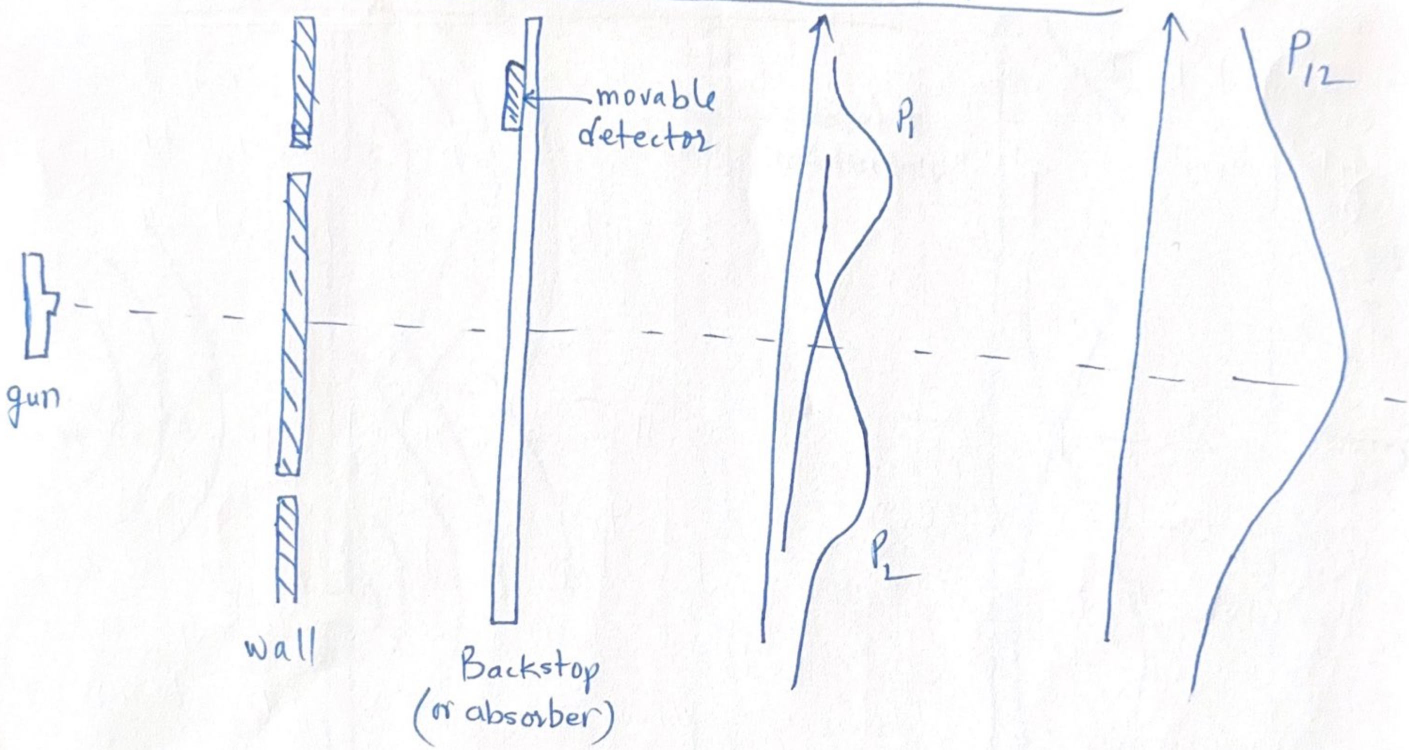
Quantum mechanics

Refers to description of behavior of matter and light at atomic scale. These behavior can not be explained by the laws of classical physics.

In today's lecture, we will examine a representative quantum phenomena, interference of electron 'waves', that can not be explained by classical interpretation of electrons as particles. We will do so by first considering double slit experiment with classical particles, waves and finally electrons.

