which diverges for $D \to 4$

Introducing a dimension for the g_s-coupeling

The action in QFT is

$$S = \int d^D x \, \mathcal{L}$$

Is dimensionless (is $c = \hbar = 1$) in natural units

$\dim[\mathcal{L}] = D$ mass dimension of the lagrangian in natural units

Compton wave length: $\lambda = \frac{\hbar}{mc}$

$\hbar = c = 1 \Rightarrow [\lambda] = [m]^{-1}$

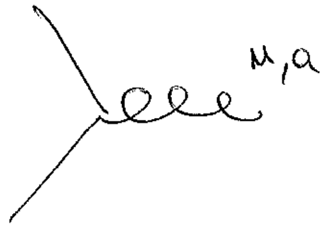
Partial derivative $\frac{\partial}{\partial x^\mu} \to$ inverse of length!

$[\partial] = 1$

$[d^4 x] = -4 \Rightarrow [d^D x] = -D$

$[\mathcal{L}] = D$ to have $[S] = 1$

Let's examine



$g_s \bar{\psi} T^a \gamma^\mu A_\mu^a$

?

↓

$\dim[g_s] + 2\dim[\psi] + \dim[A_\mu^a] = D$

$\dim[A_\mu^a] = ?$ $\quad \dim[\psi] = ?$

Let's look at the kinetic terms in the lagrangian

$$\mathcal{L} = a_1 F^{\mu\nu} F_{\mu\nu} + a_2 \bar{\psi}\gamma^\mu D_\mu \psi + a_3 \bar{\psi}\psi$$

$$+ a_4 \bar{\psi}\sigma^{\mu\nu} F_{\mu\nu}\psi + a_5 F^{\mu\nu} F_\nu{}^\lambda F_{\lambda\mu} + \cdots$$

In QED for example

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \Rightarrow$$

$$F_{\mu\nu} F^{\mu\nu} \sim [\partial_\mu A_\nu \partial^\mu A^\nu]$$

$$\Rightarrow [A] = D-2$$

$$\dim\left[\int F^{\mu\nu} F_{\mu\nu}\, d^D x\right] = 0 \Leftrightarrow [\partial^2][A^2] - D = 0$$

$$[A^2] = D-2 \Rightarrow [A] = \frac{D-2}{2}$$

By the same token

$$[\bar{\psi}\gamma^\mu \partial_\mu \psi] = D \qquad \text{(kinetic term for fermions)}$$

$$[\psi^2][\partial] = D \Rightarrow [\psi] = \frac{D-1}{2}$$

$\Rightarrow$

$$\dim[g_s] + 2\left(\frac{D-1}{2}\right) + \left(\frac{D-2}{2}\right) = D$$

We must introduce a mass scale μ and rewrite the gauge coupling constant as

$$\boxed{g_s = g_s^\circ \, \mu^{2-D/2}}$$

g_s° = dimensionless coupling

μ = DR scale parameter

We can rewrite $D = 4 - 2\varepsilon$ where ε is a small parameter

$$\varepsilon = \frac{4-D}{2}$$

$$\Sigma(p) = \frac{g_{0s}^2 C_F}{(4\pi)^2} \not{p} \left(\frac{-p^2}{4\pi\mu^2}\right)^{-\varepsilon} (1-\varepsilon)\, B(1-\varepsilon, 1-\varepsilon)\, \Gamma(\varepsilon)$$

A Laurent expansion around $\varepsilon = 0$ gives

$$\Sigma(p) = \frac{g_0^2}{(4\pi)^2} G_F \not{p} \left(\frac{1}{\varepsilon} - \gamma + 1 - \ln\left(\frac{-p^2}{4\pi\mu^2} \right) \right) + \mathcal{O}(\varepsilon)$$

where we used

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma_E + \mathcal{O}(\varepsilon)$$

$$(1-\varepsilon) B(1-\varepsilon, 1-\varepsilon) = 1 + \varepsilon + \mathcal{O}(\varepsilon^2)$$

γ_E = Euler - Mascheroni constant $= 0.57721$

<u>Note</u>: In this calculation we haven't encountered γ^5. γ^5 in D-dim requires a special treatment as it cannot be defined explicitly for arbitrary dimensions.

<u>DR conventions</u>

1. D-dim space-time metric $g^{\mu\nu} = (+,-,\dots,-)$
2. $\mathrm{Tr}[1] = 4$ in the space of gamma matrices
3. $\int \frac{d^D k}{(2\pi)^D}$ defines the integral measure
4. γ_5 is an object that satisfies $\{\gamma_5, \gamma^\mu\} = 0$