta w_1 = 0.2(-1-0.024)\ (1) = -2.048 \quad -0.2048$
$\Delta w_2 = -2.048\times\ (-1) = 2.048 \quad 0.2048$
$\Delta w_3 = (-1) = 2.048 \quad 0.2048$
$\Delta w_4 = (-1) = 2.048 \quad 0.2048$
$\Delta b = (1) = -2.048 \quad -0.2048$

$E = (-1.024)^2 = 1.048576$

$w_1 = 0.256 - 2.048 = -1.792$
$w_2 = 0.256 + 2.048 = 2.304$
$w_3 = -0.024 + 2.048 = 2.024$
$w_4 = 0.256 + 2.048 = 2.304$
$b = 0.256 - 2.048 = -1.792$

4). [-1 1 -1 -1 1] : -1

$y_{in} = 1.792 + \cancel{2.304} - 2.024 - \cancel{2.304} + 1.792$
$= 2.584 - 2.024$
$= \underline{0.560}$

$\Delta w_1 = 0.2(-1-0.560) = (-1.560)\,0.2$
$\Delta w_2 = -0.312$
$\Delta w_3 =$
$\Delta w_4 =$
$\Delta b =$

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$E = (-1.560)^2 = \underline{2.433}$

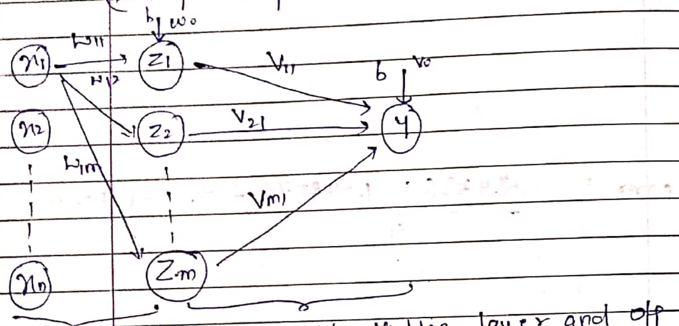
$$\text{net error} = \frac{2.4336 + 1.048576 + 0.0064 + 0.49}{4}$$

$=$

ADELINE
(ADaptive Linear Neuron)

→ wt. are adjustable in nature.

Madelines
(Multiple adaptive Linear Neuron)



wt. b/w i/p layer & Hidden layer are adjustable.

wt. b/w Hidden layer and o/p layer are fixed.

→ target o/p is 1.

$$\boxed{\Delta w = \alpha(1 - z_{in})\, x}$$

$z_{in} = x.w$

$$f(z_{in}) = \begin{cases} 1 & ; z_{in} \geq 0 \\ -1 & ; z_{in} < 0 \end{cases}$$

∵ o/p of hidden layer is used to calculate y_{in} ∴ we need to apply activation fn. to get z.

$$\boxed{y_{in} = z v}$$

To converge the system to desired o/p, we take value of wt's as small as possible to avoid overshooting.

$$\boxed{v_1 = v_2 = \text{-----} = v_n = v = \tfrac{1}{2}}$$

we try to keep them same & as small as possible.

→ Also, apply activation fn on y_{in} to get Y.

Now, compare, t and Y.

if, $t = y$; no training

if, $t \neq y$; update wt.

$\boxed{t \neq y}$

Check, $\boxed{t = 1}$ —True→ update neurons with +ve z_{in} as o/p.

false ↓

update neurons whose, z_{in} is as close to 0. (both +ve & -ve)

for randomised t

* parameter for selecting close to 0 i.e. range which we will consider depends on our coding style.

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Q. Classify ~~2D~~ i/p pattern I and F using perceptron network.

