INFINITY Workshop

July 7th, 2020

Branching Immediate Observation Petri Nets

A strong class with simple reachability

Chana Weil-Kennedy

joint work with Javier Esparza and Mikhail Raskin

European Research Council

Established by the European Commission

The project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 787367

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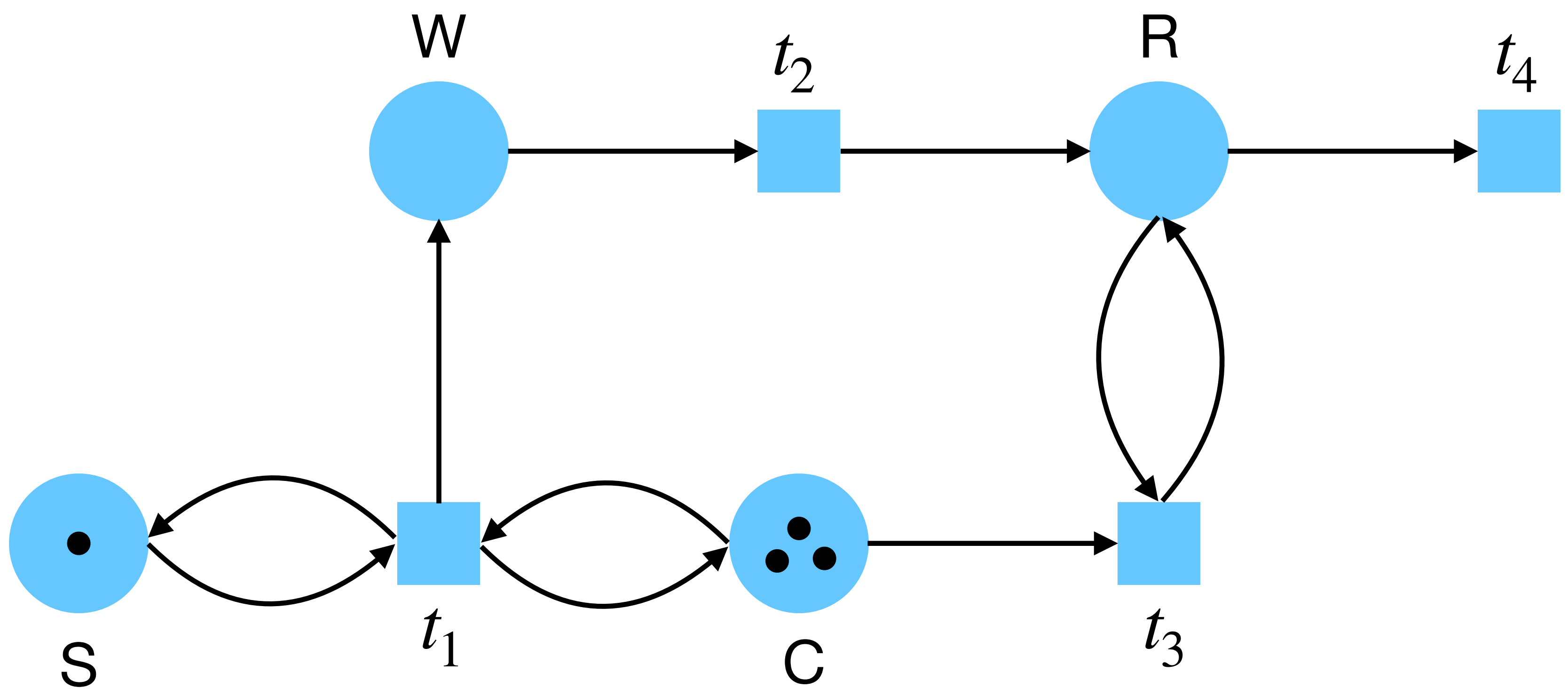

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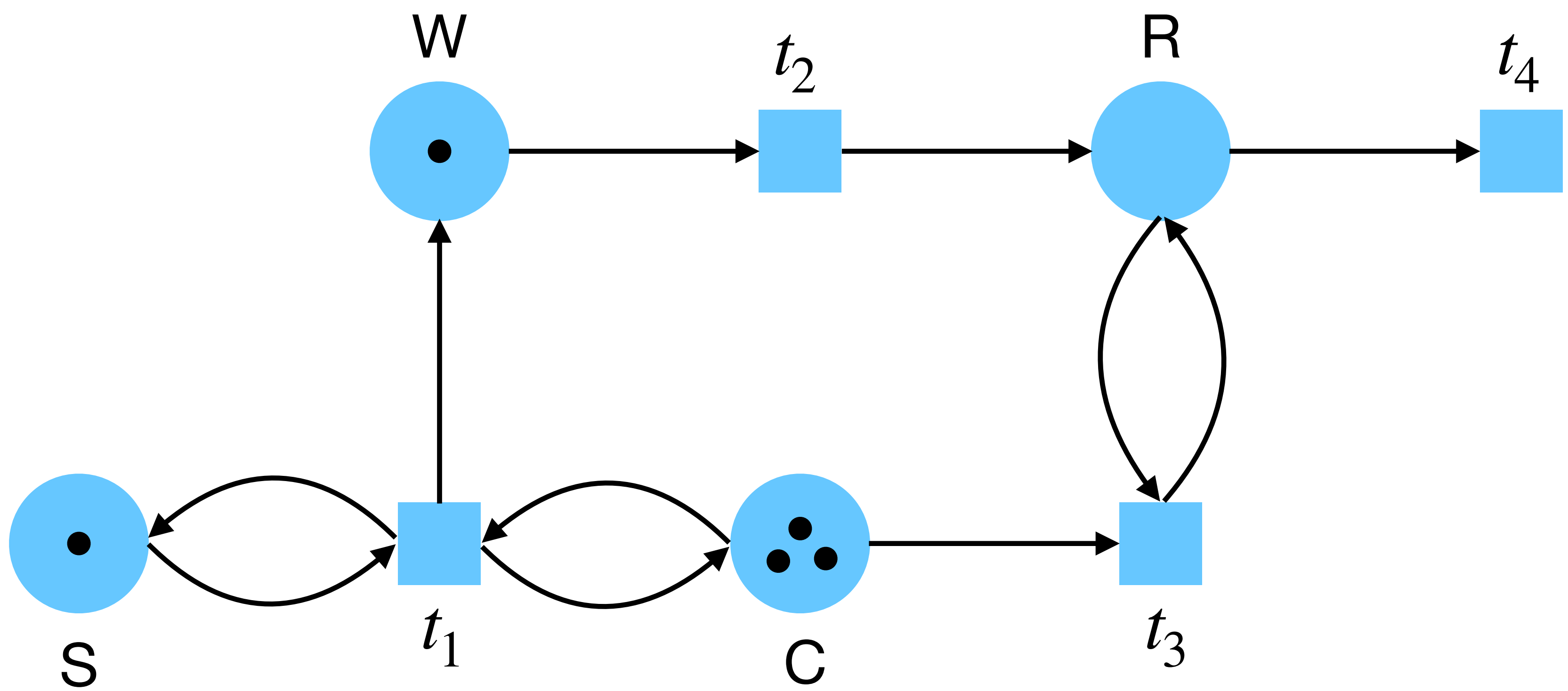

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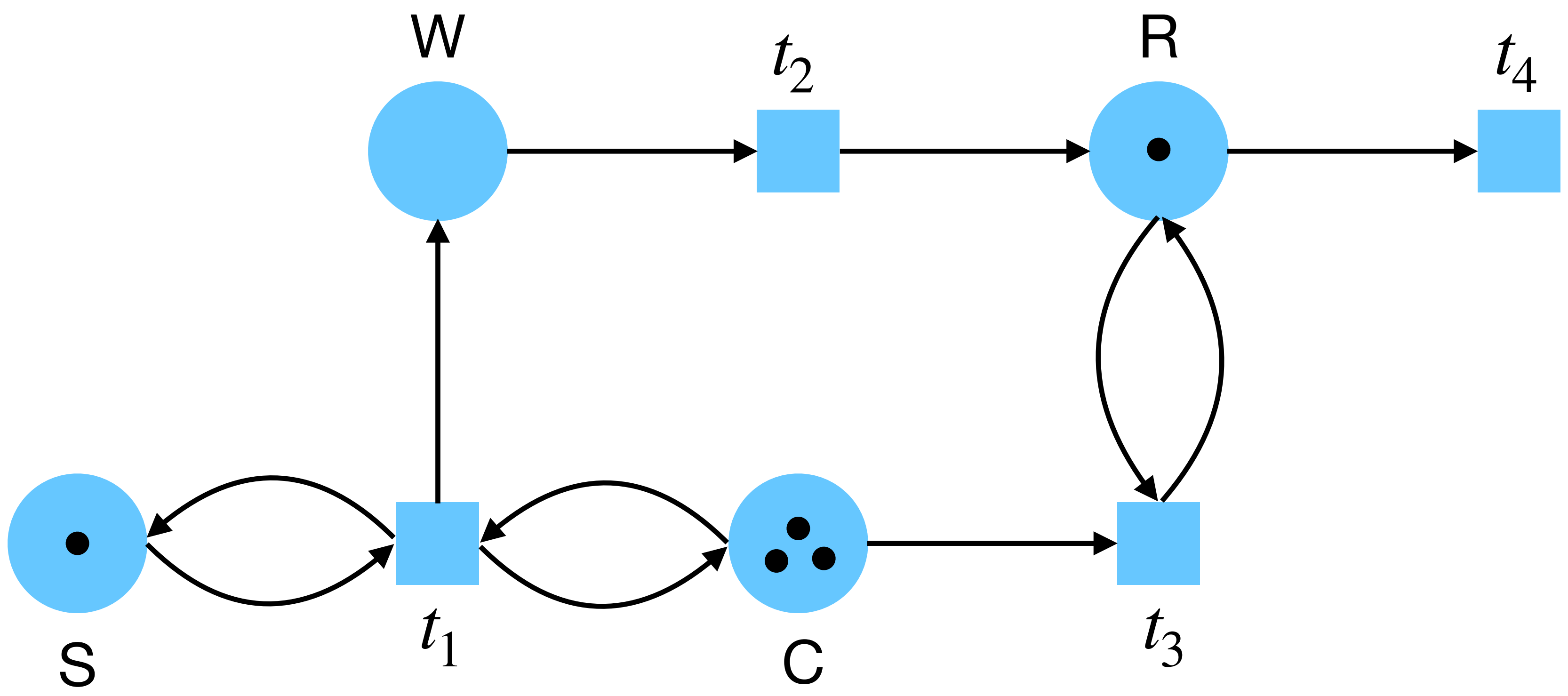

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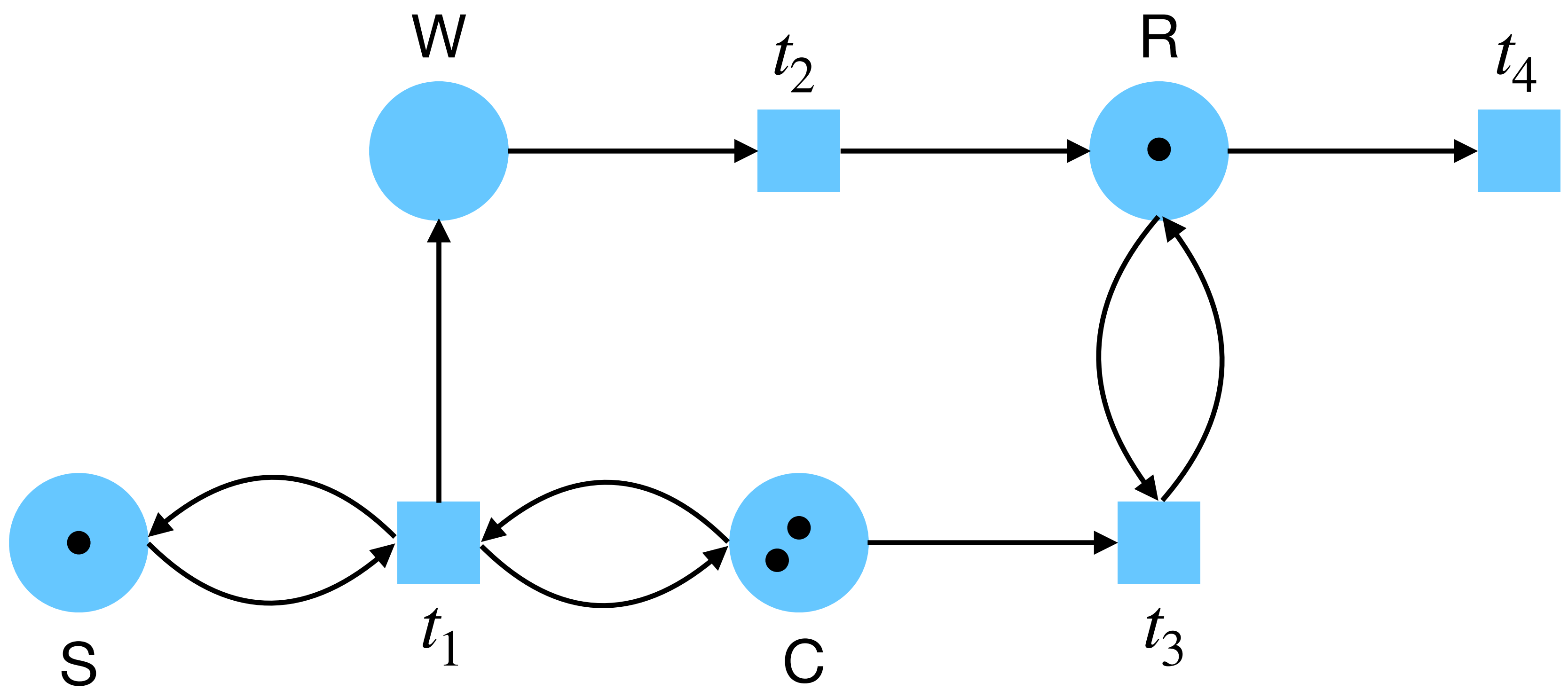

C. Weil-Kennedy, TUM

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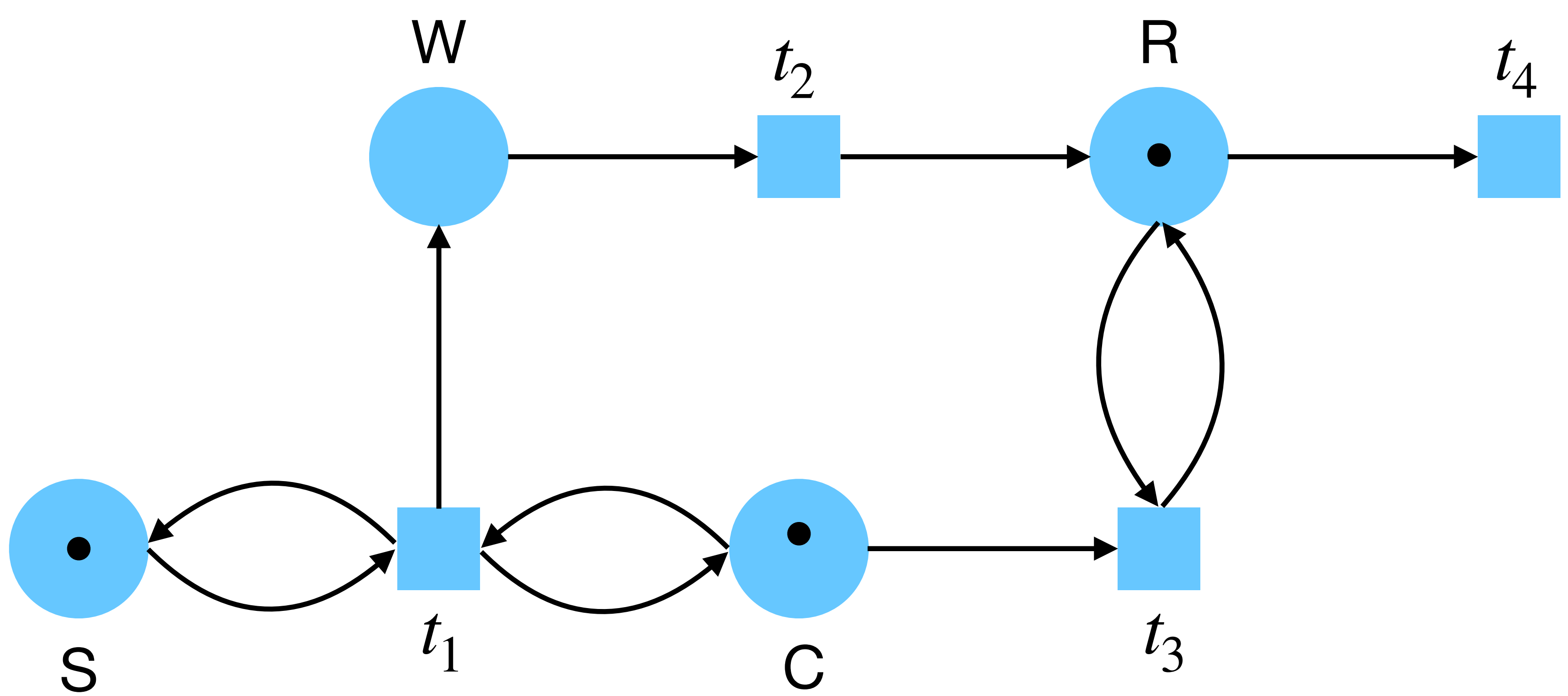

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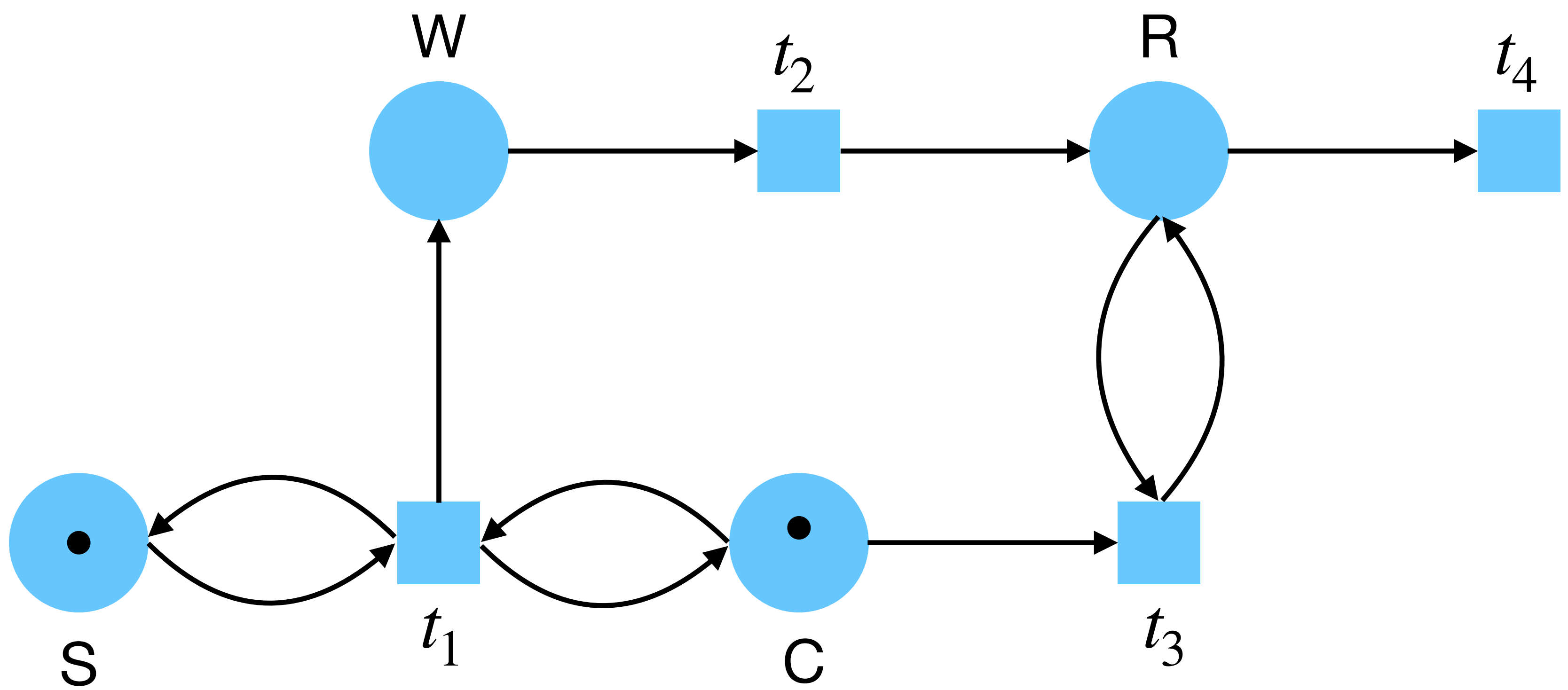

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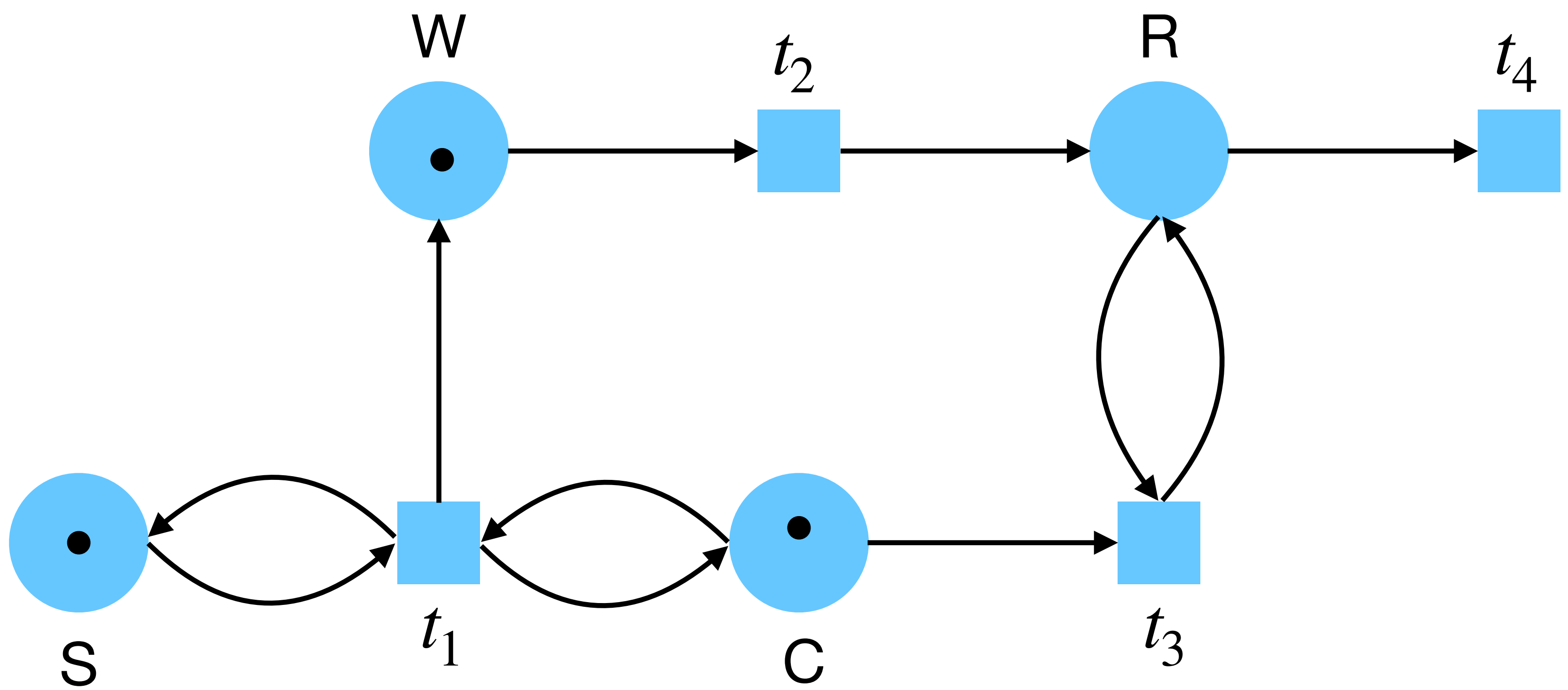

