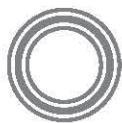


Section	1	2
Date		



Congruence and Similarity

Classwork Question

Lesson 1

Section 1 Congruent triangles

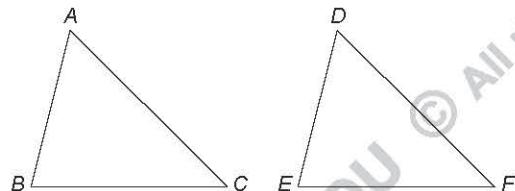
A.....

Reminder

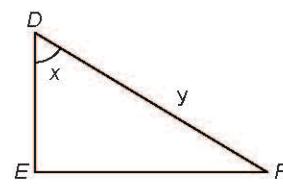
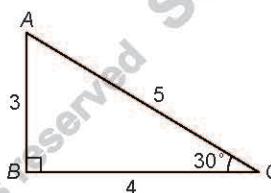
If $\triangle ABC \cong \triangle DEF$, then

$\Rightarrow AB = DE, AC = DF, BC = EF$ (corr. sides, $\cong \Delta$ s)

$\Rightarrow \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$ (corr. \angle s, $\cong \Delta$ s)

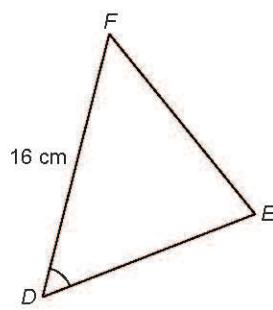
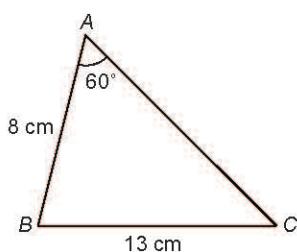


Example 1. Given that $\triangle ABC \cong \triangle DEF$. Find x and y .



2. In the figure, $\triangle ABC \cong \triangle DEF$, $AB = 8$ cm, $BC = 13$ cm, $DF = 16$ cm and $\angle A = 60^\circ$.

- Find AC, DE and EF .
- Find the perimeter of $\triangle ABC$.





Solution 1. $\angle A = 180^\circ - 90^\circ - 30^\circ$ (\angle sum of Δ)
 $= 60^\circ$

$x = \angle A = 60^\circ$ (corr. \angle s, $\cong \Delta$ s)

$y = AC = 5$ (corr. sides, $\cong \Delta$ s)

2. (a) $AC = DF = 16$ cm (corr. sides, $\cong \Delta$ s)

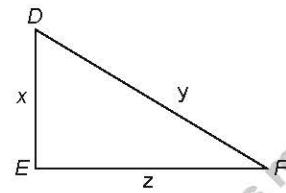
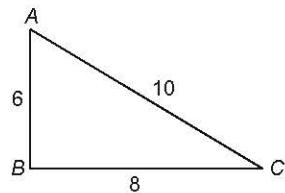
$DE = AB = 8$ cm (corr. sides, $\cong \Delta$ s)

$EF = BC = 13$ cm (corr. sides, $\cong \Delta$ s)

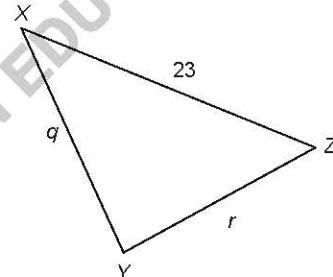
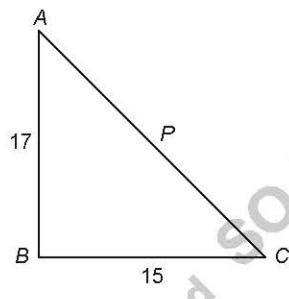
(b) perimeter of $\Delta ABC = AB + BC + AC$
 $= 8 + 13 + 16$
 $= 37$ cm

Find the unknowns in the following figures. (1-5)

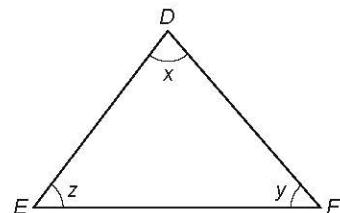
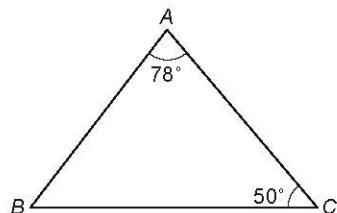
1. $\Delta ABC \cong \Delta DEF$



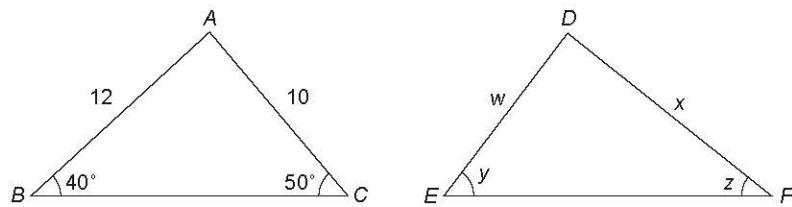
2. $\Delta ABC \cong \Delta XYZ$



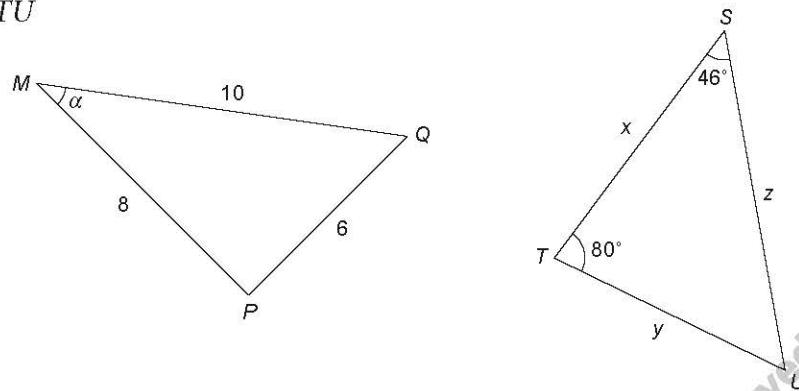
3. $\Delta ABC \cong \Delta DEF$



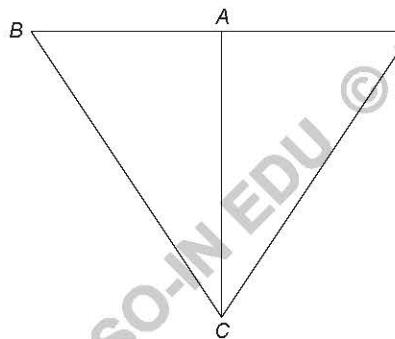
4. $\Delta ABC \cong \Delta DFE$



5. $\Delta MPQ \cong \Delta STU$



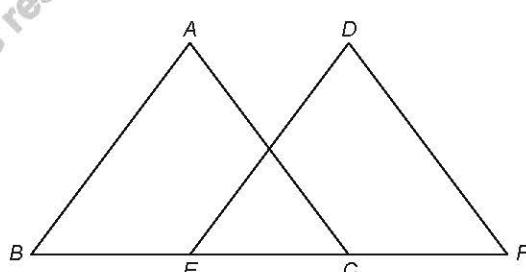
6. Given that $\Delta ABC \cong \Delta ADC$, if $\angle BAC = 90^\circ$, $AC = 12 \text{ cm}$ and $AD = 5 \text{ cm}$, find the area of ΔBCD .



7. Given that $\Delta ABC \cong \Delta DEF$, if ΔABC is an equilateral triangle, $EC = 6 \text{ cm}$ and $AB = 11 \text{ cm}$,

- (a) find BE .

- (b) find CF .



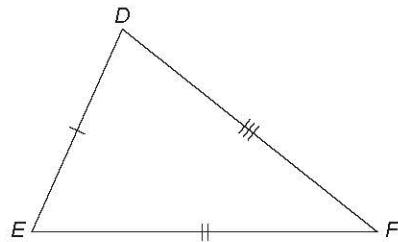
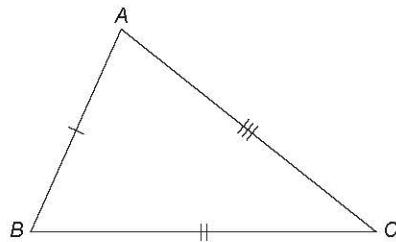


B

Q **Reminder.....**

SSS

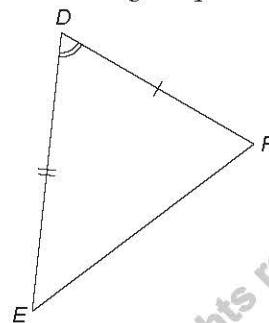
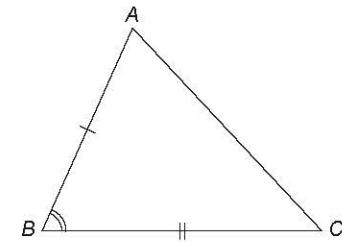
3 pairs of corresponding sides equal



$\Delta ABC \cong \Delta DEF$ (SSS)

SAS

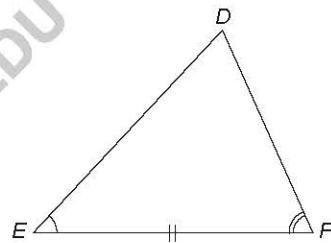
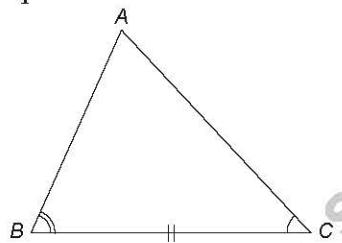
2 pairs of corresponding sides and the included angle equal.



