TM1016V

Free and Forced Vibrations

User Guide

TecQuipment has taken care to make the contents of this manual accurate and up to date. However, if there are any errors, please let us know so we can rectify the problem.

TecQuipment supplies a Packing Contents List (PCL) with the equipment. Carefully check the contents of the package(s) against the list. If any items are missing or damaged, contact TecQuipment or the local agent.

SSE/1022

TECQUIPMENT

Symbols Used in this Manual

NOTE	*Important information*
CAUTION	*Failure to carry out this instruction could cause damage to the apparatus, other equipment, personal property, or the environment.*
WARNING	***Failure to carry out this instruction could cause personal injury.***

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Introduction

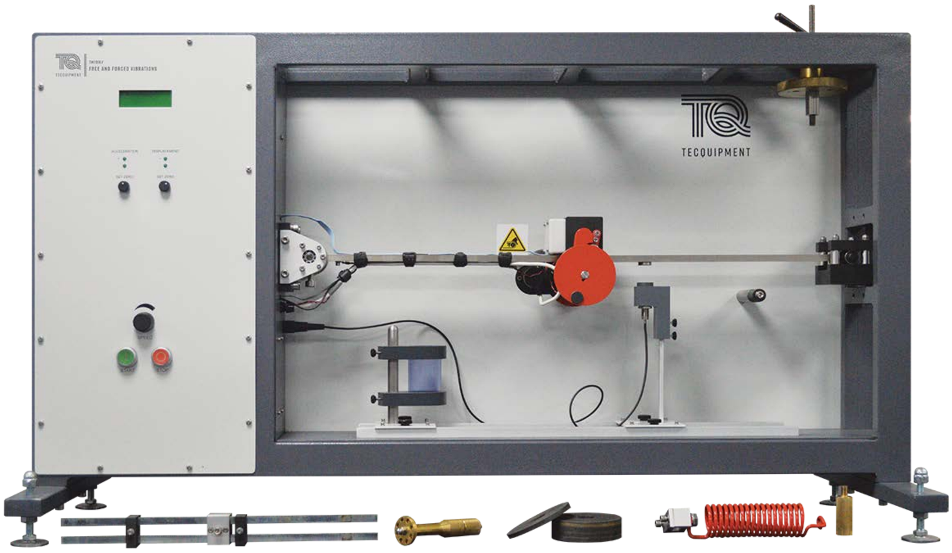

Figure 1 Free and Forced Vibrations (TM1016V)

This product has VDAS® On Baord

Free vibrations happen in many structures, where the structure vibrates at its natural frequency. Forced vibrations may occur where an external force causes the structure to vibrate at any frequency including the natural frequency of the structure. Where the forced vibration frequency equals the natural frequency, the structure will resonate at a potentially dangerous amplitude, damaging the structure. History has several examples of this problem. These include the collapse of the Tacoma Narrows Bridge in the US in 1940, and the temporary closure of the Millennium Bridge in London 2000-2002.

Designers need to understand how free and forced vibrations affect structures, the magnitude of the oscillations they can cause and how to reduce (damp) them. TecQuipment's Free and Forced Vibrations (TM1016V) uses a metal beam, held as a simply supported beam or supported by a spring. It shows the magnitudes of oscillations due to free and forced vibrations in simple structures and how damping affects their vibrations.

To automatically record the experiment results and save time, the apparatus works with TecQuipment's Versatile Data Acquisition System (VDAS®).

VDAS® is a registered trademark of TecQuipment Ltd.

Description

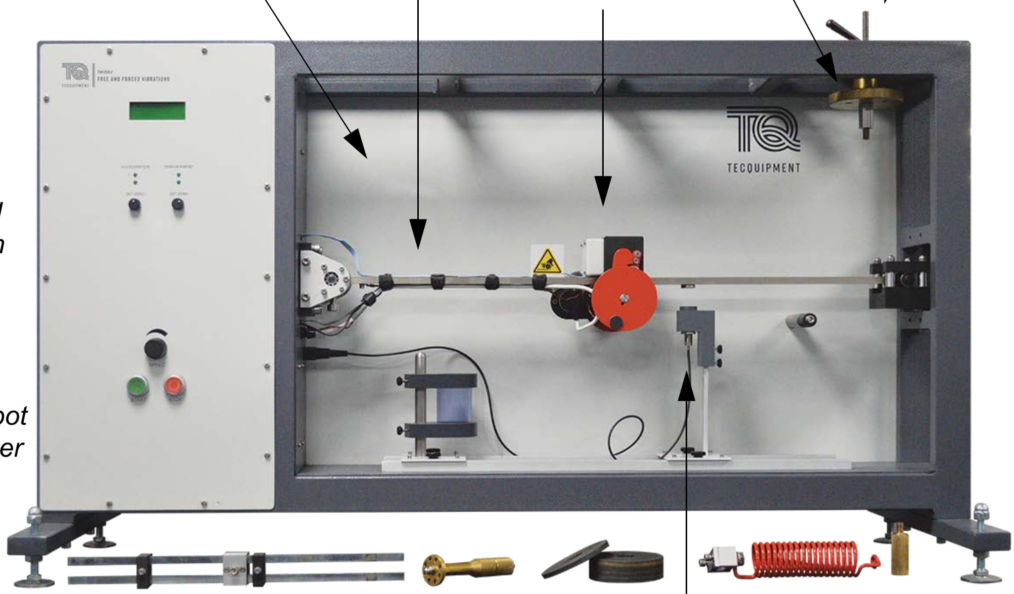

Figure 2 Free and Forced Vibrations (TM1016V)

The self-contained base unit has two areas - a left-hand section containing the controls and other electrical circuits, and a right-hand ‘window’ containing the experiment parts.

The control section has a display showing the speed of the exciter disc (in rev.min^{-1}, rad.s^{-1} and frequency in Hz) when used for forced vibrations. It also has manual controls for the servomotor. Other controls include the ‘set zero’ adjustments for the displacement and acceleration sensors.

Sockets at the back of the control section are for connection to a suitable computer (not supplied) running TecQuipment’s VDAS® software. VDAS® is needed to see the traces of the oscillations from the sensors around the beam.

The experiment window holds a metal beam. A low-friction bearing works as a pivot to hold the left-hand end of the beam. A ‘beam stop’ under the right-hand end of the beam helps to prevent damage, in case of incorrect use.

The metal beam can be set for two types of experiments:

- ‘rigid beam with spring’ - a spring supports the right-hand end of the ‘rigid’ beam. It allows the beam to vibrate through a small angle, at a frequency determined by the spring rate and beam mass. An adjuster and locking handle adjust the spring to keep the beam level.
- ‘simply supported’ - a second low-friction bearing in a holder secures the right-hand end of the beam, which can then oscillate due to its own flexural stiffness and mass.

A motor fixed to the beam has two rotating discs (for balance each side of the beam), each with an eccentric rotating mass. This combination forms an ‘exciter’, providing the forced vibrations. It uses a servometer with a controller. They work to give accurate speed control and reduced cyclical variations caused by the inertia of its rotating (and oscillating) mass.

The exciter has sensors to measure motor speed and the rotational position of the eccentric mass. These work with VDAS® to show the phase lag between the applied force and the beam displacement. An extra sensor at the exciter measures beam acceleration, so the phase difference between displacement and measured acceleration can be seen. It helps to confirm the acceleration that VDAS® derives from the displacement.

A non-contacting sensor measures the beam displacement by detecting the distance from a powerful magnet under the beam. This gives a measurement of the beam displacement at this position. Because it has no physical connection, this sensor has negligible damping effect on the oscillations.

The equipment includes a set of masses and a holder. These fix under the exciter to show how added mass affects the beam oscillations.

A dashpot damper may be connected under the beam for experiments with viscous damping of the vibrations (TecQuipment supply damping fluid with the equipment). A vibration absorber may also be attached, to experiment with an alternative method of reducing oscillation amplitude.

TecQuipment supply two metal ‘calibration’ blocks with the equipment. The shorter block is to check the displacement sensor zero. The longer block is to check the displacement sensor span calibration (see the maintenance section).

TecQuipment supply a storage tray with the equipment. This is useful to safely store the tools and other small parts.

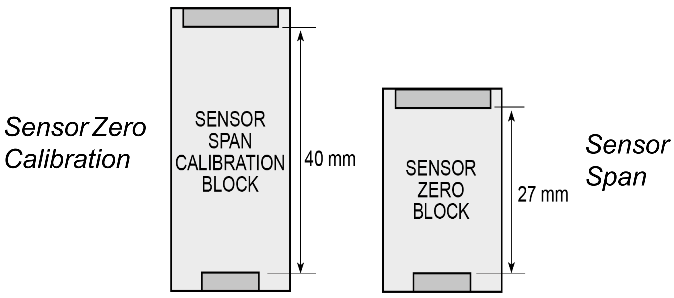

Figure 3 Calibration Blocks

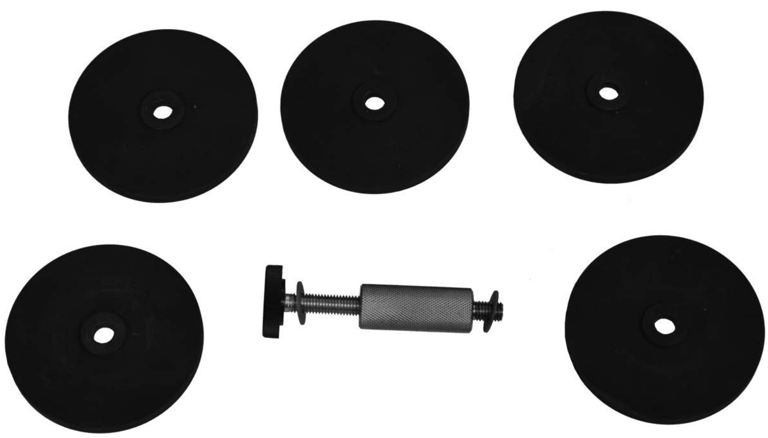

Figure 4 Masses and Mass Holder

Versatile Data Acquisition System - VDAS® On Board

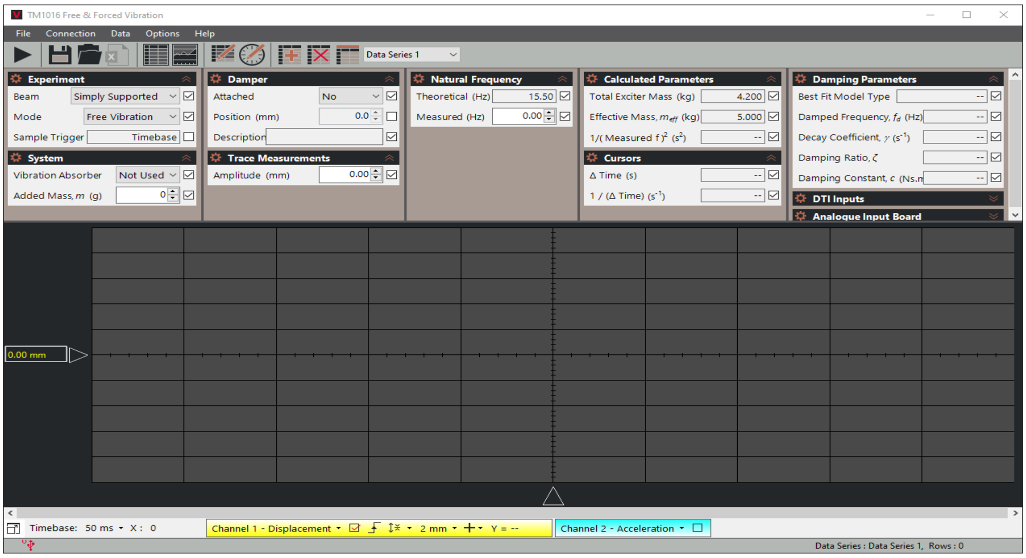

Figure 5 The VDAS® Software

The TM1016V includes VDAS® On Board data acquisition hardware that allows the unit to be directly connected to a computer via a USB cable (provided with the unit) this connects via the socket at the rear of the unit control section. no additional hardware is required. TecQuipment's VDAS® software is fully compatible and provides the following features.

- Display real time traces of the oscillations
- Automatically log data from the experiments
- Automatically calculate data
- Save time
- Reduce errors
- Create charts and tables of the data
- Export the data for processing in other software

NOTE *A suitable computer (not supplied) is needed to use TecQuipment's VDAS®.*

Technical Details

Part	Details
Operating Environment	Indoor (laboratory) Altitude up to 2000 m Temperature range 5°C to 40°C Maximum relative humidity 80% for temperatures up to 31°C, decreasing linearly to 50% relative humidity at 40°C Overvoltage category 2 (as specified in EN61010-1). Pollution degree 2 (as specified in EN61010-1).
Dimensions and Weight	1280 mm wide x 450 mm front to back x 800 mm high (assembled) and 80 kg Other parts approximately 4 kg total
Electrical Supply	Single Phase 90 - 250 VAC, 50 / 60 Hz, 1 Amp
Electrical Circuit Protection	20 mm 5 A Type T Fuse
External Connection	USB Socket
Servomotor Exciter	Nominal Maximum Speed: 17 Hz or 1020 rev.min^{-1} Mass $m_{exciter}$: 4.2 kg Offset mass: 2 x 4.5 g Offset mass radius: 0.035 mm
Displacement sensor	Distance from pivot = 0.5 m Note: Accelerometer is scaled to show acceleration at the same position as the displacement sensor.
Coiled Spring and Fixing	Nominal Rate k_{spring}: 3800 N.m^{-1} Spring Mass m_{spring}: 0.388 kg Fixing Mass m_{fixing}: 0.09 kg l_{spring} (distance from pivot): 0.75 mm
Masses and Mass Holder	5 x 400 g Masses 200 g Mass Holder
Beam	Tool Steel E = 2.0 x 10^{11} Pa 25 mm wide x 10 mm thick (nominal) l_{beam} (rigid beam and spring) = 0.815 mm Distance between pivots (simply supported) = 800 mm l_3(simply supported) = 0.375 m I_{beam} (rigid beam and spring) = 0.365 kg.m^2 I_{beam} (simply supported) = 2.083 x 10^{-9} m^4 k_{beam} (simply supported) = 4.74 x 10^4 N.m^{-1} Mass m_{beam}: 1.65 kg
Damping Fluid	500 mL non-toxic silicone oil Viscosity 200 cSt
Damper	m_{damper} (simply supported) = 0.4 kg m_{damper} (rigid beam and spring) = 0.53 kg Distance from pivot (rigid beam and spring) 200 mm to 300 mm
Calibration blocks	Sensor zero - machined to 27 mm Sensor span - machined to 40 mm

Part	Details
Vibration Absorber	Spring Steel (plated) E = 2.0 x 10^{11} Pa $I_{absorber}$ = 7.2179 x 10^{-12} m^4 $k_{absorber}$ = 1.09 x 10^3 $N.m^{-1}$ Mass (total) $m_{absorber}$ = 438 g Each moveable mass m_{mass} = 121 g Beam mass (each side) $m_{absorberbeam}$ = 0.126 $kg.m^{-1}$

Noise Levels

The maximum sound levels measured for this apparatus are lower than 70 dB(A).

Assembly and Installation

The terms **left**, **right, front** and **rear** of the apparatus refer to the operators’ position, facing the unit.

NOTE

A wax coating may have been applied to parts of this apparatus to prevent corrosion during transport. Remove the wax coating by using paraffin or white spirit, applied with either a soft brush or a cloth.

Follow any regulations that affect the installation, operation and maintenance of this apparatus in the country where it is to be used.

NOTE

Use the equipment in a clean laboratory or classroom type area.

CAUTION

The equipment uses a sensitive magnetic displacement sensor. Do not use it near any other equipment that may produce stray magnetic fields, such as mains electrical transformers or motors.

Installation

1. Put the equipment on a solid, level table or workbench.

WARNING

Use assistance to move the apparatus, it is heavy (see ***Technical Details****).*

Lift the equipment using its frame.

2. Use the adjustable feet to make sure the equipment is level.

NOTE

If the equipment is not level, this will affect the results.

3. Fit the spring adjuster to the top of the equipment.
4. Fit the threaded bar into the spring adjuster, then fit the washer, locking handle and dome shape nut (see Figure 6).

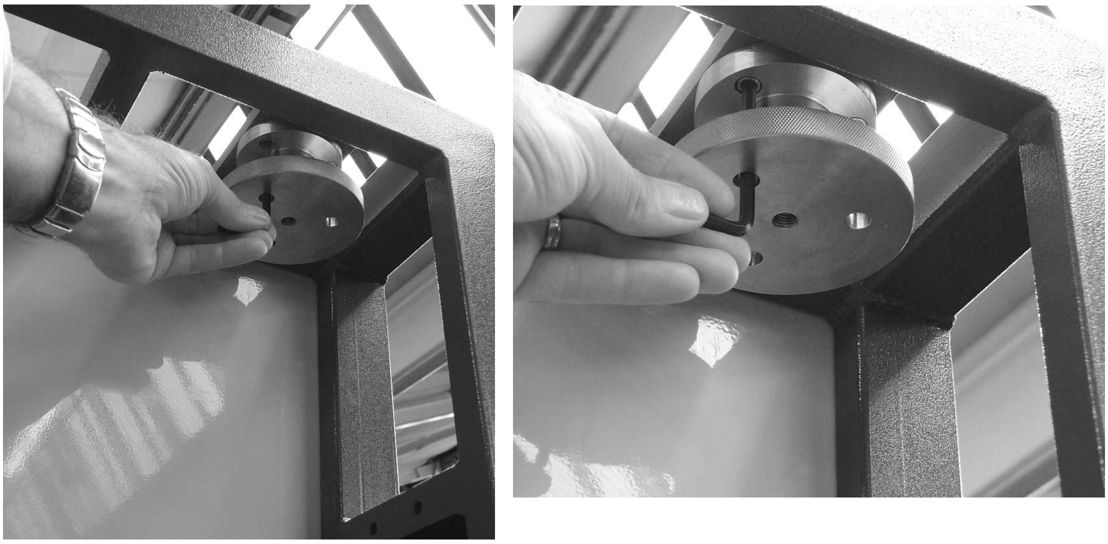

Figure 6 Fit the Spring Adjuster

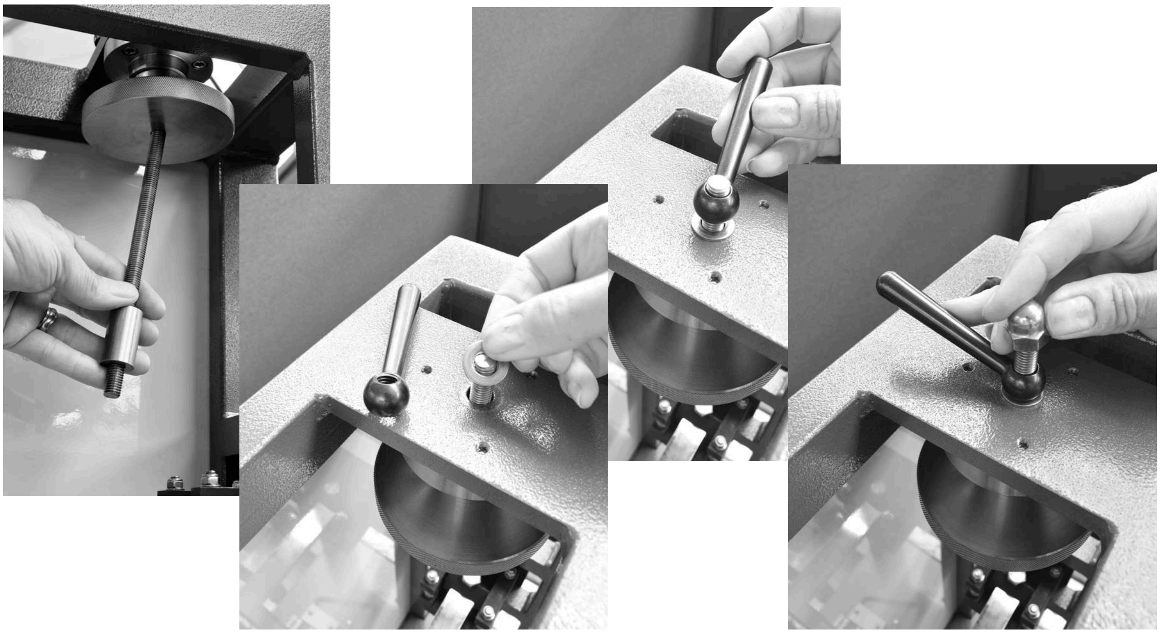

Figure 7 Fit the Bar and the Locking Handle

5. Remove the ‘keep’ from the magnet under the beam and store it safely.

WARNING

This equipment uses a powerful magnet - always put the protective metal plates (‘keeps’) back on the magnet when it is not in use. This helps to contain and keep the magnetism strong for several years.

Keep any sensitive mechanical watches or instruments away from the magnet.

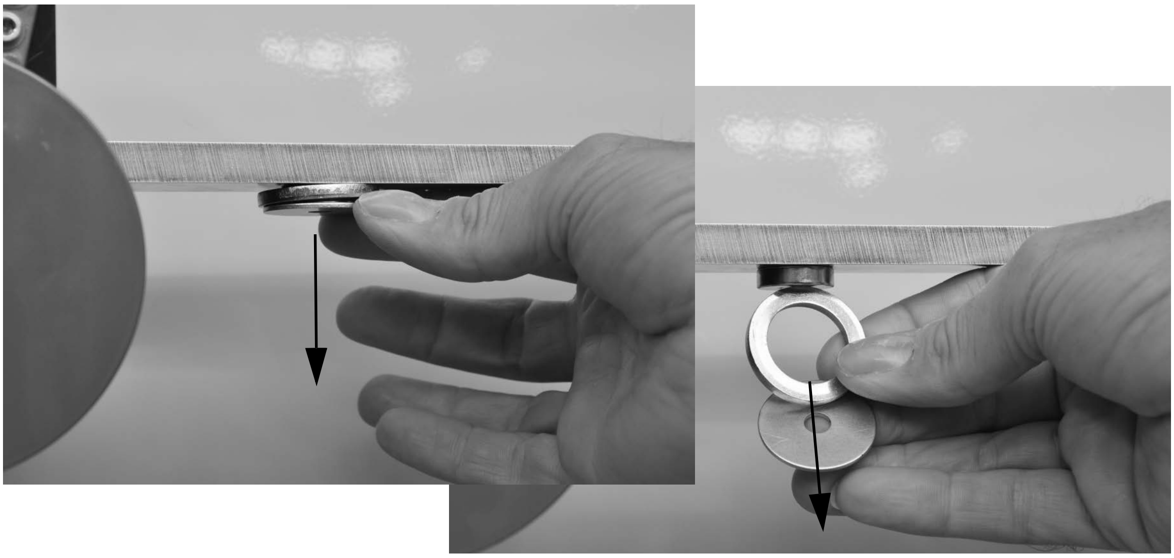

Figure 8 Remove Keep from Magnet

6. Connect the VDAS® output (at the back of the Control Section) to a suitable PC (not supplied) as shown in Figure 9.

7. Connect the equipment to the electrical supply as shown in **Electrical Connection** on page 12.

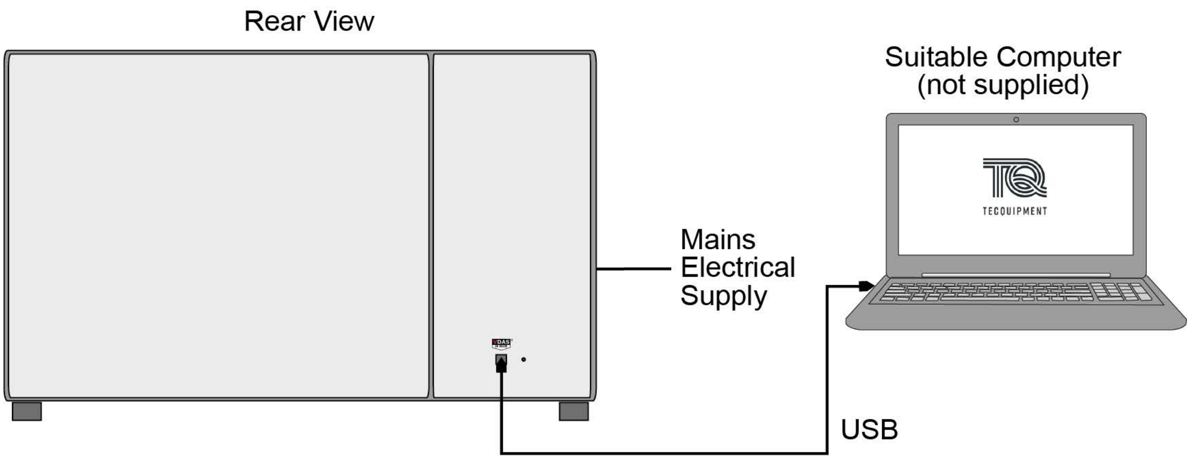

Figure 9 VDAS® and Electrical Connections

Electrical Connection

Use the cable supplied with the equipment to connect it to an electrical supply.

WARNING

Connect the equipment to the supply through a plug and socket. Connect it to earth.

The plug and socket and the mains supply connector at the side of the equipment are the mains disconnect devices. Make sure users can easily reach them.