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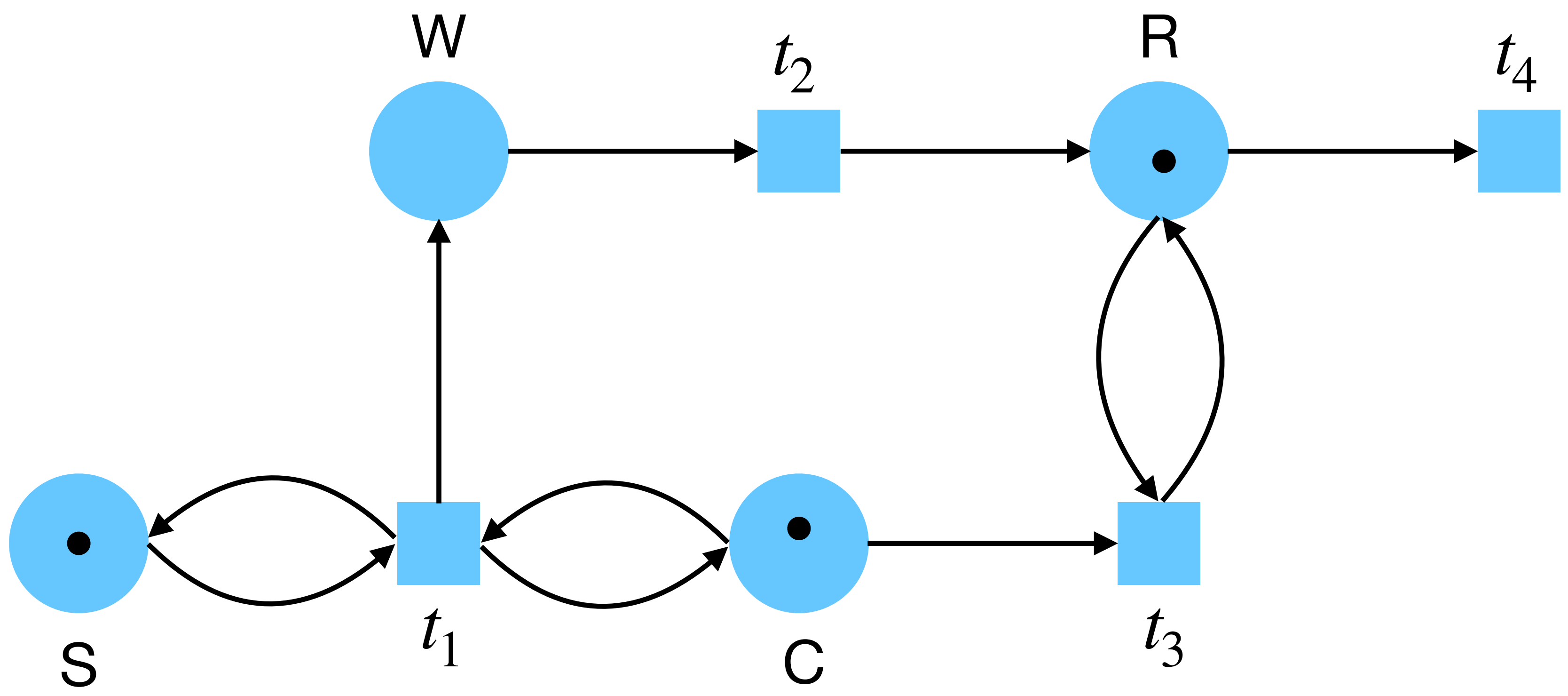

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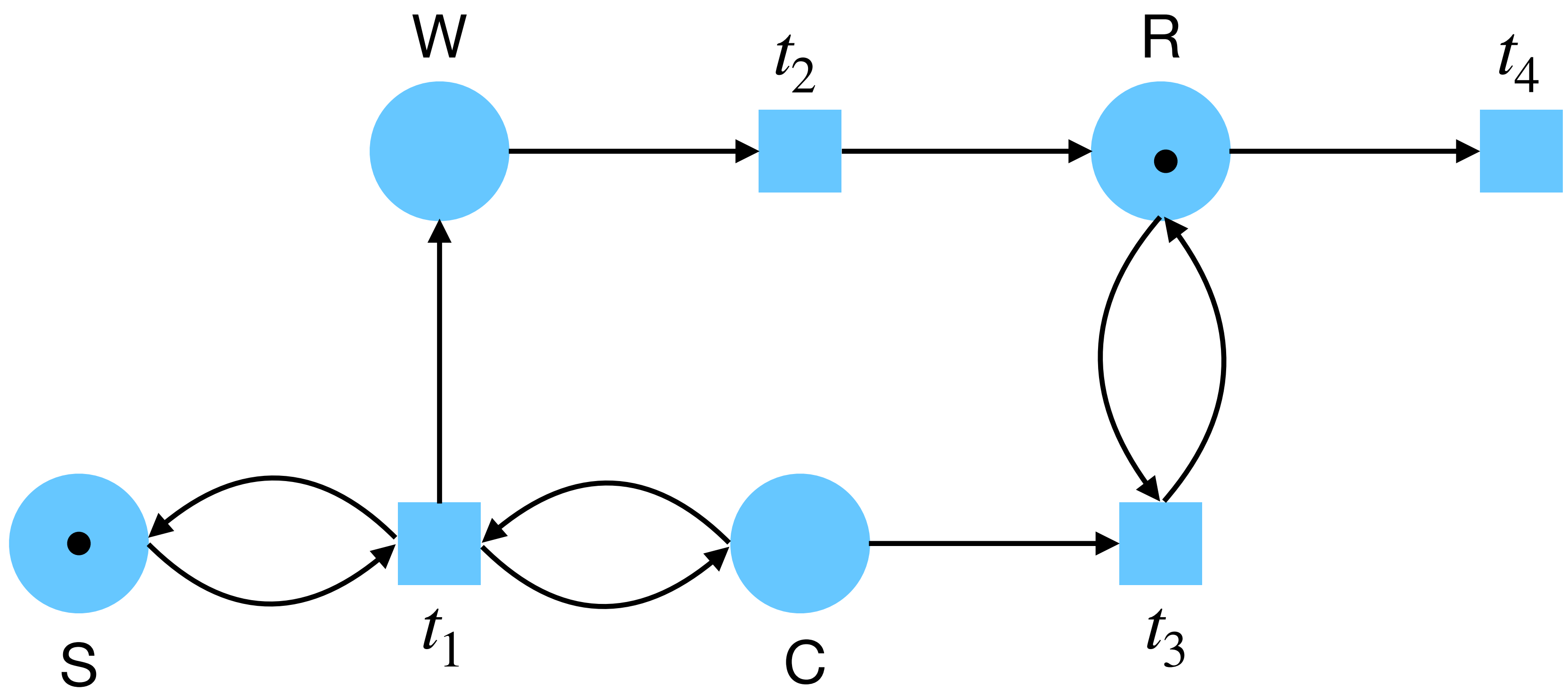

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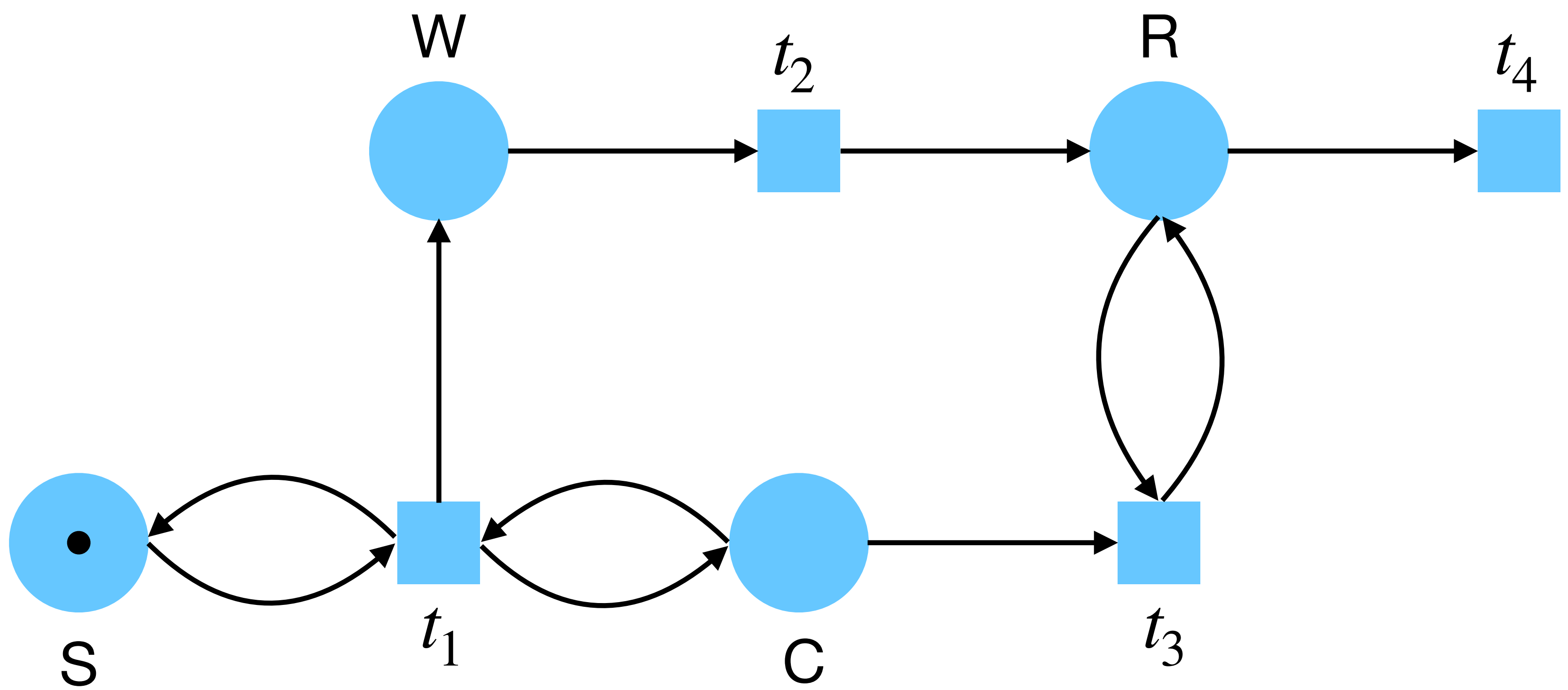

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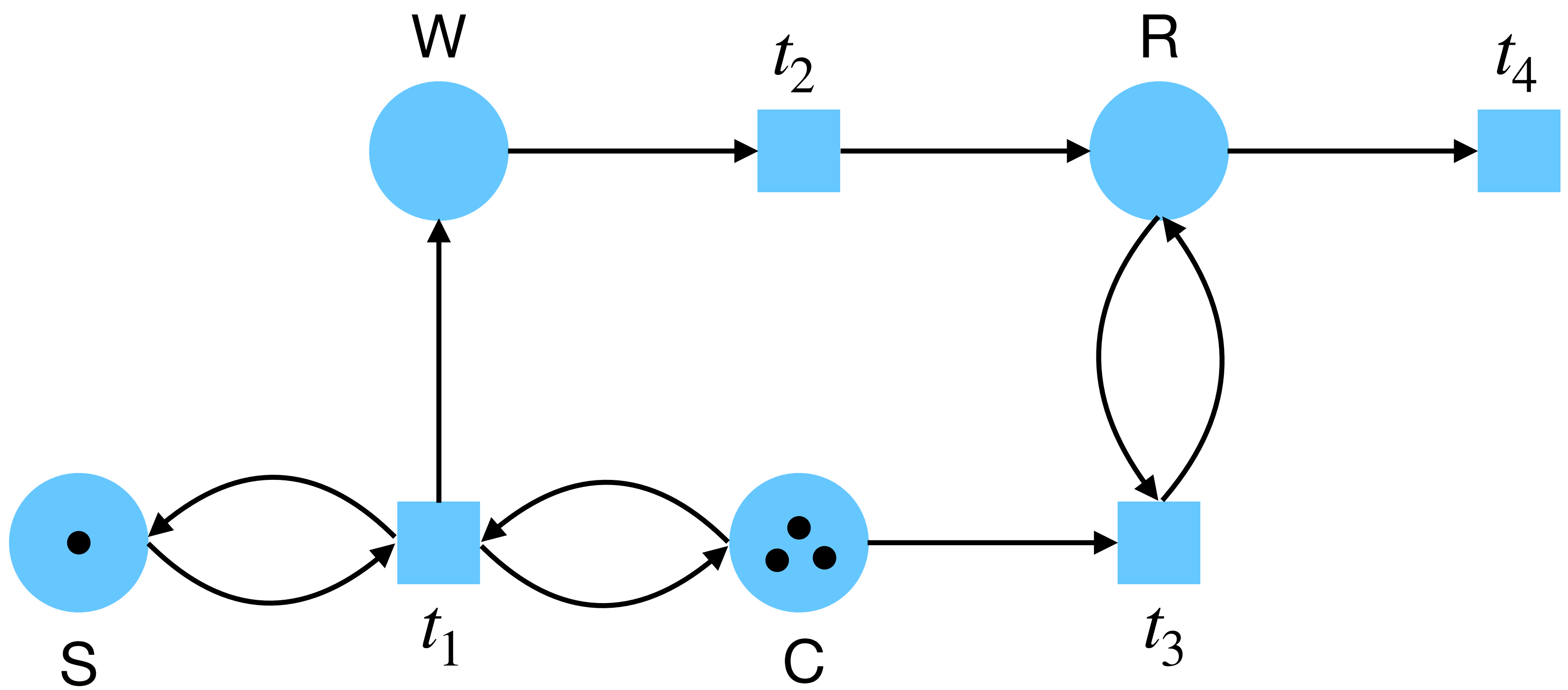

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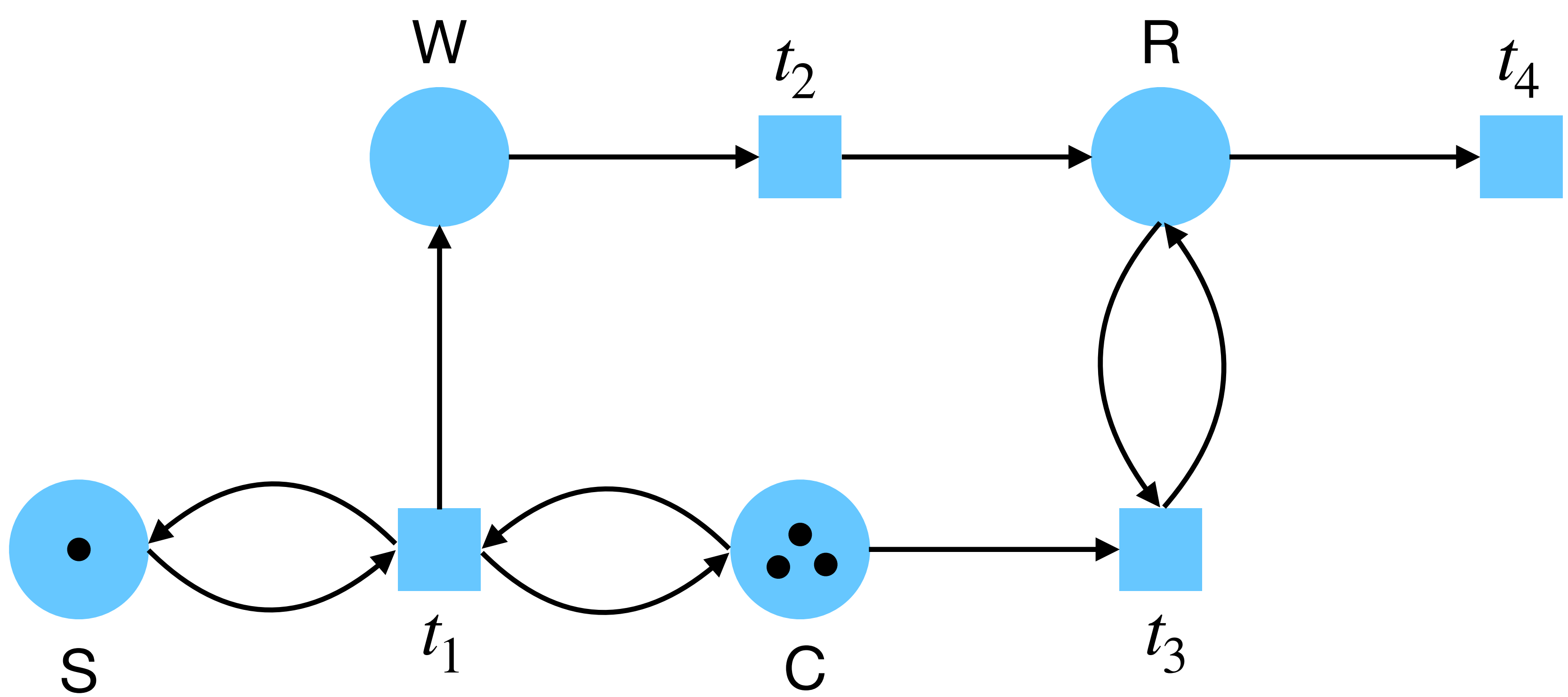

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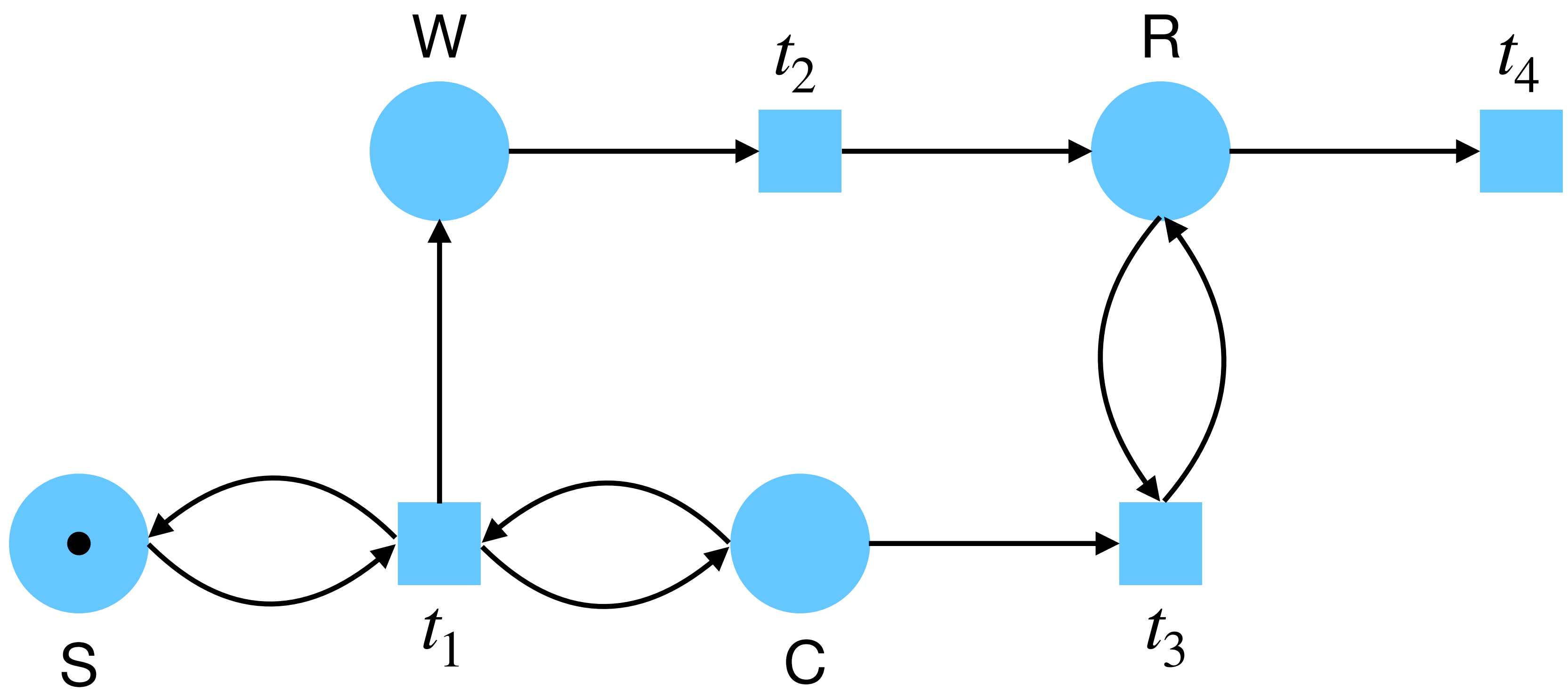

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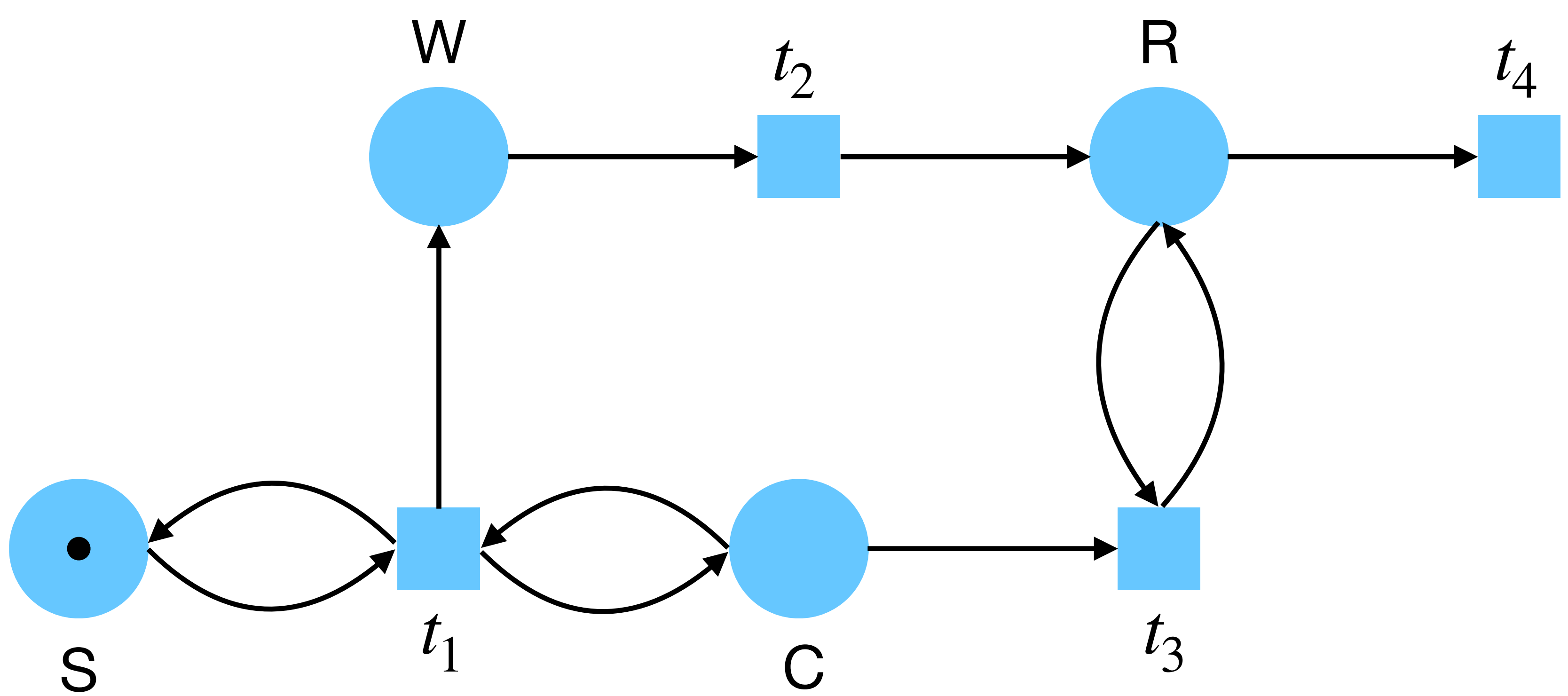

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- **verification** of systems modelled by Petri nets
- many problems are **interreducible with reachability** in Petri nets in:
  - formal languages (e.g. shuffle closure of regular language)
  - logic (e.g. logics on data words)
  - process calculi (e.g. fragment of π-calculus) [survey by S. Schmitz, '16]

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Study subclasses of Petri nets

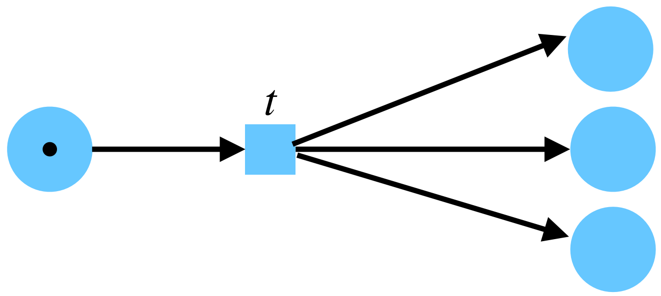
Branching immediate observation nets

[Christensen et al., ’93]

[Yen, ’97] [Lasota, ’09]

[Mayr, Weihmann, ’15]

Branching Parallel Processes (BPP)



- Token creation and destruction
- Communication-free

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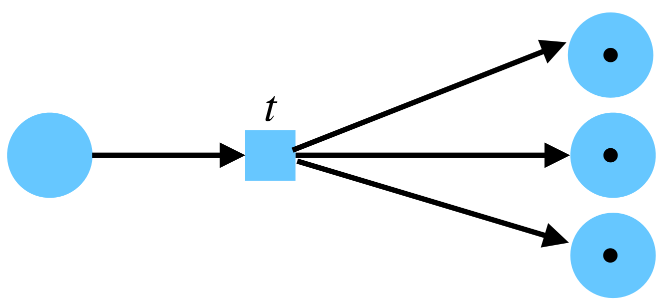

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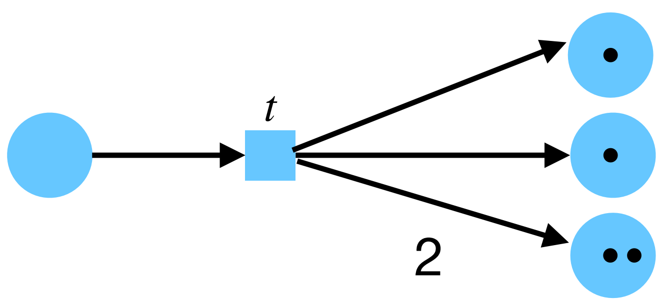