I ∈ Class 1

F ∈ Class 2

Assume, the initial wt & threshold as 0 & ($\alpha = 1$).

```
*  *  *        *  *  *
   *           *  *  *
*  *  *        *
```

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	t
1	1	1	-1	1	-1	1	1	1	1
1	1	1	1	1	1	1	-1	-1	-1

activation.

$$y = \begin{cases} 1 & ; y_{in} > 0 \\ 0 & ; y_{in} = 0 \\ -1 & ; y_{in} < 0 \end{cases}$$

(1). $y_{in} = 0$

$y = 0$

$y \neq t$

$\Delta w_1 = 1(1)(1) = 1$ $\quad \Delta w_7 = 1$

$\Delta w_2 = 1(1)(1) = 1$ $\quad \Delta w_8 = 1$

$\Delta w_3 = 1$ $\quad \Delta w_9 = 1$

$\Delta w_4 = -1$ $\quad \Delta b = 1$

$\Delta w_5 = 1$

$\Delta w_6 = -1$

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$w_1 = 1$ $\quad w_6 = -1$

$w_2 = 1$ $\quad w_7 = 1$

$w_3 = 1$ $\quad w_8 = 1$

$w_4 = -1$ $\quad w_9 = 1$

$w_5 = 1$ $\quad b = 1$

(2). $y_{in} = 1 + 1 + \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} - \cancel{1} + \cancel{1}$

$= 2$

$\boxed{y = 1}$

$y \neq t$

training

$\Delta w_1 = 1(-1)(1) = -1$ $\quad \Delta w_5 = -1$ $\quad \Delta w_9 = 1$

$\Delta w_2 = -1$ $\quad \Delta w_6 = -1$ $\quad \Delta b = -1$

$\Delta w_3 = -1$ $\quad \Delta w_7 = -1$

$\Delta w_4 = -1$ $\quad \Delta w_8 = 1$

$w_1 = 0$ $\quad w_7 = 0$

$w_2 = 0$ $\quad w_8 = 0$

$w_3 = 0$ $\quad w_9 = 0$

$w_4 = -2$ $\quad b = 0$

$w_5 = 0$

$w_6 = -2$

Q. Use adaline network to train AND NOT f^n with bipolar i/p's & targets. Assume the wt & learning rate to be (0.2).

x_1	x_2	x_0	y
1	1	1	-1
1	-1	1	1
-1	1	1	-1
-1	-1	1	-1

(1). $y_{in} = 1(0.2) + 0.2 + 0.2 = 0.6$

$\Delta w_1 = 0.2(-1-0.6)(1) = 0.2(-1.6) = -0.32$

$\Delta w_2 = 0.2(-1-0.6)(1) = 0.2(-1.6) = -0.32$

$\Delta b = 0.2(-1.6)(1) = -0.32$

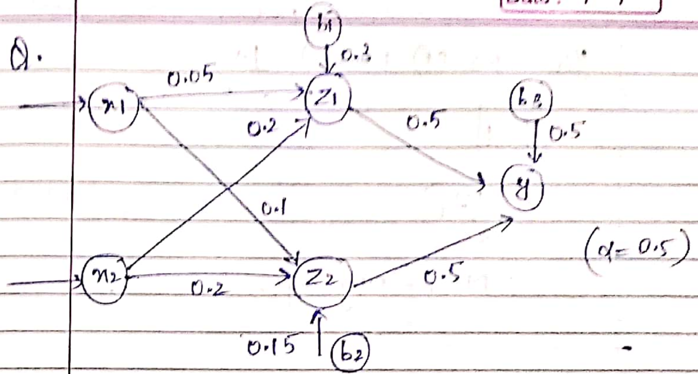
$w_1 = 0.2 - 0.32 = -0.12$

$w_2 = 0.2 - 0.32 = -0.12$

$b = 0.2 - 0.32 = -0.12$

$E = (t - y_{in})^2 = (-1.6)^2 = 2.56$

Q.



($\alpha = 0.5$)

Using MADLINE Network, implement XOR f^n with bipolar i/p's & targets.

