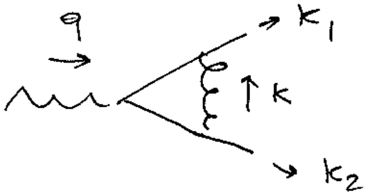
If we neglect the mass of the quarks

$$\sigma_V = \frac{1}{8s}\int \frac{d^3k_1}{(2\pi)^3 2k_{10}}\frac{d^3k_2}{(2\pi)^3 2k_{20}}(2\pi)^4\delta^{(4)}(k_1+k_2-p_1-p_2)\,F_V$$

$$F_V = \left(\sum_i Q_i\right)^2 \frac{e^4}{q^4}\,\mathrm{Tr}[\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu]\,\mathrm{Tr}[\not{k}_1\Lambda_\mu\not{k}_2\gamma_\nu] + \text{c.c.}$$

$$\Lambda_\mu = g_s^2 C_F\int\frac{d^Dk}{(2\pi)^D i}\frac{1}{k^2}\gamma_\lambda\frac{1}{\not{k}-\not{k}_1}\gamma_\mu\frac{1}{\not{k}+\not{k}_2}\gamma^\lambda$$

$\hookrightarrow$ 1-loop vertex



* We can use the Feynman gauge for this calculation

$m_q \to 0 \Rightarrow$ mass singularity $\Rightarrow$ infrared divergences
$\Rightarrow$ we need regularization!

Let's review the unrenormalized 1-loop self-energy contribution for $m_q=0$ in the Feynman gauge

singular for $p^2=0$ (mass singularity)

$$\Sigma(p) = g_{0s}^2 C_F(2-D)\int_0^1 dx\int\frac{d^Dk'}{(2\pi)^D i}\frac{\not{k}'-(1-x)\not{p}}{\{k'^2+x(1-x)p^2\}^2}$$

$$\int^{*}\frac{d^Dk}{k^2} = 0$$ can be justified in the sense of DR.

More general case in DR

$$\int \frac{d^D k}{(-k^2)^\alpha} = 0 \qquad \alpha > 0$$

with a Wick Rotation we obtain

$$\int \frac{d^D k}{(-k^2)^\alpha} = i \frac{\pi^{D/2}}{\Gamma(D/2)} \int_0^{+\infty} (\bar{k}^2)^{D/2-\alpha-1} d\bar{k}^2$$

$$\bar{k}^2 = -k^2$$

$D > 2\alpha \rightarrow$ ultraviolet divergence UV-div

$D < 2\alpha \rightarrow$ infrared divergence IR-div

- No mathematical meaningfull region in D.

We found in previous cases that the integral made sense for $D < 3$. We are going to use the same approach here and split the integration in $\bar{k}$ in two parts:

$$\begin{cases} \bar{k}^2 > \Lambda^2 & \text{ultraviolet part} \\ \bar{k}^2 < \Lambda^2 & \text{infrared part} \end{cases}$$

$$\int \frac{d^D k}{(-k^2)^\alpha} = i \frac{\pi^{D/2}}{\Gamma(D/2)} \left[\int_0^{\Lambda^2} (\bar{k}^2)^{D/2-\alpha-1} d\bar{k}^2 + \int_{\Lambda^2}^{+\infty} (\bar{k}^2)^{D/2-\alpha-1} d\bar{k}^2 \right.$$

1st integral convergent for $D > 2\alpha$

2nd integral convergente for $D < 2\alpha$

D = regulator for IR and UV divergences.

We call $D_I = D$ in the 1st integral and $D_U = D$ in the 2nd.

Integrating, we obtain

$$\int \frac{d^D k}{(-k^2)^\alpha} = i \frac{\pi^{D/2}}{\Gamma(D/2)} \left[\frac{\Lambda^{D_I - 2\alpha}}{\frac{1}{2} D_I - \alpha} - \frac{\Lambda^{D_U - 2\alpha}}{\frac{1}{2} D_U - \alpha} \right]$$

The two terms have poles at $D_I = D_U = 2\alpha$

$\frac{\Lambda^{D_I - 2\alpha}}{\frac{1}{2} D_I - \alpha} - \frac{\Lambda^{D_U - 2\alpha}}{\frac{1}{2} D_U - \alpha}$ can be continued analitically ($i\varepsilon$) and the constraints $D_I > 2\alpha$ and $D_U < 2\alpha$

$\frac{\Lambda^{D_I - 2\alpha}}{\frac{1}{2} D_I - \alpha + i\varepsilon'} - \frac{\Lambda^{D_U - 2\alpha}}{\frac{1}{2} D_U - \alpha + i\varepsilon}$ can be removed.

