4. Modified Minimal subtraction ($\overline{MS}$)

In the expression for $\Gamma(p^2)$ the pole term is accompanied by γ_E and $\ln 4\pi$

$$\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi$$

It can be shown that this combination always appears in any calculation at 1-loop order.

$\Rightarrow$ more convenient to eliminate the whole factor in the renormalization process, instead of only eliminating $1/\varepsilon$. This procedure/prescription goes under the name of modified minimal subtr.

The renormalization constant in this ($\overline{MS}$) scheme is given by

$$Z_2 = \alpha \frac{g_{0s}^2}{(4\pi)^2} C_F \left(\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi \right)$$

The renormalized propagator reads

$$\tilde{S}_{R\,ij}(p) = -\frac{\delta_{ij}}{\not p} \left\{ 1 - \alpha \frac{g_{0s}^2}{(4\pi)^2} C_F \left(-1 + \ln\left(-\frac{p^2}{\mu^2} \right) \right) \right\}^{-1}$$

- $\overline{MS}$ $\rightarrow$ many advantages $\rightarrow$ compact expression for the renormalized propagator.

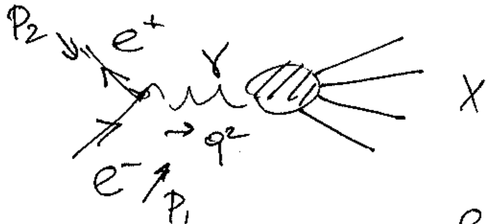
The Feynman parametrization: general formula

$$\prod_{i=1}^{n} \frac{1}{A_i^{\alpha_i}} = \frac{\Gamma(\alpha)}{\prod_{l=1}^{n} \Gamma(\alpha_i)} \int_0^1 \left(\prod_{i=1}^{n} dx_i \, x_i^{\alpha_i - 1} \right) \frac{\delta(1-x)}{\left(\sum_{i=1}^{n} x_i A_i \right)^{\alpha}}$$

Electron-positron annihilation

We'll review e^+e^- annihilation and the computation of the total X sec.

* e^+e^- annihilate through EM interactions producing hadrons in the final state
* Z_0 will not be considered for simplicity for now



X = system of hadrons

$$e^+ + e^- \to X$$

Feyman Amplitude

$$\langle X | T | e^+e^- \rangle = \bar{v}_{\lambda_2}(p_2)\, e\, \gamma^\mu\, u_{\lambda_1}(p_1) \, *$$
$$* \frac{1}{q^2} \langle X | (-e) J_\mu(0) | 0 \rangle$$

p_1, p_2 = incoming momenta of $e^+ e^-$

λ_1, λ_2 = spins of the incoming $e^+ e^-$.

$J_\mu(x)$ = quark part of the EM current.

$$\mathcal{L}_1 = (-e \bar{\psi}_e \gamma_\mu \psi_e + e J_\mu) A^\mu$$ } Little digression

$|X\rangle$ = state representing the final hadron system

Useful properties for current-current products

Completeness $\sum_X |X\rangle\langle X| = 1$

translation invariance

$$J_\mu(x) = e^{iP\cdot x} J_\mu(0)\, e^{-iP\cdot x}$$

where P is the energy-momentum operator which satisfies the eigenvalue equation

$$P^\mu |X\rangle = p_X^\mu |X\rangle$$

We indicate with q the the total momentum

$$q = p_1 + p_2$$

The total Xsec for $e^+ + e^- \to X$ can be written as

$$\sigma = \frac{1}{2s}\,\frac{1}{4}\sum_{\lambda_1,\lambda_2}\sum_X (2\pi)^4\,\delta^4(p_X - q)\,|\langle X|T|e^+e^-\rangle|^2$$

T is the operator that allows for the transition

* we are going to neglect the electron mass

$$s = q^2 = (p_1+p_2)^2 = 2m_e^2 + 2p_1\cdot p_2 \simeq 2p_1\cdot p_2$$

General formula for $\sigma \longrightarrow$

$$\sigma = \frac{1}{k(s)} \frac{1}{(2J_1+1)(2J_2+1)} \sum_{\substack{\lambda_1 \lambda_2, \mu_1 \ldots \mu_n \\ (\text{spin})}} \int \prod_{j=1}^{n} \frac{d^3 k_j}{(2\pi)^3 2k_j^0} (2\pi)^4 \delta^{(4)}\left(\sum_{j=1}^{n} k_j - p_1 - p_2\right) *$$

$$* \left| \langle k_1 \mu_1, k_2 \mu_2, \ldots | T | p_1 \lambda_1, p_2 \lambda_2 \rangle \right|^2$$

