HW13
PHYS4500: Quantum Field Theory

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Problem 1.

With the definition of our Higgs field as a doublet

$$\varphi(x) = \begin{pmatrix} 0 \\ \rho + \frac{1}{\sqrt{2}}\phi(x) \end{pmatrix}, \tag{1.1}$$

we have that the covariant derivative $D_\mu \varphi$ has the form:

$$D_\mu \varphi = \partial_\mu \varphi + \frac{i}{2} g (\sigma^i W^i_\mu) \varphi + \frac{i}{2} g' B_\mu \varphi. \tag{1.2}$$

From the form of the Pauli matrices,

$$\sigma^i W^i_\mu = \begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}. \tag{1.3}$$

So, explicitly,

$$D_\mu \varphi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\partial_\mu \phi \end{pmatrix} + \frac{ig}{2} \begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \rho + \frac{1}{\sqrt{2}}\phi \end{pmatrix} + \frac{ig'}{2} B_\mu \begin{pmatrix} 0 \\ \rho + \frac{1}{\sqrt{2}}\phi \end{pmatrix}, \tag{1.4}$$

$$= \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\partial_\mu \phi \end{pmatrix} + \frac{ig}{2} \begin{pmatrix} (W^1_\mu - iW^2_\mu)(\rho + \phi/\sqrt{2}) \\ -W^3_\mu(\rho + \phi/\sqrt{2}) \end{pmatrix} + \frac{ig'}{2} B_\mu \begin{pmatrix} 0 \\ \rho + \frac{1}{\sqrt{2}}\phi \end{pmatrix}. \tag{1.5}$$

We can consider each component of the resultant matrix (which is a 2-component column vector) individually, and try to separate out similar forms that will ultimiately be replaced by new gauge fields. The top component will read

$$\text{Top} \rightarrow \frac{i}{2} g (W^1_\mu - iW^2_\mu) \left(\rho + \frac{1}{\sqrt{2}}\phi \right) \tag{1.6}$$

$$= \frac{i}{2} (W^1_\mu - iW^2_\mu) \left(g\rho + \frac{g\phi}{\sqrt{2}} \right) \tag{1.7}$$

$$= \frac{i}{2} \left[g\rho \left(W^1_\mu - iW^2_\mu \right) + \frac{g\phi}{\sqrt{2}} \left(W^1_\mu - iW^2_\mu \right) \right]. \tag{1.8}$$

Similarly, the bottom component will read

$$\text{Bottom} \rightarrow \frac{1}{\sqrt{2}} \partial_\mu \phi - \frac{ig}{2} W^3_\mu (\rho + \phi/\sqrt{2}) + \frac{ig'}{2} B_\mu (\rho + \phi/\sqrt{2}) \tag{1.9}$$

$$= \frac{i}{2} \left(-i\sqrt{2}\partial_\mu \phi - g\rho W^3_\mu - \frac{g\phi}{\sqrt{2}} W^3_\mu + g'\rho B_\mu + \frac{g'\phi}{\sqrt{2}} B_\mu \right) \tag{1.10}$$

$$= \frac{i}{2} \left[-i\sqrt{2}\partial_\mu \phi + \rho \left(-gW^3_\mu + g'B_\mu \right) + \frac{\phi}{\sqrt{2}} \left(-gW^3_\mu + g'B_\mu \right) \right]. \tag{1.11}$$

Putting everything together then:

$$D_\mu \varphi = \frac{i}{2} \begin{pmatrix} g\rho \left(W^1_\mu - iW^2_\mu \right) + \frac{g\phi}{\sqrt{2}} \left(W^1_\mu - iW^2_\mu \right) \\ -i\sqrt{2}\partial_\mu \phi + \rho \left(-gW^3_\mu + g'B_\mu \right) + \frac{\phi}{\sqrt{2}} \left(-gW^3_\mu + g'B_\mu \right) \end{pmatrix} \tag{1.12}$$

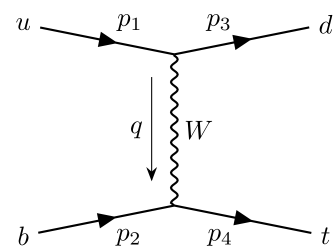

Figure 1: The Feynman diagram for the t-channel process $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$.

Problem 2.

We are considering the t-channel process for singly producing a top quark like $u(p_1) + b(p_2) \to d(p_3) + t(p_4)$

Using our Feynman rules, we can pretty easily write down the amplitude:

$$i\mathcal{M} = \bar{u}(p_3)\frac{(-ie)\gamma^\mu(1-\gamma^5)V_{ud}}{2\sqrt{2}\,\sin\theta_W}u(p_1)\frac{(-1)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}\right)}{q^2 - m_W^2 + i\epsilon}\bar{u}(p_4)\frac{(-ie)\gamma^\nu(1-\gamma^5)V_{bt}}{2\sqrt{2}\,\sin\theta_W}u(p_2). \tag{2.1}$$

$$\begin{aligned}\mathcal{M} = \frac{e^2 V_{ud}V_{bt}}{8\sin^2\theta_W[(p_1-p_3)^2 - m_W^2]}&\bar{u}(p_3)\gamma^\mu(1-\gamma^5)u(p_1) \\ &\times\left(g_{\mu\nu} - \frac{(p_1-p_3)_\mu(p_1-p_3)\nu}{m_W^2}\right)\bar{u}(p_4)\gamma^\nu(1-\gamma^5)u(p_2)\end{aligned} \tag{2.2}$$

I believe that this is as far as we can get for just the amplitude. If we want to make a similar assumption as in the notes where we consider $q^2 << m_W^2$, then this becomes

$$\mathcal{M} = -\frac{e^2 V_{ud}V_{bt}}{8\sin^2\theta_W\, m_W^2}\bar{u}(p_3)\gamma^\mu(1-\gamma^5)u(p_1)\bar{u}(p_4)\gamma_\mu(1-\gamma^5)u(p_2) \tag{2.3}$$