Spectral Curves, their trivialization and the role of Weyl groups

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The main purpose of this note is to present an algorithm for identifying weights of a representation ρ with the sheets $\{x_i\}_i$ of a spectral curve Σ_ρ above a generic point z of the base C.

1 Preliminaries

This kind of problem arises immediately in concrete approaches to the study of these covers, where one makes a choice of branch cuts, and trivializes the cover. The input data is given simply by a set of complex numbers $\{x_i\}$, marking the positions of the sheets in the fiber $\mathbb{C}$ above z, the task is then to identify consistently each of these with a weight. Once this is accomplished for a single point (the basepoint of the trivialization, in particular), the trivialization then assigns a weight to each point of a sheet uniquely.

We will discuss algorithms for accomplishing this task for all representations of ADE Lie groups (with the exception of E_8, although out methods could conceivably generalize to that group as well). A basic fact which holds in general is that the positions of the sheets are linear functions of the weights.

$$x_i = \langle \mu_i, \varphi(z) \rangle \tag{1}$$

This entails a great simplification: for each Lie algebra, it is sufficient to solve the problem for any (nontrivial) representation, the solution will then extend to all other representations by linearity.

A bit more concretely, if we solve the problem of identifying the weights of a certain rep ρ with the sheets of the cover Σ_ρ, we may then pick a basis for $\mathfrak{t}^*$ among the $\{\mu_i\}_i$, and expand all the weights of any other rep ρ' in that basis

$$\mu'_j = \sum_i c_{ij} \mu_i \tag{2}$$

this data being easily recovered from the knowledge of the weight systems themselves. By linearity, this gives the desired identification of the sheets of Σ'_ρ as

$$x'_j = \sum_i c_{ij} x_i \tag{3}$$

where x_i are the sheets of Σ_ρ.

2 A_n systems

We choose the first fundamental representation, and label the weights $\mu_1 \dots \mu_{n+1}$. The residual gauge freedom after diagonalizing $\varphi(z)$ consists in the action of the Weyl group $W = S_{n+1}$, which permutes all sheets $x_i \mapsto x_{\sigma(i)}$. Given this freedom, we can choose to assign any weight to any sheet, as long as no repetitions are made.

3 D_n systems

We choose again the first fundamental representation, a.k.a. the vector rep. This consists of $2n$ weights, subject to the relations

$$\mu_i + \mu_{i+n} = 0 \qquad (i \in \mathbb{Z}_{2n}) \,. \tag{4}$$

The Weyl group is $H_{n-1} \rtimes S_n$, of order $2^{n-1}n!$. Here $H_{n-1} \subset \mathbb{Z}_2^n$ is the kernel of the product homomorphism $\{\epsilon_1, \dots, \epsilon_n\} \to \epsilon_1 \dots \epsilon_n$ where $\epsilon_i = \pm 1$. In other words, H_{n-1} is the subgroup of elements with an even number of -1's.

The action of W is as follows: it permutes all the pairs

$$\{\mu_i, \mu_{i+n}\} \mapsto \{\mu_{\sigma(i)}, \mu_{\sigma(i)+n}\} \qquad \sigma \in S_n \tag{5}$$

and independently switches an *even* number of signs

$$\mu_i \mapsto -\mu_i = \mu_{i+n} \,. \tag{6}$$

The identification of sheets and weights can be carried out as follows: first identify the n pairs of opposite sheets such that $x + x' = 0$. Then, the permutation symmetry S_n allows us to match any pair of opposite sheets with any pair of opposite weights. The problem thus boils down to identifying consistently a 'positive' and a 'negative' sheet in *each* pair.

The choices are not independent: the H_{n-1} freedom however allows us to choose freely the positive sheet from the first $n-1$ pairs. Only the choice of positive vs negative within the last pair

$$\begin{matrix} x_n \leftrightarrow \mu_n \\ x_{2n} \leftrightarrow \mu_{2n} \end{matrix} \qquad \text{vs} \qquad \begin{matrix} x_n \leftrightarrow \mu_{2n} \\ x_{2n} \leftrightarrow \mu_n \end{matrix} \tag{7}$$

is constrained. This can be seen as follows: suppose we knew a 'reference' weight-sheet identification that works, then we can compare it to ours by first permuting the sheet pairs suitably, then by 'flipping' the positive/negative role within each sheet pair, for the first $n-1$ pairs. If we had to perform an even number of switches, then we should make the same positive/negative identification for the last pair as in the reference one; if instead we had to perform an odd number of switches, we should invert the last identification with respect to the reference one.

