Total No. of Questions—**8**] [Total No. of Printed Pages—**5**

Seat No.	

[5252]-140

S.E. (Electronics/E&TC) (Second Semester)

EXAMINATION, 2017

ENGINEERING MATHEMATICS-III

(2012 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. **1** *or* **2**, Q. **3** *or* **4**, Q. **5** *or* **6**, Q. **7** *or* **8**.

(*ii*) Neat diagram must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Use of non-programmable pocket calculator (electronic is allowed).

(*v*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^2 + 2D + 1)\, y = 2\cos x + 3x + 2$

(*ii*) $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ (by method of variation of parameter)

(*iii*) $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\log x.$

(*b*) Find Fourier cosine transform of $f(x) = \begin{cases} x, & 0 \le x \le a \\ 0, & x > a \end{cases}$ [4]

P.T.O.

Or

2. (*a*) An electric circuit consists of an inductance 0.1 henry, a resistance (R) of 20 Ω and a condenser of capacitance (C) of 25 microfarads. If the differential equation of electric circuit is $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=0$, then find the charge '$q$' and current '$i$' at any time t, given that at $t = 0$, $q = 0.05$ coulombs. $i = \frac{dq}{dt} = 0$ when $t = 0$. [4]

(*b*) Solve (any *one*) : [4]

(*i*) Find z-transfrom of $f(k) = 2^k \cos(3k + 2)$.

(*ii*) Find inverse z-transform of $\frac{1}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$ for $\frac{1}{3}<|z|<\frac{1}{2}$.

(*c*) Solve : [4]

$$f(k+1)+\frac{1}{2}f(k)=\left(\frac{1}{2}\right)^k, \quad k \geq 0, \ f(0)=0.$$

3. (*a*) Solve the equation $\frac{dy}{dx}=\sqrt{x+y}$ using fourth order Runge-Kutta method given $y(0) = 1$ to find y at $x = 0.2$ taking $h = 0.2$. [4]

(*b*) Find Lagrange's interpolating polynomial passing through set of points :

x	0	1	2
y	2	1	4

Hence find y at $x = 0.5$ and $\frac{dy}{dx}$ at $x = 2$. [4]

(*c*) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ in the direction of $2\bar{i} - 3\bar{j} + 6\bar{k}$. [4]

Or

4. (*a*) Show that (any *one*) : [4]

(*i*) For scalar functions ϕ & ψ, show that :

$$\nabla.\ (\phi\nabla\psi - \psi\nabla\phi) = \phi\nabla^2\psi - \psi\nabla^2\phi$$

(*ii*) $\nabla.\left(r\nabla\dfrac{1}{r^3}\right) = \dfrac{3}{r^4}$

(*b*) Show that the vector field $\bar{F} = (y^2\cos x + z^2)\bar{i} + 2y\sin x\,\bar{j} + 2xz\,\bar{k}$ is irrotational. Find scalar ϕ such that $\bar{F} = \nabla\phi$. [4]

(*c*) Evaluate $\int_0^{\pi/2} \dfrac{\sin x}{x}\,dx$ by using Simpson's $\left(\dfrac{1}{3}\right)^{rd}$ rule by dividing the interval into four parts. Considering the values upto four decimals. [4]

5. (*a*) Evaluate $\int_c \bar{F}.\,d\bar{r}$ for $\bar{F} = (5xy - 6x^2)\bar{i} + (2y - 4x)\bar{j}$ along the curve $c : y = x^3$ in XOY plane from (1, 1) to (2, 8). [4]

(*b*) Use divergence theorem to evaluate :

$$\iint_s (y^2z^2\bar{i} + z^2x^2\bar{j} + x^2y^2\bar{k}).\,d\bar{S}$$

where S is the upper part of the sphere $x^2 + y^2 + z^2 = 9$ above XOY plane. [5]

(c) Using Green's theorem, show that the area bounded by a simple closed curve C is given by :

$$\frac{1}{2}\int_c (x\,dy - y\,dx)$$

Hence find area of the ellipse

$x = a \cos\theta,\ y = b\sin\theta.$ [4]

Or

6. (a) Find the work done in moving a particle from A(1,0,1) to B(2,1,2) along the straight line AB in the force field $\bar{F} = x^2\bar{i} + (x-y)\bar{j} + (y+z)\bar{k}$. [4]

(b) Evaluate :

$$\iint_s (\nabla \times \bar{F}).\, d\bar{S}$$

for the vector field $\bar{F} = 4y\bar{i} - 4x\bar{j} + 3\bar{k}$ where S is a disc of radius 1 lying on the plane $z = 1$. [5]

(c) Prove that :

$$\iint_s (\phi\nabla\psi - \psi\nabla\phi).\, d\bar{S} = \iiint_V (\phi\nabla^2\psi - \psi\nabla^2\phi)\, dV$$

where S is any closed surface enclosing volume V. [4]

7. (a) If $f(z) = u + iv$ is an analytic function with $u = \cosh x \cos y$, express $f(z)$ in terms of z. [4]

(b) Evaluate :

$$\oint_c \frac{2z^2 + z + 5}{\left(z - \frac{3}{2}\right)^2}\, dz$$

where C is $\frac{x^2}{4} + \frac{y^2}{9} = 1$. [4]

(c) Find the bilinear transformation which maps the points $0, -i, -1$ from z-plane into the points $i, 1, 0$ of the w-plane. [5]

Or

8. (a) If $f(z) = u + iv = f(r\ e^{i\theta})$ is analytic, show that u satisfies the Laplace equation $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$. [4]

(b) Evaluate $\oint_c \frac{e^{2z}}{z(z-1)^2} dz$ over $c : |z| = 3$. [4]

(c) Show that the bilinear transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ into the line $4u + 3 = 0$. [5]