Feynman rules for electroweak theory

Many more interactions and rules since we have non-abelian gauge fields, symmetry breaking via a Higgs field, and massive vector bosons

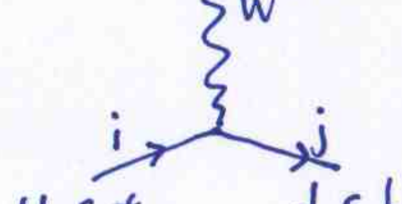
Proca equation for massive gauge bosons $\partial_\mu F^{\mu\nu} + m_A^2 A^\nu = 0$

reduces to Maxwell eq. when $m_A = 0$

Propagator for massive gauge bosons $\dfrac{-i\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}\right]}{p^2 - m^2 + i\varepsilon}$

$\dfrac{-ie}{2\sqrt{2}\sin\theta_W}\gamma^\mu(1-\gamma^5)$

same for μ^-, ν_μ
and τ^-, ν_τ



$\dfrac{-ie}{2\sqrt{2}\sin\theta_W}\gamma^\mu(1-\gamma^5)V_{ij}$

where V_{ij} is element of CKM matrix
mixing of quark generations

$\dfrac{-ie}{\sin(2\theta_W)}\gamma^\mu(c_V^f - c_A^f\gamma^5)$ where $c_V = -\frac{1}{2} + 2\sin^2\theta_W$, $c_A = -\frac{1}{2}$ for e^-, μ^-, τ^-

$c_V = c_A = \frac{1}{2}$ for ν_e, ν_μ, ν_τ

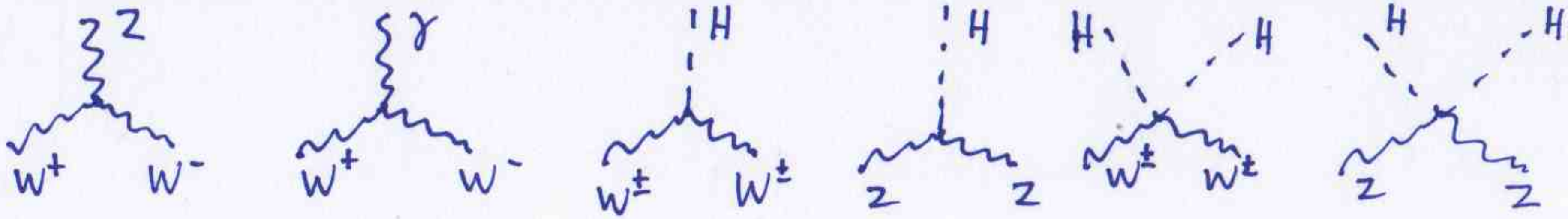
$c_V = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W$, $c_A = \frac{1}{2}$ for u, c, t

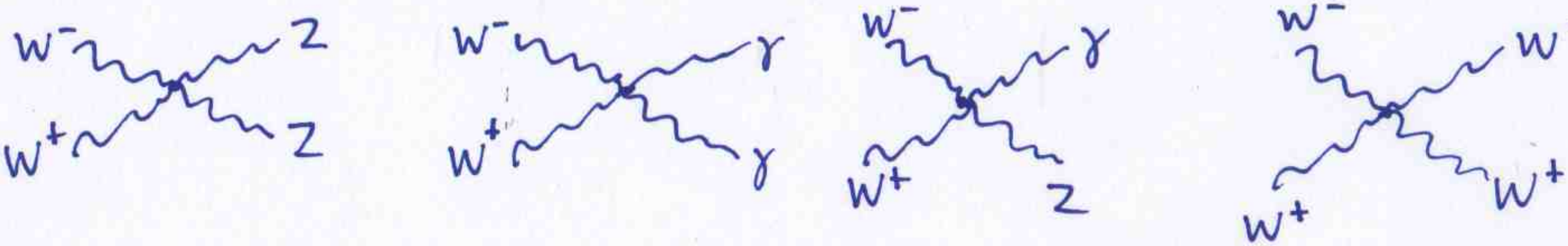
$c_V = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$, $c_A = -\frac{1}{2}$ for d, s, b

where fermion f is quark or lepton

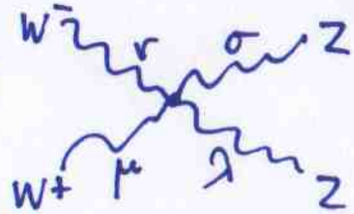
no flavor-changing neutral currents in the Standard Model