$\Delta ABC \cong \Delta FDE$ (SAS)

ASA

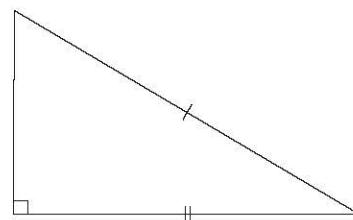
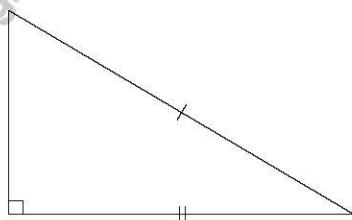
2 pairs of corresponding angles and the included pair of corresponding sides equal.



$\Delta ABC \cong \Delta DFE$ (ASA)

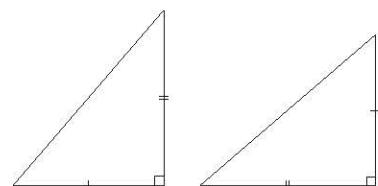
AAS

2 pairs of corresponding angles and the included pair of corresponding sides equal.

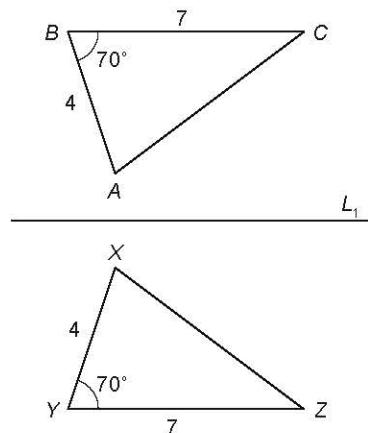


Common Mistake

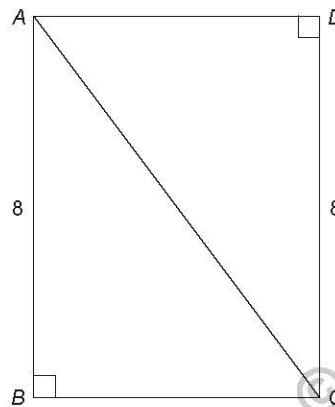
It is wrong to use RHS for the following triangles.



Example (1) $\triangle ABC$ is reflected across L_1 to obtain $\triangle XYZ$. Determine the following pair of triangles are congruent or not. If yes, give reason.



- (2) In the figure, $ABCD$ is a rectangle with length 8 and width 6. Show that $\triangle ABC \cong \triangle CDA$.



Solution 1. $\angle ABC = \angle XYZ = 70^\circ$ (given)

$AB = XY = 4$ (given)

$BC = YZ = 7$ (given)

$\therefore \triangle ABC \cong \triangle XYZ$ (SAS)

2. $\angle ABC = \angle CDA = 90^\circ$ (given)

$AB = CD = 8$ (given)

$AC = AC$ (common)

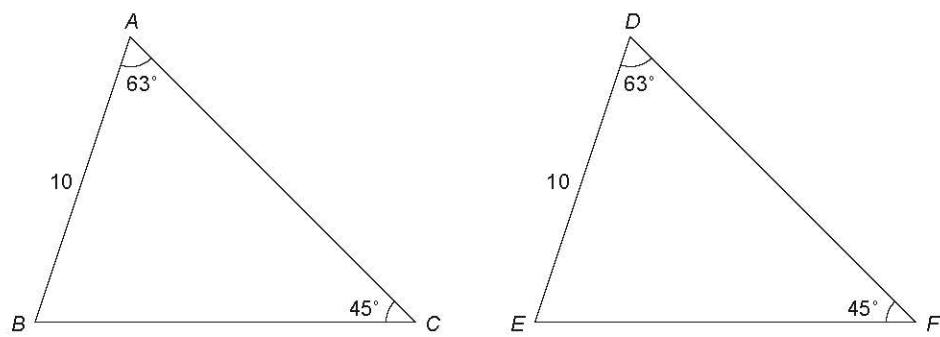
$\therefore \triangle ABC \cong \triangle CDA$ (RHS)

-
1. (a) Construct a $\angle ABC$ by ruler and protractor, with $\angle ABC = 90^\circ$, $AB = 6$ cm and $BC = 8$ cm.
 (b) Measure the length of AC .
2. Construct a $\triangle XYZ$ by ruler and protractor, with $\angle XYZ = 65^\circ$, $XY = 8$ cm and $YZ = 6$ cm.

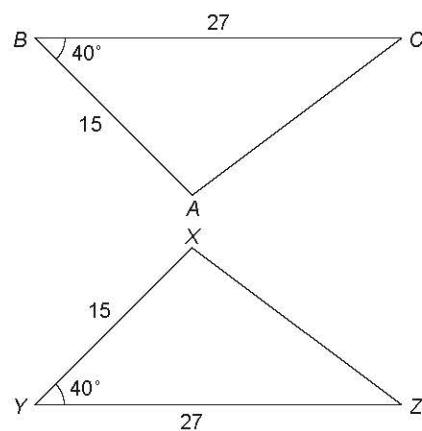


3. Name the following pairs of congruent triangles with reasons.

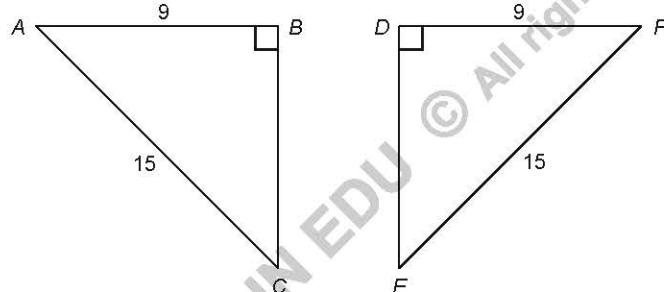
(a)



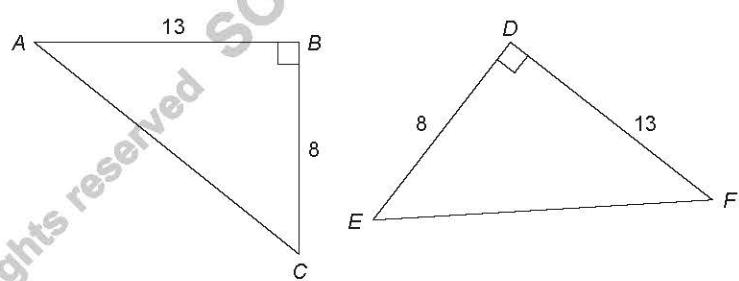
(b)



(c)

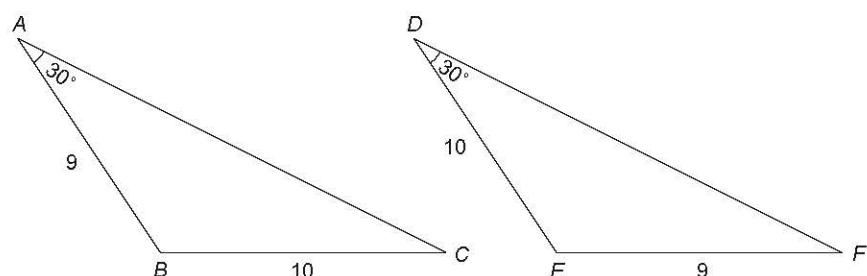


(d)



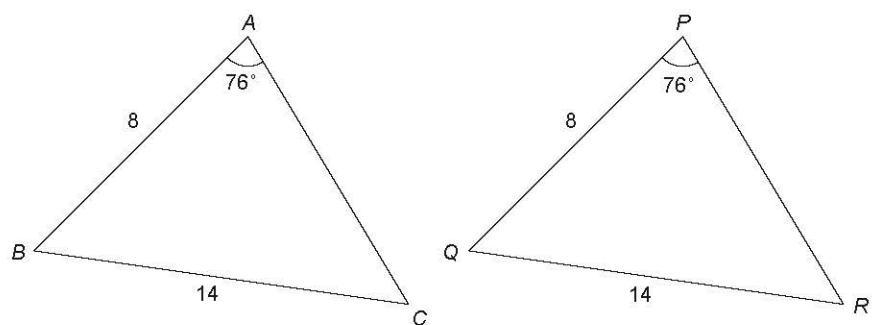
4. Determine whether the following pairs of triangles are congruent or not. If yes, give reasons.