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$$k(s) = \sqrt{\left[s - (m_1 + m_2)^2\right]\left[s - (m_1 - m_2)^2\right]}$$

Inserting the expression for $\langle X | T | e^+e^- \rangle$ into that of the X sec σ we obtain

$$\sigma = \frac{e^4}{2s^3} \, l^{\mu\nu} w_{\mu\nu}$$

$l^{\mu\nu}$ = leptonic tensor

$$l^{\mu\nu} = p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - \frac{q^2}{2} g^{\mu\nu}$$

$w^{\mu\nu}$ = hadronic tensor

$$w_{\mu\nu} = \sum_X (2\pi)^4 \delta^{(4)}(p_X - q) \langle 0 | J_\mu(0) | X \rangle \langle X | J_\nu(0) | 0 \rangle$$

This can be rewritten in a more compact form. using the completeness relation over $|X\rangle$, translation invariance and properties of the Fourier transf.

We observe that in general for a physical process

$$\int d^4x \, e^{iq\cdot x} \langle p | J_\nu(0) J_\mu(x) | p \rangle = 0$$

where E = initial energy (0-component of q^μ, or q^0)

E' = final-state energy

Physical process $\Rightarrow E > E' \Rightarrow q_0 > 0$

Using translation invariance we obtain

$$\int d^4x\, e^{iq\cdot x} \langle p|\, j_\nu(0)\, e^{i\hat{P}\cdot x} j_\mu(0)\, e^{-i\hat{P}\cdot x}\, |p\rangle$$

and using the completeness relation

$$\sum_X \int d^4x\, e^{iq\cdot x} \langle p|\, j_\nu(0)\, e^{i\hat{P}x} |X\rangle\langle X|\, j_\mu(0) e^{-i\hat{P}\cdot x}\, |p\rangle$$

Eigenvalue equs:

$$\begin{cases} e^{i\hat{P}\cdot x}|X\rangle = e^{ip_X\cdot x}|X\rangle \\ e^{-i\hat{P}\cdot x}|p\rangle = e^{-ip\cdot x}|p\rangle \end{cases}$$

Therefore we can write

$$\sum_X \int d^4x\, e^{iq\cdot x} e^{ip_X\cdot x} e^{-ip x} \langle p| j_\nu(0)|X\rangle\langle X| j_\mu(0)|p\rangle =$$

$$\sum_X (2\pi)^4 \delta^{(4)}(q-p+p_X) \langle p| j_\nu(0)|X\rangle\langle X| j_\mu(0)|p\rangle$$

For e^+e^- we found an expression with $p=0$

and with $\delta(p_X - q)$.

$$\sum_X (2\pi)^4 \delta^4(p_X - q) \langle 0 | J_\mu(0) | X \rangle \langle X | J_\nu(0) | 0 \rangle =$$

$$\sum_X \int e^{i(q - p_X)\cdot x} \langle 0 | J_\mu(0) | X \rangle \langle X | J_\nu(0) | 0 \rangle \, d^4x =$$

$$\sum_X \int e^{iq\cdot x} \langle 0 | J_\mu(0) e^{-ip_X\cdot x} | X \rangle \langle X | J_\nu(0) | 0 \rangle \, d^4x =$$

$$\langle 0 | J_\mu(x) | X \rangle = \langle 0 | e^{i0\cdot x} J_\mu(0) e^{-ip_X\cdot x} | X \rangle \Rightarrow$$

$$= \int e^{iqx} \langle 0 | J_\mu(x) J_\nu(0) | 0 \rangle \, d^4x$$

We can prove that

$$\int e^{iq\cdot x} \langle 0 | J_\nu(0) J_\mu(x) | 0 \rangle \, d^4x = 0$$

<u>Again</u>

$$\int e^{iq\cdot x} \langle 0 | J_\nu(0) J_\mu(x) | 0 \rangle \, d^4x =$$

$$\sum_X \int e^{iq\cdot x} \langle 0 | J_\nu(0) | X \rangle \langle X | e^{ip_X\cdot x} J_\mu(0) | 0 \rangle \, d^4x =$$

$$\sum_X (2\pi)^4 \delta^{(4)}(q + p_X) \langle 0 | J_\nu(0) | X \rangle \langle X | J_\mu(0) | 0 \rangle$$