Total No. of Questions—**8**] [Total No. of Printed Pages—**6**

Seat No.	

[5352]-140

S.E. (Electronics/E & TC) (I Sem.) EXAMINATION, 2018

ENGINEERING MATHEMATICS—III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

N.B. :— (*i*) Answer Q. No. **1** *or* Q. No. **2,** Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn, wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable, pocket calculator (electronic) is allowed.

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^2 - 4D + 4)\, y = 8(e^{2x} + \sin 2x)$

(*ii*) $\dfrac{d^2 y}{dx^2} + 3\dfrac{dy}{dx} + 2y = e^{e^x}$

(*iii*) $(1 + x)^2 \dfrac{d^2 y}{dx^2} + (1 + x)\dfrac{dy}{dx} + y = 2 \sin [\log(1 + x)]$.

(*b*) Solve the integral equation : [4]

$$\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda},\ \lambda > 0.$$

Or

2. (*a*) An uncharged condenser of capacity C, charged by applying an e.m.f. of value $E \sin\left(\frac{t}{\sqrt{LC}}\right)$, through the leads of inductance L and of negligible resistance. The charge Q on the plate of the condenser satisfies the differential equation : [4]

$$\frac{d^2Q}{dt^2} + \frac{Q}{LC} = \frac{E}{L}\sin\frac{t}{\sqrt{LC}}$$

Prove that :

$$Q = \frac{EC}{2}\left[\sin\left(\frac{t}{\sqrt{LC}}\right) - \frac{t}{\sqrt{LC}}\cos\left(\frac{t}{\sqrt{LC}}\right)\right].$$

(*b*) Solve any *one* : [4]

(*i*) Find z transform of $f(k) = 3^k \quad k < 0$

$= 4^k \quad k \geq 0$

(*ii*) Find z inverse of $\frac{1}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{3}\right)}, \frac{1}{3} < |z| < \frac{1}{2}$.

(c) Obtain $f(k)$, given that : [4]

$12f(k+2) - 7f(k+1) + f(k) = 0 \quad k \geq 0,$

$f(0) = 0, \quad f(1) = 3.$

3. (*a*) Solve the following differential equation to get $y(0.1)$ given that : [4]

$$\frac{dy}{dx} = x + y^2,$$

and

$y = 1$ when $x = 0$.

(*b*) Use Lagrange's interpolating polynomial passing through set of points and : [4]

x	y
0	4.3315
1	4.7046
3	5.6713
6	7.1154

find y when $x = 2$.

(*c*) Find the directional derivative of : [4]

$$\phi = x^2 - y^2 - 2z^2 \text{ at the point } P(2, -1, 3)$$

in the direction of $\overline{PQ}$ where Q is (5, 6, 4).

Or

4. (*a*) Show that (any *one*) : [4]

(*i*) $\nabla \times (\bar{r} \times \bar{u}) = \bar{r}(\nabla \cdot \bar{u}) - (\bar{r} \cdot \nabla)\, \bar{u} - 2\bar{u}$

(*ii*) $\nabla^4 \; e^r = \left(1 + \dfrac{4}{r}\right) e^r.$

(*b*) Show that $\bar{F} = \dfrac{1}{r}[r^2\bar{a} + (\bar{a}.\bar{r})\bar{r}]$ is irrotational where $\bar{a}$ is a constant vector. [4]

(c) Evaluate $\int_1^2 \left(\frac{1}{x}\right) dx$, using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule taking $h = 0.25$. [4]

5. (a) Evaluate : [4]

$$\int_C \overline{F}\, d\overline{r},$$

where

$$\overline{F} = (2xy + 3z^2)i + (x^2 + 4yz)j + (2y^2 + 6xz)k$$

and C is a curve $x = y = z$ from (0, 0, 0) to (1, 1, 1)

(b) Use Green's Lemma, to evaluate : [4]

$$\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$$

over the region bounded between $y = \sqrt{x}$ and $y = x^2$.

(c) Use Stokes' theorem to evaluate : [5]

$$\oint \overline{F} . d\overline{r},$$

where

$$\overline{F} = y^2 i + x^2 j - (x + z)k$$

over the area of triangle whose vertices are (0, 0, 0), (1, 0, 0) and (1, 1, 1).

Or

6. (a) Find the work done by a force field : [4]

$$\overline{F} = 3x^2 yi + (x^3 + 2yz)j + y^2 k$$

in moving a object from (1, –2, 1) to (3, 1, 4).

(*b*) Use Gauss divergence theorem to evaluate : [5]

$$\iint_S \left(\bar{F}.\hat{\bar{n}}\right) dS,$$

where $\bar{F} = 4xzi - y^2 j + yzk$ and S is the surface bounded by $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1.$

(*c*) If : [4]

$$\nabla.\bar{H} = 0, \nabla \times \bar{E} = -\frac{\partial \bar{H}}{\partial t}, \nabla \times \bar{H} = \frac{\partial \bar{E}}{\partial t}$$

then show that $\bar{H}$ satisfies the equation :

$$\nabla^2 u = \frac{\partial^2 u}{\partial t^2}.$$

7\. (*a*) If $f(z) = u + iv$ is analytic and $v = -\frac{y}{x^2 + y^2}$, find $f(z)$ in terms of z. [4]

(*b*) Find bilinear transformation which maps the points $z = 1, i, -1$, to the points $0, 1, \infty$, respectively. [4]

(*c*) Evaluate : [5]

$$\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$$

if C is the circle $|z| = 3$.

Or

8\. (*a*) Show that the transformation $w = \frac{z - a}{z + a}$ maps the right half of z plane into the unit circle $|w| < 1$. [4]

(*b*) If : [4]

$$f(a) = \int_C \frac{3z^2 + 5z + 2}{z - a}\, dz,$$

where 'C' is ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$, find $f'(1)$.

(*c*) If $f(z)$ is an analytic function of z . $f(z) = u + iv$, prove that : [5]

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|\operatorname{Re} f(z)|^2 = 2|f'(z)|^2.$$