These are the colours of each individual conductor:

E / Earth / Ground / ⏚	**Green and Yellow**	**OR**	**Green**
L / Live / Phase / Hot	**Brown**	**OR**	**Black**
N / Neutral	**Blue**	**OR**	**White**

Theory

Notation

Symbol	Details	Units
A	Beam displacement amplitude	m
B	Vibration absorber displacement amplitude	m
c	Viscous Damping Coefficient	$Ns.m^{-1}$ or $kg.s^{-1}$
e	An exponential function	2.718
E	Young's Modulus	MPa or $N.mm^{-2}$
f	Frequency	Hertz
g	Acceleration due to gravity	9.81 $m.s^{-2}$
I	Moment of Inertia (mass)	$kg.m^2$
	Moment of Inertia (area) or second moment of area	m^4
I_A	Total moment of inertia for a complete assembly	(as above)
k	Spring rate	$N.m^{-1}$
L or l	Length	m
m	Mass	kg
q	Forcing coefficient	$m.s^{-2}$
Q	Force due to offset mass	N
t	Time	Seconds
T	Period	Seconds
T_d	Damped period	Seconds
x_s and x_d	Spring or damper linear displacement	m
$\dot{x}$ and $\ddot{x}$	Linear velocity and acceleration	$m.s^{-1}$ and $m.s^{-2}$
y and $\dot{y}$ and $\ddot{y}$	Linear displacement, velocity and acceleration	m, $m.s^{-1}$ and $m.s^{-2}$
α	Phase Lag	Degrees
β	Magnification factor	-
δ	Logarithmic decrement	-
ϕ	Phase angle	Radians
γ	Decay coefficient	s^{-1}

Symbol	Details	Units
π	Pi	
θ and $\dot{\theta}$ and $\ddot{\theta}$	Angular displacement, velocity and acceleration	rad.s^{-1} and rad.s^{-2}
ω	Angular Frequency	radians per second (rad.s^{-1})
Ω	Forced frequency	radians per second (rad.s^{-1})
ζ	Damping ratio	-

Table 1 Notation

Constants	Details	Units
C	Constants Matrix	-
r_1 and r_2	Roots for 2nd order differential equation	-
F	Force Matrix	N
M and N	Generic Mathematical Constant	-
Y	Amplitude Matrix	m

Table 2 Constants

Units Conversions

Speed and Angular Velocity

The display shows the rotational velocity (speed) of the motor in revolutions per minute ($rev.min^{-1}$), radians per second ($rad.s^{-1}$) and frequency (Hz).

2π (6.28) radians = 360 degrees = one revolution.

One radian = 57.29578 degrees.

$$1\ rev.min^{-1} = 1/60\ rev.s^{-1}\ (0.0166\ rev.s^{-1}) = 2\pi/60\ rad.s^{-1}\ (0.1047\ rad.s^{-1})$$

$$1\ rev.min^{-1} = 1/60\ rev.s^{-1} = 0.0166\ \text{Hertz}$$

$$60\ rev.min^{-1} = 1\ rev.s^{-1} = 1\ \text{Hertz}$$

Frequency, Radians and Period

Frequency can be termed in ‘ordinary’ frequency (f), where something repetitive occurs in a given unit of time, and with units of Hertz. Where it applies to something moving with a sinusoidal motion, it may also be termed ‘angular’ frequency (ω) or (Ω). It has units of radians per second.

One revolution = 360 degrees = 2π radians

One revolution per second = $2\pi\ rad.s^{-1}$ = 1 Hertz

$$\omega \text{ or } \Omega = 2\pi f$$

The period (T) is the time taken for a complete oscillation. It is equal to the inverse of the frequency, so

$$T = 1/f = 2\pi/\omega$$

Beam and Spring - Small Angle Approximation

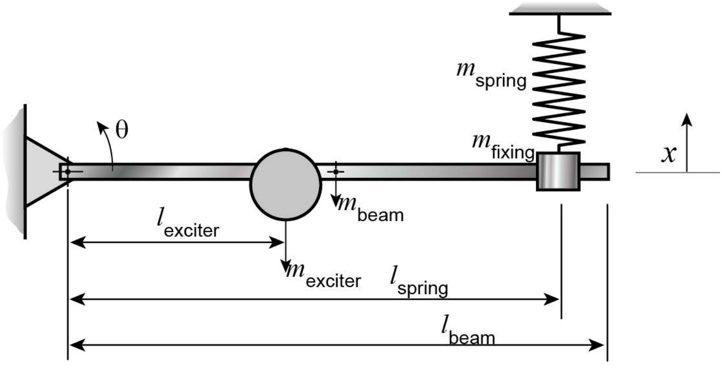

Figure 10 Beam and Spring

NOTE

When used with the spring, this theory assumes that the beam itself does not bend - it is sensibly 'rigid' in comparison with the spring.

The theory also assumes that the exciter forms a simple point mass at a given position along the beam.

The exciter remains fitted to the beam for all experiments. Its value may be increased using the added masses.

From Figure 10, it can be seen that the beam will oscillate about the left hand pivot through an angle θ. This is an angular motion. Using the small angle approximation, this angle is approximately equal to its sine in radians:

$$\sin\theta \cong \theta$$

Beam and Spring - Free Vibrations and Damping

Free Vibrations

Free vibration occurs in a system when it is given an initial input of energy. The system then vibrates at its natural frequency until the initial energy has been dissipated to friction or some other sort of damper or absorber.

From Figure 10, summing the moments about the pivot gives an equation of motion as:

$$I_A\ddot{\theta} + kx_s l_{spring} = 0$$

Theoretically, for a 'slender' beam with a uniformly distributed mass, its inertia would be based around its centre of gravity with the addition of any parallel axis components. Also, in theory it can be assumed the spring is light, having no affect on the total system inertia. However, because the beam becomes a relatively long moment arm and the spring has a relatively large mass, the spring and its fixing as a point mass must be allowed for.

The total mass moment of inertia for the complete assembly (I_A) is the sum of the individual values for the beam, the mass and the spring.

$$I_A = I_{beam} + I_{exciter} + I_{spring} \qquad (1)$$

$$I_{beam} = \frac{1}{12}m_{beam}l_{beam}^{\;2} + m_{beam}\left(\frac{l_{beam}}{2}\right)^2 = \frac{1}{3}m_{beam}l_{beam}^{\;2} \qquad (2)$$

$$I_{exciter} = m_{exciter}l_{exciter}^{\;2} \qquad (3)$$

Note: This theory assumes the central mass acts as a point load.

For accuracy, the theory allows for a proportion of the spring mass - that part which moves and contributes to the overall mass of the system. According to Rayleighs theory, this is equal to 1/3 the mass of the spring. The fixing between the spring and the beam also needs to be allowed for.

$$I_{spring} = \left(\frac{m_{spring}}{3} + m_{fixing}\right)l_{spring}^{\;2} \qquad (4)$$

Using the small angle approximation

$$x_s = \theta l_{spring}$$

So

$$I_A\ddot{\theta} + kl_{spring}^{\;2}\theta = 0 \quad \text{or} \quad \ddot{\theta} + \frac{kl_{spring}^{\;2}}{I_A}\theta = 0 \qquad (5)$$

This is a second order differential equation. A general solution of this equation gives:

$$\theta = A\cos\left(\sqrt{\frac{kl_{spring}^{2}}{I_A}}t - \alpha\right) \qquad (6)$$

Where A is the amplitude and α is the phase (two constants based on the initial conditions of the motion).

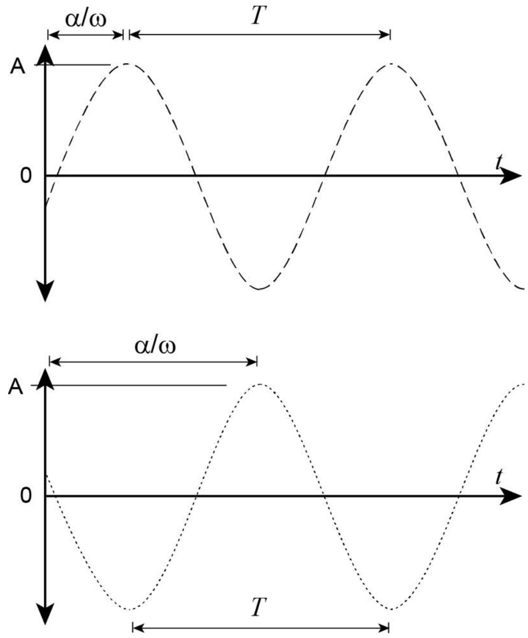

Figure 11 Amplitude and Phase for Periodic Motion

From the diagram, we can see the vibratory character of the motion. Cosine function is a periodic function that repeats every 2π. Thus our period of oscillation is:

$$\sqrt{\frac{kl_{spring}^{2}}{I_A}}T = 2\pi \text{ and therefore } T = 2\pi\sqrt{\frac{I_A}{kl_{spring}^{2}}}$$

This is the natural time period, determined by the spring rate and inertia (and therefore mass) in the system. The system will continue to vibrate with this period unless an external force influences it. The natural period in terms of frequency or angular velocity gives:

$$f = \frac{1}{2\pi}\sqrt{\frac{kl_{spring}^{2}}{I_A}} \text{ or } \omega = \sqrt{\frac{kl_{spring}^{2}}{I_A}} \text{ or } \omega^2 = \frac{kl_{spring}^{2}}{I_A} \qquad (7)$$

Inserting this into the differential Equation 5 gives:

$$\ddot{\theta} + \omega^2\theta = 0$$

From this, it can be seen that the natural frequency of any system can be found if the equation is arranged into that form.

The formula can be re-arranged to find the spring stiffness based on the resonant frequency.

$$K = \frac{I_A(2\pi f)^2}{l_{spring}^2} = \frac{I_A\omega^2}{l^2_{spring}}$$

Phase of Displacement Derivatives

To examine the phase relationships between displacement, velocity and acceleration of the beam as it vibrates through a cycle, the general equation for a sine wave must be considered (Equation 6):

$$\theta = A\cos(\omega t - \alpha)$$

Then

$$\dot{\theta} = -\omega A\sin(\omega t - \alpha) \text{ and } \ddot{\theta} = -\omega^2 A\cos(\omega t - \alpha)$$

This means that if the displacement of the beam is measured and its 2nd derivative (acceleration) found, it can be compared with the acceleration measured by the accelerometer.

We know that sin and cos have a 90 degrees ($\pi/2$ or $T/4$) phase difference, so the velocity is -90° ($-\pi/2$) out of phase to displacement. Also, acceleration is negative or 180° (π or $T/2$) out of phase with displacement.

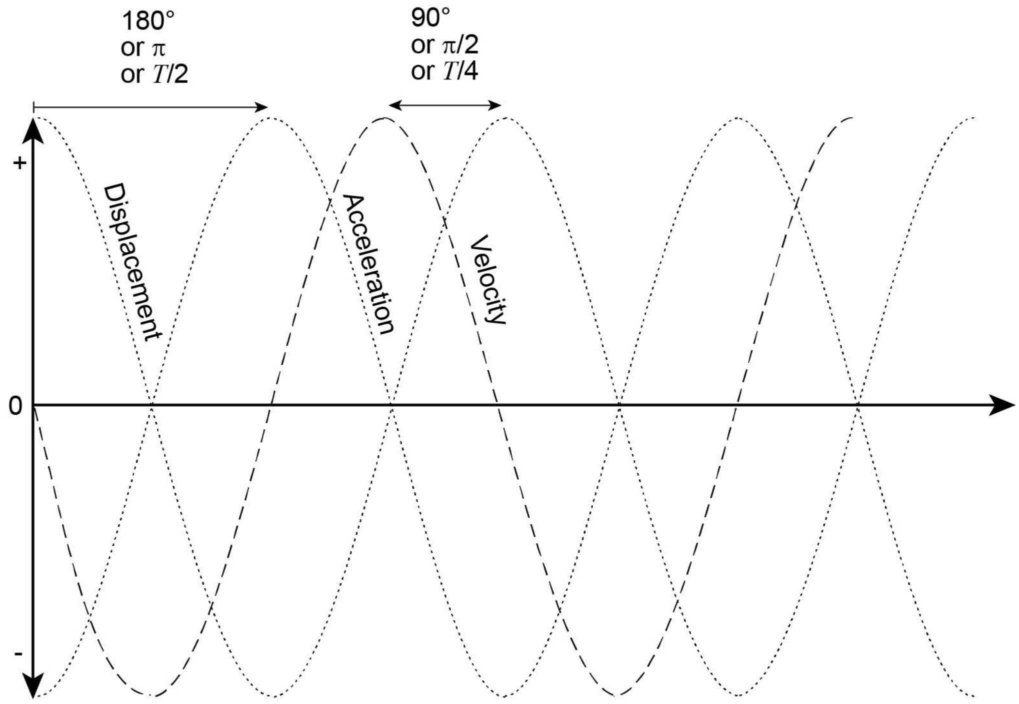

Figure 12 Phase Difference between Derivatives

Figure 12 shows that the displacement curve and its derivatives - velocity and acceleration, obey a sinusoidal shape. Also note that assuming displacement as the reference, velocity is 90 degrees out of phase and acceleration is 180 degrees out of phase.

This can be visualise by looking at the oscillation:

- Displacement is zero as the beam reaches equilibrium point and maximum as it reaches the extremes of its travel.
- Velocity slows to zero as the beam reaches the extremes of its travel, but increases to maximum as it moves past the equilibrium point.
- Acceleration (or deceleration) is zero as the beam reaches equilibrium point (velocity is constant) and maximum when the beam reaches the extremes of its travel (velocity approaches or leaves zero).

Viscous Damping (c)

No vibrating system is 'ideal', in that without continued applied energy, vibrations will eventually reduce to zero, due to friction or some other resistive method of dissipating or damping the energy of the oscillations.

If a constant external influence is added to the oscillating system, such that it causes the system to lose energy and therefore amplitude, the system has been 'damped'.

The most common form of damping is 'Viscous damping' where the damping force is proportional and in opposition to the velocity of the displacement. The dashpot damper used on the equipment uses a fluid to provide viscous damping.

In application, the amount of viscous damping is quantified as a coefficient, with the letter (c).

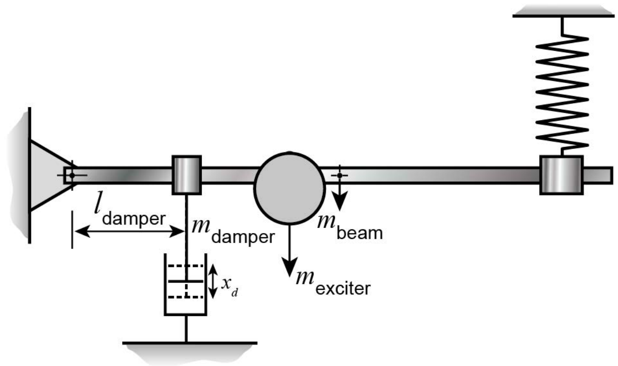

Figure 13 Damping with the Beam and Spring

As shown in Figure 13, when the damper is attached to the beam and spring, the equations of motion become:

$$I_A\ddot{\theta} = -kx_s l_{spring} - cl_{damper}\dot{x}_d$$

$$\ddot{\theta} + \frac{cl_{damper}{}^2}{I_A}\dot{\theta} + \frac{kl_{spring}{}^2}{I_A}\theta = 0$$

Introducing the decay coefficient (γ) to show the level of damping in the system:

$$\ddot{\theta} + 2\gamma\dot{\theta} + \omega^2\theta = 0 \qquad (8)$$

$$\gamma = \frac{c \times l_{damper}{}^2}{2I_A}$$

Note that ω remains the same, as it is the natural frequency.

To account for the additional inertia of the damper, it can be assumed that the damper piston forms a point mass, giving for the complete system:

$$I_A = I_{beam} + I_{spring} + I_{exciter} + I_{damper} \qquad (9)$$

Where

$$I_{damper} = m_{damper} l_{damper}^{2} \tag{10}$$

The displacement equation remains a second order linear differential equation. This time it is easier to assume a solution in the standard form:

$$\theta = Ce^{rt} \tag{11}$$

Inserting this into Equation 8 gives:

$$r^2 + 2\gamma r + \omega^2 = 0 \tag{12}$$

$$r = -\gamma \pm \sqrt{\gamma^2 - \omega^2} \tag{13}$$

At this point, there are three cases:

1. Underdamped, where $\gamma < \omega$ gives two complex roots.
2. Critically damped, where $\gamma = \omega$ gives a single root.
3. Overdamped, where $\gamma > \omega$ gives two real roots.

Damping Ratio (ζ)

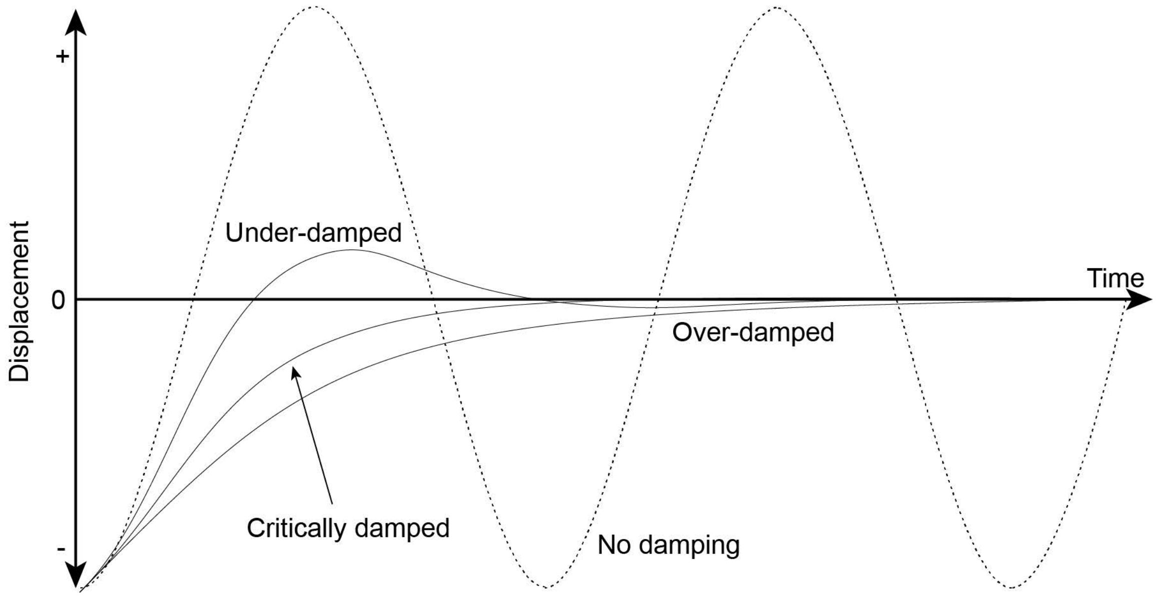

Figure 14 Damping

- An ***undamped*** system (in theory) oscillates forever at its natural frequency. In reality, it oscillates, losing energy to external factors (such as heat, sound and friction) until it reaches equilibrium.
- An ***under-damped*** system oscillates with gradually reducing amplitude until equilibrium.

- A ***critically damped*** system displaces and returns to equilibrium in the shortest possible time without oscillating.
- An ***over-damped*** system would not oscillate at all. The extra damping means that its return to equilibrium takes longer than critical damping.

The damping ratio (ζ) is a dimensionless value found from the ratio of the actual damping coefficient and the critical damping coefficient (c_c).

$\zeta = \frac{c}{c_c}$ *(14)*	Undamped	$c = 0$	$\zeta = 0$
	Under-damped	$c < c_c$	ζ = between 0 and 1
	Critically damped	$c = c_c$	$\zeta = 1$
	Over-damped	$c > c_c$	$\zeta > 1$

Table 3 Damping Coefficients and Ratios

Underdamped Oscillations

When $\gamma < \omega$ we define the two root constants

$$r_1 = -\gamma + i\omega_d$$

$$r_2 = -\gamma - i\omega_d$$

Adding these into the general solution Equation 11 and expressing in phase form, we can show:

$$\theta = Ae^{-\gamma t}\cos(\sqrt{(\omega^2 - \gamma^2)}t - \alpha_d)$$

Where A and α_d are amplitude and phase based on initial conditions. The cosine term represents the free element of the motion, and the exponential term represents the damping element. Like in free vibration, we can deduce the damped time period knowing the cosine function repeats every 2π.

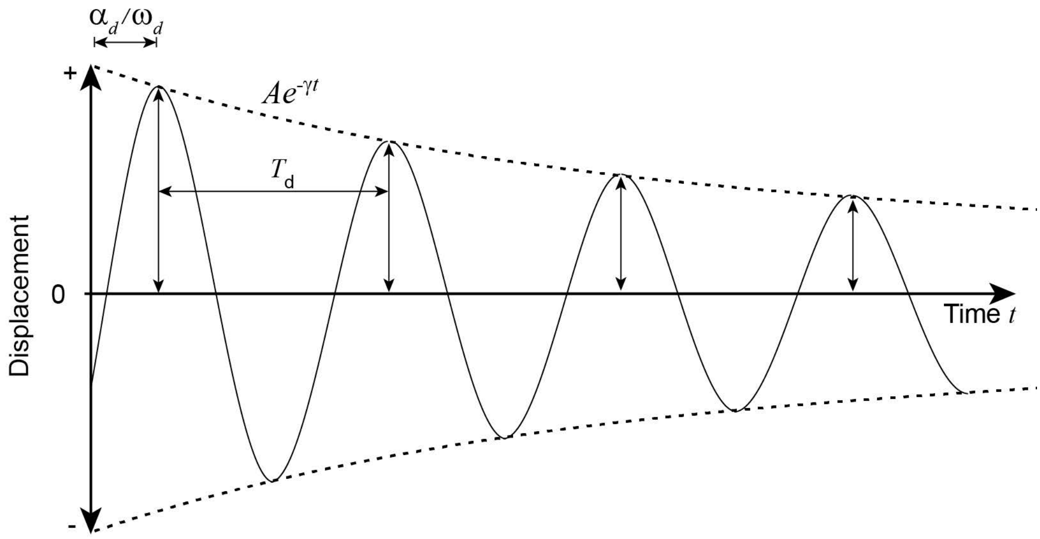

Figure 15 Underdamped Oscillations

$$\sqrt{(\omega^2-\gamma^2)}\tau_d = 2\pi \text{ or } \tau_d = 2\pi\sqrt{\frac{1}{(\omega^2-\gamma^2)}} = \frac{2\pi}{\omega_d}$$

$$\omega_d^2 = \omega^2-\gamma^2 \qquad (15)$$

Using this and Equation 14 gives:

$$\frac{\omega_d}{\omega} = \sqrt{1-\zeta^2}$$

Which is in the form of the equation of a circle. Figure 16 shows the textbook plot of this equation as the quadrant of a circle. It shows that low damping levels giving a ratio below 0.2 have minimal effect on frequency ratio (it remains near to unity). Therefore even non ideal systems oscillate near to the natural frequency.

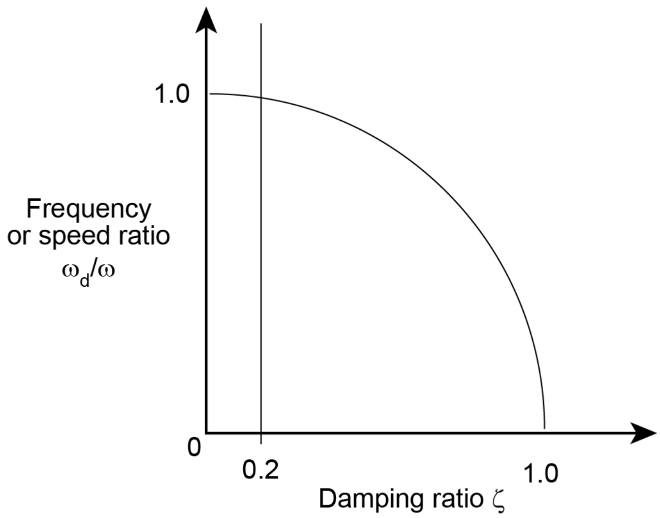

Figure 16 Speed Ratio and Damping Ratio

Critically Damped Oscillations

For critical damping $\gamma = \omega$, which gives one repeated root. The displacement equation becomes:

$$\theta = C_1 e^{-\omega t} + C_2 t e^{-\omega t}$$

Where C_1 and C_2 are constants based on initial conditions.

This equation has no cos or sin terms, showing that it does not describe an oscillatory system.

Overdamped Oscillations

For overdamped systems $\gamma > \omega$, which gives two real roots. The displacement equation becomes:

$$\theta = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

As with the critical damping, the motion is ‘aperiodic’ - meaning that it has no period of oscillation. As damping is increased, the system takes longer to reach equilibrium. Too much damping means that the system may never return to equilibrium.

