Particle content of the Standard Model

Three generations or families of quarks and leptons; gauge bosons; Higgs

Quarks	Leptons	Gauge bosons	Higgs boson
$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$	$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$	γ, W^+, W^-, Z, g	H

The quarks and the leptons are all spin $\frac{1}{2}$ fermions

The gauge bosons are all spin 1 - vector bosons

The Higgs boson has spin 0 - scalar boson

Antiquarks denoted by a bar: $\begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} \begin{pmatrix} \bar{c} \\ \bar{s} \end{pmatrix} \begin{pmatrix} \bar{t} \\ \bar{b} \end{pmatrix}$

Antiparticles of the leptons: $\begin{pmatrix} e^+ \\ \bar{\nu}_e \end{pmatrix} \begin{pmatrix} \mu^+ \\ \bar{\nu}_\mu \end{pmatrix} \begin{pmatrix} \tau^+ \\ \bar{\nu}_\tau \end{pmatrix}$

Electromagnetic interactions are mediated by virtual photons γ for all particles with electric charge: $u, c, t: +\frac{2}{3}e$; $d, s, b: -\frac{1}{3}e$; $e^-, \mu^-, \tau^-: -e$; $\nu_e, \nu_\mu, \nu_\tau: 0$; $\gamma, Z, g, H: 0$; $W^+: +e$; $W^-: -e$

Weak interactions are mediated by virtual W^+, W^-, and Z bosons for all particles with weak charge: all quarks and leptons (left-handed)

Stong interactions are mediated by virtual gluons g (there are eight of them) for all particles with color charge: all quarks and gluons

The Standard Model of Particle Physics

$$SU(3)_c \otimes \underbrace{SU(2)_L \otimes U(1)_{Y_W}}_{\substack{\text{electroweak} \\ \text{weak isospin} \\ \text{and hypercharge}}} \xrightarrow[\substack{\text{symmetry} \\ \text{breaking} \\ \text{- Higgs} \\ \text{mechanism}}]{\text{electroweak}} SU(3)_c \otimes U(1)_Q$$

$SU(3)_c$ ↑ QCD color charge

$SU(3)_c$ ↑ QCD color-charge

$U(1)_Q$ ↑ QED electric charge

These are all gauge theories: abelian $U(1)$ and nonabelian $SU(3), SU(2)$

Color charge and electric charge conservation.

Baryon number and lepton number conservation.

<u>charge conjugation</u> C: $\psi \to -i\gamma^2\psi^*$

<u>parity</u> P: $(t, \vec{x}) \to (t, -\vec{x})$

<u>time reversal</u> T: $(t, \vec{x}) \to (-t, \vec{x})$

strong and e&m interactions invariant under C or P or T

weak interactions violate C and P; also CP violation

But CPT obeyed in the Standard Model.

The <u>CPT theorem</u> applies to any QFT with Lorentz invariance and unitarity

Non-abelian gauge fields — Yang-Mills theory

Consider two spinor fields ψ_a and ψ_b. The Dirac Lagrangian (no interactions) is $\mathcal{L} = i\bar{\psi}_a \gamma^\mu \partial_\mu \psi_a - m_a \bar{\psi}_a \psi_a + i\bar{\psi}_b \gamma^\mu \partial_\mu \psi_b - m_b \bar{\psi}_b \psi_b$

ψ_a and ψ_b are Dirac spinors, each one having four components

We can combine ψ_a and ψ_b into a two-component vector $\psi = \begin{pmatrix} \psi_a \\ \psi_b \end{pmatrix}$

and thus $\bar{\psi} = (\bar{\psi}_a \; \bar{\psi}_b)$

Then we can rewrite $\mathcal{L}$ as $\mathcal{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - \bar{\psi} M \psi$ with $M = \begin{pmatrix} m_a & 0 \\ 0 & m_b \end{pmatrix}$

Next we consider a <u>global</u> gauge transformation $\psi \to U\psi$

where U is a 2×2 unitary matrix $U^\dagger U = 1$

Also $U = e^{iH}$ where H is a Hermitian 2×2 matrix $H^\dagger = H$

Then $\bar{\psi} \to \bar{\psi} U^\dagger$ If we choose $m_a = m_b = m$ then $\mathcal{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$

and under the transformation $\mathcal{L} \to i\bar{\psi} U^\dagger \gamma^\mu \partial_\mu (U\psi) - m\bar{\psi} U^\dagger U \psi = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi = \mathcal{L}$

So Lagrangian is invariant under a global gauge transformation

Furthermore, we can write $H = \theta 1 + \vec{\sigma}\cdot\vec{\theta} = \theta 1 + \sigma_1\theta_1 + \sigma_2\theta_2 + \sigma_3\theta_3$

where 1 is the unit 2×2 matrix and $\sigma_1 = \sigma_x$, $\sigma_2 = \sigma_y$, $\sigma_3 = \sigma_z$ are the Pauli spin matrices

We already studied $U(1)$ transformations $\psi \to e^{i\theta}\psi$ in QED

So now we consider $H = \vec{\sigma}\cdot\vec{\theta}$ so $U = e^{i\vec{\sigma}\cdot\vec{\theta}}$ and $\psi \to e^{i\vec{\sigma}\cdot\vec{\theta}}\psi$ $SU(2)$ transformation

(special unitary since here U has determinant 1)

This is the 2×2 case of the special unitary group $SU(N)$. Here $N=2$.

Next we consider a <u>local</u> $SU(2)$ gauge transformation, i.e. with $\vec{\theta}(x^\mu)$

Then $\mathcal{L} \to i\bar{\psi}U^\dagger\gamma^\mu\partial_\mu(U\psi) - m\bar{\psi}U^\dagger U\psi = i\bar{\psi}U^\dagger\gamma^\mu(\partial_\mu U)\psi + i\bar{\psi}U^\dagger\gamma^\mu U\partial_\mu\psi - m\bar{\psi}\psi$

$= \mathcal{L} + i\bar{\psi}U^\dagger\gamma^\mu(\partial_\mu U)\psi$ so not invariant

Try to find an invariant Lagrangian. Write $\theta_i(x) = q\lambda_i(x)$, $i=1,2,3$,

and add a term $-q(\bar{\psi}\gamma^\mu\vec{\sigma}\psi)\cdot\vec{A}_\mu$ where $\vec{A}^\mu = (A_1^\mu, A_2^\mu, A_3^\mu)$ with $A_1^\mu, A_2^\mu, A_3^\mu$ vector fields

and under the transformation $\vec{\sigma}\cdot\vec{A}_\mu \to U\vec{\sigma}\cdot\vec{A}_\mu U^{-1} + \frac{i}{q}(\partial_\mu U)U^{-1}$

<u>check</u>: $-q(\bar{\psi}\gamma^\mu\vec{\sigma}\psi)\cdot\vec{A}_\mu \to -q\,\bar{\psi}U^\dagger\gamma^\mu U\vec{\sigma}\cdot\vec{A}_\mu U^{-1}U\psi - q\bar{\psi}U^\dagger\gamma^\mu\frac{i}{q}(\partial_\mu U)U^{-1}U\psi$

$= -q\,\bar{\psi}\gamma^\mu\vec{\sigma}\cdot\vec{A}_\mu\psi - i\bar{\psi}U^\dagger\gamma^\mu(\partial_\mu U)\psi$

This term cancels the extra term above

So the extended Lagrangian $i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi - q\bar{\psi}\gamma^\mu\vec{\sigma}\cdot\vec{A}_\mu\psi$ is locally gauge invariant