Given however that we don't have a reference identification to compare with, we cannot determine how to make the positive/negative identification in the last pair. All we can do is evaluate the consequence of a wrong choice. As it turns out,the wrong choice corresponds to acting with the outer automorphism which exchanges the spinor weights in the D_n Dynking diagram. We can therefore choose the identification for the last pair at will, the price for such "unconscious" choice will be that we cannot tell whether a given spinor cover Σ_ρ corresponds to one spinor rep, or the other.

This last statement requires a clarification: in *loom* we currently *build a ρ-cover Σ_ρ on the vector-cover*. This means that, if we wish to study a spectral network for a certain spinor cover Σ_ρ, we start by studying the corresponding vector-rep cover Σ_{ρ_1}; then we use the above algorithm to identify sheets of this with the weights of ρ_1, and finally we use the linear relation between the weights of ρ and those of ρ_1 to *construct* the sheets of Σ_ρ from those of Σ_{ρ_1}. So, if we "switch" the identification of positive/negative in the last pair of sheets for Σ_{ρ_1}, we end up switching the spinor weights; in turn, this affects the linear map between the weights of ρ_1 and weights of ρ, which we will use to construct the spinor cover. In this sense, we may end up building either spinor cover with our procedure, and we cannot tell which one we actually build.

4 E_n systems

4.1 E_6

We choose to work with the ρ_1 representation, with Dynkin indices $(1,0,0,0,0,0)$, of dimension 27. The Weyl group is of order $51840 \ll 27!$. The precise goal will be to "label" all the sheets $x_0 \dots x_{26}$ so that $x_i = \langle \mu_i, \varphi(z) \rangle$.

As a preliminary, we state the following empyrical observation. The 27 weights of ρ_1 can be arranged into *null triples*, obeying

$$\mu_i + \mu_j + \mu_k = 0 \tag{8}$$

There are 45 such triples (up to $S_3^{\times 45}$ permutations within each triple), each weight appears in exactly 5 of them. Similarly, by linearity, the sheets must also organize into 45 null triples, and each sheet will feature in exactly 5 of them.

We can now start identifying sheets and weights. Choose any sheet, then by using part of the Weyl freedom (to be quantified more precisely below) we can label it by x_0, thus identifying $\mu_0 \leftrightarrow x_0$. Then consider the quintet Q_0 of null triples to which μ_0 belongs, they are:

$t_1^{(0)}$	$t_2^{(0)}$	$t_3^{(0)}$	$t_4^{(0)}$	$t_5^{(0)}$
$(0,12,26)$	$(0,15,26)$	$(0,17,24)$	$(0,19,23)$	$(0,21,22)$

(9)

In notation where (i,j,k) stands for (μ_i, μ_j, μ_k); here we stick to weight-labels as they are given by the standard ordering employed by SAGE. In appendix we provide explicit expressions.

Our choice to identify x_0, μ_0 cost us part of the Weyl freedom, but we still have the remaining freedom given by the stabilizer subgroup

$$W_0 := \{ w \in W \mid w(\mu_0) = \mu_0 \} \tag{10}$$

It's a standard fact about Weyl groups that $W_0 \simeq W(D_5)$: this follows from the fact that μ_0 can be taken to be the first fundamental weight $\mu_0 = \omega_1$, and that the stabilizer of a dominant weight $\mu = \sum_i m_i \omega_i$ in W is generated by the simple reflections for which $m_i = 0$ [1].

[1] *C.Procesi, "Lie Groups: An Approach through Invariants and Representations", p.324.*

The order of W_0 is 1920. The action of W_0 on the *quintet of ordered triples* $(t_1^{(0)}, \ldots, t_5^{(0)})$ gives 1920 distinct quintets of ordered triples. For clarity, two quintets

$$
\begin{array}{c}
(t_1^{(0)}, t_2^{(0)}, t_3^{(0)}, t_4^{(0)}, t_5^{(0)}) \\
(t_2^{(0)}, t_1^{(0)}, t_3^{(0)}, t_4^{(0)}, t_5^{(0)})
\end{array}
\tag{11}
$$

are considered different. Moreover, also two quintets differing by a triplet changing from $(0, 12, 26)$ to $(0, 26, 12)$ are considered different in our count. Note that the index 0 is left invariant by W_0, by definition.

