PHYS 4210/ Fall 2024
Midterm Exam 2

Kennesaw State University
College of Science and Mathematics
Department of Physics

Student Name: ________________________________

This exam consists of six questions, each worth 25 points. Answer them on the provided sheets. You have 70 minutes to complete the exam. You may use a calculator and your own integration formula sheet. All other work must be your own, without assistance from peers, notes, books, or online resources.

After grading your answers to all six questions, your two lowest scores will be dropped, i.e. the maximum possible score for this exam is 100.

1. The solutions to time-independent Schrodinger equation for a particle in infinite square well potential of width a are given as:

$$\psi_n(x) = \sqrt{\frac{2}{a}}\, sin\left(\frac{n\pi}{a}x\right)$$

   Where n =1, 2, 3, …
   Show that these solutions are mutually orthogonal.

2. The ground state of quantum harmonic oscillator of mass m is

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$$

Assume that the ladder operators have the form

$$\hat{a}_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp i\hat{p} + m\omega x)$$

Find the first excited state and normalize it.

3. Using the definition of ladder operator from question 2, show that $\hat{a}_-$ is the Hermitian conjugate of $\hat{a}_+$.

4. The state of a quantum particle is written as

$$|\psi\rangle = \sqrt{\frac{7}{15}}\,|\phi_1\rangle + \sqrt{\frac{1}{3}}\,|\phi_2\rangle + \sqrt{\frac{1}{5}}\,|\phi_3\rangle$$

Where $|\phi_1\rangle$, $|\phi_2\rangle$, and $|\phi_3\rangle$ are eigenstates to a Hermitian operator $\hat{B}$ such that $\hat{B}|\phi_n\rangle = (3n^2 - 1)|\phi_n\rangle$

1) For operator $\hat{B}$, what is the probability of getting an eigen value of 80? (10 credits)
2) Find the expectation value of $\hat{B}$ for the state $|\psi\rangle$. (15 credits)

5. Evaluate

(a) $\int_0^{2\pi} cosx\, \delta(x^2 - \pi^2)dx$ (12.5 credits)

(b) $\int_{-3}^{1} f(x)\, \delta(x + 2)\, dx$,

where $f(x) = 10x - 1$ for $0 \leq x < \infty$ and $f(x) = x^3 - 3x^2 + 2x - 1$ for $-\infty < x \leq 0$

(12.5 credits)

6. In this question, I am gauzing your understanding of basic ideas in this course. Short answers explaining basic math will get you full credit. Show your work.

(a) Show that $[Ae^{ikx} + Be^{-ikx}]$ and $[C\ cos\ kx\ +\ D\ sin\ kx]$ are equivalent ways of writing the same function of x, and determine the constants C and D in terms of A and B, and vice versa. (10 points)

(b) Why is $\psi(x) = Ae^{-kx} + Be^{kx}$ a general solution to the equation to the equation $\frac{d^2\psi}{dx^2} = k^2\psi$? Assume k is real and positive. (10 points)

(c) Explain the fundamental difference between the solutions of type (a) and type (b)? (5 points)

Potentially Useful Identities

$$cos(A+B) = cosA\ cosB - sinA\ sinB$$
$$cos(A-B) = cosA\ cosB + sinA\ sinB$$

$$sin^2x = \frac{1-cos2x}{2}$$

$$\int dx\ = x + C$$

$$\int coskx\ dx\ = \frac{1}{k}\ sinkx\ +\ C$$

Odd functions are those which satisfy f(-x) = -f(x). Integral of an odd function in symmetric limit is zero.

$$\int_{-\infty}^{\infty} e^{-x^2/k^2} dx\ =\ k\sqrt{\pi}$$

$$\int_{-\infty}^{\infty} x^2 e^{-x^2/k^2} dx\ =\ \frac{k^3\sqrt{\pi}}{2}$$

$$\frac{d(e^{kx})}{dx}\ =\ ke^{kx}$$

Chain rule of derivatives: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

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Additional Answer Sheet

Student Name: ___________________________________

Answer to Question# : _______________