

## Chapter 8 - Recursion

- \* ) Recursion is defining a function in terms of itself via self-referential expressions, i.e. the function continues to call itself and repeat its behaviour until some condition is met to return a result.
- \* ) Being able to write recursive functions is essential for Turing completeness. Lambda calculus does not seem on the surface to have any means of recursion, because of the anonymity of expressions. But, we use a combinator -  $Y$  combinator or fixed-point combinator to write recursive functions in the lambda calculus. Haskell has native recursion ability based on the same principle as the  $Y$  combinator.

→ Factorial

Factorial  $\therefore \text{Integer} \rightarrow \text{Integer}$

factorial  $x$

$$| x \leq 0 = 1$$

$$| \text{otherwise} = (*) \times \text{factorial } (x-1)$$

→ Recursion can be seen as a special case of function composition, where rather than passing the result of the first fn. to a different fn., we pass it to the same fn., till the base case is hit. Recursion is self-referential composition. We apply a fn. to an argument,

then pass that result on as argument to a second application of the same function & so on.

→ Bottom: term used in Haskell to refer to computations that do not successfully result in a value. The two main varieties of bottom are computations that failed with an error or those that failed to terminate.

\*) maybe datatype (making a partial fn into a total one)

data maybe a = Nothing | Just a → allows us to take an argument and allows us to return the data we're wanting

one way of saying that there is no result or data from the function without hitting bottom

\*) makes all uses of nil values and most uses of bottom unnecessary.

\*) for example

$f :: \text{Bool} \rightarrow \text{Maybe Int}$  → type defn change

$f \text{ False} = \text{Just } 0$  → needs to be wrapped in Just

$f \text{ _} = \text{Nothing}$

data constructor

→ Fibonacci Numbers (returning the x<sup>th</sup> member of the fibonacci series)

$\text{Fibonacci} :: \text{Integer} \rightarrow \text{Integer}$

or

$\text{fibonacci} :: \text{Integral } a \Rightarrow a \rightarrow a$

fibonacci 0 = 0

fibonacci 1 = 1

fibonacci x = fibonacci (x-1) + fibonacci (x-2)

→ Integral division from scratch

type  
synonym  
or  
alias.

type Numerator = Integer

type Denominator = Integer

type Quotient = Integer

something that can  
be done to make the  
code more readable

↑↑

dividedBy :: Numerator → Denominator → Quotient

---

dividedBy :: Integral a ⇒ a → a → (a, a)

dividedBy num denom = go num denom 0

changed the  
type signature  
to make it more  
polymorphic,  
and also to return  
a tuple

where go n d count

| n < d = (count, n)

| otherwise =

go (n-d) d (count+1)

quotient

remainder

allows us to define a function via a  
where-clause that can accept more  
arguments than the top-level function  
dividedBy does.

↓

Here the top-level fn. takes two arguments, num & denom,  
but we need a third argument in order to keep track

of how many times we do subtraction. That argument is count & is defined by a starting value of zero & is incremented by 1 everytime the otherwise case is invoked.

↓

It is not significant that we changed the argument names from num & denom to n and d. The go function has already been applied to them in the definition of dividedBy so that the num, denom & 0 are bound to n, d & count in the where clause.