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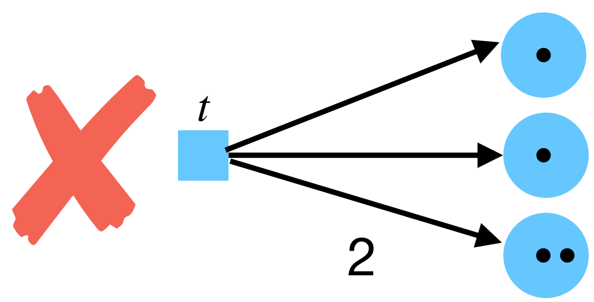

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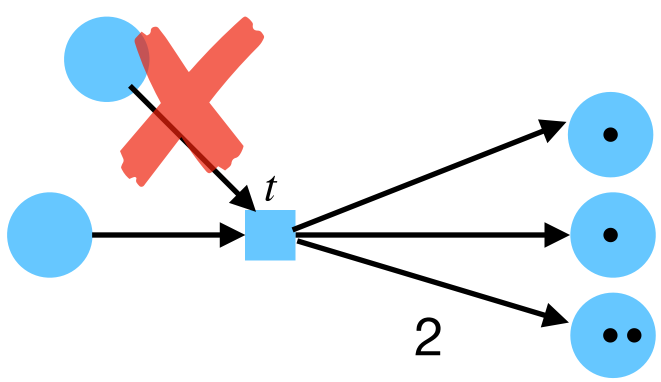

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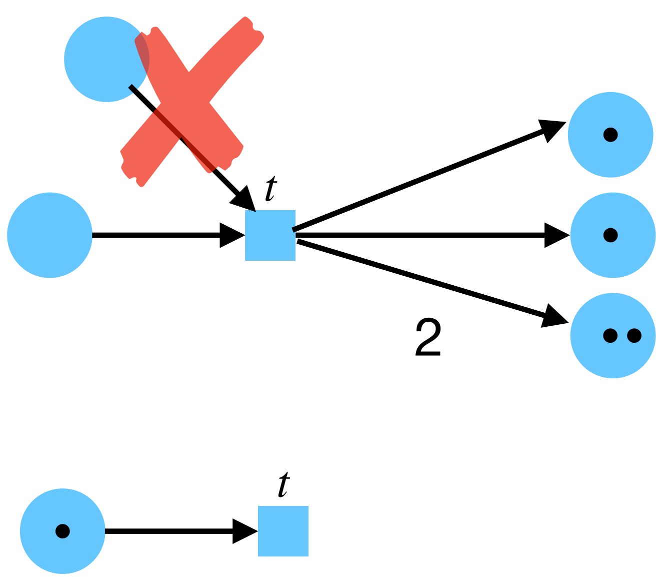

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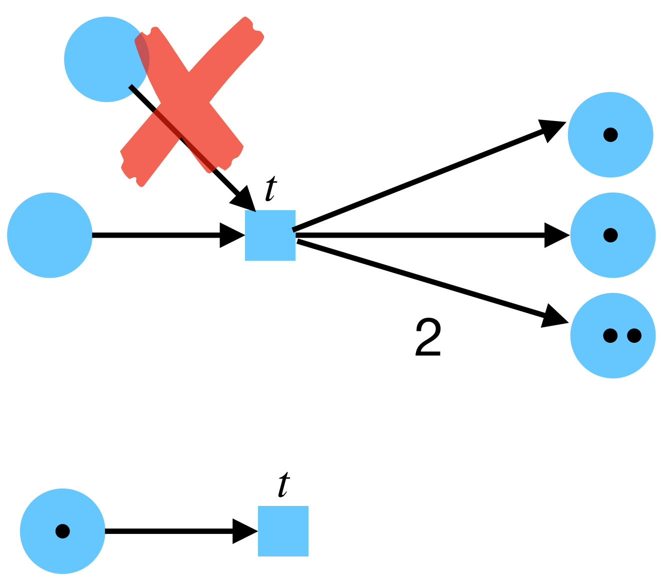

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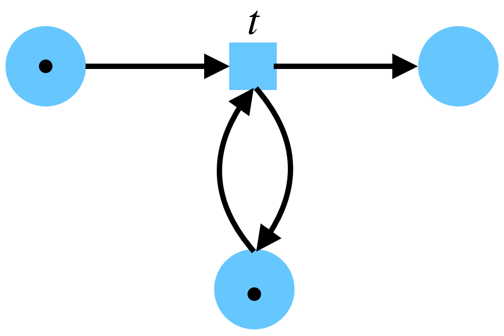

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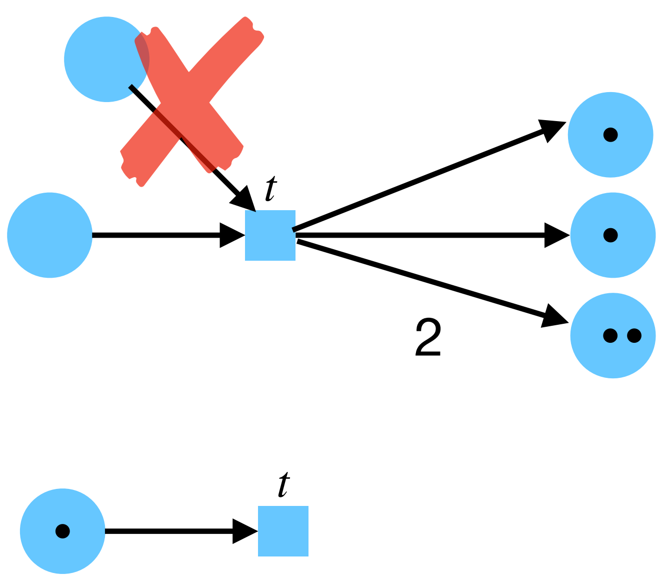

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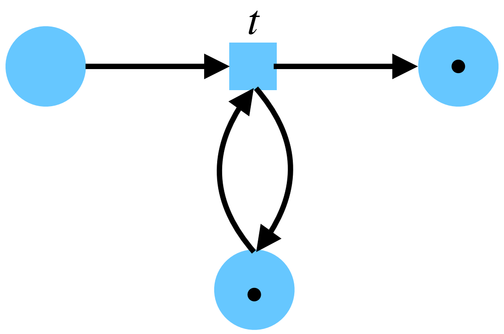

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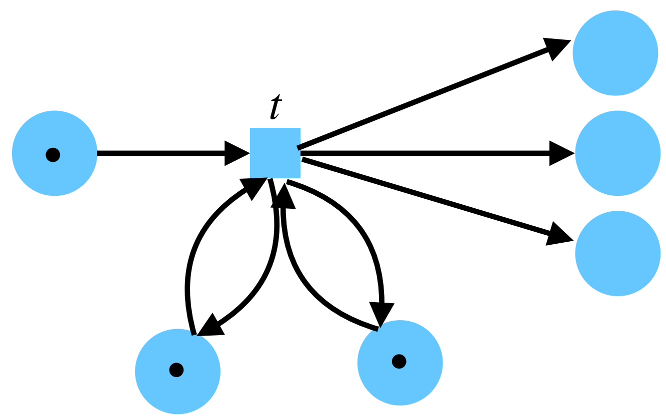

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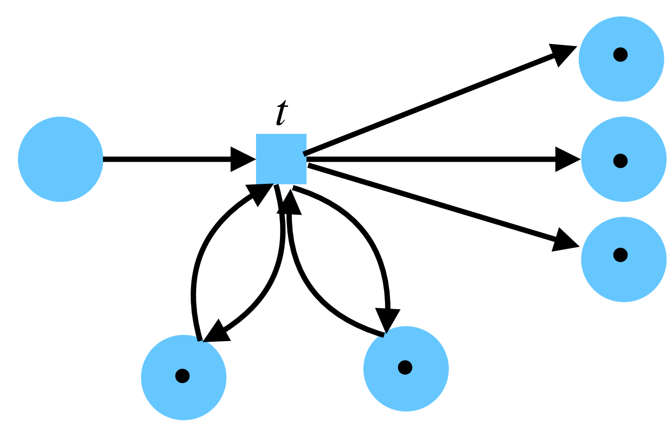

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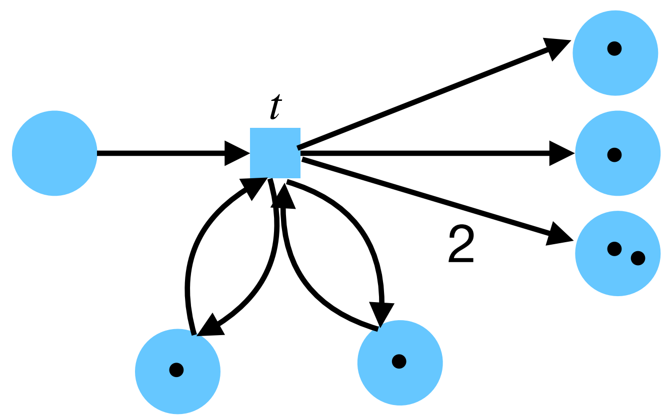

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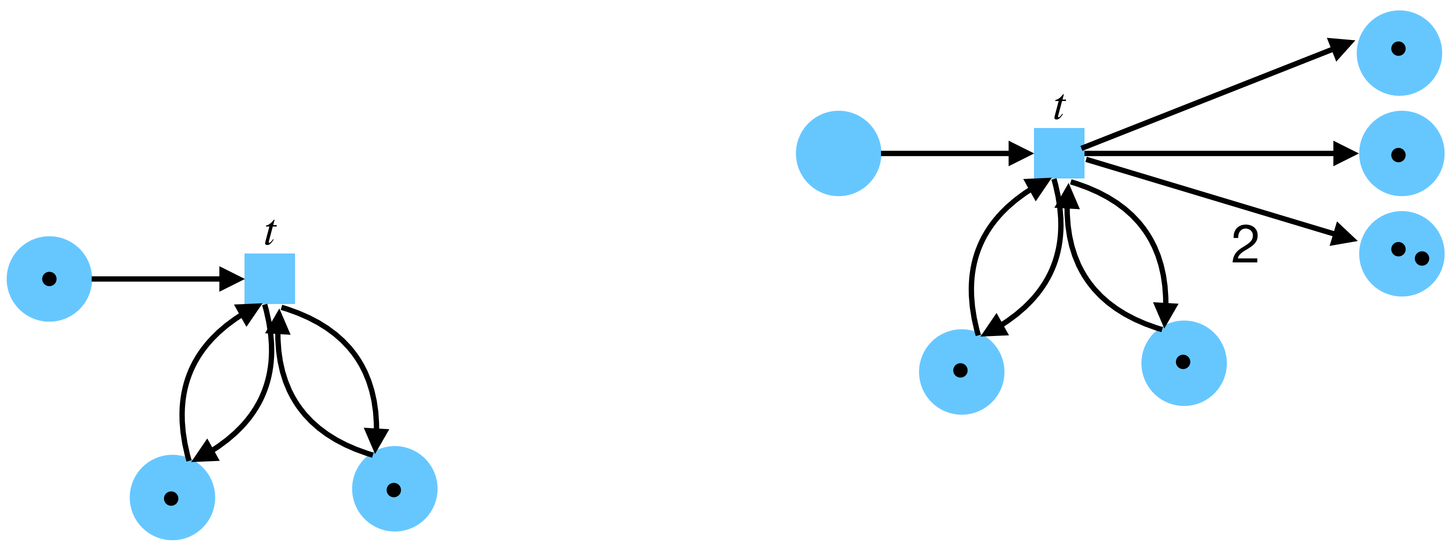

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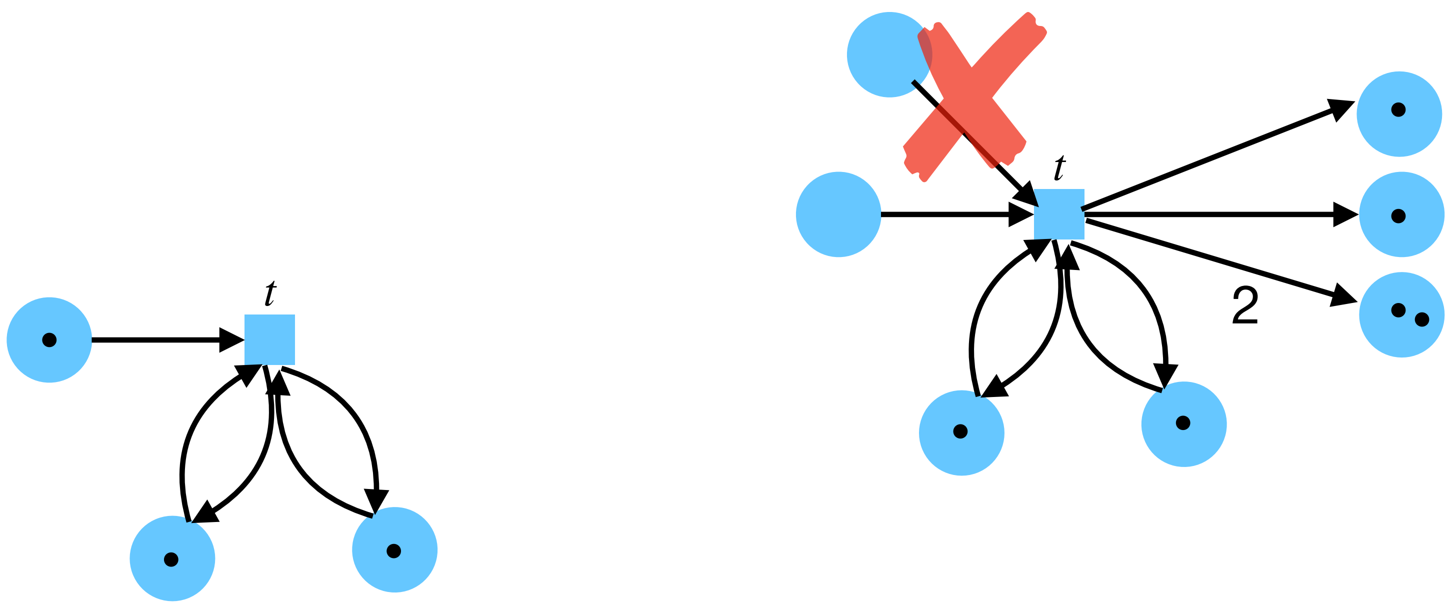

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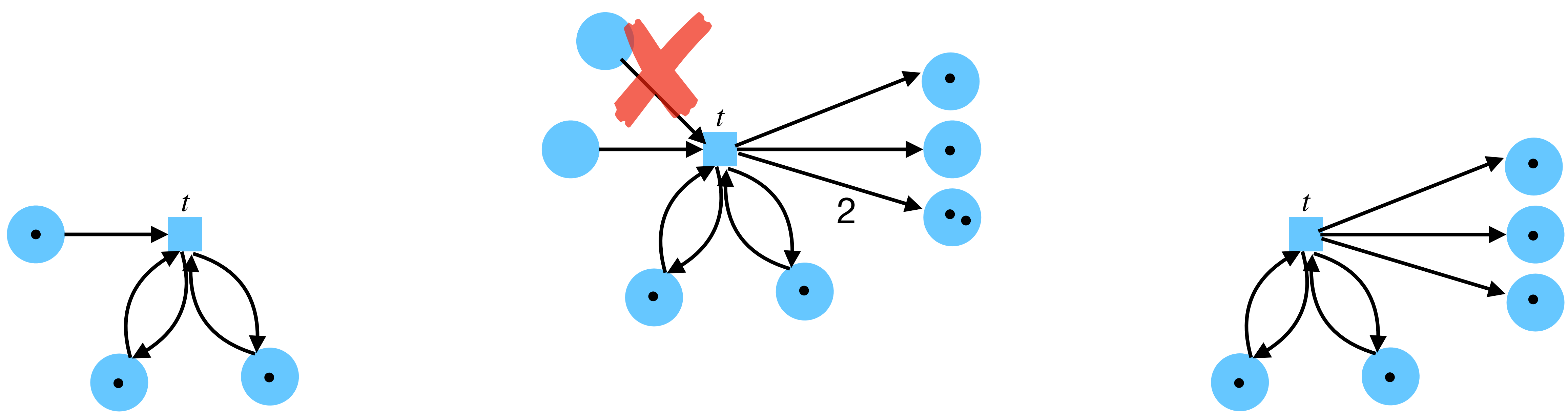

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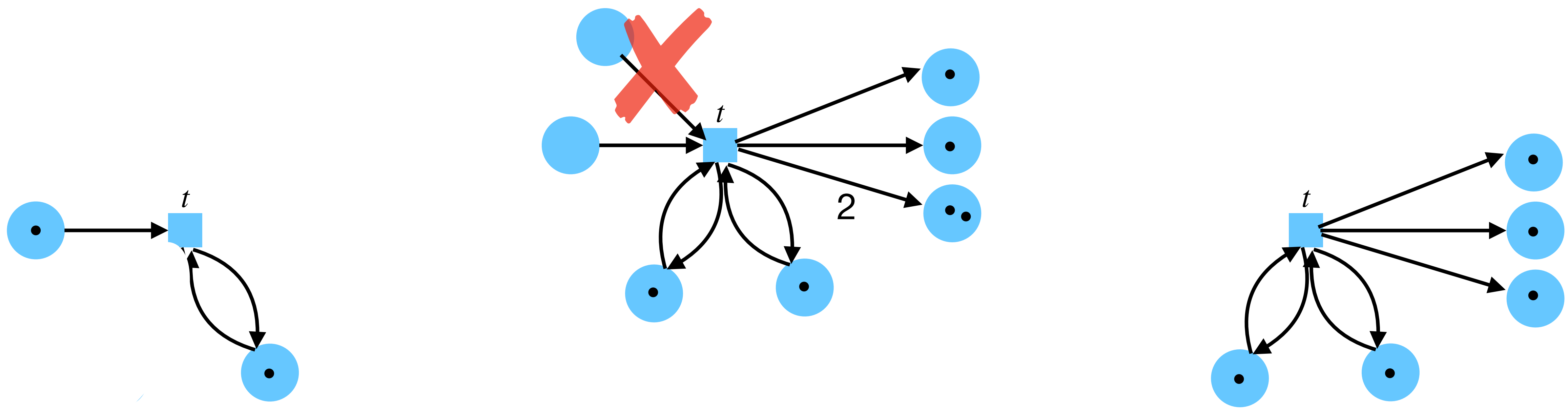

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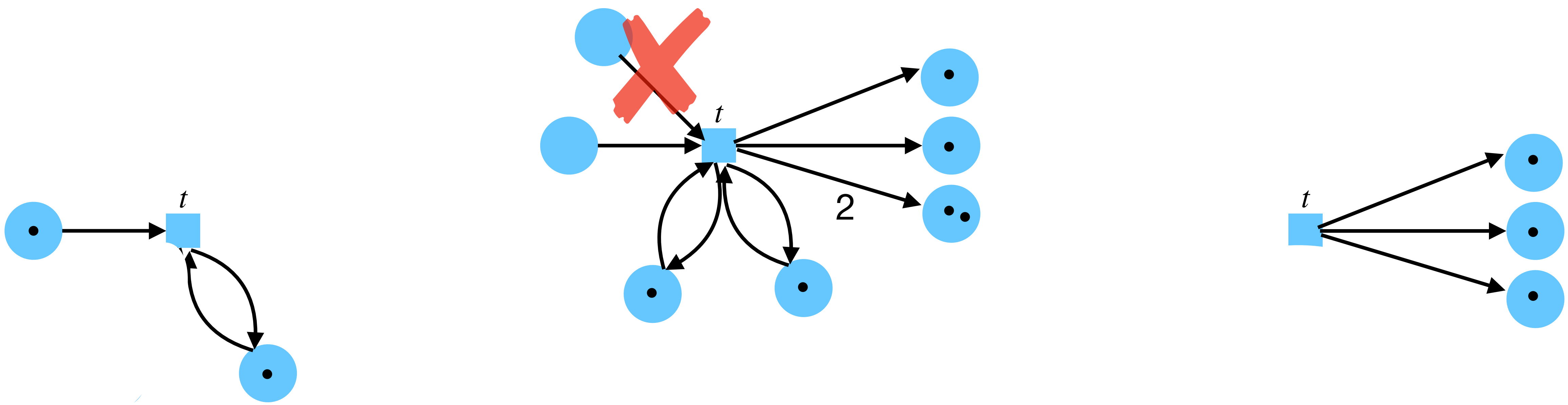

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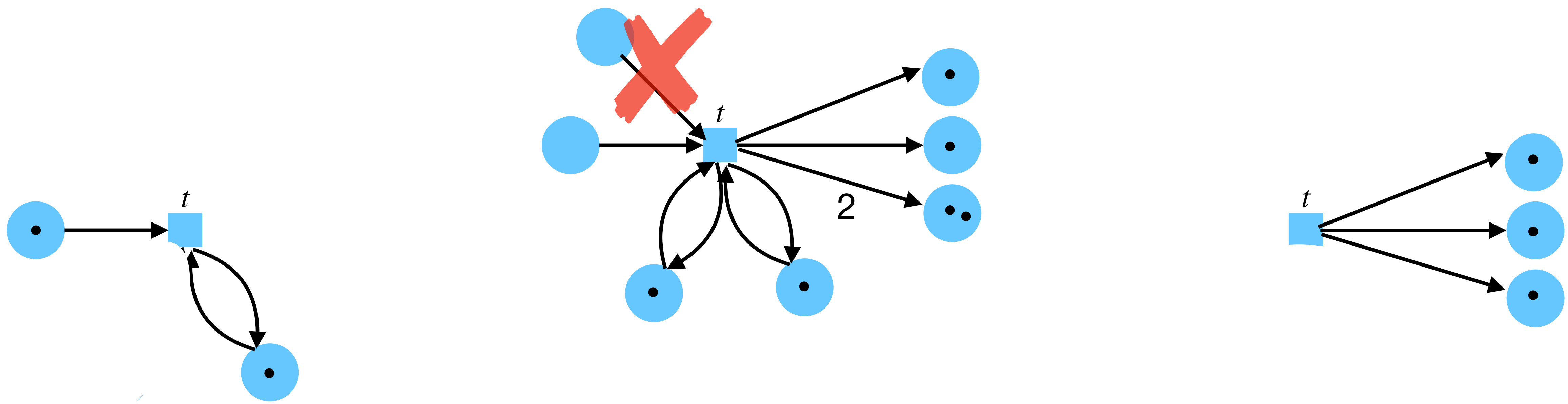

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$$\boxed{Card({}^{\bullet}t - t^{\bullet}) \leq 1}$$

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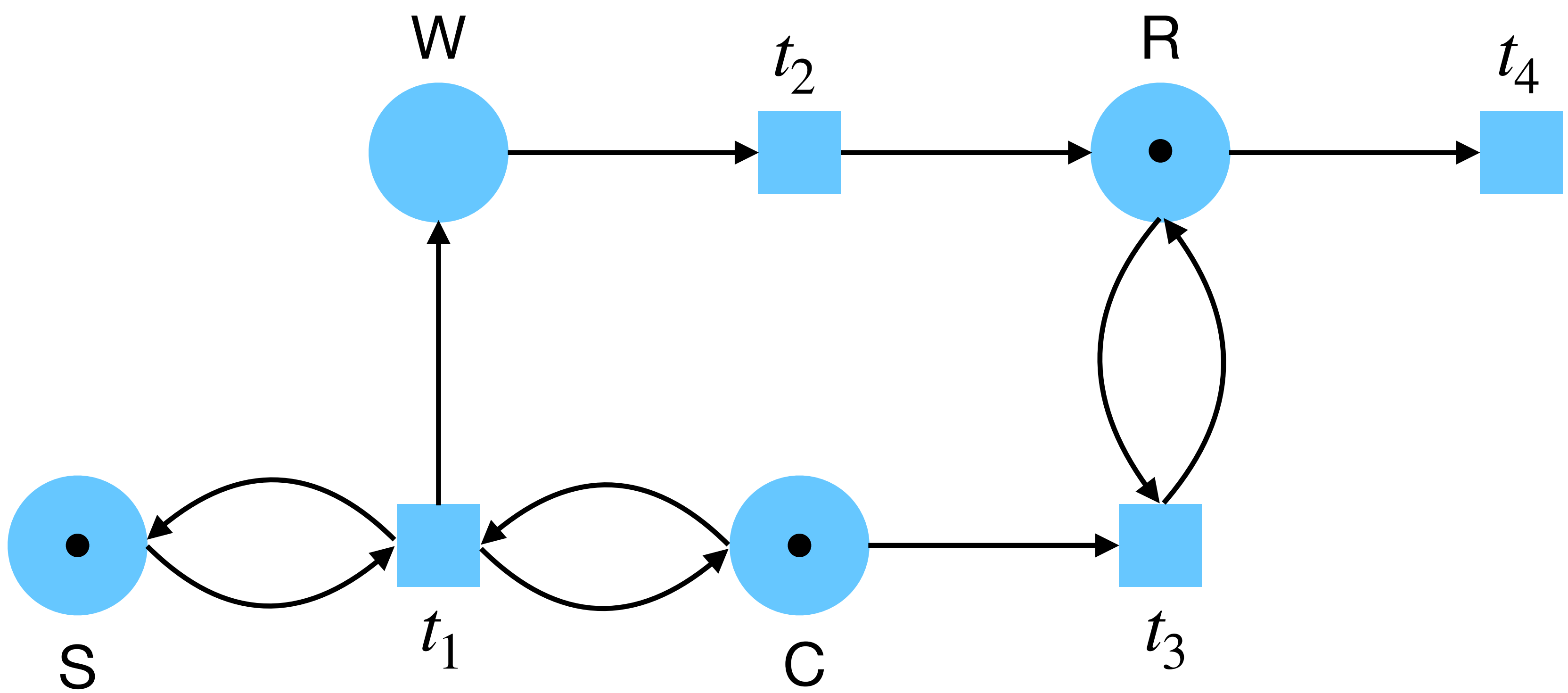

