PHYS 4500 Fall 2024

Test 3

You may use the lecture notes, your homework, and the textbooks but no other resources or materials.

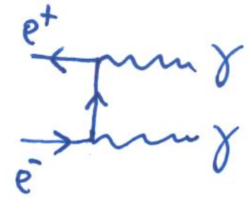
1) Show that the gauge transformation in Yang-Mills theory

$$\vec{\sigma}\cdot\vec{A}_\mu \rightarrow U\vec{\sigma}\cdot\vec{A}_\mu U^{-1} + \frac{i}{q}(\partial_\mu U)U^{-1}$$

where $U = e^{i\vec{\sigma}\cdot\vec{\theta}} = e^{iq\vec{\sigma}\cdot\vec{\lambda}} = 1 + iq\vec{\sigma}\cdot\vec{\lambda} + \dots$ can be written for small $\vec{\lambda}$ as $\vec{A}_\mu \rightarrow \vec{A}_\mu - \partial_\mu\vec{\lambda} - 2q\vec{\lambda}\times\vec{A}_\mu$

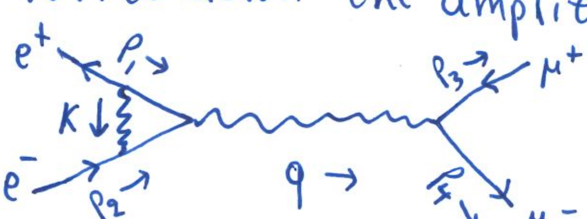
[Hint: for vectors $\vec{B}$ and $\vec{C}$, we have $(\vec{\sigma}\cdot\vec{B})(\vec{\sigma}\cdot\vec{C}) = \vec{B}\cdot\vec{C} + i\vec{\sigma}\cdot(\vec{B}\times\vec{C})$]

2) The leading-order diagram for $e^+ + e^- \rightarrow \gamma + \gamma$ is



Draw all the NLO diagrams for this process.

3) Write down the amplitude for the diagram below.



Note that this is a NLO diagram with a vertex correction for the process $e^+(p_1) + e^-(p_2) \rightarrow \mu^+(p_3) + \mu^-(p_4)$.

4) Use Feynman parameters to rewrite the amplitude in problem 3 in a form such that we can use the integrals in dimensional regularization. You do not need to calculate any integrals.

5) In the path integral formalism for spinor fields, calculate

$$-\frac{\delta^2 Z_0[\eta,\bar{\eta}]}{\delta\bar{\eta}(x)\,\delta\eta(y)}$$

Then show that this gives $S(x-y)$ if we set $\eta = \bar{\eta} = 0$.