(a)

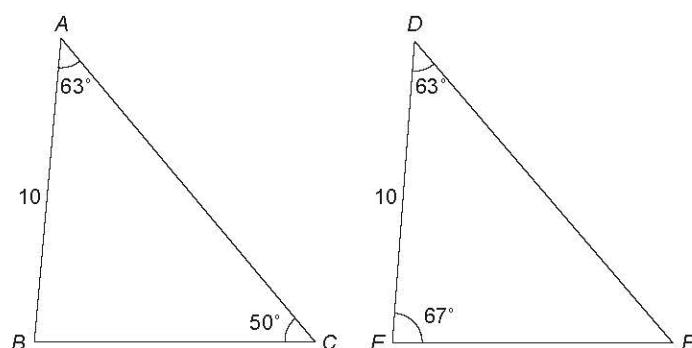




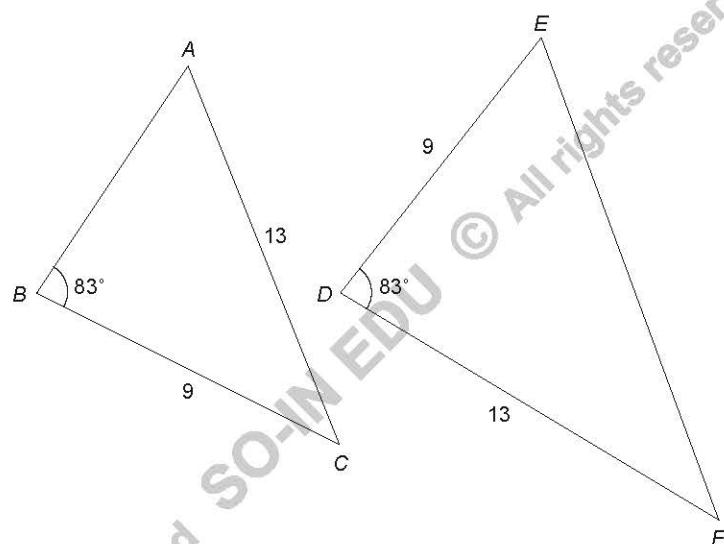
(b)



(c)

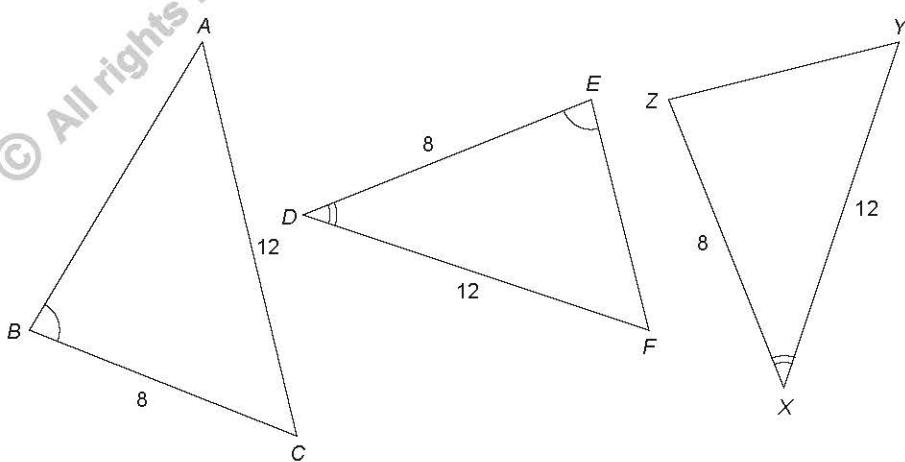


(d)



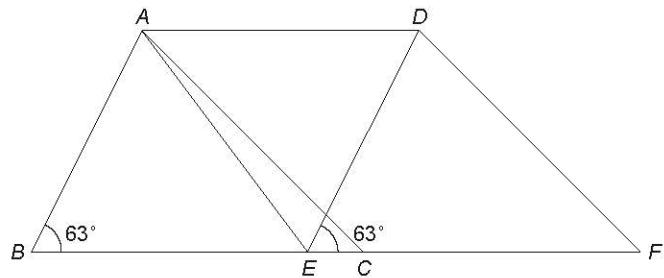
5. In each of the following, find the pair of congruent triangles and give reasons.

(a)

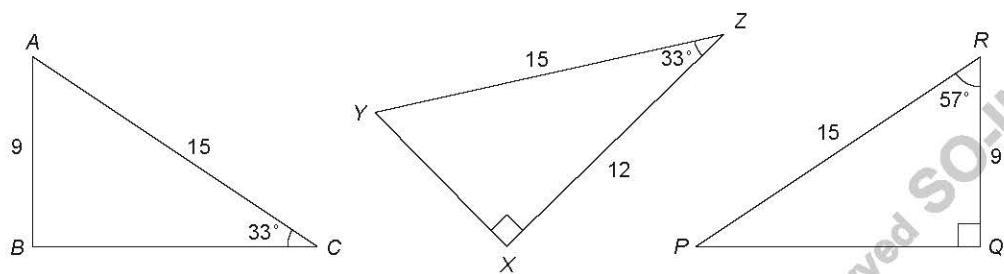




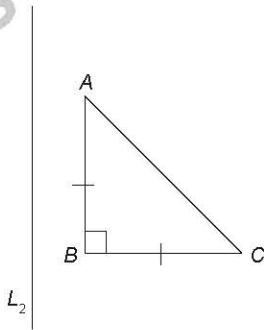
- (b) $AB = 7$, $DE = 7$, $EC = 1.5$, $BE = 6$ and $CF = 6$



- (c)



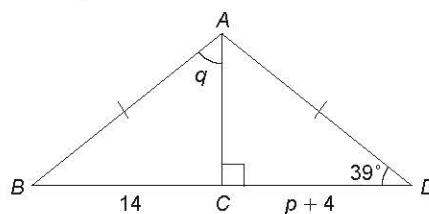
6. Determine whether $\triangle ABC$ and the images are congruent triangles or not if the image undergoes the following transformations.
- Translate $\triangle ABC$ to the left
 - Reflect $\triangle ABC$ across L_2
 - Enlarge $\triangle ABC$ 2 times
 - Rotate $\triangle ABC$ 90° anti-clockwise through A





C

Example In the figure, BCD is a straight line.



- Find a pair of congruent triangles and give reason.
- Find p and q .

Solution (a) $\angle ACB = \angle ACD = 180^\circ$ (given)

$AC = AC$ = (common)

$AB = AC$ = (given)

$\therefore \Delta ABC \cong \Delta ADC$ (RHS)

(b) $p + 4 = 14$ (corr. sides, $\cong \Delta$ s)

$$p = 10$$

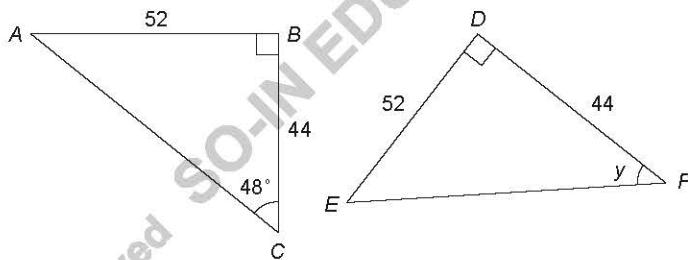
$$\begin{aligned} \angle ACB &= \angle ACD = 180^\circ \text{ (given)} \\ &= 51^\circ \end{aligned}$$

$$q = 51^\circ \text{ (corr. } \angle \text{s, } \cong \Delta \text{s)}$$

- Refer to the figure below,

(a) prove that $\Delta ABC \cong \Delta EDF$.

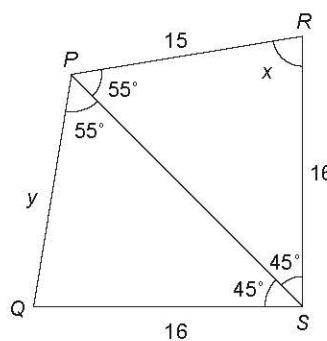
(b) find y .



- Refer to the figure below,

(a) name a pair of congruent triangles with reason.

(b) find x and y .

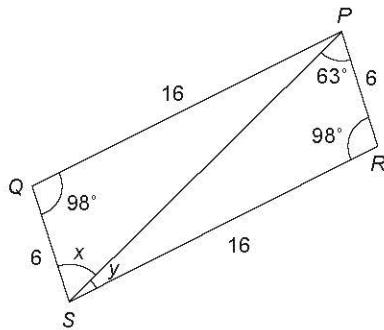




3. Refer to the figure below,

(a) Name a pair of congruent triangles with reason.