Many diagrams with interactions (including self-interactions) of the electroweak gauge bosons with each other and with the Higgs boson



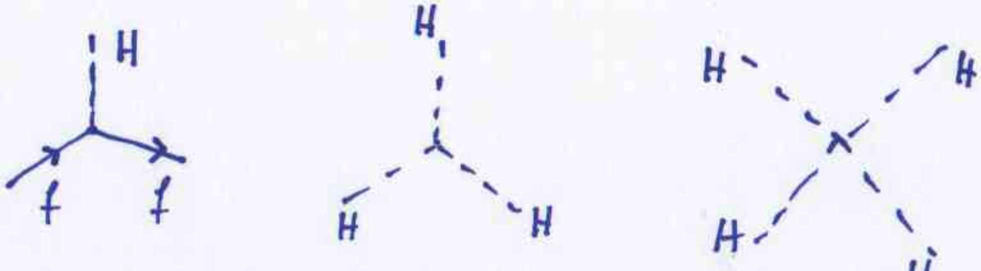


They are complicated. For example



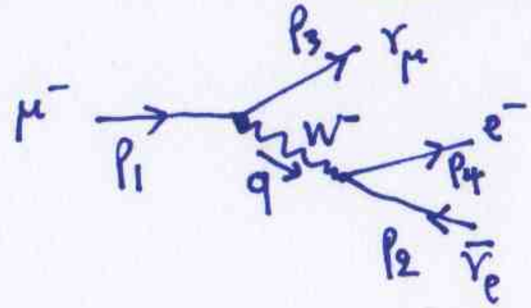
$$-ie^2\cot^2\theta_W\left(2g_{\mu\nu}g_{\lambda\sigma}-g_{\mu\lambda}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\lambda}\right)$$

Also



We note that lepton number is conserved in all Standard Model processes.

Muon decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$



$p_1 = p_2 + p_3 + p_4$ and $q = p_1 - p_3 = p_2 + p_4$

If $\mathcal{M}$ is the amplitude for this decay

then
$$i\mathcal{M} = \bar{u}(p_3)\frac{(-ie)\gamma^\mu(1-\gamma^5)}{2\sqrt{2}\sin\theta_W}u(p_1)\,(-i)\frac{\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}\right)}{q^2 - m_W^2}\,\bar{u}(p_4)\frac{(-ie)\gamma^\nu(1-\gamma_5)}{2\sqrt{2}\sin\theta_W}v(p_2)$$

$$\Rightarrow \mathcal{M} = \frac{e^2}{8\sin^2\theta_W\,[(p_1-p_3)^2 - m_W^2]}\,\bar{u}(p_3)\gamma^\mu(1-\gamma^5)u(p_1)\left(g_{\mu\nu} - \frac{(p_1-p_3)_\mu(p_1-p_3)_\nu}{m_W^2}\right)\bar{u}(p_4)\gamma^\nu(1-\gamma_5)v(p_2)$$

For $q^2 \ll m_W^2$ we can write the W^- propagator as $i g_{\mu\nu}/m_W^2$

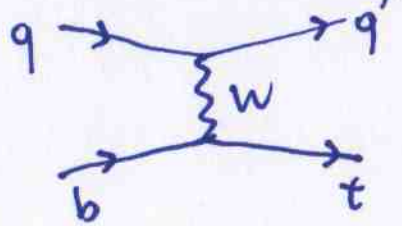
Then
$$\mathcal{M} = -\frac{e^2}{8\sin^2\theta_W\,m_W^2}\,\bar{u}(p_3)\gamma^\mu(1-\gamma^5)u(p_1)\,\bar{u}(p_4)\gamma_\mu(1-\gamma^5)v(p_2)$$