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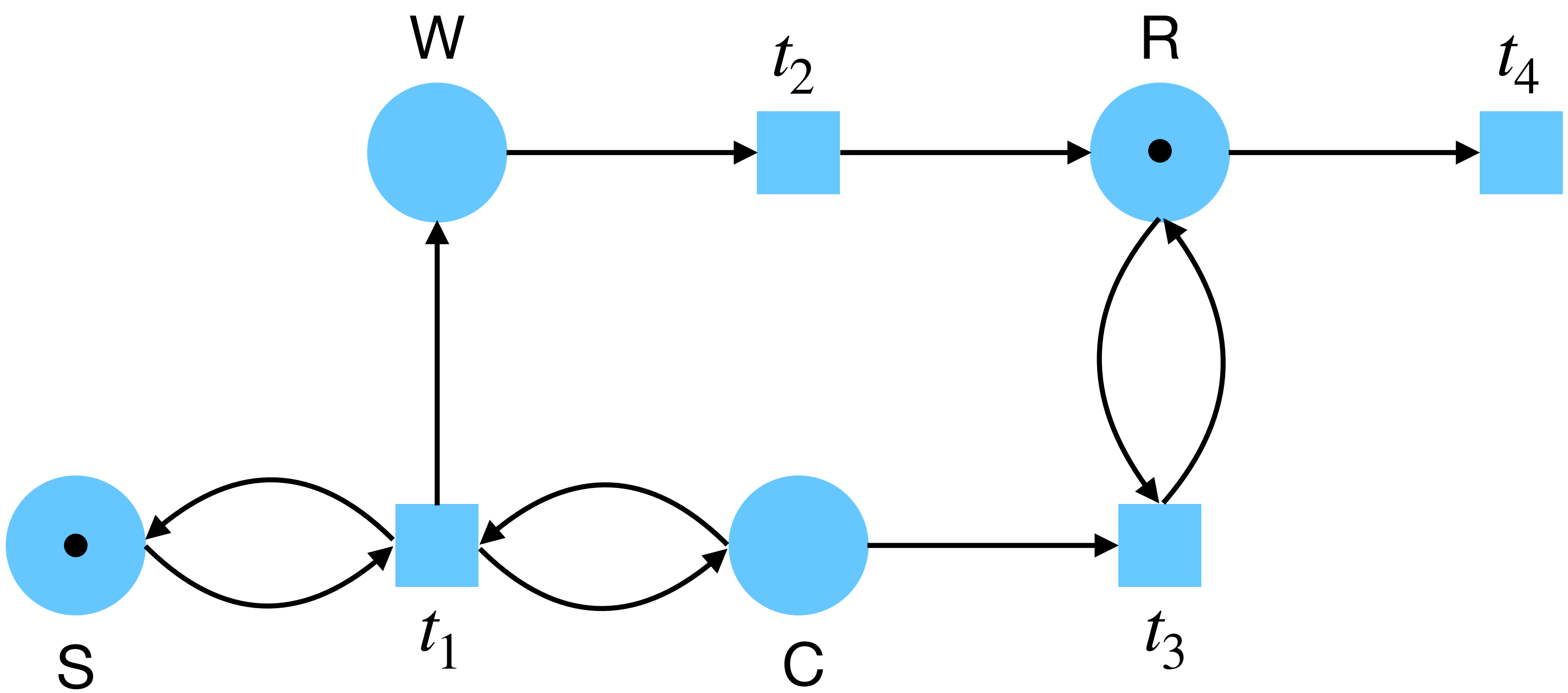
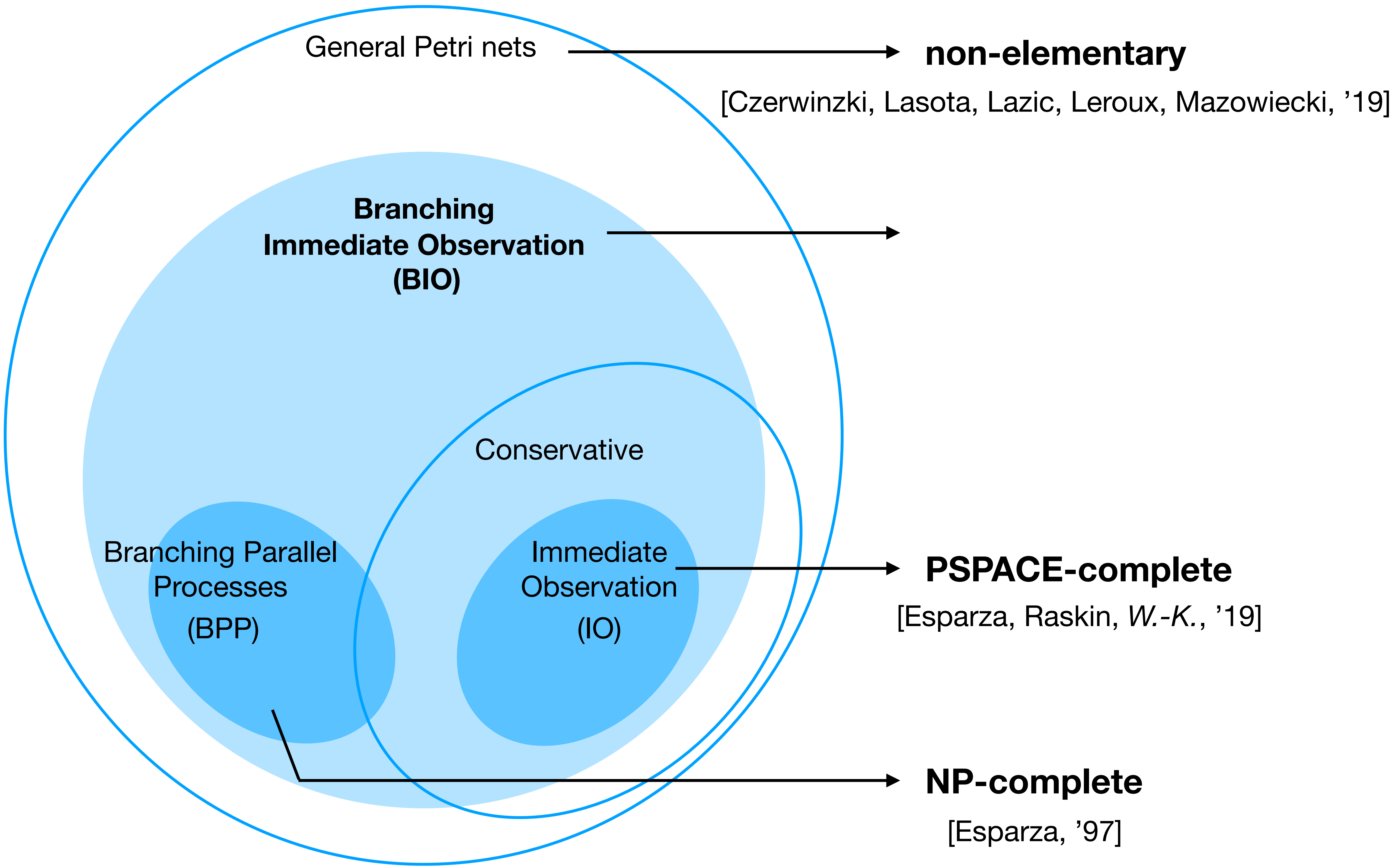


A strong class with simple reachability



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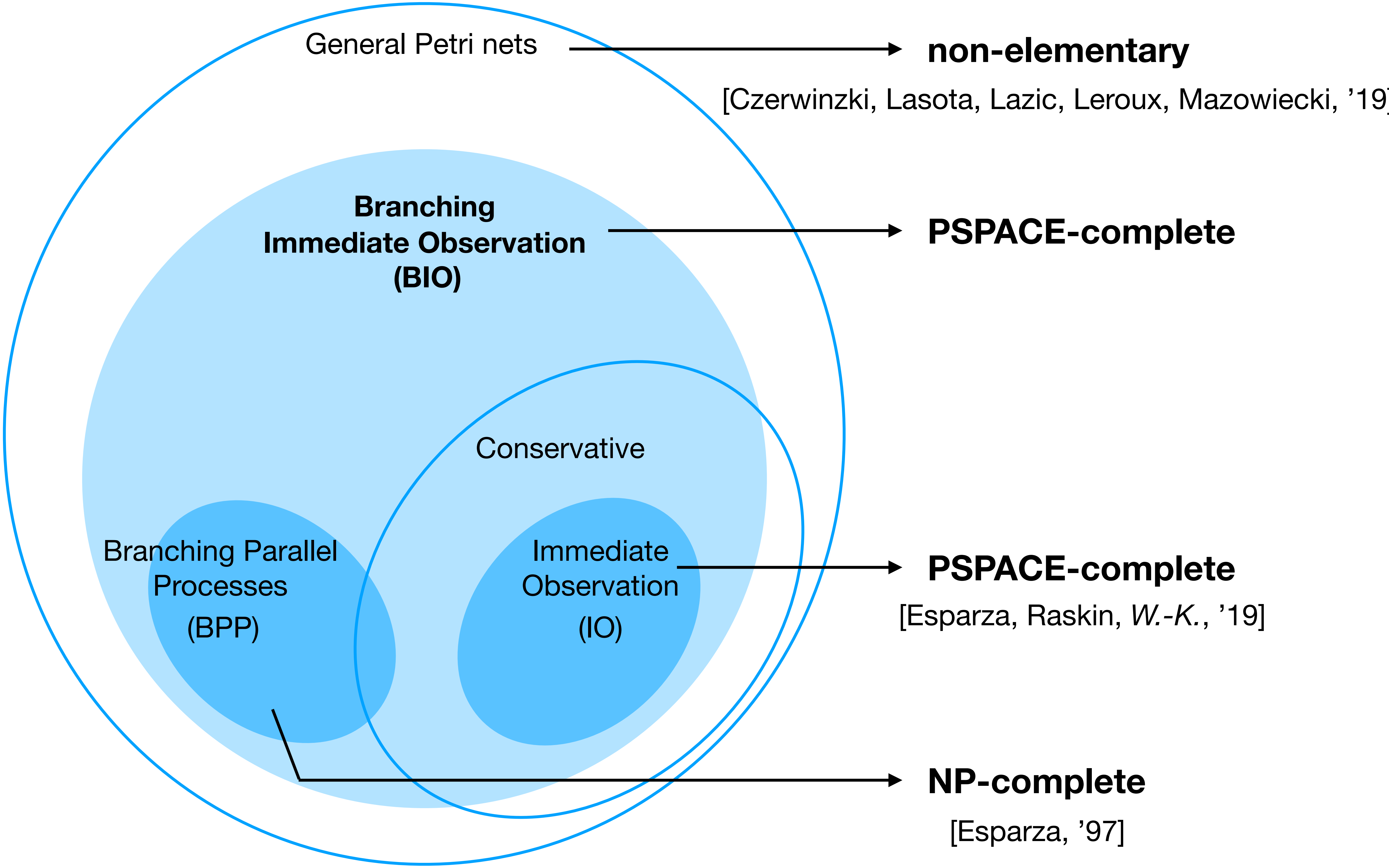

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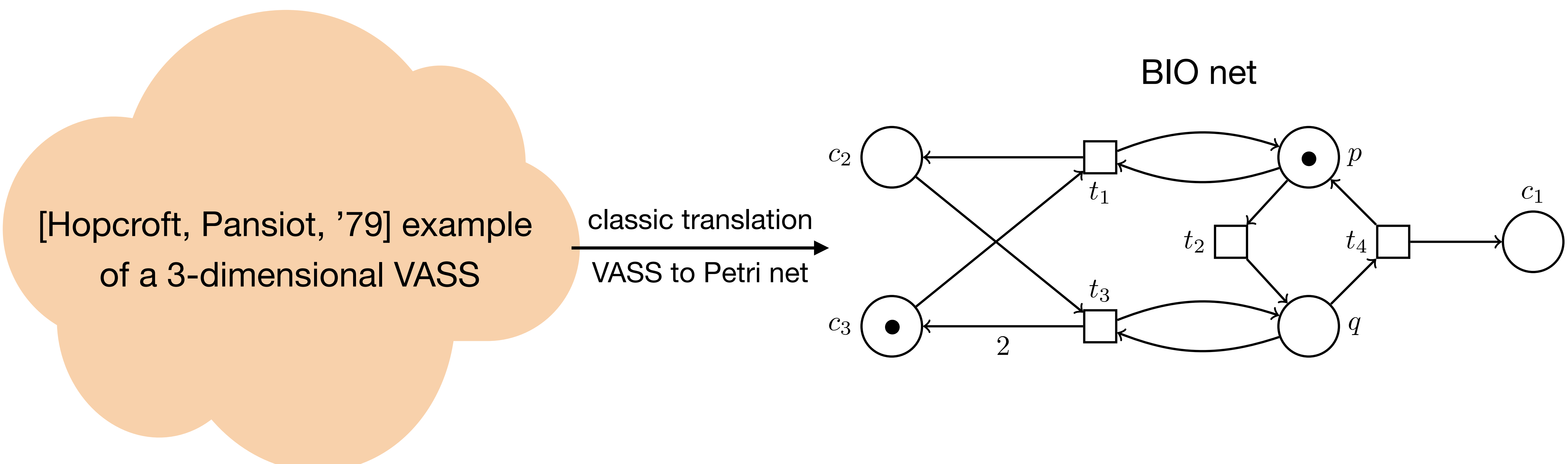

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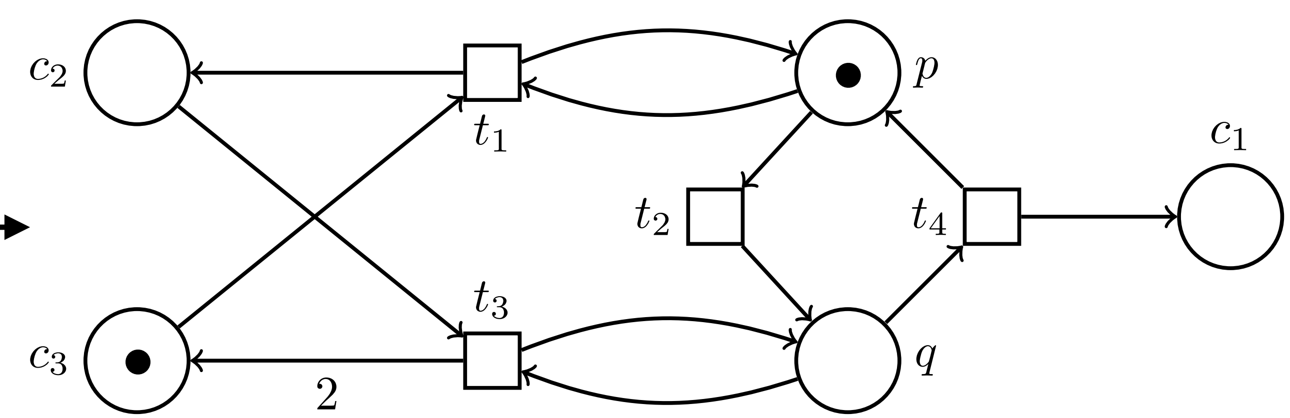
[Hopcroft, Pansiot, ’79] example of a 3-dimensional VASS

classic translation

VASS to Petri net

BIO net



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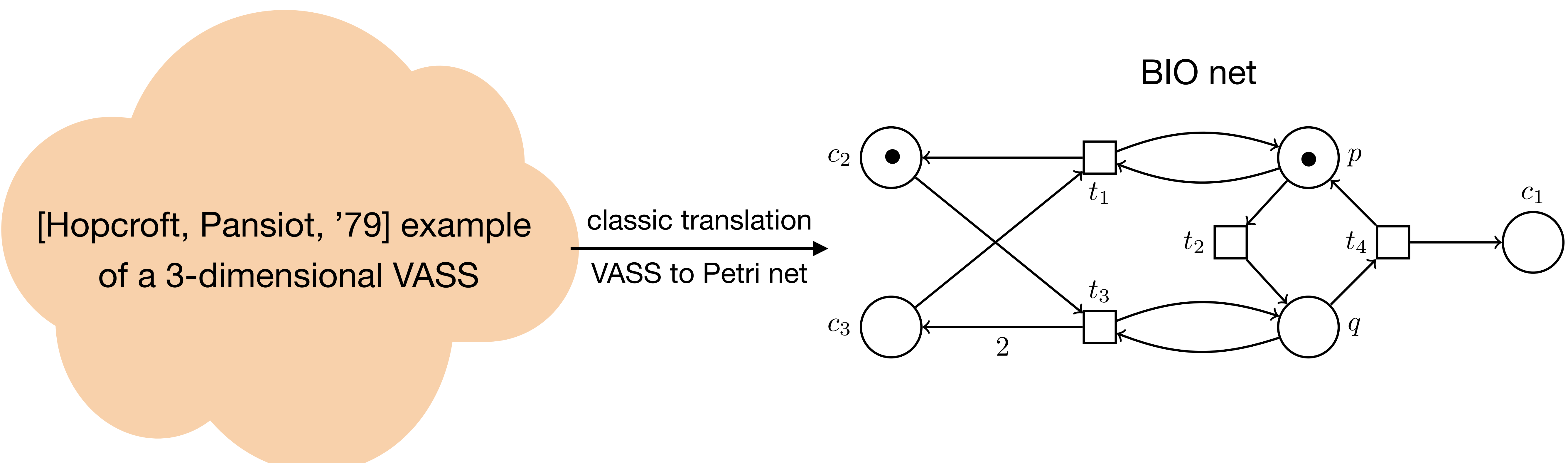

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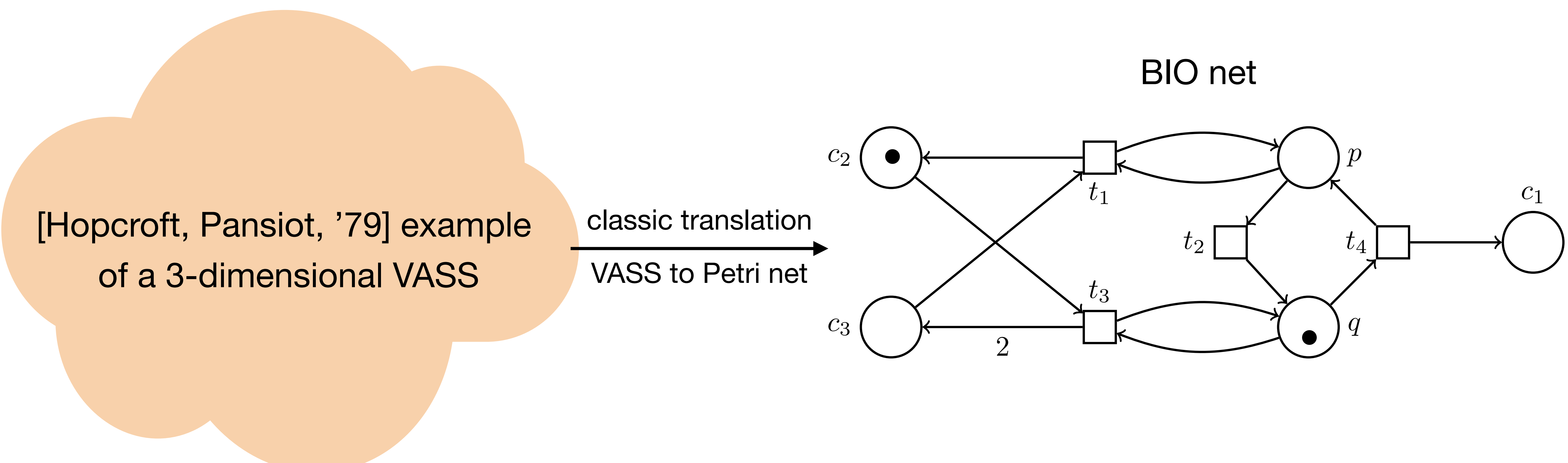

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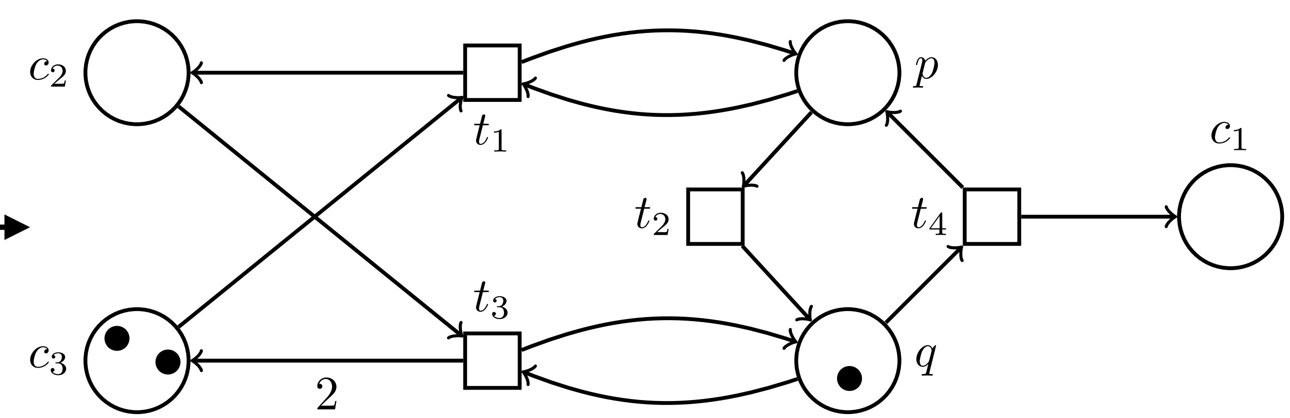

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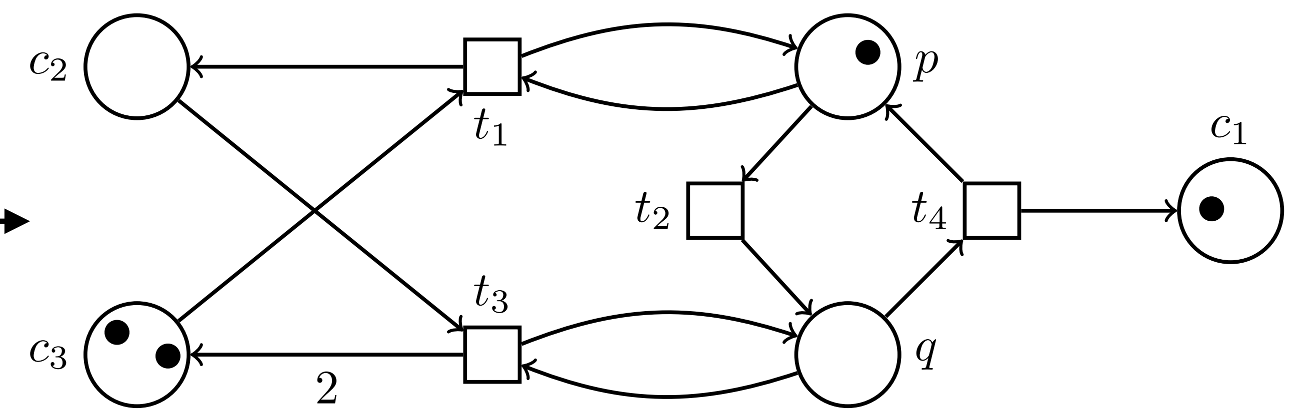

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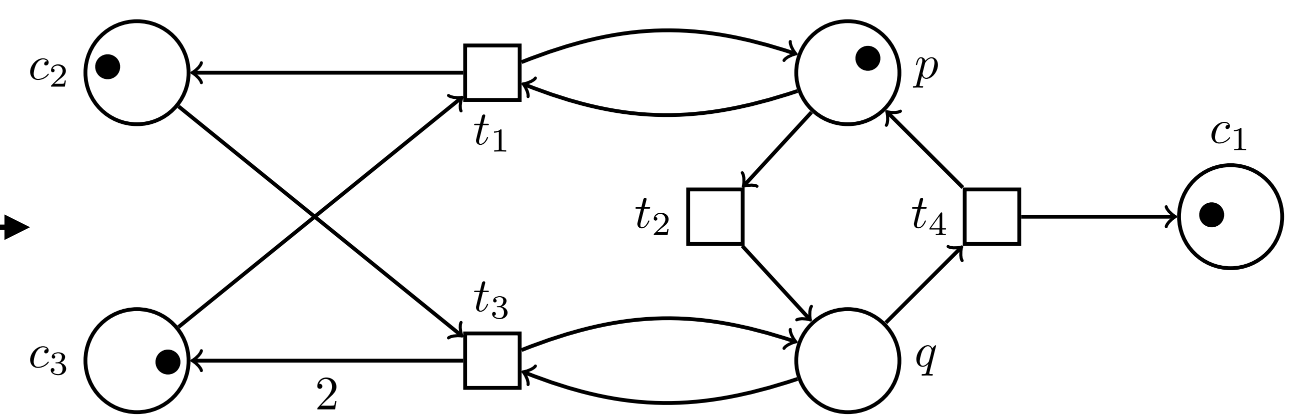