(XOR - Table)

x_1	x_2	x_0	t
1	1	1	-1
1	-1	1	+1
-1	1	1	+1
-1	-1	1	-1

$$f(z_{in}) = \begin{cases} 1 ; & z_{in} \geq 0 \\ -1 ; & z_{in} < 0 \end{cases}$$

1). [1 1 1]:-1

$\boxed{\Delta w = \alpha(1 - z_{in}) x}$ → perform, when $y \neq t$ not equal.

for z_1

$z_{in} = 1(0.05) + 1(0.2) + 0.3 = 0.05 + 0.2 + 0.3 = 0.55$

$\boxed{Z_1 = 1}$

for z_2

$z_{in} = 1(0.1) + 1(0.2) + 0.15 = 0.1 + 0.2 + 0.15 = 0.45 \Rightarrow Z_2 = 1$

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$y_{in} = 0.5(1) + 0.5(1) = 1.5$

$\Rightarrow \boxed{y=1}$

$y \neq t$

$\therefore$ No training req.

Q). [1 -1 1] : -1

Z1. $z_{in} = 1(0.05) + (-1)(0.2) + 0.3$
$= 0.05 + 0.3 - 0.2$
$= 0.05 + 0.1 = 0.15 > 0$

0.10
0.15
0.05

$Z_1 = 1$

Z2. $z_{in} = 1(0.1) + (-1)(0.2) + 0.15$
$= 0.1 - 0.2 + 0.15$
$= -0.1 + 0.15$
$= 0.05 > 0$

$Z_2 = 1$

$y_{in} = 1(0.5) + 1(0.5) = 1$

$\Rightarrow y = 1$

$y \neq t$

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Perform training

$w_{11} = 0.05$, $w_{21} = 0.2$, $w_{22} = 0.2$, $w_{12} = 0.2$

Now, (Training)

$\because \boxed{t \neq 1}$ $(t=-1)$

$\Rightarrow$ Training Let's close to 0.

$w_{11} = 0.05$, $w_{12} = 0.1$, $w_{21} = 0.2$, $w_{22} = 0.2$

Now, train, (Z2). z_{in} of Z_2 is close.
$\Rightarrow$ train w_{12} & w_{22}.

$\Delta w_{12} = \alpha(t - z_{in}) \cdot x_i$

$= 0.5(-1 - 0.45)(1) = 0.5(-1.45)$
$= -0.725$

$\Delta w_{22} = 0.5(-1 - 0.45)(1) = 0.5(-1.45)$
$= -0.725$

$w_{12} = 0.1 - 0.725 = -0.625$

$w_{22} = 0.2 - 0.725 = -0.525$

$\Delta b_2 = \alpha(-1.45) = 0.5(-1.45) = -0.725$
$b_2 = 0.15 - 0.725$
$= -0.575$

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$w_{12} = -0.625$

$w_{22} = -0.625$

$\Delta b_2 = -0.575$

2). [1 -1 1] : +1

(z1) $z_{in} = 1(0.05) + (-1)(0.2) + 0.3$

$= 0.05 - 0.2 + 0.3$

$= 0.05 + 0.1$

$= 0.15$

$z_1 = 1$

(z2). $z_{in} = 1(-0.625) + (-1)(-0.625) + 1(-0.575)$

$= -0.625 + 0.625 - 0.575$

$= -0.575$

$z_2 = -1$

$y_{in} = 1(0.5) + (-1)(0.5) + 0.5$

$= 0.5$

$y = 1$

$y = t$

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3). [-1 1 1] : 1

(z1). $z_{in} = (-1)(0.05) + 1(0.2) + 0.3$

$= -0.05 + 0.5$

$= 0.45$

$z_1 = 1$

$$\begin{array}{r} 0.50 \\ -0.05 \\ \hline 0.45 \end{array}$$

(z2) $z_{in} = -1(-0.625) + (1)(-0.625) + 1(-0.575)$

$= 0.625 - 0.625 - 0.575$

$= -0.575$

$z_2 = -1$

$y_{in} = 1(0.5) + (-1)(0.5) + 0.5$

$= 0.5$

$y = 1$

$y = t$.