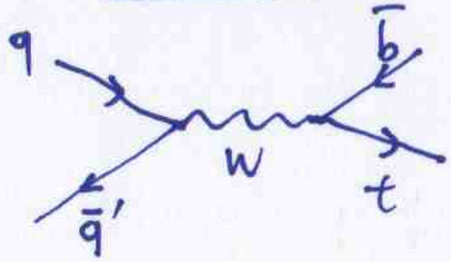
After some work we find
$$|\mathcal{M}|^2 = \frac{2e^4}{\sin^4\theta_W\,m_W^4}(p_1\cdot p_2)(p_3\cdot p_4)$$

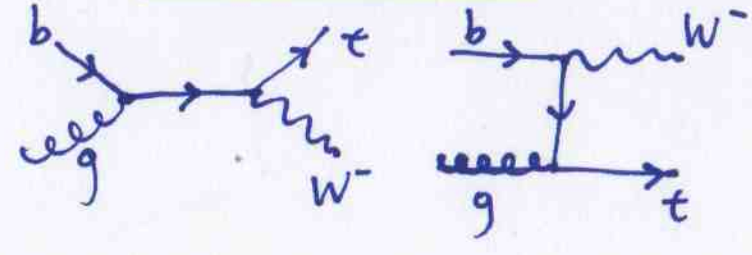
Decay rate
$$d\Gamma = \frac{|\mathcal{M}|^2}{2E_1}(2\pi)^4\delta^4(p_1 - p_2 - p_3 - p_4)\prod_{i=2}^{4}\frac{d^3p_i}{(2\pi)^3 2E_i}$$

Single top-quark production

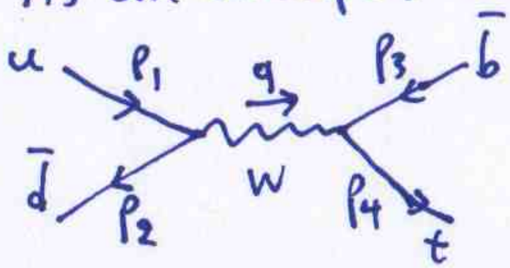
t-channel   **s-channel**   **tW production**







As an example, consider the s-channel process $u(p_1) + \bar{d}(p_2) \to \bar{b}(p_3) + t(p_4)$



$p_1 + p_2 = p_3 + p_4$ and $q = p_1 + p_2 = p_3 + p_4$

For the amplitude $\mathcal{M}$ for this process we have

$$i\mathcal{M} = \bar{u}(p_4)\,\frac{(-ie)\,\gamma^\mu(1-\gamma^5)\,V_{tb}}{2\sqrt{2}\,\sin\theta_W}\,v(p_3)\,(-i)\,\frac{\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}\right)}{q^2 - m_W^2}\,\bar{v}(p_2)\,\frac{(-ie)\,\gamma^\nu(1-\gamma^5)\,V_{ud}}{2\sqrt{2}\,\sin\theta_W}\,u(p_1)$$

$$\Rightarrow \mathcal{M} = \frac{e^2 V_{tb} V_{ud}}{8\sin^2\theta_W\,[(p_1+p_2)^2 - m_W^2]}\,\bar{u}(p_4)\gamma^\mu(1-\gamma^5)v(p_3)\left(g_{\mu\nu} - \frac{(p_1+p_2)_\mu (p_1+p_2)_\nu}{m_W^2}\right)\bar{v}(p_2)\gamma^\nu(1-\gamma^5)u(p_1)$$

After some work, we find $|\mathcal{M}|^2 = 4\pi^2 \frac{\alpha^2 V_{tb}^2 V_{ud}^2}{\sin^4\theta_W}\,\frac{t(t-m_t^2)}{(s-m_W^2)^2}$ with $\alpha = \frac{e^2}{4\pi}$

and $\frac{d\sigma}{dt} = \frac{|\mathcal{M}|^2}{16\pi s^2}$