

Chapter 18 : Monad : Applicative functors

*) Monads can be thought of as another way of applying functions over structure, with a couple of additional features.

*) defn: class Applicative m \Rightarrow monad m where

$(\gg=) :: ma \rightarrow (a \rightarrow mb) \rightarrow mb$

$(\gg) :: ma \rightarrow mb \rightarrow mb$

$\text{return} :: a \rightarrow ma$

*) Applicative m: monad is stronger than Applicative functor, and we can derive Applicative functor in terms of monad. just as we can derive functor in terms of Applicative.

↓

This means that we can write fmap using monadic operations.

↓

$a \rightarrow b$

$\text{fmap } f \times s = xs \gg= \text{return} . f$ $[+]$

↓ example

$\gg \text{fmap } (+1) [1..3]$

$[2, 3, 4]$

$a \rightarrow ma$

$\gg [1..3] \gg= \text{return} . (+1)$
 $[2, 3, 4]$

|

↓

This happens to be a law. It is important to understand this chain of dependency:

functor \rightarrow Applicative \rightarrow monad.

i.e. when we implement an instance of monad for a type, we necessarily have an applicative and a functor as well.

* Core operations : three core ops, although only ($>>=$) needs to be defined for a minimally complete monad instance.

($>>=$) :: $ma \rightarrow (a \rightarrow mb) \rightarrow mb$

($>>$) :: $ma \rightarrow mb \rightarrow mb$

return :: $a \rightarrow ma$: same as pure . Takes a return . (+1) value & return it inside a structure, whether the structure is list

↓ Just or IO

$>>$: Mr. Pointy (Some \rightarrow sequencing operator)

sequences two actions while discarding any resulting value of the first action.

↓

$>>=$: bind operator , makes the monad special .

(*) The novel part of monad : Conventionally when we use monads , we use the bind fn. Sometimes directly , and sometimes indirectly via do syntax.

\downarrow

$\langle \$ \rangle :: \text{functor } F$

$\Rightarrow (a \rightarrow b) \rightarrow Fa \rightarrow Fb$

$\langle * \rangle :: \text{Applicative } F$

$\Rightarrow F(a \rightarrow b) \rightarrow Fa \rightarrow Fb$

$\gg= :: \text{Monad } F$

$\Rightarrow Fa \rightarrow (a \rightarrow Fb) \rightarrow Fb$

\downarrow

The idea of mapping a fn. over a value while bypassing its surrounding structure is not unique to Monad.

\downarrow

Can be demonstrated by fmapping a fn. of type

$(a \rightarrow mb)$, to make it more like $\gg=$.

\downarrow

$fmap :: \text{functor } F$

$\Rightarrow (a \rightarrow Fb) \rightarrow Fa \rightarrow F(Fb)$

$\downarrow \text{example}$

$\gg \text{let andOne } x = [x, 1]$

$\gg \text{andOne } 10$

$[10, 1]$

$\gg :t \text{ fmap andOne } [4, 5, 6]$

$\text{fmap andOne } [4, 5, 6] :: \text{num t} \Rightarrow [[t]]$

$\gg \text{ fmap andOne } [4, 5, 6]$

$[[4, 1], [5, 1], [6, 1]]$

\downarrow

was clear from the type signature that we would reach

here , i.e. is an extra layer of structure . Now what if want to remove the extra layer of structure or to discard it in favour of one layer of that structure .



can be done for lists using concat .



concat & fmap and one [4, 5, 6]

[4, 1, 5, 1, 6, 1]



a less general type on concat :

concat :: [[a]] → [a]



Monad in a sense is a generalisation of concat . The unique part of Monad is the following func .

import Control.Monad (join)

join :: Monad m ⇒ m (ma) → ma



We can inject more structure with the std . fmap if needed , however the ability to flatten those two layers of structure into one , makes Monad special . And it's by putting join + mapping fn. together that we get the bind , also known as >> =

→ Exercise: bind in terms of fmap & join

bind :: Monad m \Rightarrow (a \rightarrow mb) \rightarrow ma \rightarrow mb

bind g x = join \$ fmap g x

fmap :: (a \rightarrow b) \rightarrow fa \rightarrow fb.

(a \rightarrow fb) \rightarrow fa \rightarrow f(fb)

join :: f(fb) \rightarrow f.b

*> What monad is not?

a) Impure. Monadic functions are pure functions. IO is an abstract datatype, that allows for impure, or effectful, actions and it has a monad instance. But there's nothing impure about Monads.

b) An embedded lang. for imperative programming

↓

While monads are used for sequencing actions in a way that looks like imperative programming, there are also commutative monads that do not order actions.

c) A value. The type class describes a specific relationship between elements in a domain and define operations over them, same way we refer to monoids, functors or applicatives for that manner.

d) About strictness: monadic ops of bind and return

are nonstrict. Some ops can be made strict within a specific instance.

