- general covariant gauge:

$$\Sigma(p) = g_s^2 C_F \int \frac{d^D k}{(2\pi)^D i} \frac{1}{k^2 (k-p)^2} \left\{ \gamma_\mu (\not{k} - \not{p}) \gamma^\mu - (1-\alpha) \frac{\not{k}(\not{k}-\not{p})\not{k}}{k^2} \right\}$$

- two components, one proportional to $(1-\alpha)$, the other we already calculated:

$$\Sigma(p) = \Sigma_1(p) - (1-\alpha)\Sigma_2(p),$$

where

$$\Sigma_2(p) = g_s^2 C_F \int \frac{d^D k}{(2\pi)^D i} \frac{\not{k}(\not{k}-\not{p})\not{k}}{(k^2)^2 (k-p)^2}$$

- We use a Feynman Parametrization:

$$\frac{1}{AB^2} = 2 \int_0^1 dx \frac{(1-x)}{\{xA + (1-x)B\}^3}$$

here, $k^2 = B$ $\left(\text{so } (k^2)^2 = B^2\right)$, and $(k-p)^2 = A$:

$$\frac{1}{(k^2)^2 (k-p)^2} = 2 \int_0^1 dx \frac{(1-x)}{\underbrace{[x(k-p)^2 + (1-x)k^2]}_{\text{just examine this guy}}{}^3}$$

$$= x(k^2 - 2kp + p^2) + k^2 - xk^2$$

$$= xk^2 - 2kxp + xp^2 + k^2 - xk^2$$

$$= k^2 - 2kxp + xp^2$$

$$= k^2 - 2kxp + x^2p^2 - x^2p^2 + xp$$

$$= (k - xp)^2 - x(x-1)p^2$$

$$= -\left[-(k-xp)^2 - x(1-x)p^2\right]$$

- with $L = -x(1-x)p^2$, we have (we can pull the minus out since denominator is cubed:

$$\frac{1}{(k^2)^2 (k-p)^2} = -2 \int_0^1 dx \frac{(1-x)}{[-(k-xp)^2 + L]^3}$$

- so,

$$\Sigma_2(p) = -2g_s^2 C_F \int_0^1 dx\,(1-x) \int \frac{d^D k}{(2\pi)^D i} \frac{\not{k}(\not{k}-\not{p})\not{k}}{[-(k-xp)^2+L]^3}$$

- after $k \to k' = k - xp$, some tricks, and a Wick rotation, we arrive at:

$$\Sigma_2(p) = 2g_s^2 C_F \not{p} \int_0^1 dx\,(1-x) \int \frac{d^D \bar{k}}{(2\pi)^D}$$

$$\times \left\{ \frac{L}{(\bar{k}^2+L)^3}\left[\frac{2(1-x)}{D} - 1 - 2x\right] - \frac{1}{(\bar{k}^2+L)^2}\left[\frac{2(1-x)}{D} - 1 - x\right] \right\}$$

- we have 2 $\bar{k}$ integrations. We will need the general result

$$\int \frac{d^D \bar{k}}{(2\pi)^D} \frac{1}{(\bar{k}^2+L)^a} = \frac{\Gamma(a - D/2)}{(4\pi)^{D/2}\Gamma(a)} L^{D/2-a}$$

for $a \in \mathbb{C}$, $\mathrm{Re}[a] > 0$. For $a = 2, 3$, this is satisfied (imaginary part is just zero), so

$$\int \frac{d^D \bar{k}}{(2\pi)^D} \frac{1}{(\bar{k}^2+L)^2} = \frac{\Gamma(2 - D/2)}{(4\pi)^{D/2}\Gamma(2)} L^{D/2-2}$$

and
$$\int \frac{d^D \bar{k}}{(2\pi)^D} \frac{1}{(\bar{k}^2+L)^3} = \frac{\Gamma(3 - D/2)}{(4\pi)^{D/2}\Gamma(3)} L^{D/2-3}$$

now, $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\,dt$, so

$$\Gamma(2) = \int_0^\infty t e^{-t}\,dt = -te^{-t}\Big]_0^\infty + \int_0^\infty e^{-t}\,dt$$
$$= -\left[e^{-t}\right]_0^\infty = 1$$

and $\Gamma(3) = \int_0^\infty t^2 e^{-t}\,dt = -t^2 e^{-t}\Big]_0^\infty + 2\int_0^\infty t\, e^{-t}\,dt = 2.$

- Further, $\Gamma(3 - D/2) = (2 - D/2)\Gamma(2 - D/2)$ (from $\Gamma(z+1) = z\Gamma(z)$)

- Putting this together,

$$\Sigma_2(p) = \frac{2g_s^2}{(4\pi)^{D/2}} C_F \not{p}\, \Gamma\left(2-\frac{D}{2}\right) \int_0^1 dx\, (1-x)$$

$$\times \left\{ \frac{(2-D/2)\, L^{D/2-2}}{2} \left[\frac{2(1-x)}{D} - 1 - 2x\right] - L^{D/2-2} \left[\frac{2(1-x)}{D} - 1 - x\right] \right\}$$

