$$\int_0^{\pi} \sin^n \theta \, d\theta$$

$$\int_0^{\pi} \sin(\theta)\, d\theta = -\left[\cos\theta\right]_0^{\pi} = -[-1-1] = 2$$

$$\int_0^{\pi} \sin^2\theta \, d\theta = \frac{1}{2}\int_0^{\pi} (1-\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} d\theta - \frac{1}{2}\int_0^{\pi} \cos 2\theta \, d\theta$$

$$= \frac{\pi}{2} - \frac{1}{4}\left[\sin 2\theta\right]_0^{\pi}$$

$$= \frac{\pi}{2}$$

$$\int_0^{\pi} \sin^3\theta \, d\theta = \frac{1}{2}\int_0^{\pi} \sin\theta(1-\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} \sin\theta \, d\theta - \frac{1}{2}\int_0^{\pi} \sin\theta \cos 2\theta \, d\theta$$

$u = \sin\theta \quad dv = \cos 2\theta \, d\theta$

$du = \cos\theta \quad v = \frac{1}{2}\sin(2\theta)$

$$= \frac{1}{2}\cdot 2$$

$$- \frac{1}{2}\left\{\cancelto{0}{\left[\frac{1}{2}\sin\theta \sin 2\theta\right]_0^{\pi}} - \frac{1}{2}\int_0^{\pi} \cos\theta \sin 2\theta \, d\theta\right.$$

$\frac{1}{2}\cdot x = \frac{1}{3}$

$x = \frac{2}{3}$

$$= 1 + \frac{1}{4}\int_0 \cos\theta\,(2\sin\theta\cos\theta)\, d\theta$$

$$= 1 + \frac{1}{2}\int_0^{\pi} \sin\theta \cos^2\theta \, d\theta$$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$$= 1 - \frac{1}{2}\int_1^{-1} u^2 \, du$$

$$= 1 + \frac{1}{6}\left[u^3\right]_{-1}^{1} = 1 + \frac{1}{6}[1-(-1)]$$

$$= 1 + \frac{1}{3} = \frac{4}{3}$$

even will have π's, odds no.

$$\int_0^{\pi} \sin^4\theta \, d\theta = \frac{1}{4}\int (1-\cos 2\theta)^2 \, d\theta$$

$$= \frac{1}{4}\left\{\int_0^{\pi} d\theta - 2\int_0^{\pi} \cos 2\theta + \int_0^{\pi} \cos^2 2\theta \, d\theta\right]$$

$$= \frac{1}{4}\left[\pi - \cancelto{0}{\left[\sin 2\theta\right]_0^{\pi}} + \frac{1}{2}\int_0^{\pi} (1+\cos 4\theta)\, d\theta\right]$$

$$= \frac{1}{4}\left[\pi + \frac{1}{2}\int_0^{\pi} d\theta + \frac{1}{2}\int_0^{\pi} \cancelto{0\ \text{(turns into sin)}}{\cos 4\theta \, d\theta}\right]$$

$$= \frac{1}{4}\left[\pi + \frac{\pi}{2}\right] = \underline{\frac{3\pi}{8}}$$

$$\int_0^\pi \sin^5\theta\, d\theta = \int_0^\pi \sin\theta \sin^4\theta = \frac{1}{4}\int_0^\pi \sin\theta\,(1-\cos 2\theta)^2\, d\theta$$

$$= \frac{1}{4}\left[\int_0^\pi \sin\theta\, d\theta - 2\int_0^\pi \sin\theta\cos 2\theta\, d\theta + \int_0^\pi \sin\theta\cos^2 2\theta\, d\theta\right]$$

$$= \frac{1}{4}\left[2 - 2\left(-\frac{2}{3}\right) + \underbrace{\int_0^\pi \sin\theta\cos^2 2\theta}\right]$$

from earlier

$$\int_0^\pi \sin\theta\cos^2 2\theta\, d\theta = \frac{1}{2}\int_0^\pi \sin\theta\,(1+\cos 4\theta)\, d\theta$$

$$= \frac{1}{2}\left[\int_0^\pi \sin\theta\, d\theta + \int_0^\pi \sin\theta\cos 4\theta\, d\theta\right]$$

