HW10 PHYS4260 Quantum Mechanics II.

Due date: Thu April 24

1. Solve Problem 11.17 of Ch 11 of the textbook.
2. Show the relation between the Field strength $F^{\mu\nu}$ and the components of the electric and magnetic fields respectively.
3. Prove the following equality

$$\frac{d^4k}{(2\pi)^4}(2\pi)\delta(k^2-m^2)\theta(k_0)=\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2\sqrt{|\vec{k}|^2+m^2}}$$

4. In the radiation-field quantization procedure, we have introduced the functions

$$f_k(r)=\frac{e^{-ikr}}{\left[(2\pi)^3 2k_0\right]^{1/2}}$$

and its complex conjugate $f_k^*(r)$. The product $kr=k_0r^0-\vec{k}\cdot\vec{r}$ is between 4-vectors. Prove that they satisfy the orthonormality condition below

$$\int f_k^*(r)\; i\overleftrightarrow{\partial_0}\; f_{k'}(r)\; d^3\vec{r}=\delta^{(3)}(\vec{k}-\vec{k}')$$

5. In the quantization procedure for the radiation field, the vector potential is expressed in terms of plane waves and polarization vectors as follows:

$$\vec{A}(r)=\int\frac{d^3k}{\left[(2\pi)^3 2k_0\right]^{1/2}}\sum_{\lambda=0}^{2}\vec{\varepsilon}^{(\lambda)}(k)\left[f_k(r)a_k^{(\lambda)}+f_k^*(r)a_k^{(\lambda)\dagger}\right]$$

where the coefficients $a_k^{(\lambda)}=a^{(\lambda)}(k)$ and $a_k^{(\lambda)\dagger}=a^{(\lambda)\dagger}(k)$ are functions of the wave number k and the functions $f_k(r)$ which contain the plane waves are defined as

$$f_k(r)=\frac{e^{-ikr}}{\left[(2\pi)^3 2k_0\right]^{1/2}}$$

The product $kr=k_0r^0-\vec{k}\cdot\vec{r}$ is between 4-vectors. Use the orthonormality condition for the f_k functions:

$$\int f_k^*(r)\; i\overleftrightarrow{\partial_0}\; f_{k'}(r)\; d^3\vec{r}=\delta^{(3)}(\vec{k}-\vec{k}')$$

(see Problem 4.), and the orthonormality of the polarization vectors

$$\vec{\varepsilon}^{(\lambda)}(k)\cdot\vec{\varepsilon}^{(\lambda')}(k)=\delta_{\lambda\lambda'}$$

to calculate the coefficient $a_k^{(\lambda)\dagger}$. (NB.: you are allowed to use natural units here $\hbar=c=1$.)