PHYS 4500 Fall 2024
Final Exam

You may use the lecture notes, your homework, and the textbooks but no other resources or materials.

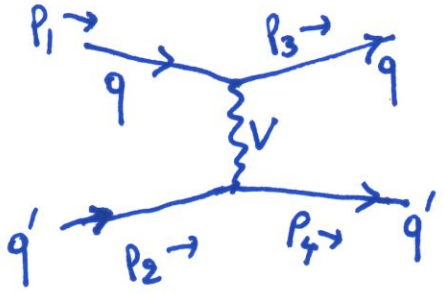
1) Write the Euler-Lagrange equation for the Dirac spinor $\bar{\psi}$ using (a) the QED Lagrangian, (b) the electroweak theory Lagrangian, and (c) the QCD Lagrangian.

2) Show that under local SU(3) gauge transformations, i.e. $\psi \to e^{\frac{i}{2}\lambda^a\theta^a}\psi$ and (for small θ) $G^a_\mu \to G^a_\mu - \frac{1}{g_s}\partial_\mu\theta^a - f^{abc}\theta^b G^c_\mu$, the terms in the QCD Lagrangian involving ψ, i.e $\mathcal{L}_\psi = i\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{g_s}{2}\bar{\psi}\gamma^\mu\lambda^a G^a_\mu\psi - m\bar{\psi}\psi$, remain invariant.

Also show that under the same transformation, the covariant derivative $D_\mu\psi = \partial_\mu\psi + \frac{i}{2}g_s\lambda^a G^a_\mu\psi$ transforms as

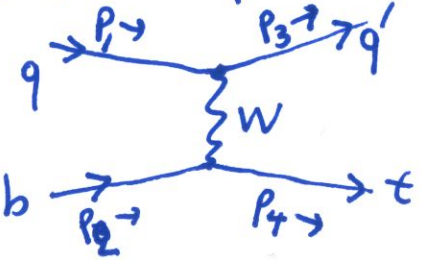
$$D_\mu\psi \to e^{\frac{i}{2}\lambda^a\theta^a} D_\mu\psi$$

3) Write the amplitudes for the processes



where q and q' are quarks and where V can be a photon, a Z boson, or a gluon.

Also, write the amplitude for the process



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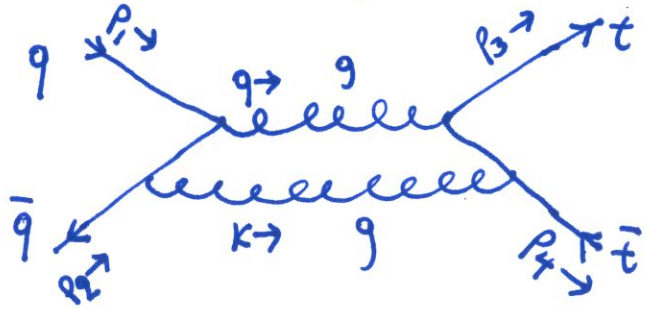
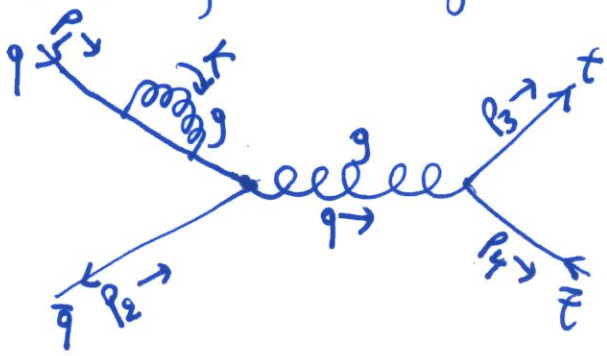
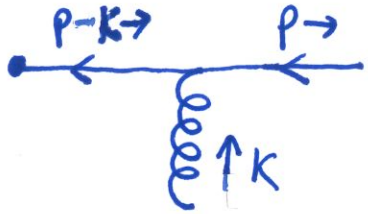
4) Write the amplitudes for the following two diagrams:



and



5) Derive the eikonal rule for an outgoing antiquark, i.e. for



and write the integral (do not calculate it) using eikonal rules for the one-loop cusp diagram