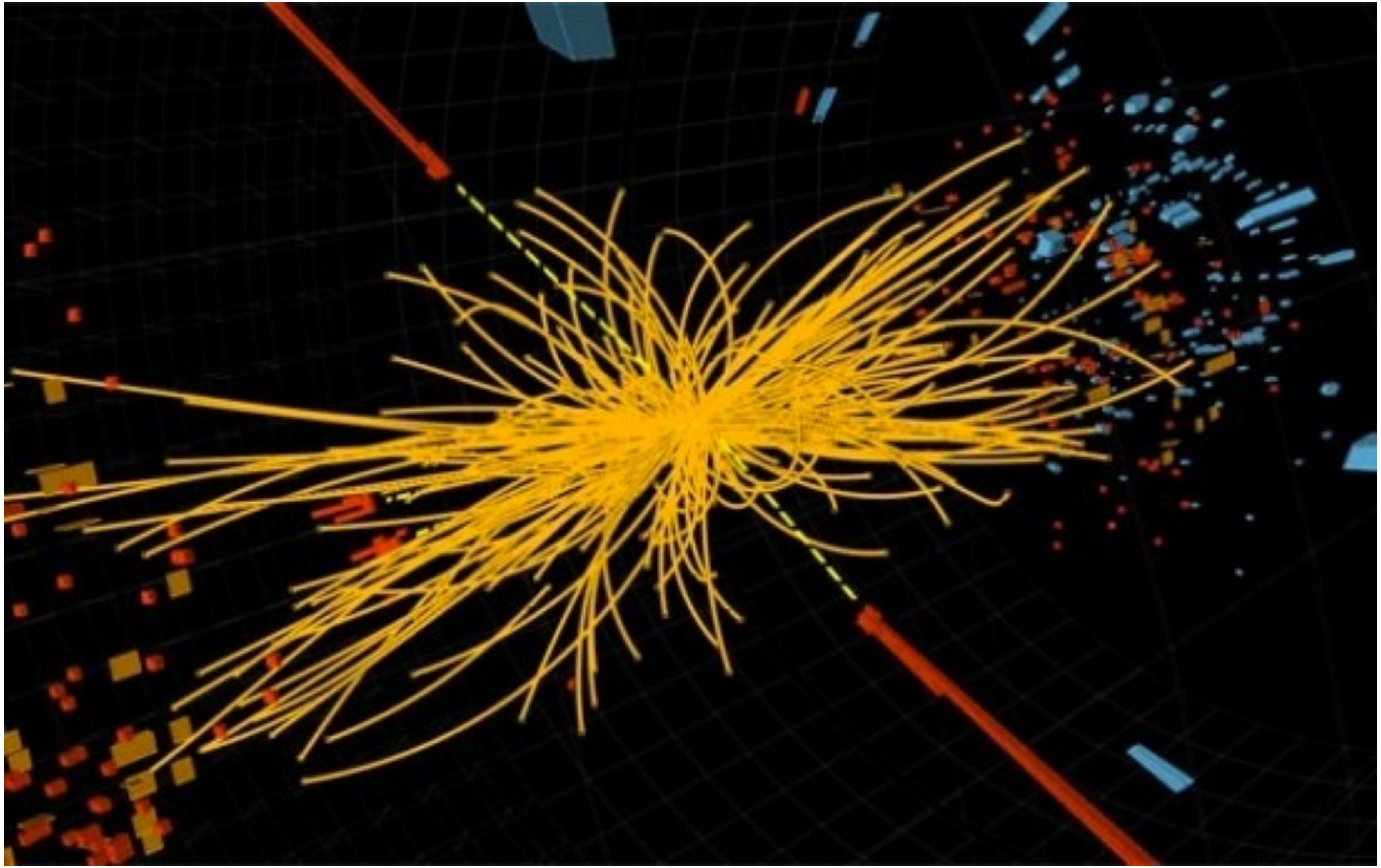
At colliders, particles are smashed into each other at very high energy. A very complicated final state is produced, populated by thousands of particles



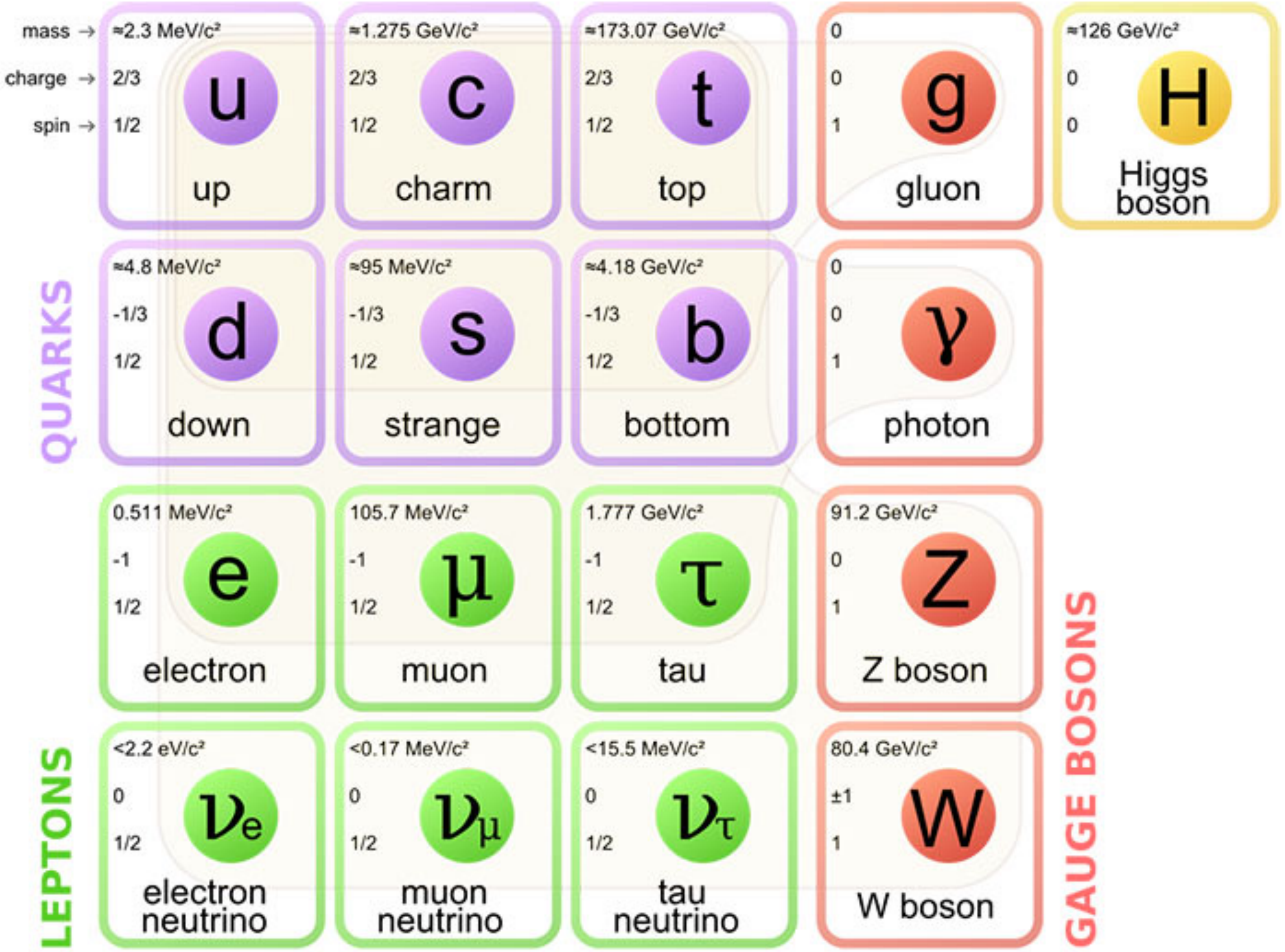
Detectors, plus sophisticated algorithmic procedures allow us to discriminate the ``signal'' from ``noise'' (background)

Modern particle accelerators provide us with a great opportunity to answer some of the most fundamental open questions in Physics

- Higgs boson’s properties
- Electroweak Symmetry breaking
- SUSY?
- Origin of mass
- Origin of proton’s spin
- New physics interactions
- Dark Matter, dark energy
- ….

A future synergy between the LHC and EIC will shed new light on them.

Standard Model: where we stand



Discovered at the LHC in 2012 after being theoretically predicted (at the same time) by theoretical physicists Brout, Englert, and Higgs in 1964.