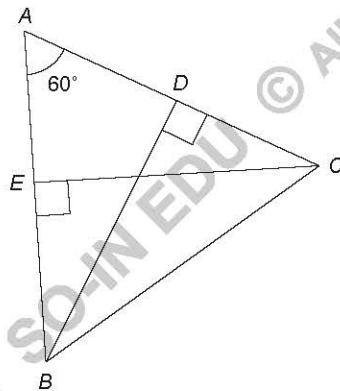
(b) Find x and y .



4. In the figure below, $BD = EC$

(a) Write down a pair of congruent triangles and state the reason.

(b) Find $\angle EBC$.





Lesson 2

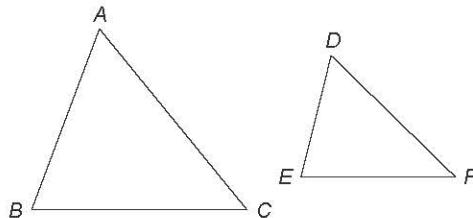
Section 2 Similar triangles

Q Reminder...

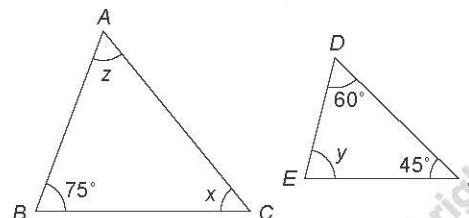
If $\Delta ABC \sim \Delta DEF$, then

$$\Leftrightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \text{ (corr. sides, } \sim \Delta \text{s)}$$

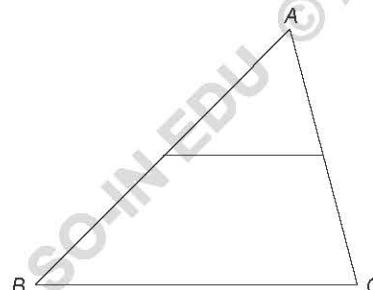
$$\Leftrightarrow \angle A = \angle D, \angle B = \angle E, \angle C = \angle F \text{ (corr. } \angle \text{s, } \sim \Delta \text{s)}$$



Example 1. In the figure $\Delta ABC \sim \Delta DEF$. Find x , y and z .



2. In the figure, $\Delta ABC \sim \Delta AKH$. Find HC if $AH = 8$, $AK = 12$ and $KB = 4$.



Solution 1. $x = 45^\circ$ (corr. \angle s, $\sim \Delta$ s)

$y = 75^\circ$ (corr. \angle s, $\sim \Delta$ s)

$z = 60^\circ$ (corr. \angle s, $\sim \Delta$ s)

$$2. \quad \frac{AB}{AK} = \frac{AC}{AH} \text{ (corr. sides, } \sim \Delta \text{s)}$$

$$\frac{AK + KB}{AK} = \frac{AH + HC}{AH}$$

$$\frac{12 + 4}{12} = \frac{8 + HC}{8}$$

$$\frac{16}{12} = \frac{8 + HC}{8}$$

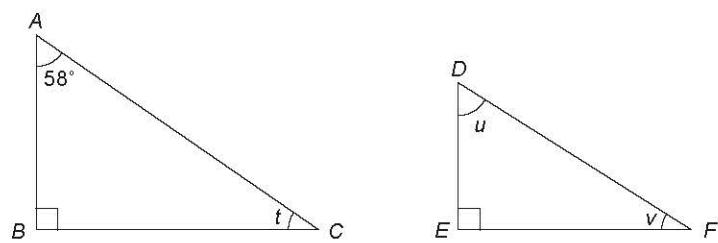
$$8 + HC = \frac{32}{3}$$

$$HC = \frac{8}{3}$$

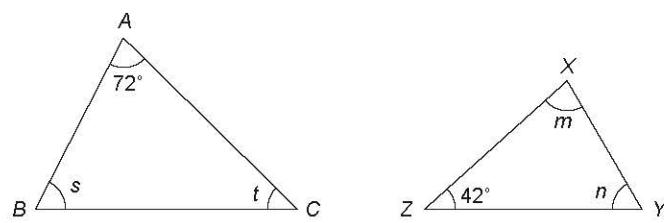


Find the unknowns in the following figures. (1–5)

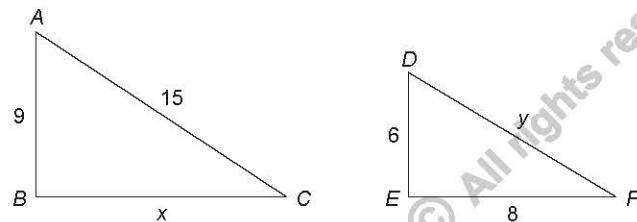
1. $\Delta ABC \sim \Delta DEF$



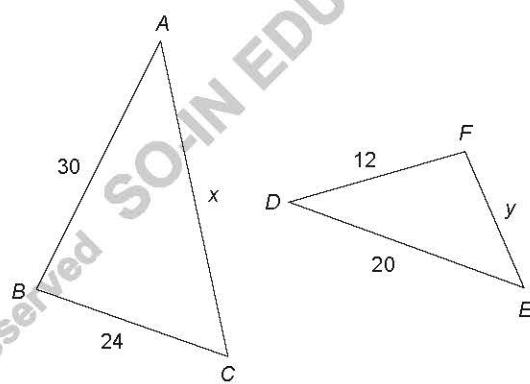
2. $\Delta ABC \sim \Delta XYZ$



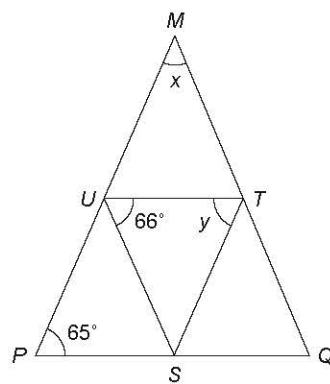
3. $\Delta ABC \sim \Delta DEF$



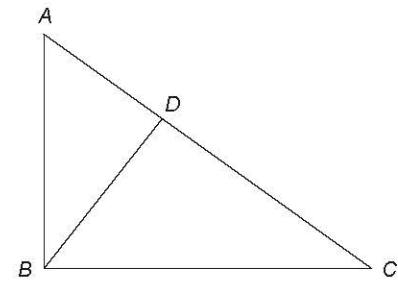
4. $\Delta ABC \sim \Delta DFE$



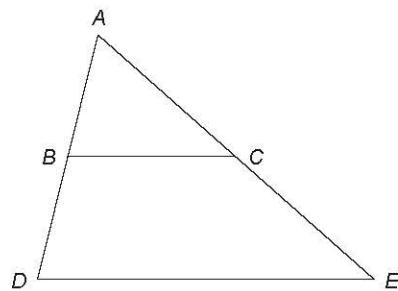
5. $\Delta MPQ \sim \Delta STU$



6. Given that $\triangle ABC \sim \triangle BDC$, if $\angle ABC = 90^\circ$, $BD = 5.7$ cm, $AD = 7$ cm and $DC = 9$ cm,
- find BC .
 - find AB .
 - find the area of $\triangle ABC$.



7. Given that $\triangle ABC \sim \triangle ADE$, $AB = 6$ cm, $BD = 6$ cm, $BC = 10$ cm and $CE = 7$ cm.
- Find AC .
 - Find DE .



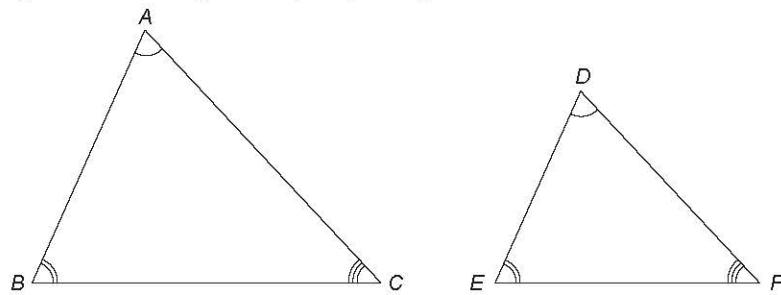


B

Q **Reminder**...

1. AAA

3 pairs of corresponding angles equal.

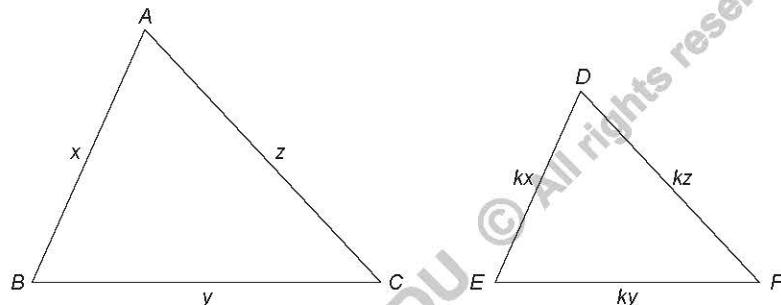


$$\Delta ABC \sim \Delta DEF \text{ (AAA)}$$

[(AA) or (equiangular) may be used instead]

2. 3 sides prop.