- *) Monad also lifts : includes some lift fns that are same as the ones we already seen in Applicative, only present for compatibility reasons, and were used before Applicative were discovered.

↓

liftA :: Applicative f

$$\Rightarrow (a \rightarrow b) \rightarrow f a \rightarrow f b$$

lift m :: monad m

$$\Rightarrow (a l \rightarrow s) \rightarrow m a l \rightarrow m r$$

↓

similarly for liftA₂ & liftm₂, same for liftA₃ & liftm₃.

↓

zipWith is liftA₂ or liftm₂ specialized to lists.

↓

>> : t zipWith

zipWith :: (a → b → c)

$$\rightarrow [a] \rightarrow [b] \rightarrow [c]$$

>> zipWith (+) [3, 4] [5, 6]

[8, 10]

>> liftA₂ (+) [3, 4] [5, 6]

[8, 9, 9, 10]

Different results,
although same types,
is because of the
different monoids
being used.

→ Do syntax and monads: do syntax as introduced in modules chapter was being used in the context of IO as syntactic sugar that allowed us to easily sequence actions by feeding the result of one action as the i/p value to the next.

↓

(*>) :: Applicative $f \Rightarrow fa \rightarrow fb \rightarrow fb$

(>>) :: monad $m \Rightarrow ma \rightarrow mb \rightarrow mb$

(should in all cases do the same thing, just the underlying constraints are different)

↓

>> putStrLn "Hello, " >> putStrLn "World!"

Hello,

World!

↓ do degugared using (>> & *>)

sequencing :: IO()

sequencing = do

putStrLn "blah"

putStrLn "another-thing"

sequencing' :: IO()

sequencing' =

putStrLn "blah" ss

putStrLn "ano"

↓ variable binding ex

binding :: IO()

binding = do

binds it to
a variable

← name ← getLine
putStrLn name

binding' :: IO()

binding' = do

getLine >>= putStrLn

passes it directly to
putStrLn (tn).

*) when fmap alone isn't enough : fmap putStrLn over getLine won't do anything.

putStrLn < \$ > getLine

↓

This does look for an input, but does not print it upon entering. i.e. putStrLn doesn't successfully map over getLine.

↓

» : t putStrLn < \$ > getLine

putStrLn < \$ > getLine :: $\text{IO}(\text{IO}())$

↓

Now getLine :: IO String, whereas

putStrLn :: String \rightarrow IO()

↓

in terms of fmap:

$\langle \$ \rangle :: \text{Functor } f \Rightarrow (a \rightarrow b) \rightarrow fa \rightarrow fb$

now, putStrLn :: String \rightarrow IO()

$(a \rightarrow fb) \rightarrow fa \rightarrow f(fb)$

$a \rightarrow b$
↓

gets specialized to IO()

↓
This situation is similar to the situation when we used fmap to map a fn. with type $a \rightarrow mb$ instead of $a \rightarrow b$.

↓

$(\text{String} \rightarrow \text{IO}()) \rightarrow \text{IO String} \rightarrow \text{IO}(\text{IO}())$

effects that would be performed if putStrLn

represents the effects
getline must perform to
get us a string that the
user typed in.

↓
was evaluated
The unit that
putStrLn
returns.

- * we are able to evaluate I/O actions multiple times.
- * we can refer to, compose & map over effectful computations.
without performing them. example:

```
>> let printOne = putStrLn "1"  
>> let printTwo = putStrLn "2"  
>> let twoActions = (printOne, printTwo)  
>> :t twoActions  
twoActions :: (IO(), IO())
```

- * To solve the issue with getline & putStrLn, we can use join
join \$ putStrLn <\$> getline.

↓
monadic actions are pure, and the sequencing
operations we use here are ways of nesting lambdas.

Sometimes it is valuable to suspend or otherwise perform
an I/O action until some determination is made, so
types like $\text{IO } (\text{IO}())$ aren't necessarily invalid.

- * Back to desugaring do syntax

binding And sequencing :: IO()

binding And sequencing = do

putStrLn "name pls:"

String (name) ← getLine → IO String

putStrLn ("y halo thar :" ++ name)

part of
do
syntax

binding And Sequencing' :: IO()

binding And Sequencing' =

putStrLn "name pls:" >> sequencing

getLine >>

\name → putStrLn ("y halo thar :" ++ name)

won't be able to
use `<`

↓
do avoid

*) As the nesting intensifies , do syntax can make things a bit cleaner and easier to read.

→ Examples of Monad use

*) List

>> = :: Monad m

⇒ m a → (a → mb) → mb

>>= :: [a] → (a → [b]) → [b]

return :: a → [a]

↓

Can generate more list (or an empty list) inside of our mapped fn, examples.