The Logarithmic Decrement (δ) and Calculating Damping

NOTE *VDAS® has a tool to automatically calculate these values.*

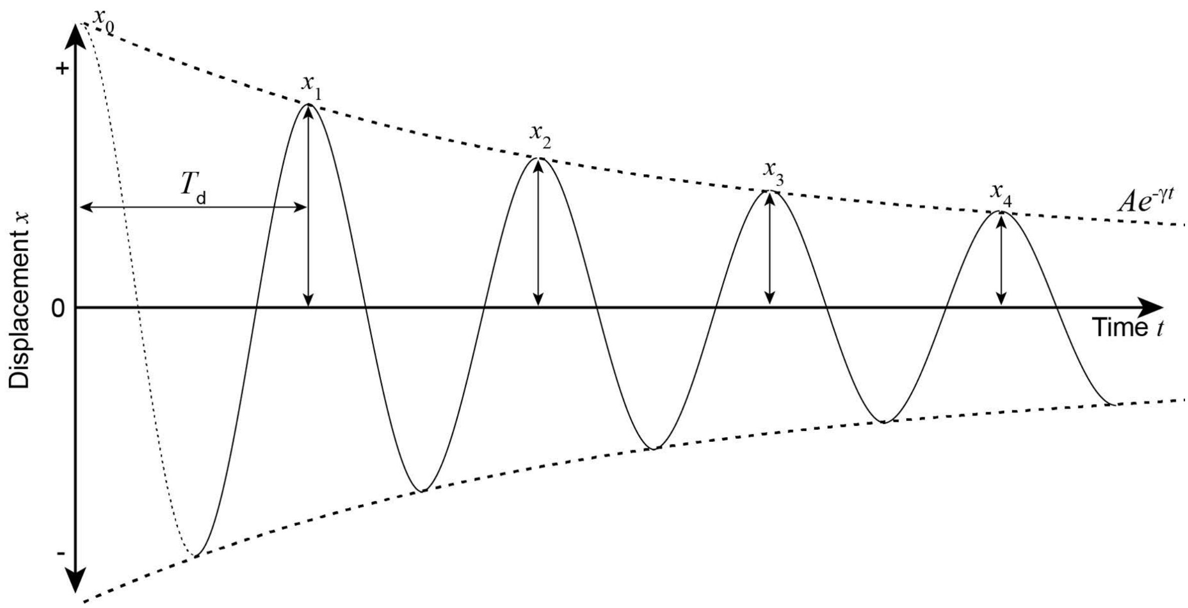

Figure 17 Calculating Damping

When damping is applied to a system, the oscillation amplitude decays at an ***exponential*** rate (see Figure 17. Equation 16 describes the curve that follows the amplitude decay.

$$Ae^{-\gamma t} \qquad (16)$$

To calculate a damping value based on physical properties needs so many variables that only complex equations or computer analysis can give an answer. The answer will be very approximate because of the many possible differences between theory and a real application. A better method is to measure the oscillations of a system and find its damped period (T_d) and ***logarithmic decrement***. This is the ***natural log*** of the ratio of amplitude of successive peaks of j time periods apart, where:

$$\frac{x_0}{x_{0+j}} = \frac{Ae^{-\gamma t_0}}{Ae^{-\gamma(t_0 + jt_d)}} = e^{\gamma j T_d} = e^{j\delta} \qquad (17)$$

and

$$\delta = \frac{1}{j}\ln\frac{x_0}{x_{0+j}} = \gamma \times T_d = \frac{2\pi\gamma}{\omega_d} \approx \frac{2\pi\gamma}{\omega} \text{ for low damping levels} \qquad (18)$$

For the best accuracy, an average of the log of three or more ratios of successive peaks is better, unless the amplitude reduces too quickly to obtain more than two peaks. For example, for the five peaks of Figure 17:

$$\delta = \frac{1}{4}\ln\frac{x_0}{x_{(0+4)}} = \gamma \times T_d \qquad (19)$$

Overdamped Systems

From Equation 13:

$$r_1 = -\gamma \pm \sqrt{\gamma^2 - \omega^2} \text{ and } r_2 = -\gamma \pm \sqrt{\gamma^2 - \omega^2}$$

Critically Damped Systems

For critical damping, the overdamped model can be used, as it assumes the aperiodic motion. The VDAS® tool may predict damping ratio near to 1, where we can assume critical damping.

Beam and Spring - Forced Vibrations

NOTE *These derivations include damping*

When the exciter turns it provides an oscillating force (Q) that causes the beam to oscillate at a frequency that may not be the natural frequency ω.

The equation is:

$$I_A\ddot{\theta} + cl_{damper}^2\dot{\theta} + kl_{spring}{}^2\theta = l_{mass}Q\sin\Omega t$$

$$Q = \Omega^2 m_{offsetmass} r_{offsetmass}$$

$$\ddot{\theta} + \left(\frac{cl_{damper}^2}{I_A}\right)\dot{\theta} + \left(\frac{kl_{spring}{}^2}{I_A}\right)\theta = \left(\frac{l_{exciter}Q}{I_A}\right)\sin\Omega t$$

$$\ddot{\theta} + 2\gamma\dot{\theta} + \omega^2\theta = q\sin\Omega t$$

Where

$$\gamma \text{ and } \omega \text{ remain the same and } q = \frac{l_{exciter}Q}{I_A}$$

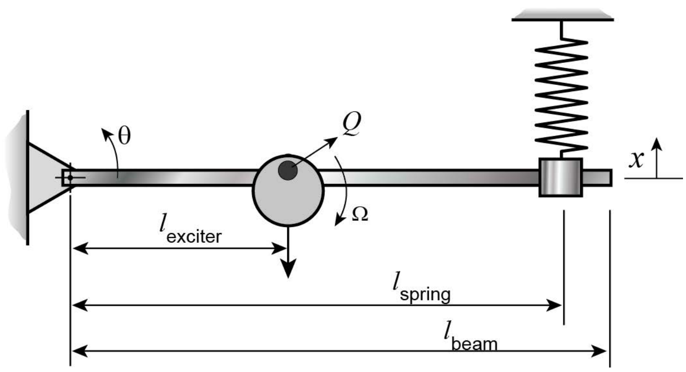

Figure 18 Oscillating Force Q

One solution to this equation gives:

$$\theta = M\cos(\Omega t) + N\sin(\Omega t)$$

Differentiating this and substituting it into the base equations will lead two linear algebraic equations to calculate the generic mathematical constants M and N.

Writing in the phase angle form, and introducing the magnification factor:

$$\theta = \frac{q}{\omega^2}\beta \sin(\Omega t - \phi)$$

Where

$$\beta = \frac{1}{\sqrt{(1 - \Omega^2/\omega^2)^2 + (2\zeta\Omega/\omega)^2}} \qquad (20)$$

and phase lag (ϕ) of the offset mass is:

$$\tan\phi = \frac{2\zeta\Omega/\omega}{1 - \Omega^2/\omega^2}$$

The phase relationships between displacement, velocity and acceleration remain the same for forced vibrations.

Magnification Factor (β)

Under forced vibration, the vibrations of the structure are considered to be subject to a ‘magnification factor’ (β). This is a dynamic increase in amplitude compared to the amplitude caused by a statically applied force. This value is based on the relationship between the damping ratio and the forced and natural frequency ratio Ω/ω.

$$\beta = \frac{1}{\sqrt{(1-\Omega^2/\omega^2)^2 + (2\zeta\Omega/\omega)^2}} \qquad (21)$$

In an ideal system with no damping, C is zero, then γ and the damping ratio ζ are both zero. The magnification simplifies to:

$$\beta = \frac{1}{(1-\Omega^2/\omega^2)} \qquad (22)$$

This shows that for an ideal undamped system, when the forced frequency matches the natural frequency (resonance), the magnification factor and vibration amplitude become infinite. Looking at Equation 21, even small amounts of damping (as in a real system) prevent the magnification factor from reaching infinite.

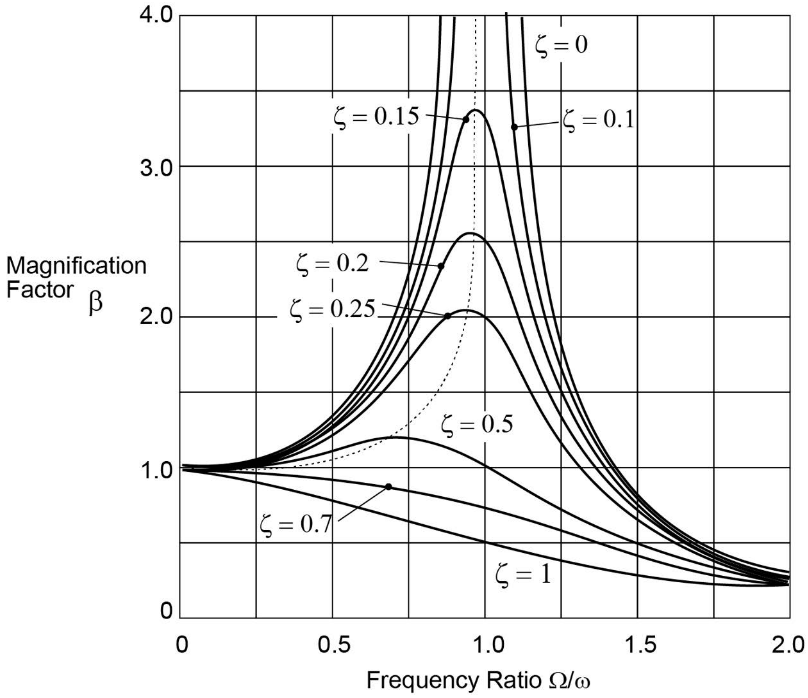

Figure 19 Textbook Curves

Figure 19 shows the typical textbook curves for the relationship between magnification factor and frequency ratio for a range of damping ratios. The dotted line shows that the peak of the magnification factor moves to a lower frequency ratio with increased damping ratio. Equation 23 shows the relationship between the frequency ratio and the damping ratio.

$$\frac{\Omega}{\omega} = \sqrt{1-2\zeta^2} \qquad (23)$$

Phase Lag (ϕ)

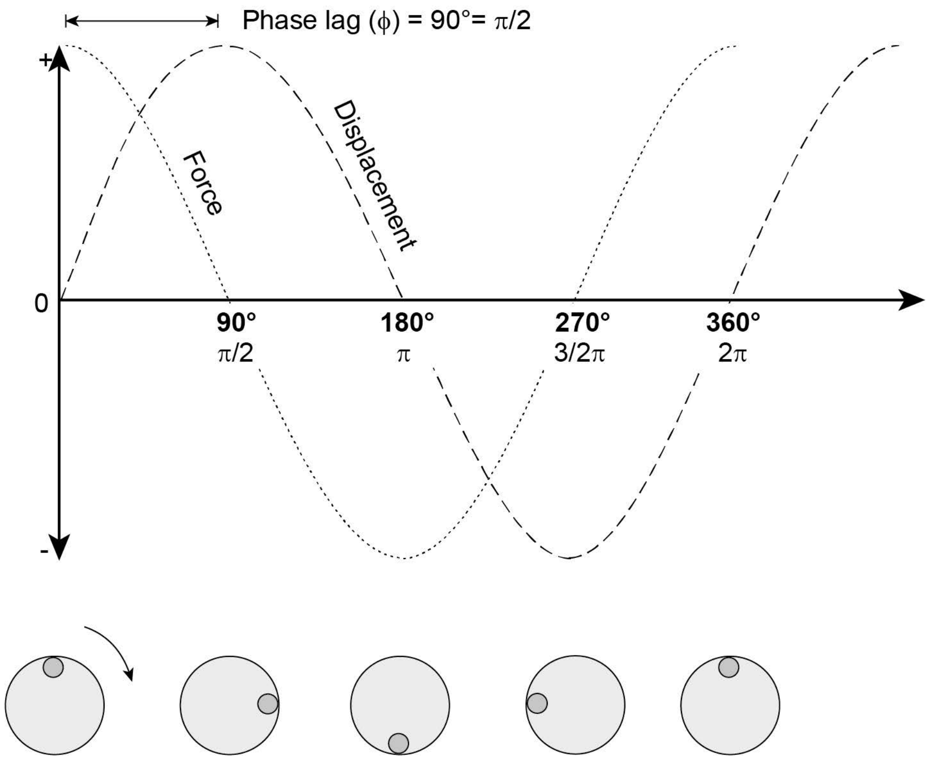

Figure 20 Forced Vibration Phase Lag

It would be expected that if an external force of a given sinusoidal angular frequency (Ω) causes a structure to vibrate, then the waveform of the applied force would be in phase with the displacement waveform of the structure. In practice, the two waveforms are not in phase. The phase of the displacement waveform of the structure ***lags*** behind the force waveform. Figure 20 shows a phase lag of 90 degrees as an example.

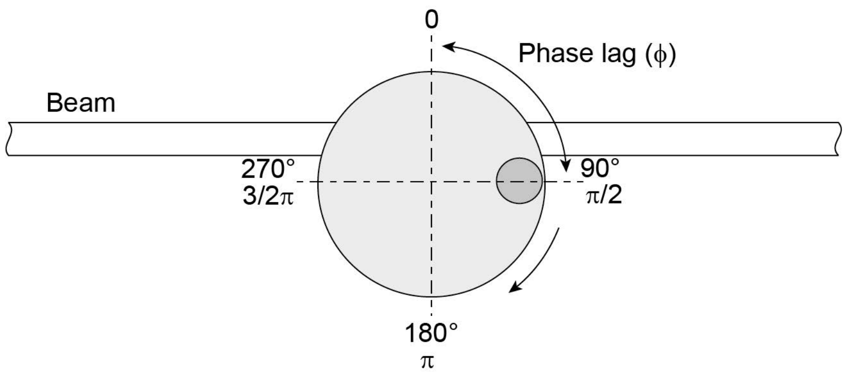

Figure 21 Exciter Phase Lag

On the equipment, the exciter uses an eccentric mass to create the forced vibrations. If the angular position of the eccentric mass is known and compared with the displacement of the beam, the forced vibration phase lag in the system will be seen. Figure 21 shows a sketch of the exciter on the beam with the eccentric mass at roughly 90 degrees with respect to the top position. If the displacement is at its peak with the mass at 90 degrees, then the phase lag would be 90 degrees, as shown in Figure 20.

Phase Lag and Damping

Equation 24 gives the phase lag of the offset mass on the exciter, allowing for damping:

$$\tan\phi = \frac{2\zeta\Omega/\omega}{1-\Omega^2/\omega^2} \qquad (24)$$

From this, an ideal system with no damping gives:

$$\tan\phi = \frac{0}{1-\Omega^2/\omega^2} \qquad (25)$$

The value of the frequency ratio and any damping affect the phase lag. Figure 22 shows the textbook curves of phase lag against frequency ratio with different damping. It shows that in an ideal system with no damping (ζ = 0), there is no lag until resonance, where the lag changes quickly to 90° precisely, then a full 180° past resonance. As damping increases, the change from 0 to 180° becomes slower.

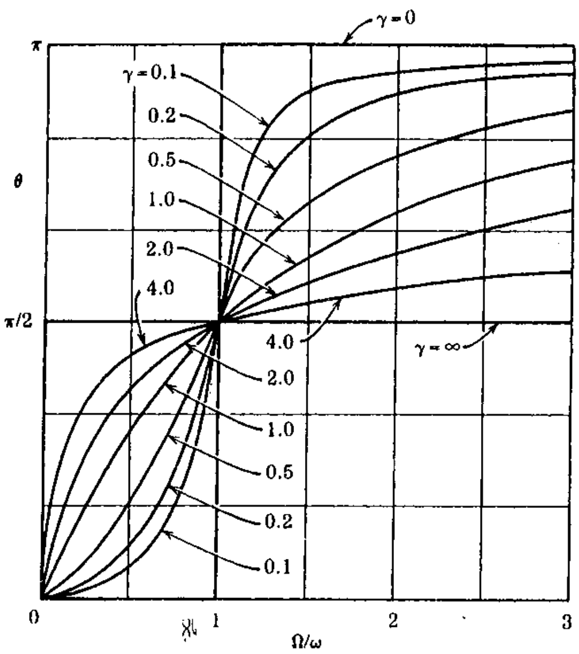

FIG. 1.34

Figure 22 Phase Lag against Frequency Ratio

Figure 23 shows the textbook plot of a tangent function to help understanding of the theory.

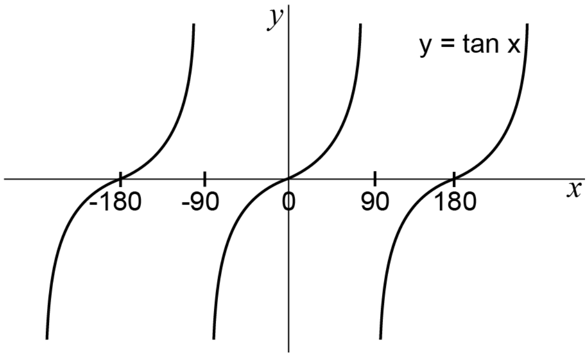

Figure 23 Tan Plot

Simply Supported Beam

Basic Theory

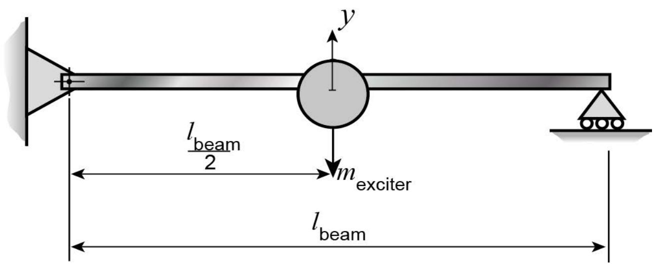

Figure 24 Simply Supported Beam

The exciter remains fitted to the beam for all experiments. It forms a simple point mass at mid-span of the beam. Its value may be increased using the added masses.

From Figure 24, the beam will be seen to flex so that its mid point will move up and down in oscillation. This is a linear motion.

NOTE

This basic theory assumes that the beam mass is negligible compared to the point mass.

It also assumes that the exciter forms a point mass.

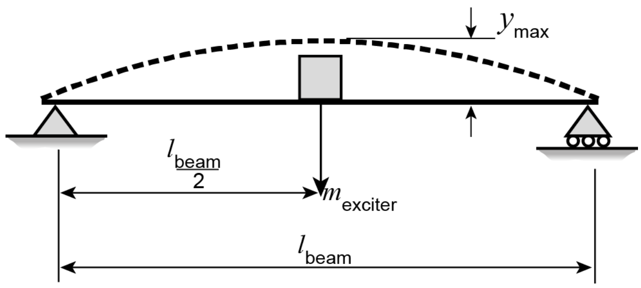

Figure 25 Deflection in a Simply Supported Beam

From simply supported beam deflection theory, the static deflection depends on the flexural rigidity of the beam, its length and the applied mass:

$$\delta_{st} = \frac{m_{exciter} g l_{beam}}{48 E I_{beam}}$$

$$I_{beam} = \frac{bd^3}{12}$$

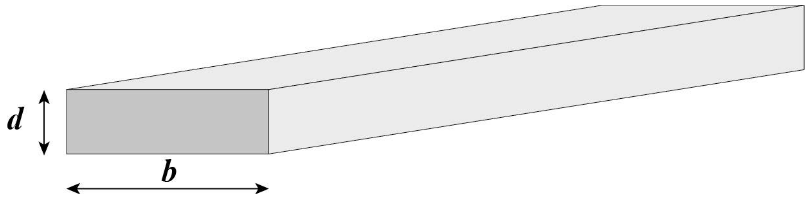

Figure 26 Beam Dimensions

The flexural rigidity of the beam is directly comparable to a spring, such that it has an effective spring constant (k_{beam}).

So:

$$F = k_{beam}x \text{ and } mg = k_{beam}\delta_{st}$$

Where

$$k_{beam} = \frac{48EI_{beam}}{l_{beam}{}^3}$$

The beam deflection along the length of the beam is symmetrical about the centre and we assume its curve obeys the equation:

$$y = y_{max}\left(\frac{3xl_{beam}{}^2 - 4x^3}{l_{beam}{}^3}\right)\left\{\frac{x}{l} \le \frac{1}{2}\right\}$$

The motion is linear, so we can examine the forces:

$$m_{exciter}\ddot{y} + k_{beam}y = 0$$

$$\ddot{y} + \frac{k_{beam}}{m}y = 0$$

Using the steps described for the rigid beam, it can be seen that:

$$\omega^2 = \frac{k_{beam}}{m_{exciter}}$$

$$\ddot{y} + \omega^2 y = 0$$

This equation is in the same form as the one derived for the rigid beam system. Using the same steps as before, we can show the period of oscillation is:

$$T = 2\pi\sqrt{\frac{m_{exciter}}{k_{beam}}} = 2\pi\sqrt{\frac{m_{exciter}l_{beam}{}^3}{48EI_{beam}}}$$

and:

$$f = \frac{1}{2\pi}\sqrt{\frac{48EI_{beam}}{m_{exciter}l_{beam}{}^{3}}} \qquad (26)$$

Also we can show our displacement equation in the phase form:

$$y = A\cos(\omega t - \alpha)$$

This is in the same form as before. The same relationship between displacement, velocity and acceleration is proven using the same method.

Improved Theory (Rayleigh)

The basic theory assumes two things:

1. That '***the beam is light in comparison with the central load'***.

This is acceptable in most applications, but for accuracy the beam mass and inertia must be allowed for.

An English physicist - John William Strutt (Lord Rayleigh) experimented with frequencies of vibration and conservation of energy. His work led to a correction of the standard beam theory when used in vibrational analysis.

This gives a corrected mass that allows for the mass of the simply supported beam of $\frac{17}{35}m_{beam}$.

Adding the corrected mass of the beam to the central mass (of the exciter), gives an 'effective' mass. The equations then apply to the complete assembly:

$$m_{effective} = m_{exciter} + \frac{17}{35}m_{beam}$$

then, from the basic equations:

$$f = \frac{1}{2\pi}\sqrt{\frac{48EI_{beam}}{m_{effective}l_{beam}{}^{3}}}$$

2. That '***the central mass (exciter) is a point load'***.

However, in the application, it clamps a central portion of the beam, increasing its stiffness, affecting its flexural rigidity. To allow for this it can be seen that the beam actually forms two cantilevers, one each side of the exciter.

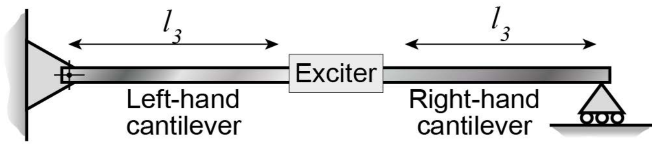

Figure 27 Effective Mass for Simply Supported Beam

The static deflection then becomes

$$\delta_{st} = \frac{(m_{effective}/2)gl_3^{\ 3}}{3EI_{beam}} = \frac{m_{effective}gl_3^{\ 3}}{6EI_{beam}} \text{ and } k_{beam} = \frac{6EI_{beam}}{l_3^{\ 3}}$$

Compiling these improvements gives:

$$f = \frac{1}{2\pi}\sqrt{\frac{6EI_{beam}}{m_{effective}l_3^{\ 3}}} \tag{27}$$

$$\omega = \sqrt{\frac{k_{beam}}{m_{eff}}} \tag{28}$$

$$T = 2\pi\sqrt{\frac{m_{eff}}{k_{beam}}} \tag{29}$$

Dunkerley's Method

Stanley Dunkerley - an engineer in mechanics during the late 19th Century, worked with vibrations and whirling shafts. He discovered that the addition of the (reciprocal) angular velocities of individual elements of a whirling shaft give the total (reciprocal) angular velocity of the shaft. Vibrating beams are directly analogous to a whirling shaft, as both obey simple harmonic motion.

The reciprocal of the total oscillating frequency of the beam assembly (f_{total}) is due to the sum of the reciprocal oscillating frequencies of the beam (f_{beam}) and any masses (f_{masses}) that are clamped to it.

$$\frac{1}{f_{total}} = \frac{1}{f_{beam}} + \frac{1}{f_{masses}} \tag{30}$$

Finding the natural frequency of a beam by itself is very difficult, as all the items attached would need to be removed.

So, to find the natural frequency of just the beam, simply find its vibration frequency for a range of additional masses and extrapolate the results down to zero mass. The intercept on a $1/f$ axis for zero mass would produce $1/f$ of the beam itself. However the frequency and mass relationship is not linear, so to make it linear for an easier prediction, $1/f^2$ must be plotted against mass. See Figure 28. Then simply find the square root of $1/(1/f^2)$.

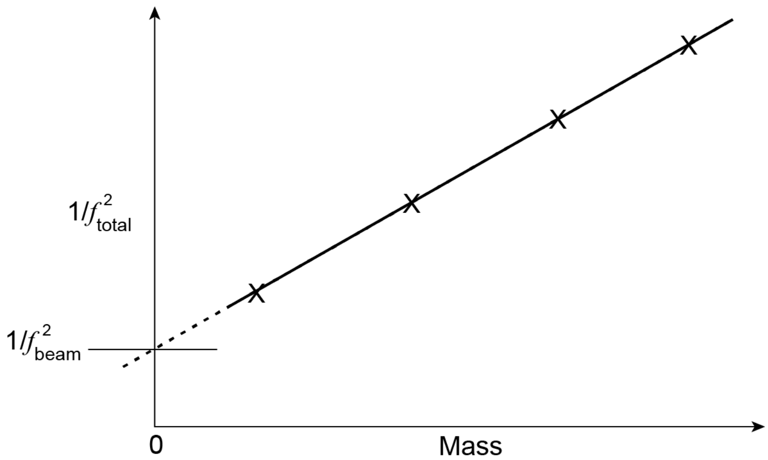

Figure 28 Finding Beam Frequency

Equation 31 gives the theoretical frequency ***for just the beam itself***.

$$f_{beam} = \frac{\pi}{2}\sqrt{\frac{EI_{beam}}{m_{beam}(2l_3)^3}} \qquad (31)$$

When simplified gives:

$$f = 0.5554\sqrt{\frac{EI_{beam}}{m_{beam}l_3^{\,3}}} \qquad (32)$$

Simply Supported Beam - Damping

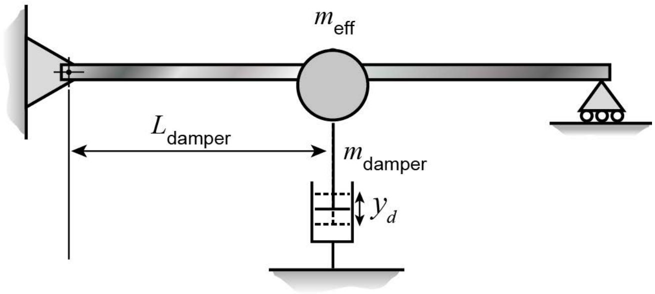

Figure 29 Damping with the Simply Supported Beam

NOTE *All other equations and methods are the same as for the beam and spring, but may need changes in notation for the different systems.*

Writing our displacement equation of motion:

$$m_{eff}\ddot{y} + c\dot{y} + k_{eff} = 0$$

Where:

$$m_{eff} = m_{mass} + \frac{17}{35}m_{beam} + m_{damper}$$

$$\ddot{y} + 2\gamma\dot{y} + \omega^2 y = 0$$

This time

$$\gamma = \frac{c}{2m_{eff}}$$

$$y = Ae^{-\gamma t}\cos(\omega_d t - \alpha_d)$$

Again, the correlated equations mean that the relationships remain as with the rigid beam.