- one integration term for x will be

① $$\frac{(2-D/2)}{2} \int_0^1 dx\, (1-x)\, L^{D/2-2} \left[\frac{2(1-x)}{D} - 1 - 2x\right]$$

which will have 3 more terms:

$$\underset{①}{\frac{2}{D}\int_0^1 dx\, (1-x)^2 L^{D/2-2}} - \underset{②}{\int_0^1 dx\, (1-x) L^{D/2-2}} - \underset{③}{2\int_0^1 dx\, x(1-x) L^{D/2-2}}$$

① $$\frac{2(-p^2)^{D/2-2}}{D} \int_0^1 dx\, x^{D/2-2} (1-x)^{D/2} = \frac{2}{D}(-p^2)^{D/2-2} B\left(\frac{D}{2}-1, \frac{D}{2}+1\right)$$

② $$-(-p^2)^{D/2-2} \int_0^1 dx\, x^{D/2-2} (1-x)^{D/2-1} = -(-p^2)^{D/2-2} B\left(\frac{D}{2}-1, \frac{D}{2}\right)$$

③ $$-2(-p^2)^{D/2-2} \int_0^1 dx\, x^{D/2-1} (1-x)^{D/2-1} = -2(-p^2)^{D/2-2} B\left(\frac{D}{2}, \frac{D}{2}\right)$$

so,

① $$= \frac{2-D/2}{2} (-p^2)^{D/2-2} \left[\frac{2}{D} B\left(\frac{D}{2}-1, \frac{D}{2}+1\right) - B\left(\frac{D}{2}-1, \frac{D}{2}\right) - 2B\left(\frac{D}{2}, \frac{D}{2}\right)\right]$$

The other main integration term is nearly identical:

② $$\int_0^1 dx\, (1-x)\, L^{D/2-2} \left[\frac{2(1-x)}{D} - 1 - x\right]$$

$$= (-p^2)^{D/2-2} \left[\underbrace{\frac{2}{D} B\left(\frac{D}{2}-1, \frac{D}{2}+1\right) - B\left(\frac{D}{2}-1, \frac{D}{2}\right)} - B\left(\frac{D}{2}, \frac{D}{2}\right)\right]$$

let's fix ↗ to be in terms of ↗

$$B\left(\frac{D}{2}-1, \frac{D}{2}+1\right) = \frac{\Gamma\left(\frac{D}{2}-1\right)\Gamma\left(\frac{D}{2}+1\right)}{\Gamma\left(\frac{D}{2}+\frac{D}{2}\right)} = \frac{\frac{D}{2}}{\frac{D}{2}-1} \frac{\Gamma\left(\frac{D}{2}\right)\Gamma\left(\frac{D}{2}\right)}{\Gamma\left(\frac{D}{2}+\frac{D}{2}\right)}$$

$$= \frac{D/2}{D/2-1} B\left(\frac{D}{2}, \frac{D}{2}\right)$$

$$B\left(\frac{D}{2}-1, \frac{D}{2}\right) = \frac{\Gamma\left(\frac{D}{2}-1\right)\Gamma\left(\frac{D}{2}\right)}{\Gamma\left(\frac{D}{2}+\frac{D}{2}-1\right)} = \frac{D-1}{\frac{D}{2}-1} \frac{\Gamma\left(\frac{D}{2}\right)\Gamma\left(\frac{D}{2}\right)}{\Gamma\left(\frac{D}{2}+\frac{D}{2}\right)}$$

$$= \frac{D-1}{\frac{D}{2}-1} B\left(\frac{D}{2}, \frac{D}{2}\right)$$

So,

$$① = \frac{2-D/2}{2}(-p^2)^{D/2-2} B\left(\frac{D}{2},\frac{D}{2}\right)\left[\frac{1}{D/2-1} - \frac{D-1}{D/2-1} - 2\right] \quad \text{and}$$

$$② = (-p^2)^{D/2-2} B\left(\frac{D}{2},\frac{D}{2}\right)\left[\frac{1}{D/2-1} - \frac{D-1}{D/2-1} - 1\right]$$

So,

$$\Sigma_2(p) = \frac{2g_s^2}{(4\pi)^{D/2}} C_F \not{p}\, \Gamma\left(2-\frac{D}{2}\right)(-p^2)^{D/2-2} B\left(\frac{D}{2},\frac{D}{2}\right)$$
$$\times \left[\frac{2-D/2}{2}\left(\frac{1}{D/2-1} - \frac{D-1}{D/2-1} - 2\right) - \left(\frac{1}{D/2-1} - \frac{D-1}{D/2-1} - 1\right)\right]$$

$$\frac{2-D}{D/2-1} - \frac{(D-2)}{D/2-1} = \frac{4(1-D/2)}{D/2-1} = -4$$

$$\frac{2-D}{D/2-1} - \frac{D/2-1}{D/2-1} = \frac{3\left(1-\frac{D}{2}\right)}{D/2-1} = -3$$

$$= \frac{2g_s^2}{(4\pi)^{D/2}} C_F \not{p}(-p^2)^{D/2-2} B\left(\frac{D}{2},\frac{D}{2}\right)\Gamma\left(2-\frac{D}{2}\right) \cdot \left[-2(2-D/2)+3\right]$$

$$-4+D+3 = D-1$$

$$\Rightarrow \boxed{\Sigma_2(p) = \frac{2g_s^2}{(4\pi)^{D/2}} C_F \not{p}(-p^2)^{D/2-2}(D-1) B\left(\frac{D}{2},\frac{D}{2}\right)\Gamma\left(2-\frac{D}{2}\right)}$$