$u = \sin\theta \quad dv = \cos 4\theta\, d\theta$

$du = \cos\theta \quad v = \frac{1}{4}\sin(4\theta)\, d\theta$

$$= \frac{1}{2}\left\{2 + \left[\frac{1}{4}\cancel{\sin\theta\sin 4\theta}\right]_0^\pi - \frac{1}{4}\underbrace{\int_0^\pi \cos\theta\sin 4\theta\, d\theta}\right\}$$

$$\int_0^\pi \cos\theta\sin 4\theta\, d\theta = 2\int_0^\pi \cos\theta\sin 2\theta\cos 2\theta\, d\theta$$

$$= 4\int_0^\pi \cos^2\theta\sin\theta\,(2\cos^2\theta - 1)\, d\theta$$

$$= 8\int_0^\pi \cos^4\theta\sin\theta\, d\theta - 4\int_0^\pi \cos^2\theta\sin\theta\, d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$= 8\int_{-1}^1 u^4\, d\theta - 4\int_{-1}^1 u^2\, d\theta$$

$$= \frac{16}{5} - \frac{8}{3} = \frac{48-40}{15} = \frac{8}{15}$$

so,

$$\int_0^\pi \sin\theta\cos^2 2\theta\, d\theta = \frac{1}{2}\left(2 - \frac{1}{4}\left(\frac{8}{15}\right)\right) = \frac{1}{2}\left(\frac{30-2}{15}\right) = \frac{14}{15}, \text{ and}$$

$$\int_0^\pi \sin^5\theta\, d\theta = \frac{1}{4}\left[2 + 2\left(\frac{2}{3}\right) + \frac{14}{15}\right]$$

$$= \frac{1}{4}\left[2 + \frac{4}{3} + \frac{14}{15}\right] = \frac{1}{4}\left[\frac{30+20+14}{15}\right] = \underline{\underline{\frac{16}{15}}}$$

$$\int_0^\pi \sin^6\theta\, d\theta = \frac{1}{8}\int_0^\pi (1-\cos 2\theta)^3\, d\theta$$

$$= \frac{1}{8}\int_0^\pi \left(1 - 3\cos 2\theta + 3\cos^2 2\theta - \cos^3 2\theta\right) dx$$

$$= \frac{1}{8}\left[\int_0^\pi d\theta - 3\int_0^\pi \cancel{\cos 2\theta\, d\theta} + 3\underbrace{\int_0^\pi \cos^2 2\theta\, d\theta}_{\#1} - \underbrace{\int_0^\pi \cos^3 2\theta\, d\theta}_{\#2}\right.$$

#1 $$\int_0^\pi \cos^2 2\theta\, d\theta = \frac{1}{2}\int_0^\pi (1+\cos 4\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^\pi d\theta + \frac{1}{2}\int_0^\pi \cancelto{0}{\cos 4\theta\, d\theta}$$

$$= \frac{\pi}{2}$$

#2 $$\int_0^\pi \cos^3 2\theta\, d\theta = \frac{1}{2}\int_0^\pi \cos 2\theta\,(1+\cos 4\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} \cancel{\cos 2\theta}^{\,0}\, d\theta + \frac{1}{2}\int_0^{\pi} \cos 2\theta \cos 4\theta \, d\theta$$

$u = \cos 2\theta \quad dv = \cos 4\theta$

$du = 2\sin 2\theta \quad v = \frac{1}{4}\sin 4\theta$

$$= \frac{1}{2}\left\{ \left[\frac{1}{4}\cancel{\cos 2\theta \sin 4\theta}^{\,0} \right]_0^{\pi} + \frac{1}{2}\int_0^{\pi} \sin 2\theta \sin 4\theta \, d\theta \right\}$$

0, lims become 0

$$= \frac{1}{2}\int_0^{\pi} \cancel{\sin^2 2\theta \cos 2\theta \, d\theta} \qquad u = \sin 2\theta \quad du = 2\cos 2\theta$$

so,

$$\int_0^{\pi} \sin^6\theta \, d\theta = \frac{1}{8}\left[\pi + 3\left(\frac{\pi}{2}\right)\right]$$

$$= \frac{1}{8}\left[\frac{(2+3)\pi}{2}\right] = \underline{\underline{\frac{5\pi}{16}}}$$

n	$\int_0^{\pi} \sin^n\theta \, d\theta$
1	2
2	$\pi/2$
3	$4/3$
4	$3\pi/8$
5	$16/15$
6	$5\pi/16$

look at even n first:

$$\int_0^{\pi} \sin^n\theta \, d\theta = \frac{1}{2^{n/2}}\int_0^{\pi} \left[1 - \cos^{(n/2)}(2\theta)\right] d\theta$$

$$= \frac{1}{2^{n/2}}\int_0^{\pi} \left[\sum_{k=0}^{n/2} (-1)^k \binom{n/2}{k} \cos^k 2\theta \right] d\theta$$

$$= \frac{1}{2^{n/2}} \sum_{k=0}^{n/2} (-1)^k \binom{n/2}{k} \underline{\int_0^{\pi} \cos^k 2\theta \, d\theta}$$

$$\int_0^{\pi} \cos^k 2\theta \, d\theta$$

$$\to \int_0^{\pi} \cos 2\theta \, d\theta = 0$$

$$\int_0^{\pi} \cos^2(2\theta)\, d\theta = \frac{1}{2}\int_0^{\pi} [1 - \cos(4\theta)]\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi} d\theta - \frac{1}{2}\int_0^{\pi} \cos 4\theta \, d\theta$$