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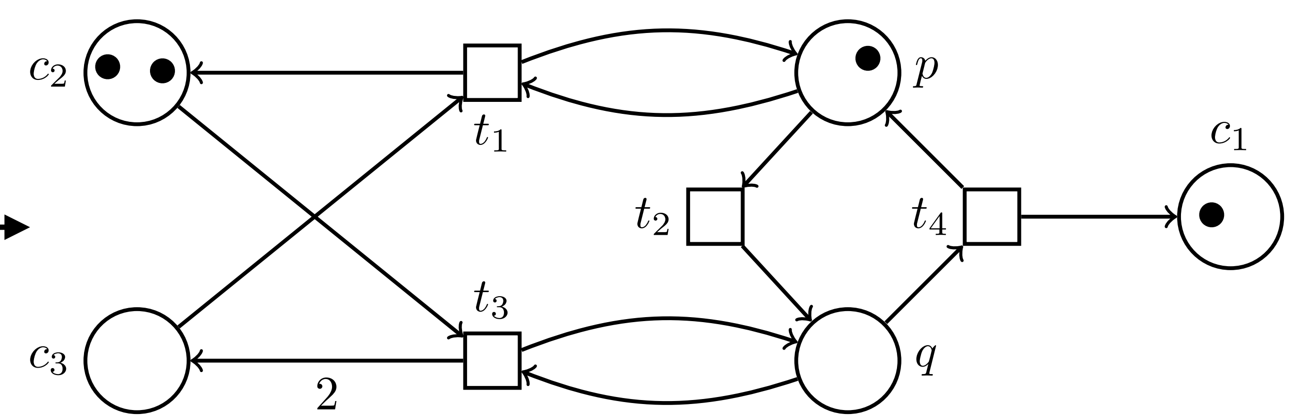

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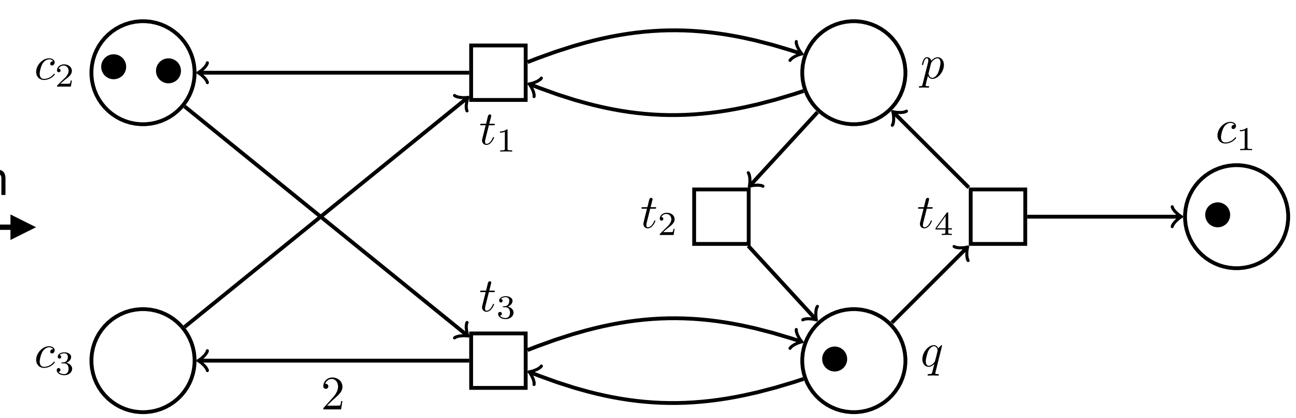

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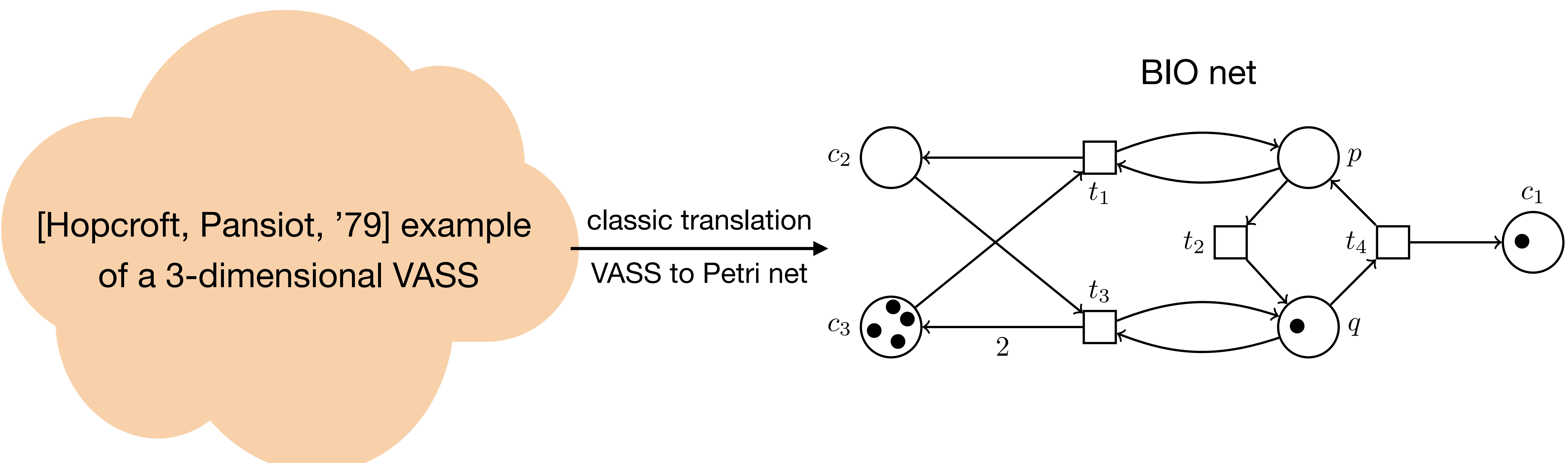

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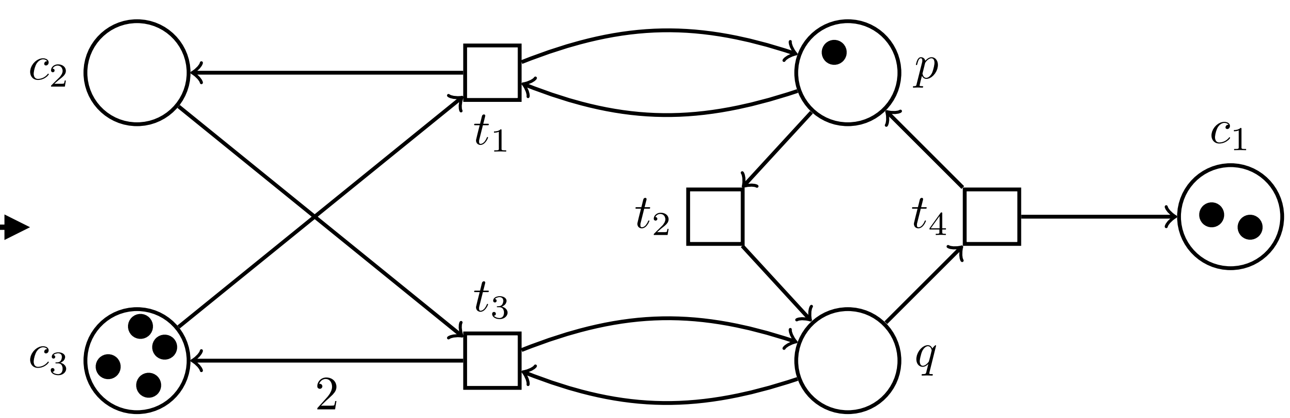

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VASS to Petri net

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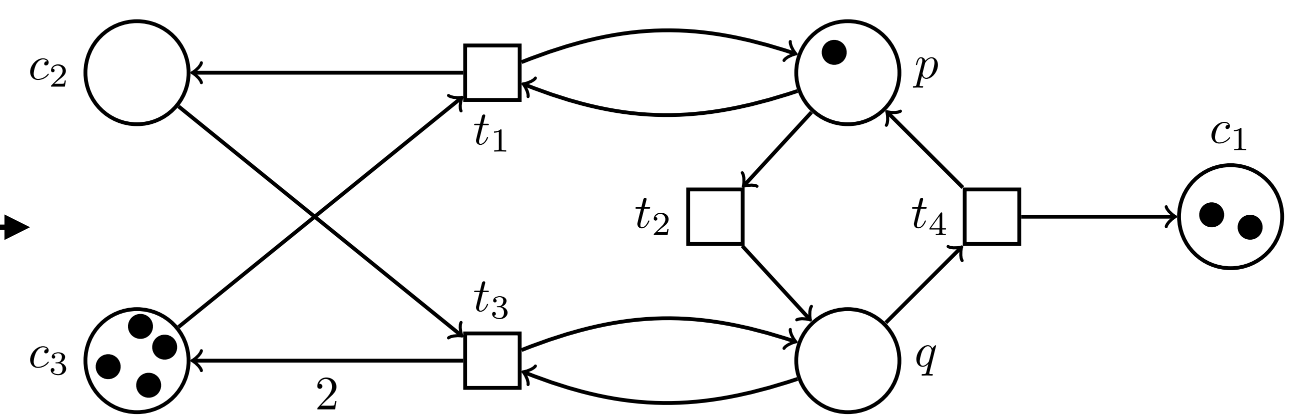

$$c_2 + c_3 \leq 2^{c_1}$$

PSPACE reachability

BIO nets reachability is a **PSPACE-complete** problem

- **PSPACE-hard** by weakly simulating bounded tape Turing machines

PSPACE reachability

BIO nets reachability is a **PSPACE-complete** problem

- **PSPACE-hard** by weakly simulating bounded tape Turing machines
- Solvable in **PSPACE** via a main theorem which provides firing sequences of **bounded length** and **bounded token count**.

PSPACE reachability

PSPACE reachability

Main Theorem

In a BIO net with n places, and transitions producing $\leq \gamma$ *tokens*

$$If \quad M_0 \xrightarrow{*} M$$

then $\exists$ *markings* $M_1, M_2, \ldots, M_l$

$\exists$ *transitions* $t_1, t_2, \ldots, t_l$ $\qquad M_0 \xrightarrow{t_1^{k_1}} M_1 \xrightarrow{t_2^{k_2}} M_2 \to \ldots \xrightarrow{t_l^{k_l}} M_l = M$

$\exists$ *constants* $k_1, k_2, \ldots k_l \geq 0$

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such that $l \in O(|M| n)^n$ bound on (accelerated) length

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and $\forall i,\ M_i \in O(|M_o||M|n\gamma)^n$ bound on token count

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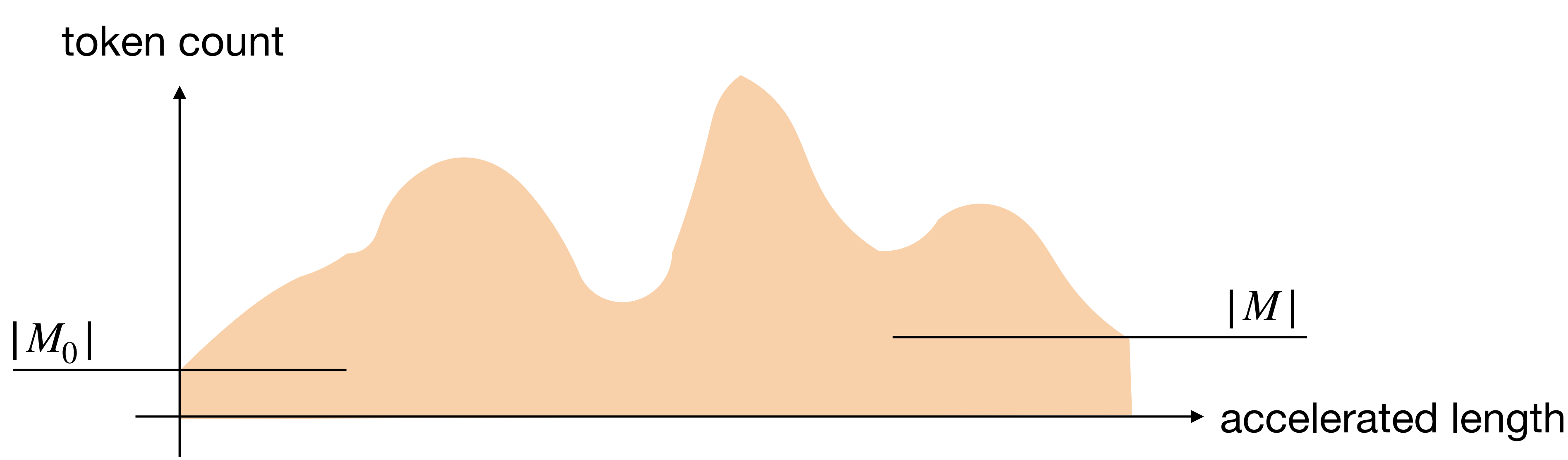

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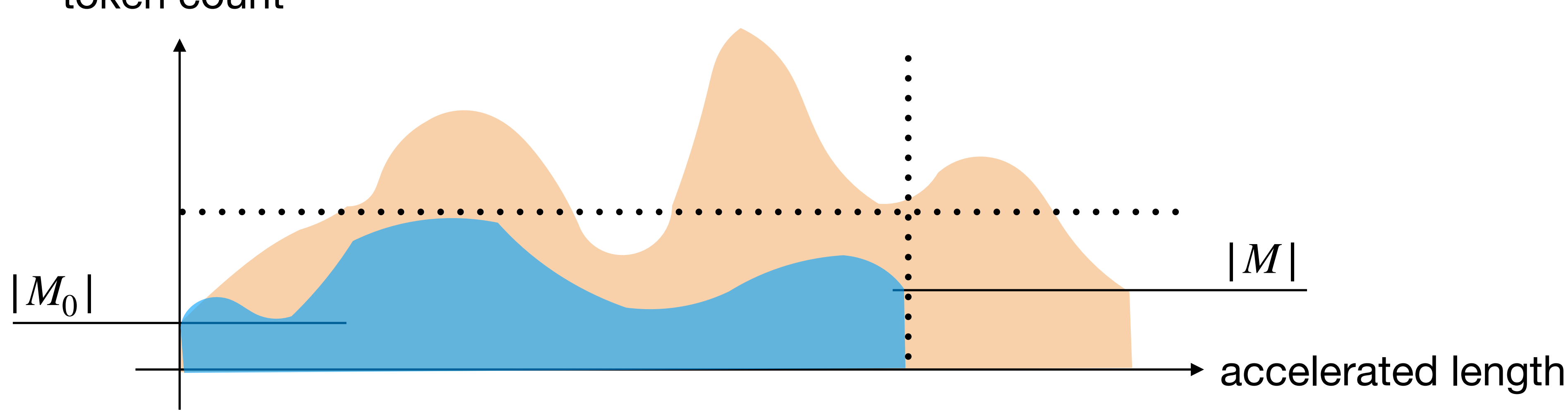

PSPACE reachability

Main Theorem

In a BIO net with n places, and transitions producing $\leq \gamma$ *tokens*

$$If \;\; M_0 \xrightarrow{*} M$$

$$then \;\; \exists \; markings \; M_1, M_2, \ldots, M_l \quad \exists \; transitions \; t_1, t_2, \ldots, t_l \quad \exists \; constants \; k_1, k_2, \ldots k_l \geq 0$$

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$$such \; that \;\; l \in O(|M|n)^n \;\; and \;\; \forall i, \; M_i \in O(|M_o||M|n\gamma)^n$$

NPSPACE algorithm for reachability

$$M_0 \xrightarrow{*} M \;\; ?$$

PSPACE reachability

Main Theorem

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NPSPACE algorithm for reachability

$$M_0 \xrightarrow{*} M\ ?$$

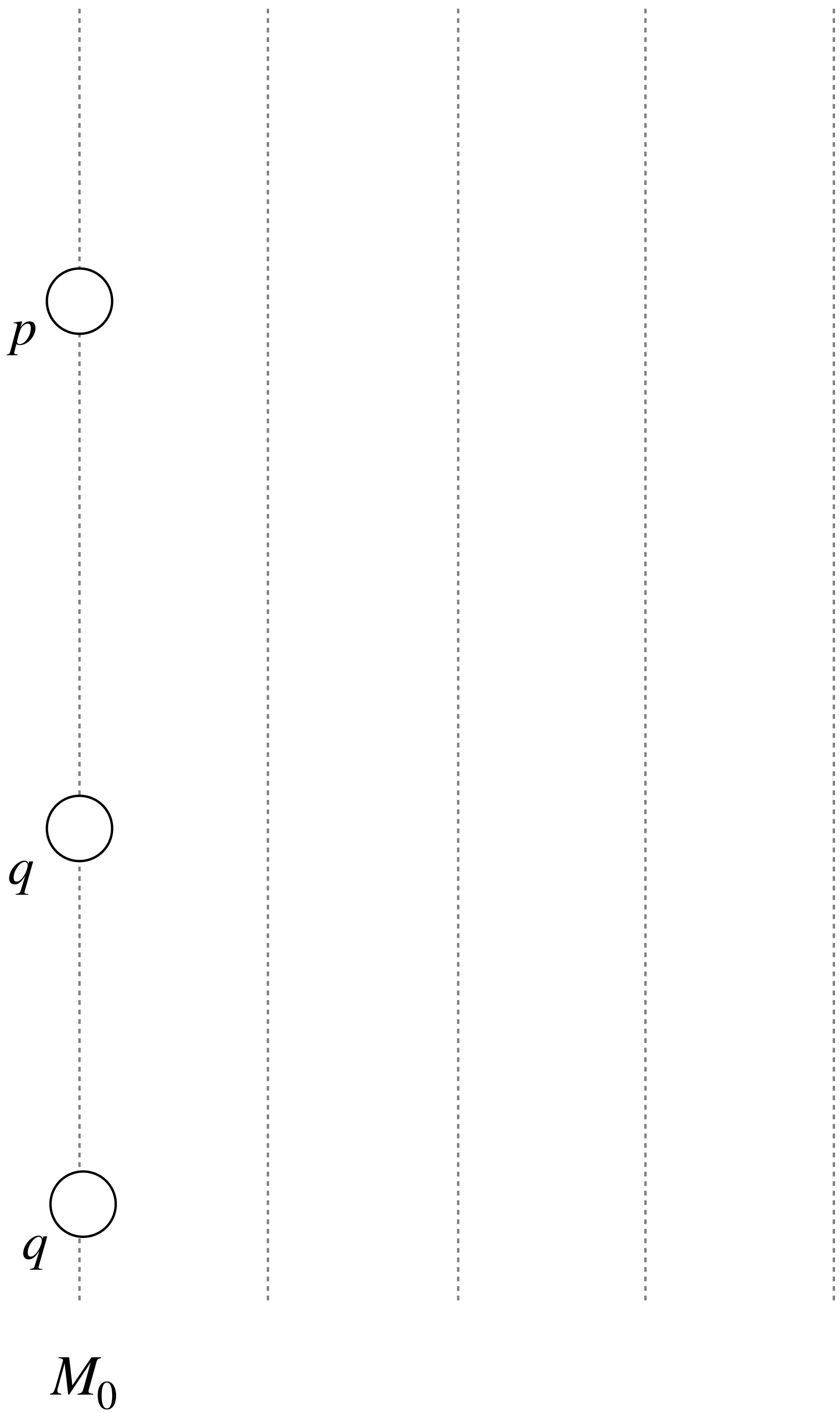
Guess the first marking M_1

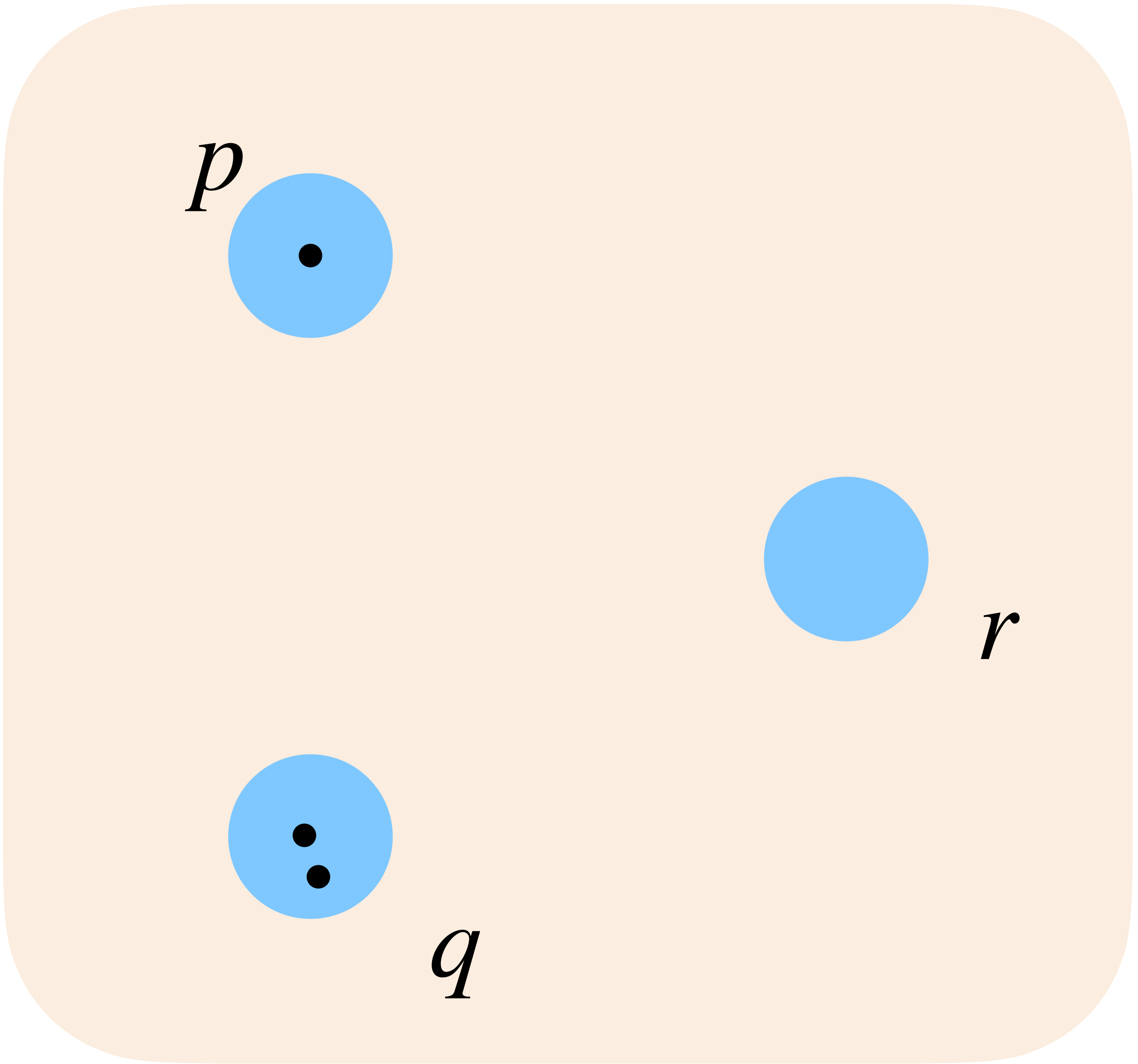
Check that there $\exists t, k$ such that $M_0 \xrightarrow{t^k} M_1$

Guess the next marking M_2

...