Double slit experiment with "classical" particle



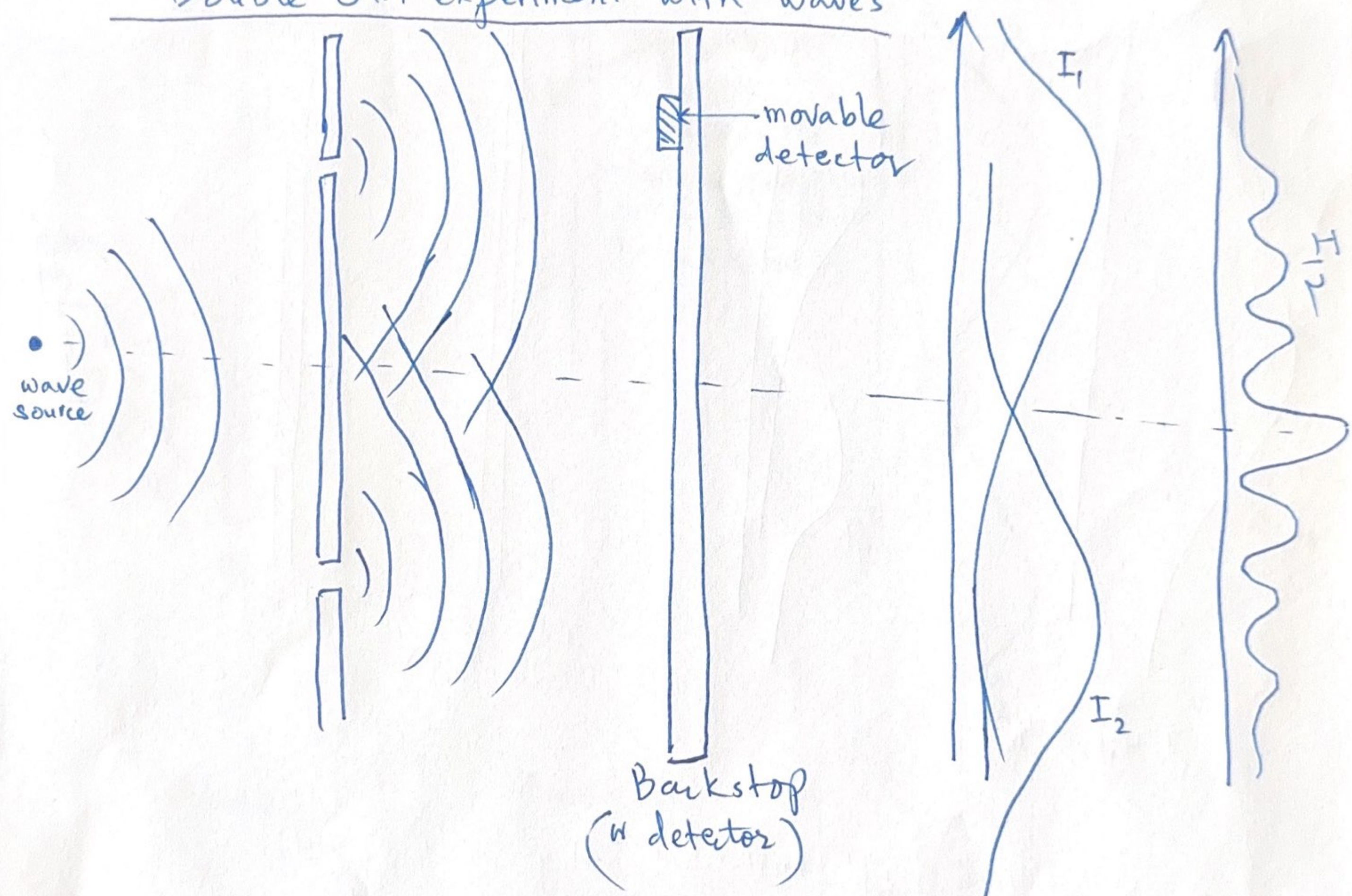
$$P_{12} = P_1 + P_2$$

Let us take idealized "indestructible" bullets as an example of "classical" object.

$$P_{12} = P_1 + P_2$$

Probability of arrival shows no interference.