4). [-1 -1 1] : -1

$$\begin{array}{r} 0.1 \\ 0.05 \\ \hline 0.5 \end{array}$$

(z1) $z_{in} = -1(0.05) + (-1)(0.2) + 0.3$

$= -0.05 - 0.2 + 0.3$

$= -0.05 + 0.1$

$= 0.05$

$z_1 = 1$

(z2) $z_{in} = 0.625 + 0.625 - 0.575$

$= 1.250 - 0.575 = 0.675$

$z_2 = 1$

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$y_{in} = 1(-1).5 + 1(0.5) + 0.5$

$= 1.5$

$y = 1$

$y \neq t$ ✓

↓ training

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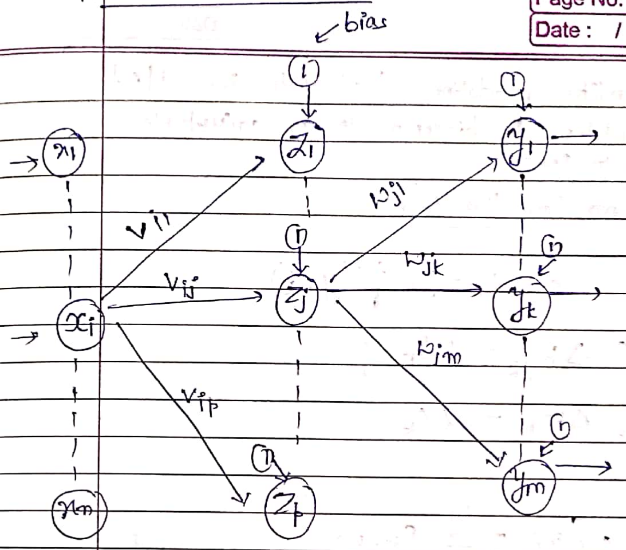
BACKPROPOGATION



$$Z_{in\,j} = V_{0j} + \sum_{i=1}^{n} x_i v_{ij}$$

$$Z_j = f(Z_{inj})$$

feedforward phase

$$y_{ink} = w_{0k} + \sum_{j=1}^{p} Z_j w_{jk}$$

$$y_k = f(y_{ink})$$

✓ applied to multilayer feedforward architecture.

✓ continuous activation fn → sigmoidal

✓ works in two phase → feed forward, → back propogation

* unlike madaline, both wts i.e. i/p to hidden & hidden to o/p are adjustable.

Back Propagation Phase:

Error Correction Term

$$\delta_k = (t_k - y_k) f'(y_{ink})$$

$$\Delta w_{jk} = \alpha \cdot \delta_k \cdot Z_j$$

$$\Delta w_{ok} = \alpha \, \delta_k$$

$$\delta_{inj} = \sum_{k=1}^{m} \delta_k \cdot w_{jk}$$

$$\delta_j = (\delta_{inj}) f'(z_{inj})$$

derivative of activation fn.

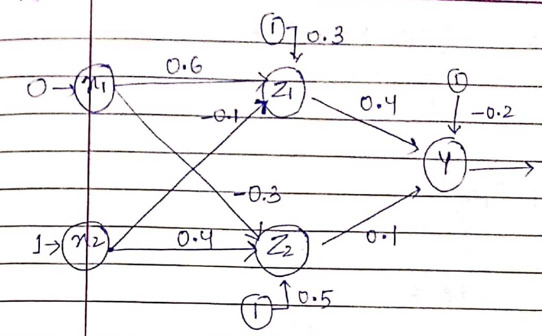
Error from o/p & hidden layer is backpropagated to hidden & i/p layer.

$$\Delta v_{ij} = \alpha \cdot \delta_j \cdot x_i$$

$$\Delta v_{oj} = \alpha \cdot \delta_j$$

all new wts w & v are calculated i.e. updated at the end at the same time.

Que.