↓

twice when even : [Integer] → [Integer]

twice when even = $\lambda xs \dots$

$x \leftarrow xs$ → binds individual values out of the list

if even x $\quad \quad \quad$ if p., like for list

then $[x^*x, x^*x]$ comprehension

else $[x^*x]$

*) Maybe monad

($\gg=$) :: Maybe a → (a → Maybe b) → Maybe b

(return) :: a → Maybe a

↓

data Cow = Cow {

 name :: String

 , age :: Int

 , weight :: Int

} deriving (Eq, Show)

noEmpty :: String → MaybeString

nonnegative :: Int → MaybeInt

weightcheck :: Cow → Maybe Cow

mksphericalCow :: String → Int → Int → Maybe Cow

mksphericalCow name' age' weight' =

case noEmpty name' of

 Nothing → Nothing

 Just nammy →

 case nonnegative age' of

 Nothing → Nothing

 Just aggy →

 case nonnegative weight' of

nothing → nothing

Just weighty →

weight check (cow

nammy aggy weighty)

↓ cleaning up using 'do'

mkspherical cow name 'age' weight' = do

nammy ← noEmpty name '

aggy ← noNegative age '

weight ← noNegative weight '

weightcheck (cow nammy aggy weight)

↓ >> =

mkspherical cow' name' age' weight' =

'noEmpty name' >> = produces maybe String but
`nammy → nammy gets bound to string.

'noNegative age' >> =

'aggy →

mx >> = k = kx

'noNegative weight' >> =

'weighty →

weightcheck (cow nammy aggy weighty)

↓

we can't do the above using Applicative . because our weightcheck fn . depends on the prior existence of a cow value and returns more monadic structure in its return type Maybe Cow . The prior existence of cow is also dependent on other arguments to the fn , the sequential processing

doSomething = do

$a \leftarrow f$

$b \leftarrow g$

$c \leftarrow h$

Pure(a, b, c)

of which yields a w.w.

→ This can be rewritten using
Applicative

$f(a \rightarrow b) \rightarrow fa \rightarrow fb$

↓ but

doSomething = do

$a \leftarrow fn$

$b \leftarrow ga$

$c \leftarrow hb$

Pure(a, b, c)



g and h are
producing monadic
structure that
can only be obtained
by depending on values
generated from monadic
structure.

$f :: \text{Integer} \rightarrow \text{Maybe Integer}$

$f 0 = \text{Nothing}$

$f n = \text{Just } n$

$g :: \text{Integer} \rightarrow \text{Maybe Integer}$

$g i$

| even $i = \text{Just } (i+1)$

| otherwise = Nothing

$h :: \text{Integer} \rightarrow \text{Maybe String}$

$h i = \text{Just } ("10191" ++ show i)$

do something 'n = do

```

a ← fn
b ← ga
c ← hb
pure (a, b, c)

```

- *) with the maybe applicative , each Maybe computation fails or succeeds independently of each other. Lifting funcs. that are also Just or Nothing over Maybe values.
- *) with the maybe monad , computations contributing to the final computation can choose to return nothing based on previous computations .
- *) On providing a Nothing value on mkSphericalCow' , we get Nothing straightaway due to the following defn :

func. arg transfer

$$\text{Just } x \gg= k = k(x) \rightarrow \begin{array}{l} \text{when provided we move on,} \\ \text{else} \end{array}$$

$$\text{Nothing } \gg= - = \text{Nothing} \leftarrow$$

- *) Nothing here even gets used as indicated above and this can be demonstrated using :

```

>> Nothing >>= undefined
Nothing

```

- *) we use Maybe monad & applicative because they make the computation convenient to code , and more readable as well .

→ Either :

$(>>=)$:: monad m

$\Rightarrow ma$

$\rightarrow (a \rightarrow mb)$

$\rightarrow mb$

$(>>=)$:: Either e a

$\rightarrow (a \rightarrow \text{Either } e b)$

$\rightarrow \text{Either } e b$

return :: Monad m $\Rightarrow a \rightarrow ma$

return ::
 $a \rightarrow \begin{cases} \text{Either } e \\ \downarrow \end{cases} a$

part of the structure

module EitherMonad where

type founded = Int

type coders = Int

data SoftwareShop =

Shop {

 founded :: founded

 `Programmers :: coders

} deriving (Eq, Show)

data foundedError =

 NegativeYears founded

 | TooManyYears founded

 | NegativeCoders coders

 type of associated error

| TooManycoders Coders
| TooManycoders for years founded coders
deriving (Eq, Show)

validateFounded :: Int → Either FoundedError Founded

validateCoders :: Int → Either FoundedError Coders

mkSoftware :: Int → Int → Either FoundedErrors Coders

mkSoftware years coders = do

 founded ← validateFounded years.

