

**MULTIRATE AND ADAPTIVE SIGNAL PROCESSING**  
**(2012 Pattern)(Elective-II)**

*Time : 2½Hours]**[Max. Marks : 70**Instructions to the candidates:*

- 1) *Neat diagrams must be drawn wherever necessary.*
- 2) *Use of logarithmic tables slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.*
- 3) *Assume suitable data, if necessary.*

**Q1)** a) Verify Parsevals theorem for  $x(t) = e^{-6t} \cdot u(t)$  **[6]**

b) For a given signal,  $x(t)$

$$x(t) = 1+t \quad -1 \leq t \leq 0$$

$$= 1-t \quad 0 \leq t \leq 1$$

find- i) average time **[2]**

ii) energy in  $x(t)$  **[2]**

iii) variance in time domain **[4]**

iv) Energy in  $\frac{d}{dt}x(t)$  **[2]**

v) Variance in frequency domain **[4]**

OR

**Q2)** a) Design at a block diagram level, a two stage decimator that down samples an audio signal by a factor 30 and satisfies the following specifications-

- i) ilp sampling frequency  $f_s \rightarrow 240$  KHz
- ii) Highest frequency  $f_o$  interest in the  $\rightarrow 3.4$  KHz data
- iii) Pass band ripple,  $\delta_p \rightarrow 0.05$
- iv) Stop band ripple,  $\delta_s \rightarrow 0.01$

$$\text{filter length, } N = \frac{-10 \log(\delta_p \delta_s) - 13}{14.6 \Delta f} + 1$$

Where  $\Delta f$  = normalized transistion width assume decimation factors of 10&3 for stages 1&2 respectively. **[16]**

b) For the decimator in part a) calculate the total number of multiplications per second (MPS) and the total storage requirements(TSR) **[4]**

**P.T.O.**

**Q3) a)** Derive the conditions of alias cancellation for a Harr 2 band filter bank structure [8]

**b)** Find out the magnitude and phase response of the systems represented by following i/p o/p relations

i)  $Y(n) = \frac{1}{2}[x(n) + x(n-1)]$  [5]

ii)  $Y(n) = \frac{1}{2}[x(n) - x(n-1)]$  [5]

OR

**Q4)** For the signal,  $y(t)$  shown in fig-1

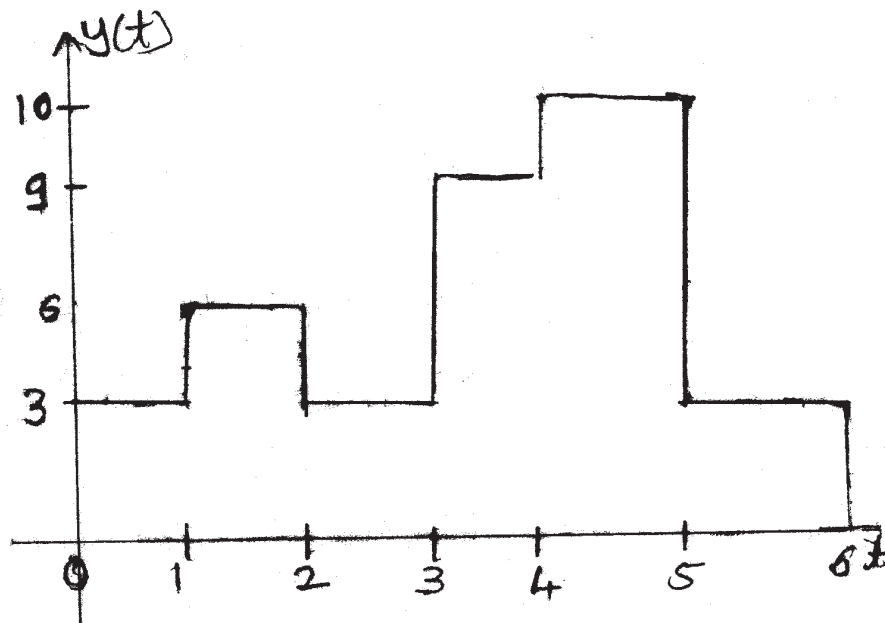


fig ①

**a)** State which V subspace  $Y(t)$  belongs to and why [2]

**b)** Calculate the piecewise constants such that  $y(t)$  belongs to  $V-1$  &  $W-1$  subspace [6]

**c)** Using Harr  $\phi\left(\frac{t}{2}\right)$ , plot projections and span of  $y(t)$  on  $V-1$  and using Harr  $\psi\left(\frac{t}{2}\right)$ , plot projections & span as  $y(t)$  on  $W-1$  [4]

**d)** Reconstruct the original signal. Show that [6]

$$V_0 = V_{-1} \oplus W_{-1}$$

**Q5)** For an adaptive filter, inputs  $X_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$  and  $X_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$  have target(desired)

values,  $Y_1 = -1$   $Y_2 = 1$  respectively.

The convergence factor  $\mu = 0.3$ . The initial weights of the filter are  $W = [0 \ 0 \ 0]$ .

The filter is trained using LMS algorithm, for four iterations. The inputs applied to the filters follow the sequence,  $X_1, X_2, X_1 \& X_2$ .

Find-

- a) Find the weight vector at the end of each iteration [8]
- b) Also find the error at the end of each iteration [4]
- c) Find mean square error at the end of second and fourth iteration [4]

OR

**Q6)** a) Prove that cost function of an adaptive filter is given by

$$J(W) = E[d^2(n)] - 2W^T P_{dx} + W^T R_x W$$

Where  $d(n)$  is the desired signal

$P_{dx}$  is the cross correlation vector

$R_x$  is the auto correlation matrix

$W$  is the weight vector. [8]

b) For an adaptive filter is

$$R_x = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} P_{dx} = [-1 \ 1]^T \& E[d^2(n)] = 4$$

Find-

- i) Optimum weight vector by solving wiener hoff equation [6]
- ii) minimum value of the cost function [2]

**Q7)**  $X[n]=\{40, 10, 36, 4, 48, 2, 10, 0\} \in V_3$

- a) Show smoothing effect [8]
- b) Reconstruct after suppressing coefficients in  $W_j$  subspaces [8]

OR

**Q8)** Write a notes on: [16]

- a) Wavelet lifting scheme
- b) Any one application of Adaptive filters

