HW9
PHYS4500: Quantum Field Theory

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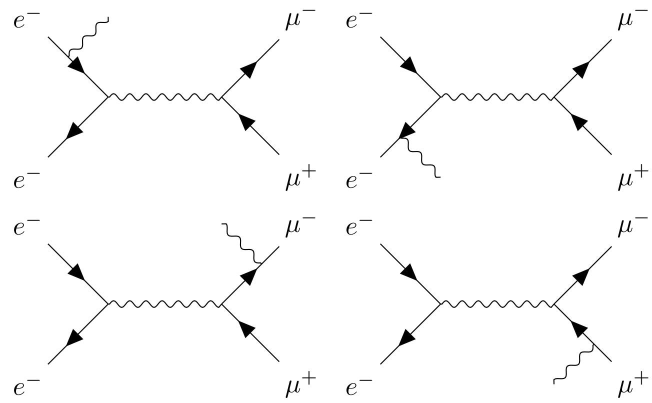
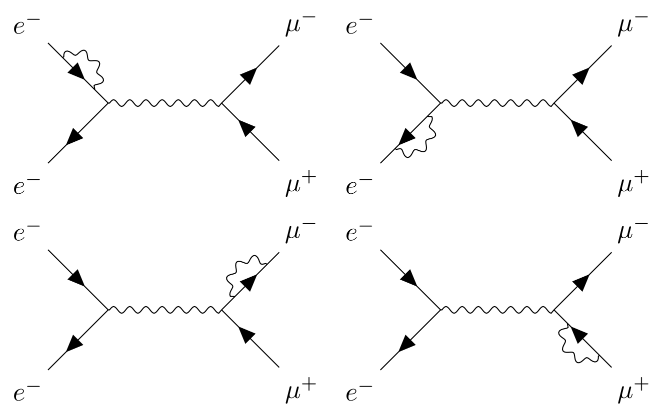
October 14, 2024

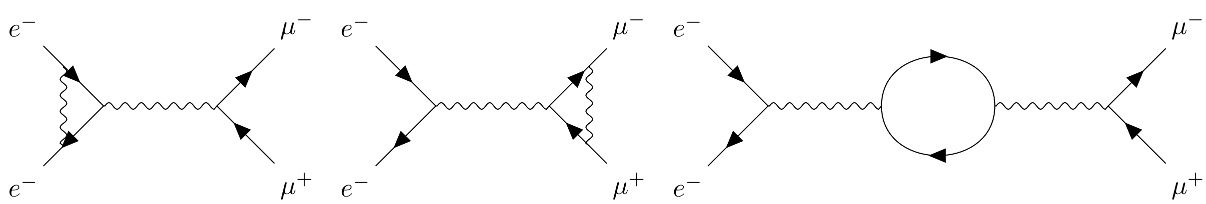
Problem 1.

There are four real-emission diagrams since any of the four external particles can emit a photon.

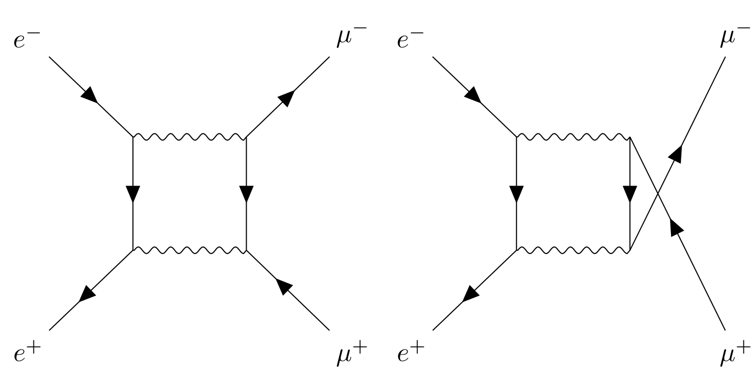


There are four fermion propagator self-energies, as well as two vertex corrections and a photon vacuum-polarization diagram:





There are two "box" diagrams where either the electron or positron emits a virtual photon that is then absorbed by either the muon or anti-muon:



Problem 2.

The Feynman parametrization for $1/AB$ is given by:

$$\frac{1}{AB} = \int_0^1 \frac{\mathrm{d}x}{[Ax + (1-x)B]^2}.$$

With a u-substitution of $u \equiv Ax + (1-x)B$ where $\mathrm{d}u = (A-B)\mathrm{d}x$ or $1/(A-B)\mathrm{d}u = \mathrm{d}x$, we have that

$$\begin{aligned}\frac{1}{AB} &= \frac{1}{A-B}\int_B^A \frac{1}{u^2}\mathrm{d}u = \frac{1}{A-B}\left[\frac{1}{u}\right]_A^B \\ &= \frac{1}{A-B}\left(\frac{1}{B} - \frac{1}{A}\right) = \frac{1}{A-B}\left(\frac{A-B}{AB}\right) = \frac{1}{AB}.\end{aligned}$$

Problem 3.

We are to show that

$$\gamma^\mu \gamma_\mu = n \tag{3.1}$$

in n dimensions. First:

$$\gamma^\mu \gamma_\mu = g_{\mu\nu}\gamma^\mu\gamma^\nu = g_{\mu\nu}\left(\{\gamma^\mu, \gamma^\nu\} - \gamma^\nu\gamma^\mu\right).$$

The anti-commutator of the gamma matrices is independent of the dimension - it's just $2g^{\mu\nu}$, so

$$\begin{aligned}\gamma^\mu\gamma_\mu &= g_{\mu\nu}\left(2g^{\mu\nu} - \gamma^\nu\gamma^\mu\right) = 2g_{\mu\nu}g^{\mu\nu} - \gamma^\mu\gamma_\mu \\ 2\gamma^\mu\gamma_\mu &= 2g^\mu_\mu \\ \gamma^\mu\gamma_\mu &= n.\end{aligned}$$