For the given network, apply backpropagation algo to calculate the change in wt from i/p to hidden layer & from hidden layer to o/p layer.

$$\alpha = 0.25, \quad t = 1$$

$$f(x) = \frac{1}{1+e^{-\lambda x}} \; ; \; \lambda = 1$$

$$f'(x) = f(x)\,[1 - f(x)]$$

x_1	x_2	x_0	t
0	1	1	1

[0, 1, 1] : 1

$$z_{in1} = 0(0.6) + 1(-0.1) + 0.3$$
$$= -0.1 + 0.3$$
$$= 0.2$$

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$$z_{2in} = 0(-0.3) + 0.4(1) + 0.5$$
$$= 0.4 + 0.5$$
$$= 0.9$$

$$z_1 = f(z_{1in})$$
$$= \frac{1}{1+e^{-z_{1in}}}$$
$$= \frac{1}{1+e^{-0.2}} = 0.55$$

$$z_2 = f(z_{2in})$$
$$= \frac{1}{1+e^{-0.9}} = 0.71$$

$$y_{in} = 0.55(0.4) + 0.71(0.1) - 0.2$$
$$= 0.22 + 0.071 - 0.2$$
$$= 0.02 + 0.071$$
$$= 0.091$$

$$y = \frac{1}{1+e^{-0.091}} = 0.52$$

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Error - Correction

$$\delta_k = (t_k - y_k) f'(y_{ink})$$

$$\delta = (1 - 0.52)(0.25) = 0.48(0.25)$$
$$= 0.12$$

$$f'(y_{ink}) = f(y_{ink})[1 - f(y_{ink})]$$
$$= 0.52[1 - 0.52]$$
$$= 0.52(0.48)$$
$$= 0.25$$

$$\boxed{\delta_k = 0.12}$$

$$\Delta w_{11} = \alpha \cdot (0.12)(z_1)$$
$$= 0.25(0.12)(0.55)$$
$$= \cancel{0.016}\ 0.0165$$

$$\Delta b = 0.25(0.12)$$
$$= 0.03$$

$$\Delta w_{21} = 0.25(0.12)(0.71)$$
$$= 0.0213$$

$$\cancel{\delta_{inj} = 0.0165(0.12) + 0.0213(0.12)}$$
$$\cancel{= 0.12(0.0165 + 0.0213)}$$
$$\cancel{= 0.0045}$$

$\delta_{in1} = \cancel{0.0165\,(0.1)}$

$\delta_k w_1 = 0.12\,(0.4)$

$= \underline{\underline{0.048}}$

$\delta_{in2} = \delta_k w_2 = 0.12\,(0.1)$

$= \underline{\underline{0.012}}$

$\delta_1 = \delta_{in1}\, f'(0.2)$

$= 0.048\left[f(0.2)\left(1 - f(0.2)\right)\right]$

$= 0.048\left[0.55\,(1 - 0.55)\right]$

$= \underline{\underline{0.0118}}$

$\delta_2 = 0.012\left[0.71\,(1 - 0.71)\right]$

$= \underline{\underline{0.00247}}$

$\Delta v_{11} = \alpha\, \delta_1 (x_1) = 0.25\,(0.0118)(0)$

$= 0$

$\Delta v_{12} = \alpha\, \delta_2 (x_1) = 0$

$\Delta v_{21} = 0.25\,(0.0118)(1) = \underline{\underline{0.003}}$

$\Delta v_{22} = 0.25\,(\cancel{0.0118})(1) = \underline{\underline{0.000617}}$

0.00247

$\Delta v_{01} = 0.25\,(0.0118) = 0.003$

$\Delta v_{02} = 0.25\,(0.00247) = 0.000617$

Updated Weights :-

$v_{11} = 0.6 + 0 = 0.6$

$v_{12} = -0.3 + 0 = -0.3$

$v_{21} = -0.1 + 0.003 = -0.097$

$v_{22} = 0.4 + 0.0006 = 0.4006$

$v_{01} = 0.3 + 0.003 = 0.303$

$v_{02} = 0.5 + 0.0006 = 0.5006$

$w_{11} = 0.4 + 0.0165 = 0.4165$

$w_{21} = 0.1 + 0.0213 = 0.1213$

$w_0 = -0.2 + 0.03 = \underline{-0.17}$

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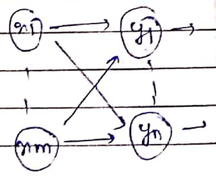
Associative Memories

two phases
1. Store → [w] (we need to find wt.)
   ↓ ← x.
2. Recall → y
   we need to find out o/p

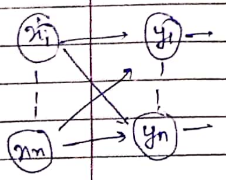
Two types of memories :- (based on structure)

Autoassociative Memories
↓
(Same i/p & o/p) Exactly same.
⇒ no. of i/p's = no. of o/p's.
→ No self loop.
∴ Diagonal elements in a matrix are zero.

