PHYS 4210/ Fall 2024
Midterm Exam 1

Kennesaw State University
College of Science and Mathematics
Department of Physics

Student Name: ________________________________

This exam consists of four questions, each worth 25 points. Answer them on the provided sheets. You have 70 minutes to complete the exam. You may use a calculator and your own integration formula sheet. All other work must be your own, without assistance from peers, notes, books, or online resources.

1. The ground state of a particle in one-dimensional infinite square well is given by

$$\Psi(x,t) = \sqrt{\frac{2}{a}}\, sin\frac{\pi x}{a}\; e^{-iEt/\hbar}$$

   where a and E are positive.

   What is the probability of finding the particle in the range $0 \leq x \leq a/4$?

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2. A particle is represented by a wave function ϕ(x) that obeys the time-independent Schrödinger equation as:

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2} + V\phi = E\phi$$

(a) Show that the variance of Hamiltonian operator is zero (i.e. ${\sigma_H}^2 = 0$). (20 credit)
(b) Discuss the significance of the result in (a). (5 credit)

Note: Hamiltonian operator is defined as $\hat{\mathrm{H}} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V$

3. Show that the uncertainty principle holds for a particle described by wave function:

$$\Psi(x) = \left(\frac{2a}{\pi}\right)^{\frac{1}{4}} \; e^{-ax^2} \qquad -\infty \leq x \leq \infty.$$

For full credit, show all your work.
For a NEGATIVE credit of 5 points (*i.e.* points deducted), you may use $< \mathrm{p}^2 >= \mathrm{a}\hbar^2$

4. For a particle in time-independent potential field, the ground state and the first excited state can be written as

$$\psi_1(x,t) = \phi_1(x)\, e^{-iE_1 t/\hbar}$$
$$\psi_2(x,t) = \phi_2(x)\, e^{-iE_2 t/\hbar}$$

where $\phi_1 and\ \phi_2$ are the solutions of corresponding time-independent Schrödinger equation. Their linear combination can be written as

$$\psi = c_1\, \psi_1 + c_2\, \psi_2$$

Assume that $c_1, c_2, \phi_1\ and\ \phi_2\ are\ real$.

(a) For either $\psi_1 or\ \psi_2$ (not both), show that the expectation value of x, i.e. $\langle x \rangle$, is independent of time. (10 credits)
(b) What is the implication of (a)? (5 credits)
(c) Show that the probability density of ψ, $i.e.$ $|\psi|^2$, is NOT independent of time. (15 credits)

Potentially Useful Identities

$$sin^2x = \frac{1 - cos2x}{2}$$

$$\int dx \ = x + C$$

$$\int coskx\, dx \ = \frac{1}{k}\, sinkx \ + \ C$$

Odd functions are those which satisfy f(-x) = -f(x). Integral of an odd function in symmetric limit is zero.

$$\int_{-\infty}^{\infty} e^{-x^2/k^2} dx \ = \ k\sqrt{\pi}$$

$$\int_{-\infty}^{\infty} x^2 e^{-x^2/k^2} dx \ = \ \frac{k^3\sqrt{\pi}}{2}$$

$$\frac{d(e^{kx})}{dx} \ = \ ke^{kx}$$

Chain rule of derivatives: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

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Additional Answer Sheet

Student Name: ___________________________________

Answer to Question# : _______________