The LHC already brought us in a new realm of precision

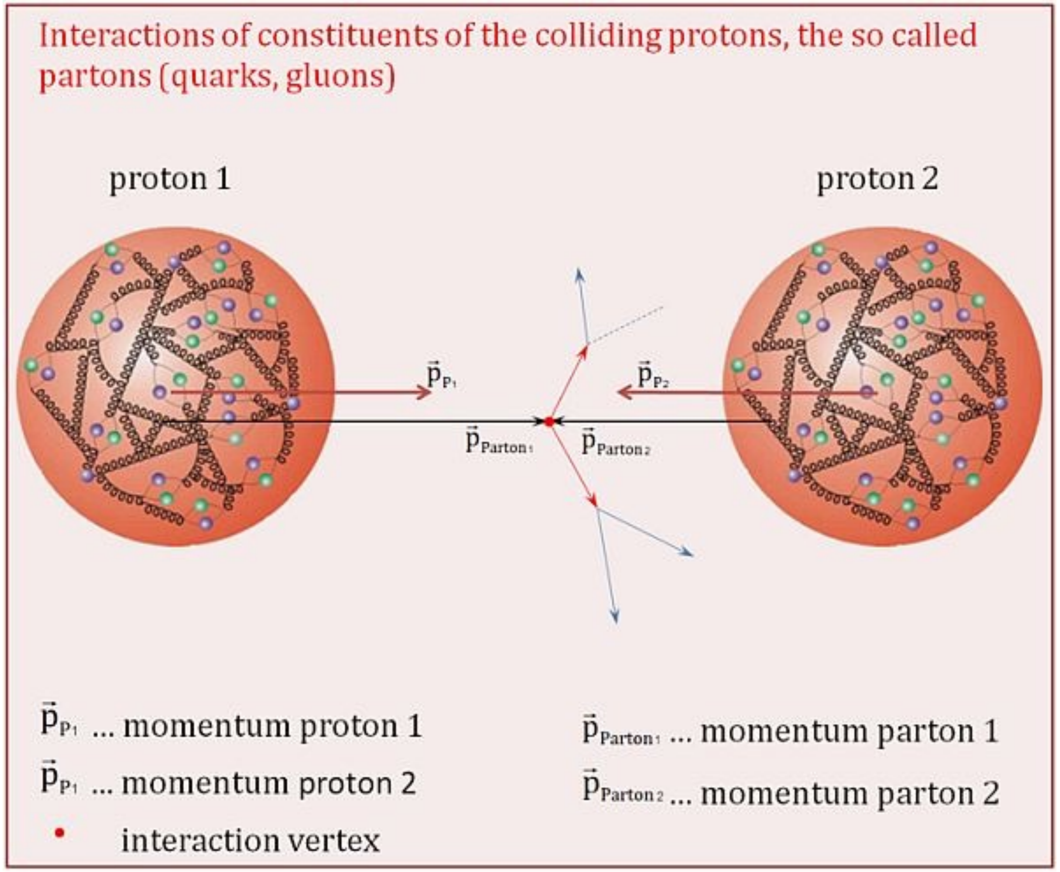
LHC collisions are produced at unprecedented energy. Physics observables are measured with unmatched precision.

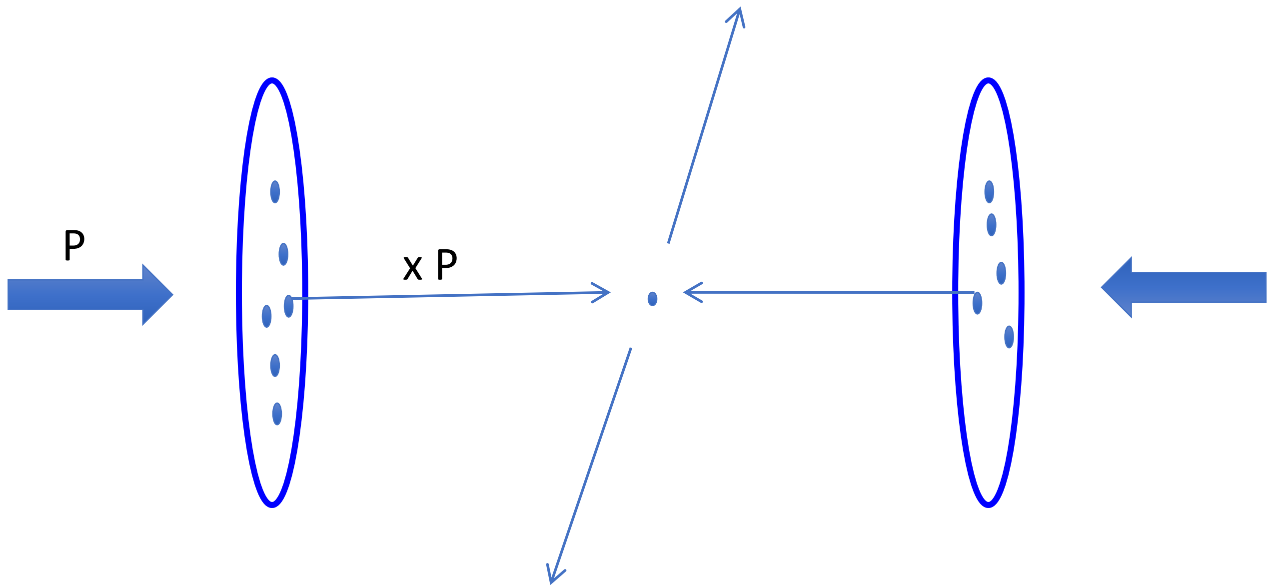
Theory predictions with comparable precision and accuracy are indispensable to:

- set stringent tests on the Standard Model (SM),
- search for signatures of new physics Beyond the Standard Model (BSM)

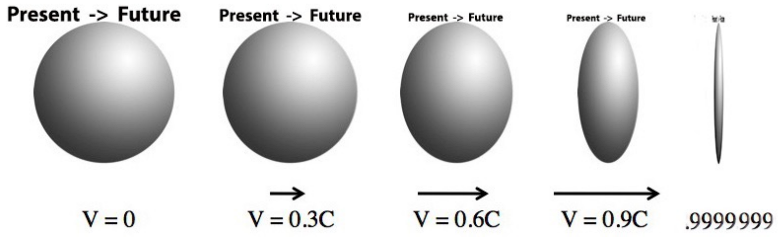
When two protons collide

…it's like smashing two pancakes with chocolate chips onto each other





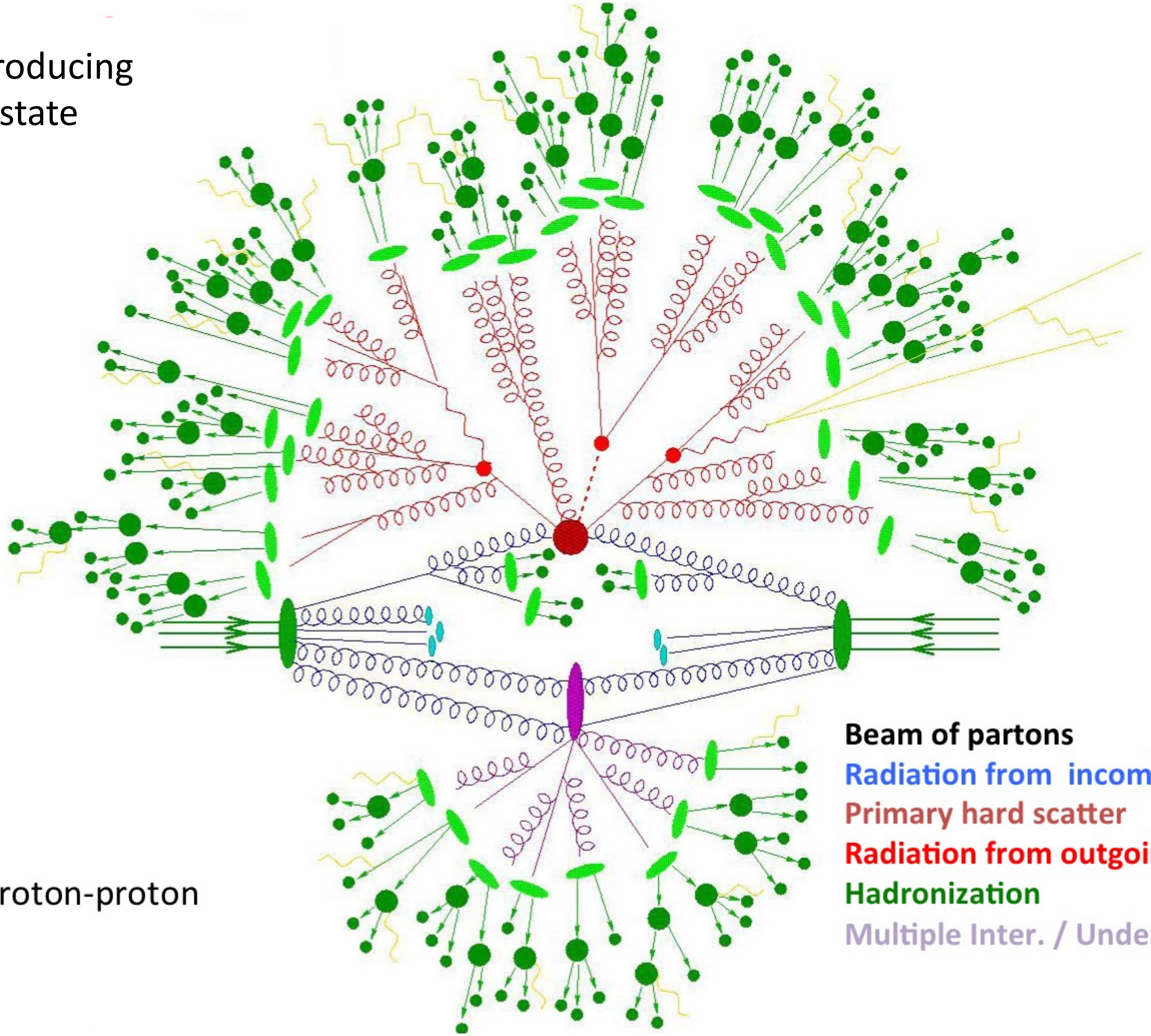
x = fraction of the initial proton's longitudinal momentum



Protons appear like disks because of Lorentz contraction.
They travel at 99.9999991% the speed of light.
The constituent partons are spread on each disk.

