

Total No. of Questions—8]

[Total No. of Printed Pages—4+1

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**[4957]-1050**

**S.E. (Electronics/E&TC) (Second Semester) EXAMINATION, 2016**  
**ENGINEERING MATHEMATICS-III**  
**(2012 PATTERN)**

**Time : Two Hours**

**Maximum Marks : 50**

- N.B. :—** (i) Neat diagrams must be drawn wherever necessary.  
(ii) Figures to the right indicate full marks.  
(iii) Use of logarithmic tables, electronic pocket calculator is allowed.  
(iv) Assume suitable data, if necessary.

**1. (a) Solve (any two) :** [8]

(i)  $(D^3 + 4D) y = \sin 5x \cos 3x$

(ii)  $(D^2 + 2D + 1)y = \frac{e^{-x}}{x + 2}$

(iii)  $(x + 2)^2 \frac{d^2 y}{dx^2} + 3(x + 2) \frac{dy}{dx} + y = \cos \log (x + 2)$

**(b) Find Fourier sine transform of :** [4]

$$f(x) = \frac{e^{-ax}}{x}$$

*Or*

**2. (a) An e.m.f. 200 V is in series with a 10  $\Omega$  resistor, a 1 henry inductor and 0.02 farad capacitor. At  $t = 0$ ,  $Q = I = 0$ . Find charge  $Q$  and current,  $I$  at any time  $t$ .** [4]

P.T.O.

(b) Solve (any one) : [4]

(i) Find  $z$ -transform of :

$$f(k) = k^2 e^{-3k}, k \geq 0$$

(ii) Find inverse  $z$ -transform of :

$$F(z) = \frac{z^3}{(z-1)(z-2)^2}, |z| > 2$$

(c) Solve : [4]

$$f(k+1) + \frac{1}{2}f(k) = \left(\frac{1}{2}\right)^k, k \geq 0, f(0) = 0$$

3. (a) Using fourth order Runge-Kutta method, solve the differential equation : [4]

$$\frac{dy}{dx} = \sqrt{x+y}$$

with  $y(0) = 1$ , and find  $y(0.2)$  taking  $h = 0.2$ .

(b) Find Lagrange's interpolating polynomial passing through set of points : [4]

$x$	$y$
0	4
1	3
2	6

(c) Find the directional derivative of :

$$\phi = e^{2x} \cos yz$$

at the origin in the direction tangent to the curve  
 $x = a \sin t, y = a \cos t, z = at$  at  $t = \pi/4$ . [4]

Or

4. (a) With usual notations prove (any one) : [4]

(i)  $\nabla \times (\bar{a} \times \bar{r}) = 2\bar{a}$ ,  $\bar{a}$  is constant vector

(ii)  $\nabla^2 \left( \frac{\bar{a} \cdot \bar{b}}{r} \right) = 0$

(b) Show that :

$$\bar{F} = \frac{xi + yj}{x^2 + y^2}$$

is solenoidal as well as irrotational. [4]

(c) Evaluate : [4]

$$\int_0^1 \frac{1}{1+x} dx$$

by Simpson's  $\left(\frac{3}{8}\right)$ th rule.

5. (a) If : [4]

$$\bar{F} = (2x + y^2)\bar{i} + (3y - 4x)\bar{j},$$

then evaluate  $\int_C \bar{F} \cdot d\bar{r}$  around the parabolic curve  $y = x^2$  joining

(0, 0) and (1, 1) [4]

(b) Show that : [4]

$$\iint \frac{\bar{r}}{r^3} \cdot \hat{n} dS = 0$$

(c) Evaluate :

$$\iint (\nabla \times \bar{F}) \cdot \hat{n} dS$$

where 'S' is the curved surface of the paraboloid :

$$x^2 + y^2 = 2z$$

bounded by the plane  $z = 2$ , where : [5]

$$\bar{F} = 3(x - y) \bar{i} + 2xz \bar{j} + xy \bar{k}$$

*Or*

6. (a) Using Green's theorem show that : [4]

$$\int_C \bar{F} \cdot d\bar{r} = 0$$

where :

$$\bar{F} = x\bar{i} + y^2\bar{j},$$

over the first quadrant of the circle :

$$x^2 + y^2 = a^2.$$

(b) If [4]

$$\bar{E} = \nabla\phi \text{ and } \nabla^2\phi = -4p\pi,$$

prove that :

$$\iint_S \bar{E} \cdot d\bar{s} = -4p\pi \iiint_V dv$$

(c) Evaluate : [5]

$$\iint_S \text{curl } \bar{F} \cdot \hat{n} ds$$

for the surface of paraboloid

$$z = 9 - (x^2 + y^2), \text{ where } \bar{F} = (x^2 + y - 4) \bar{i} + 3xy \bar{j} + (2xz + z^2) \bar{k}$$

7. (a) Obtain the analytic function  $f(z)$  such that : [4]

$$\operatorname{Im} \{f(z)\} = \tan^{-1} y / x \text{ and } f(1) = 0$$

- (b) Evaluate : [4]

$$\int_C \frac{z^4}{z - 3i} dz$$

where :

$$C = \{z \mid |z - 2| < 5\}.$$

- (c) Find the bilinear transformation which sends the points  $1, i, -1$  from  $z$ -plane into the points  $i, 0, -i$  of the  $w$ -plane. [5]

*Or*

8. (a) If  $f(z)$  is an analytic function with constant modulus then prove that  $f(z)$  is a constant function. [4]

- (b) Evaluate : [4]

$$\int_C \frac{z dz}{(z - 1)(z - 2)^2},$$

where  $C$  is the circle  $|z - 2| = \frac{1}{2}$ .

- (c) Show that under the transformation  $w = z + \frac{4}{z}$  the circle  $|z| = 3$  is mapped onto the ellipse. [5]