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**[4757]-1041**

**S.E. (Electronics/E & TC) (Second Semester)**

**EXAMINATION, 2015**

**ENGINEERING MATHEMATICS-III**

**(2012 Pattern)**

**Time : Two Hours**

**Maximum Marks : 50**

**N.B. :—** (i) Answer Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4,  
Q. No. 5 or Q. No. 6 and Q. No. 7 or Q. No. 8.

(ii) Neat diagrams must be drawn wherever necessary.

(iii) Figures to the right indicate full marks.

(iv) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(v) Assume suitable data, if necessary.

1. (a) Solve any two : [8]

(i)  $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x} \sin 2x$

P.T.O.

(ii)  $(D^2 - 4D + 4)y = e^{2x}x^{-2}$  (by variation of parameters)

(iii)  $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = x + \frac{1}{x}$

(b) Solve : [4]

$$f(k) - 4f(k - 2) = \left(\frac{1}{2}\right)^k, \quad k \geq 0$$

Or

2. (a) The charge  $Q$  on the plate of condenser satisfies the differential equation : [4]

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E}{L} \sin \frac{t}{\sqrt{LC}}$$

Assuming  $\frac{1}{LC} = \omega^2$  find the charge  $Q$  at any time ' $t$ '.

(b) Find the Fourier sine integral representation for the function : [4]

$$f(x) = \begin{cases} \frac{\pi}{2} & ; 0 < x < \pi \\ 0 & ; x > \pi \end{cases}$$

(c) Attempt any one : [4]

(i) Find  $z$ -transform of  $f(k) = ke^{-3k}; k \geq 0$

(ii) Find :

$$z^{-1} \left[ \frac{z^2}{z^2 + 1} \right]$$

3. (a) Given :

$$\frac{dy}{dx} = 3x + \frac{y}{2}; \quad y(0) = 1 \quad h = 0.1$$

Evaluate  $y(0.1)$  by using Runge-Kutta method of fourth order. [4]

(b) The distance travelled by a point  $p$  in XY – plane in a mechanism is given by  $y$  in the following table. Estimate distance travelled by  $p$  when  $x = 4.5$ .

$x$	$y$
1	14
2	30
3	62
4	116
5	198

[4]

- (c) Find the directional derivative of function  $\phi = xy^2 + yz^3$  at  $(1, -1, 1)$  along the direction normal to the surface  $2x^2 + y^2 + 2z^2 = 9$  at  $(1, 2, 1)$ . [4]

Or

4. (a) Prove that (any one) : [4]

$$(i) \quad \bar{a} \cdot \nabla \left[ \bar{b} \cdot \nabla \frac{1}{r} \right] = - \frac{(\bar{a} \cdot \bar{b})}{r^3} + \frac{3(\bar{b} \cdot \bar{r})(\bar{a} \cdot \bar{r})}{r^5}$$

$$(ii) \quad \nabla \cdot \left[ r \nabla \frac{1}{r^5} \right] = \frac{15}{r^6}.$$

- (b) Use Trapezoidal Rule to estimate the value of :

$$\int_0^2 \frac{x}{\sqrt{2+x^2}} dx$$

by taking  $h = 0.5$ . [4]

- (c) Show that the vector field  $f(r)\bar{r}$  is always irrotational and then determine  $F(r)$  such that vector field  $f(r)\bar{r}$  is solenoidal. [4]

5. (a) Evaluate :

$$\int_C \left[ (2x^2y + y + z^2)i + 2(1 + yz^3)j + (2z + 3y^2z^2)k \right] \cdot d\bar{r}$$

along the curve  $C : y^2 + z^2 = a^2 \quad x = 0$  [4]

(b) Find

$$\iint_S \bar{F} \cdot \hat{n} \, ds.$$

where  $s$  is the sphere  $x^2 + y^2 + z^2 = 9$  and

$$\bar{F} = (4x + 3yz^2)\hat{i} - (x^2z^2 + y)\hat{j} + (y^3 + 2z)\hat{k} \quad [4]$$

(c) Evaluate :

$$\iint_S \nabla \times \bar{F} \cdot \hat{n} \, ds$$

for the surface of the paraboloid  $z = 4 - x^2 - y^2$ ; ( $z \geq 0$ ) and

$$\bar{F} = y^2\hat{i} + z\hat{j} + xy\hat{k}. \quad [5]$$

*Or*

6. (a) Find the total work done in moving a particle is a force field

$$\bar{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k} \quad \text{along the curve } x = t^2 + 1, \\ y = 2t^2, \, z = t^3 \text{ from } t = 1 \text{ and } t = 2. \quad [5]$$

(b) Using divergence theorem to evaluate the surface integral

$$\iint_S \bar{F} \cdot \hat{n} \, ds \quad \text{where } \bar{F} = \sin x \hat{i} + (2 - \cos x)\hat{j} \text{ and } S \text{ is the total}$$

surface area of the parallelepiped bounded by  $x = 0$ ,  $x = 3$ ,

$$y = 0, \, y = 2, \, z = 0 \text{ and } z = 1. \quad [4]$$

(c) Equations of electromagnetic wave theory are given by :

$$(i) \quad \nabla \cdot \bar{D} = \rho$$

$$(ii) \quad \nabla \cdot \bar{H} = 0$$

$$(iii) \quad \nabla \times \bar{D} = \frac{-1}{C} \frac{\partial \bar{H}}{\partial t} \quad \text{and}$$

$$(iv) \quad \nabla \times \bar{H} = \frac{1}{C} \left[ \frac{\partial \bar{D}}{\partial t} + \rho \bar{v} \right]$$

Prove that :

$$\nabla^2 \bar{D} - \frac{1}{C} \frac{\partial^2 \bar{D}}{\partial t^2} = \nabla \rho + \frac{1}{C^2} \frac{\partial}{\partial t} (\rho \bar{v}) \quad [4]$$

7. (a) Find the analytic function  $f(z) = u + iv$  if  $2u + v = e^x (\cos y - \sin y)$ . [5]

(b) Evaluate :

$$\int_C \frac{e^{2z}}{(z-1)(z-2)} dz,$$

where C is circle  $|z| = 3$ . [4]

(c) Find the bilinear transformation which maps the points

$z = -1, 0, 1$  of  $z$ -plane into the points  $w = 0, i, 3i$  of

$w$ -plane. [4]

Or

8. (a) Find the analytic function  $f(z) = u + iv$

where

$$u = r^3 \cos 3\theta + r \sin \theta. \quad [4]$$

- (b) Evaluate :

$$\int_C \frac{1 - 2z}{z(z - 1)(z - 2)} dz$$

where

$$C \text{ is } |z| = 1.5. \quad [4]$$

- (c) Find the map of the straight line  $y = 2x$  under the transformation : [5]

$$w = \frac{z - 1}{z + 1}$$