Total No. of Questions : 9]

SEAT No. :

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S.E. (Electronics/E&TC)/(Electronics & Computer Engineering)
ENGINEERING MATHEMATICS-III
(2019 Pattern) (Semester-III) (207005)

Time : 2½ Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Q.1 is compulsory.***
2) ***Attempt Q2 or Q3, Q4 or Q5, Q6 or Q7, Q8 or Q9.***
3) ***Neat diagrams must be drawn wherever necessary.***
4) ***Figures to the right indicate full marks.***
5) ***Use of electronic pocket calculator is allowed.***
6) ***Assume suitable data, if necessary.***
7) ***Write numerical calculations correct upto four decimal places.***

Q1) Write the correct options for the following multiple choice questions. **[2]**

a) For $f(x) = x^2$, $h=2$, second forward difference $\Delta^2 f(x)$ is given by

i) 6 ii) 12

iii) 4 iv) 8

b) Unit vector in the direction normal to the surface $x^2 + y^2 + z^2 = 9$ at (1,2,2) is _______ **[2]**

i) $\frac{1}{3}\left(\bar{i} + 2\bar{j} + 2\bar{k}\right)$ ii) $\frac{1}{3}\left(\bar{i} - 2\bar{j} - 2\bar{k}\right)$

iii) $\frac{1}{3}\left(\bar{i} + \bar{j} + \bar{k}\right)$ iv) $\frac{1}{9}\left(\bar{i} + 2\bar{j} + 2\bar{k}\right)$

c) The value of $\oint_C \frac{4z^2 + z}{(z-1)} dz$ where C is $|z| = 2$ **[2]**

i) 5πi ii) 10πi

iii) –10πi iv) –5πi

d) For $\overline{F} = x^2\bar{i} + xy\bar{j}$ the value of $\oint_C \overline{F}.\overline{dr}$ for the curve $y^2 = x$ joining the points (0,0) and (1,1) is **[2]**

i) $\frac{1}{2}$ ii) $\frac{7}{12}$

iii) $\frac{5}{12}$ iv) $\frac{2}{3}$

e) The Cauchy integral formula for analytic function $f(z)$ is **[1]**

i) $\oint_c \frac{f(z)}{(z-a)}dz$ ii) $\oint_c \frac{f(z)}{(z+a)}dz$

iii) $\oint_c \frac{f(z)}{(z-a)^2}dz$ iv) $\frac{1}{2\pi i}\oint_c \frac{f(z)}{(z-a)}dz$

f) Given equation is $\frac{dy}{dx} = f(x,y)$ with initial condition $x = x_0$, $y = y_0$ and h is step size. Euler's formula to calculate y_1 at $x = x_0 + h$, is given by **[1]**

i) $y_1 = y_0 + h\,f(x_0, y_0)$ ii) $y_1 = y_0 + hf(x_1, y_1)$

iii) $y_1 = y_1 + h\,f(x_0, y_0)$ iv) $y_1 = hf(x_0, y_0)$

Q2) a) Find value of y for x=0.5 using newton's forward difference formula for following data **[5]**

x	0	1	2	3	4
y	1	5	25	100	250

b) By using simpson's $\left(\frac{3}{8}\right)^{th}$ rule, find the value of $\int_1^7 f(t)dt$ for following data **[5]**

t	1	2	3	4	5	6	7
f(t)	81	75	80	83	78	70	60

c) Given $\frac{dy}{dx} = x^2 + y, y(0) = 1$. Determine y(0.02) by using modified Euler's method, take h=0.02 **[5]**

OR

Q3) a) Find longrange's interpolation polynomial for following data. **[5]**

x	0	1	2
y	7	–1	–7

b) By trapezoidal Rule, find the value of $\int_0^1 \frac{1}{1+x^2} dx$ by taking h=0.25 **[5]**

c) Use Runge-Kutta method of fourth order to solve $\frac{dy}{dx} = x^2 + y^2, y(1) = 1.5$ in the interval (1,1.5) with h=0.1 **[5]**

Q4) a) Find the directional derivative of the function $\phi = x^2y + xyz + z^3$ at (1,2,–1) in the direction $-8\bar{i} - 8\bar{j} + 4\bar{k}$ **[5]**

b) Show that

$\bar{F} = (2xz^3 + 6y)\bar{i} + (6x - 2yz)\bar{j} + (3x^2z^2 - y^2)\bar{k}$ is irrotational. Find scalar potential ϕ such that $\bar{F} = \nabla\phi$. **[5]**

c) If $\bar{r} \times \frac{d\bar{r}}{dt} = 0$,them show that $\bar{r}$ has a constant direction. **[5]**

OR

Q5) a) Find the directional derivative of the function $\phi = 4xz^3 - 3x^2y^2z$ at (2,–1,2) in the direction $2\bar{i} - 3\bar{j} + 6\bar{k}$ **[5]**

b) Prove that $\bar{F} = \frac{x\bar{i} + y\bar{j}}{x^2 + y^2}$ is solenoidal **[5]**

c) Prove that $\nabla \circ \left[r\nabla\left(\frac{1}{r^n}\right)\right] = \frac{n(n-2)}{r^{n+1}}$ **[5]**

Q6) a) Find the work done in moving a particle once round the ellipse x=5 cosθ,y=4 sinθ, z=0 under the field of force.

$\overline{F} = (2x - y + z)i + (x + y - z)j + (3x^2 - 2y^2 + z^2)K$ **[5]**

b) Evaluate $\iint_s \overline{r}.\hat{n}\, ds$ over the surface of a sphere of radius 4 with centre at origin. **[5]**

c) Apply stoke's theorem to evaluate $\int_C (yi + zj + xk).d\overline{r}$ where C is the circle given by $x^2 + y^2 + z^2 = 4$, $x + z = 2$ **[5]**

OR

Q7) a) Evaluate $\int_C \overline{F}.d\overline{r}$ for

$\overline{F} = 3x^2 i + (2xz - y)j + zk$ along the straight line joining O(0,0,0) and A (1,1,1). **[5]**

b) Apply stoke's theorem to evaluate $\int_C 4y dx + 2z dy + 6y dz$ where C is the circle $x^2 + y^2 + z^2 - 6z = 0$, $x - z + 3 = 0$ **[5]**

c) Use divergence theorem to evaluate $\iint_s (xi + yj + z^2 k).d\overline{s}$ where S is the surface of the cylinder $x^2 + y^2 = 4$ bounded by the planes $z = 0$ and $z = 2$ **[5]**

Q8) a) If $f(z)$ is analytic function of z, and $f(z)$ = u + iv prove that

$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|\text{Re } f(z)|^2 = 2|f'(z)|^2$ **[5]**

b) Evaluate $\oint_c \log z$ dz where C is the circle |Z|=1 **[5]**

c) Find the bilinear transformation which maps the points 0,–1, i of the z-plane onto the points $2, \infty, \frac{1}{2}(5 + i)$ of w-plane **[5]**

OR

Q9) a) Determine K such that the function $f(z)=\frac{1}{2}\log(x^2+y^2)+i\tan^{-1}\left(\frac{ky}{x}\right)$ is analytic. **[5]**

b) Evaluate $\oint_C \cot z\, dz$ where C is circle |z|=4 by residue theorem. **[5]**

c) Show that under transformation $w=\frac{i-z}{i+z}$, x–axis in z-plane is mapped on to the circle |w|=1 **[5]**

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