# Spontaneous symmetry breaking

The Higgs mechanism: consider first a simplified model

We have a scalar field $\varphi$ and a gauge field $A^\mu$

$\varphi$ is complex $\varphi = (\varphi_1 + i\varphi_2)/\sqrt{2}$

The Lagrangian terms with $\varphi$ are $\mathcal{L}_\varphi = (D_\mu\varphi)^* D^\mu\varphi - m^2\varphi^*\varphi - \lambda(\varphi^*\varphi)^2$

with $D_\mu = \partial_\mu + iqA_\mu$

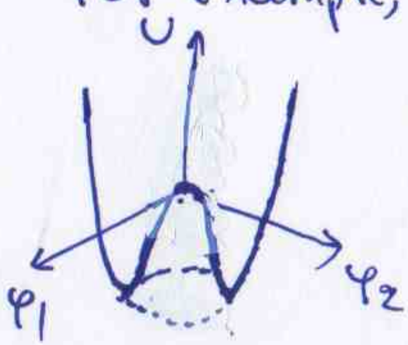
If $m^2 < 0$ then define $\mu^2 = -m^2$ and $\mathcal{L}_\varphi = (D_\mu\varphi)^* D^\mu\varphi + \mu^2\varphi^*\varphi - \lambda(\varphi^*\varphi)^2$

Then $\varphi = 0$ is not the ground state (it is actually a local maximum)

The potential term is $U = -\mu^2\varphi^*\varphi + \lambda(\varphi^*\varphi)^2$ Then $\dfrac{dU}{d\varphi} = 0 \Rightarrow -\mu^2\varphi^* + 2\lambda(\varphi^*)^2\varphi = 0$

$$\Rightarrow \varphi^*\varphi = \frac{\mu^2}{2\lambda} \quad \Rightarrow \frac{1}{2}(\varphi_1^2 + \varphi_2^2) = \frac{\mu^2}{2\lambda} \quad \Rightarrow \varphi_1^2 + \varphi_2^2 = \frac{\mu^2}{\lambda}$$

For example, we can choose $\varphi_2 = 0$ and $\varphi_1 = \mu/\sqrt{\lambda}$




$$U_{min} = -\mu^2\frac{\mu^2}{2\lambda} + \lambda\frac{\mu^4}{4\lambda^2} = -\frac{\mu^4}{4\lambda}$$

We can pick new fields $\eta = \varphi_1 - \frac{\mu}{\sqrt{\lambda}}$ and $\chi = \varphi_2$

so $\varphi = \frac{1}{\sqrt{2}}\left(\eta + \frac{\mu}{\sqrt{\lambda}} + i\chi\right)$ Then $U$ is minimum when $\eta = 0$ and $\chi = 0$

Then $\mathcal{L} = \frac{1}{2}\left[(\partial_\mu - iqA_\mu)(\eta + \frac{\mu}{\sqrt{\lambda}} - i\chi)\right]\left[(\partial^\mu + iqA^\mu)(\eta + \frac{\mu}{\sqrt{\lambda}} + i\chi)\right]$

$$+ \frac{\mu^2}{2}(\eta + \frac{\mu}{\sqrt{\lambda}} - i\chi)(\eta + \frac{\mu}{\sqrt{\lambda}} + i\chi) - \frac{\lambda}{4}\left[(\eta + \frac{\mu}{\sqrt{\lambda}} - i\chi)(\eta + \frac{\mu}{\sqrt{\lambda}} + i\chi)\right]^2$$

or $\mathcal{L} = \frac{1}{2}\left[\partial_\mu\eta - i\partial_\mu\chi - iqA_\mu\eta - iqA_\mu\frac{\mu}{\sqrt{\lambda}} - qA_\mu\chi\right]\left[\partial^\mu\eta + i\partial^\mu\chi + iqA^\mu\eta + iqA^\mu\frac{\mu}{\sqrt{\lambda}} - qA^\mu\chi\right]$

$$+ \frac{\mu^2}{2}\left[(\eta + \frac{\mu}{\sqrt{\lambda}})^2 + \chi^2\right] - \frac{\lambda}{4}\left[(\eta + \frac{\mu}{\sqrt{\lambda}})^2 + \chi^2\right]^2$$

or $\mathcal{L} = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{1}{2}q^2A_\mu A^\mu\left[(\eta + \frac{\mu}{\sqrt{\lambda}})^2 + \chi^2\right] + qA^\mu(\eta + \frac{\mu}{\sqrt{\lambda}})\partial_\mu\chi$

$$- qA^\mu\chi\partial_\mu\eta + \frac{\mu^2}{2}\left[(\eta + \frac{\mu}{\sqrt{\lambda}})^2 + \chi^2\right] - \frac{\lambda}{4}\left[(\eta + \frac{\mu}{\sqrt{\lambda}})^2 + \chi^2\right]^2$$