Heteroassociative Memories.
↓
— no. of i/p's ≠ no. of o/p's.
→ nature of i/p is diff^n from nature of o/p.



Uses :-
Ram And Rom In computers.



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On addressing mode :-

Memories

Content addressable Memory (CAM)
Eg. → (naming in Phonebook)

Address addressable Memory (AAM)
Eg. → Connecting via address.
Example → Ptr.

• Store Phase

Calculating Values Of Wt

1. Hebb Rule

$W_{new} = W_{old} + \Delta w$

$\Delta w = \alpha \cdot x \cdot y$

↓ Here, we always assume. * $\alpha = 1$

⇒ $\boxed{\Delta w = xy}$ [updates wt. individually]

2. Outer Product Rule

s t → o/p pattern.
↓ store pattern

[updates wt. in matrix form]

$\boxed{W = s^T t}$ → o/p pattern matrix
↓
transpose of store matrix

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• RECALL PHASE

activation f^n used strictly in auto associative memory,

$$Y = \begin{cases} 1 & ; \; y_{in} \geq 0 \\ -1 & ; \; y_{in} < 0 \end{cases}$$

In case of Heteroassociative memory

if, binary i/p's are there i.e. 0 and 1 then,

$$Y = \begin{cases} 1 & ; \; y_{in} > 0 \\ -1 & ; \; y_{in} \leq 0 \end{cases}$$

else,

$$Y = \begin{cases} 1 & ; \; y_{in} > 0 \\ 0 & ; \; y_{in} = 0 \\ -1 & ; \; y_{in} < 0 \end{cases}$$

Que Train autoassociative network for the i/p pattern,

$[-1 \;\; 1 \;\; 1 \;\; 1]$

and test network,

a) for same i/p vector

b) with one missing entry

c) with one mistaken entry

d) two missing entries

e) two mistaken entries

* If, it is not mentioned about self loop, then do not make diagonal matrix $=0$ IMP.

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Ans: i/p & o/p are same.

Using Outer product Rule.

$$W = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}_{4\times1} [-1 \;\; 1 \;\; 1 \;\; 1]_{1\times4}$$

$$W = \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix}_{4\times4}$$

(a) $i/p = [-1 \;\; 1 \;\; 1 \;\; 1]$

$$= [-1 \;\; 1 \;\; 1 \;\; 1]_{1\times4} \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix}_{4\times4}$$

$$= \begin{bmatrix} -1-1-1-1 \\ 1+1+1+1 \\ 1+1+1+1 \\ 1+1+1+1 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ 4 \\ 4 \end{bmatrix} = [-4 \;\; 4 \;\; 4 \;\; 4]$$

$$= [-1 \;\; 1 \;\; 1 \;\; 1]$$

(same o/p)

$\Rightarrow$ training is correct

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Missing

one missing entry.
⇒ ~~if bipolar i/p's~~
(a) ⇒ we can't take bipolar
⇒ we'll take binary i.e. 0

Mistaken

any one bit with its wrong entry
i.e. if bipolar i/p's
then replace 1 with −1 or
−1 with 1

** Important

original → [−1 1 1 1]

(b) 1st bit = 0

[0 1 1 1]

$$y_{in} = [0\ 1\ 1\ 1] \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 3 & 3 & 3 \end{bmatrix}$$

$$y = [-1\ 1\ 1\ 1]$$

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(c) 1st bit = 1

[1 1 1 1]

$$y_{in} = [1\ 1\ 1\ 1] \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix}$$

$$= [-2\ 2\ 2\ 2]$$

$$y = [-1\ 1\ 1\ 1]$$

(d) 1st = 0 | last = 0

[0 1 1 0]

$$y_{in} = [0\ 1\ 1\ 0] \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix}$$

$$= [-2\ 2\ 2\ 2]$$

$$y = [-1\ 1\ 1\ 1]$$

(e) 1st = 1, last = −1

[1 1 1 −1]