… or a bit more realistically, in terms of elementary processes

Protons break up producing
a complicated final state
with thousands
of particles



Beam of partons
Radiation from incoming partons
Primary hard scatter
Radiation from outgoing partons
Hadronization
Multiple Inter. / Underlying event

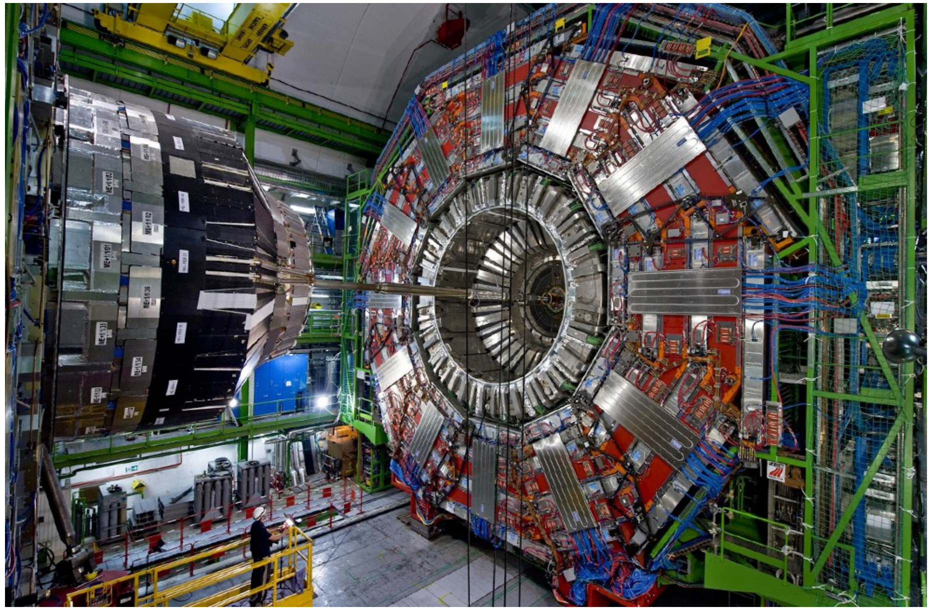
Typical proton-proton
collision

Image by F. Krauss

Quest for precision: what does it take to make a precise theoretical prediction at the LHC?

``Precisely'', what do we calculate? What do we observe?

- In particle collision experiments we count (observe) events relative to a specific collision (process) in our detector.
- The theory calculation predicts how many events are expected to be seen in the detector.



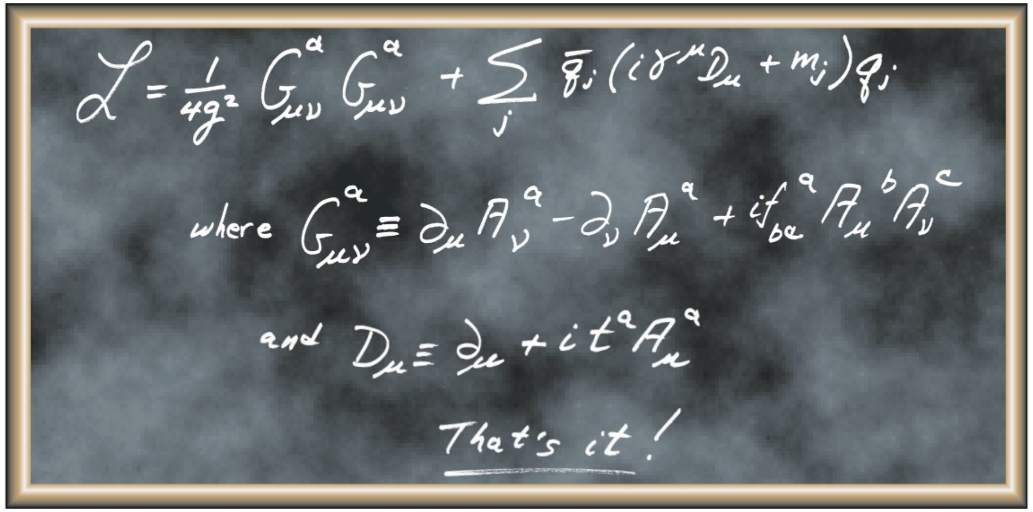


FIGURE 1. THE QCD LAGRANGIAN $\mathcal{L}$ displayed here is, in principle, a complete description of the strong interaction. But, in practice, it leads to equations that are notoriously hard to solve. Here m_j and q_j are the mass and quantum field of the quark of jth flavor, and A is the gluon field, with spacetime indices μ and ν and color indices a, b, c. The numerical coefficients f and t guarantee SU(3) color symmetry. Aside from the quark masses, the one coupling constant g is the only free parameter of the theory.

What does it take to make a precise theoretical prediction at the LHC?

How do we precisely estimate the theory rate at which events occur? This is given by a simple calculation

$$N_{\mathrm{ev}}/\mathrm{sec} = L \cdot \sigma$$

Can theoretically
be calculated

N_{ev}/sec = number of events per second

L = luminosity: number of collisions that can be produced in a detector per cm^2 and per second.
At the LHC, L =10^(34) cm^(-2) s^(-1) .

σ = cross section: describes the probability that two particles collide.
(It´s measured in "barn": 1 barn = 10^(-24) cm^2)

The theory cross section is (approximately) the product of two quantities!
Then we can compare to the experimental measurements

cross section

collisions between
protons constituents

$$\sigma = \sum_{\text{partons}=i,j} f_i \otimes f_j \otimes \hat{\sigma}_{ij}$$

+ tiny corrections if energy is high

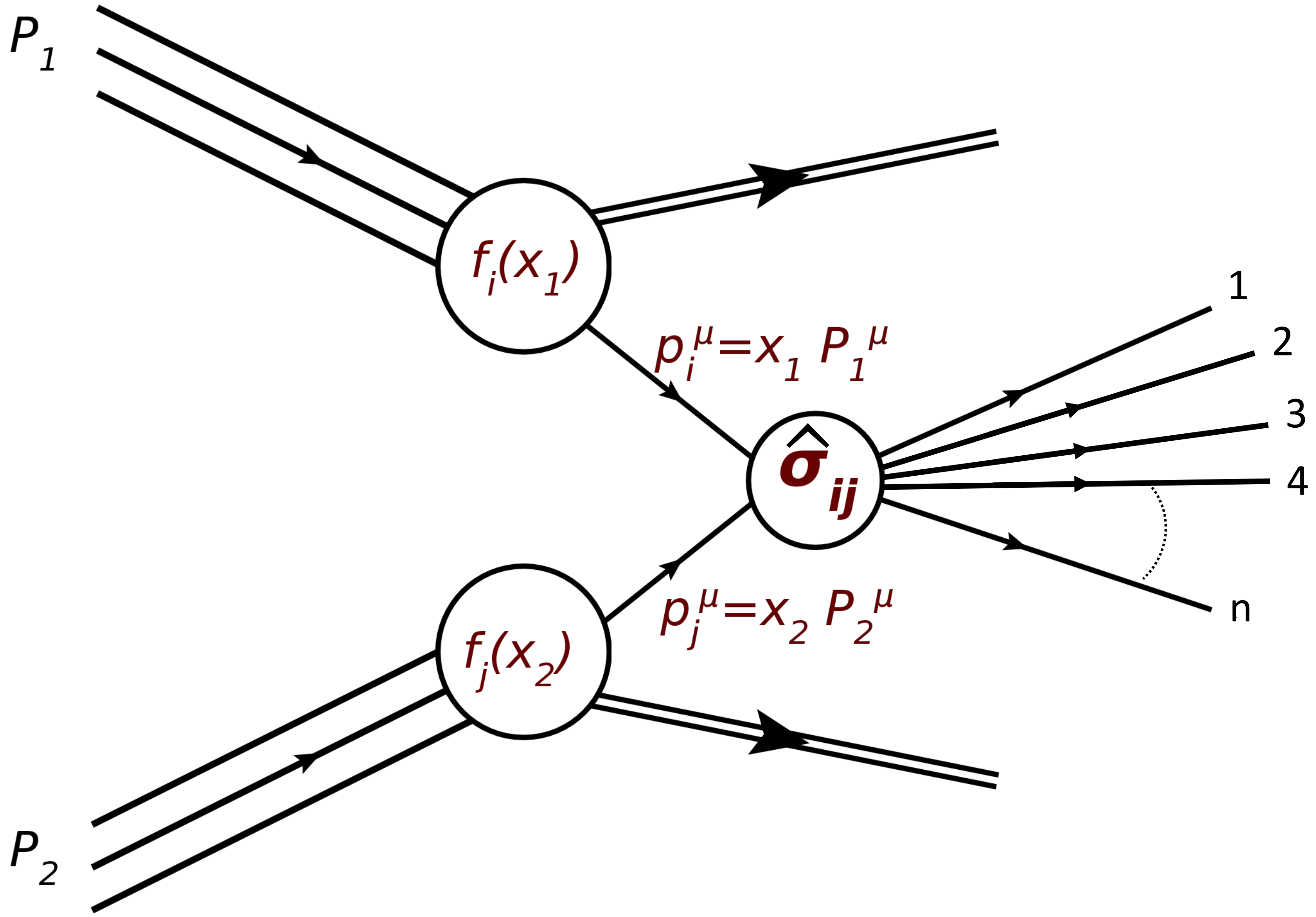
Inner structure of the proton

partons = proton's constituent quarks and gluons

f_i = parton distribution functions (PDFs) of the proton: universal probability that parton i is emitted by the proton at a certain energy scale: $f_i(x, Q^2)$