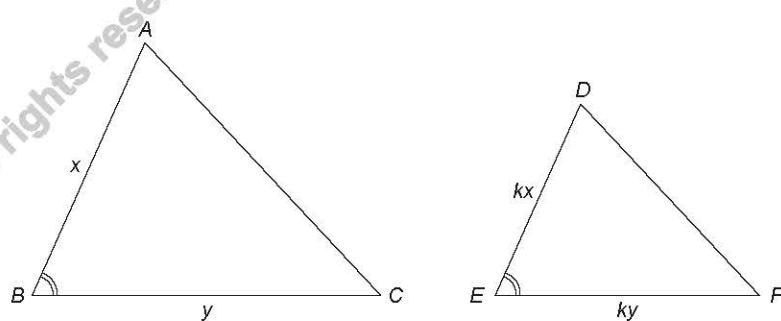
3 pairs of corresponding sides are proportional.



$$\Delta ABC \sim \Delta DEF \text{ (3 sides prop.)}$$

3. 2 sides prop. inc. \angle equal

2 pairs of corresponding sides are proportional and the pair of included angle equal.

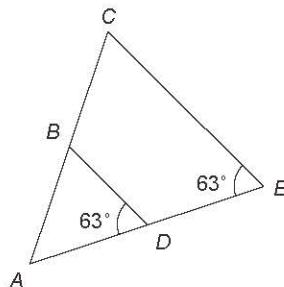


$$\Delta ABC \sim \Delta DEF \text{ (2 sides prop., inc. } \angle\text{)}$$

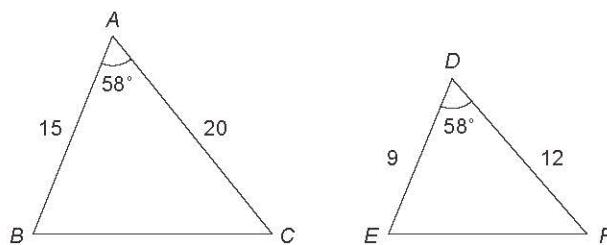
([(ratio of 2 sides inc. \angle) may be used instead])



- Example** 1. (a) Determine the following pair ΔABD and ΔACE are similar or not. If yes, give reason.
 (b) How is ΔABD transformed to be ΔACE ?



2. Show that $\Delta ABC \sim \Delta DEF$.



- Solution** 1. (a) $\angle ADB = \angle AEC = 63^\circ$ (given)

$\angle A = \angle A$ (common)

By \angle sum of Δ , $\angle ABD = \angle ACE$

$\therefore \Delta ABD \sim \Delta ACE$ (AAA)

- (b) ΔABD is enlarged through A to obtain ΔACE .

2. $\angle A = \angle D = 58^\circ$ (given)

$$\frac{AB}{AC} = \frac{15}{20}$$

$$= \frac{3}{4}$$

$$\frac{DE}{DF} = \frac{9}{12}$$

$$= \frac{3}{4}$$

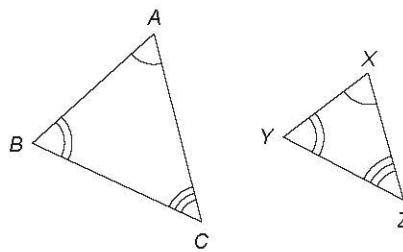
$$\text{Hence } \frac{AB}{AC} = \frac{DE}{DF}$$

$\Delta ABD \sim \Delta DEF$ (2 sides prop., inc. \angle)

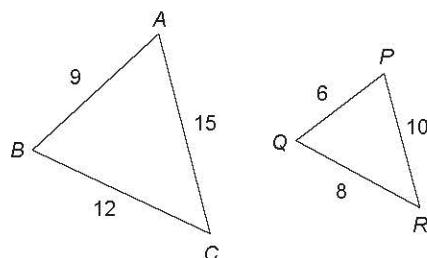


1. Name the following pairs of similar triangles with reasons.

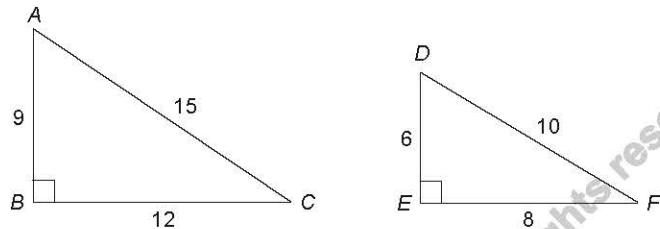
(a)



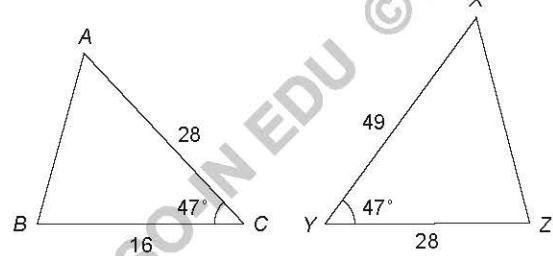
(b)



(c)

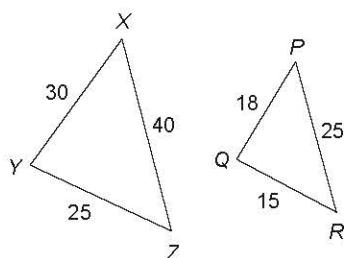


(d)



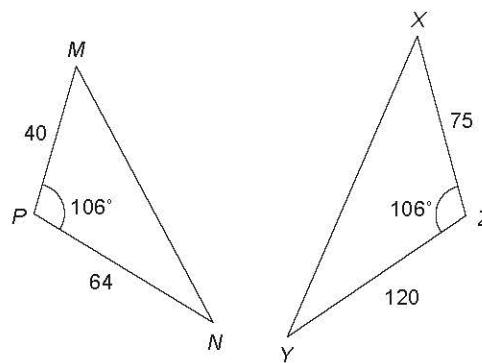
2. Determine the following pairs of triangles are similar or not. If yes, give reasons.

(a)

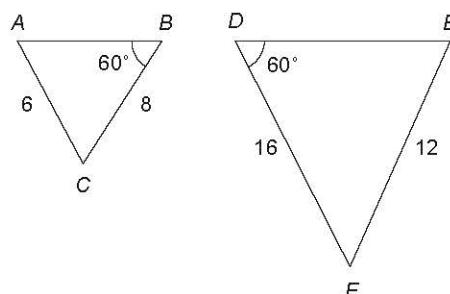




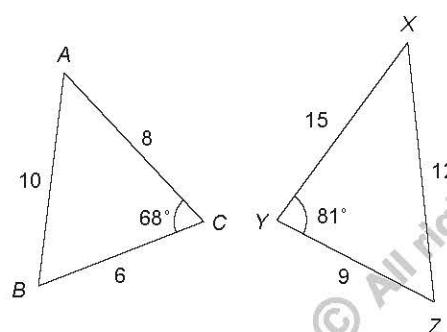
(b)



(c)

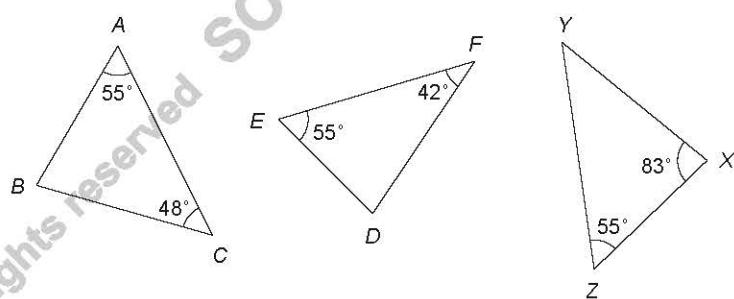


(d)

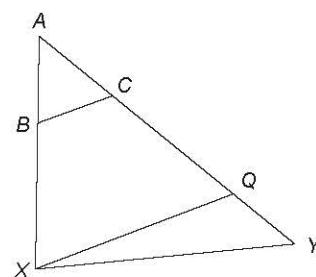


3. In each of the following, find the pair of similar triangles and give reasons.

(a)

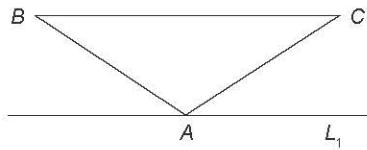


- (b) $AB = 4$, $BX = 8$, $AC = 3$, $CQ = 5$ and $QY = 10$





4. Determine whether $\triangle ABC$ and the images are similar triangles or not if the image undergoes the following transformations.

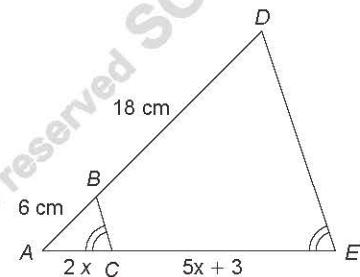


- (a) Translate $\triangle ABC$ downward
- (b) Reflect $\triangle ABC$ across L_1
- (c) Enlarge $\triangle ABC$ 2 times
- (d) Rotate $\triangle ABC$ 90° anti-clockwise about A

C

Example Refer to the figure below,

- (a) find a pair of similar triangles and give reasons.
- (b) solve for x .
- (c) express BC in terms of DE .