 programmers ← validateCoders coders

 if programmers > div founded 10

 then Left \$

 TooManycoders for years founded programmers

 else Right \$ Shop founded programmers

*) so there is no monad for validation. Applicative + monad instances must have the same behaviour . expressed in the form :

import Control.Monad (ap)

(<*>) == ap (way of saying that the Applicative apply for a type must not change behaviour if derived from the Monad instance's bind operation)

<*> :: Applicative F

⇒ f(a → b) → fa → fb

$\text{ap} :: \text{Monad } m$

$\Rightarrow m(a \rightarrow b) \rightarrow ma \rightarrow mb$

↓

Deriving applicative from the stronger instance :

$\text{ap} :: (\text{monad } m) \Rightarrow m(a \rightarrow b) \rightarrow ma \rightarrow mb$

$\text{ap } m \ m' = \text{do}$

$x \leftarrow m$

$x' \leftarrow m'$

$\text{return } (x \ x')$

↓

We can't have a Monad instance of validation that accumulates errors like the Applicative does. Instead any Monad instance for validation would be identical to Either's Monad instance.

Exercise : Either Monad

$\text{data Sum } a \ b =$

 first a

$f(a \rightarrow b) \rightarrow fa \rightarrow fb$

 | second b

 der ()

instance functor (Sum a) where

$\text{fmap } f \ (\text{second } b) = \text{second } (fb)$

$\text{fmap } - \ (\text{first } a) = \text{first } a$

instance Applicative (Suma) where

pure b = Second b.

(Second f) \star (Second b) = Second (f b)

- \star (first a) = first \tilde{a}

(first a) \star — = first \tilde{a} → won't be applicative values or functions, would be monoidal values.

instance Monad (Suma) where

return = pure

Second b \gg = f = f b

first a \gg = — = first a

→ Monad laws :

*) Identity laws → i) $m \gg \text{return} = m$ (right identity)
ii) $\text{return } x \gg f = fx$ (left identity)

like pure, return shouldn't change any of the behaviour of the rest of the fn., it is only there to put things into structure when we need to, and the existence of the structure should not affect the computation.

*) Associativity laws → The grouping of the fns. should not have any impact on the final result.

$(m \gg f) \gg g = m \gg (\lambda x \rightarrow fx \gg g)$

↓

Keeping in mind that " \gg " allows the result value of

One fn. to be passed as i/p to the next , when we reassociate the fn. , we need to apply f so that g has an i/p value of type ma to start the whole thing off .

→ Application & Composition

*) with monad the composition looks less neat at first .

$mcomp :: \text{Monad } m$

$\Rightarrow (b \rightarrow mc)$

$\rightarrow (a \rightarrow mb)$

$\rightarrow (a \rightarrow mc)$

$mcomp f g a = f(g a) \rightarrow$ will give an error , couldn't match b with mb .

↓

f expects b , but g is passing a type of mb to f .

$m(mb) \rightarrow mb$

↖

Applying a fn. in presence of some context , structure :

$fmap$ -

↓

that will further give us $m(mc)$ at the end

↑

to resolve that we use $\text{join} (m(mc) \rightarrow mc)$

↓

$mcomp f g a = \text{join} (f <\$> (g a))$

↓ Joining fmap & join & using $>>=$

`mcomp f g a = (g a) >>= f`

* nothing special needs to be written to make monadic functions compose (as long as the monadic contexts are the same Monad)

↓

Kleisli composition

↓

`(.) :: (b → c) → (a → b) → a → c`

`(>>=) :: Monad m`

$\Rightarrow ma \rightarrow (a \rightarrow mb) \rightarrow mb$

↓

kicking Kleisli composition off the ground, flip some arguments around to make types work:

↓

`(>=>) :: Monad m`

Kleisli comp. op. $\Rightarrow (a \rightarrow mb) \rightarrow (b \rightarrow mc) \rightarrow a \rightarrow mc$

↓

function composition with monadic structure hanging off the fns we're composing.

↓

`import Control.Monad ((>=>))`

SayHi :: String → IO String

SayHi greeting = do
putStrLn greeting

getline

readM :: Read a \Rightarrow String \rightarrow IO a

readM = return. read \rightarrow provides monadic structure

getAge :: String \rightarrow IO Int

after being bound over
the output of sayHi

getAge = sayHi ($>= >$) readM

askForAge :: IO Int

askForAge = getAge "Age ?"

$(a \rightarrow mb)$
sayHi :: String \rightarrow IO String

The String i/p from the user that sayHi returns.

IO sayHi performs to get i/p.

$(b \rightarrow mc)$
readM :: String \rightarrow IO a

The string that read expects, and which sayHi will produce

return/pure perform no IO values.

$(a \rightarrow mc)$
getAge :: [String] \rightarrow IO a

int that is finally returned on composition

greeting for the user

Combined IO action which performs all effects necessary to produce the final result.