In fact $1920 = 5! \cdot 2^4$ is the number of such ordered quintets obtained by considering all their permutations by S_5, and by flipping an *even* number of pairs within each triple (in accordance with the fact that $W_0 = H_4 \rtimes S_5 \simeq W(D_5)$).

This freedom means the following. Consider the null triples of *sheets* in which x_0 features. This singles out 5 pairs of sheets (x_i, x_j) such that $x_i + x_j = -x_0$. We choose to label the sheets of those five triples as follows:

$$
\tilde{Q}_0 := \left\{(x_0, x_{12}, x_{26})\ (x_0, x_{15}, x_{26})\ (x_0, x_{17}, x_{24})\ (x_0, x_{19}, x_{23})\ (x_0, x_{21}, x_{22})\right\} \tag{12}
$$

where the identification of x_0 is fixed by our initial choice, and the W_0 gauge freedom accounts for possible permutations among the five pairs of other sheets, as well as for an *even* number of 'flips' of the pairs of sheets $x_i \leftrightarrow x_j$ within each triple (x_0, x_i, x_j). More precisely, we have the freedom to choose who is x_i vs x_j (where we associate $x_{i,j} \to \mu_{i,j}$), in 4 out of 5 pairs, but the last choice is constrained by the first four ones (the reasoning is the same as for D_n, and likewise a wrong choice on the last pair corresponds to the $\mathbb{Z}_2$ outer automorphism of the Lie algebra).

Having identified the first 11 sheets, the subgroup of W which stabilizes all of our choices is precisely of order 1. Thus, the Weyl freedom has been used exhaustively at this point.

How to identify the remaining 16 sheets? We have identified the 11 sheets

$$
x_0, x_{12}, x_{26}, x_{15}, x_{25}, x_{17}, x_{24}, x_{19}, x_{23}, x_{21}, x_{22} \tag{13}
$$

with the corresponding weights $(x_i \leftrightarrow \mu_i)$. We can therefore construct their quintets:

$$
\tilde{Q}_0, \tilde{Q}_{12}, \tilde{Q}_{26}, \tilde{Q}_{15}, \tilde{Q}_{25}, \tilde{Q}_{17}, \tilde{Q}_{24}, \tilde{Q}_{19}, \tilde{Q}_{23}, \tilde{Q}_{21}, \tilde{Q}_{22} \tag{14}
$$

Again, each quintet will contain 5 triples of the form (x_i, x_j, x_k) with x_i being one of the sheets we identified, and x_j, x_k being generally sheets that we have yet to identify with a weight.

Now, from the weight data, we know that each one of the missing weights has a unique pattern of whether it belongs or not to each of these quintets. The same must be true for the weights, by linearity [2], and therefore the problem of identifying the remaining weights and sheets is completely solved. We give below the data table: each row tells whether one of the missing weights belongs or not to the quintets which label the columns. A '0' stands for 'not contained', while a '1' stands for 'contained'. The same analysis can be carried out for the remaining sheets, since we know their coordinates, and we know the

[2] Expand a bit here on this assertion...

sheet quintets corresponding to the weight quintets $Q_i \leftrightarrow \tilde{Q}_i$. The requirement that the pattern in table of $(x_i, \tilde{Q}_j)$ matches with the pattern of the table (μ_i, Q_j) uniquely fixes all the remaining x_i's.