Solution (a) $\angle ACB = \angle AED$ (given)

$\angle A = \angle A$ (common)

$\therefore \triangle ACB \sim \triangle AED$ (AAA)

$$(b) \frac{AB}{AC} = \frac{AD}{AE} \text{ (corr. sides, } \sim \text{ } \Delta\text{s)}$$

$$\frac{6}{2x} = \frac{6 + 18}{2x + 5x + 3}$$

$$\frac{6}{24} = \frac{2x}{7x + 3}$$

$$\frac{1}{4} = \frac{2x}{7x + 3}$$

$$7x + 3 = 8x$$

$$x = 3$$

$$(c) \frac{BC}{AB} = \frac{DE}{AD} \text{ (corr. sides, } \sim \text{ } \Delta\text{s)}$$

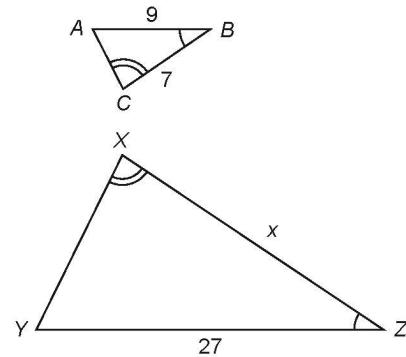
$$\frac{BC}{6} = \frac{DE}{6 + 18}$$

$$BC = \frac{1}{4}DE$$



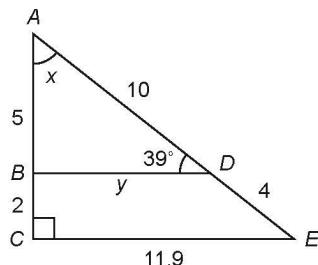
1. Refer to the figure below,

- (a) prove that $\triangle ABC \sim \triangle YZX$.
 (b) find x .

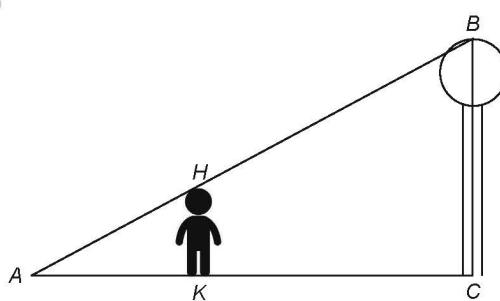


2. Refer to the figure below,

- (a) name a pair of similar triangles with reason.
 (b) find x and y .



3. Fiona is standing on the ground to observe the street lamp. The situation is captured by painter Jimmy. He wants to find the height of street lamp by the height of Fiona. Given that $\triangle AHK \sim \triangle ABC$, the distance between Fiona and the street lamp is 22 m and the distance between point A and Fiona is 13 m. If the height of Fiona is 1.3 m, find the height of street lamp.





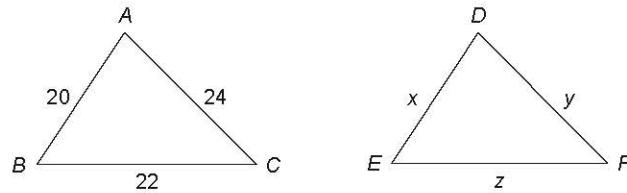
Homework Question

Lesson 1

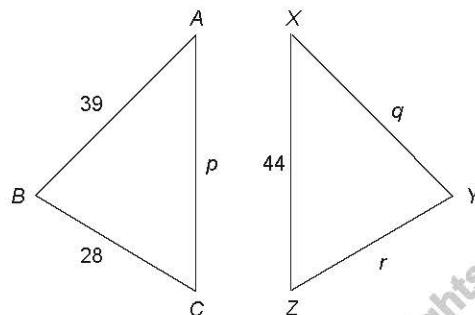
Section 1 Congruent triangles

- A** Find the unknowns in the following figures. (1–5)

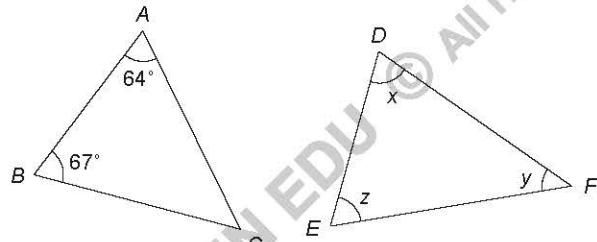
1. $\Delta ABC \cong \Delta DEF$.



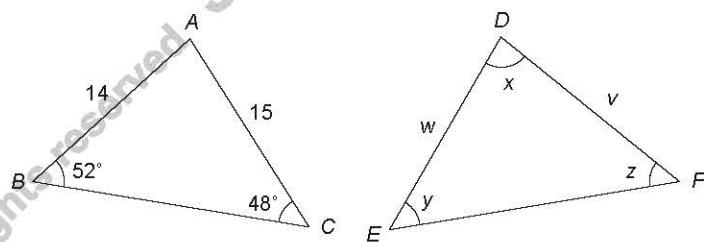
2. $\Delta ABC \cong \Delta XYZ$.



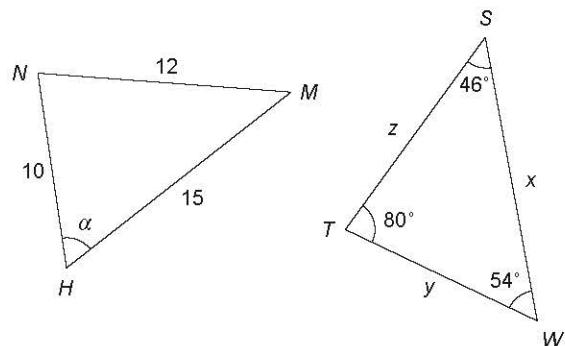
3. $\Delta ABC \cong \Delta DEF$



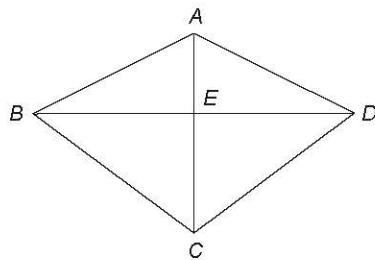
4. $\Delta ABC \cong \Delta DFE$



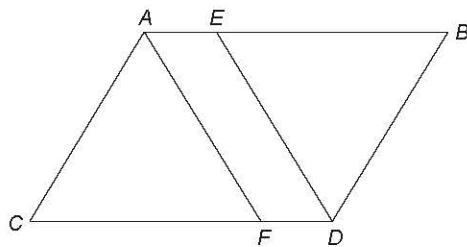
5. $\Delta MNH \cong \Delta STW$



6. Given that $\triangle ABC \cong \triangle ADC$ and $\triangle BEC \cong \triangle DEC$, if $\angle AEB = \angle AEC = 90^\circ$, $\angle AC = 11.5$ cm and $\angle BD = 16.4$ cm, find the area of $\triangle ACD$.

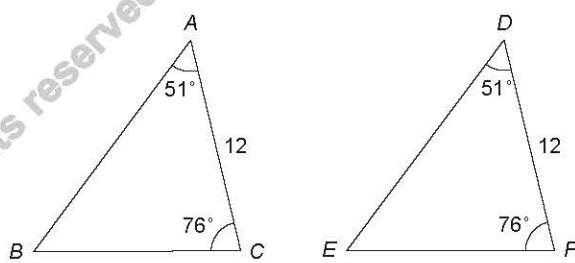


7. Given that $ABDC$ is a parallelogram and $\triangle ACF \cong \triangle DBE$. If $\triangle ACF$ is an equilateral triangle, $AF = 13$ cm and the perimeter of parallelogram $AEDF = 34$ cm,
- find AE .
 - find the perimeter of parallelogram $ABDC$.

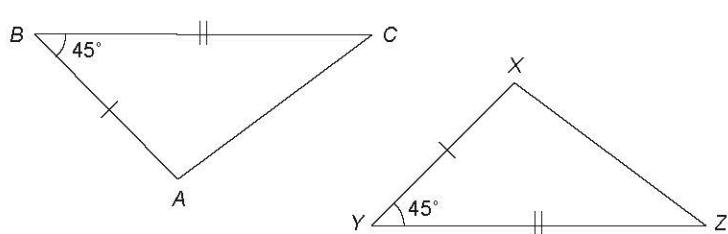


- B**
- (a) Construct a $\triangle XYZ$ by ruler and protractor, with $\angle XYZ = 50^\circ$, $XY = 8$ cm and $YZ = 8$ cm.
(b) Measure $\angle XZY$.
 - Construct a $\triangle HKT$ by ruler and protractor, with $\angle KHT = 30^\circ$, $HK = 6$ cm and $KT = 3$ cm.
 - Name the following pairs of congruent triangles with reasons.

