PHYS 4500 Fall 2024
Test 2

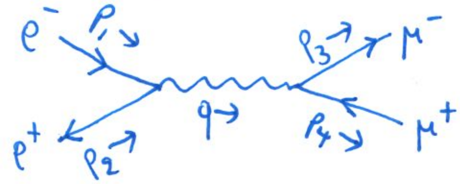
You may use the lecture notes, your homework, and the textbooks but no other resources or materials.

1) Show that for the Lorenz gauge Lagrangian, $\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial_\mu A^\mu)^2$, the zeroth component of the conjugate momentum is given by $\pi^0 = -\partial_\mu A^\mu$.

2) Show explicitly that the QED Lagrangian, $\mathcal{L}_{QED} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi - q\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, is invariant under the local gauge transformation $\psi(x) \to e^{i\theta(x)}\psi(x)$, $A_\mu(x) \to A_\mu(x) - \frac{1}{q}\partial_\mu\theta(x)$

3) Using the standard representation for the Dirac γ matrices, and the expressions for the Dirac spinors $u^{(1)}$ and $u^{(2)}$, show explicitly that $\sum_{s=1,2} u^{(s)}(p)\,\bar{u}^{(s)}(p) = \not{p} + m$

4) For the process $e^-(p_1) + e^+(p_2) \to \mu^-(p_3) + \mu^+(p_4)$ with Feynman diagram



a) write down the amplitude $\mathcal{M}$

b) write the squared amplitude $|\mathcal{M}|^2$ and do the sum over final spins and average over initial spins to get an expression involving the product of two traces

c) Calculate the traces and write $|\mathcal{M}|^2$ in terms of scalar products of 4-momenta and the masses of the electron and muon.

d) Write $d\sigma/dt$ for this process in terms of the Mandelstam variables.