Double slit experiment with waves



$I_1 = |h_1|^2$

$I_2 = |h_2|^2$

$I_{12} = |h_1 + h_2|^2$

$I_{12} \neq I_1 + I_2$

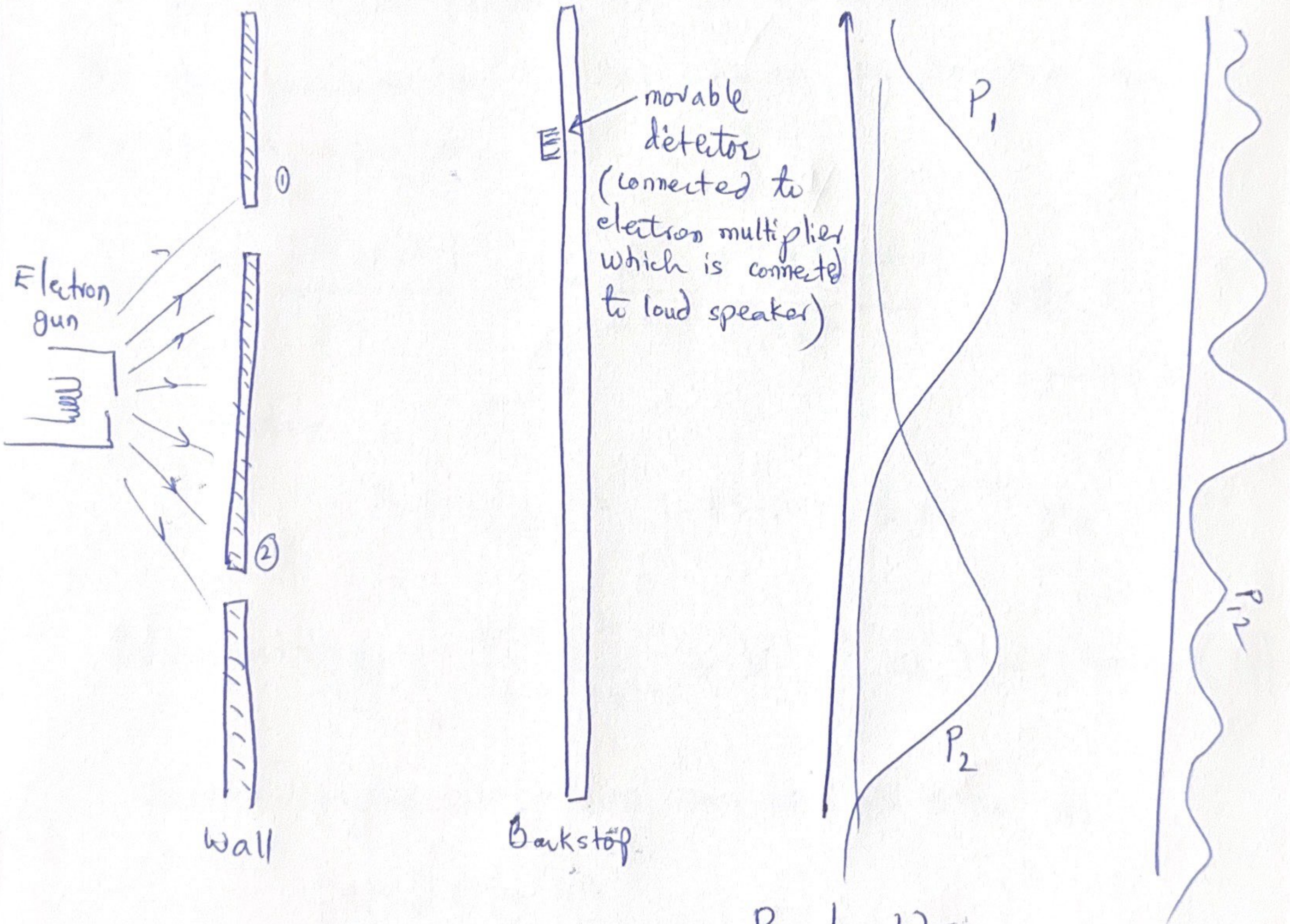
$$I_{12} = |h_1 + h_2|^2 = |h_1|^2 + |h_2|^2 + 2|h_1||h_2| \cos\delta$$

$$= I_1 + I_2 + 2\sqrt{I_1 \cdot I_2} \cos\delta$$

where δ is the phase difference between h_1 and h_2

An interference pattern is observed.

Double slit experiment with electrons:



$$P_1 = |\phi_1|^2$$
$$P_2 = |\phi_2|^2$$
$$P_{12} = |\phi_1 + \phi_2|^2$$

Two important results

1. Electrons arrive in lumps: all "clicks" are of same intensity.
2. Probability of arrival at a point when both slits are open is <u>not</u> a simple sum of probabilities P_1 and P_2; as would be expected for classical processes.

Schrödinger Equation:

A particle wave function in 1D is written as $\psi(x,t)$.
The wave function can be determined by solving the following equation:

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} + V\psi(x,t) \quad \text{—— (1)}$$

where $i = \sqrt{-1}$

$$\hbar = \frac{h}{2\pi} = 1.054573 \times 10^{-34} \text{ Js}$$

Designating $\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right)$ as Hamiltonian (H)

Schrödinger equation can be written as

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = H\psi(x,t) \quad \text{—— (2)}$$

In practice, SE can only be solved for handful of idealized cases - some of these cases we will examine in this course.

When the system's Hamiltonian is time-independent: leading to stationary states with fixed energy levels (E), We can write:

$$H\psi(x) = E\psi(x) \quad \text{—— (3)}$$

Equation (3) is called time-independent Schrödinger equation.