PHYS 4500 Fall 2024

Test 1

You may use the lecture notes, your homework, and the textbooks but no other resources or materials

1) The matrix γ^5 is defined as $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. Calculate γ^5 in the chiral representation of the Dirac matrices.

2) Show by explicit calculation that the Dirac spinor $v^{(1)}$ satisfies the Dirac equation $\gamma^\mu p_\mu v^{(1)} = -mc\, v^{(1)}$ using the standard representation of the Dirac matrices.

3) For the process $a+b \to 1+2$ we define the Mandelstam variables $s=(p_a+p_b)^2$, $t=(p_a-p_1)^2$, $u=(p_b-p_1)^2$. Show that $s+t+u = m_a^2+m_b^2+m_1^2+m_2^2$.

4) The Lagrangian for a scalar field with a quartic interaction is $\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4!}\varphi^4$ with λ a constant. Write the Euler-Lagrange equation, and calculate the conjugate momentum, the stress-energy tensor, and the Hamiltonian density.

5) Given that $a^\dagger(p) = \int d^3x\,(2p^0)^{-1/2}\, i\,[\varphi(x)\,\partial_0 e^{-ip\cdot x} - (\partial_0\varphi(x))\,e^{-ip\cdot x}]$, and similarly for $a^\dagger(q)$, show explicitly that $[a^\dagger(p), a^\dagger(q)] = 0$ by using the equal-time commutation relations for φ and π. You are reminded that $\pi = \partial_0\varphi$ for the real scalar field.

Hint: use a dummy variable y (instead of x) for $a^\dagger(q)$, and note the relation $\int d^3x\, e^{-i(p+q)\cdot x} = (2\pi)^3\delta^3(\vec{p}+\vec{q})$