	Q_0	Q_{12}	Q_{26}	Q_{15}	Q_{25}	Q_{17}	Q_{24}	Q_{19}	Q_{23}	Q_{21}	Q_{22}
μ_1	0	0	1	0	1	0	1	0	1	0	1
μ_2	0	0	1	0	1	0	1	1	0	1	0
μ_3	0	0	1	0	1	1	0	0	1	1	0
μ_4	0	0	1	0	1	1	0	1	0	0	1
μ_5	0	0	1	1	0	0	1	0	1	1	0
μ_6	0	0	1	1	0	0	1	1	0	0	1
μ_7	0	1	0	0	1	0	1	0	1	1	0
μ_8	0	0	1	1	0	1	0	0	1	0	1
μ_9	0	1	0	0	1	0	1	1	0	0	1
μ_{10}	0	0	1	1	0	1	0	1	0	1	0
μ_{11}	0	1	0	0	1	1	0	0	1	0	1
μ_{13}	0	1	0	0	1	1	0	1	0	1	0
μ_{14}	0	1	0	1	0	0	1	0	1	0	1
μ_{16}	0	1	0	1	0	0	1	1	0	1	0
μ_{18}	0	1	0	1	0	1	0	0	1	1	0
μ_{20}	0	1	0	1	0	1	0	1	0	0	1

(15)

4.2 E_7

We choose to work with the ρ_7 representation, with Dynkin indices $(0,0,0,0,0,0,1)$, of dimension 56. The Weyl group is of order $2903040 \ll 56!$.

The 56 weights of ρ_7 can be arranged into 28 *null pairs*, obeying

$$\mu_i + \mu_j = 0 \tag{16}$$

The 56 weights of ρ_7 can be also arranged into *null quartets*, obeying

$$\mu_i + \mu_j + \mu_k + \mu_l = 0 \tag{17}$$

There are 1008 such quartets, each weight appears in exactly 72 of them. But, if we exclude the 378 quartets obtained from combining null pairs, we are left with 630 genuine null quartets, and each weight appears in 45 of them.

Similarly, by linearity, the sheets must also organize into 28 null pairs, as well as 630 "genuine" null quartets, and each sheet will feature in exactly 45 of them.

Choose a labeling for the sheets $x_0, \ldots, x_{55}$, and make an ansatz by identifying $\mu_0 \leftrightarrow x_0$. The Weyl group is

$$W(E_7) = \mathbb{Z}_2 \times PSp_6(2) \tag{18}$$

where (CHECK) the $\mathbb{Z}_2$ takes $\mu \to -\mu\,,\ \forall \mu \in \mathfrak{t}^*$.

We consider the 45-plet Q_0 of null quartets to which μ_0 belongs, they are:

$$Q_0 \ : \quad \begin{array}{c|c|c} q_1^{(0)} & \ldots & q_{45}^{(0)} \\ \hline (0,2,18,54) & \ldots & (0,40,52,53) \end{array} \tag{19}$$

In notation where (i,j,k,l) stands for $(\mu_i,\mu_j,\mu_k,\mu_l)$. It turns out that all the weights appearing in Q_0 come from all the 28 distinct null pairs. To be explicit, these 28 weights are:

$$\mathcal{W}_0 = \{0,2,18,54,20,34,25,30,40,50,48,55,3,13,19,21,29,32,7,27,43,51,53,49,15,42,22,52\} \tag{20}$$

On the side of sheets, having fixed the correspondence $\mu_0 \leftrightarrow x_0$, we will obtain by the same procedure an unordered set of $28-1=27$ sheets, which we call $\mathcal{S}_0$. The next task is to understand how to identify each sheet in $\mathcal{S}_0$ with a weight from $\mathcal{W}_0 - \{0\}$.

From the 45 quartets in Q_0 we can extract 45 triplets, in the obvious way: these will be (i,j,k) such that $\mu_i+\mu_j+\mu_k=-\mu_0$. Now, taken any $i \in \mathcal{W}_0$, with $i \neq 0$, it turns out that it appears in *exactly* 5 triplets. The same story, by linearity, must be true of the sheets x_i: the sheets in $\mathcal{S}_0$ must arrange in triples (x_i,x_j,x_k) such that $x_i+x_j+x_k=-x_0$; moreover there will be 45 triples, and each x_i will appear in precisely 5 of them.

The similarity with the previous section is now evident: in fact, the stabilizer of μ_0 is $W_0 \simeq W(E_6)$, following a reasoning analogous to that employed in the previous section (where the stabilizer was found to be $W(D_5)$).

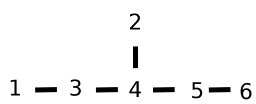
So we know from the E_6 case how to proceed now: the residual Weyl symmetry can be used to uniquely fix all the 27 weights/sheet pairs in $\mathcal{W}_0 - \{0\}$, precisely following the algorithm devised in the previous section (i.e., choose the subgroup of W_0 which stabilizes, say, μ_2, it will be of order 1920, etc etc). The remaining 28 sheets are related to these by the $\mu \to -\mu$ $\mathbb{Z}_2$ symmetry, the same relation extends by linearity to the sheets $x \to -x$. This completely fixes the sheet/weight identification.