$$= \frac{\pi}{2}$$

$$\int_0^{\pi} \cos^3(2\theta)\, d\theta = \frac{1}{2}\int_0^{\pi} \cos 2\theta \left(1 - \sin^2 2\theta\right) d\theta \qquad u = \sin 2\theta$$

$$= -\frac{1}{2}\int_0^{\pi} \cos 2\theta \sin^2 2\theta \, d\theta \qquad du = 2\cos 2\theta \, d\theta$$

$$= -\frac{1}{4}\int_0^{0} \sim \quad = 0$$

$$\int_0^{\pi} \cos^4(2\theta)\, d\theta = \frac{1}{4}\int_0^{\pi}\left[1+\cos(4\theta)\right]^2 d\theta$$
$$= \frac{1}{4}\int_0^{\pi} d\theta + \frac{1}{2}\int_0^{\pi}\cos(4\theta)\,d\theta + \frac{1}{4}\int_0^{\pi}\cos^2(4\theta)\,d\theta$$
$$= \frac{\pi}{4} + \frac{1}{4}\left(\frac{\pi}{2}\right) = \frac{3\pi}{8}$$

odd k are zero $\rightarrow (-1)^k = 1$

$$= \frac{1}{2^{n/2}} \sum_{k=\text{even}}^{n/2} \binom{n/2}{k} \underbrace{\int_0^{\pi} \cos^k 2\theta\, d\theta}$$

had the same thing:

$$\int_0^{\pi} \cos^k 2\theta\, d\theta = \frac{1}{2^{k/2}} \int_0^{\pi} \left[1+\cos^{(k/2)}(2\theta)\right] d\theta$$
$$= \frac{1}{2^{k/2}} \sum_{l=\text{even}}^{k/2} \binom{k/2}{l} \int_0^{\pi} \cos^l(4\theta)\, d\theta$$

so, $$\int_0^{\pi} \sin^n\theta\, d\theta = \frac{1}{2^{n/2}} \sum_{k=\text{even}}^{n/2} \frac{1}{2^{k/2}} \binom{n/2}{k} \sum_{l=\text{even}}^{k/2} \binom{k/2}{l} \int_0^{\pi} \cos^l(4\theta)\, d\theta$$

at the end, the $\cos^l$ integral will be π, because it'll be $l=0$

$$= \frac{\pi}{2^{n/2}} \left[\sum_{k=\text{even}}^{n/2} \frac{1}{2^{k/2}} \binom{n/2}{k}\right] \left[\sum_{l=\text{even}}^{k/2} \binom{k/2}{l}\right]$$

product of these

$$\int_0^{\pi} \sin^n\theta\, d\theta = \frac{\pi}{2^{n/2}} \prod_{l=0}^{n/2} \left[\sum_{k=\text{even}}^{l} \frac{1}{2^l} \binom{l}{k}\right] = \frac{\pi}{2^{n/2}} \prod_{l=0}^{n/2} \left[\sum_{k=\text{even}}^{l} \frac{1}{2^l} \frac{l!}{k!(l-k)!}\right]$$

even n

$$n=4 \quad = \frac{\pi}{2^2} \prod_{l=0}^{2} \left[\sum_{k=\text{even}}^{l} \frac{1}{2^l} \binom{l}{k}\right]$$

$$= \frac{\pi}{4}\left[1\right]\left[1\right]\left[1+\frac{1}{2}\right] = \frac{\pi}{4}\left(\frac{3}{2}\right) = \frac{3\pi}{8}$$

odd n $$\int_0^{\pi} \sin^n\theta\, d\theta = \int_0^{\pi} \sin\theta \sin^{(n-1)}(\theta)\, d\theta$$

$u = \sin^{(n-1)}(\theta)$ $dv = \sin\theta\, d\theta$
$du = (n-1)\sin^{(n-2)}\theta \cdot \cos\theta$
$v = -\cos\theta$

$$= -(n-1)\left[\cos\theta \sin^{(n-1)}\theta\right]_0^{\pi} + (n-1)\int_0^{\pi} \sin^{(n-2)}\theta \cos^2\theta\, d\theta$$

this is zero because of sine

$$= (n-1)\int_0^{\pi} \sin^{(n-2)}(\theta)(1-\sin^2\theta)$$

$$\int_0^{\pi} \sin^n\theta\, d\theta = (n-1)\int_0^{\pi} \sin^{(n-2)}\theta - (n-1)\int_0^{\pi} \sin^n(\theta)\, d\theta$$

$$\int_0^{\pi} \sin^n\theta\, d\theta + (n-1)\int_0^{\pi} \sin^n\theta\, d\theta \sim$$

$$n\int_0^{\pi} \sin^n\theta\, d\theta = (n-1)\int_0^{\pi} \sin^{(n-2)}\theta$$

$$\int_0^{\pi} \sin^n\theta\, d\theta = \frac{n-1}{n}\int_0^{\pi} \sin^{(n-2)}\theta$$

recursion relation for odd n?