Some notes on the fibrations

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Some notes on the conventions used for fibration data within our code.

Contents

1 $SU(2)\ N_f = 1$

From section 2 of 9706145 (Bilal-Ferrari)

$$y^2 = x^2(x-u) + \frac{m\Lambda^3}{4}x - \frac{\Lambda^6}{64} \tag{1}$$

shifting $z = x + u/3$ brings this into Weierstrass normal form

$$\begin{aligned} y^2 &= z^3 + f(u)z + g(u) \\ f(u) &= \frac{\Lambda^3 m}{4} - \frac{u^2}{3} \\ g(u) &= -\frac{\Lambda^6}{64} + \frac{\Lambda^3 m u}{12} - \frac{2u^3}{27} \end{aligned} \tag{2}$$

which is the form appearing in the code.

2 $SU(2)\ N_f = 2$

From section 2 of 9706145 (Bilal-Ferrari)

$$y^2 = x^2(x-u) - \frac{\Lambda^4}{64}(x-u) + \frac{\Lambda^2}{4}m_1 m_2 x - \frac{\Lambda^4}{64}(m_1^2 + m_2^2) \tag{3}$$

shifting $z = x + u/3$ brings this into Weierstrass normal form

$$\begin{aligned} y^2 &= z^3 + f(u)z + g(u) \\ f(u) &= -\frac{\Lambda^4}{64} + \frac{1}{4}\Lambda^2 m_1 m_2 - \frac{u^2}{3} \\ g(u) &= \frac{1}{12}\Lambda^2 m_1 m_2 u - \frac{2u^3}{27} + \frac{\Lambda^4 u}{96} - \frac{1}{64}\Lambda^4\left(m_1^2 + m_2^2\right) \end{aligned} \tag{4}$$

3 $SU(2)$ $N_f = 3$

From section 2 of 9706145 (Bilal-Ferrari)

$$\begin{aligned} y^2 = x^2(x-u) - \frac{\Lambda^2}{64}(x-u)^2 - \frac{\Lambda^2}{64}(x-u)(m_1^2 + m_2^2 + m_3^2) \\ + \frac{\Lambda}{4} m_1 m_2 m_3 x - \frac{\Lambda^2}{64}(m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2) \end{aligned} \tag{5}$$

shifting $z = x + u/3$ brings this into Weierstrass normal form

$$\begin{aligned} y^2 &= z^3 + f(u)z + g(u) \\ f(u) &= -\frac{\Lambda^4}{12288} - \frac{1}{64}\Lambda^2\left(m_1^2 + m_2^2 + m_3^2\right) + \frac{1}{4}\Lambda m_1 m_2 m_3 - \frac{u^2}{3} + \frac{\Lambda^2 u}{48} \\ g(u) &= -\frac{1}{27}\left(2u^3\right) - \frac{5\Lambda^2 u^2}{576} + \frac{\Lambda^4 u}{9216} - \frac{\Lambda^6}{3538944} + \frac{1}{12}\Lambda m_1 m_2 m_3 u + \frac{1}{768}\Lambda^3 m_1 m_2 m_3 \\ &\quad + \frac{1}{96}\Lambda^2\left(m_1^2 + m_2^2 + m_3^2\right) u - \frac{\Lambda^4\left(m_1^2 + m_2^2 + m_3^2\right)}{12288} - \frac{1}{64}\Lambda^2\left(m_2^2 m_1^2 + m_3^2 m_1^2 + m_2^2 m_3^2\right) \end{aligned} \tag{6}$$