The theory remains the same for underdamped, critically damped and overdamped.

Simply Supported Beam - Forced Vibrations

$$m_{eff}\ddot{y} + c\dot{y} + k_{eff}y = Q\sin\Omega t$$

$$\ddot{y} + 2\gamma\dot{y} + \omega^2 y = q\sin\Omega t$$

Where

$$q = \frac{Q}{m_{eff}}$$

This becomes

$$y = \frac{q}{\omega^2}\beta\sin(\Omega t - \alpha) = \left(\frac{Q}{k_{eff}}\right)\beta\sin(\Omega t - \alpha) \tag{33}$$

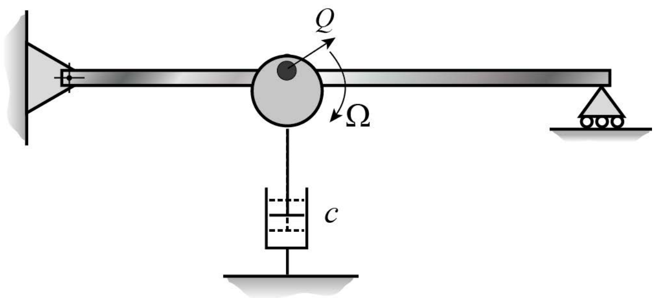

Figure 30 Force Q on the Simply Supported Beam

NOTE *All other equations remain the same as for the beam and spring.*

The Undamped Vibration Absorber (or Neutralizer)

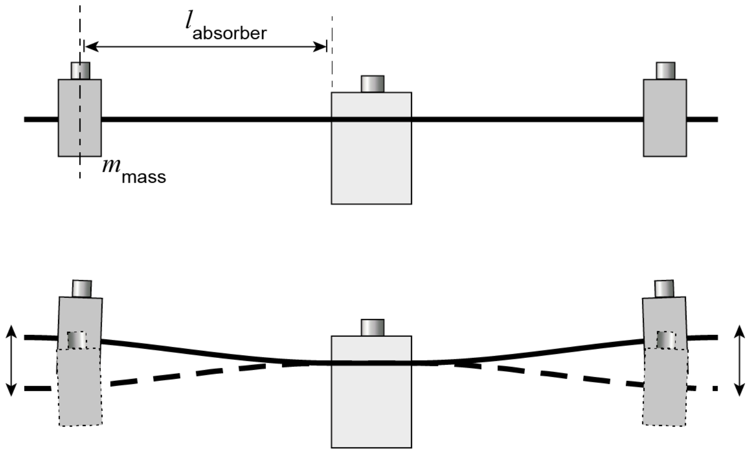

Figure 31 The Vibration Absorber

The vibration absorber works exactly as its name suggests. It absorbs the energy of the oscillations in the simply supported beam assembly. In doing this, its own flexible beam vibrates and dissipates or neutralizes the energy from the main beam, mainly to air movement (friction). It can be considered as an auxiliary oscillating system.

Made of a centrally clamped flexible beam, the absorber has two opposing and symmetrical cantilevers (for balance), each with a mass attached. The position of the masses may be adjusted to ‘tune’ the absorber until it absorbs or cancels the vibrations in the main assembly.

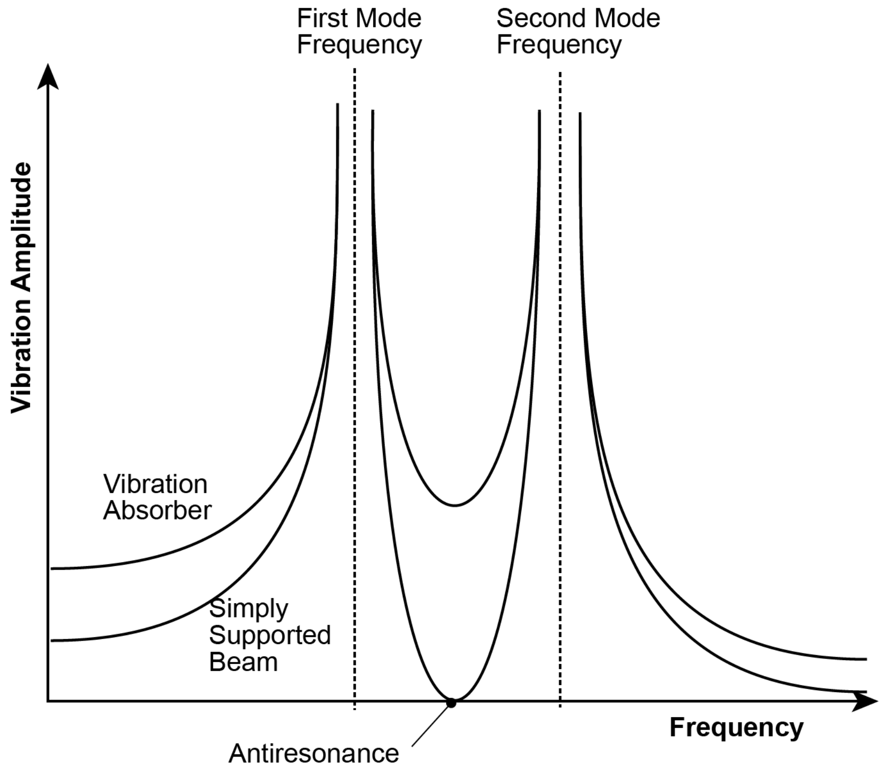

Figure 32 Amplitude and Frequency Chart for Two Degrees of Freedom

Both the rigid beam and spring and simply supported beam are examples of 'one degree of freedom' (1 DOF) systems. They have ***one*** natural frequency. The addition of the vibration absorber to the simply supported beam creates a system of two degrees of freedom (2 DOF). These systems have ***two*** natural frequencies. They obey an amplitude and frequency relationship as shown in Figure 32. At some point of 'antiresonance' between the two frequencies, the amplitude of the simply supported beam reduces to zero. At this point, the vibration absorber will still vibrate, but at a lower value than at either of the two natural frequencies.

NOTE *If the masses on the vibration absorber are not at equal distances, then this can create a 3DOF system.*

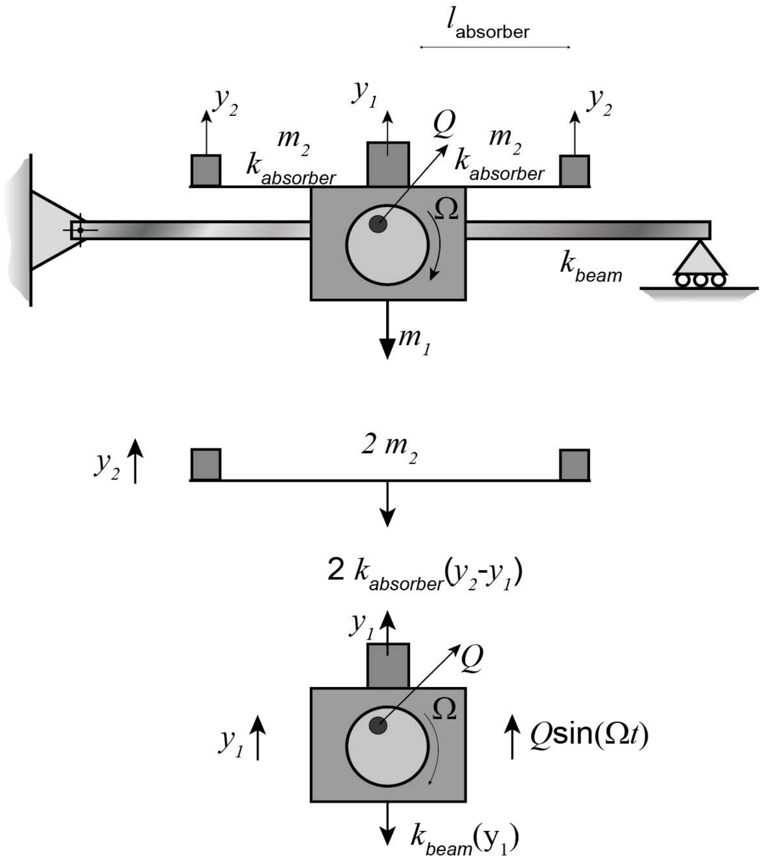

Figure 33 Simplified Sketch

Figure 33 shows a simplified sketch of the vibration absorber on the main beam assembly. It can be broken into two parts, giving two equations of motion.

$$m_1\ddot{y}_1 + (k_{beam} + 2k_{absorber})y_1 - 2k_{absorber}y_2 = Q\sin(\Omega t) \tag{34}$$

$$2m_2\ddot{y}_2 - 2k_{absorber}y_1 + 2k_{absorber}y_2 = 0 \tag{35}$$

Using the same methods as shown earlier in the theory, and using solutions:

$$y_1 = A\sin(\omega t) \quad \text{and} \quad \ddot{y}_1 = A\omega^2\sin(\omega t)$$

$$y_2 = B\sin(\omega t) \quad \text{and} \quad \ddot{y}_1 = B\omega^2\sin(\omega t)$$

Where A and B are oscillation amplitudes, we can compute:

$$[(k_{beam} + 2k_{absorber} - m_1\omega^2)A - 2k_2B]\sin(\omega t) = Q\sin(\Omega t)$$

$$[-2k_{absorber}A + (2k_{absorber} - 2m_2\omega)^2B]\sin(\omega t) = 0$$

Writing in matrix form

$$\begin{bmatrix} k_{beam} + 2k_{absorber} - m_1\omega^2 & -2k_{absorber} \\ -2k_{absorber} & 2k_{absorber} - 2m_2\omega^2 \end{bmatrix} \begin{bmatrix} A\sin(\omega t) \\ B\sin(\omega t) \end{bmatrix} = \begin{bmatrix} Q\sin(\Omega t) \\ 0 \end{bmatrix}$$

$$CY = F$$

Natural frequencies are determined with external forces set to zero. From this, the determinate of the coefficient matrix (C) must be zero.

$$\begin{bmatrix} k_{beam} + 2k_{absorber} - m_1\omega^2 & -2k_{absorber} \\ -2k_{absorber} & 2k_{absorber} - 2m_2\omega^2 \end{bmatrix} = 0$$

$$2m_1m_2\omega^4 - [2m_1k_{absorber} + 2m_2Ck_{beam} + 2k_{absorber}]\omega^2 + 2k_{beam}k_{absorber} = 0$$

$$\omega^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (36)$$

Where:

$$a = 2m_1m_2 \qquad (37)$$

$$b = -(2m_1k_{absorber} + 2m_2(k_{beam} + 2k_{absorber})) \qquad (38)$$

$$c = 2k_{beam}k_{absorber} \qquad (39)$$

The purpose of the vibration absorber is to reduce the amplitude of the main beam (A) to zero.

$$CY = F$$

$$Y = C^{-1}F = \frac{adj(C)}{det(C)}F$$

$$\begin{bmatrix} A\sin(\omega t) \\ B\sin(\omega t) \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} 2k_2 - 2m_2\omega^2 & 2k_2 \\ 2k_2 & k_1 + 2k_2 - m_1\omega^2 \end{bmatrix}\begin{bmatrix} Q\sin(\Omega t) \\ 0 \end{bmatrix}$$

$$A = \frac{1}{\Delta}[(2k_2 - 2m_2\omega^2)Q + 2k_2(0)]$$

$$B = \frac{1}{\Delta}[2k_2 Q + (k_1 + 2k_2 - m_1\omega^2)(0)]$$

The undamped vibration absorber is a special case where its job is to reduce amplitude A to zero. If the value of A is set to zero, then the frequency for the vibration absorber can be found.

$$0 = \frac{1}{\Delta}(2k_2 - 2m_2\omega^2)Q$$

$$2k_2 = 2m_2\omega^2 \text{ and } \omega = \sqrt{\frac{k_2}{m_2}}$$

This is the natural frequency of the vibration absorber.

We can also calculate the expected amplitude of the vibration absorber (B).

$$B = \frac{1}{\Delta}2k_2 Q$$

$$\Delta = (2k_2 - 2m_2\omega^2)(k_1 + 2k_2 - m_1\omega^2) - 4k_2^2$$

$$\text{and } \omega^2 = \frac{k_2}{m_2}$$

$$\Delta = \left(2k_2 - \frac{2m_2k_2}{m_2}\right)\left(k_1 + 2k_2 - \frac{m_1k_2}{m_2}\right) - 4k_2^2$$

$$= 4k_2^2$$

$$B = -\frac{2k_2}{4k_2^2}Q = -\frac{Q}{2k_2}$$

This shows that as the amplitude of the vibration absorber is 180° out of phase compared with the main beam. It creates an opposing force equal and opposite to the excitation force, cancelling it out and reducing the amplitude to zero.

Based on beam theory for a cantilever, the effective stiffness is:

$$k_2 = \frac{3EI_{absorber}}{l_{absorber}{}^3} \qquad (40)$$

Where the E and I values are for the material and beams of the vibration absorber.

In a similar manner to the theory for the main beam, according to Rayleighs formula, the total value of the mass (m_2) on each cantilever of the vibration absorber is the sum of the applied mass and a portion of the vibration absorber beam mass, so that:

$$m_2 = m_{mass} + \frac{33}{140} m_{absorberbeam}$$

and

$$m_1 = m_{exciter} + \frac{17}{35} m_{beam} + m_{absorber}$$

NOTE

It is only necessary to calculate values for one side of the absorber, as the other half is identical and is only for balance. However, the total weight of the absorber must be allowed for when calculating the complete beam assembly equations.

The vibration absorber only works for the simply supported beam.

From this, the frequency equation is:

$$f = \frac{1}{2\pi}\sqrt{\frac{3EI_{absorber}}{m_2 l_{absorber}{}^3}}$$

rearranging this to find the position for the masses on the beam gives:

$$l_{absorber} = \left(\frac{3EI_{absorber}/m_2}{(2\pi f)^2}\right)^{\frac{1}{3}}$$

Experiments

Safe Use

WARNING

If the equipment is not used as described in these instructions, its protective parts may not work correctly.

Never use the equipment with any of its safety guards removed.

WARNING

Always supervise students when using this apparatus.

The supervisor must check that all persons operating the apparatus are deemed as competent and are dressed suitably for laboratory experiments. This includes tying back long hair and securing loose clothing such as ties and jewellery.

Setting as a Beam and Spring

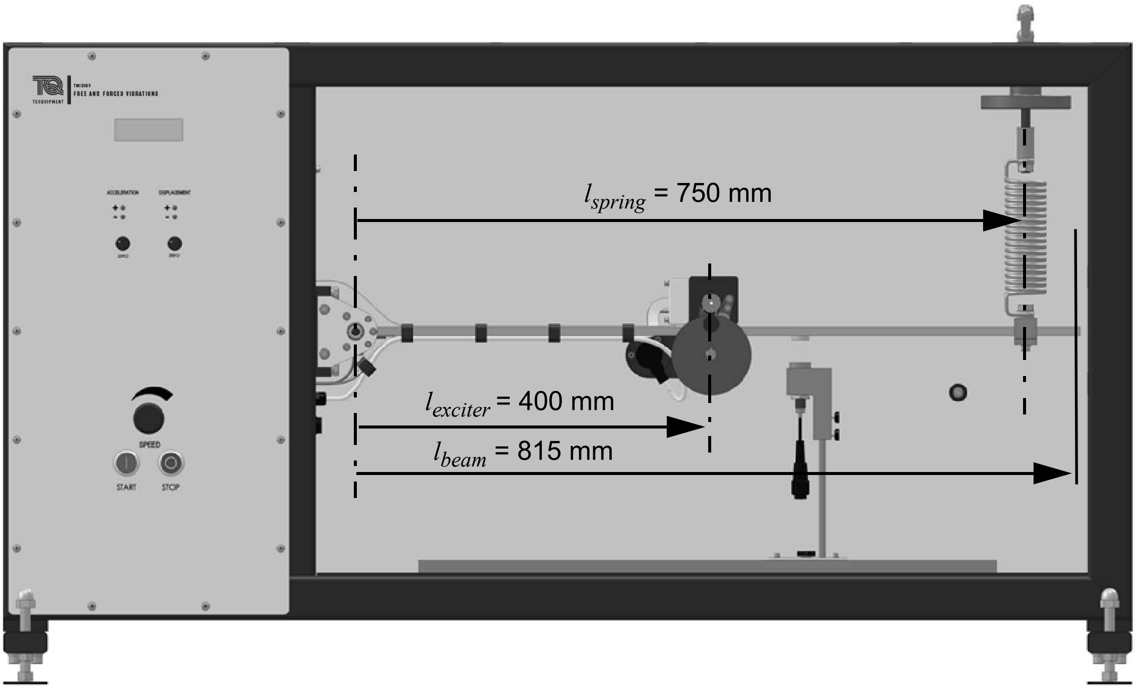

Figure 34 Setup for a Beam and Spring

Setting as a Simply Supported Beam

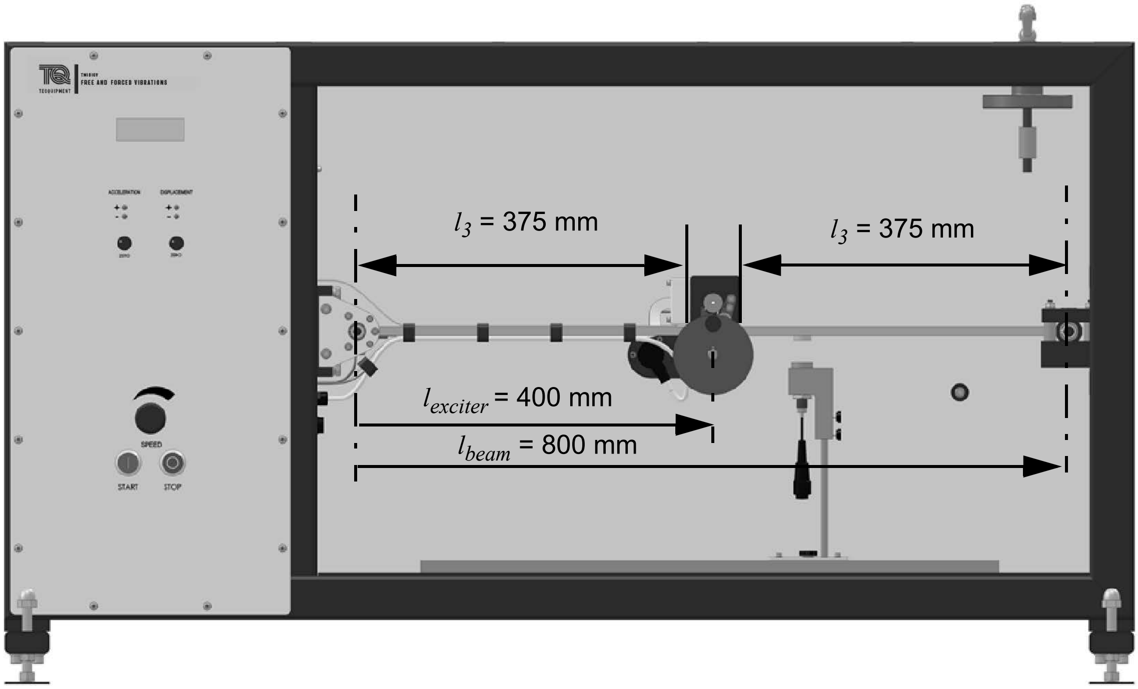

Figure 35 Setup for a Simply Supported Beam

Fitting or Removing the Beam Holder

Leave the beam holder in position to hold the beam when fitting or removing the spring. Leave the spring in position to hold the beam while the beam holder is fitted or removed. Make sure the beam holder sits centrally, then use a spanner to tighten the fixing.

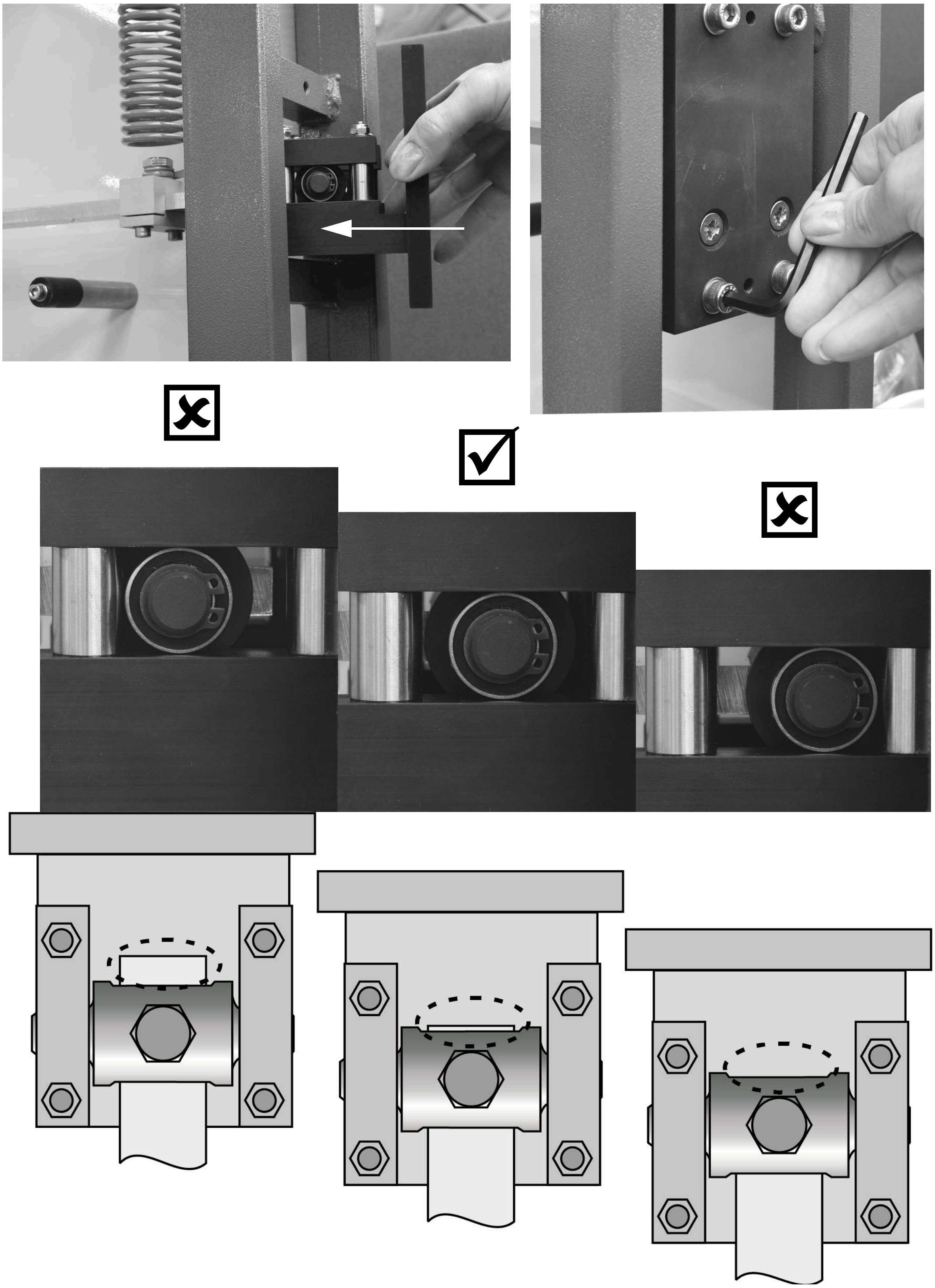

Figure 36 Fitting or Removing the Beam Holder

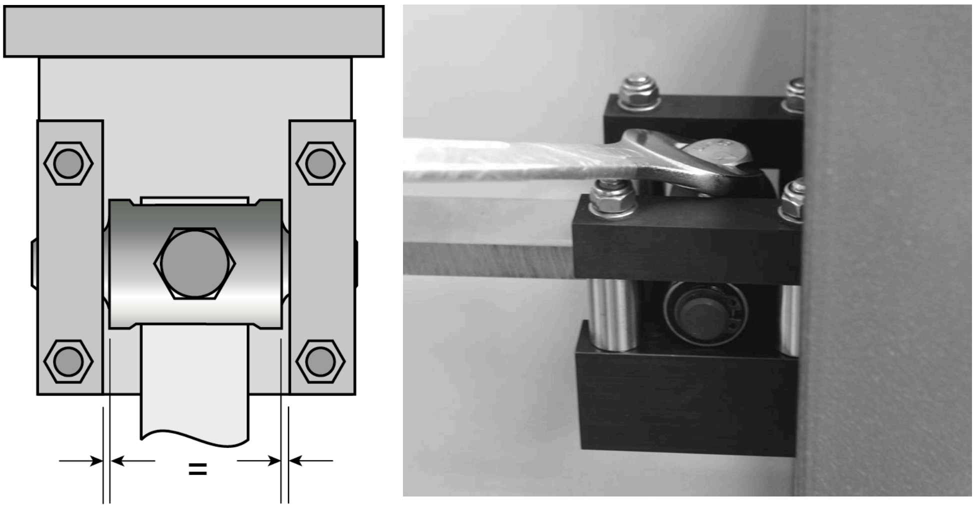

Figure 37 Make sure the Beam Holder sits Centrally, then Tighten the Fixing

Fitting or Removing the Spring

Leave the beam holder in position to hold the beam when fitting or removing the spring. Leave the spring in position to hold the beam while fitting or removing the beam holder.

