Total No. of Questions—8]

[Total No. of Printed Pages—6

Seat	
No.	20

[5352]-140

S.E. (Electronics/E & TC) (I Sem.) EXAMINATION, 2018 ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time: Two Hours

Maximum Marks: 50

- N.B. :— (i) Answer Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.
 - Neat diagrams must be drawn, wherever necessary. (ii)
 - Figures to the right indicate full marks. (iii)
 - Use of non-programmable, pocket calculator (electronic) is (iv)allowed.
 - Assume suitable data, if necessary.

Solve any two 1. (a)

Solve any
$$two$$
:

(i) $(D^2 - 4D + 4) y = 8(e^{2x} + \sin 2x)$

$$d^2 y = dy$$

$$(ii)$$
 $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$

(iii)
$$(1 + x)^2 \frac{d^2 y}{dx^2} + (1 + x) \frac{dy}{dx} + y = 2 \sin [\log(1 + x)].$$

(*b*)

[4]

Solve the integral equation :
$$\int_{0}^{\infty} f(x) \cos \lambda x \ dx = e^{-\lambda}, \ \lambda > 0.$$

An uncharged condenser of capacity C, charged by applying 2. (a) an e.m.f. of value $E \sin\left(\frac{t}{\sqrt{LC}}\right)$, through the leads of inductance

> L and of negligible resistance. The charge Q on the plate of the condenser satisfies the differential equation: [4]

$$rac{d^2 ext{Q}}{dt^2} + rac{ ext{Q}}{ ext{LC}} = rac{ ext{E}}{ ext{L}}\sinrac{t}{\sqrt{ ext{LC}}}$$

Prove that:

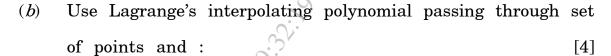
$$Q = \frac{EC}{2} \left[\sin \left(\frac{t}{\sqrt{LC}} \right) - \frac{t}{\sqrt{LC}} \cos \left(\frac{t}{\sqrt{LC}} \right) \right].$$

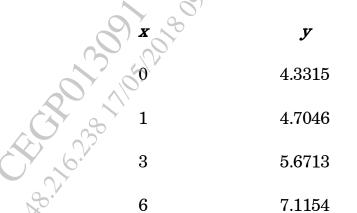
- Solve any one: (*b*) [4]
 - Find z transform of $f(k) = 3^k k < 0$ (1)
 - $= 4^k \quad k \ge 0$ Find z inverse of $\frac{1}{\left(z \frac{1}{2}\right)\left(z \frac{1}{3}\right)}, \frac{1}{3} < |z| < \frac{1}{2}.$ (ii)
- Obtain f(k), given that: (c) 12f(k+2) - 7f(k+1) + f(k) = 0 $k \ge 0$, f(0) = 0, f(1) = 3.
- Solve the following differential equation to get y(0.1) given 3. (a) $\frac{dy}{dx} = x + y^2,$ x = 0.[4]that:

$$\frac{dy}{dx} = x + y^2,$$

and

y = 1 when x = 0.

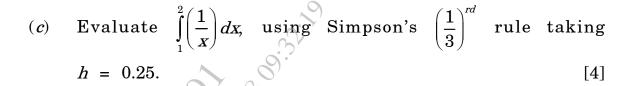




find y when x = 2.

Or

- **4.** (a) Show that (any *one*):
 - $(i) \qquad \nabla \times (\overline{r} \times \overline{u}) = \overline{r}(\nabla \cdot \overline{u}) (\overline{r} \cdot \nabla) \ \overline{u} 2\overline{u}$
 - (ii) $\nabla^4 e^r = \left(1 + \frac{4}{r}\right)e^r$.
 - (b) Show that $\overline{F} = \frac{1}{r} [r^2 \overline{a} + (\overline{a}. \overline{r}) \overline{r}]$ is irrotational where \overline{a} is a constant vector. [4]



$$\int_{\mathcal{C}} \overline{\mathbf{F}} \ d\overline{r}$$

$$\overline{\mathbf{F}} = (2xy + 3z^2)i + (x^2 + 4yz)j + (2y^2 + 6xz)k$$

and C is a curve x = y = z from (0, 0, 0) to (1, 1, 1)

(b) Use Green's Lemma, to evaluate : [4]
$$\int (2z^2 + 8z^2) dz = 6z^2 dz$$

$$\int_{C} (3x^2 - 8y^2) dx + (4y - 6xy) dy$$

over the region bounded between $y = \sqrt{x}$ and $y = x^2$.

(c)Use Stokes' theorem to evaluate: [5]

$$\oint \overline{\mathbf{F}} \cdot d\overline{r}$$

where

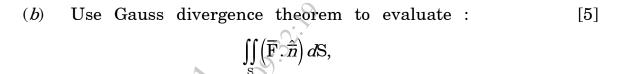
$$\oint \overline{F} \cdot d\overline{r},$$

$$\overline{F} = y^2 i + x^2 j - (x + z)k$$

over the area of triangle whose vertices are (0, 0, 0), (1, 0, 0) and (1, 1, 1).

Find the work done by a force field 6. (a) [4] $\overline{F} = 3x^2yi + (x^3 + 2yz)j + y^2k$

in moving a object from (1, -2, 1) to (3, 1, 4).



where $\overline{F} = 4xzi - y^2j + yzk$ and S is the surface bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.

(c) If :
$$\nabla . \overline{\mathbf{H}} = 0, \ \nabla \times \overline{\mathbf{E}} = -\frac{\partial \overline{\mathbf{H}}}{\partial t}, \ \nabla \times \overline{\mathbf{H}} = \frac{\partial \overline{\mathbf{E}}}{\partial t}$$

then show that \overline{H} satisfies the equation:

$$\nabla^2 u = \frac{\partial^2 u}{\partial t^2}.$$

- $\nabla^2 u = \frac{3u}{\partial t^2}.$ If f(z) = u + iv is analytic and $v = -\frac{y}{x^2 + y^2}$, find f(z) in terms **7**. of z. [4]
 - Find bilinear transformation which maps the points z = 1, (*b*) i, -1, to the points 0, 1, ∞ , respectively.
 - Evaluate: [5] (c)

$$\int_{C} \frac{e^{2z}}{(z-1)(z-2)} dz$$

if C is the circle |z| = 3.

 $\frac{z-a}{z+a}$ maps the right half Show that the transformation 8. (a) of z plane into the unit circle |w| < 1. [4]

P.T.O. [5352]-140 5

(*b*) If: [4]

$$f(a) = \int_{C} \frac{3z^2 + 5z + 2}{z - a} dz,$$

If: $f(a) = \int_{C} \frac{3z^{2} + 5z + 2}{z - a} dz,$ where 'C' is ellipse $\frac{x^{2}}{4} + \frac{y^{2}}{9} = 1$, find f(1).

If f(z) is an analytic function of z. f(z) = u + iv, prove that : $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |\operatorname{Re} f(z)|^2 = 2|f'(z)|^2.$ [5] (c)

Chino 20 The Parish of the Chino 20 The Parish o