Seat
No.

[4657]-542

S.E. (E&TC/Electronics) (First Semester) EXAMINATION, 2014

SIGNALS AND SYSTEMS

(2012 PATTERN)

Time : Two Hours   **Maximum Marks : 50**

N.B. :— (*i*)   Attempt four questions as Q. No. **1** *or* Q. No. **2**,

Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7**

or Q. No. **8**.

(*ii*)   Answer any *three* questions from each Section.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*)   Use of calculator is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1.   (*a*)   Perform the following operations on the given signal $x(t)$ which

is defined as :   [4]

$$x(t) = u(t) - u(t - 4)$$

(*i*)   Sketch $z(t) = x(-t - 1)$

(*ii*) Sketch $y(t) = x(t) + z(t)$.

P.T.O.

(b)   Determine whether the following signals are Energy or Power, and find energy or time averaged power of the signal : [4]

(i) $x(t) = 5 \cos(\pi t) + \sin(5\pi t) \; ; \; -\infty \leq t \leq \infty$

(ii) $x[n] = n, \qquad\qquad 0 \leq n < 5$

$\qquad\qquad = 10 - n, \qquad 5 \leq n \leq 10$

$\qquad\qquad = 0, \qquad\qquad\text{otherwise}$

(c)   Determine whether the following system is Static/Dynamic, Causal/Non-causal and Stable/Unstable and justify :   [4]

$$h(t) = e^{-5t} u(t).$$

Or

2.   (a)   Compute the convolution integral by graphical method and sketch the output for the following signals :   [6]

$$x(t) = u(t) - u(t - 2)$$

$$h(t) = e^{-2t} \, u(t)$$

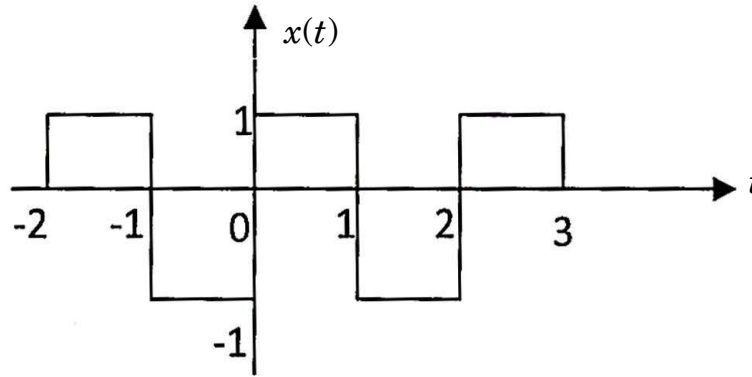(b)   Evaluate the following integrals :   [4]

(i) $\displaystyle\int_{0}^{\infty} t^2 \, \delta(t - 10) \, dt$

(ii) $\displaystyle\int_{0}^{10} \delta(t) \sin(2\pi t) \, dt$.

(c) Determine whether the following signal is periodic or not, if periodic, find the fundamental period of the signal :   [2]

$$x(t) = \cos^2(2\pi t).$$

3. (a) Find the trigonometric Fourier series for the periodic signal $x(t)$ shown in the following figure and sketch the amplitude and phase spectra :   [6]

(b) Find the inverse Laplace transform of :   [6]

$$X(s) = \frac{2}{(s + 4)(s - 1)}.$$

If the Region of convergence is :

(i) $-4 \leq \text{Re}(s) < 1$

(ii) $\text{Re}(s) > 1$

(iii) $\text{Re}(s) < -4.$

4. (*a*) Find the Fourier transform of the following signals : [6]

   (*i*) $x(t) = \text{sng}(t)$

   (*ii*) $x(t) = u(t)$

   (*iii*) $x(t) = e^{-at} \sin(\omega_0 t) u(t)$.

(*b*) Find the initial and final value of the following signal : [4]

$$X(s) = \frac{2s + 3}{s^2 + 5s - 7}.$$

(*c*) State the relationship between Fourier transform and Laplace transform. [2]

SECTION II

5. (*a*) Find the following for the given signal $x(t)$: [6]

   (*i*) Autocorrelation

   (*ii*) Energy from Autocorrelation

   (*iii*) Energy Spectral Density.

$$x(t) = e^{-10t} u(t).$$

(*b*) Determine the cross-correlation between two sequences which are given below : [4]

$$x_1(n) = \{1 \ 2 \ 3 \ 4\}$$
$$x_2(n) = \{3 \ 2 \ 1 \ 0\}.$$

(*c*) State and describe any *three* properties of Power Spectral Density (PSD). [3]

Or

6. (*a*) Prove that autocorrelation function and energy spectral density form Fourier transform pair of each other and verify the same for : [9]

$$x(t) \ = \ e^{-10t}u(t).$$

(*b*) State and describe any *four* properties of Energy Spectral Density (ESD). [4]

7. (*a*) Explain Exponential probability model with respect to its density and distribution function. [4]

(*b*) Two cards are drawn from a 52 card deck successively without replacing the first : [4]

   (*i*) Given the first one is heart, what is the probability that second is also a heart ?

   (*ii*) What is the probability that both cards will be hearts ?

(*c*) A coin is tossed three times. Write the sample space which gives all possible outcomes. A random variable X, which represents the number of heads obtained on any double toss. Draw the mapping of S on to real line. Also find the probabilities of X and plot the C.D.F. [5]

Or

8. (*a*) PDF of a random variable X is : [6]

$$f_x(x) = ke^{-10x}, \qquad x > 0 \text{ and}$$

$$f_x(x) = 0, \qquad x \leq 0.$$

Find :

(*i*) value of k

(*ii*) $P(1 \leq X \leq 2)$

(*iii*) $P(X \geq 3)$.

(*b*) State the properties of Cumulative probability distribution function. [3]

(*c*) Find the mean standard deviation and variance of the uniform random variable. [4]