$\hat{\sigma}$ = hard scattering cross section: it depends on the elementary process we consider

Representation of a collision between protons at high energy

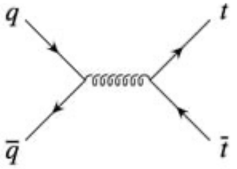


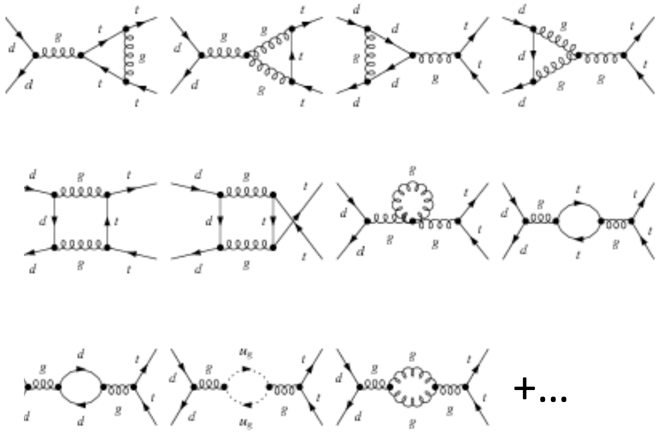
$f_i(x_1)$ = ? $\hat{\sigma}_{ij}$= ?

A precise theory prediction at parton level

$\hat{\sigma}$: perturbatively calculable as a series expansion in a small parameter (coupling).
As the perturbative order increases, the number of Feynman diagrams and topologies get more involved

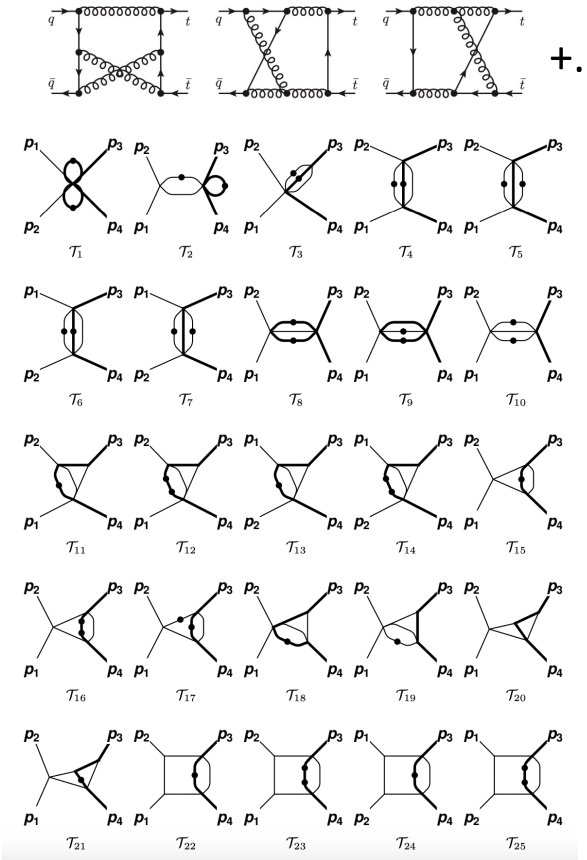
$q\bar{q} \rightarrow t\bar{t}$ @$NNLO$





+…

Part of the NLO contributions



+…

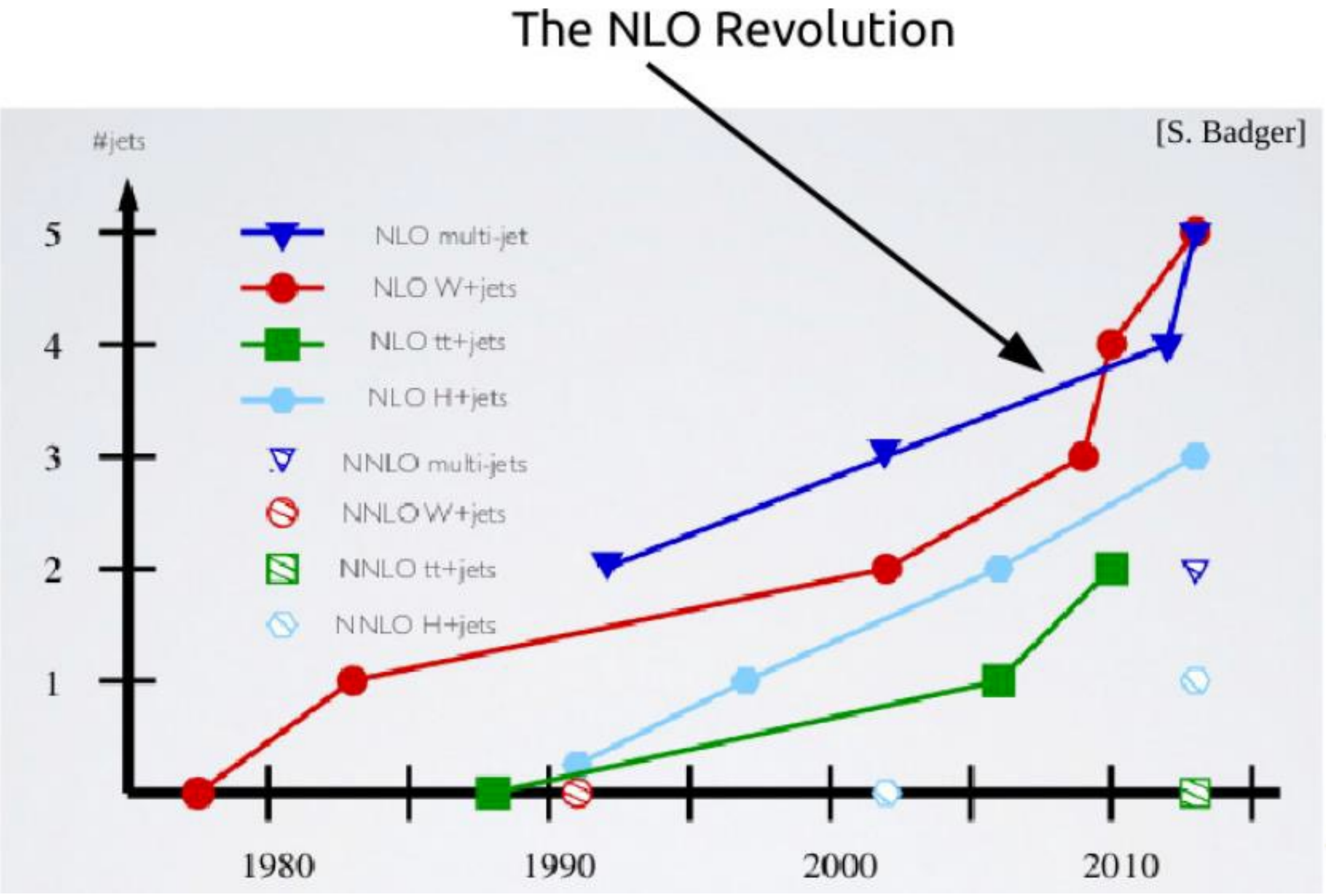
Di Vita, et al. JHEP 2019

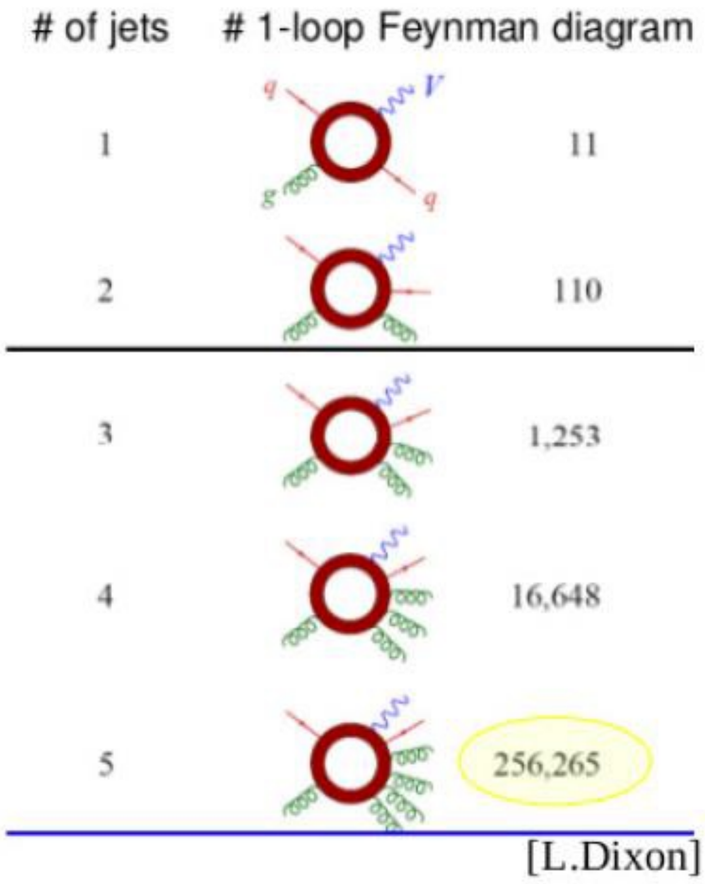
LO + NLO + NNLO + N^3LO + …

$$\hat{\sigma} = \hat{\sigma}_0 + \alpha\hat{\sigma}_1 + \alpha^2\hat{\sigma}_2 + \alpha^3\hat{\sigma}_3 + \ldots$$