After some further algebra, we find

$$\mathcal{L} = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \mu^2\eta^2 + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{q^2\mu^2}{2\lambda}A_\mu A^\mu + \frac{\mu^4}{4\lambda} + \frac{1}{2}q^2A_\mu A^\mu\left[\eta^2 + \frac{2\eta\mu}{\sqrt{\lambda}} + \chi^2\right]$$

$$+ qA^\mu(\eta\partial_\mu\chi - \chi\partial_\mu\eta + \frac{\mu}{\sqrt{\lambda}}\partial_\mu\chi) - \frac{\lambda}{4}\left[\eta^4 + \chi^4 + 4\eta^3\frac{\mu}{\sqrt{\lambda}} + 2\eta^2\chi^2 + 4\eta\frac{\mu\chi^2}{\sqrt{\lambda}}\right]$$

No mass term for $\chi$. So $\chi$ is Goldstone boson: the spontaneous breaking of a symmetry entails the existence of a massless particle

But the gauge field $A^\mu$ has acquired a mass: $\frac{1}{2}\frac{q^2\mu^2}{\lambda}A_\mu A^\mu \Rightarrow m_A = \frac{q\mu}{\sqrt{\lambda}}$

We can use local gauge invariance to eliminate the field $\chi$

$$\varphi \to e^{i\theta}\varphi = (\cos\theta + i\sin\theta)\frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2) = \frac{1}{\sqrt{2}}\left[(\varphi_1\cos\theta - \varphi_2\sin\theta) + i(\varphi_1\sin\theta + \varphi_2\cos\theta)\right]$$

If we pick $\tan\theta = -\frac{\varphi_2}{\varphi_1}$ then the imaginary part vanishes so $\varphi_2 = \chi = 0$

Then $\mathcal{L} = \frac{1}{2}\partial_\mu\eta\,\partial^\mu\eta - \mu^2\eta^2 + \frac{1}{2}m_A^2 A_\mu A^\mu + \frac{q^2}{2}A_\mu A^\mu\left(\eta^2 + 2\frac{\mu}{\sqrt{\lambda}}\eta\right) - \frac{\lambda}{4}\left(\eta^4 + 4\frac{\mu}{\sqrt{\lambda}}\eta^3\right) + \frac{\mu^4}{4\lambda}$

So we eliminated the massless Goldstone boson $\chi$, and the gauge field $A^\mu$ became massive: the gauge field "ate" the Goldstone boson and got mass

We also have a massive scalar Higgs particle $\eta$

The mechanism outlined in this simple model generalizes to electroweak theory: the $W^+$, $W^-$, and $Z$ gauge fields become massive. However, the photon remains massless.

Furthermore, the quarks and leptons also acquire masses.

# Higgs mechanism in electroweak GWS model

Write the Higgs field in the form $\varphi(x) = \begin{pmatrix} 0 \\ \rho + \frac{1}{\sqrt{2}}\phi(x) \end{pmatrix}$ with $\rho = \frac{\mu}{\sqrt{2\lambda}}$

Then $D_\mu \varphi = \partial_\mu \varphi + \frac{i}{2} g \vec{\sigma} \cdot \vec{W}_\mu \varphi + \frac{i}{2} g' B_\mu \varphi$

$$= \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \partial_\mu \phi \end{pmatrix} + \frac{i}{2} g \begin{pmatrix} W_\mu^3 & W_\mu^1 - i W_\mu^2 \\ W_1^\mu + i W_\mu^2 & -W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ \rho + \frac{\phi}{\sqrt{2}} \end{pmatrix} + \frac{i}{2} g' B_\mu \begin{pmatrix} 0 \\ \rho + \frac{\phi}{\sqrt{2}} \end{pmatrix}$$

$$= \frac{i}{2} \begin{pmatrix} g\rho (W_\mu^1 - i W_\mu^2) + \frac{g\phi}{\sqrt{2}} (W_\mu^1 - i W_\mu^2) \\ -i\sqrt{2}\, \partial_\mu \phi + \rho(-g W_\mu^3 + g' B_\mu) + \frac{\phi}{\sqrt{2}} (-g W_\mu^3 + g' B_\mu) \end{pmatrix}$$

and $(D_\mu \varphi)^\dagger (D^\mu \varphi) = \frac{1}{2} \partial_\mu \phi^\dagger \partial^\mu \phi + \frac{1}{4} g^2 \rho^2 (W_\mu^1 W^{\mu 1} + W_\mu^2 W^{\mu 2})$

$$+ \frac{\rho^2}{4} (g W_\mu^3 - g' B_\mu)(g W^{\mu 3} - g' B^\mu) + \dots$$

Next we define $Z_\mu = \dfrac{g W_\mu^3 - g' B_\mu}{(g^2 + g'^2)^{1/2}} = \cos\theta_W\, W_\mu^3 - \sin\theta_W\, B_\mu$

and $A_\mu = \dfrac{g' W_\mu^3 + g B_\mu}{(g^2 + g'^2)^{1/2}} = \sin\theta_W\, W_\mu^3 + \cos\theta_W\, B_\mu$ where $\tan\theta_W = \frac{g'}{g}$ Weinberg angle $\theta_W$

Then $W_\mu^1, W_\mu^2, Z_\mu$ become massive while $A_\mu$ (photon) is massless

$m_{W_1} = m_{W_2} = g\rho/\sqrt{2}$ and $m_Z = m_W / \cos\theta_W$ Also $W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \pm i W_\mu^2)$