Bessel function $j_\ell(x)$: $x \to 0$ limit

$$j_\ell(x) = \sqrt{\frac{\pi}{2x}}\; J_{\ell+1/2}(x)$$

$$J_\alpha(x) = \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!\,\Gamma(n+\alpha+1)} \left(\frac{x}{2}\right)^{\alpha+2n} \xrightarrow[x\to 0]{} \left(\frac{x}{2}\right)^{\alpha} \frac{1}{\Gamma(\alpha+1)}$$

$$j_\ell(x) \simeq \frac{\pi^2}{\sqrt{2}\,\sqrt{x}} \; \frac{x^{\ell+1/2}}{2^{\ell+1/2}\,\Gamma(\ell+1/2+1)} = \frac{\pi^{1/2}\, x^\ell}{2^{\ell+1}\,\Gamma(\ell+1/2+1)}$$

Properties of the Γ (Gamma) function

$$\Gamma(1+x) = x\,\Gamma(x)$$

$$\Gamma(\ell+1/2+1) = (\ell+1/2)\,\Gamma(\ell+1/2)\ldots = 2^{-(\ell+1)}(2\ell+1)!!\,\pi^{1/2}$$

$\Rightarrow$

$$\lim_{x\to 0} j_\ell(x) = \frac{x^\ell}{(2\ell+1)!!} \overset{(*)}{=} \frac{2^\ell\,\ell!\,x^\ell}{(2\ell+1)!}$$

(*) $\quad (n+1)!! = \dfrac{(n+1)!}{n!!} \Rightarrow (2\ell+1)!! = \dfrac{(2\ell+1)!}{(2\ell)!!} = \dfrac{(2\ell+1)!}{2^\ell\,\ell!}$