A Explicit conventions for E_6

The Cartan Matrix is

$$\begin{pmatrix} 2 & 0 & -1 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 \\ -1 & 0 & 2 & -1 & 0 & 0 \\ 0 & -1 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix} \tag{21}$$

Thus the Dynkin diagram and the corresponding simple roots read



The weights of the representation we study read, explicitly

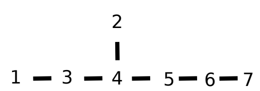
$$
\begin{array}{cc}
0 & (0,0,0,0,0,-2/3,-2/3,2/3) \\
1 & (-1/2,1/2,1/2,1/2,1/2,-1/6,-1/6,1/6) \\
2 & (1/2,-1/2,1/2,1/2,1/2,-1/6,-1/6,1/6) \\
3 & (1/2,1/2,-1/2,1/2,1/2,-1/6,-1/6,1/6) \\
4 & (-1/2,-1/2,-1/2,1/2,1/2,-1/6,-1/6,1/6) \\
5 & (1/2,1/2,1/2,-1/2,1/2,-1/6,-1/6,1/6) \\
6 & (-1/2,-1/2,1/2,-1/2,1/2,-1/6,-1/6,1/6) \\
7 & (1/2,1/2,1/2,1/2,-1/2,-1/6,-1/6,1/6) \\
8 & (-1/2,1/2,-1/2,-1/2,1/2,-1/6,-1/6,1/6) \\
9 & (-1/2,-1/2,1/2,1/2,-1/2,-1/6,-1/6,1/6) \\
10 & (1/2,-1/2,-1/2,-1/2,1/2,-1/6,-1/6,1/6) \\
11 & (-1/2,1/2,-1/2,1/2,-1/2,-1/6,-1/6,1/6) \\
12 & (0,0,0,0,1,1/3,1/3,-1/3) \\
13 & (1/2,-1/2,-1/2,1/2,-1/2,-1/6,-1/6,1/6) \\
14 & (-1/2,1/2,1/2,-1/2,-1/2,-1/6,-1/6,1/6) \\
15 & (0,0,0,1,0,1/3,1/3,-1/3) \\
16 & (1/2,-1/2,1/2,-1/2,-1/2,-1/6,-1/6,1/6) \\
17 & (0,0,1,0,0,1/3,1/3,-1/3) \\
18 & (1/2,1/2,-1/2,-1/2,-1/2,-1/6,-1/6,1/6) \\
19 & (0,1,0,0,0,1/3,1/3,-1/3) \\
20 & (-1/2,-1/2,-1/2,-1/2,-1/2,-1/6,-1/6,1/6) \\
21 & (-1,0,0,0,0,1/3,1/3,-1/3) \\
22 & (1,0,0,0,0,1/3,1/3,-1/3) \\
23 & (0,-1,0,0,0,1/3,1/3,-1/3) \\
24 & (0,0,-1,0,0,1/3,1/3,-1/3) \\
25 & (0,0,0,-1,0,1/3,1/3,-1/3) \\
26 & (0,0,0,0,-1,1/3,1/3,-1/3)
\end{array}
\tag{22}
$$

B Explicit conventions for E_6

The Cartan Matrix is

$$
\begin{pmatrix}
2 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & -1 & 0 & 0 & 0 \\
-1 & 0 & 2 & -1 & 0 & 0 & 0 \\
0 & -1 & -1 & 2 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 2 & -1 & 0 \\
0 & 0 & 0 & 0 & -1 & 2 & -1 \\
0 & 0 & 0 & 0 & 0 & -1 & 2
\end{pmatrix}
\tag{23}
$$