(leading order + next-to-leading order + next-to-next-to ...). More higher orders -> more precision

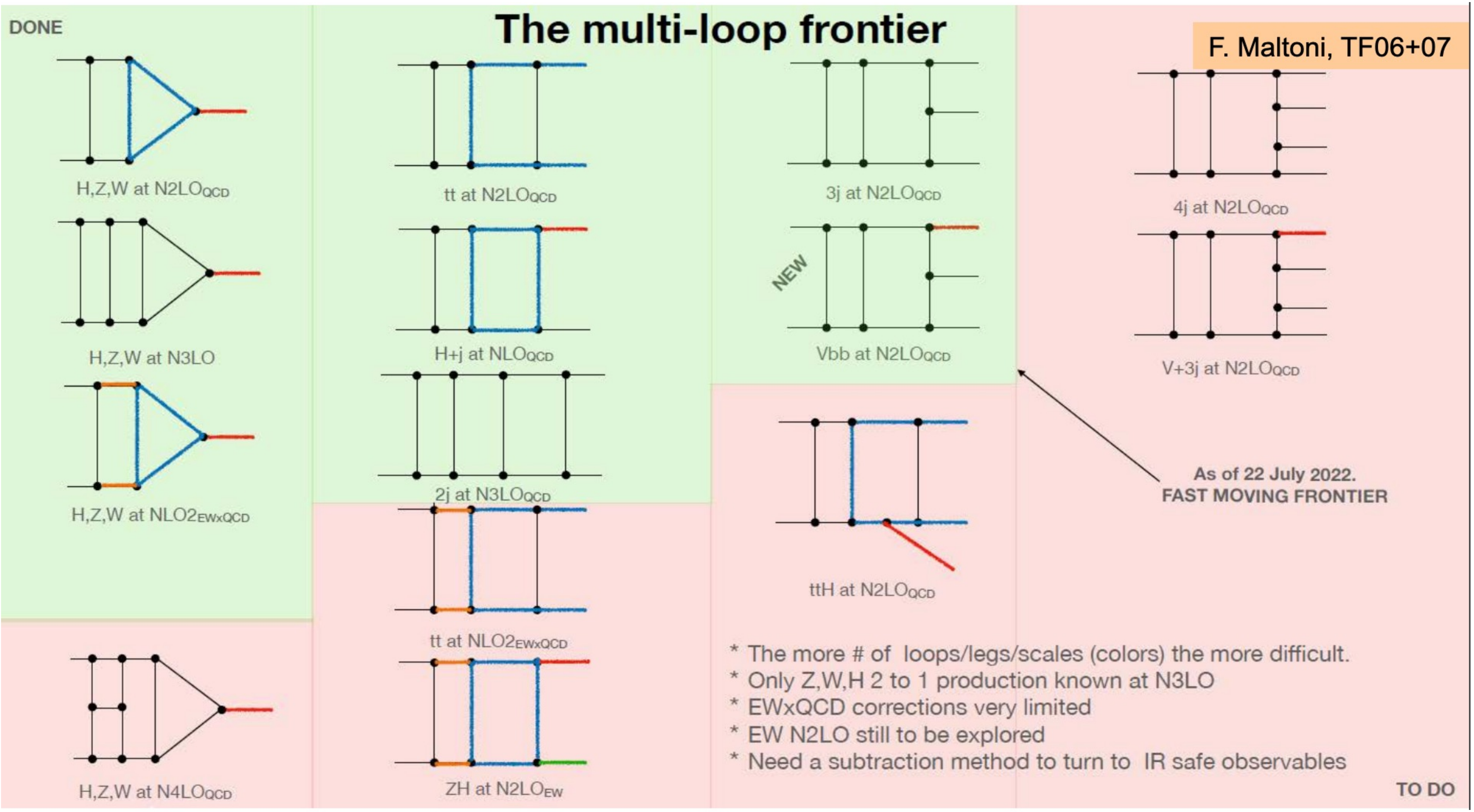
Remarkable progress in the past few years in Feynman diagrams calculations with many loops….and legs!





See related discussion in Gehrmann, Glover, et al. arXiv:2310.19757 for progress toward the next revolution at NNLO.

The multi-loop frontier



Dramatic advances in **perturbative** computations of NLO/NNLO/N3LO hard cross sections $\hat{\sigma}$.

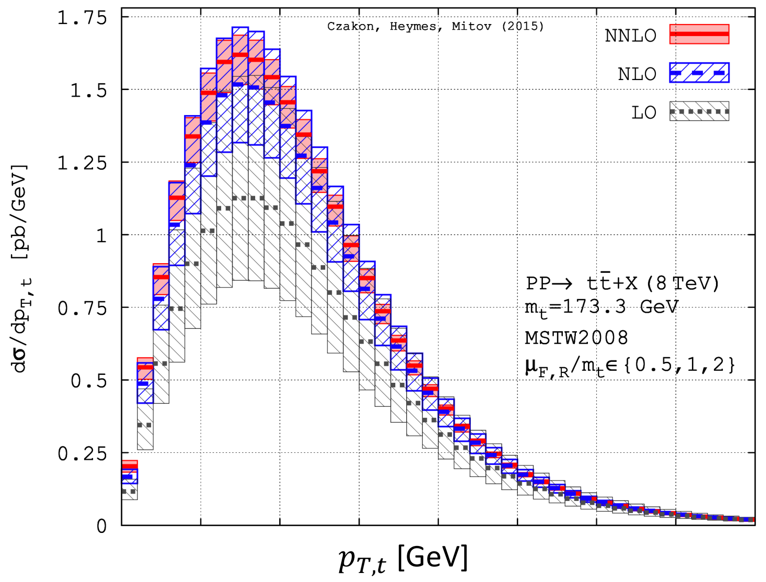
The High Energy Theory group at MSU played an important role in this

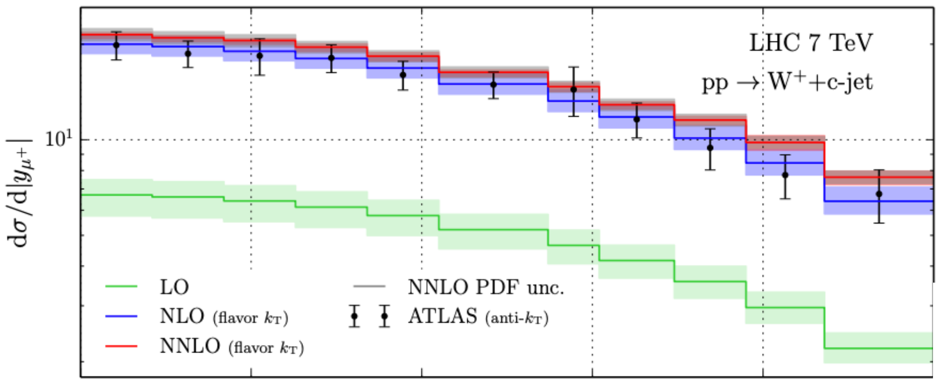
Many important processes have recently been obtained at high perturbative order, e.g., NNLO and N^3LO

Theory uncertainty due to scale dependence is greatly reduced including higher orders!!

→ $\hat{\sigma}$ can be computed with very high precision!