(a)

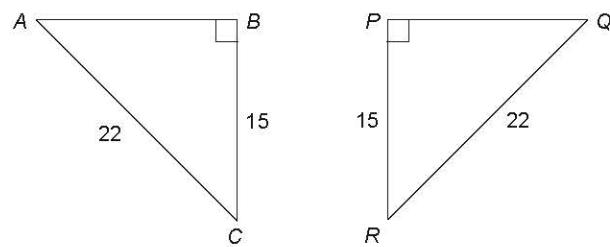


(b)

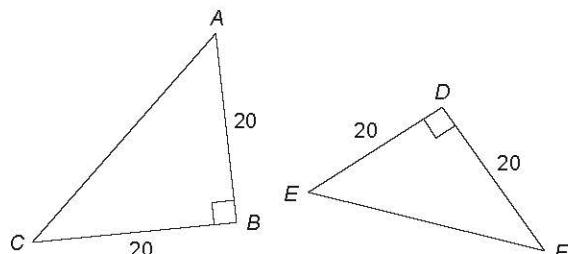




(c)

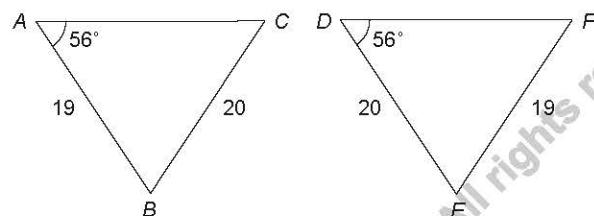


(d)

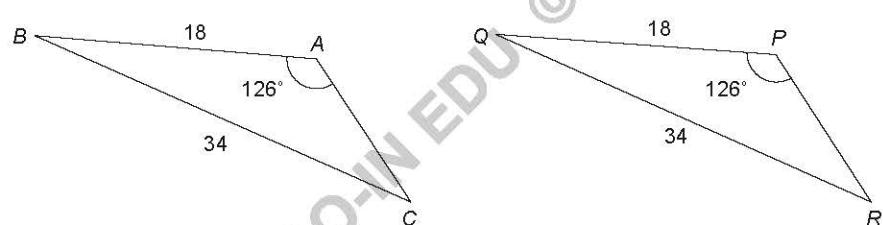


4. Determine whether the following pairs of triangles are congruent or not. If yes, give reasons.

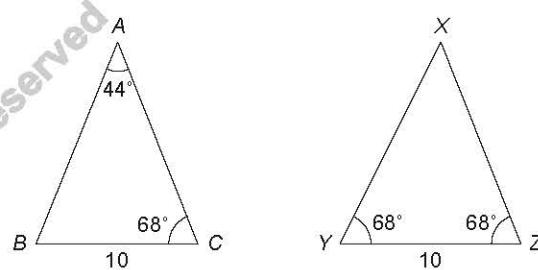
(a)



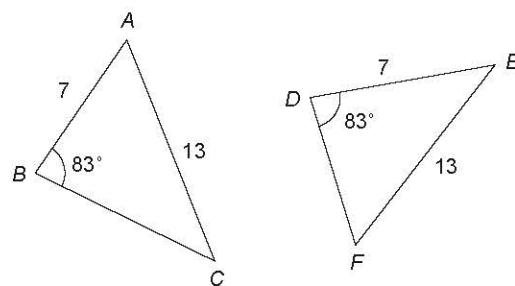
(b)



(c)

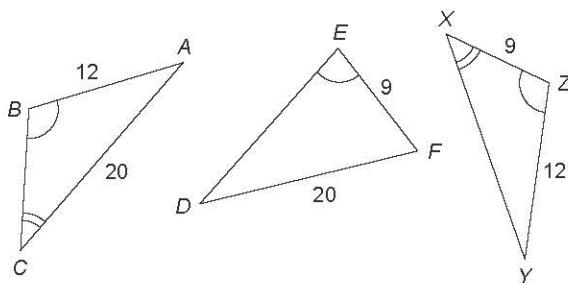


(d)

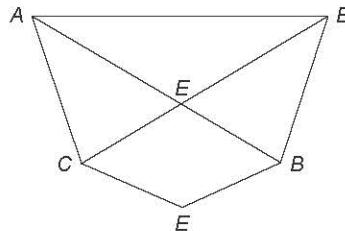


5. In each of the following, find the pair of congruent triangles and give reasons.

(a)

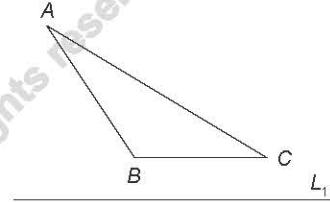


(b) $AC = BD$, $AE = BE$, $\angle AEC = \angle BED$ and $\angle ACE = \angle BDE$



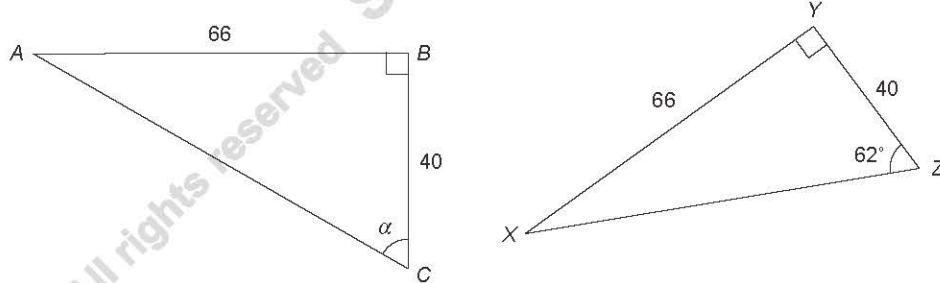
6. Determine whether $\triangle ABC$ and the images are congruent triangles or not if the image undergoes the following transformations.

- (a) Translate $\triangle ABC$ downward
- (b) Reflect $\triangle ABC$ across L_1
- (c) Reduce $\triangle ABC$ by $\frac{1}{2}$ across L_1
- (d) Rotate $\triangle ABC$ 90° clockwise through B



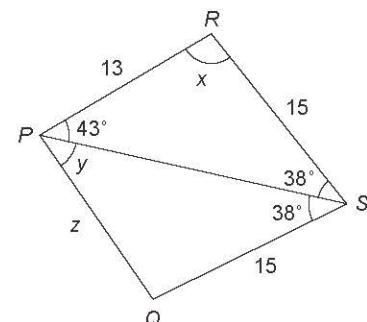
- C 1. Refer to the figure below,

- (a) prove that $\triangle ABC \cong \triangle XYZ$.
- (b) find α .



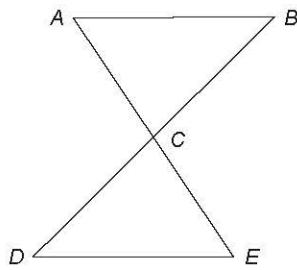
2. Refer to the figure below,

- (a) name a pair of congruent triangles with reason.
- (b) find all the unknowns.





3. In the figure below, $\angle A = \angle E$ and $\angle B = \angle D$, $AB = DE = 10$ cm, $AC = 8$ cm and $CD = 9$ cm.
- (a) Write down a pair of congruent triangles and state the reason.
- (b) Find the perimeter of the figure.



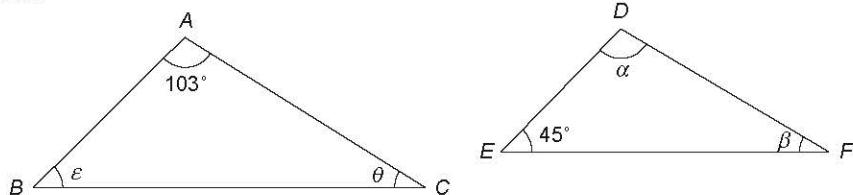


Lesson 2

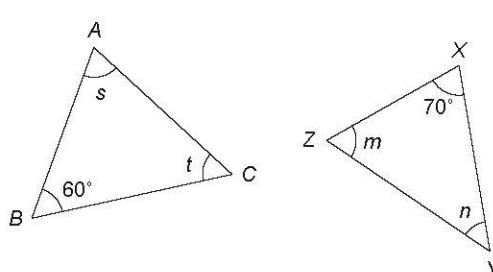
Section 2 Similar triangles

A Find the unknowns in the following figures. (1–5)