Thus the Dynkin diagram and the corresponding simple roots read



The weights of the representation we study read, explicitly

$$\begin{gathered}
(-1/2,-1/2,1/2,1/2,-1/2,-1/2,0,0)\\
(-1/2,1/2,1/2,1/2,-1/2,1/2,0,0)\\
(0,0,0,0,0,1,1/2,-1/2)\\
(-1/2,1/2,1/2,-1/2,1/2,1/2,0,0)\\
(0,0,0,0,-1,0,-1/2,1/2)\\
(-1/2,1/2,-1/2,1/2,-1/2,-1/2,0,0)\\
(0,0,1,0,0,0,1/2,-1/2)\\
(-1/2,-1/2,-1/2,-1/2,1/2,1/2,0,0)\\
(1/2,1/2,1/2,1/2,-1/2,-1/2,0,0)\\
(1/2,-1/2,1/2,-1/2,-1/2,-1/2,0,0)\\
(0,0,0,1,0,0,1/2,-1/2)\\
(0,0,0,0,0,-1,-1/2,1/2)\\
(-1/2,1/2,1/2,1/2,1/2,-1/2,0,0)\\
(1/2,-1/2,-1/2,-1/2,-1/2,1/2,0,0)\\
(0,0,1,0,0,0,-1/2,1/2)\\
(1/2,-1/2,1/2,-1/2,1/2,1/2,0,0)\\
(-1/2,-1/2,-1/2,-1/2,-1/2,-1/2,0,0)\\
(-1,0,0,0,0,0,1/2,-1/2)\\
(0,0,-1,0,0,0,-1/2,1/2)\\
(1,0,0,0,0,0,1/2,-1/2)\\
(0,1,0,0,0,0,-1/2,1/2)\\
(1/2,1/2,-1/2,1/2,-1/2,1/2,0,0)\\
(0,0,0,-1,0,0,1/2,-1/2)\\
(1/2,-1/2,-1/2,1/2,-1/2,-1/2,0,0)\\
(-1/2,-1/2,-1/2,1/2,1/2,-1/2,0,0)\\
(-1/2,1/2,-1/2,-1/2,1/2,-1/2,0,0)
\end{gathered} \tag{24}$$

$$
\begin{array}{c}
(0,-1,0,0,0,0,1/2,-1/2)\\
(1/2,1/2,1/2,-1/2,-1/2,1/2,0,0)\\
(1/2,-1/2,1/2,1/2,-1/2,1/2,0,0)\\
(0,0,-1,0,0,0,1/2,-1/2)\\
(1,0,0,0,0,0,-1/2,1/2)\\
(-1/2,-1/2,-1/2,1/2,-1/2,1/2,0,0)\\
(1/2,-1/2,-1/2,1/2,1/2,1/2,0,0)\\
(-1/2,-1/2,1/2,-1/2,-1/2,1/2,0,0)\\
(1/2,-1/2,-1/2,-1/2,1/2,-1/2,0,0)\\
(-1/2,-1/2,1/2,1/2,1/2,1/2,0,0)\\
(0,-1,0,0,0,0,-1/2,1/2)\\
(0,0,0,1,0,0,-1/2,1/2)\\
(0,0,0,0,0,-1,1/2,-1/2)\\
(-1/2,-1/2,1/2,-1/2,1/2,-1/2,0,0)\\
(1/2,1/2,-1/2,-1/2,-1/2,-1/2,0,0)\\
(-1,0,0,0,0,0,-1/2,1/2)\\
(-1/2,1/2,-1/2,-1/2,-1/2,1/2,0,0)\\
(0,1,0,0,0,0,1/2,-1/2)\\
(0,0,0,0,-1,0,1/2,-1/2)\\
(1/2,-1/2,1/2,1/2,1/2,-1/2,0,0)\\
(1/2,1/2,-1/2,-1/2,1/2,1/2,0,0)\\
(-1/2,1/2,1/2,-1/2,-1/2,-1/2,0,0)\\
(1/2,1/2,-1/2,1/2,1/2,-1/2,0,0)\\
(-1/2,1/2,-1/2,1/2,1/2,1/2,0,0)\\
(0,0,0,0,1,0,-1/2,1/2)\\
(1/2,1/2,1/2,1/2,1/2,1/2,0,0)\\
(0,0,0,0,0,1,-1/2,1/2)\\
(0,0,0,0,1,0,1/2,-1/2)\\
(1/2,1/2,1/2,-1/2,1/2,-1/2,0,0)\\
(0,0,0,-1,0,0,-1/2,1/2)
\end{array}
\tag{25}
$$