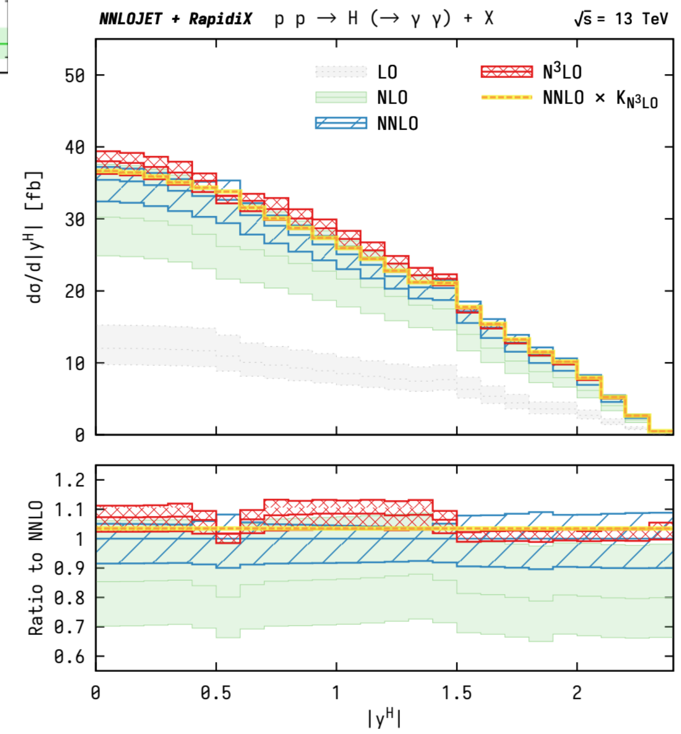
Czakon et al. PRL2016

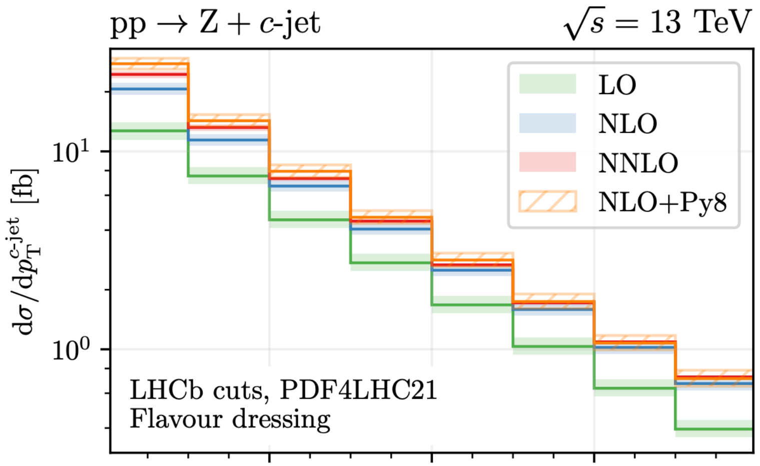


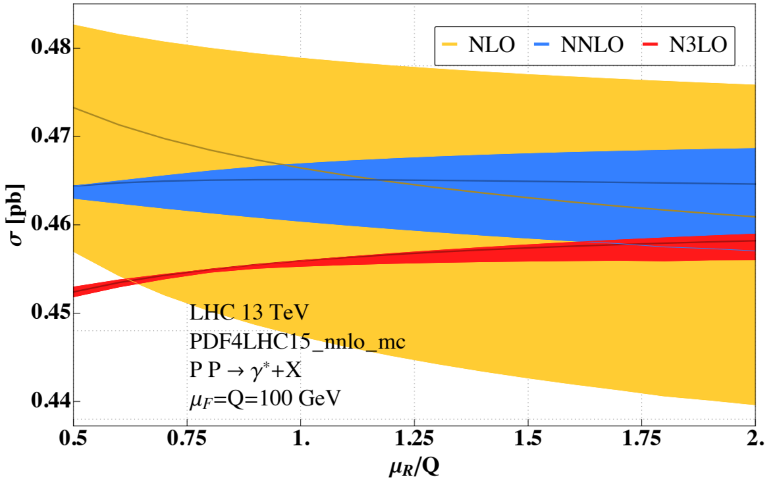


Czakon et al. JHEP2021 $|y_{\mu^+}|$

This list is of course not exhaustive.







Baglio et al. JHEP2022, Duhr et al. JHEP2022

Gauld et al. EPJC2023

Chen et al. PRL2021

What about proton PDFs?

$$\sigma = \sum_{\text{partons}=i,j} f_i \otimes f_j \otimes \hat{\sigma}_{ij}$$

Inside a proton

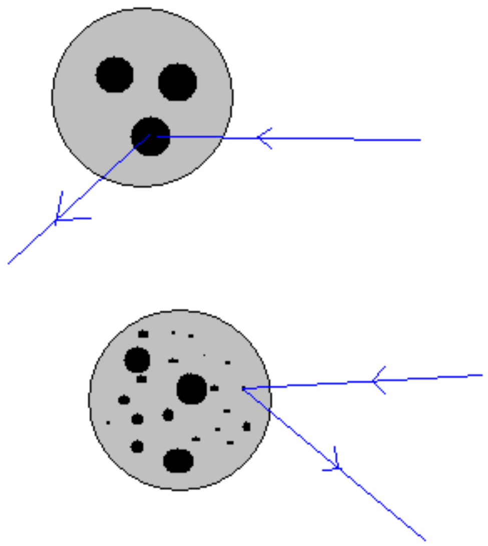
What about proton PDFs?

Parton Distribution Functions (PDFs) of the proton map out the longitudinal momentum distribution of proton's constituent quarks and gluons.

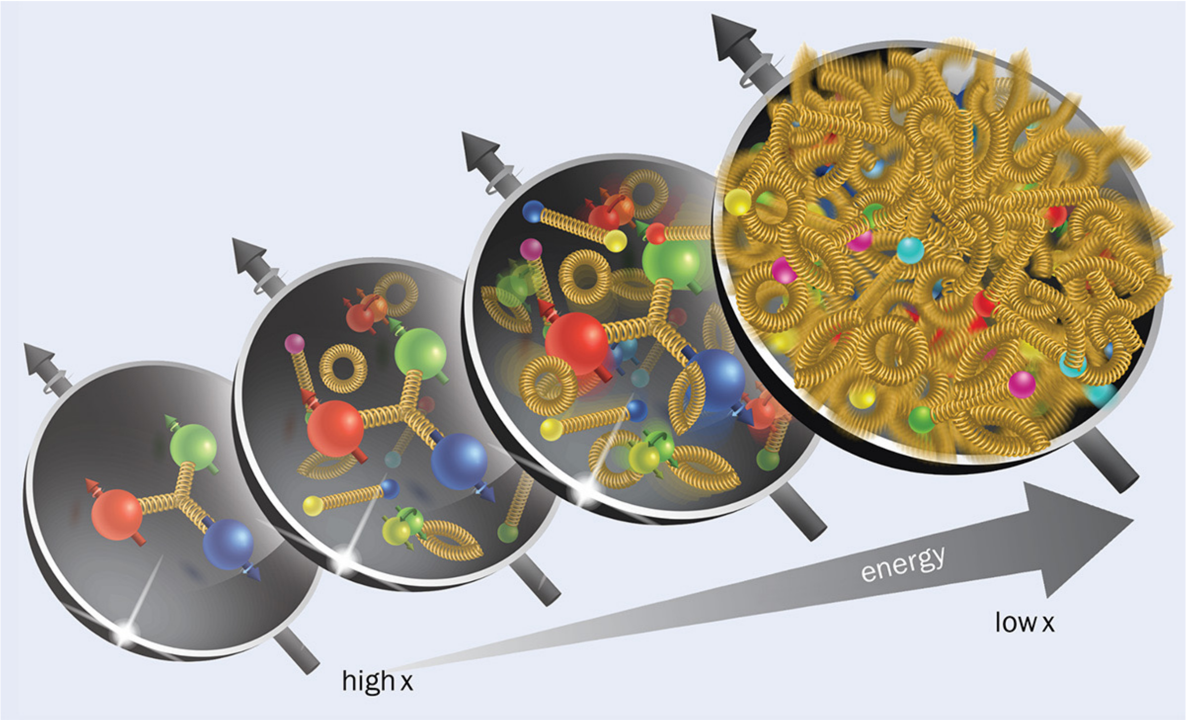
In collinear factorization, PDFs are defined as probability densities for finding a parton *i* with a certain longitudinal momentum fraction *x* at resolution scale *Q*.

$$f_i(x, Q^2)$$

PDFs are nonperturbative universal objects that cannot be fully predicted in pQCD.



We can only predict their energy behavior (Q-dependence).
The *x*-dependence must be extracted from experiments.

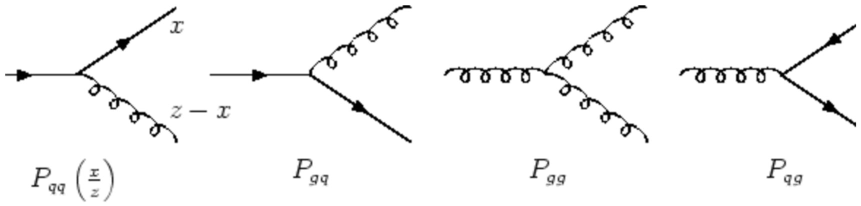


at scale Q_0

at scale $Q > Q_0$

$$\frac{d}{d\ln Q^2} f_a(x, Q^2) = \sum_{b=q,\bar{q},g} \int_x^1 \frac{dz}{z} P_{ab}\left(\frac{x}{z}, Q^2\right) f_b(z, Q^2)$$

DGLAP RG Equations
Dokshitzer-Gribov-Lipatov-Altarelli-Parisi



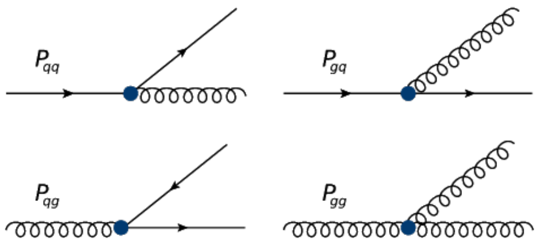
$P(x, Q^2)$ =splitting functions

Splitting functions at high perturbative order are very difficult to calculate!

$$P_{\mathrm{ns}}^{(0)}(x) = C_F\,(2p_{\mathrm{qq}}(x)+3\delta(1-x))$$
$$P_{\mathrm{ps}}^{(0)}(x) = 0$$
$$P_{\mathrm{qg}}^{(0)}(x) = 2n_f\,p_{\mathrm{qg}}(x)$$
$$P_{\mathrm{gq}}^{(0)}(x) = 2C_F\,p_{\mathrm{gq}}(x)$$
$$P_{\mathrm{gg}}^{(0)}(x) = C_A\Big(4p_{\mathrm{gg}}(x)+\frac{11}{3}\delta(1-x)\Big)-\frac{2}{3}n_f\,\delta(1-x)$$

