PHYS 4210/ Fall 2024
Midterm Exam 3

KENNESAW STATE UNIVERSITY

**Kennesaw State University**
**College of Science and Mathematics**
**Department of Physics**

Student Name: _____

This exam consists of four questions, each worth 25 points. Answer them on the provided sheets. You have 70 minutes to complete the exam. You may use a calculator and your own integration formula sheet. All other work must be your own, without assistance from peers, notes, books, or online resources.

As I aim to distinguish between top scorers, I will assess your answer based on its quality and completeness. While unnecessary details are not required, all essential elements to support your answer must be included.

1. For a spin $s = 1/2$ particle, Hamiltonian is given by

$$\widehat{H} = \frac{\alpha}{\hbar^2}\left(\hat{S}_x^2 + \hat{S}_y^2 - 2\hat{S}_z^2\right) - \frac{\beta}{\hbar}\hat{S}_z$$

where $\alpha$ and $\beta$ and constants.

a) Find the energy levels? (20 points)
b) Are these levels degenerate. (5 points)

2. Energies and wavefunction of particle of mass $m$ in infinite square well of size $a$ are

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2a^2 m}$$

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right)$$

If three spinless particles are in such a well, then

(a) Write the energies and wave functions of the ground state and the first excited state of the system if the masses of the particles are $m1 > m2 > m3$. (10+10)

(b) Write the energies and wave functions of the first excited state of the system if the masses of the particles are $m1 = m2 = m3$, but somehow the particles are still distinguishable. (5)

3. If it is possible to write the following *simultaneously*,

$$\hat{H}\psi_{nlm} = E_n\psi_{nlm}$$
$$\hat{L}_z\psi_{nlm} = m\hbar\psi_{nlm}$$
$$\hat{L}^2\psi_{nlm} = l(l+1)\hbar^2\psi_{nlm}$$

where the symbols have their usual meanings, answer the following questions:

a) What commutation relation(s) does this imply? Write *all* of those that are implied and do not write any that is not *directly* implied. (10)
b) What type of symmetry does the system have? Additionally, is it continuous or discrete? Write down the Hamiltonian with the implied symmetry built in. (5)
c) Does this imply any conservation laws? Prove it. (Yes – I am expecting you to recall Ehrenfest's theorem.) (10)

4. (a) Show that the parity transformation of $\hat{p}_x$ gives -$\hat{p}_x$. (10)
   (b) What is the electronic configuration of Al (Z=13)? Explain your answer using $n, l$ and $m$ values and their relations. (10)
   (c) Bloch's theorem for a wave function of a particle under discrete translational symmetry of period $a$ (e.g. Dirac comb with width of $a$) is

$$\psi(x) = e^{iqx}u(x)$$

   What is the condition $u(x)$ must satisfy? Write in an equation form. (5)

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# Potentially Useful Identities

$$sin^2 x = \frac{1 - cos2x}{2}$$

$$\int dx = x + C$$

$$\int coskx\, dx = \frac{1}{k}\, sinkx + C$$

Odd functions are those which satisfy f(-x) = -f(x). Integral of an odd function in symmetric limit is zero.

$$\int_{-\infty}^{\infty} e^{-x^2/k^2} dx = k\sqrt{\pi}$$

$$\int_{-\infty}^{\infty} x^2 e^{-x^2/k^2} dx = \frac{k^3\sqrt{\pi}}{2}$$

$$\frac{d(e^{kx})}{dx} = ke^{kx}$$

Chain rule of derivatives: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

Scratch:

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**Additional Answer Sheet**

Student Name: _____

Answer to Question# : _____