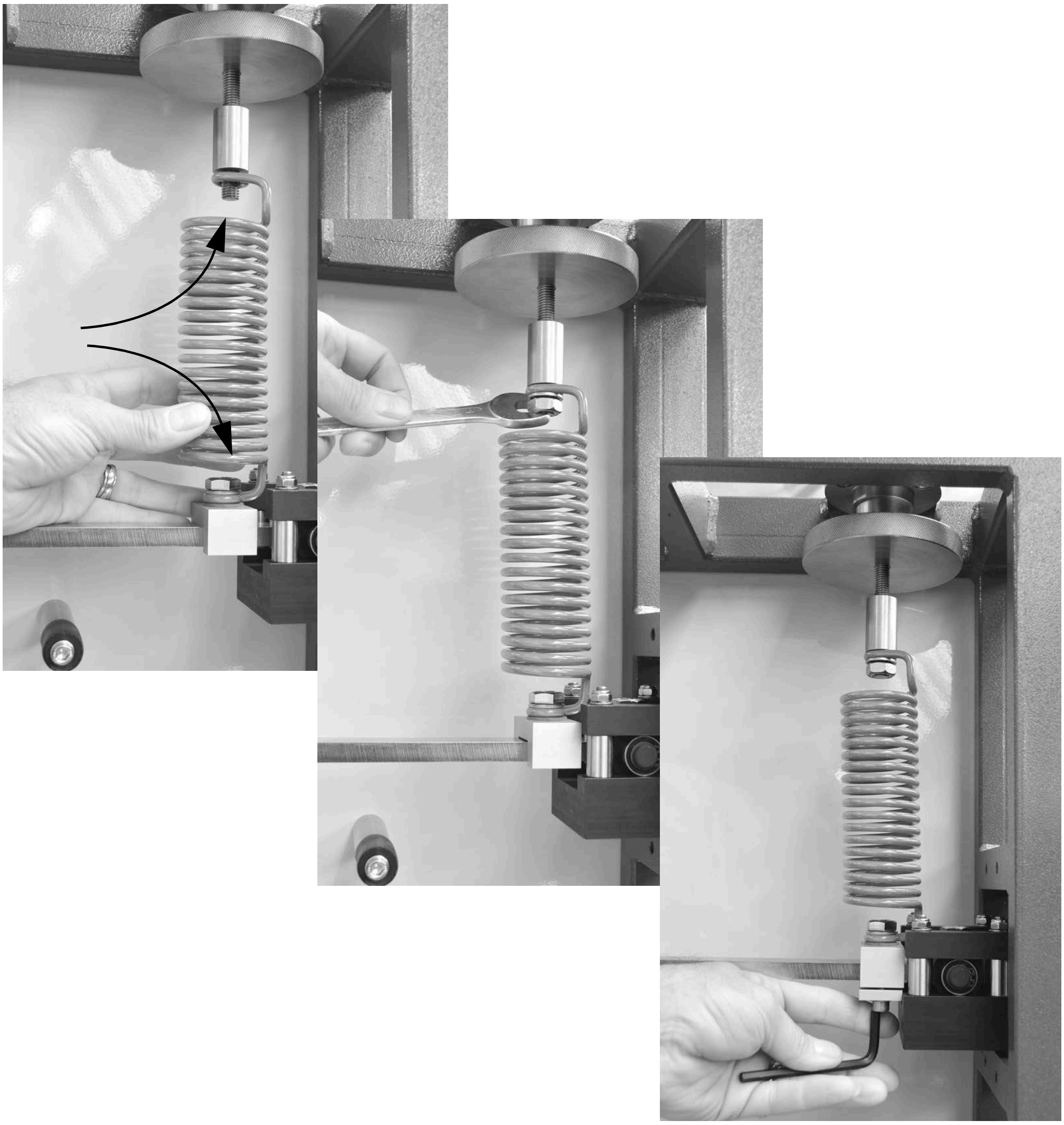

Figure 38 Fitting or Removing the Spring

Adding Masses

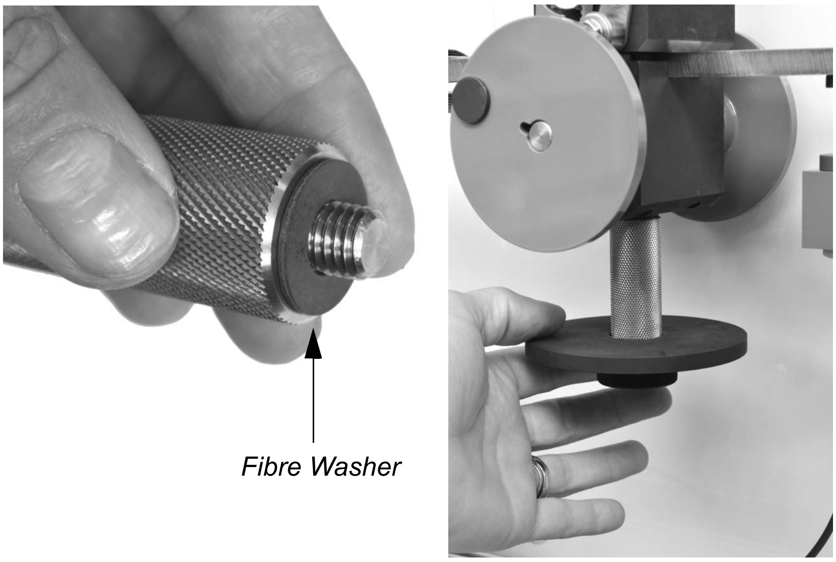

Figure 39 Adding Masses

To add mass to the exciter, simply screw the Mass and Holder to the hole underneath the exciter. Remember to fit the Fibre Washer.

Fitting the Dashpot Damper

The Dashpot Damper fits in two different places, one for each experiment. Fit it as shown, so the damper disc is roughly half way down the clear section, then add fluid to just fill the clear section.

Rigid Beam and Spring Experiments

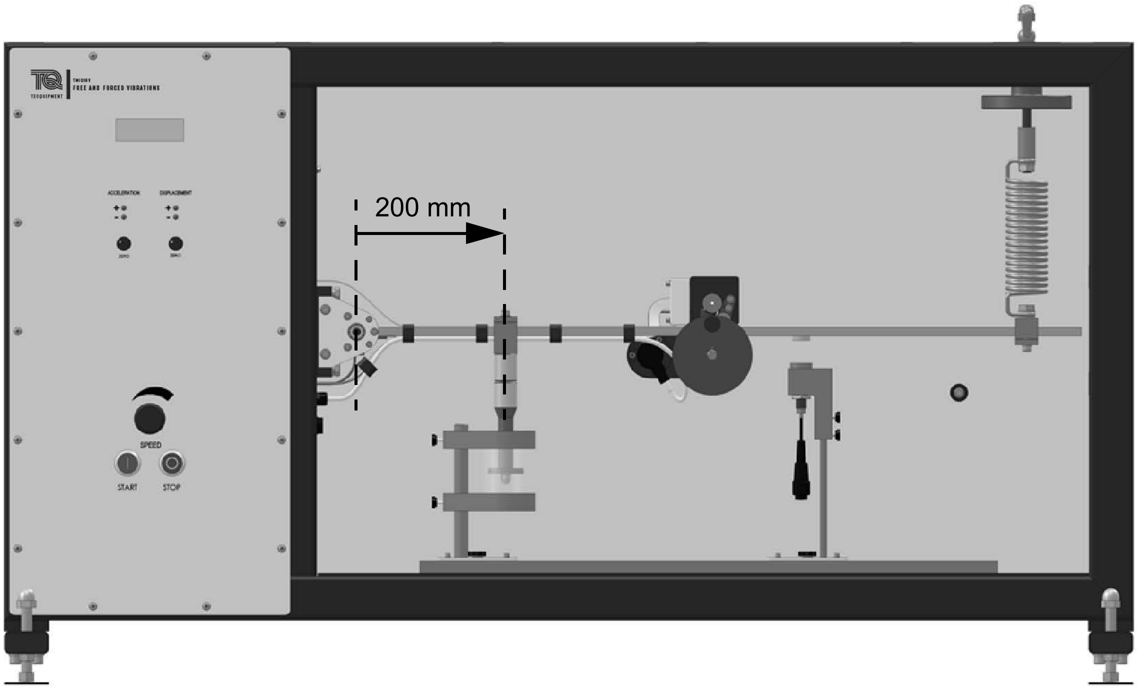

Figure 40 Dashpot Damper Position for Rigid Beam and Spring Experiment

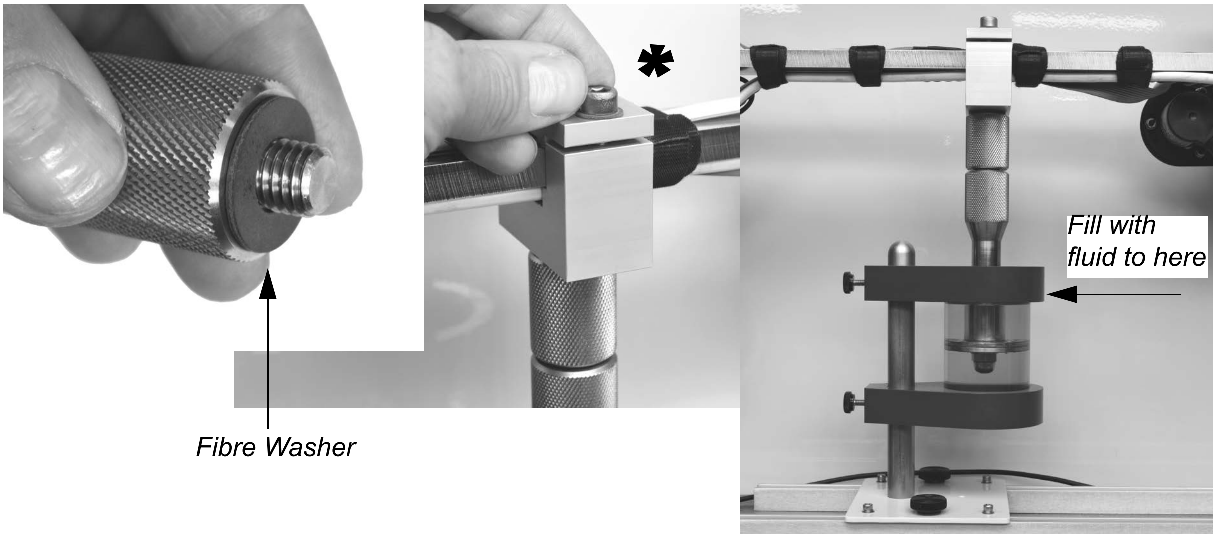

Figure 41 Fitting Dashpot Damper for Rigid Beam and Spring Experiments

CAUTION **Take care not to grip or damage the cables when fitting the clamp to the beam.*

Simply Supported Beam Experiments

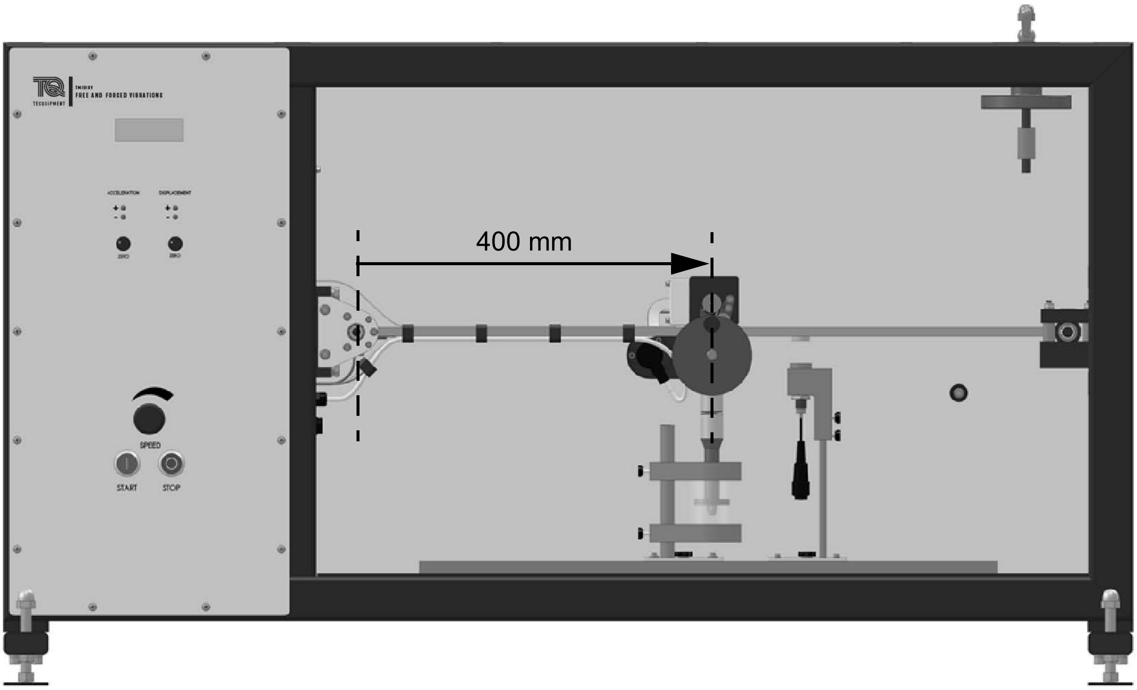

Figure 42 Dashpot Damper Position for Simply Supported Beam Experiment

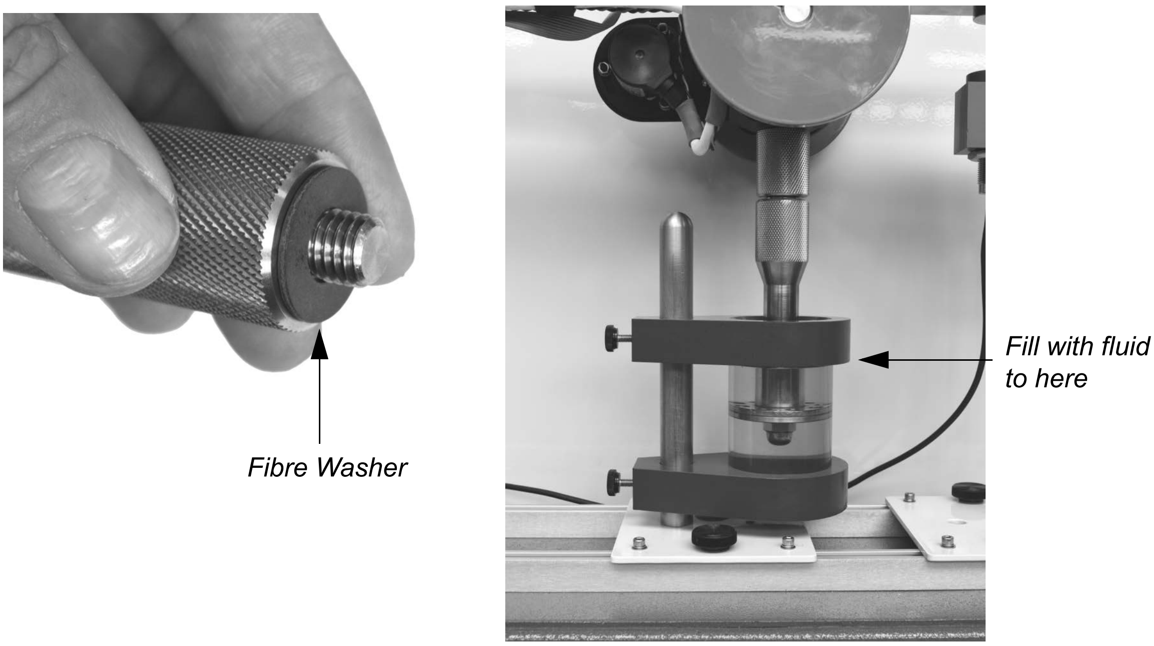

Figure 43 Fitting Dashpot for Beam and Spring Experiments

Adjusting The Damper Disc

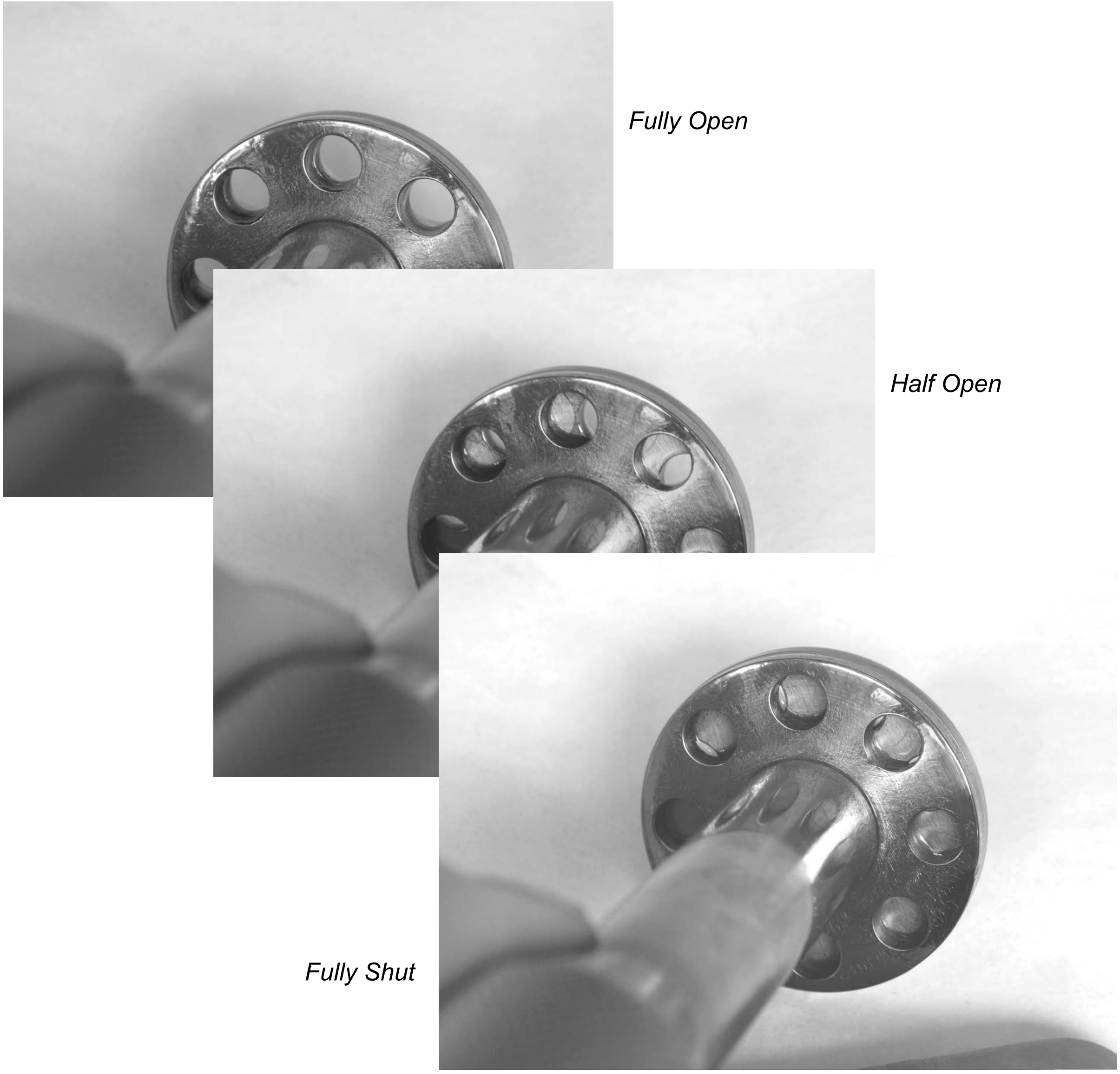

Figure 44 Adjusting the Damper Disc

The Dashpot Damper Disc has two sets of holes. As its metal shaft is turned, the two sets of holes move relative to each other. This is done to adjust the damping. When the holes are aligned, they are fully open and the damper produces minimum damping, as the fluid can easily pass through the holes. When the holes are fully shut it produces maximum damping. When the holes are half open, it produces a medium amount of damping.

Fitting the Vibration Absorber

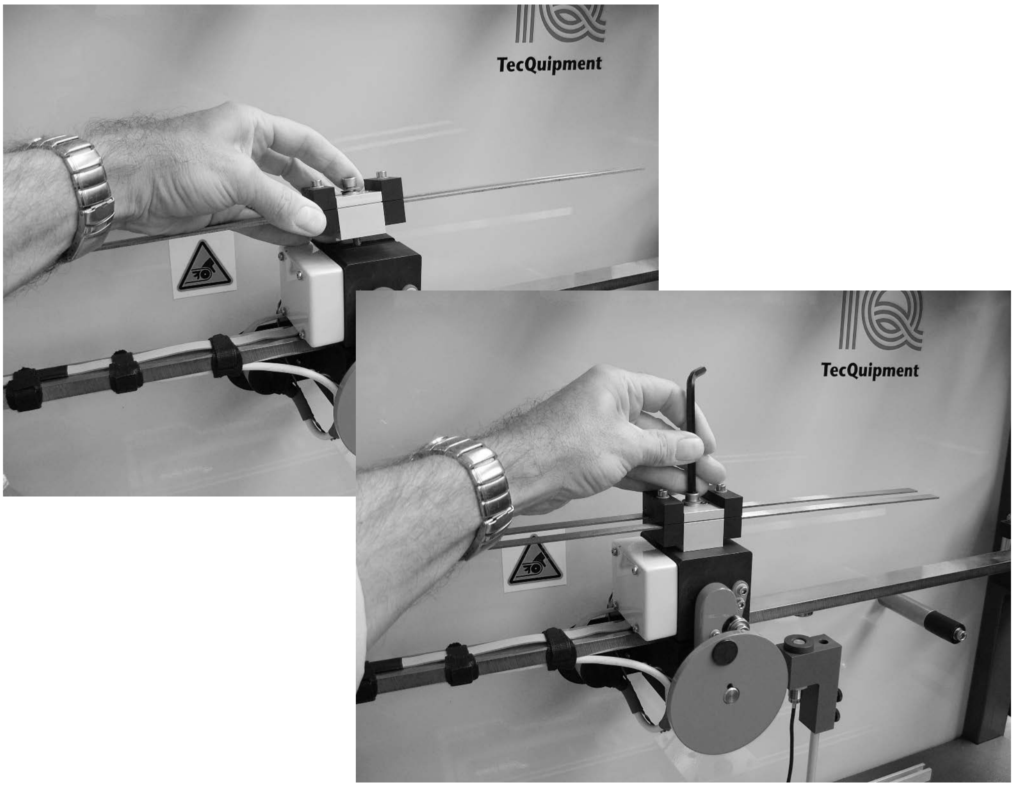

Figure 45 Fitting the Vibration Absorber

Using VDAS® with the TM1016V

Refer to the VDAS® User Guide for full details.

Useful Notes

Use both analogue channels with this equipment. Channel 1 is the displacement. Channel 2 is the accelerometer.

The VDAS® chart area is in two styles, based on the experiment mode:

Free vibration experiments use the ‘Timebase’ trigger. The trace window shows amplitude against a timebase. The vertical lines indicate a time determined by the timebase. If the X and Y cursors are moved to measure differences, ΔY and ΔX will show amplitude and time.

Forced vibration experiments use the ‘Digital Encoder’ trigger from the encoder of the exciter. The trace window shows amplitude against a complete rotation of the exciter, broken into 360 divisions of data, so each division is 1 degree. The vertical lines indicate 45 degree divisions. If the mouse cursor is positioned at 90 degrees, the X cursor value will show 90. If the X and Y cursors are moved to measure differences, ΔY and ΔX will show amplitude and degrees.

When recording chart data, record the normal VDAS® data field values plus 360 or 1250 data values. The VDAS® data table can produce plots of each trace ready for print, with the horizontal axis converted into degrees or time.

Setting Zero

Before each test, use the non-magnetic spirit level to check that the frame is level, then for the rigid beam and spring, check that the beam is level.

Now use the SET ZERO controls to set the zero (equilibrium value) for the acceleration and displacement sensors. Both lights above the controls will go out when zero has been set. It may also be necessary to make some more small adjustments to set the trace in VDAS® to the centre of the chart.

NOTE

The simply supported beam cannot be adjusted to make it level, just use the SET ZERO control.

Use the spring adjuster and locking handle to adjust the beam level for the rigid beam and spring.

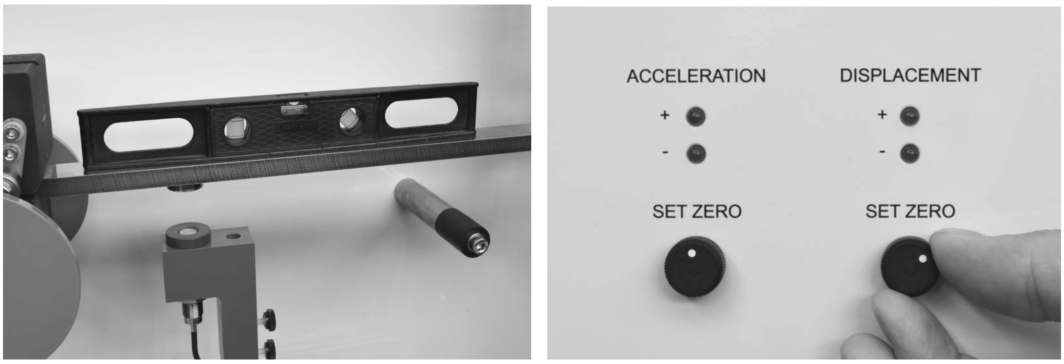

Figure 46 Setting Zero

Setting Sensor Distance

Check the sensor distance each time the experiment setup is changed from beam and spring to simply supported (or the other way around), or if it is believed that the sensor has moved.

To do this:

1. Use the non-magnetic spirit level (supplied) to check that the equipment and the beam is level. See Figure 47.

2. Remove the protective ring from around the sensor. See Figure 48.

3. Adjust the Sensor so that it is possible to just slide the sensor zero block into position as shown in Figure 49. It should just slide into position, without lifting the beam.