LO 1973

Here is the result for the quantum corrections to the splitting functions in QCD



LO

$$\begin{aligned}
P_{\mathrm{ns}}^{(1)+}(x) = {} & 4C_AC_F\Big(p_{\mathrm{qq}}(x)\Big[\frac{67}{18}-\zeta_2+\frac{11}{6}\mathrm{H}_0+\mathrm{H}_{0,0}\Big]+p_{\mathrm{qq}}(-x)\Big[\zeta_2+2\mathrm{H}_{-1,0}-\mathrm{H}_{0,0}\Big] \\
& +\frac{14}{3}(1-x)+\delta(1-x)\Big[\frac{17}{24}+\frac{11}{3}\zeta_2-3\zeta_3\Big]\Big)-4C_Fn_f\Big(p_{\mathrm{qq}}(x)\Big[\frac{5}{9}+\frac{1}{3}\mathrm{H}_0\Big]+\frac{2}{3}(1-x) \\
& +\delta(1-x)\Big[\frac{1}{12}+\frac{2}{3}\zeta_2\Big]\Big)+4C_F^{\,2}\Big(2p_{\mathrm{qq}}(x)\Big[\mathrm{H}_{1,0}-\frac{3}{4}\mathrm{H}_0+\mathrm{H}_2\Big]-2p_{\mathrm{qq}}(-x)\Big[\zeta_2+2\mathrm{H}_{-1,0} \\
& -\mathrm{H}_{0,0}\Big]-(1-x)\Big[1-\frac{3}{2}\mathrm{H}_0\Big]-\mathrm{H}_0-(1+x)\mathrm{H}_{0,0}+\delta(1-x)\Big[\frac{3}{8}-3\zeta_2+6\zeta_3\Big]\Big)
\end{aligned}$$

$$\begin{aligned}
P_{\mathrm{ns}}^{(1)-}(x) = {} & P_{\mathrm{ns}}^{(1)+}(x)+16C_F\Big(C_F-\frac{C_A}{2}\Big)\Big(p_{\mathrm{qq}}(-x)\Big[\zeta_2+2\mathrm{H}_{-1,0}-\mathrm{H}_{0,0}\Big]-2(1-x) \\
& -(1+x)\mathrm{H}_0\Big)
\end{aligned}$$

$$P_{\mathrm{ps}}^{(1)}(x) = 4C_Fn_f\Big(\frac{20}{9}\frac{1}{x}-2+6x-4\mathrm{H}_0+x^2\Big[\frac{8}{3}\mathrm{H}_0-\frac{56}{9}\Big]+(1+x)\Big[5\mathrm{H}_0-2\mathrm{H}_{0,0}\Big]\Big)$$

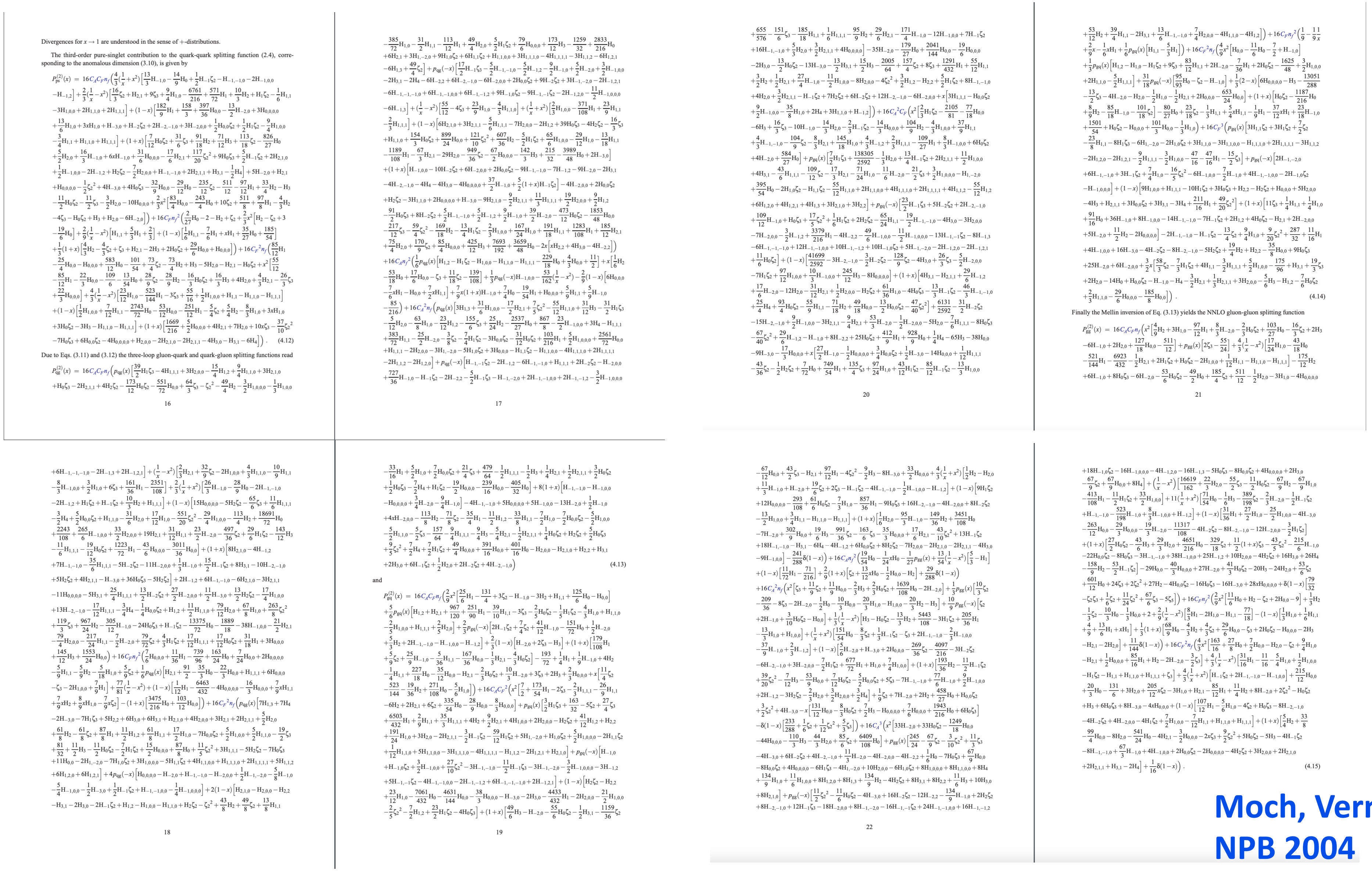
$$\begin{aligned}
P_{\mathrm{qg}}^{(1)}(x) = {} & 4C_An_f\Big(\frac{20}{9}\frac{1}{x}-2+25x-2p_{\mathrm{qg}}(-x)\mathrm{H}_{-1,0}-2p_{\mathrm{qg}}(x)\mathrm{H}_{1,1}+x^2\Big[\frac{44}{3}\mathrm{H}_0-\frac{218}{9}\Big] \\
& +4(1-x)\Big[\mathrm{H}_{0,0}-2\mathrm{H}_0+x\mathrm{H}_1\Big]-4\zeta_2x-6\mathrm{H}_{0,0}+9\mathrm{H}_0\Big)+4C_Fn_f\Big(2p_{\mathrm{qg}}(x)\Big[\mathrm{H}_{1,0}+\mathrm{H}_{1,1}+\mathrm{H}_2 \\
& -\zeta_2\Big]+4x^2\Big[\mathrm{H}_0+\mathrm{H}_{0,0}+\frac{5}{2}\Big]+2(1-x)\Big[\mathrm{H}_0+\mathrm{H}_{0,0}-2x\mathrm{H}_1+\frac{29}{4}\Big]-\frac{15}{2}-\mathrm{H}_{0,0}-\frac{1}{2}\mathrm{H}_0\Big)
\end{aligned}$$

$$\begin{aligned}
P_{\mathrm{gq}}^{(1)}(x) = {} & 4C_AC_F\Big(\frac{1}{x}+2p_{\mathrm{gq}}(x)\Big[\mathrm{H}_{1,0}+\mathrm{H}_{1,1}+\mathrm{H}_2-\frac{11}{6}\mathrm{H}_1\Big]-x^2\Big[\frac{8}{3}\mathrm{H}_0-\frac{44}{9}\Big]+4\zeta_2-2 \\
& -7\mathrm{H}_0+2\mathrm{H}_{0,0}-2\mathrm{H}_1x+(1+x)\Big[2\mathrm{H}_{0,0}-5\mathrm{H}_0+\frac{37}{9}\Big]-2p_{\mathrm{gq}}(-x)\mathrm{H}_{-1,0}\Big)-4C_Fn_f\Big(\frac{2}{3}x \\
& -p_{\mathrm{gq}}(x)\Big[\frac{2}{3}\mathrm{H}_1-\frac{10}{9}\Big]\Big)+4C_F^{\,2}\Big(p_{\mathrm{gq}}(x)\Big[3\mathrm{H}_1-2\mathrm{H}_{1,1}\Big]+(1+x)\Big[\mathrm{H}_{0,0}-\frac{7}{2}+\frac{7}{2}\mathrm{H}_0\Big]-3\mathrm{H}_{0,0} \\
& +1-\frac{3}{2}\mathrm{H}_0+2\mathrm{H}_1x\Big)
\end{aligned}$$

