But this cannot be satisfied:

$\delta^{(4)}(q + p_x) \Rightarrow \quad q = -p_x \quad$ with $q_0 < 0$

$q^\mu = (q_0, \vec{0}) \qquad q^2 = (p_1 + p_2)^2 = 2 p_1 \cdot p_2 = s > 0$

$p_1^\mu = (p_1^0, \vec{0}_\perp, p_z)$

$p_2^\mu = (p_2^0, \vec{0}_\perp, -p_z)$

$p_1^\mu + p_2^\mu = q^\mu = (p_1^0 + p_2^0, \vec{0})$ CoM system

$q^2 \cong 2 p_{10} p_{20} = s > 0 \qquad \searrow \quad q_0 > 0 \Rightarrow$ contradiction

$\Rightarrow$

$$W_{\mu\nu} = \int d^4x \, e^{iq \cdot x} \langle 0 | [J_\mu(x), J_\nu(0)] | 0 \rangle$$

In high-energy annihilations, q_0 is large.

$\Rightarrow q_0 \to +\infty \Rightarrow e^{iq \cdot x}$: rapid oscillatory behavior with no bounds.

Therefore only the $x_0 \sim 0$ makes a major contribution to the integral.

The integrand on the other hand, has support only for $x^2 \geq 0$ due to causality requirement. $[J_\mu(x), J_\nu(0)] = 0$ for $x^2 < 0$

Therefore, $X_0 \sim 0 \Rightarrow X \sim 0$ and we conclude that the total Xsec for e^+e^- annihilation is governed by short-distance current commutators.

$W_{\mu\nu}$ can be written even more generally following Lorentz invariance and current conservation

→ only 1 invariant amplitude needed

$$W_{\mu\nu} = \left(q_\mu q_\nu - q^2 g_{\mu\nu}\right) \frac{1}{6\pi} \, W(q^2)$$

$1/(6\pi)$ is for convenience (we shall see this later)

Therefore σ is obtained as

$$\sigma = \frac{4\pi \alpha_{em}^2}{3s} \, W(s)$$

$$\alpha_{em} = \frac{e^2}{4\pi}$$

We have studied for example that $e^+e^- \to \mu^+\mu^-$

$$\sigma_{\mu\mu}^{(LO)} = \frac{4\pi \alpha_{em}^2}{3s}$$

$$R = \frac{\sigma}{\sigma_{\mu\mu}} = W(s) = \text{Drell ratio}$$

$$R = \frac{4\pi \alpha_{em}^2}{3s} W(s) \frac{3s}{4\pi \alpha_{em}^2} = W(s)$$

which can be written as

$$R = -\frac{2\pi}{q^2} \int d^4x \, e^{iq\cdot x} \langle 0 | J_\mu(x) J^\mu(0) | 0 \rangle$$

In a $e^+ + e^- \to X$ process at very high COM (i.e., large $\sqrt{q^2}$) masses of the quarks are negligible

$$R = R(s=q^2, \underbrace{g}_{\text{renormalized coupling constant}}, \underbrace{\mu}_{\text{renormalization scale}})$$

$R = R(s/\mu^2, g)$: rearranged for dimensional reasons.

R : does not contain any hadronic state in its expression. It consists only of the short distance piece for large s

According to the renormalization group equation for R (which we will not discuss here for now) we can write

$$\left[\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right] R\left(\frac{s}{\mu^2}, g\right) = 0$$

The general solution is given by

$$R\left(\frac{s}{\mu^2}, g\right) = R\left(1, \bar{g}(s)\right)$$

where $\bar{g}(s)$ is defined in terms of the β-function $\ni$'

$$\frac{d\bar{g}}{dt} = \beta(g) \qquad \bar{g}(\mu^2) = g$$

with $t = 1/2 \ln(s/\mu^2)$.

This means that the explicit s-dependence of R computed by using the coupling constant g can be completely absorbed in the s-dependence of the "running coupling" constant $\bar{g}(s)$

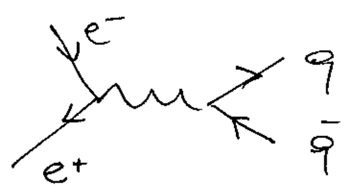
Asymptotically free field theories $\Rightarrow$ $\bar{g}(s) \to 0$ when $s \to \infty$

<u>Perturbative expansion for R</u>

$$R\left(\frac{s}{\mu^2}, g\right) = \sum_i Q_i^2 \left[1 + a(s/\mu^2) g^2 + b(s/\mu^2) g^4 + \cdots\right]$$

The first term in the expansion refers to

$e^+ + e^- \to q + \bar{q}$



$\sigma = Q^2 \frac{4\pi\alpha_{em}^2}{3s}$

$\sum_i Q_i$ $\to$ the index i runs over colors and flavors of quarks

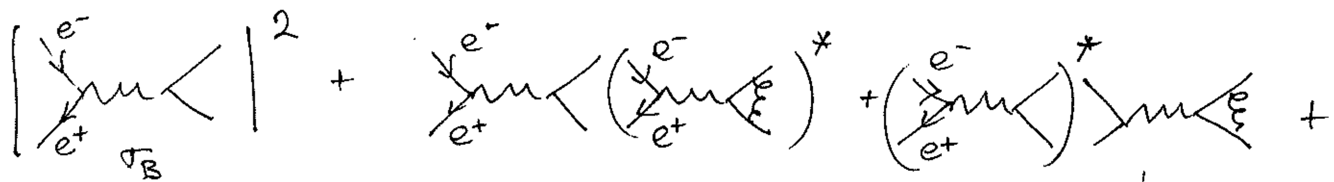
The other terms represent radiative corrections

The coefficients a, b, etc, contain large logs $\ln(s/\mu^2)$

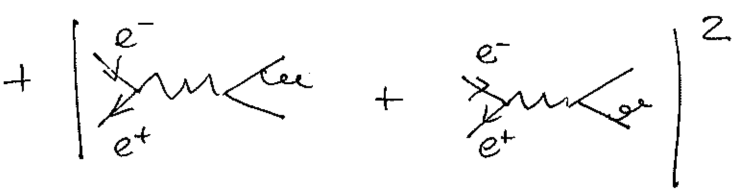
$$R\left(s/\mu^2, \bar{g}(s)\right) = R(1, \bar{g}(s)) \Rightarrow R\left(\frac{s}{\mu^2}, g\right) = \sum_i Q_i^2 \left[1 + a(1)\bar{g}(s)^2 + b(1)\bar{g}(s)^4 + \dots\right]$$

where $g \to \bar{g}(s)$ indicates that the large logs in the coeffic. disappear. The latter expression is more stable in perturbation theory because the expansion coefficients are smaller, and the expansion parameter $\bar{g}(s)$ is smaller than g for $s \gg \mu^2$ (according to asymptotic freedom)

Radiative corrections to e^+e^- annihilation

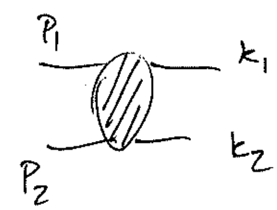


↳ virtual contributions



→ real emission contributions

 $\sim O(g_s)$ $\qquad g_s = \sqrt{4\pi\alpha_s}$



$$\sigma_B = \frac{4\pi\alpha_{em}^2}{s} \sum_i Q_i$$

$$\sigma = Z_2^2\, \sigma_B + \sigma_V + \sigma_R = \sigma_B + \tilde{\sigma}_V + \sigma_R$$

where $\tilde{\sigma}_V = \sigma_V + (Z_2^2 - 1)\sigma_B$

Z_2 = field renormalization constant