4. Refit the Protective Ring.

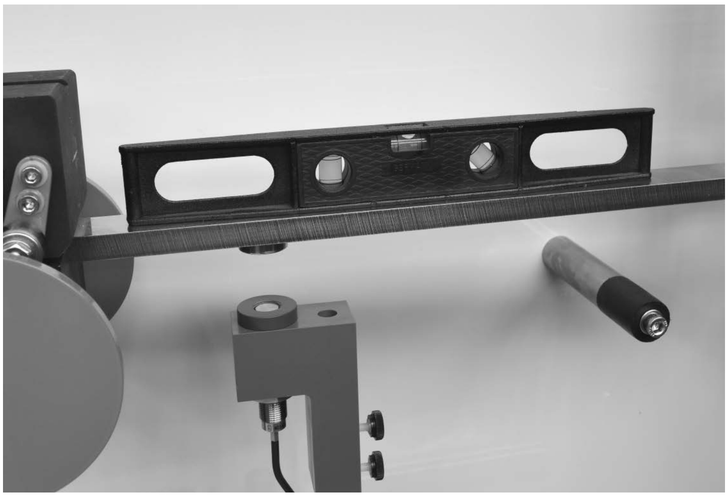

Figure 47 Use the Level

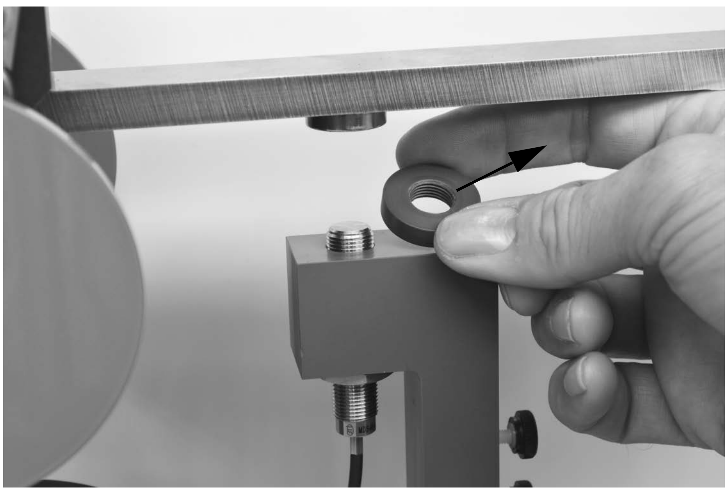

Figure 48 Remove Protective Ring

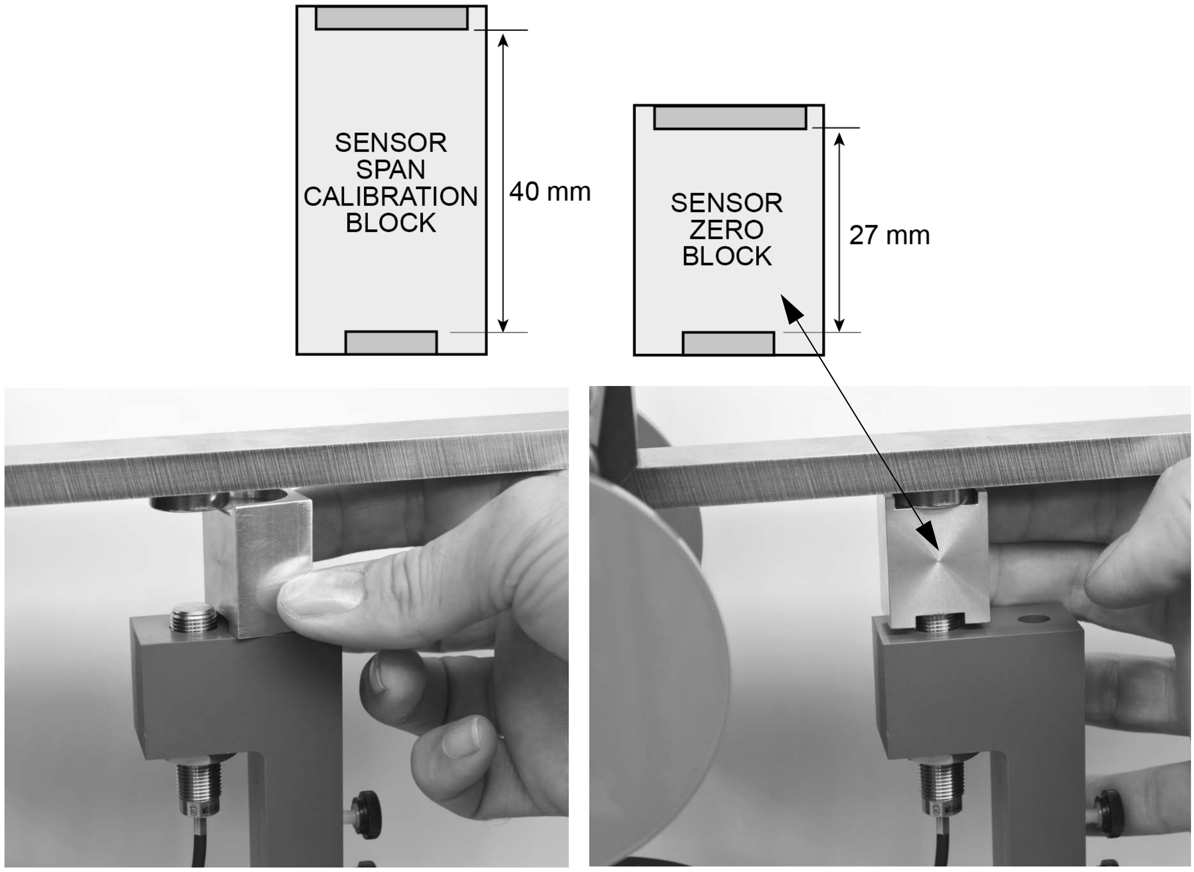

Figure 49 Fit the Zero Block

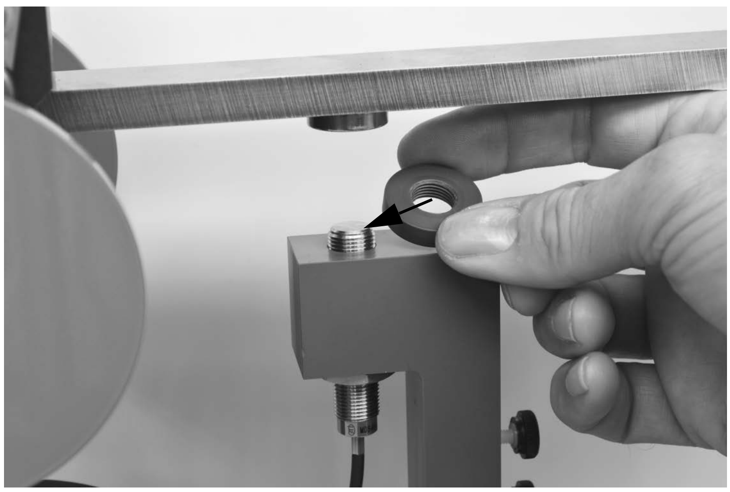

Figure 50 Refit the Protective Ring

Experiment 1: Introduction to the Equipment

Aims

To use the rigid beam and spring as an introduction to help show how the equipment works.

NOTE *This experiment will work for either setup, but uses the rigid beam and spring because it has a simpler arrangement.*

Procedure 1 - Free Vibration Displacement

Beam	Mode	Channel 1 Displacement	Timebase	Channel 2 Acceleration
Rigid	Free Vibration	2 mm	50 ms	Not Visible

Table 4 Recommended VDAS® Settings

1. Set the beam as shown in **Setting as a Beam and Spring** on page 45.
2. Set VDAS® as shown in Table 4.
3. Set the zero for the displacement and acceleration sensors. It may be necessary to make some small adjustment to the SET ZERO controls so the VDAS® trace is at the centre of the chart.
4. Start VDAS® and press down the beam a few millimetres, then release start the displacement oscillations (see Figure 51). The oscillations will slowly reduce after a few seconds. Check that it is possible to see them on the computer screen. They should be similar to Figure 52.

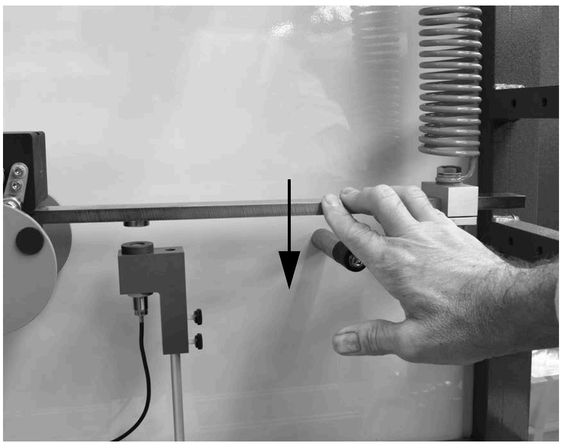

Figure 51 Press Down and Release the Beam to Start Oscillations

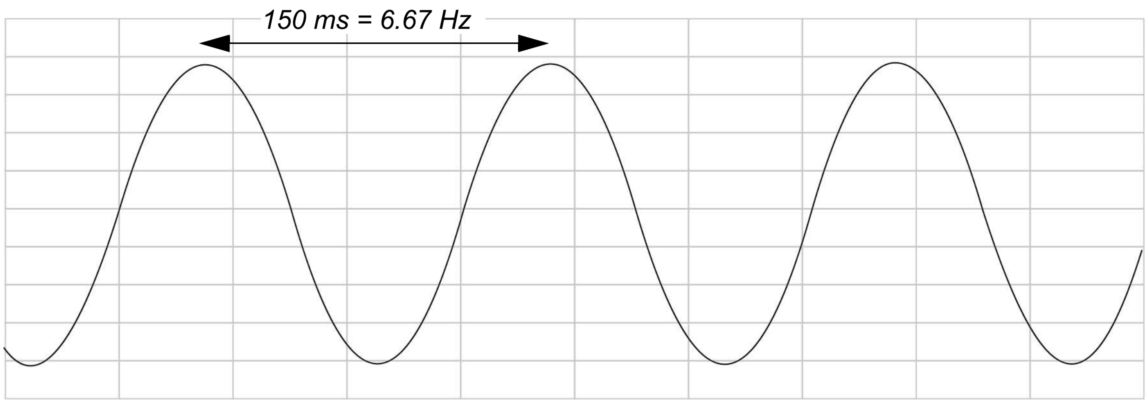

Figure 52 Typical Displacement Curves

If necessary, press down and release the beam again, then stop VDAS® to freeze the oscillation on the screen. Use the cursor tools in VDAS® to measure the distance between oscillations as shown in Figure 52. This gives the measured natural frequency of the rigid beam with spring.

Procedure 2 - Displacement 1st Derivative (Velocity)

Beam	Mode	Channel 1 Displacement	Timebase	Channel 1 1st Derivative (velocity)	Channel 2 Acceleration
Rigid	Free Vibration	2 mm	50 ms	100 $mm.s^{-1}$	Not Visible

Table 5 Recommended VDAS® Settings

1. Set VDAS® as shown in Table 5.

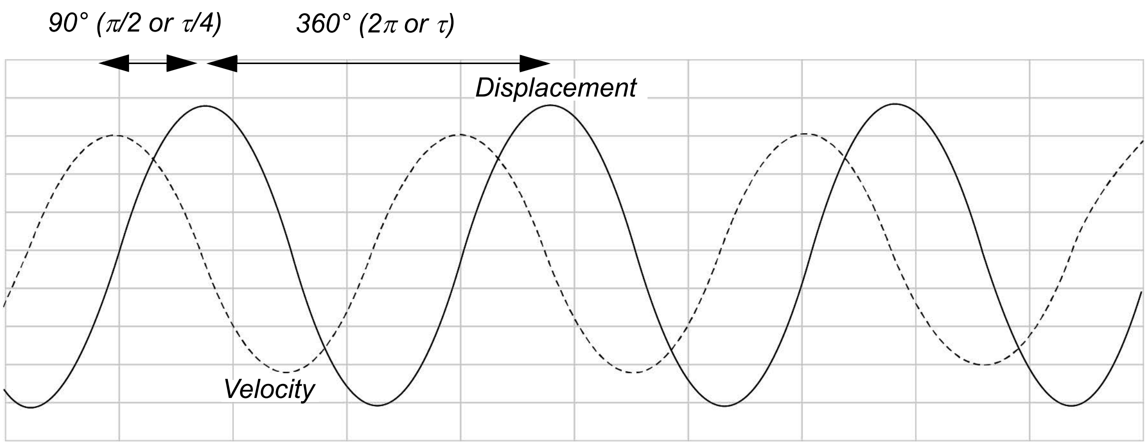

Figure 53 Typical Displacement and First Derivative (velocity) Curves

2. Start VDAS® and press down the beam to start the oscillations. Adjust the filter of the derivative to produce a smoothed sinusoidal trace without reducing its amplitude too much. The traces should be

similar to Figure 53, showing that the velocity is 90 degrees out of phase from displacement. The relative amplitudes are not important and will not be equal.

Procedure 3 - Displacement 2nd Derivative (Acceleration)

Beam	Mode	Channel 1 Displacement	Timebase	Channel 1 1st Derivative (velocity)	Channel 1 2nd Derivative (acceleration)	Channel 2 Acceleration
Rigid	Free Vibration	2 mm	50 ms	100 $mm.s^{-1}$	5 $m.s^{-2}$	Not Visible

Table 6 Recommended VDAS® Settings

1. Set VDAS® as shown in Table 6.
2. Start VDAS® and press down the beam to start the oscillations. Adjust the filter of each derivative to produce smoothed sinusoidal traces without reducing their amplitude too much.They should be similar to Figure 54, showing that the derived velocity is 90 degrees out of phase from displacement and derived acceleration is 180 degrees out of phase from displacement. The relative amplitudes are not important and will not be equal.

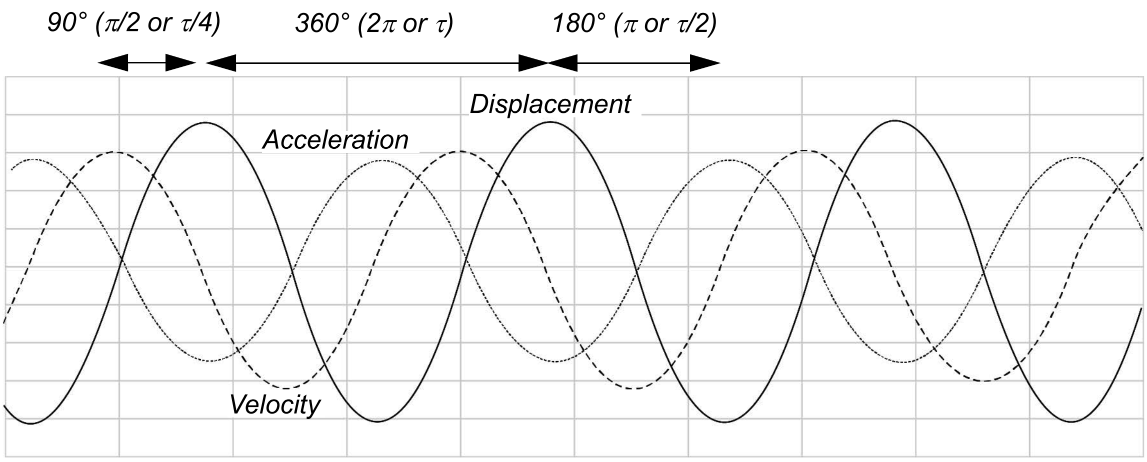

Figure 54 Typical Displacement with First Derivative (velocity) and Second Derivative Curves

Procedure 4 - Measured Acceleration

Beam	Mode	Channel 1 Displacement	Timebase	Channel 1 1st Derivative (velocity)	Channel 1 2nd Derivative (acceleration)	Channel 2 Acceleration
Rigid	Free Vibration	Not Visible	50 ms	Not Visible	5 $m.s^{-2}$	5 $m.s^{-2}$

Table 7 Recommended VDAS® Settings

1. Set VDAS® as shown in Table 7.

2. Start VDAS® and press down the beam to start the oscillations. Adjust the filter of the derivative to produce a smoothed sinusoidal trace without reducing its amplitude too much. The traces should be similar to Figure 53, showing that the derived acceleration from channel 1 is similar to the real acceleration from channel 2. The relative amplitudes and phase should be similar, confirming that the software derived version can be trusted.

NOTE

The channel 2 acceleration may show some initial distortion - determined by how 'cleanly' the oscillations are started. The accelerometer has to be sensitive to work correctly, so it can measure stray oscillations. Hitting the beam or not releasing it correctly will add extra harmonics in the beam and spring - distorting its signal. Try hitting the beam or exciter to see how these extra harmonics affect the channel 2 waveform.

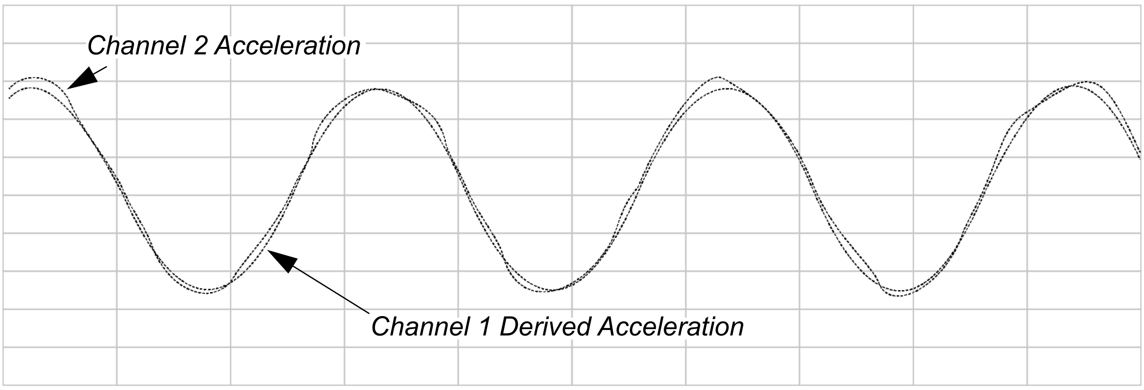

Figure 55 Typical Second Derivative and Channel 2 Curves

Procedure 5 - Forced Vibration Displacement

Beam	Mode	Channel 1 Displacement	Timebase
Rigid	Forced Vibration	5 mm	-

Table 8 Recommended VDAS® Settings

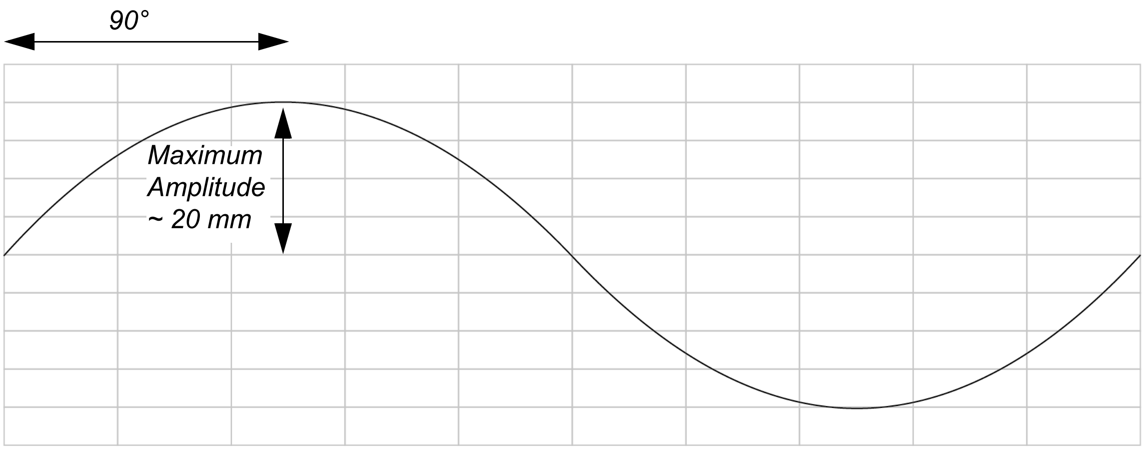

Figure 56 Typical Forced Vibration Curve

1. Set VDAS® as shown in Table 8.

2. Start VDAS® and press the exciter motor START button. Adjust the exciter motor speed to the same value as the natural frequency of the beam that was found earlier. Slowly adjust it up or down by a until the displacement reaches maximum amplitude. Wait a few seconds for the oscillations to stabilize. The oscillations should look like Figure 56. Use the cursors in VDAS® to measure the amplitude and the angle from the start of the trace to the peak of the maximum amplitude. It should show a phase lag of around 90 degrees from the start of the chart - triggered when the exciter mass is at its top position. The value of the amplitude will be around 20 mm.

Procedure 6 - Damping Ratio

Beam	Mode	Channel 1 Displacement	Timebase
Rigid	Free Vibrations	2 mm	500 ms

Table 9 Recommended VDAS® Settings

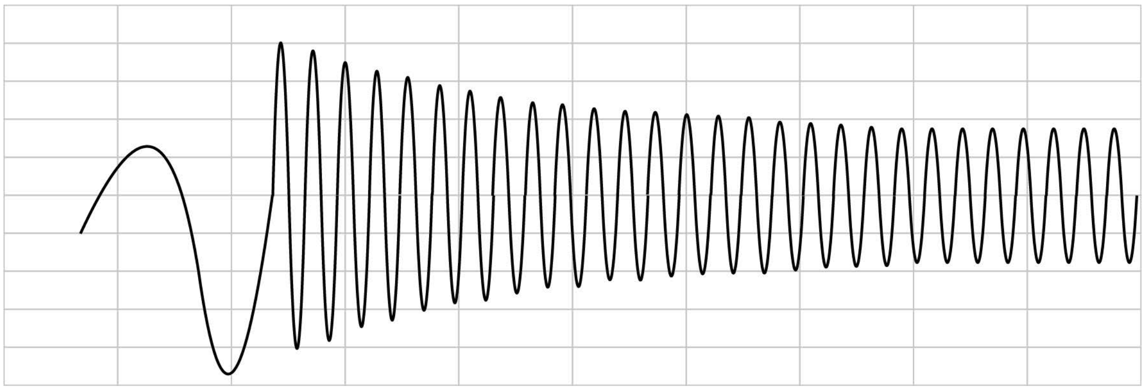

Figure 57 Typical Free Vibration Curve

1. Set VDAS® as shown in Table 9. Enter the natural frequency found earlier.

2. Start VDAS® and press down the beam to start the oscillations, which should be similar to Figure 57. Allow VDAS® to record at least six seconds of oscillations (just over one full screen), then stop VDAS®. Use the horizontal scroll bar on the chart to show only the cleanest section of the waveform (as in Figure 58).

3. It should be seen that the oscillations are underdamped. Use the channel 1 tools to choose the underdamped model, and fit the data. VDAS® will update its damping parameter data fields including the damping ratio. It should produce a model with the lines fitting the waveform as shown in Figure 58. The damping model should give a value in VDAS® of around $\zeta = 0.002$. This should be a very low number, proving an underdamped response.

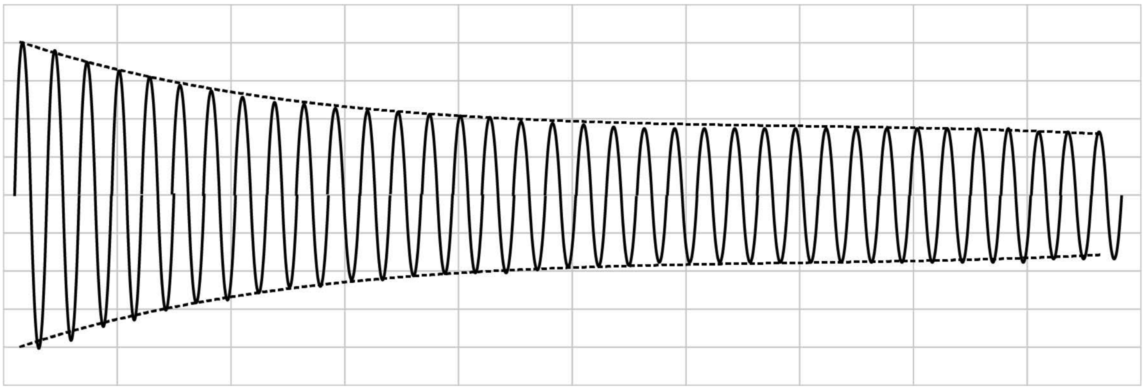

Figure 58 Typical Free Vibration Curve Showing Damping Model

Experiment 2: Rigid Beam and Spring - Added Mass

Aim

To show how added mass affects the natural frequency of the Rigid Beam and Spring.

Procedure

Added Mass (kg)	Total Exciter Mass (kg)	I_{mass} (kg.m^2)	I_A (kg.m^2)	Natural Frequency f (Hz)	
				Measured	Theoretical
0	4.2				
Mass Holder = 0.2 kg	4.4				
400 g + 0.2 kg = 0.6 kg	4.8				
800 g + 0.2 kg = 1.0 kg	5.2				
1200 g + 0.2 kg = 1.4 kg	5.6				
1600 g + 0.2 kg = 1.8 kg	6.0				
2000 g + 0.2 kg = 2.2 kg	6.4				
m_{beam} = 1.65 kg l_{beam} = 0.815 m $l_{exciter}$ = 0.4 m		m_{spring} = 0.388 kg l_{spring} = 0.75 m m_{fixing} = 0.09 kg		I_{beam} = 0.365 kg.m^2 I_{spring} = 0.123 kg.m^2 k = 3800 N.m^{-1}	

Table 10 Blank Results Table

1. Create a Blank results table similar to Table 10. Alternatively, VDAS® automatically creates its own data table as data is recorded.
2. Set the beam as shown in **Setting as a Beam and Spring** on page 45.
3. With no additional mass, find the measured natural frequency as shown in Experiment 1.
4. Add the mass holder with no masses. If necessary, readjust the spring to bring the beam back to a level position and readjust the SET ZERO control.
5. In VDAS®, enter the mass holder value into the Added Mass field.
6. Again, find the natural frequency.