PSPACE reachability





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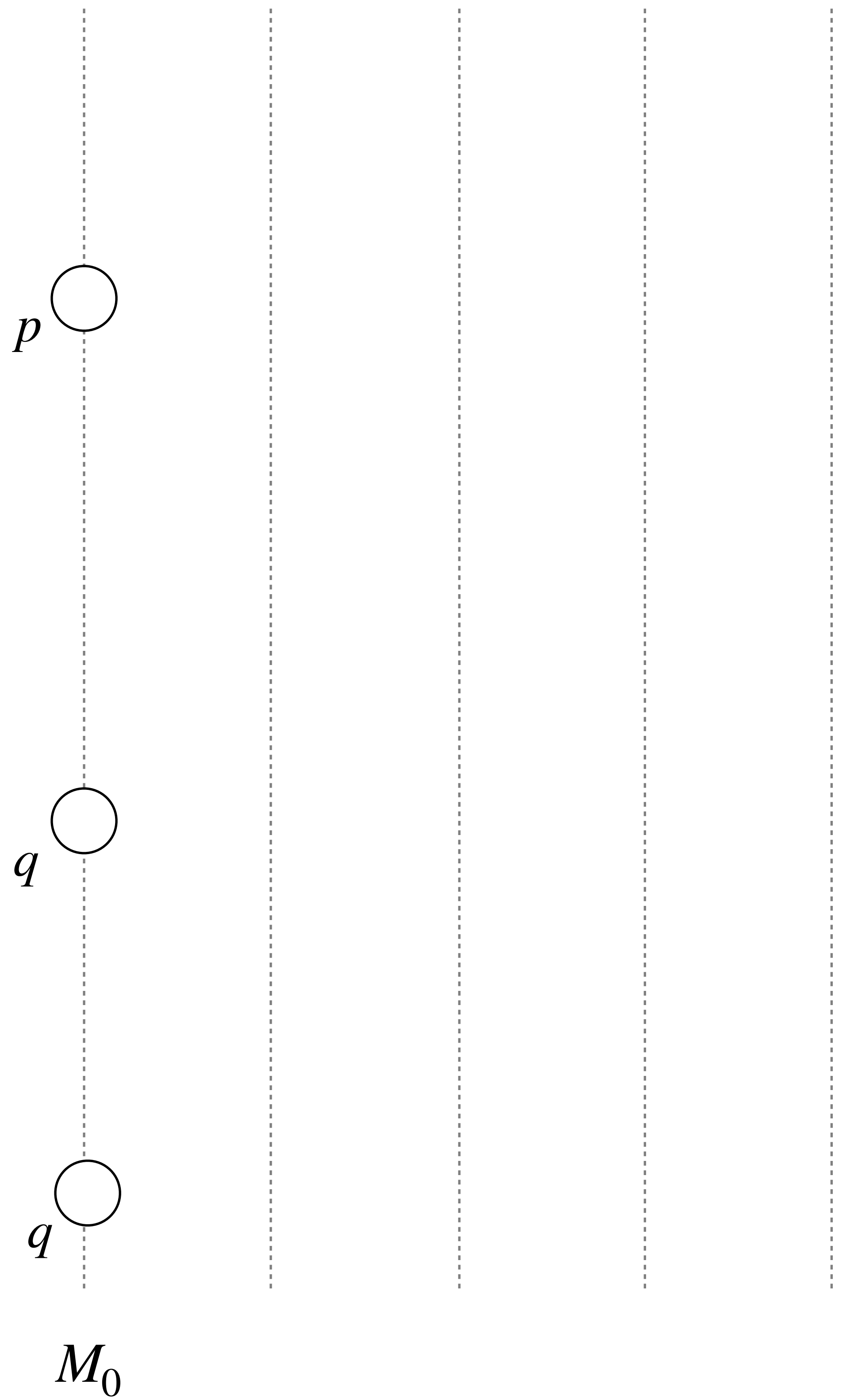

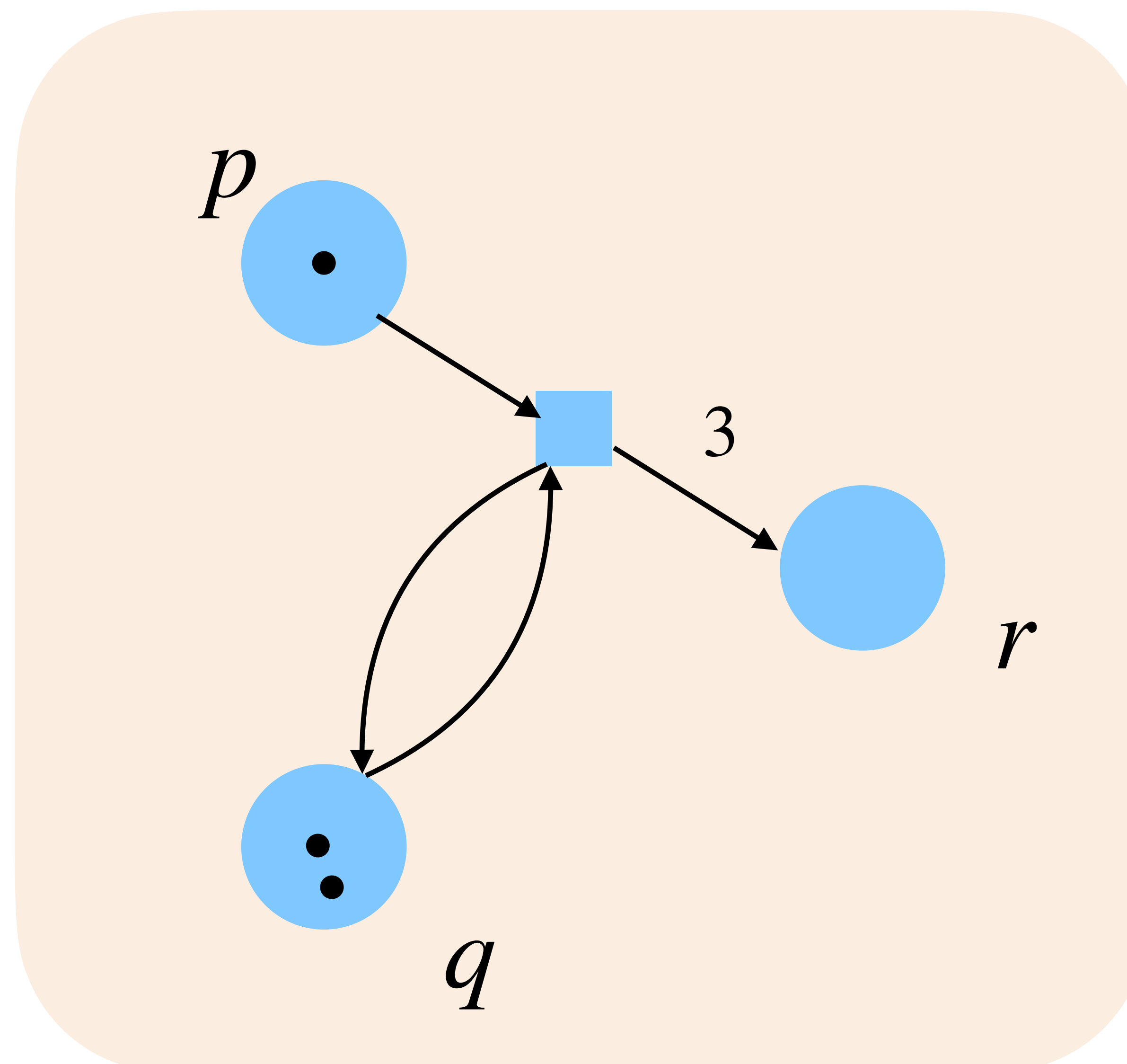

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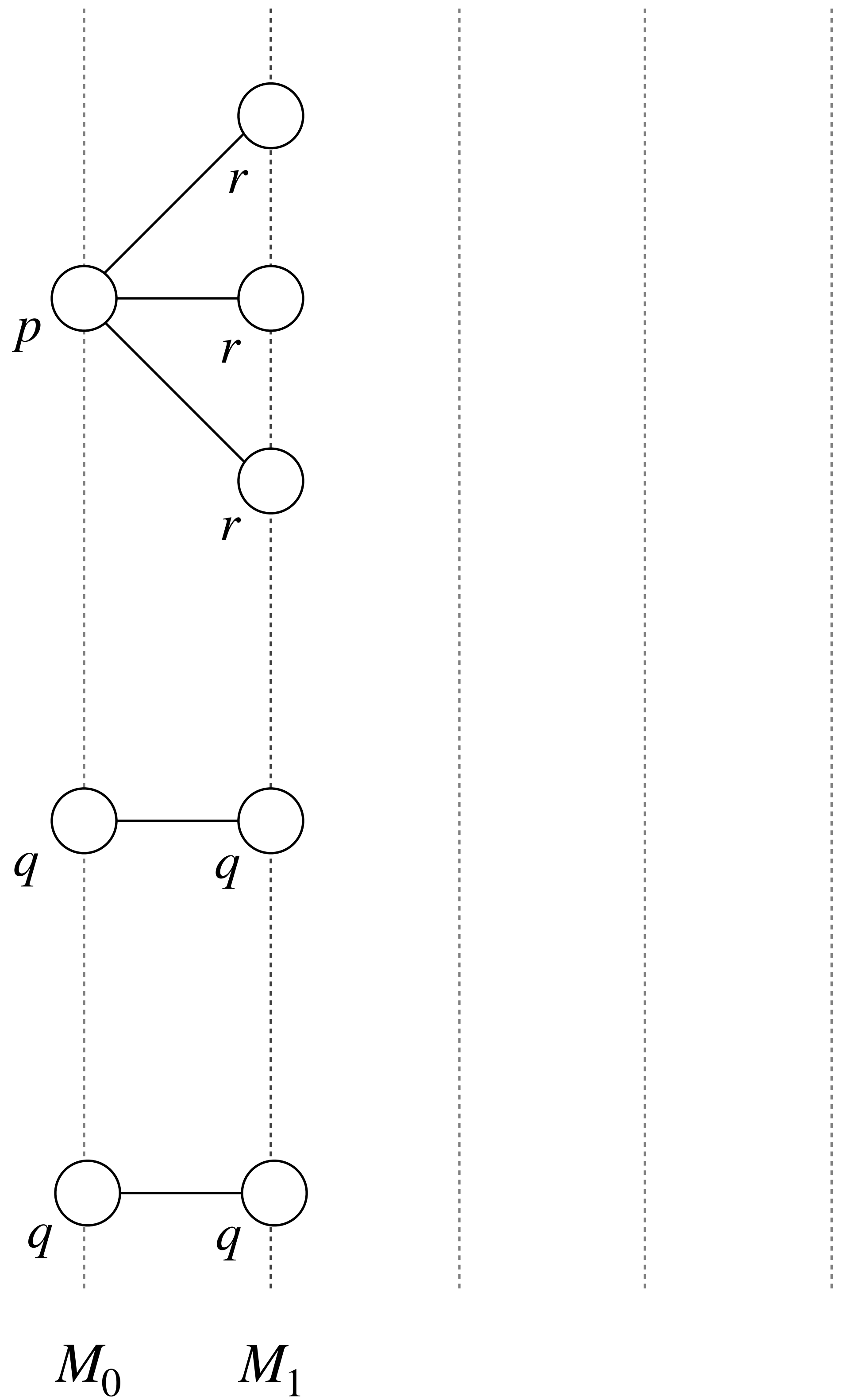
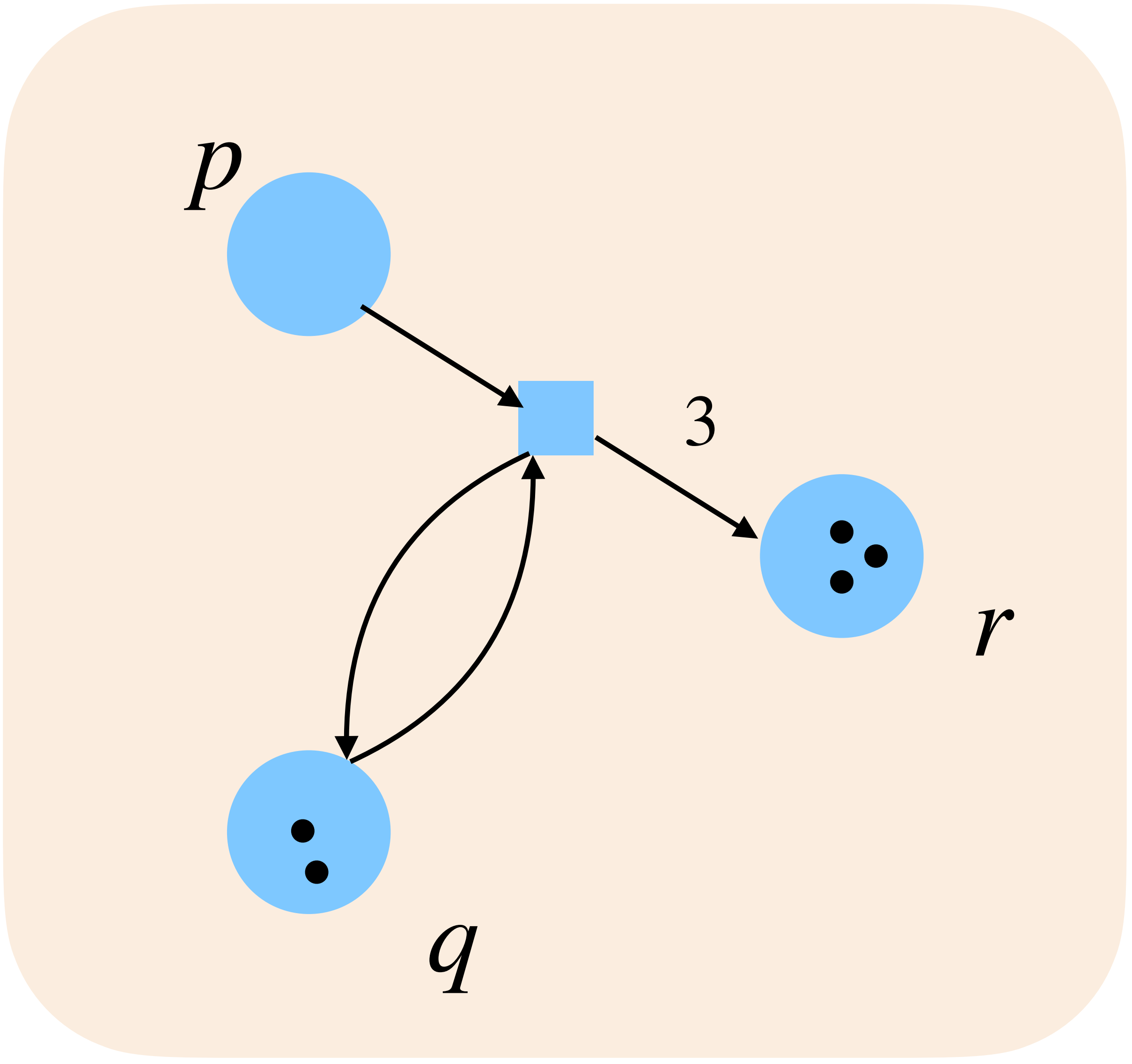
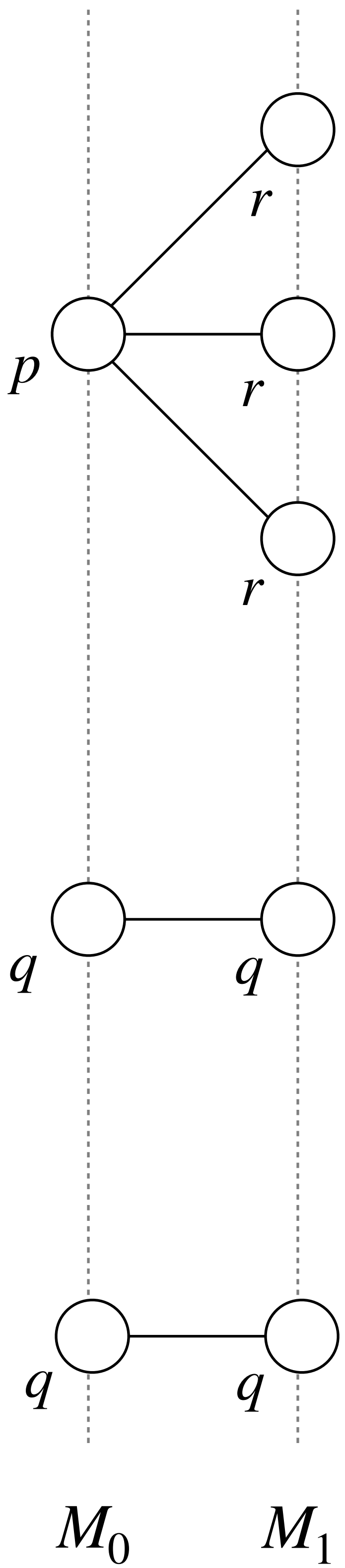
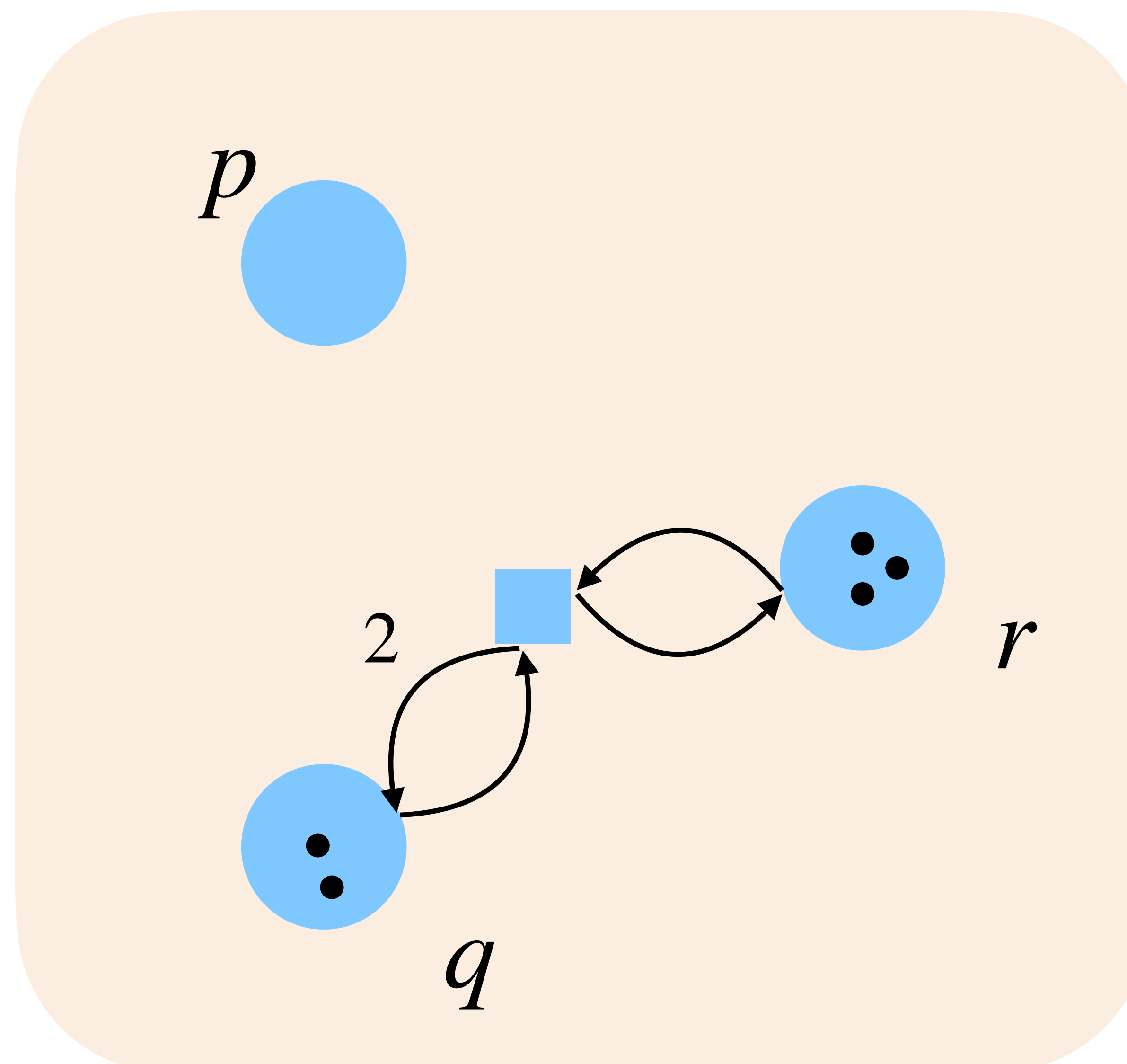




C. Weil-Kennedy, TUM

PSPACE reachability





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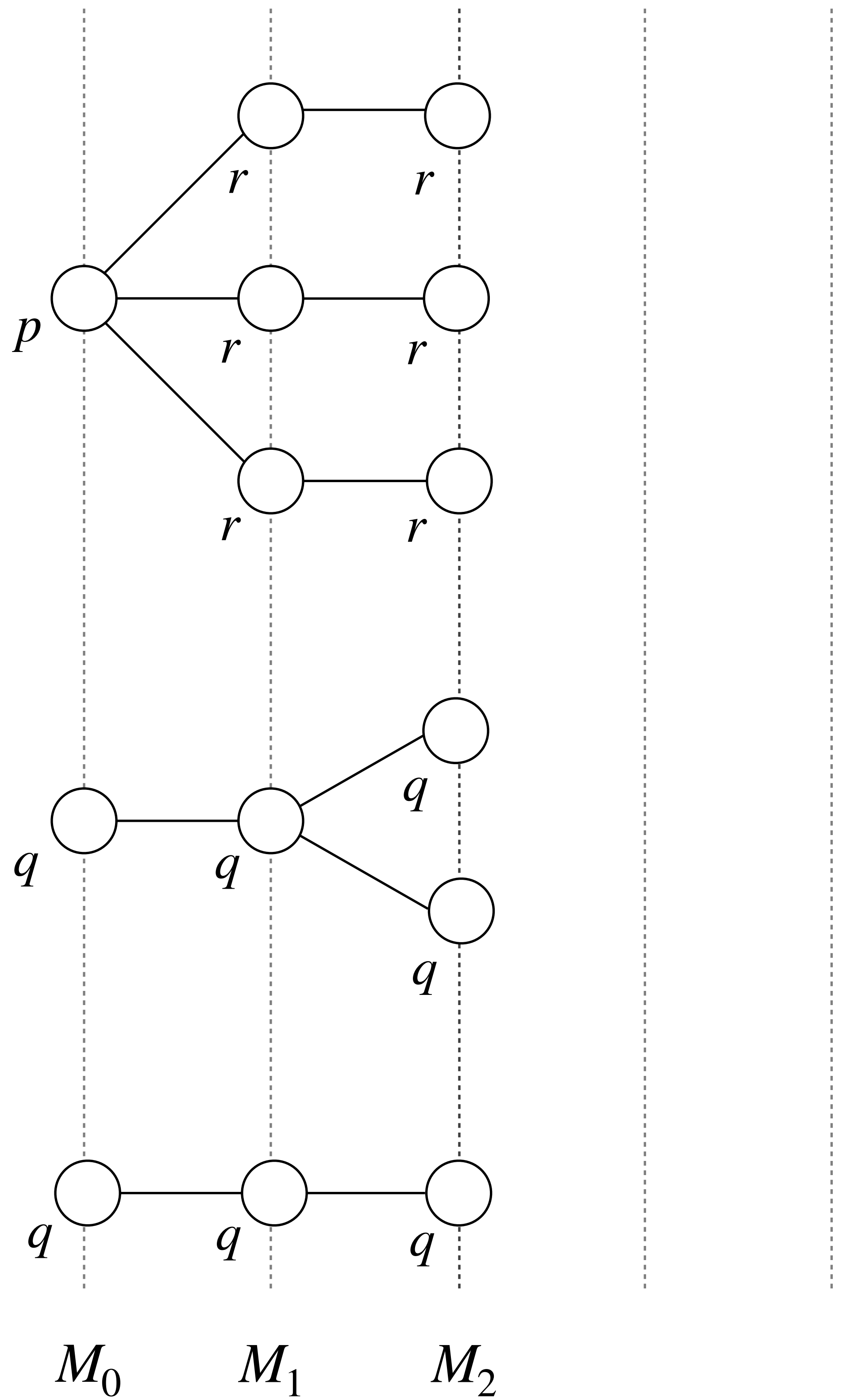

