PHYS 4210/Fall 2024
Class Quiz (graded)
Date: Oct 24, 2024
Time: 10 minutes

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(a) For the same question shown in class, find $< \hat{s}_y >$.

In this basis $\rightarrow \langle \hat{S}_y \rangle = \langle \chi | \hat{S}_y | \chi \rangle$

where $\chi = a\chi_+ + b\chi_-$ w/ $\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Now,

$$\langle \chi | \hat{S}_y | \chi \rangle = \frac{\hbar}{2} \begin{pmatrix} a^* & b^* \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$= \frac{\hbar}{2} \begin{pmatrix} ib^* & -ia^* \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$= \frac{\hbar}{2} i \left(ab^* - a^* b \right)$$

- For a generic complex number $z = a + ib$, we know

$$-i \frac{z - z^*}{2} = b = \mathrm{Im}(z),$$

so $$-i \frac{ab^* - a^* b}{2} = \mathrm{Im}(ab^*).$$

Thus, $$\boxed{\langle \hat{S}_y \rangle = -\hbar \, \mathrm{Im}(ab^*)}$$