This is a companion note to the notebook

z-pure-SU-2-mod.nb

(1) $$Y^2 = (X-1)(X+1)(X-U)$$

is equivalent to

(2) $$y^2 = (x-e_1)(x-e_2)(x-e_3) = x^3+fx+g$$

Through $$X = \frac{1}{e_2-e_1}(2x-e_2-e_1)$$

$$Y = \left(\frac{2}{e_2-e_1}\right)^{3/2} y$$

$$U = \frac{e_1+e_2-2e_3}{e_1-e_2}$$

Fixing the explicit form:

$$f = \frac{1}{4}\left(1 - 4\frac{u^2}{3}\right) \qquad g = \frac{1}{4}\left(u/3 - \frac{8}{27}u^3\right)$$

we find roots of $x^3+fx+g=0$

$$\alpha = -\tfrac{1}{3}u$$

$$\beta = \tfrac{1}{6}\left(u - 3\sqrt{u^2-1}\right)$$

$$\gamma = \tfrac{1}{6}\left(u + 3\sqrt{u^2-1}\right)$$

However, since f, g are sym. in e_i, it's unclear how to identify α, β, γ with $\{e_i\}$:

To do so, note that $U = \pm 1$ is the disc. locus of (1), and $u = \pm 1$ is that of (2).

Moreover,

$$X = -1 \quad \longleftrightarrow \quad x = e_1$$

$$X = +1 \quad \longleftrightarrow \quad x = e_2$$

$$X = U \quad \longleftrightarrow \quad x = e_3$$

So, when $U = -1$, we need $e_1 = e_3$.

Therefore near $U = -1$,

$$\begin{aligned} e_2 &= \alpha \\ e_1 &= \beta \\ e_3 &= \gamma \end{aligned} \qquad \text{or} \qquad \begin{aligned} e_2 &= \alpha \\ e_1 &= \gamma \\ e_3 &= \beta \end{aligned}$$

pick the left option:

(3) $$U = \frac{\beta + \alpha - 2\gamma}{\beta - \alpha} = - \frac{u + 3\sqrt{u^2 - 1}}{u - \sqrt{u^2-1}} \xrightarrow{u \to -1} -1 \quad \underline{\text{ok}}$$

The other option would just correspond to choosing the other branch of $\sqrt{u^2-1}$.

Similarly, near $U=+1$ we must have $e_2 = e_3$. Therefore

$$\alpha = e_1 \qquad\qquad \alpha = e_1$$

$$\beta = e_2 \qquad \text{OR} \qquad \beta = e_3$$

$$\gamma = e_3 \qquad\qquad \gamma = e_2$$

The first option gives

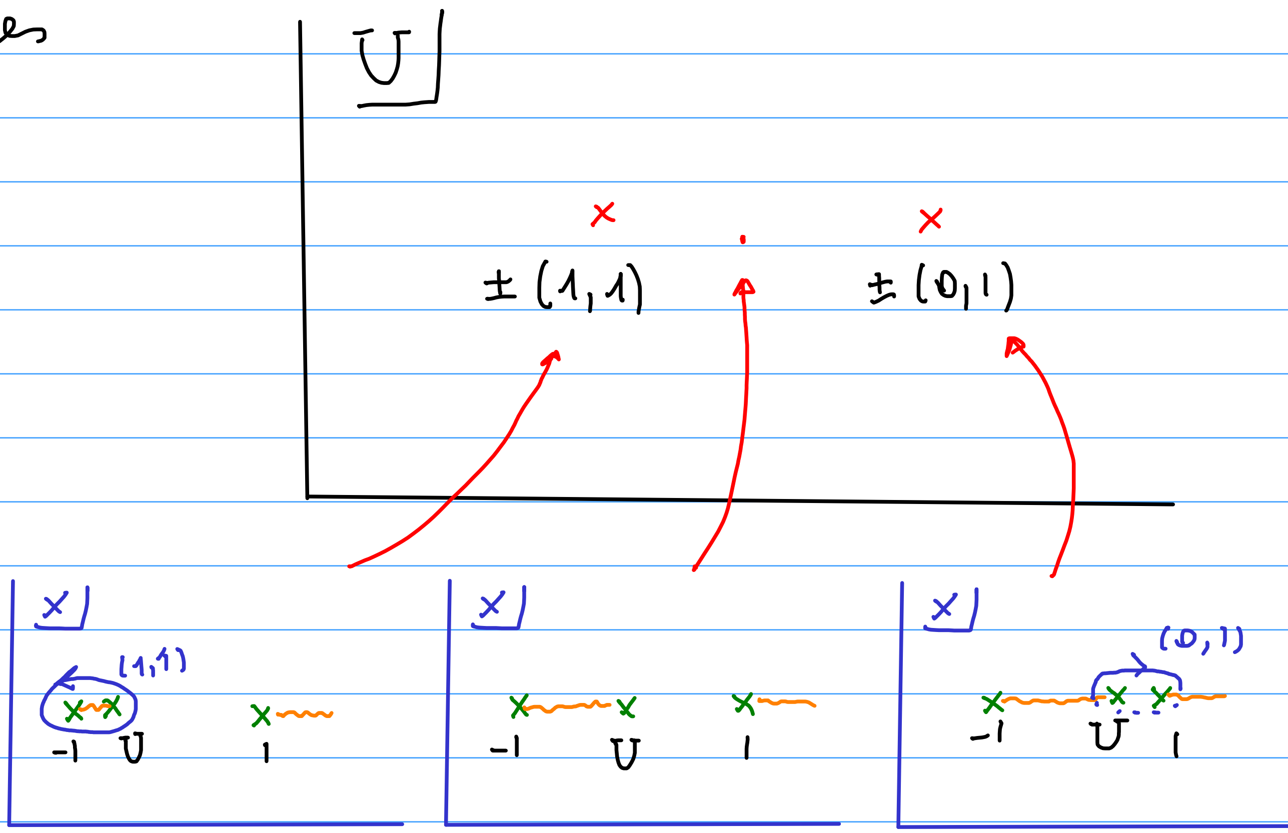
(4) $$U = \frac{\alpha+\beta-2\gamma}{\alpha-\beta} = \frac{u+3\sqrt{u^2-1}}{u-\sqrt{u^2-1}} \xrightarrow{u\to 1} +1 \qquad \underline{\text{ok}}$$

Using these identifications, we find that

$$a_D(U=+1) = 0$$

$$a(U=-1) + a_D(U=-1) = 0$$

So the Coulomb branch has vanishing cycles



see (3), (4)

The u plane is a covering of the U plane

for this reason we can't just write U as a function of α, β, γ.

Indeed we wrote U as a function of the e_i which are placeholders, and the map from $\{e_i\} \to \{\alpha(u), \beta(u), \gamma(u)\}$ jumps as we move around.