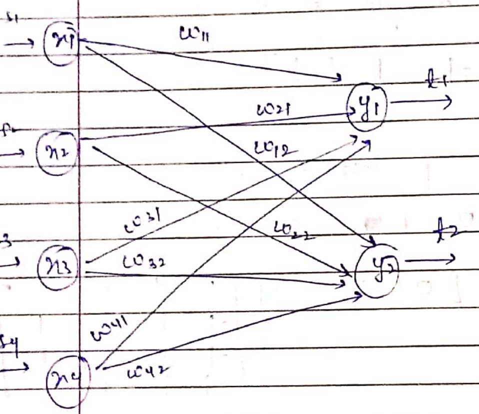
$$\cancel{y_{in} = [1\ 1\ 1\ -1]} \begin{bmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix}$$

$$y_{in} = [0\ 0\ 0\ 0] \quad y = [-1\ -1\ -1\ -1]$$

* Mid Sem - till, Hetero associative Memory

Q. Train and test the following network using hetero associative memory.

s_1	s_2	s_3	s_4	t_1	t_2
1	0	0	0	0	1
1	1	0	0	0	1
0	0	0	1	1	0
0	0	1	1	1	0



(i) (store phase)

$$W = \begin{bmatrix}1&1&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&1\end{bmatrix}_{4\times4}^{T}\begin{bmatrix}0&1\\0&1\\1&0\\1&0\end{bmatrix}_{4\times2}$$

$$W = \begin{bmatrix}0&2\\0&1\\1&0\\2&0\end{bmatrix}_{4\times2} = \begin{bmatrix}w_{11}&w_{12}\\w_{21}&w_{22}\\w_{31}&w_{32}\\w_{41}&w_{42}\end{bmatrix}$$

$$\cancel{W_1 = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}_{4\times1}\begin{bmatrix}0&1\end{bmatrix}_{1\times2} = \begin{bmatrix}\ \end{bmatrix}_{4\times2}}$$

Method → corresponding transpose i/p matrix, multiplied with o/p matrix & then sum all of them to get final matrix.

* Alternate

Acc. to book * for Binary

$$y = \begin{cases}1 & y_{in} > 0\\ 0 & y_{in} \le 0\end{cases}$$

Now,

(I). $y_{in} = \begin{bmatrix}1&0&0&0\end{bmatrix}_{1\times4}\begin{bmatrix}0&2\\0&1\\1&0\\2&0\end{bmatrix}_{4\times2}$

$y_{in} = \begin{bmatrix}0&2\end{bmatrix}_{1\times2}$

$Y = \begin{bmatrix}0&1\end{bmatrix}$

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(2). $y_{in} = [1\ 1\ 0\ 0] \begin{bmatrix} 0 & 2 \\ 0 & 1 \\ 1 & 0 \\ 2 & 0 \end{bmatrix}$

$= [0 \quad 3]$

$y = [0 \quad 1]$ ✓

(3). $y_{in} = [0\ 0\ 0\ 1] \begin{bmatrix} 0 & 2 \\ 0 & 1 \\ 1 & 0 \\ 2 & 0 \end{bmatrix}$

$y_{in} = [2 \quad 0]$

$y = [1 \quad 0]$

(4). $y_{in} = [0\ 0\ 1\ 1] \begin{bmatrix} 0 & 2 \\ 0 & 1 \\ 1 & 0 \\ 2 & 0 \end{bmatrix}$

$y_{in} = [3 \quad 0]$

$y = [1 \quad 0]$ ✓

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(b)

** IMP

For the above mentioned problem, test the heteroassociative memory with similar & unsimilar test vectors.