7. Add the 5 x 400 g masses in steps. Remember to update the Added Mass field in VDAS®. At each step, readjust the level of the beam and find the measured natural frequency.

Results Analysis

Find the total mass using the added mass and the mass of the exciter, use this with the other values to find the total moment of inertia of the system for each added mass.

Calculate the theoretical natural frequency of the rigid beam and spring for each additional mass.

Create a chart of frequency (vertical axis) against added mass. Add the measured and theoretical results to compare them. Can any errors be explained?

Experiment 3: Rigid Beam and Spring - Damping

Aims

To show the relationship between oscillation amplitude, phase lag, magnification factor and speed ratio.

To understand how damping affects the oscillations in a beam and spring assembly.

Procedure

Damping Condition	Natural Frequency (Hz)	Damping Ratio z	Magnification factor b
Undamped			
Fully Open			
Half open			
Fully Shut			

Table 11 Blank Results Table

Beam	Mode	Channel 1 Displacement	Timebase	Damper
Rigid	Free Vibration	2 mm	50 ms	Attached

Table 12 Recommended VDAS® Settings

1. Set the beam as shown in **Setting as a Beam and Spring** on page 45.
2. Fit the damper as described in **Fitting the Dashpot Damper** on page 51. However, do not add fluid yet, so that there is the additional mass of the damper piston, but no damping.
3. If necessary, readjust the spring to bring the beam back to a level position and readjust the SET ZERO control.
4. Find the measured natural frequency (free vibration) as shown in Experiment 1 and enter into VDAS®.
5. Find the damping ratio as shown in Experiment 1.
6. Change VDAS® to FORCED VIBRATIONS and make sure it has the measured natural frequency, so it can calculate the speed ratio.

Beam	Mode	Channel 1 Displacement	Timebase	Damper
Rigid	Forced Vibration	0.1 mm	-	Attached

Table 13 Recommended VDAS® Settings

7. Start VDAS® and press the exciter motor START button.
8. As shown in Figure 59, adjust the speed slowly in steps of 0.25 Hz from a speed ratio of 0.75. As the natural frequency is approached (speed ratio of 1.0), adjust the speed in 0.1 Hz steps. This will produce better results. At each step, wait for up to a minute for the oscillations to stabilize then use VDAS® to measure and record the displacement amplitude, phase lag and all other values.

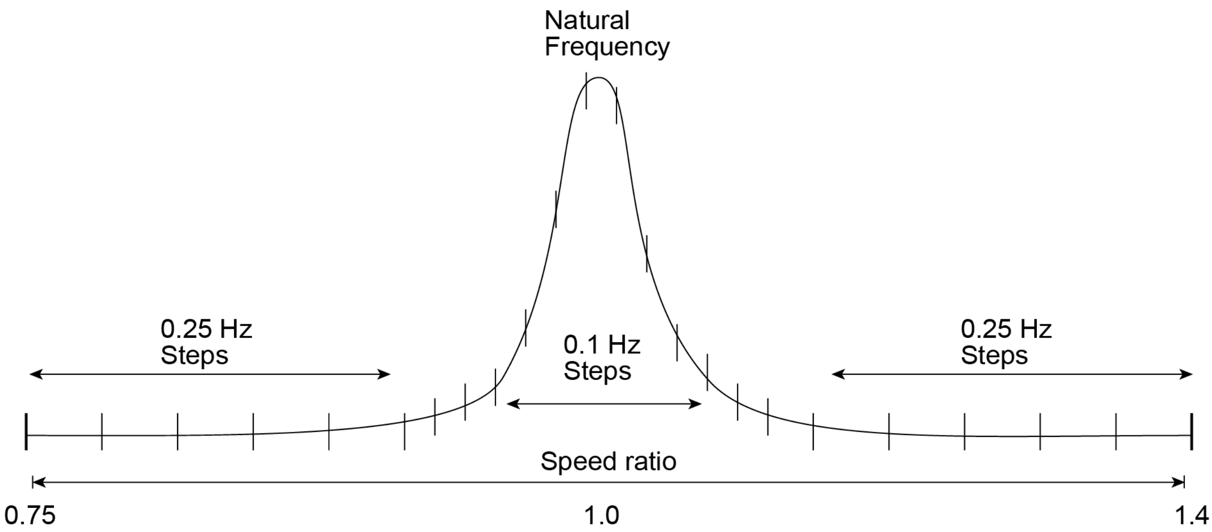

Figure 59 Results Method

9. Now add fluid to the Dashpot Damper as shown and adjust the piston disc to fully open for the lowest damping coefficient and repeat the test.
10. Repeat the test for piston disc settings of half open and fully shut. In VDAS®, start a new data series between each level of damping coefficient used.

Results Analysis

Compare the natural frequency and damping ratio for all four conditions.

Calculate the magnification factor for a speed ratio of 1.0 for all four sets of results to compare.

Produce charts of Amplitude and Phase Lag (vertical axis) against speed ratio.

Do the results compare well with those shown in the theory? Can any differences be explained?

Experiment 4: Simply Supported Beam - Added Mass

Aims

To show how added mass affects the natural frequency of the Simply Supported Beam.

Procedure

Beam	Mode	Channel 1 Displacement	Timebase	Channel 2 Acceleration
Simply Supported	Free Vibration	0.5 mm	20 ms	Not Visible

Table 14 Recommended VDAS® Settings

Added Mass (kg)	Total Exciter Mass (kg)	Effective Mass (kg)	Natural Frequency f (Hz)		$1/f^2$
			Measured	Theoretical	
0	4.2				
Mass Holder = 0.2 kg	4.4				
400 g + 0.2 kg = 0.6 kg	4.8				
800 g + 0.2 kg = 1.0 kg	5.2				
1200 g + 0.2 kg = 1.4 kg	5.6				
1600 g + 0.2 kg = 1.8 kg	6.0				
2000 g + 0.2 kg = 2.2 kg	6.4				
m_{beam}= 1.65 kg 17/35 m_{beam} = 0.8 kg l_3 = 0.375 m			I_{beam} = 2.083 x 10^{-9} m^4 E = 2.00 x 10^{11} Pa $6EI_{beam}$ = 2.50 x 10^3		

Table 15 Blank Results Table

1. Create a Blank results table similar to Table 15. Alternatively, VDAS® automatically creates its own data table as data is recorded.

2. Set the beam as shown in **Setting as a Simply Supported Beam** on page 46.

3. With no additional mass, find the natural frequency as shown in Experiment 1.

NOTE *To create free vibrations with clear waveforms in the simply-supported beam use the flat of the hand to ‘slap’ the top of the exciter. This system is less sensitive to stray harmonics and pressing the beam does not create large enough oscillations.*

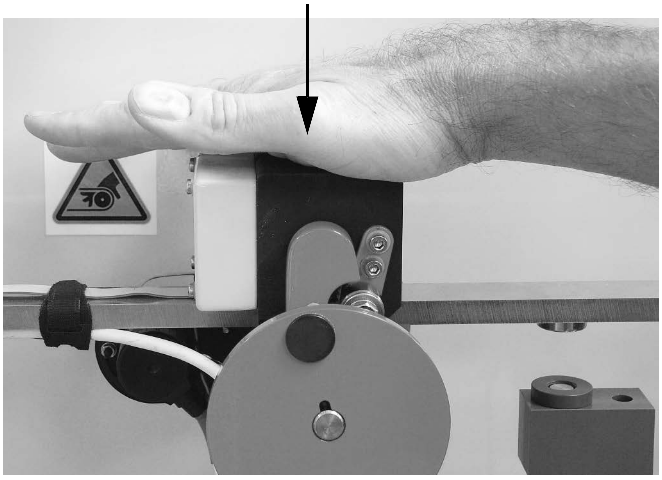

Figure 60 Slap the Top of the Exciter

4. Add the mass holder with no masses. If necessary, readjust the SET ZERO control. In VDAS®, enter the mass holder value into the Added Mass field.

5. Again, find the measured natural frequency.

6. Add the 5 x 400 g masses in steps. Remember to update the Added Mass field in VDAS®. At each step, readjust the SET ZERO and find the natural frequency.

Results Analysis

Use Rayleighs theory to calculate the effective mass and then find the theoretical oscillation frequency. Compare with the measured value from experiment.

As shown in the Dunkerley’s theory, plot a chart of $1/f^2$ (measured natural frequency) as a vertical axis against total mass. Extend the line of the chart to cut the vertical axis and find the theoretical frequency for just the beam.

Compare that with the value from the theory. Remember that the frequency of two cantilevers is being found (375 mm) and not the entire beam.

Experiment 5: Simply Supported Beam - Damping

Aim

To show the relationship between oscillation amplitude, phase lag, magnification factor and speed ratio.

To understand how damping affects the oscillations in a simply supported beam assembly.

Procedure

Damping Condition	Natural Frequency (Hz)	Damping Ratio z	Magnification factor b
Undamped			
Fully Open			
Half open			
Fully Shut			

Table 16 Blank Results Table

Beam	Mode	Channel 1 Displacement	Timebase	Damper
Simply Supported	Free Vibration	0.5 mm	20 ms	Attached

Table 17 Recommended VDAS® Settings

1. Set the beam as shown in **Setting as a Simply Supported Beam** on page 46.
2. Fit the damper as described in **Fitting the Dashpot Damper** on page 51. However, do not add fluid yet, so that there is the additional mass of the damper piston, but no damping.
3. If necessary, readjust the SET ZERO control.
4. Find the measured natural frequency (free vibration) as shown in Experiment 1 and enter into VDAS®.
5. Find the damping ratio as shown in Experiment 1.
6. Change VDAS® to FORCED VIBRATIONS and make sure it has the measured natural frequency, so it can calculate the speed ratio.

Beam	Mode	Channel 1 Displacement	Timebase	Damper
Simply Supported	Forced Vibration	0.1 mm	-	Attached

Table 18 Recommended VDAS® Settings

7. Start VDAS® and press the exciter motor START button.

8. As shown in Figure 59, adjust the speed slowly in steps of 0.25 Hz from a speed ratio of 0.85. As the natural frequency is approached (speed ratio of 1.0), adjust the speed in 0.1 Hz steps. This will produce better results. At each step, wait for up to a minute for the oscillations to stabilize then use VDAS® to measure and record the displacement amplitude, phase lag and all other values.

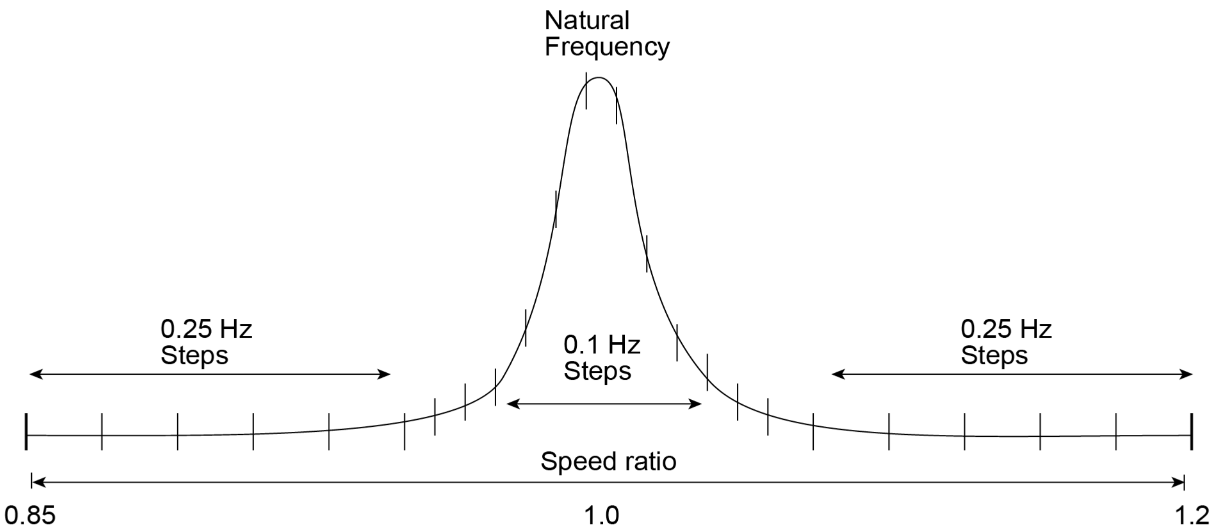

Figure 61 Results Method

9. Now add fluid to the Dashpot Damper as shown and adjust the piston disc to fully open for the lowest damping coefficient and repeat the test.

10. Repeat the test for piston disc settings of half open and fully shut.

Results Analysis

Compare the natural frequency and damping ratio for all four conditions.

Calculate the magnification factor for a speed ratio of 1.0 for all four sets of results to compare.

Produce charts of Amplitude and Phase Lag (vertical axis) against speed ratio.

Do the results compare well with those shown in the theory? Can any differences be explained?

Experiment 6: Simply Supported Beam - Vibration Absorber

Aims

To show how an auxiliary oscillating system can help to absorb the vibrations of the main assembly (a simply supported beam).

To show the three key frequencies of the two degree of freedom (2DOF) system - antiresonance and upper and lower system natural frequencies.

Procedure 1 - Antiresonance

Beam	Mode	Channel 1 Displacement	Timebase	Vibration Absorber
Simply Supported	Forced Vibration	2 mm	-	Attached

Table 19 VDAS® Settings

Vibration Absorber Mass centre Position (mm)	Frequency (Hz)	Amplitude (mm)
7 mm (innermost)	Natural =	
	Antiresonance =	
	First =	
	Second =	

Table 20 Blank Results Table

1. Set the beam as shown in **Setting as a Simply Supported Beam** on page 46.
2. Fit the vibration absorber as shown in **Fitting the Vibration Absorber** on page 54.
3. Set the masses of the vibration absorber to their innermost position (see Figure 62), so they form a basic load, but cannot vibrate. Each mass is 14 mm thick, so their mass centre is at 7 mm.

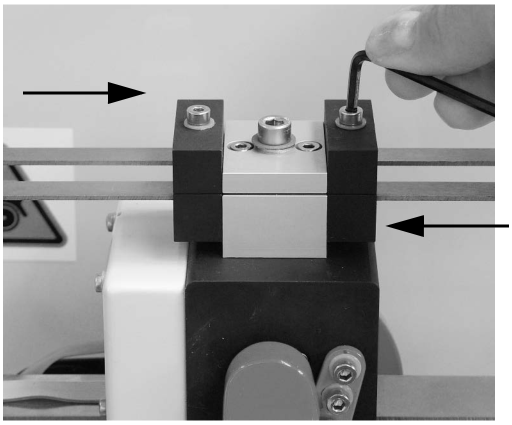

Figure 62 Set Masses to Innermost Position

4. Using forced vibration, slowly increase the speed to find the natural frequency of the system with the added mass of the vibration absorber. If necessary, readjust the SET ZERO control.

5. Note the frequency and oscillation amplitude.

6. Calculate the theoretical position of the masses to cancel the natural frequency of the system.

NOTE *Remember that the theory calculates distance to the centre of the mass.*

7. Adjust the masses of the vibration absorber to the calculated position.

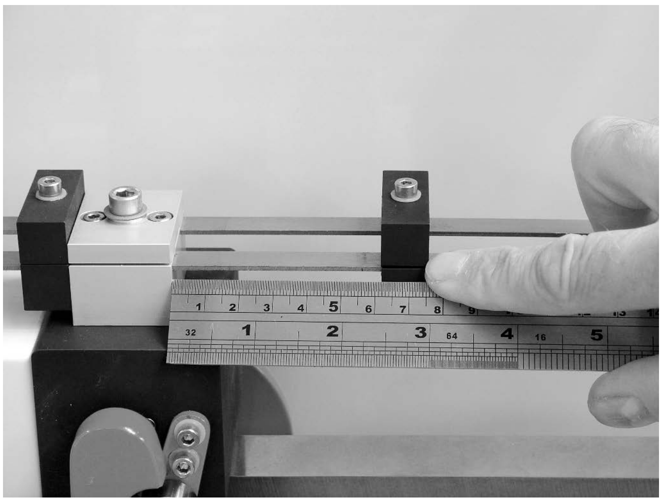

Figure 63 Adjust the Masses to the Calculated Position

8. Using forced vibration, slowly adjust the speed again until the natural frequency is reached. Note that the main beam oscillation amplitude is near zero. The system is now at antiresonance. Look at the vibration absorber to see the amplitude of its oscillations (they cannot be measured).

9. Slowly adjust the speed up and down from antiresonance to see the two new system frequencies where both the main beam and vibration absorber oscillate at relatively high amplitudes. Make a note of these two frequencies and their amplitudes

Procedure 2 - 2DOF Chart

1. With the masses set at the distance for antiresonance, adjust the exciter speed slowly in steps of around 0.2 Hz from around 12 Hz to the maximum speed that the exciter can deliver. At each step, record the frequency and amplitude of oscillation.

2. Plot a chart of the oscillation amplitude against frequency to compare with the 2DOF chart in the theory.

Advanced Analysis

If comfortable with the theory, try to use the quadratic equation (36) to calculate the two new system frequencies and compare with those found by experiment.

Typical Results

Note: These results are typical only. Actual results may differ slightly.

Experiment 1: Introduction to the Equipment

Results are explained in the experiment procedure.

Experiment 2: Rigid Beam and Spring - Added Mass

Added Mass (kg)	Total Exciter Mass (kg)	I_{mass} ($kg.m^2$)	I_A ($kg.m^2$)	Natural Frequency f (Hz)	
				Measured	Theoretical
0	4.2	0.672	1.161	6.58	6.83
Mass Holder = 0.2 kg	4.4	0.704	1.193	6.46	6.74
400 g + 0.2 kg = 0.6 kg	4.8	0.768	1.257	6.23	6.56
800 g + 0.2 kg = 1.0 kg	5.2	0.832	1.321	6.10	6.39
1200 g + 0.2 kg = 1.4 kg	5.6	0.896	1.385	5.92	6.25
1600 g + 0.2 kg = 1.8 kg	6.0	0.96	1.449	5.77	6.11
2000 g + 0.2 kg = 2.2 kg	6.4	1.024	1.513	5.59	5.98
m_{beam} = 1.65 kg l_{beam} = 0.815 m l_{mass} = 0.4 m		m_{spring} = 0.388 kg l_{spring} = 0.75 m m_{fixing} = 0.09 kg		I_{beam} = 0.365 $kg.m^2$ I_{spring} = 0.123 $kg.m^2$ k = 3800 $N.m^{-1}$	

Table 21 Typical Results

Typical calculation

Mass Holder 0.2 kg + 800 g = 1.0 kg added Mass

Exciter mass 4.2 kg, so total mass = 1.0 + 4.2 = 5.2 kg

I_{mass} = 5.2 x 0.4^2 = 0.832 $kg.m^2$

I_{beam} = (1.65/3) x 0.815^2 = 0.365 $kg.m^2$

I_{spring} = [0.09 + (0.388/3)] x 0.75^2 = 0.123 $kg.m^2$

I_A = 0.832 + 0.365 + 0.123 = 1.328 kg.m^2

f = 0.159 x [(3800 x 0.75^2)/1.328]$^{0.5}$ = 0.159 x 1609.6$^{0.5}$ = 6.39 Hz

Note that some values may have been rounded up compared to the typical results.

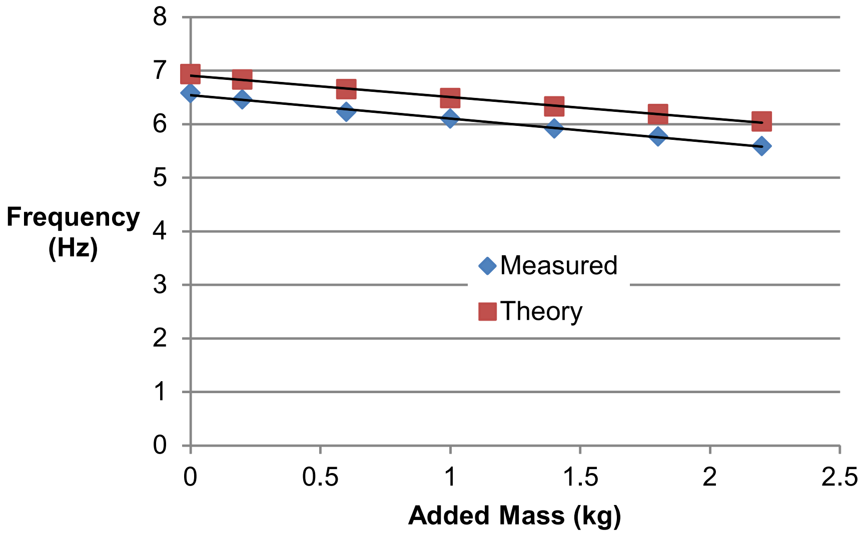

Figure 64 Typical Results

The theoretical values should be similar to the measured values (within roughly 10%). The main causes of error are the accuracy of the spring rate (normally +/- 10% as suggested by manufacturers) and the assumption of a sensibly rigid beam.

Experiment 3: Rigid Beam and Spring - Damping

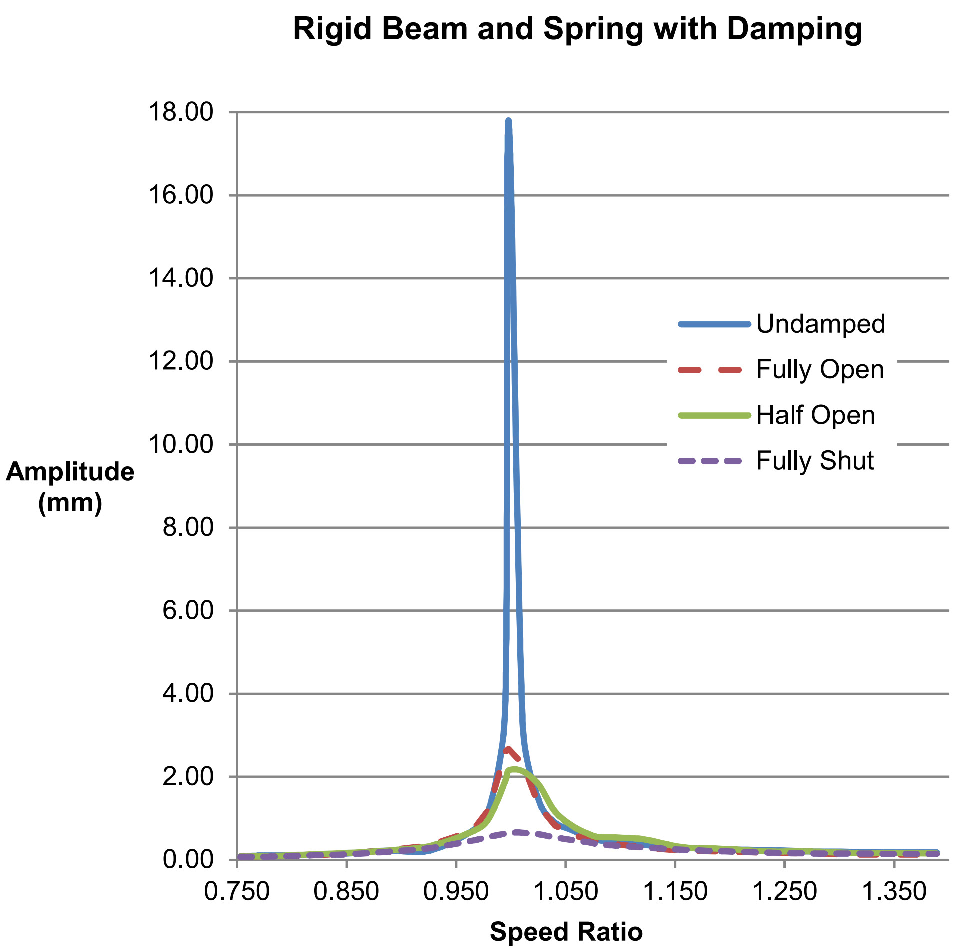

Figure 65 Typical Results

The results should confirm the theory with reasonable accuracy. Expect low damping ratios for the equipment, as all conditions give an underdamped response using the fluid supplied. This means that the damping should not affect the natural frequency.

A large reduction in amplitude should be seen and therefore magnification factor between undamped and the three other conditions.

Finally, the phase lag results should be seen to be becoming more curved as the damping levels are increased. The undamped phase lag is similar to that of the ideal system with no damping, with friction due to air movement being the most likely cause of any difference.

