Total No. of Questions : 9]

SEAT No. :

**PA-1189**

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# [5925]-211

## S.E. (Electronics/E&TC) (Electronics & Computer)

## ENGINEERING MATHEMATICS - III

## (2019 Pattern) (Semester - III) (207005)

*Time : 2½ Hours]* *[Max. Marks : 70*

***Instructions to the candidates:***

1) *Q.1 is compulsory.*
2) *Attempt Q2 or Q3, Q4 or Q5, Q6 or Q7, Q8 or Q9.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data, if necessary.*
7) *Write numerical calculations correct upto four decimal places.*



***Q1)*** Write the correct option for the following multiple choice questions :

i) The divergence of vector field

$\vec{F} = 3x^2\bar{i} + 3y^2\bar{j} + 2xz\bar{k}$ at point (1,1,1) is **[2]**

a) 14 b) 2

c) 12 d) 8

ii) If $f(x) = x^2$, $h = 1$, $\Delta\nabla f(x)$ is given by **[2]**

a) $-2$ b) 1

c) 2 d) $-1$

iii) The value of $\int_C \frac{z^2+1}{z-2}dz$ where C is $|z| = 1$, **[2]**

a) 0 b) $2\pi i$

c) $4\pi i$ d) $\frac{\pi i}{2}$

***P.T.O.***

iv) By Gauss - Divergence theorem $\iint_S \bar{r}_0 \hat{n} ds$ is equal to **[2]**

a) $3\iiint_v dv$

b) $\iiint_v \frac{1}{r^2} dv$

c) $\iiint_v dv$

d) 0

v) Inverse shifting operator is equivalent to. **[1]**

a) $1 - \delta$

b) $1 + \delta^2$

c) $1 + \delta$

d) $1 - \nabla$

vi) If $f(z)$ is analytic on and within a closed conter C then by Cauchy's Integral theorem $\oint_C f(z)dz$ is equal to **[1]**

a) $2\pi i$

b) $\pi i$

c) 0

d) 1

***Q2)*** a) Using Newton's forward difference formula, find a polynomial passing through the points (0,1), (1,1), (2,7), (3,25), (4,61), (5,121). Hence find $y$ and $\frac{dy}{dx}$ at $x = 0.5$. **[5]**

b) The speed (km/hr) of a train which starts from rest is given by the following table, the time being recorded in minutes. **[5]**

| t (minutes) | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| v = ds/dt (km/hr) | 0 | 10 | 18 | 25 | 29 | 32 | 20 | 11 | 5 | 2 | 0 |

Find approximately the total distance run in 20 minutes using Simpson's $\frac{1}{3}$rd rule.

c) Determine using modified Euler's method the value of $y$ at $x = 0.1$, given **[5]**

$$\frac{dy}{dx} = x^2 + y, y(0) = 1.$$

Take h = 0.1. (Two iterations only)

OR

*Q3)* a) Given the table of square roots, calculate the value of $\sqrt{155}$ by Newton's backward difference formula. **[5]**

| x | 150 | 152 | 154 | 156 |
|---|---|---|---|---|
| $y=\sqrt{x}$ | 12.247 | 12.329 | 12.410 | 12.490 |

b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule taking h = $\frac{1}{4}$. **[5]**

c) Using fourth order Range-Kutta method, solve $\frac{dy}{dx}=\sqrt{(x+y)},\ \ y(0)=1,$ to find $y$ at $x$ = 0.2 taking h = 0.2. **[5]**

*Q4)* a) Find angle between the normal to the surface $xy = z^2$ at (1,4,2) and (–3,–3,3). **[5]**

b) Find the directional derivative of $\varphi = x^2 - y^2 - 2z^2$ at the point p(2,–1,3), in the direction PQ where Q is (5,6,4). **[5]**

c) Show that the vector field $\bar{F} = (8xy + z^4)\ \bar{i}\ \ (4x^2 - 2)\ \bar{j}\ + (4xz^3 - y)\ \bar{k}$ is irrotational. Find scalar potential function φ. **[5]**

OR

*Q5)* a) Find directional derivative of $\varphi = e^{2x} \cos (yz)$ at the origin in the direction tangent to the curve $\bar{r} = a \sin t + \bar{i} + a \cos t\ \bar{j} + at\ \bar{k}$ at $t = \frac{\pi}{4}$. **[5]**

b) Prove that $\bar{b}\times\nabla(\bar{a}\cdot\nabla\log r)=\frac{\bar{b}\times\bar{a}}{r^2}-\frac{2(\bar{a}\cdot\bar{r})}{r^4}(\bar{b}\times\bar{r})$. **[5]**

c) Find angle between the tangents to the curve $\bar{r} = t^2\bar{i} + 2t\bar{j} - t^3\bar{k}$ at the points $t$ = 1 and $t$ = –1. **[5]**

***Q6)*** a) Apply Green's theorem to evaluate :

$$\int_C (x^2 dx + xy dy)$$

Where C is the curve of region enclosed by $y = x^2$ and the line $y = x$. **[5]**

b) Using Gauss - Divergence theorem, evaluate : **[5]**

$$\iint_S (x^3 \overline{i} + y^3 \overline{j} + z^3 \overline{k}) \circ d\,\overline{s}$$

Over the surface of $x^2 + y^2 + z^2 = 1$.

c) Using Stoke's theorem, evaluate : **[5]**

$$\iint_s (\nabla \times \overline{F}) \circ \hat{n}\, ds$$

for $\overline{F} = (x^2 + y - 4)\overline{i} + 3xy\,\overline{j} + (2xz + z^2)\overline{k}$ over the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above the XOY - plane.

OR

***Q7)*** a) Find the work done in moving a particle once around the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1,\ z = 0$ under the field of force given by

$\overline{F} = (2x - y + z)\overline{i} + (x + y - z^2)\overline{j} + (3x - 2y + 4z)\overline{k}$. **[5]**

b) Using Gauss - Divergence Theorem, show that **[5]**

$$\iiint_V \frac{1}{r^2} dV = \iint_S \frac{\overline{r}}{r^2} \circ \hat{n}\ ds$$

c) Using stoke's theorem, evaluate **[5]**

$$\iint_s (\nabla \times \overline{F}) \circ d\,\overline{s}$$

Where $\overline{F} = (x^3 - y^3)\overline{i} - xyz\overline{j} + y^3\overline{k}$ and S is the surface

$x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$.

***Q8)*** a) If $u = \frac{1}{2}\log(x^2 + y^2)$, find V such that $f(z) = u + iv$ is analytic function. Express $f(z)$ in terms of $z$. **[5]**

b) Use Cachy's integral formula to evaluate $\oint_C \frac{e^z}{z+2} dz$ where C is the circle $|z + 2| = 2$. **[5]**

c) Find the bilinear transformation which maps the points 0,1,2 from z plane on to the points $1, \frac{1}{2}, \frac{1}{3}$ of the W - plane. **[5]**

OR

***Q9)*** a) Show that the analytic function $f(z)$ with constant modulus is constant. **[5]**

b) Use residue theorem to evaluate $\oint_C \frac{e^z}{(z+1)(z+2)} dz$ where C is the contour $|z + 1| = \frac{1}{2}$. **[5]**

c) Show that the transformation $w = z + \frac{1}{z} - 2i$ maps the circle $|z| = 2$ into an ellipse. Find centre, semi-major and semi-minor axes of ellipse **[5]**

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