A Laurent expansion around $\varepsilon = 0$ gives

$$\Sigma(p) = \frac{g_0^2}{(4\pi)^2} C_F \not{p} \left(\frac{1}{\varepsilon} - \gamma + 1 - \ln\left(\frac{-p^2}{4\pi\mu^2} \right) \right) + \mathcal{O}(\varepsilon)$$

where we used

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma_E + \mathcal{O}(\varepsilon)$$

$$(1-\varepsilon)\, B(1-\varepsilon, 1-\varepsilon) = 1 + \varepsilon + \mathcal{O}(\varepsilon^2)$$

γ_E = Euler - Mascheroni constant $= 0.57721$

<u>Note</u>: In this calculation we haven't encountered γ^5. γ^5 in D-dim requires a special treatment as it cannot be defined explicitely for arbitrary dimensions.

DR conversions

1. D-dim space-time metric $g^{\mu\nu} = (+, -, \dots, -)$
2. $\mathrm{Tr}[1] = 4$ in the space of gamma matrices
3. $\int \frac{d^D k}{(2\pi)^D}$ defines the integral measure
4. γ_5 is an object that satisfies $\{\gamma_5, \gamma^\mu\} = 0$

A Laurent expansion around $\epsilon = 0$ gives

$$\Sigma(p) = \frac{g_0^2}{(4\pi)^2} C_F \not{p} \left(\frac{1}{\varepsilon} - \gamma + 1 - \ln\left(\frac{-p^2}{4\pi\mu^2} \right) \right) + \mathcal{O}(\epsilon)$$

where we used

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma_E + \mathcal{O}(\varepsilon)$$

$$(1-\varepsilon)\, B(1-\varepsilon, 1-\varepsilon) = 1 + \varepsilon + \mathcal{O}(\varepsilon^2)$$

γ_E = Euler - Mascheroni constant $= 0.57721$

<u>Note</u>: In this calculation we haven't encountered γ^5. γ^5 in D-dim requires a special treatment as it cannot be defined explicitly for arbitrary dimensions.

<u>DR conventions</u>

1. D-dim space-time metric $g^{\mu\nu} = (+,-,\dots,-)$
2. $\mathrm{Tr}[1] = 4$ in the space of gamma matrices
3. $\int \frac{d^D k}{(2\pi)^D}$ defines the integral measure
4. γ_5 is an object that satisfies $\{\gamma_5, \gamma^\mu\} = 0$

Self energy correction in an arbitrary covariant gauge

$$\Sigma(\not p) = g_s^2 C_F \int \frac{d^D k}{(2\pi)^D i} \frac{1}{k^2 (k-p)^2} \left\{ \gamma_\mu (\not k - \not p) \gamma^\mu - (1-\alpha) \frac{\not k (\not k - \not p) \not k}{k^2} \right\}$$

we need to calculate the term prop to $(1-\alpha)$. We write

$$\Sigma(\not p) = \Sigma_1(\not p) - (1-\alpha) \Sigma_2(\not p)$$

$$\Sigma_1(\not p) = \frac{g_0^2}{(4\pi)^2} C_F \not p \left(\frac{1}{\varepsilon} - \gamma + 1 - \ln\left(\frac{-p^2}{4\pi\mu^2}\right)\right) + O(\varepsilon)$$

calculated at pag 16.

$$\Sigma_2(\not p) = g_s^2 C_F \int \frac{d^D k}{(2\pi)^D} \frac{\not k (\not k - \not p) \not k}{i (k^2)^2 (k-p)^2}$$

Now we use the following Feynman parametr.

$$\frac{1}{AB^2} = 2 \int_0^1 dx \frac{(1-x)}{\{xA + (1-x)B\}^3}$$

Again $L = -x(1-x)p^2$

At this point we need to "massage" the denominator in such a way that we can use a shift in the k momentum.

$$k \to k - xp + xp$$

The $\Sigma_2(p)$ expression then becomes

$$\Sigma_2(p) = -2g_s^2 C_F \int_0^1 dx\,(1-x) \int \frac{d^Dk}{(2\pi)^D i} \frac{\not{k}(\not{k}-\not{p})\not{k}}{\left\{-(k-xp)^2+L\right\}^3}$$

At this point we shift the k-momentum

$$k' = k - xp$$

and discard all the terms odd in k'

$$\Sigma_2(p) = 2g_s^2 C_F \int_0^1 dx\,(1-x) \int \frac{d^Dk'}{(2\pi)^D} \frac{(1-x)\not{k}'\not{p}\not{k}' - 2xk'^2\not{p} - xL\not{p}}{\left(-k'^2+L\right)^3}$$

For any integrable function $f(k'^2)$ we use the following formula

$$\int d^Dk'\, k'_\mu k'_\nu\, f(k'^2) = \frac{1}{D} g_{\mu\nu} \int d^Dk'\, k'^2 f(k'^2)$$

In fact, remember that $g^{\mu\nu} g_{\mu\nu} = D \Rightarrow$

$$\frac{D}{D} \int d^Dk'\, k'_\mu k'_\nu\, f(k'^2) = \frac{g_{\mu\nu}}{D} \int d^Dk'\, k'^2 f(k'^2)$$

We obtain

$$\Sigma_2(p) = 2g_s^2 C_F \not{p} \int_0^1 dx\,(1-x) \int \frac{d^Dk'}{(2\pi)^D i} \frac{1}{[-k'^2+L]^3} \left\{\left(\frac{2(1-x)}{D} - 1 - x\right)k'^2 - xL\right\}$$

which becomes, after a Wick rotation,

$$\Sigma_2(p) = 2 g_s^2 C_F \not{p} \int_0^1 dx\,(1-x) \int \frac{d^D K}{(2\pi)^D} \left\{ \frac{1}{(K^2+L)^3} \left[\left(\frac{2(1-x)}{D} - 1 - 2x \right) L \right. \right.$$

$$\left. \left. - \frac{1}{(K^2+L)^2} \left[\frac{2(1-x)}{D} - 1 - x \right] \right] \right\}$$

We use the generalized result for the K integration

- $$\int \frac{d^D K}{(2\pi)^D} \frac{1}{(K^2+L)^a} = \frac{\Gamma(a - D/2)}{(4\pi)^{D/2}\,\Gamma(a)} L^{D/2 - a} \qquad a \in \mathbb{C} \ni' \mathrm{Re}[a] > 0$$

We use the following relations for the x integration

- $$B(p,q) = \int_0^1 dx\, x^{p-1} (1-x)^{q-1}$$

- $$B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

- $$\Gamma(z+1) = z\,\Gamma(z)$$

With these results the expression for $\Sigma_2(p)$ is given by

$$\Sigma_2(p) = 2 g_s^2 C_F \not{p} (-p^2)^{D/2-2} (D-1)\, B(D/2, D/2)\, \Gamma(2 - D/2)$$

which is exactly equal to $\Sigma(p)$ calculated at pag 16!