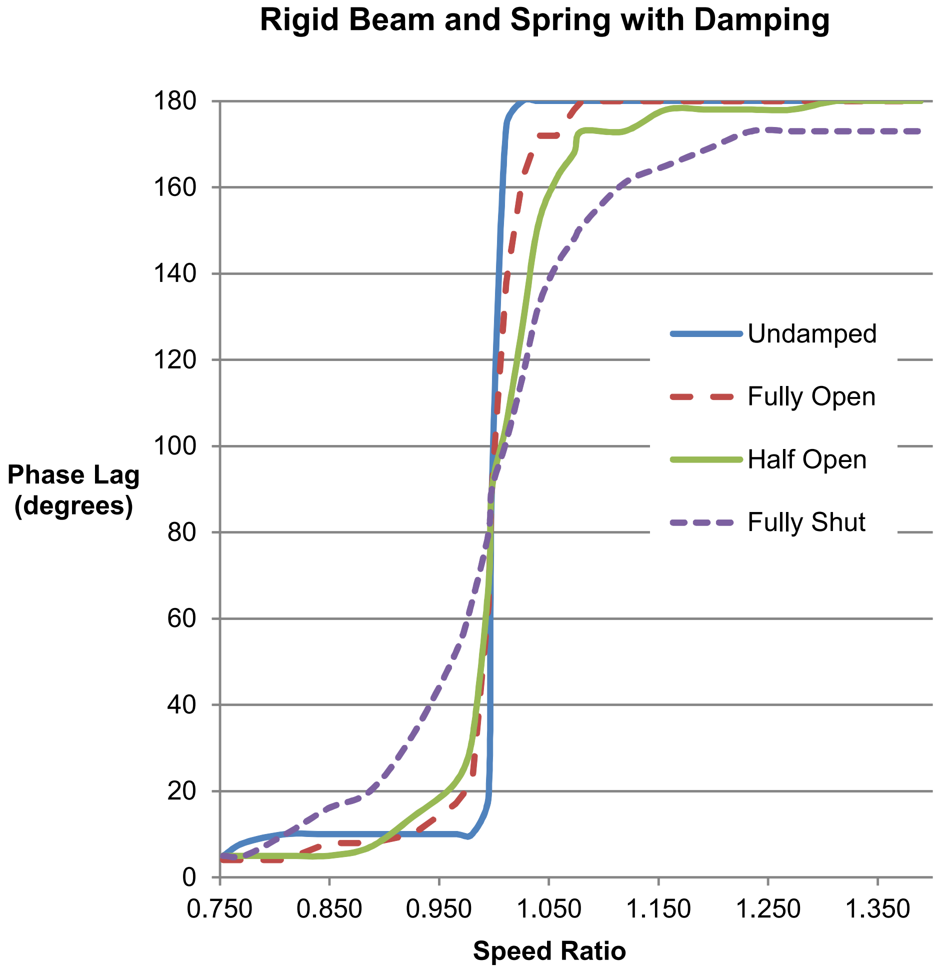

Figure 66 Typical Results

Damping Condition	Natural Frequency (Hz)	Damping Ratio z	Magnification factor b
Undamped	6.48	0.002	250
Fully Open	6.48	0.008	62.5
Half open	6.48	0.018	27.8
Fully Shut	6.48	0.044	11.36

Table 22 Typical Results

Typical Calculations (magnification factor)

$$\beta = \frac{1}{\sqrt{(1-\Omega^2/\omega^2)^2 + (2\zeta\Omega/\omega)^2}}$$

At resonance $\Omega/\omega = 1$, so

$$\beta = \frac{1}{\sqrt{(2\zeta 1)^2}}$$

For Undamped $\zeta = 0.002$, so:

$$\beta = \frac{1}{\sqrt{0.000016}} = 250$$

For Fully open $\zeta = 0.008$, so:

$$\beta = \frac{1}{\sqrt{0.000256}} = 62.5$$

For Half Open $\zeta = 0.018$, so:

$$\beta = \frac{1}{\sqrt{0.001296}} = 27.8$$

For Fully Shut $\zeta = 0.044$, so:

$$\beta = \frac{1}{\sqrt{0.007744}} = 11.36$$

Experiment 4: Simply Supported Beam - Added Mass

Added Mass (kg)	Total Exciter Mass (kg)	Effective Mass (kg)	Natural Frequency f (Hz)		$1/f^2$
			Measured	Theoretical	
0	4.2	5.00	15.59	15.5	0.004114
Mass Holder = 0.2 kg	4.4	5.20	15.28	15.2	0.004283
400 g + 0.2 kg = 0.6 kg	4.8	5.60	14.57	14.64	0.004711
800 g + 0.2 kg = 1.0 kg	5.2	6.00	14.11	14.15	0.005023
1200 g + 0.2 kg = 1.4 kg	5.6	6.40	13.68	13.7	0.005344
1600 g + 0.2 kg = 1.8 kg	6.0	6.80	13.35	13.29	0.005611
2000 g + 0.2 kg = 2.2 kg	6.4	7.20	12.86	12.91	0.006047
m_{beam} = 1.65 kg 17/35 m_{beam} = 0.801 kg l_3 = 0.375 m			I_{beam} = 2.083 x 10^{-9} m^4 E = 2.00 x 10^{11} Pa $6EI_{beam}$ = 2.50 x 10^3		

Table 23 Typical Results

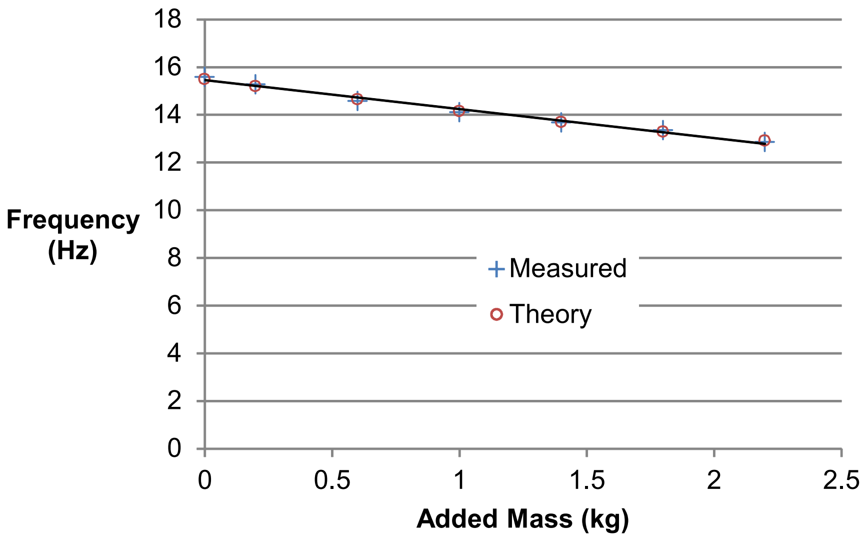

Figure 67 Typical Results

Typical calculation (theory frequency)

Mass Holder = 0.2 kg + 800 g = 1.0 kg added Mass

Exciter mass = 4.2 kg, so total mass = 1.0 + 4.2 = 5.2 kg

17/35 m_{beam} = 0.801 kg

Effective mass = 5.2 + 0.801 = 6 kg

f = 0.1592 x [2.50 x 10^3/(6 x 0.375^3)]$^{0.5}$ = 0.1592 x [2.50 x 10^3/0.3164]$^{0.5}$ = 14.15 Hz

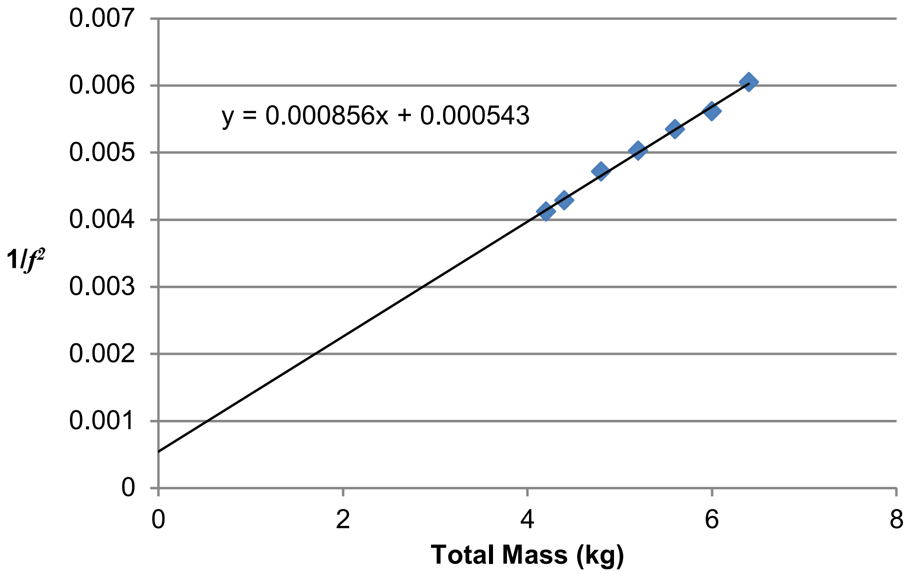

Figure 68 Typical Results

The chart shows an intercept of 0.000543, so from Dunkerley's theory, the frequency of the beam without additional mass is 1/0.000543$^{0.5}$ = 42.9 Hz.

Using the equations in the theory:

$$f_{beam} = \frac{\pi}{2}\sqrt{\frac{EI_{beam}}{m_{beam}(2l_3)^3}}$$

= 1.57 x [(2.00 x 10^{11} x 2.083 x 10^{-9})/1.65(2 x 0.375)^3]$^{0.5}$

= 1.57 x [416.6/0.696]$^{0.5}$ = 38.4 Hz

Giving an error of less than 11%, which is acceptable when considering the inaccuracy of extending the chart to the vertical axis and the approximation of stiffness of the beam and effective central mass.

Experiment 5: Simply Supported Beam - Damping

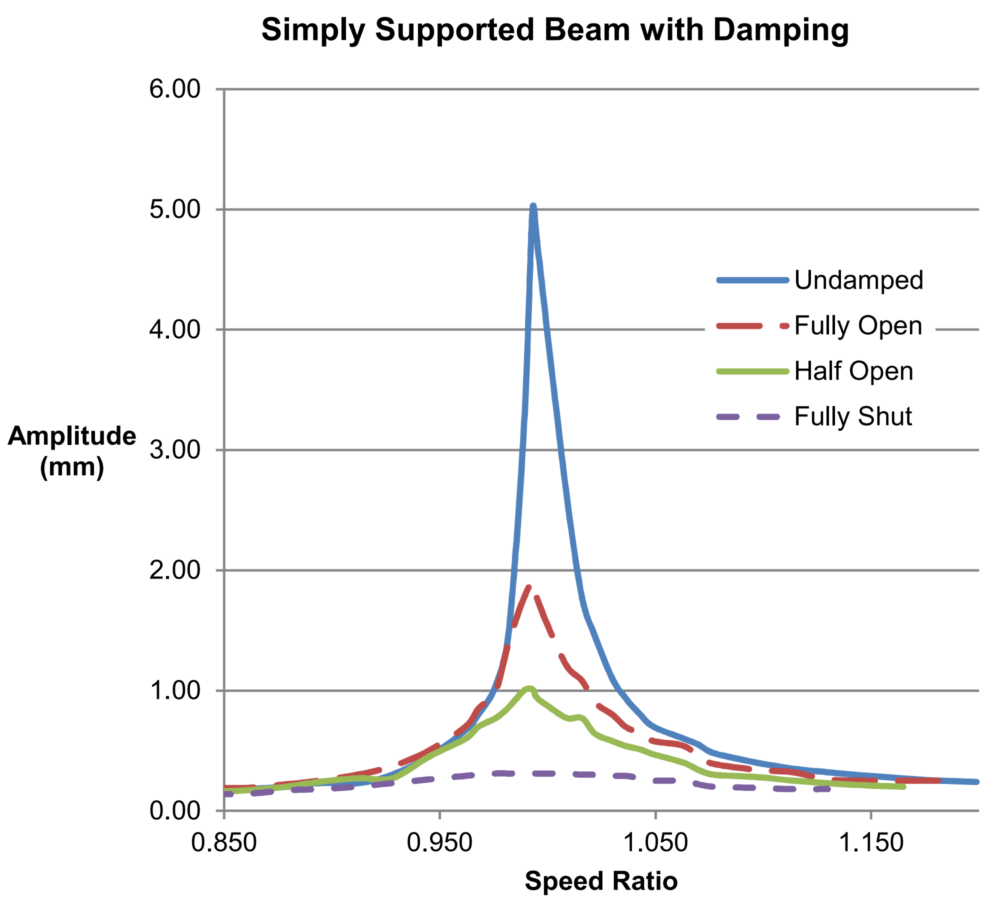

Figure 69 Typical Results

The results should confirm the theory with reasonable accuracy. Expect low damping ratios for the equipment, as all conditions give an underdamped response using the fluid supplied. This means that the damping should not affect the natural frequency.

Note the lower magnitude (amplitude) of the oscillations compared to the rigid beam. Also note that the damping ratios for this experiment are higher than for the rigid beam and spring, mainly due to the different system, but also due to the position of the dashpot damper.

A large reduction in amplitude and therefore magnification factor between undamped and the three other conditions should be seen.

Finally, the phase lag results should be seen to be becoming more curved as the damping levels are increased. The undamped phase lag is similar to that of the ideal system with no damping, with friction due to air movement being the most likely cause of any difference.

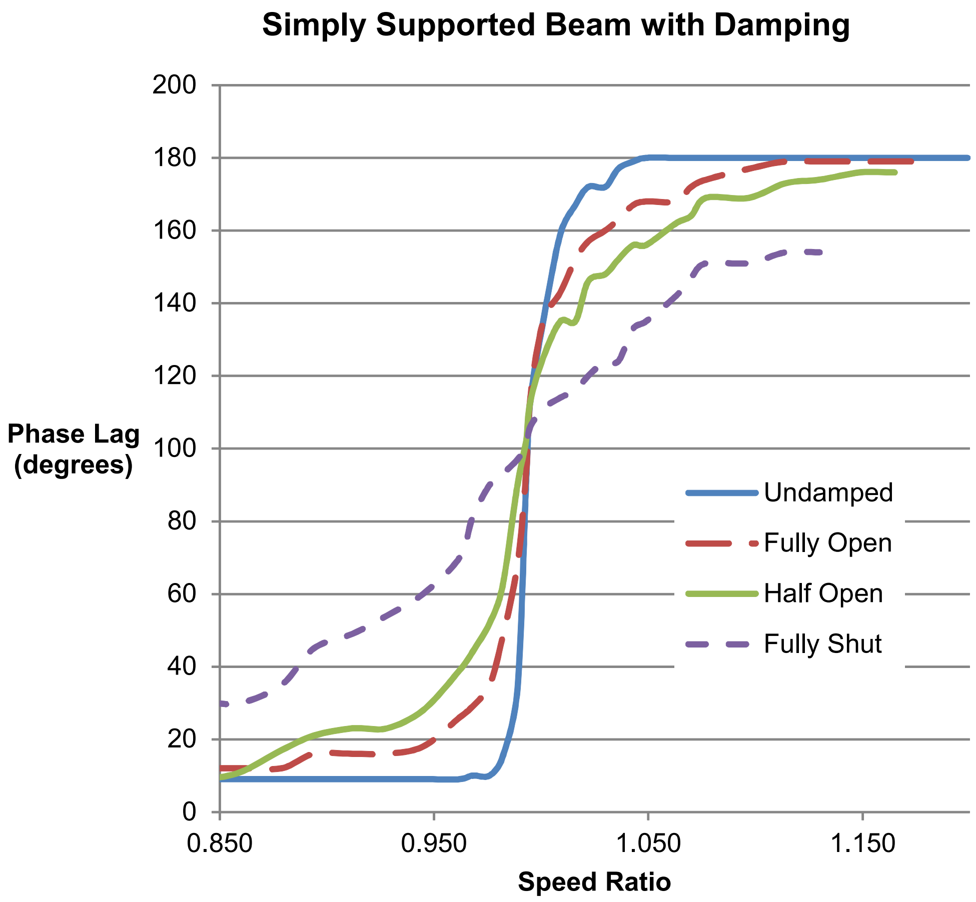

Figure 70 Typical Results

Damping Condition	Natural Frequency (Hz)	Damping Ratio z	Magnification factor b
Undamped	14.81	0.005	100
Fully Open	14.81	0.016	31.25
Half open	14.81	0.029	17.24
Fully Shut	14.81	0.086	5.8

Table 24 Typical Results

Typical Calculations (magnification factor)

$$\beta = \frac{1}{\sqrt{(1 - \Omega^2/\omega^2)^2 + (2\zeta\Omega/\omega)^2}}$$

At resonance $\Omega/\omega = 1$, so

$$\beta = \frac{1}{\sqrt{(2\zeta 1)^2}}$$

For Undamped ζ = 0.005, so:

$$\beta = \frac{1}{\sqrt{0.0001}} = 100$$

For Fully open ζ = 0.016, so:

$$\beta = \frac{1}{\sqrt{0.001024}} = 31.25$$

For Half Open ζ = 0.029, so:

$$\beta = \frac{1}{\sqrt{0.003364}} = 17.24$$

For Fully Shut ζ = 0.086, so:

$$\beta = \frac{1}{\sqrt{0.02958}} = 5.8$$

Experiment 6: Simply Supported Beam - Vibration Absorber

Vibration Absorber Mass centre Position (mm)	Frequency (Hz)	Amplitude (mm)
7 mm (innermost)	Natural = 14.6	5.286
141 mm	Antiresonance = 14.6	0.1
	First = 13.03	1.94
	Second = 16.5	3.17

Table 25 Typical Results

$$l_{absorber} = \left(\frac{3EI_{absorber}/m_2}{(2\pi f)^2}\right)^{\frac{1}{3}}$$

$$m_2 = m_{mass} + \frac{33}{140}m_{absorberbeam} = 125 + \{(33/140) \times 74.4)\} = 142.5 \text{ g } (0.1425 \text{ kg})$$

$$l_{absorber} = \left(\frac{3 \times 1.125/0.1425}{(91.73)^2}\right)^{\frac{1}{3}} = \left(\frac{23.68}{8414.4}\right)^{\frac{1}{3}} = 0.141 \text{ m } (141 \text{ mm})$$

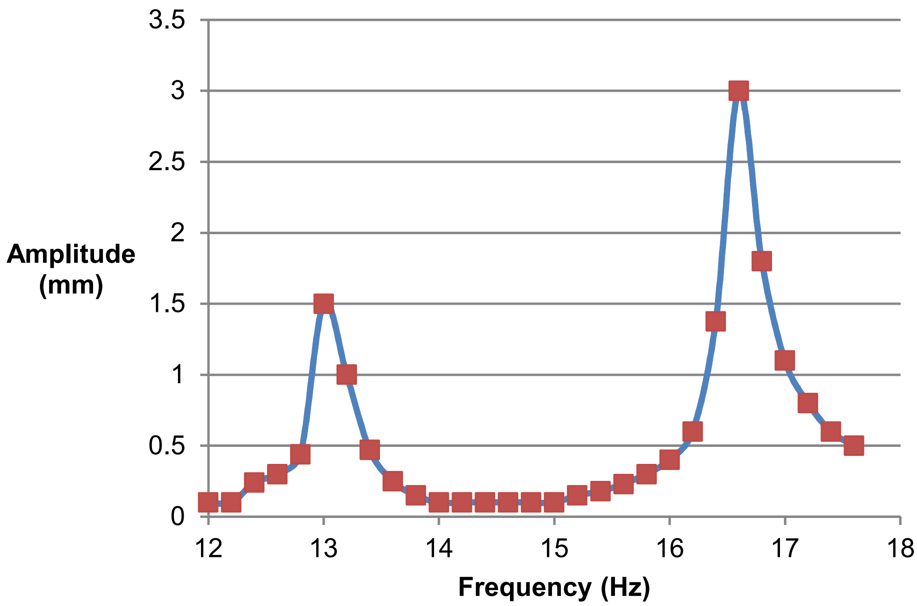

Table 26 Typical Results

Advanced Analysis

m_1 = 4.2 + (17/35 x 1.5) + 0.438 = 5.368 kg

m_2 = 0.142 kg

k_{beam} = 4.74 x 10^4 N.m^{-1}

$k_{absorber}$ = 1.2 x 10^3 N.m^{-1}

$$a = 2m_1m_2$$

= 2 x 5.368 x 0.142 = 1.52

$$b = -(2m_1k_{absorber} + 2m_2(k_{beam} + 2k_{absorber}))$$

= -(2 x 5.368 x 1.2 x 10^3 + 2 x 0.142 (4.74 x 10^4 + 2 x 1.2 x 10^3))

= -(10.736 x 1.2 x 10^3 + 0.284 (49800))

= - (12883.2 + 14143.2)

= - 27026.4

$$c = 2k_{beam}k_{absorber}$$

= 2 x 4.74 x 10^4 x 1.2 x 10^3

= 1.14 x 10^8

For the new higher system frequency $\omega^2 = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ = 103.97 rad.s^{-1}

f = 103.97/2π = 16.55 Hz

For the lower system frequency $\omega^2 = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$ = 82.78 rad.s^{-1}

f = 82.78/2π = 13.18 Hz

With the vibration absorber mass at their innermost position, the vibration absorber is simply an extra mass, as its beam are relatively light. The main beam oscillation at the natural frequency reaches over 5 mm.

With the masses of the vibration absorber at the calculated position, the main oscillation reduces to almost zero at the same frequency. The beams are now at antiresonance. The vibration absorber oscillates with an amplitude of around 5 mm at its tips, but this can be subjective as the equipment does not measure the value.

The chart of amplitude against frequency shows a similar trend to the 2DOF chart in the theory, with two new natural frequencies for the system. Note that neither of the two new frequencies reach the same amplitude as the initial value with the vibration absorber masses set innermost.

The quadratic equation from the theory should predict the two new system frequencies with reasonable accuracy, allowing for tolerances in materials and masses.

Useful Textbooks

Vibration Problems in Engineering

Published by Wiley

ISBN 978 81 265 4079 2

W Weaver

S P Timoshenko

D H Young

Advanced Level Physics

Published by Heinemann

ISBN 978 043 592 3037

Nelkon and Parker

Maintenance, Spare Parts and Customer Care

Maintenance

General

Regularly check all parts of the equipment for damage, renew if necessary.

When not in use, store the equipment in a dry dust-free area, preferably covered with a plastic sheet.

If the equipment becomes dirty, wipe the surfaces with a damp, clean cloth. Do not use abrasive cleaners.

Regularly check all fixings and fastenings for tightness; adjust where necessary.

NOTE *Renew or replace faulty or damaged parts or detachable cables with an equivalent item of the same type or rating.*

Exciter Belt Guard

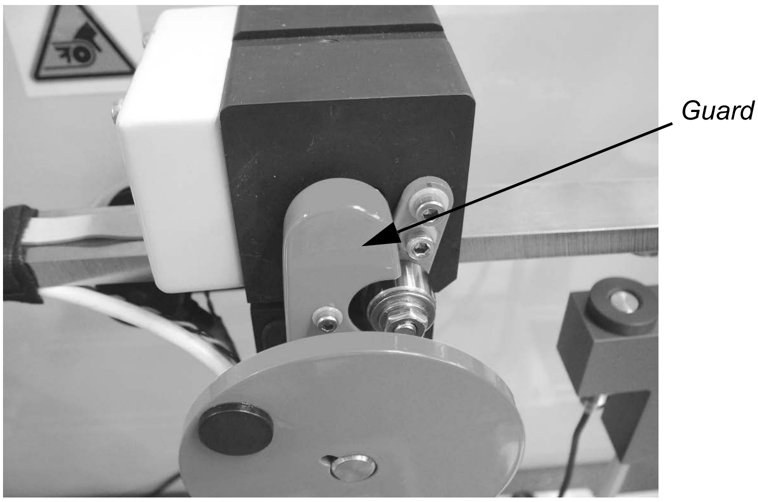

Figure 71 Exciter Belt Guard

The belt of the exciter has a guard (see Figure 71). Regularly check that the guard is secure. Tighten its fixings if necessary.

Displacement Span Check (needs VDAS®)

TecQuipment calibrate the non-contacting displacement sensor. It should not need adjustment when the equipment is supplied as new. However, variances in the strength of the magnet under the beam and the local magnetic field may cause the sensor to become less accurate over time.

The sensor has good a resolution and dynamic response, with an overall accuracy of better than 5% of the reading. If it is suspected that the readings are not within this accuracy, then check the sensor calibration. To do this:

1. Set the the beam as a simply supported beam (see **Setting as a Simply Supported Beam** on page 46).

2. Set the sensor distance as shown in **Setting Sensor Distance** on page 56.

NOTE *Do this carefully. Any forced deflection of the beam or sensor holder will give inaccurate readings!*

3. Set the zero as shown in **Setting Zero** on page 55.

4. Now carefully lower the sensor down its mounting by around 20 to 30 mm.

5. Exchange the zero block for the longer span calibration block and repeat the procedure. This time do not zero the reading, simply remove the block and note the displacement value from the VDAS® software. This value should be 13 mm ±0.5 mm (as this block is 13 mm longer than the zero block).

6. Contact TecQuipment if the reading is not within this tolerance.

Electrical

WARNING ***Only a qualified person may carry out electrical maintenance.***

Obey these procedures:

- Assume the apparatus is energised until it is known to be isolated from the electrical supply.
- Use insulated tools where there are possible electrical hazards.
- Confirm that the apparatus earth circuit is complete.
- Identify the cause of a blown fuse before renewing.

To renew a broken fuse

- Isolate the apparatus from the electrical supply.
- Renew the fuse.
- Reconnect the apparatus to the electrical supply and switch on.
- If the apparatus fails again, contact TecQuipment Ltd or the agent for advice.

Fuse Location

The main fuse is on the side of the unit at the IEC inlet. Use a flat-blade screwdriver to remove the fuseholder(s).

Spare Parts

Check the Packing Contents List to see what spare parts we send with the apparatus.

If technical help or spares are needed, please contact the local TecQuipment Agent, or contact TecQuipment direct.

When asking for spares, please tell us:

- Contact Name
- The full name and address of the college, company or institution
- Contact email address
- The TecQuipment product name and product reference
- The TecQuipment part number (if known)
- The serial number
- The year it was bought (if known)

Please give us as much detail as possible about the parts needed and check the details carefully before contacting us.

If the product is out of warranty, TecQuipment will advise know the price of the spare parts for confirmation.

Customer Care

We hope our products and manuals are liked. If there are any questions, please contact our Customer Care department:

Telephone: +44 115 9722611

Fax: +44 115 973 1520

email: **customer.care@tecquipment.com**

For information about all TecQuipment Products and Services, visit:

www.tecquipment.com