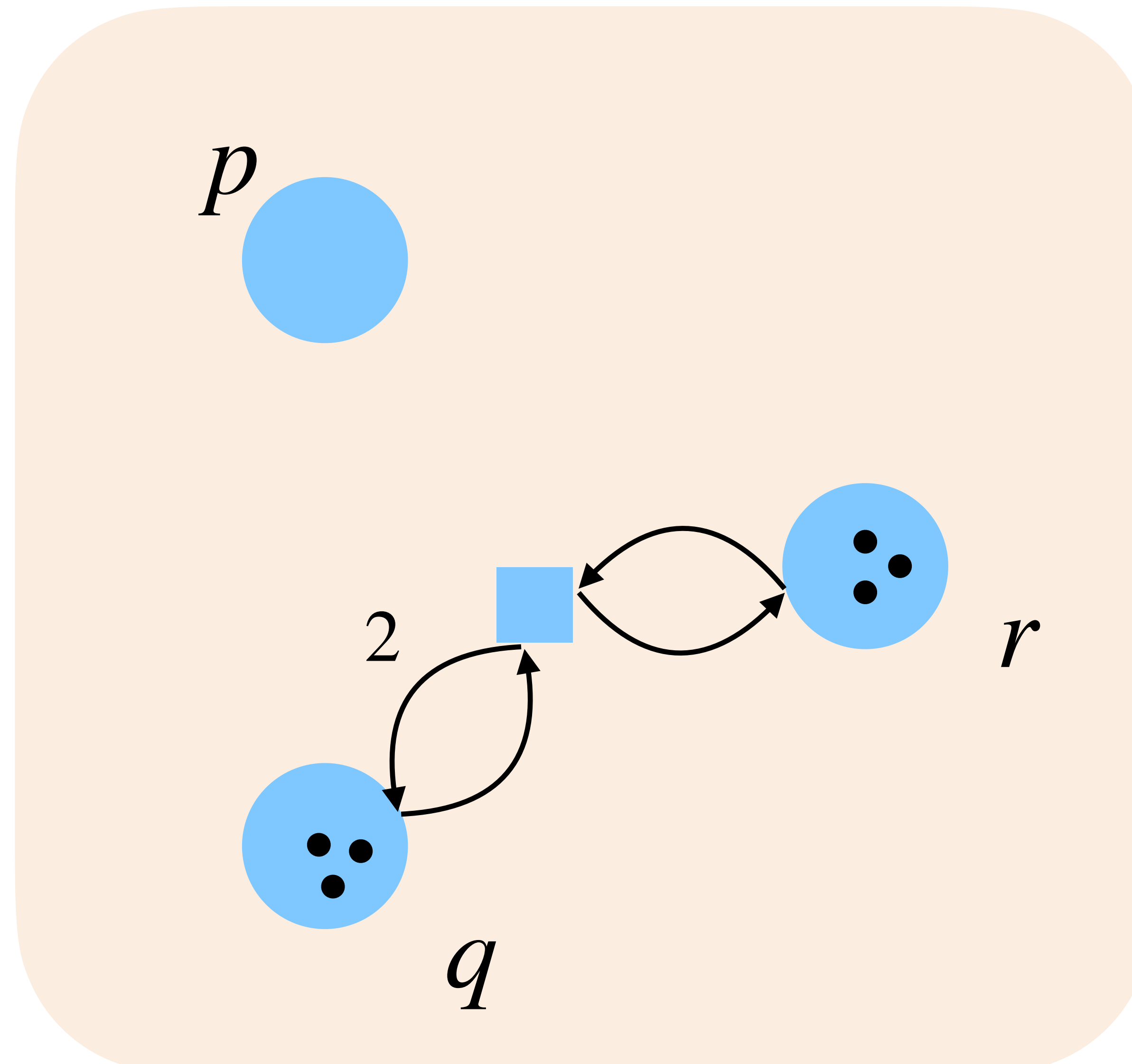

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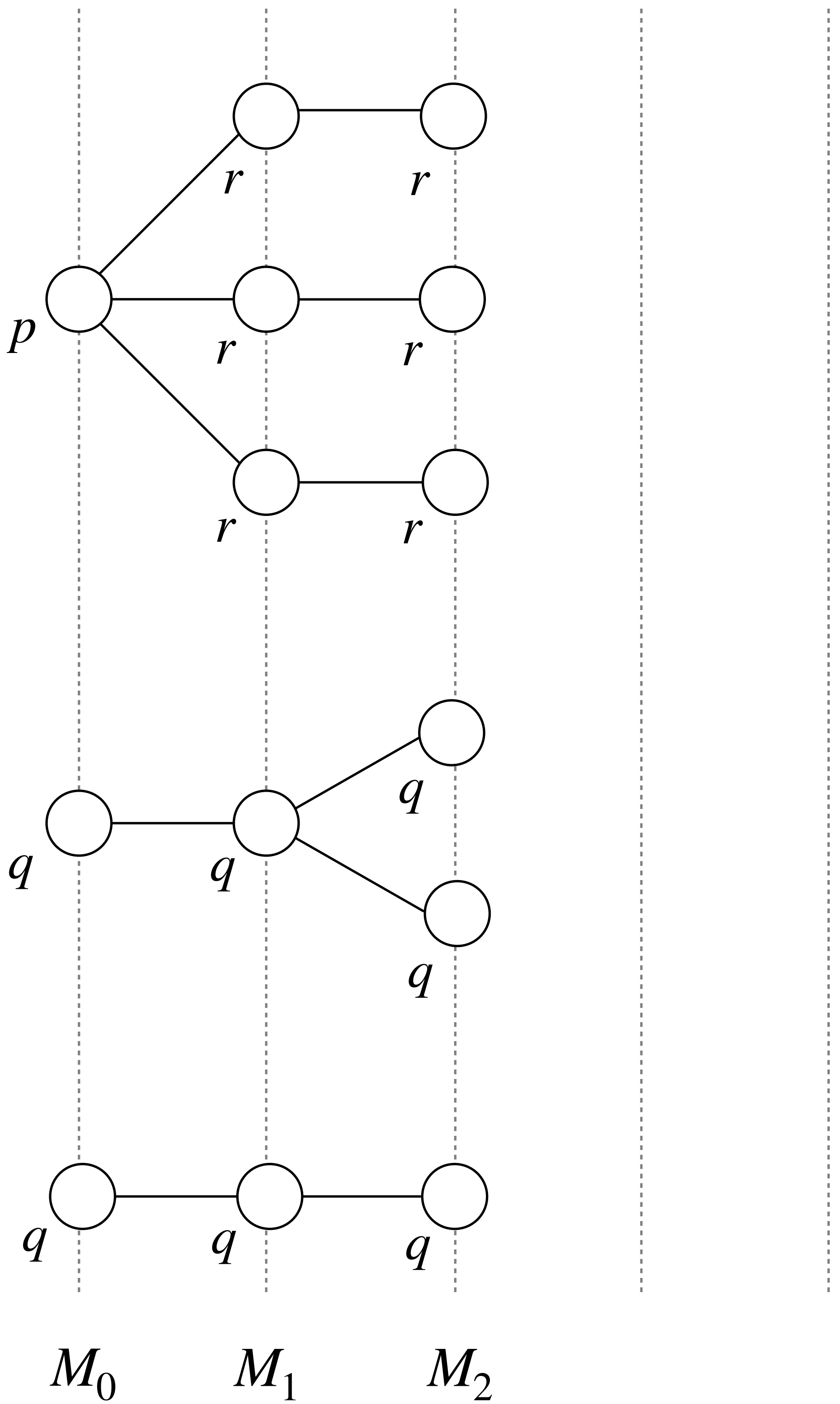

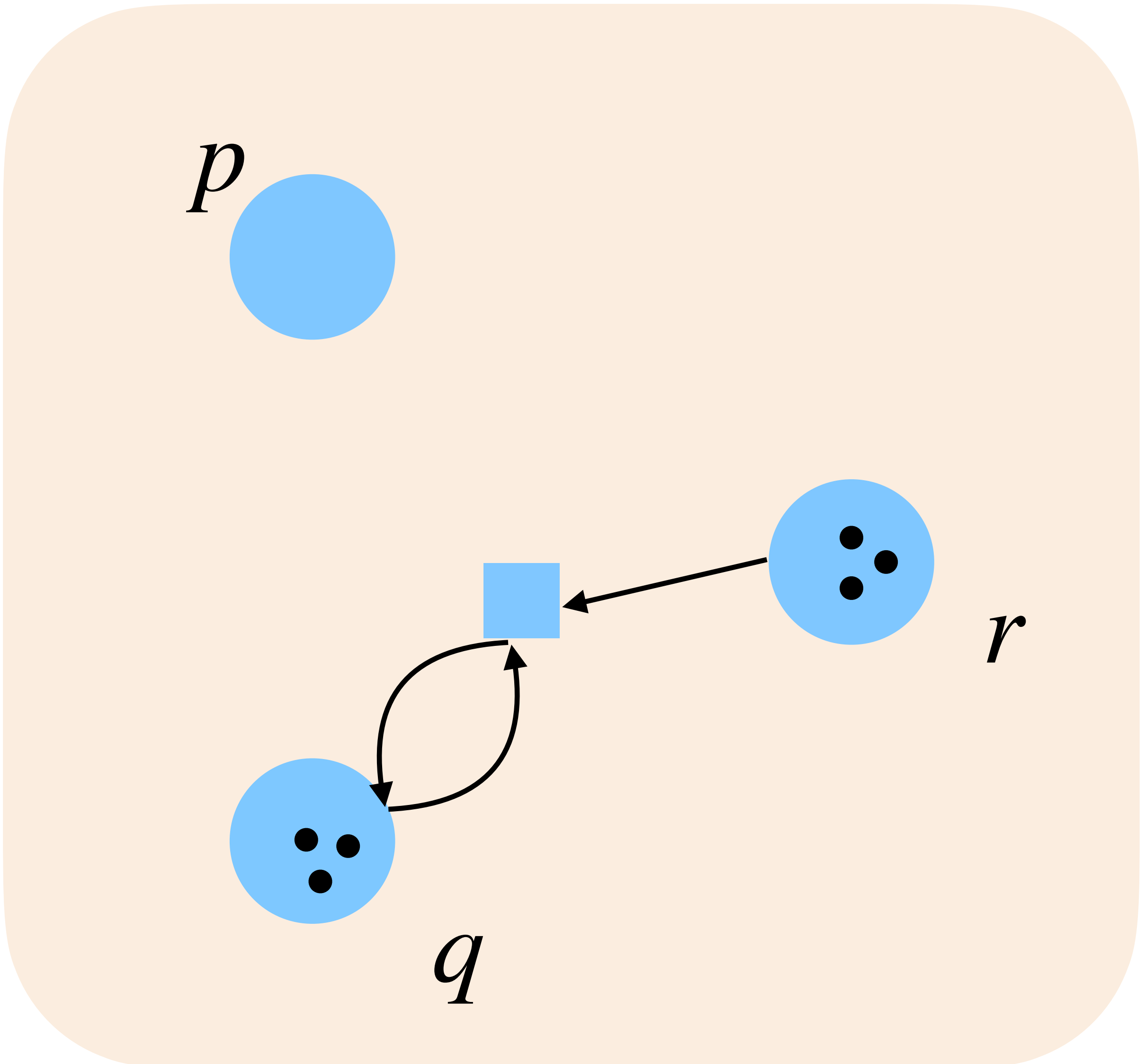

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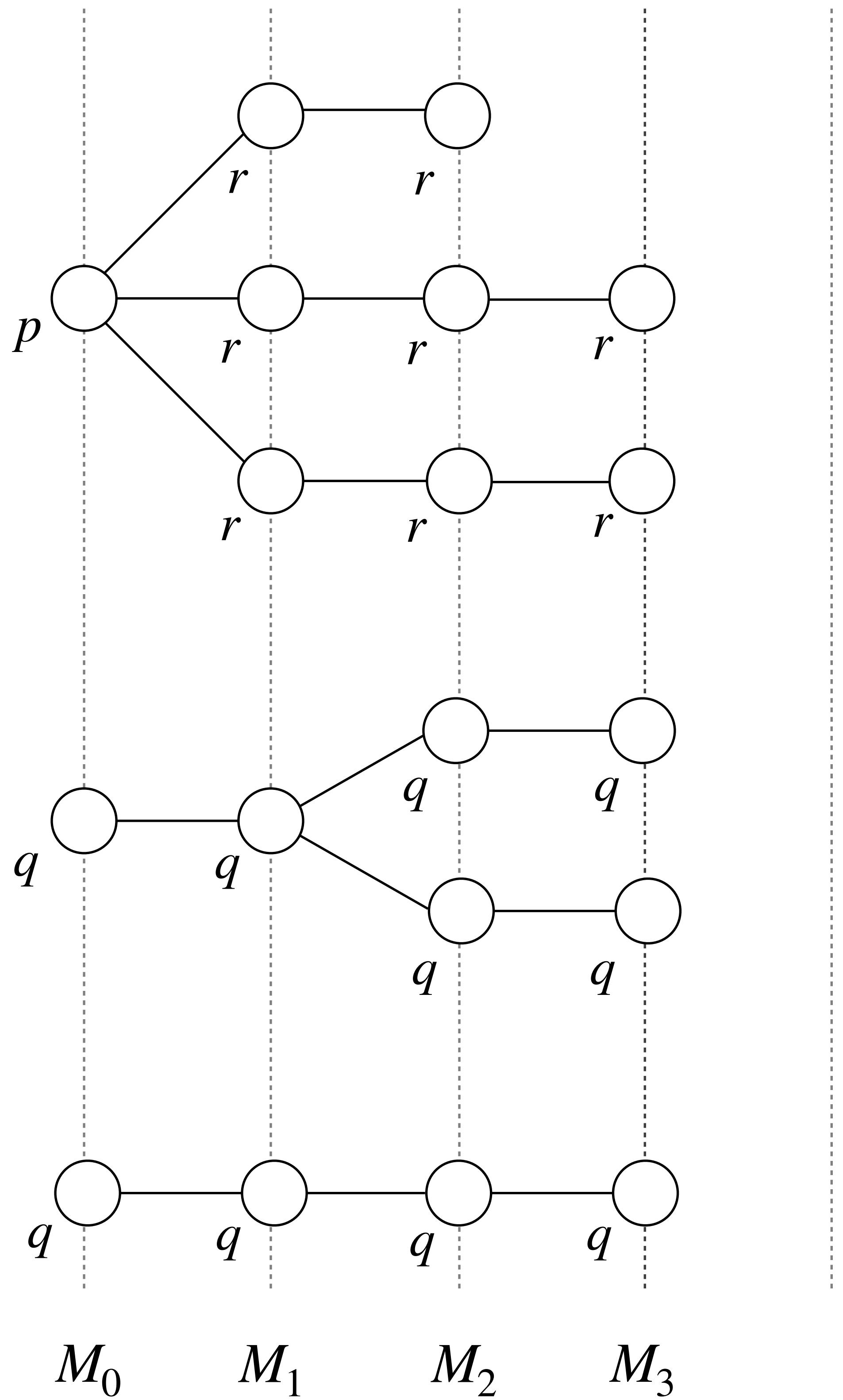

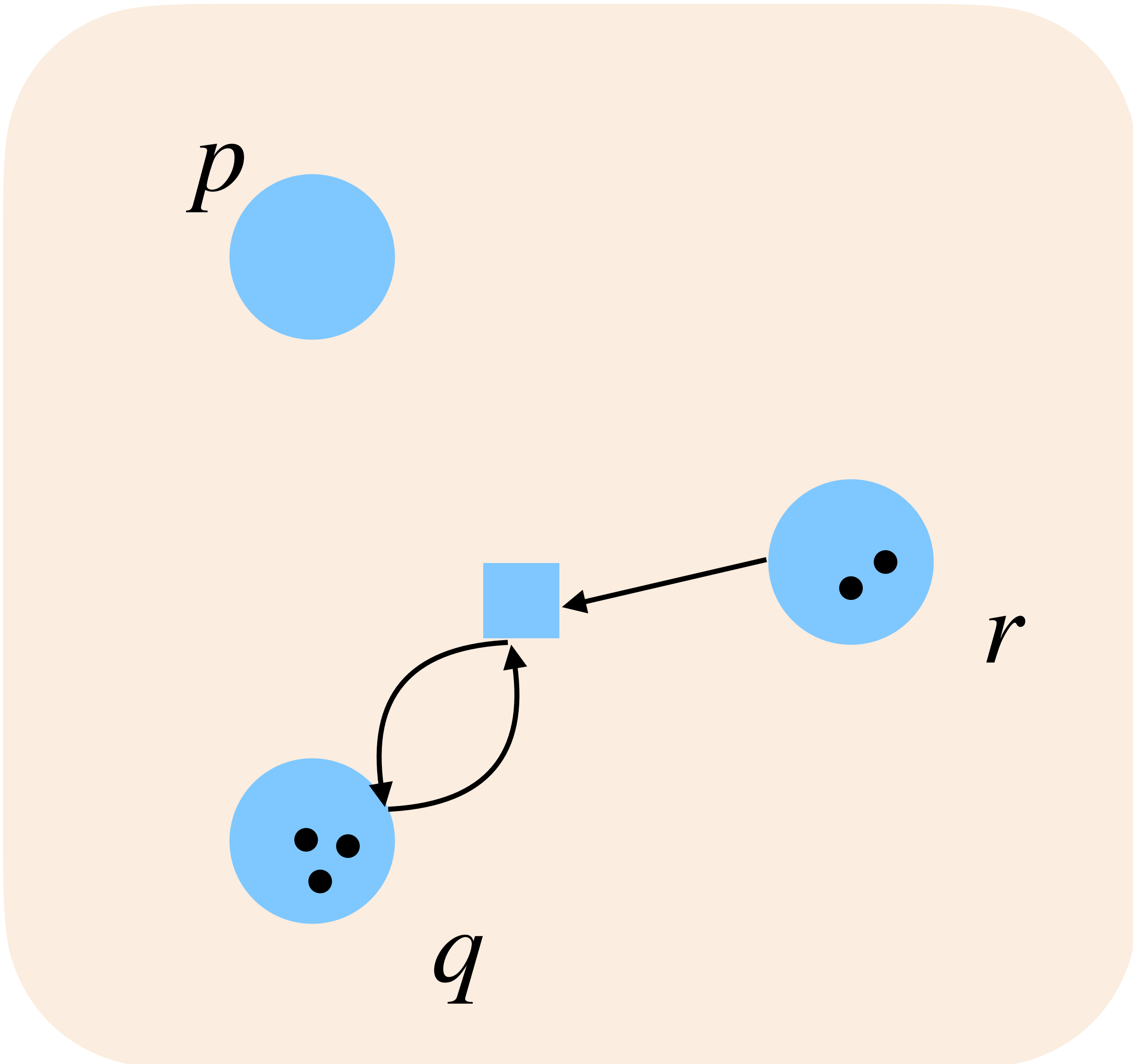

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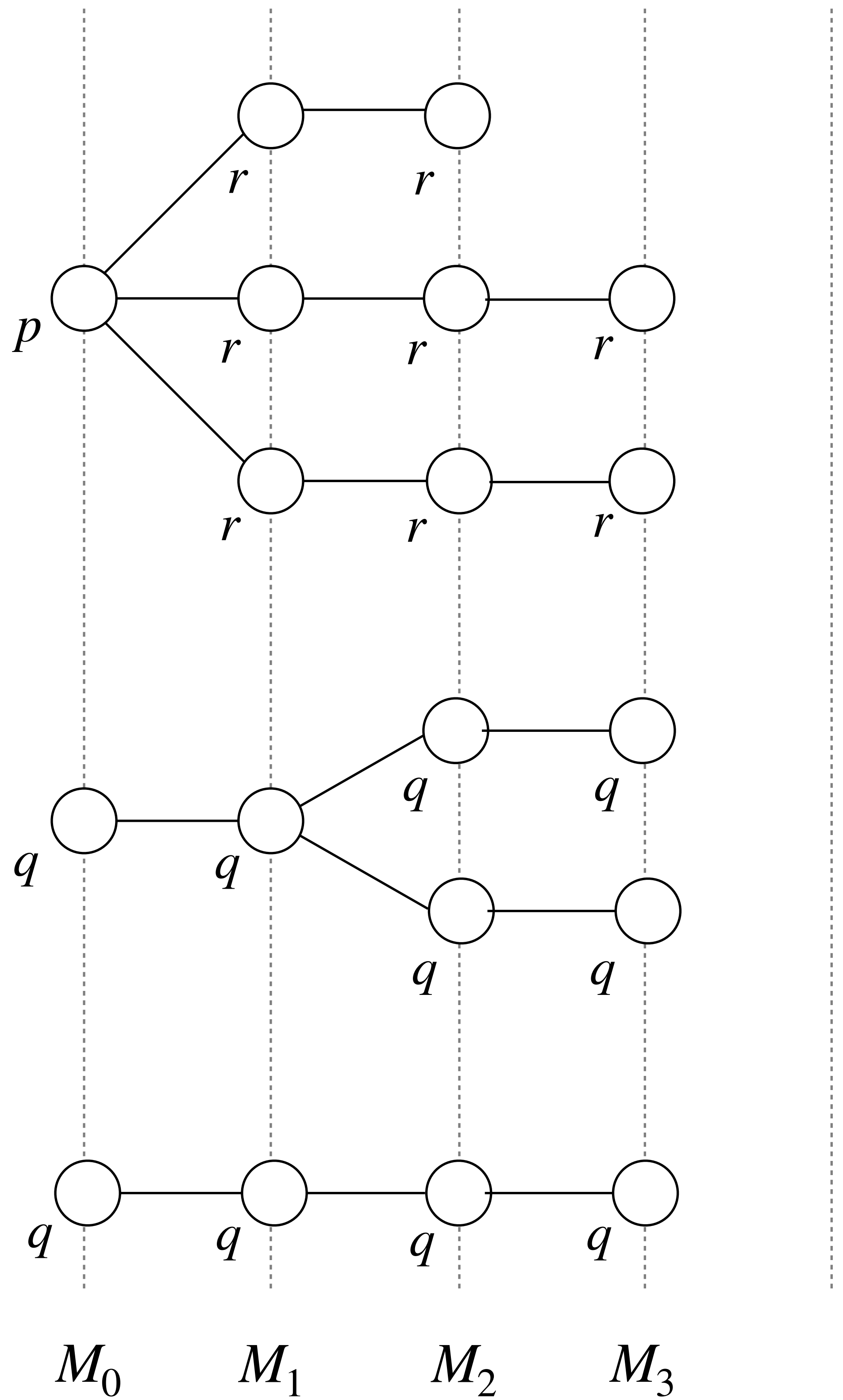

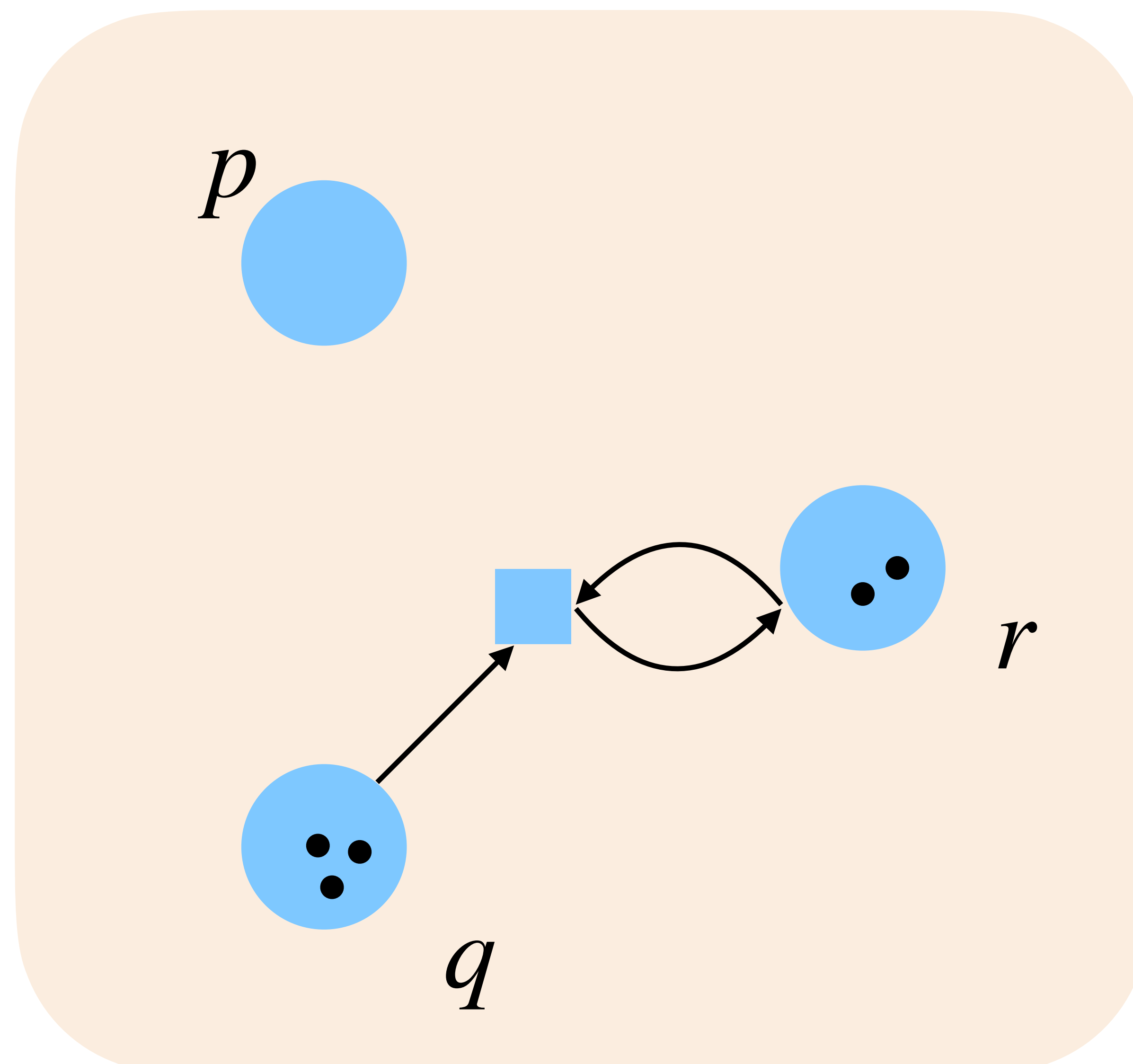

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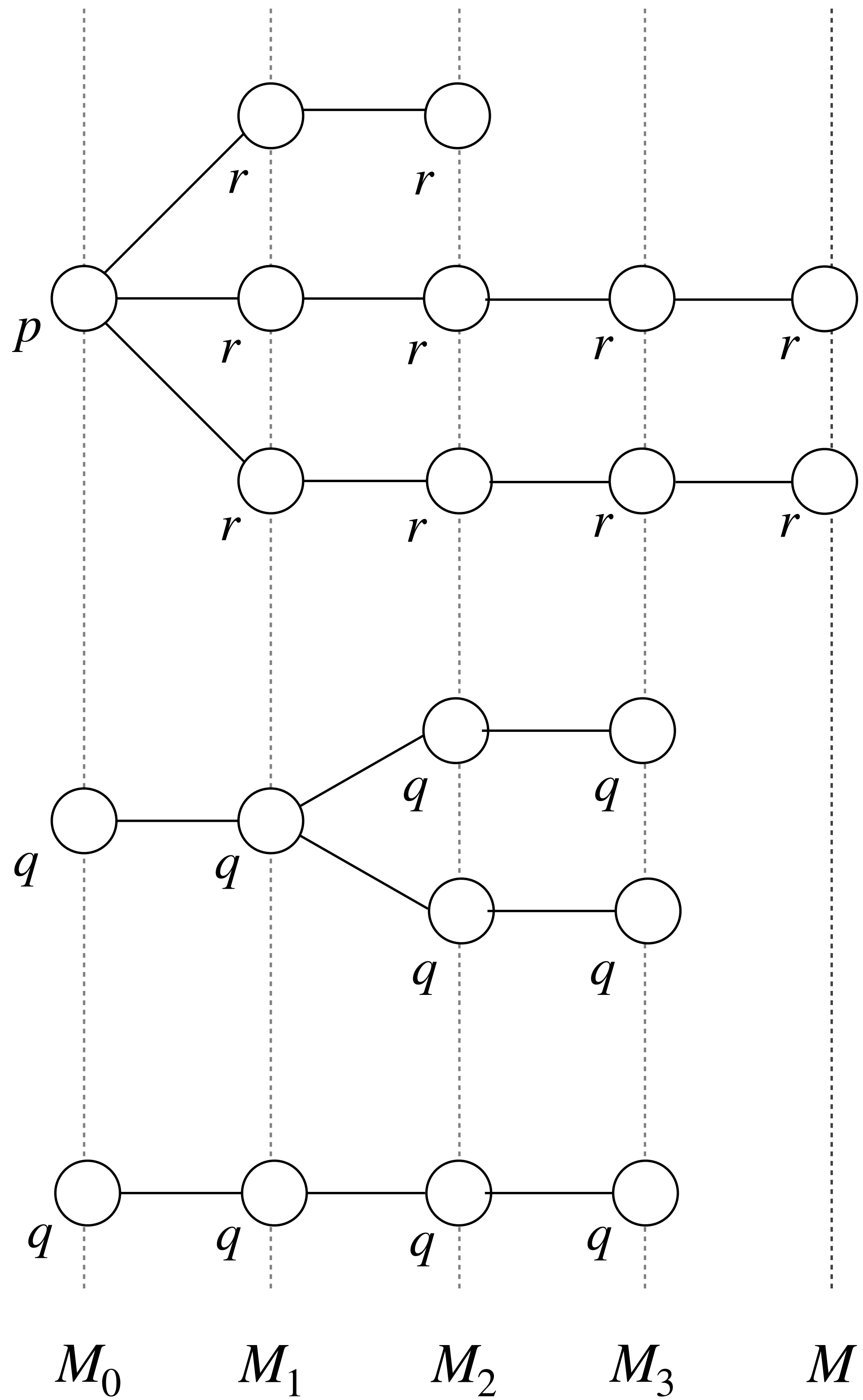