$$\begin{aligned}
P_{\mathrm{gg}}^{(1)}(x) = {} & 4C_An_f\Big(1-x-\frac{10}{9}p_{\mathrm{gg}}(x)-\frac{13}{9}\Big(\frac{1}{x}-x^2\Big)-\frac{2}{3}(1+x)\mathrm{H}_0-\frac{2}{3}\delta(1-x)\Big)+4C_A^{\,2}\Big(27 \\
& +(1+x)\Big[\frac{11}{3}\mathrm{H}_0+8\mathrm{H}_{0,0}-\frac{27}{2}\Big]+2p_{\mathrm{gg}}(-x)\Big[\mathrm{H}_{0,0}-2\mathrm{H}_{-1,0}-\zeta_2\Big]-\frac{67}{9}\Big(\frac{1}{x}-x^2\Big)-12\mathrm{H}_0 \\
& -\frac{44}{3}x^2\mathrm{H}_0+2p_{\mathrm{gg}}(x)\Big[\frac{67}{18}-\zeta_2+\mathrm{H}_{0,0}+2\mathrm{H}_{1,0}+2\mathrm{H}_2\Big]+\delta(1-x)\Big[\frac{8}{3}+3\zeta_3\Big]\Big)+4C_Fn_f\Big(2\mathrm{H}_0 \\
& +\frac{2}{3}\frac{1}{x}+\frac{10}{3}x^2-12+(1+x)\Big[4-5\mathrm{H}_0-2\mathrm{H}_{0,0}\Big]-\frac{1}{2}\delta(1-
\end{aligned}$$

NLO: 1980

It took several years to calculate higher orders in QCD perturbative expansions



Moch, Vermaseren, Vogt
NPB 2004

NNLO: next-to-next-to leading order. N^3LO: is currently in progress. 2023/24 ?

QCD GLOBAL ANALYSIS OF DATA in a nutshell:

Initialisation

Input Data

Data Type

- Collider ep
- Collider pp, ppbar
- Fixed Target data

χ^2

Theory Predictions

Factorisation Theorem

- PDF Parametrisation
- QCD Evolution
- Cross Section Calculation

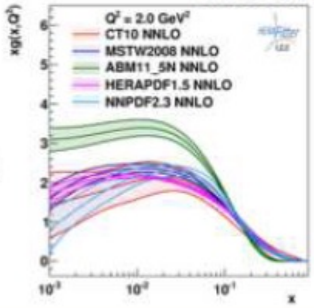
Minimisation (MINUIT)

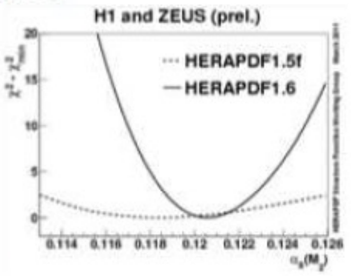
Treatment of the uncertainties:

- Nuisance parameters
- Covariance Matrix
- Monte Carlo method

Results

- PDF LHgrids
- alphas, mc, ..
- Data vs Predictions
- Chi2, pulls, shifts





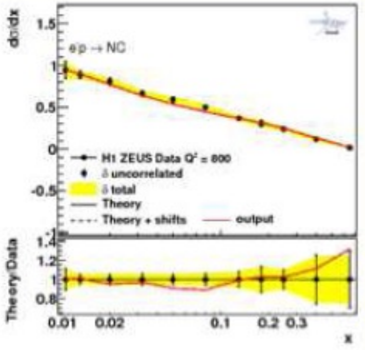
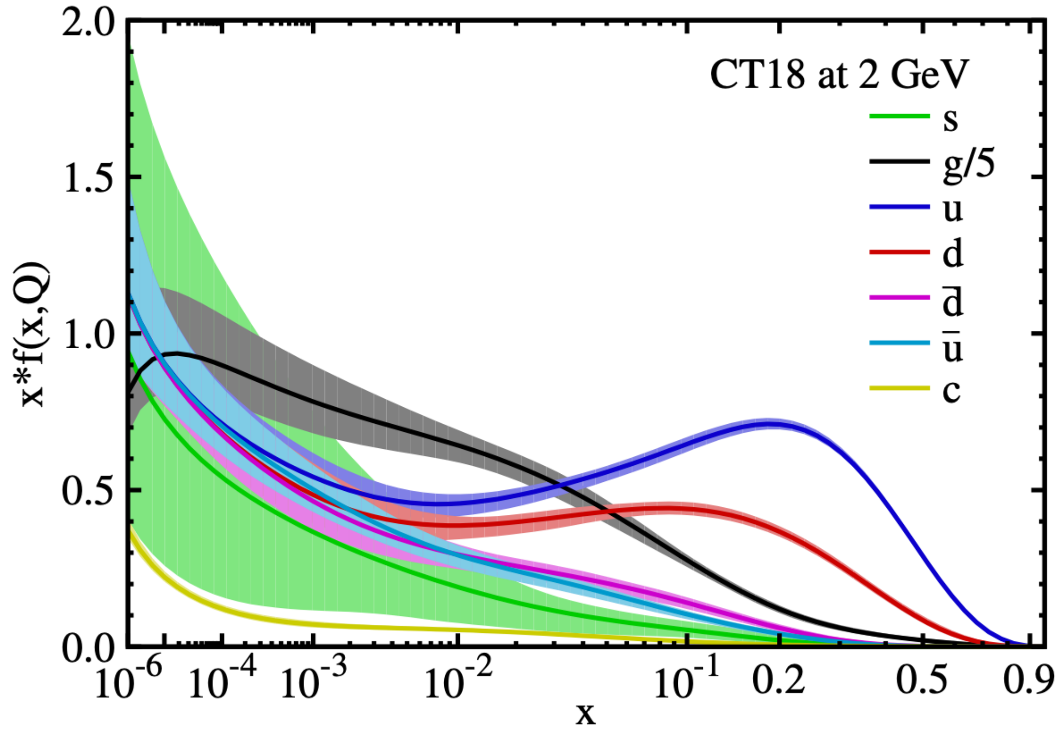
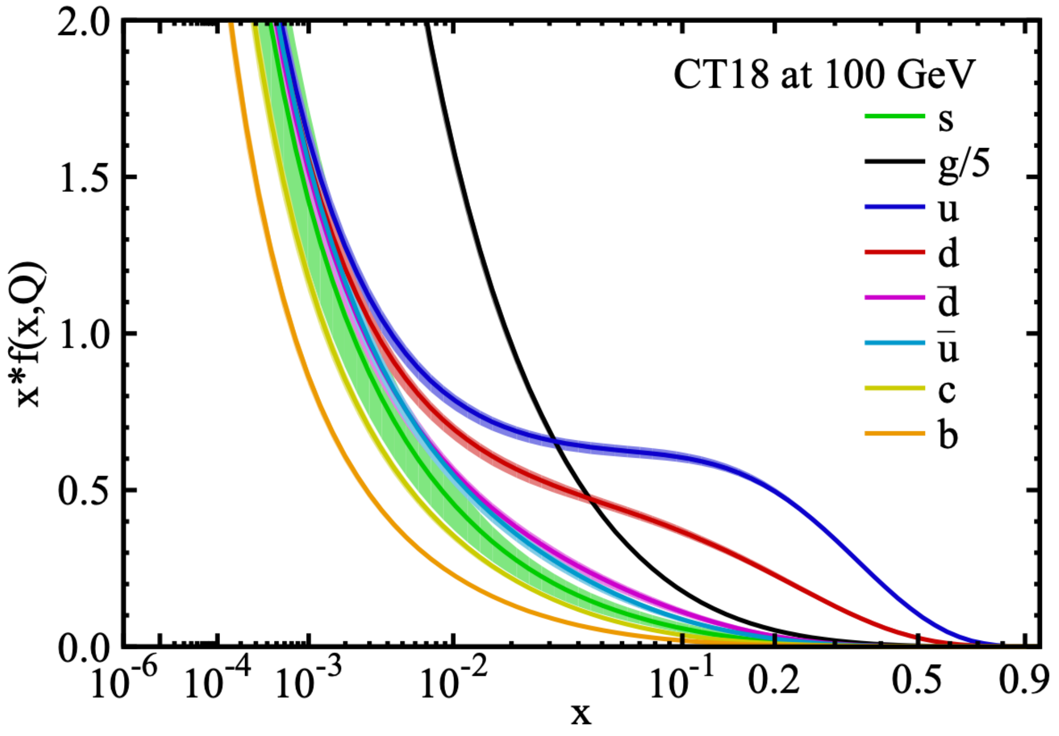


Image: The xFitter Coll.

CT18 NNLO PDFs (PRD 2021)





PDFs are determined by global analyses of world hadron data using a variety of analytical and statistical methods.

They still represent one of the major sources of uncertainties for theory predictions and simulations at hadron colliders: Bottleneck for precision!

The High Energy Theory group at MSU hosts the CTEQ group which is world leading in PDFs determinations