If D_I and D_U are identified after this analytic continuation, then when $D_I = D_U$ on the right hand side of the equation is zero. $\Rightarrow$

$$\int \frac{d^D k}{(-k)^\alpha} = 0$$

This result is justified within DR.

Therefore, if we look at $\Sigma(p)$ for $p^2=0$

$$\Sigma(p) = g_{s0}^2 C_F (D-2) \not{p} \int_0^1 dx(1-x) \int \frac{d^D k'}{(2\pi)^D i} \frac{1}{\{k'^2 + x(1-x)p^2\}^2}$$

after we have removed $\left(\int d^D k \, k_\mu f(k^2) = 0 \right)$ the linear terms in k_μ,

$$\Sigma(p)\Big|_{p^2=0} = g_{0s}^2 C_F (D-2) \not{p} \underbrace{\int_0^1 d(1-x)}_{1/2} \int \frac{d^D k}{(2\pi)^D i} \frac{1}{k^4}$$

This is like the case we analyzed before with $\alpha = 2$

$$\Sigma(p)\Big|_{p^2=0} = \frac{g_{0s}^2}{(4\pi)^2} C_F \not{p} \left[\frac{1}{\varepsilon'} - \frac{1}{\varepsilon} \right] \Rightarrow \boxed{Z_2 = 1 + \frac{g^2}{(4\pi)^2} C_F \left(\frac{1}{\varepsilon'} - \frac{1}{\varepsilon} \right)}$$

where ε' and ε are equal to the $(4-D)/2$ param.

* We do not set $\varepsilon' = \varepsilon$ here because we want to distinguish between the two divergences

* The integral for Λ_μ can be treated in the same way

$$\Lambda_\mu = \gamma_\mu \frac{g_{0s}^2 C_F}{8\pi^2} \left(\frac{4\pi\mu^2}{-q^2} \right)^\varepsilon \Gamma(1+\varepsilon) B(1-\varepsilon, 2-\varepsilon) \left(\frac{1}{\varepsilon'} - \frac{2}{\varepsilon^2} - 2 \right)$$

$\mu^2 \rightarrow$ mass scale to make g dimensionless.

$q = p_1 + p_2$

The knowledge of the singular structure of $\Sigma(p)$, allows us to determine the field renormalization const. Z_2

The remarkable task of renormalization is accomplished in this case:

$\tilde{\sigma}_V = \sigma_V + \left(Z_2^2 - 1\right)\sigma_B$ all UV div cancel out in $\tilde{\sigma}_V$!

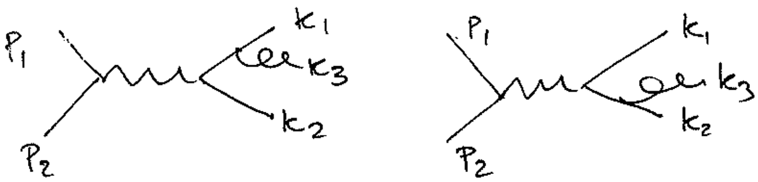
Putting all the ingredients together we obtain

$$\tilde{\sigma}_V = A_V \sigma_B$$

$$A_V = \frac{\alpha_s}{\pi} C_F \left(\frac{4\pi\mu^2}{s}\right)^{\varepsilon} \frac{\cos(\pi\varepsilon)}{\Gamma(1-\varepsilon)} \left[-\frac{1}{\varepsilon^2} - \frac{3}{2\varepsilon} - 4 + O(\varepsilon)\right]$$

where $\alpha_s = \dfrac{g_s^2}{4\pi}$

Real emission corrections



$$\sigma_R = \frac{1}{8s}\int \prod_{i=1}^{3} \frac{d^{D-1}k_i}{(2\pi)^{D-1}\, 2k_{i0}} (2\pi)^D \delta^{(D)}\left(\sum_{i=1}^{3} k_i - p_1 - p_2\right) F_R$$

$$F_R = -\left(\sum_i Q_i^2\right)\frac{e^4}{q^2} g_s^2 C_F \underbrace{\mathrm{Tr}\left[\not{p}_2 \gamma^\mu \not{p}_1 \gamma^\nu\right]}_{L^{\mu\nu}} \underbrace{\mathrm{Tr}\left[\not{k}_1 S_{\lambda\mu} \not{k}_2 S^{\lambda}{}_{\nu}\right]}_{G_{\mu\nu}}$$

$$S_{\mu\nu} = \gamma_\mu \frac{-1}{\not{k}_1 + \not{k}_3}\gamma_\nu + \gamma_\nu \frac{1}{\not{k}_2 + \not{k}_3}\gamma_\mu$$

$$L^{\mu\nu} = \left[4\, p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - \frac{q^2}{2} g^{\mu\nu}\right]$$