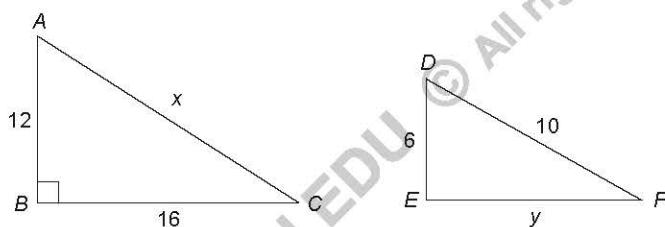
1. $\Delta ABC \sim \Delta DEF$



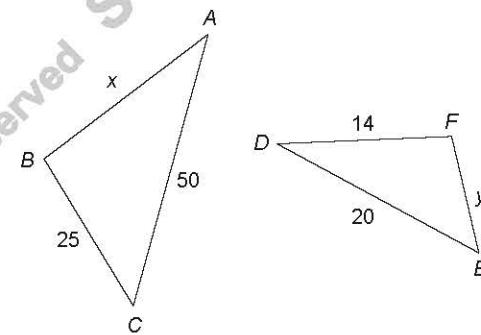
2. $\Delta ABC \sim \Delta XYZ$



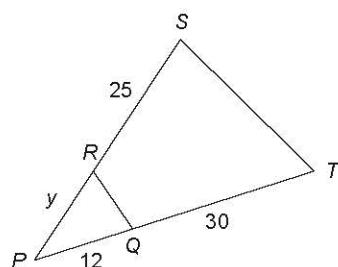
3. $\Delta ABC \sim \Delta DEF$



4. $\Delta ABC \sim \Delta DFE$

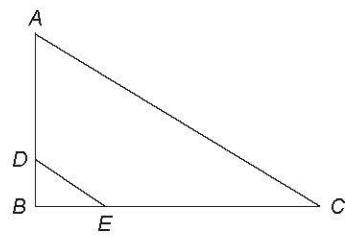


5. $\Delta PRQ \sim \Delta PST$





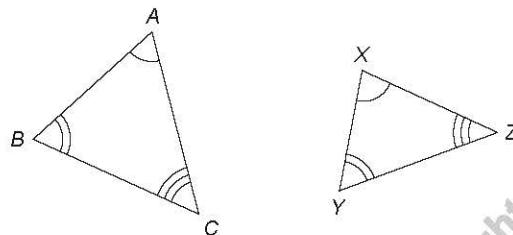
6. Given that $\Delta ABC \sim \Delta DBE$, if $AC = 28$ cm, $DE = 7$ cm, $BE = 5$ cm and $BC = 20$ cm,
- find AB .
 - find EC .
 - find the perimeter of ΔABC .



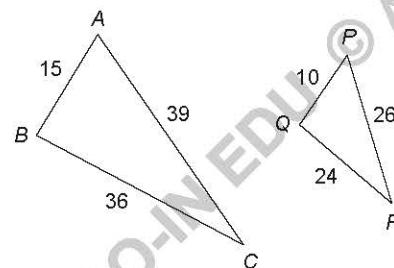
7. Given that $\Delta ABC \sim \Delta TSP$, $\angle B = 63^\circ$, $BC = 14$ cm, $AB = 8$ cm and $SP = 8$ cm.
- Find $\angle TSP$.
 - Find TS .

- B** 1. Name the following pairs of similar triangles with reasons.

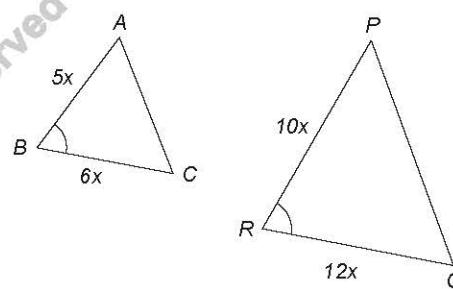
(a)



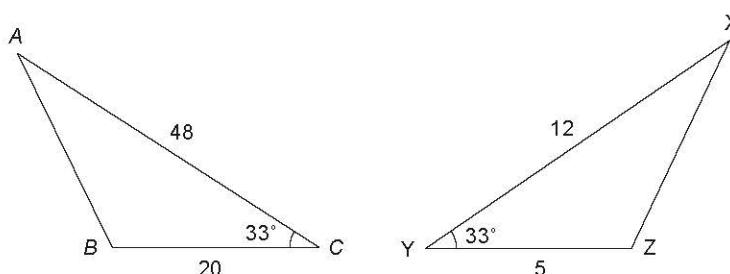
(b)



(c)

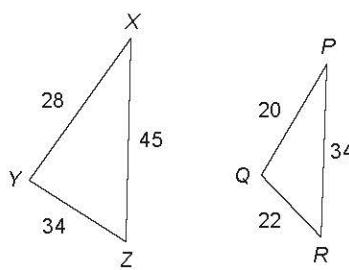


(d)

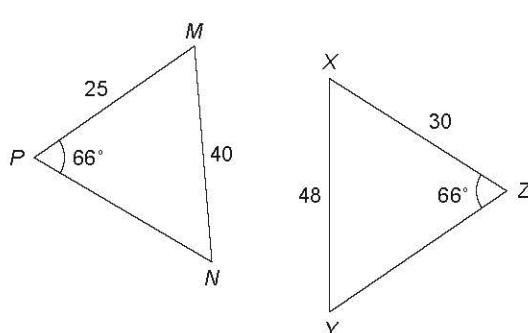


2. Determine the following pairs of triangles are similar or not. If yes, give reasons.

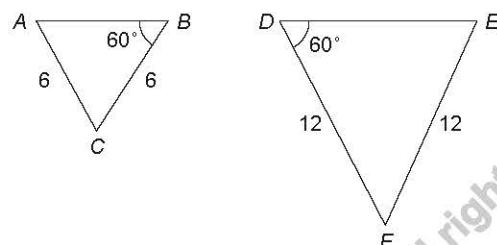
(a)



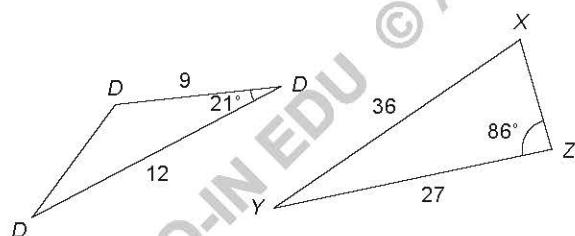
(b)



(c)

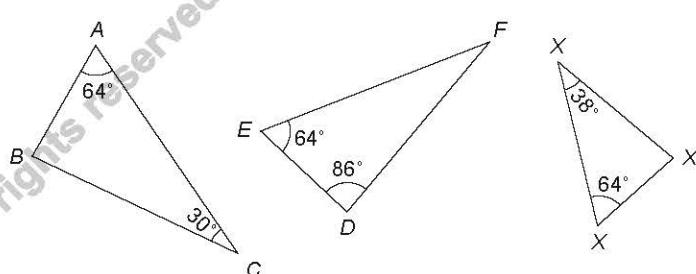


(d)

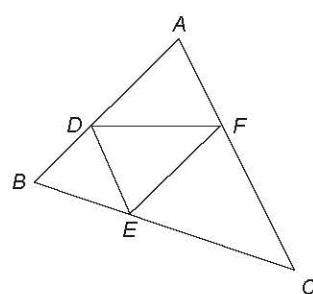


3. In each of the following, find the pair of similar triangles and give reasons.

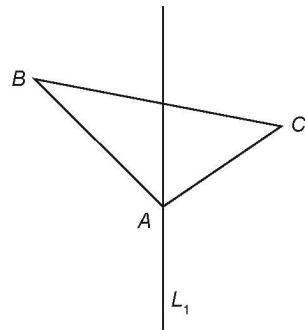
(a)



- (b) $AD = 16$, $BD = 8$, $BE = 6$, $EC = 12$, $AF = 10$ and $FC = 24$.

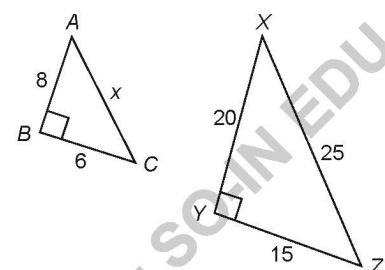


4. Determine whether $\triangle ABC$ and the images are similar triangles or not if the image undergoes the following transformations.
- Translate $\triangle ABC$ to the right
 - Reflect $\triangle ABC$ across L_1
 - Enlarge $\triangle ABC$ 2 times
 - Rotate $\triangle ABC$ 90° clockwise about B



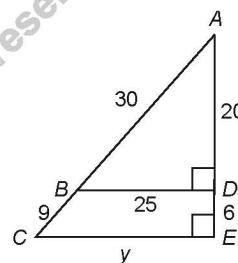
- C 1. Refer to the figure below,

- prove that $\triangle ABC \sim \triangle XYZ$.
- find x .



2. Refer to the figure below,

- name a pair of similar triangles with reason.
- find y .



3. Fiona is standing on the ground to observe the street lamp. Given that $\triangle AHK \sim \triangle ABC$, the distance between Fiona and the street lamp is 22 m and the distance between point A and Fiona is 13 m. The height of Fiona is 1.3 m, the height of street lamp is 3.5 m. If Fiona's friend Lily stand next to Fiona so that the distance between Lily and the street lamp is x m, where $x < 22$, express the height of Lily in terms of x .