(more than one bit say (2 bits)) ✓

changing one bit only, like mistaken. (in each i/p ~~test~~ set)

A. Similar (1st bit)

s_1	s_2	s_3	s_4	t_1	t_2
0	0	0	0	0	1
0	1	0	0	0	1
1	0	0	1	1	0
1	0	1	1	1	0

$$y_{in} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 0 & 1 \\ 1 & 0 \\ 2 & 0 \end{bmatrix}$$

$$y_{in} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 2 & 2 \\ 3 & 2 \end{bmatrix} \Rightarrow y = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{array}{l} \text{y not} \\ \checkmark \text{correct} \\ \text{y not} \\ \text{y not} \end{array}$$

unsimilar (2nd & 3rd bit)

$$y_{in} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 0 & 1 \\ 1 & 0 \\ 2 & 0 \end{bmatrix}$$

$$y_{in} = \begin{bmatrix} 1 & 3 \\ 1 & 2 \\ 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$y = \begin{array}{c|cc} & t_1 & t_2 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \\ 4 & 1 & 1 \end{array}$$

not matching in all training pair

s_1	s_2	s_3	s_4	t_1	t_2
1	-1	-1	-1	-1	1
1	1	-1	-1	-1	1
-1	-1	-1	1	1	-1
-1	-1	1	1	1	-1

Test
(i) for same data
(ii) Test with missing data.
(iii) Test with mistaken data.

$$W = \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}_{4\times4} \begin{bmatrix} -1 & 1 \\ -1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}_{4\times2}$$

$$W = \begin{bmatrix} -4 & 4 \\ -2 & 2 \\ 2 & -2 \\ 4 & -4 \end{bmatrix}_{4\times2}$$

$1+\cancel{1}-\cancel{1}+1$
$-1-1+\cancel{1}-\cancel{1}$
$1+1+1+1$
$-1-1-1-1$

(i)

$$y_{in} = \begin{bmatrix} 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -4 & 4 \\ -2 & 2 \\ 2 & -2 \\ 4 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 8 \\ -12 & 12 \\ 8 & -8 \\ 12 & -12 \end{bmatrix}$$

$-4+\cancel{2}-\cancel{2}-4$
$4-\cancel{2}+\cancel{2}+4$
$-4-2-2-4$
$4+2+2+4$

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Using activation fn

$$y = \begin{bmatrix} -1 & 1 \\ -1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$$ ✓

(ii) 1st bit → 0

$$y_{in} = \begin{bmatrix} 0 & -1 & -1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -4 & 4 \\ -2 & 2 \\ 2 & -2 \\ 4 & -4 \end{bmatrix}$$

$$y_{in} = \begin{bmatrix} -4 & 4 \\ -8 & 8 \\ 4 & -4 \\ 8 & -8 \end{bmatrix}$$

2 −2 −4
−2 +2 +4
−2 −2 −4
−4−4=−8
2 +2 +4
2 −2 +4   −2 +2 −4
2 +2 +4   −2 −2 −4

$$y = \begin{bmatrix} -1 & 1 \\ -1 & 1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$$ ✓ Correct.

(iii) 2nd bit mistaken.

✓ if not mentioned then by default, only one bit

$$y_{in} = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & 1 \\ -1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -4 & 4 \\ -2 & 2 \\ 2 & -2 \\ 4 & -4 \end{bmatrix}$$

$$y_{in} = \begin{bmatrix} -12 & 12 \\ -8 & 8 \\ 4 & -4 \\ 8 & 0 \end{bmatrix}$$

−4 −2 −2 −4
4 +2 +2 +4
−4 +2 −2 −4
4 −2 +2 +4
4 −2 −2 +4
2 +2 = 4
−4 +2 +2 −4
−2 −2
4 −2 +2 +4
−4 +2 −2 −4

$$y = \begin{bmatrix} -1 & 1 \\ -1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix}$$

→ not matching.

Syllabus Midterm

- Ch 1 — 1.1, 1.2, 1.6 (Flowchart also)
- Ch 2 —
- ✓ Ch 3 — (3.1, 3.2, 3.5, 3.3, 3.4) (Algo, Short-notes.)
- Ch 4 — (4.1 to 4.4)

(draw architecture. write activation fn.)