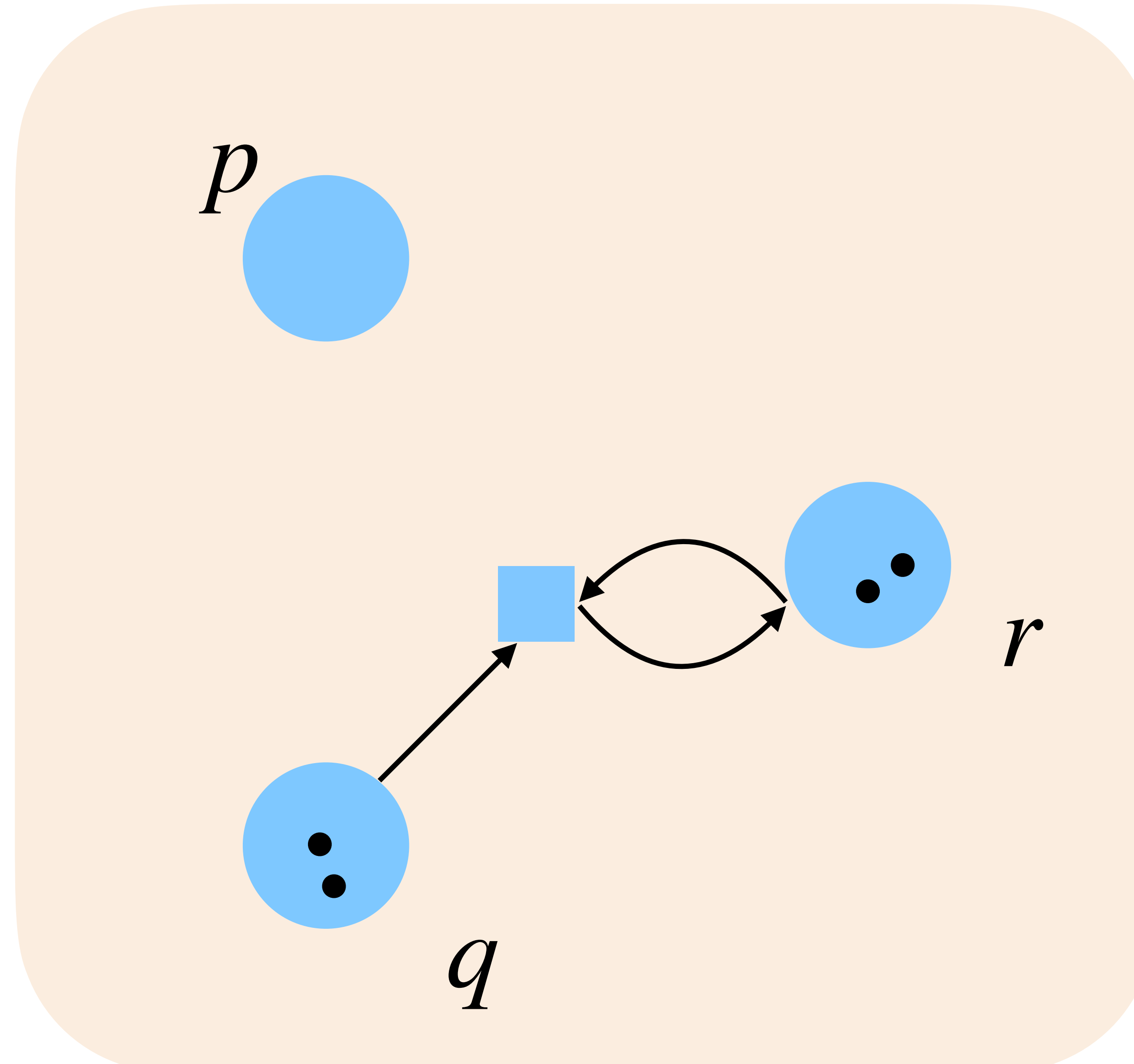

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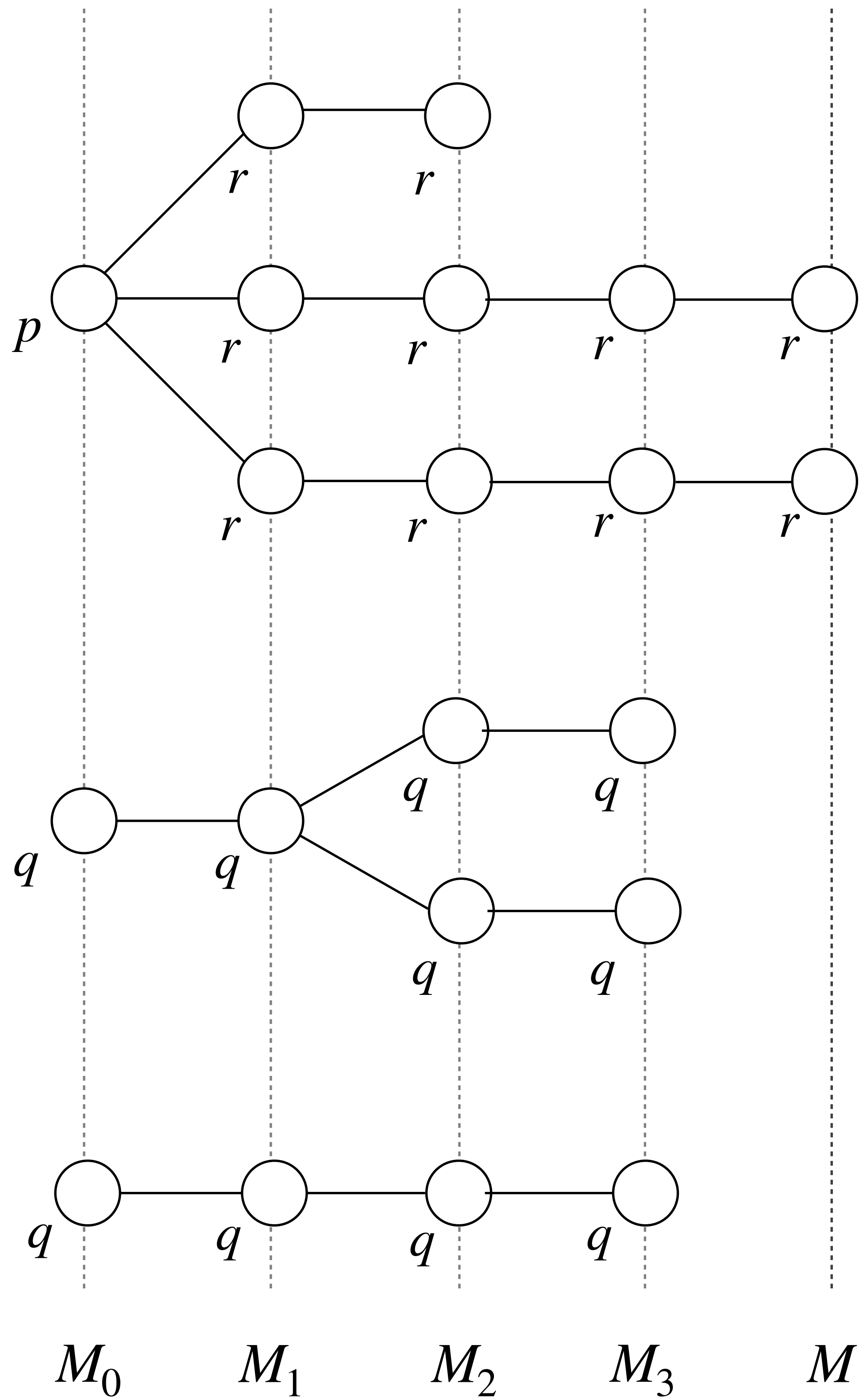

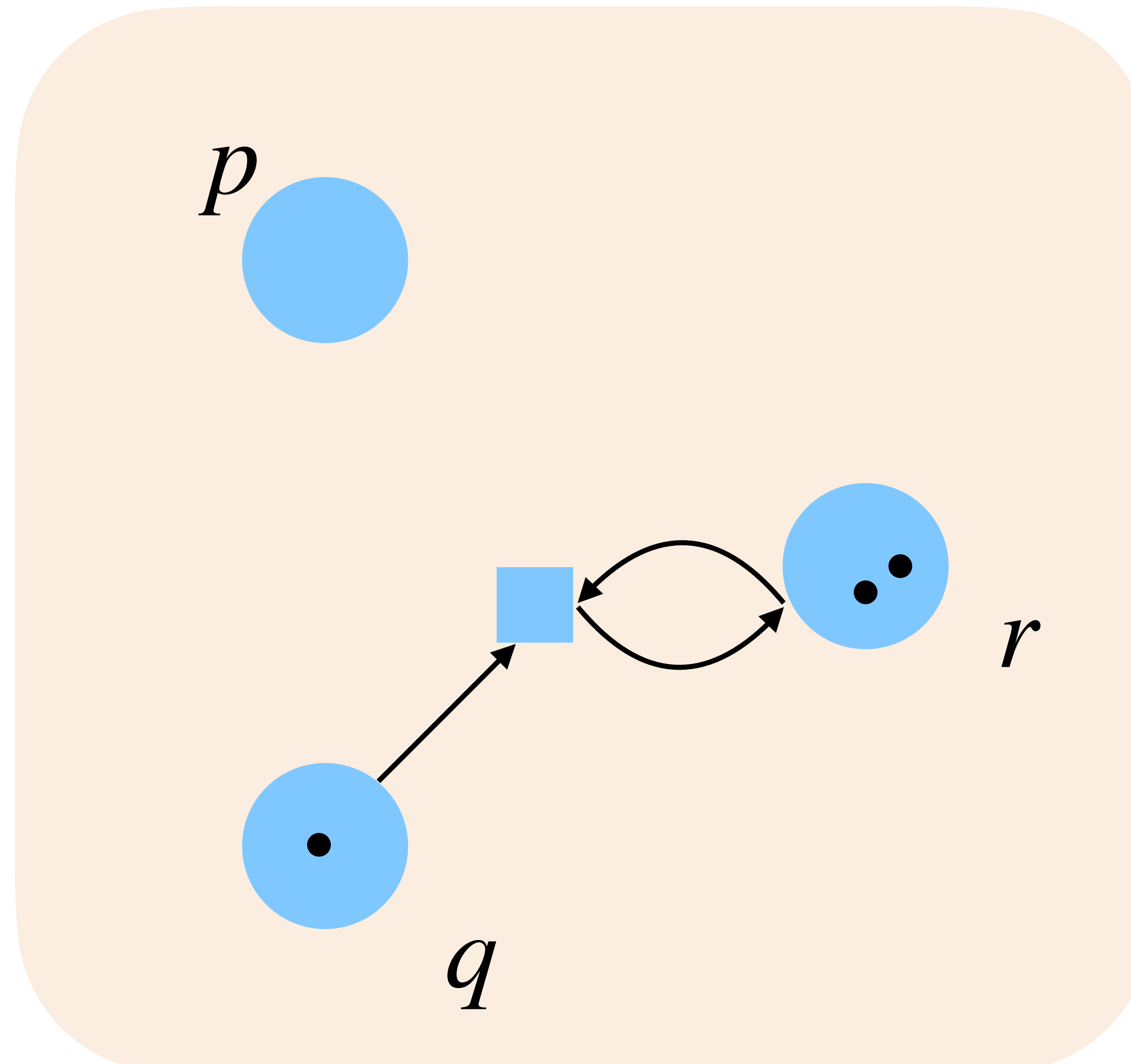

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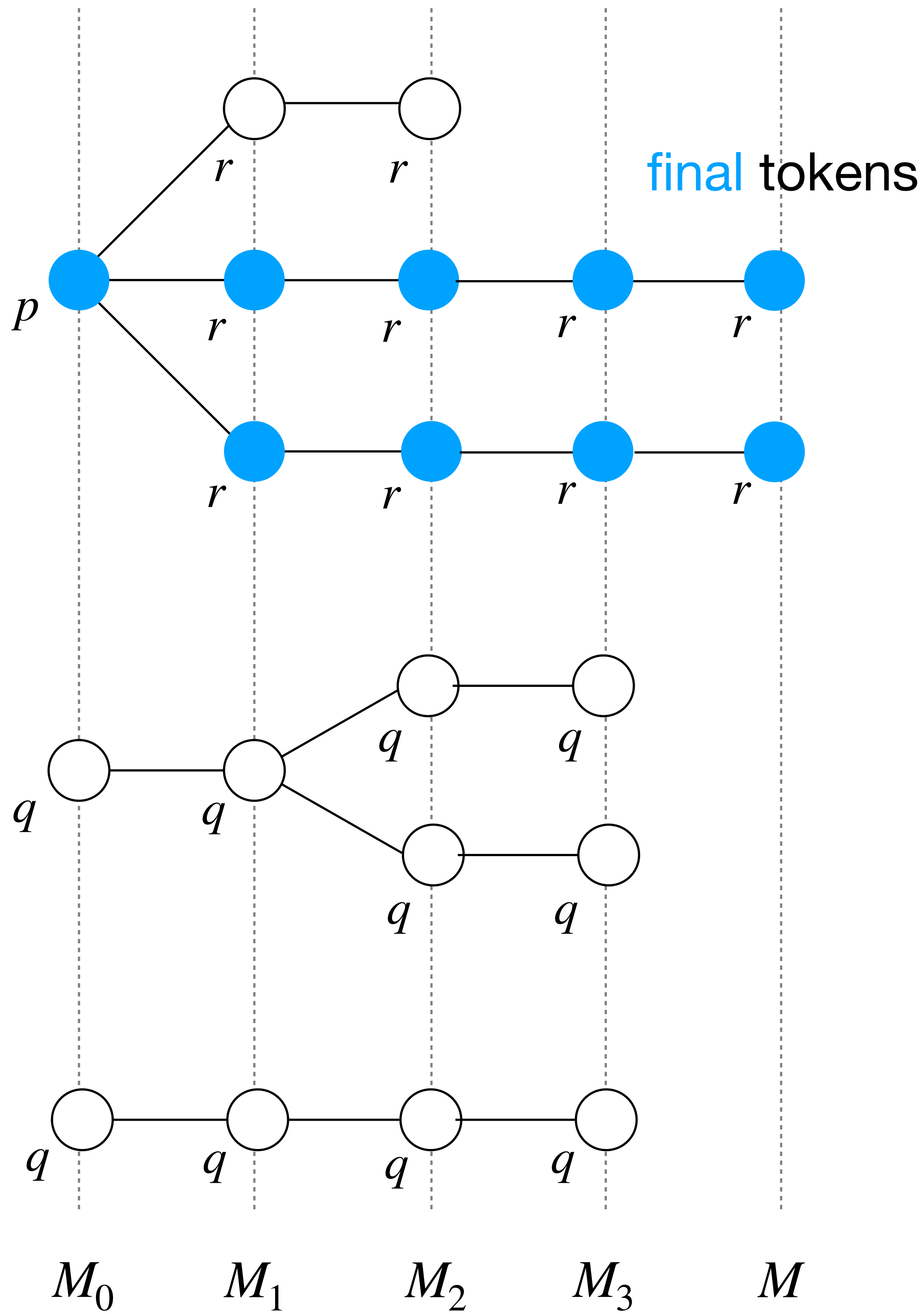

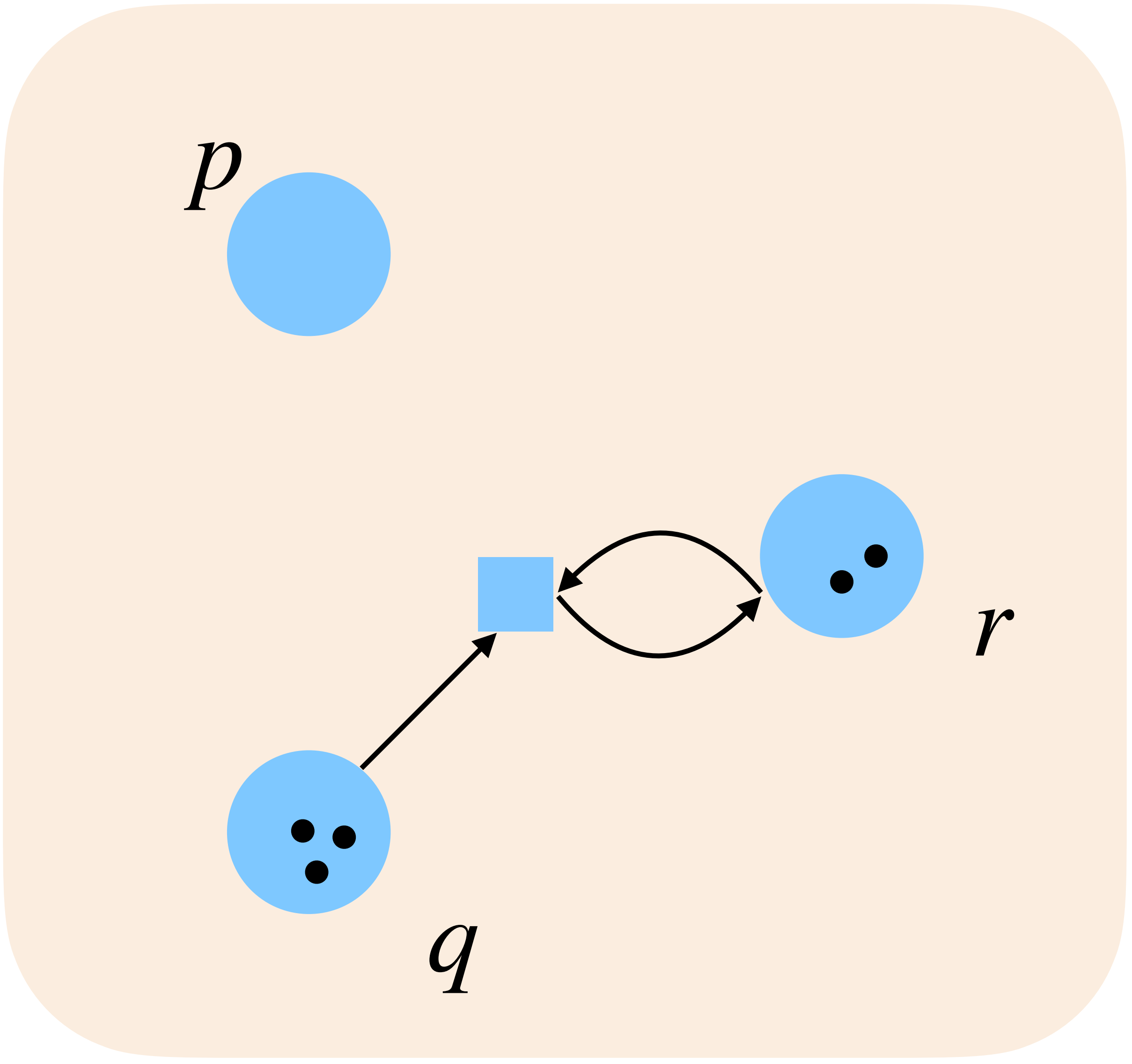

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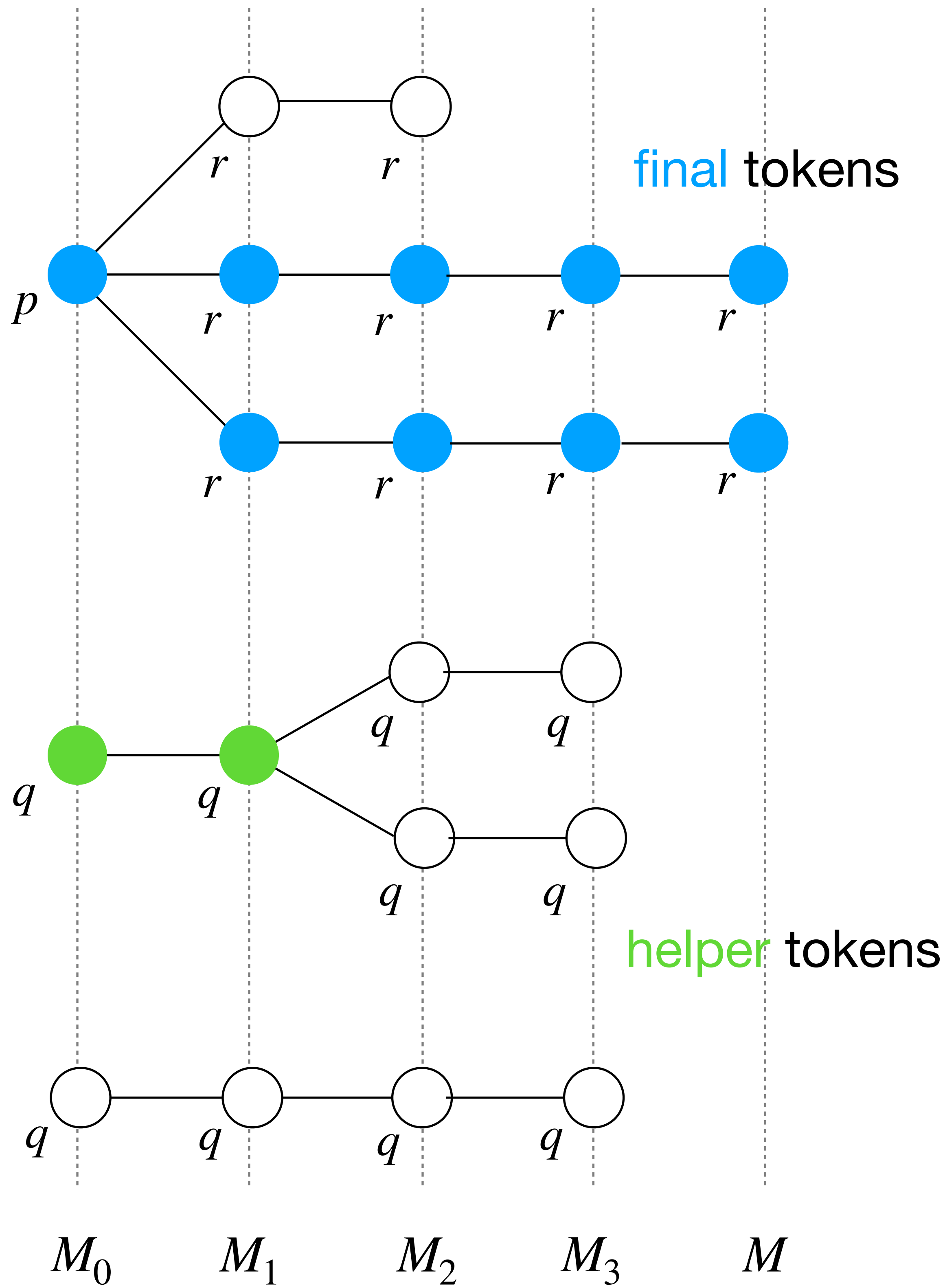

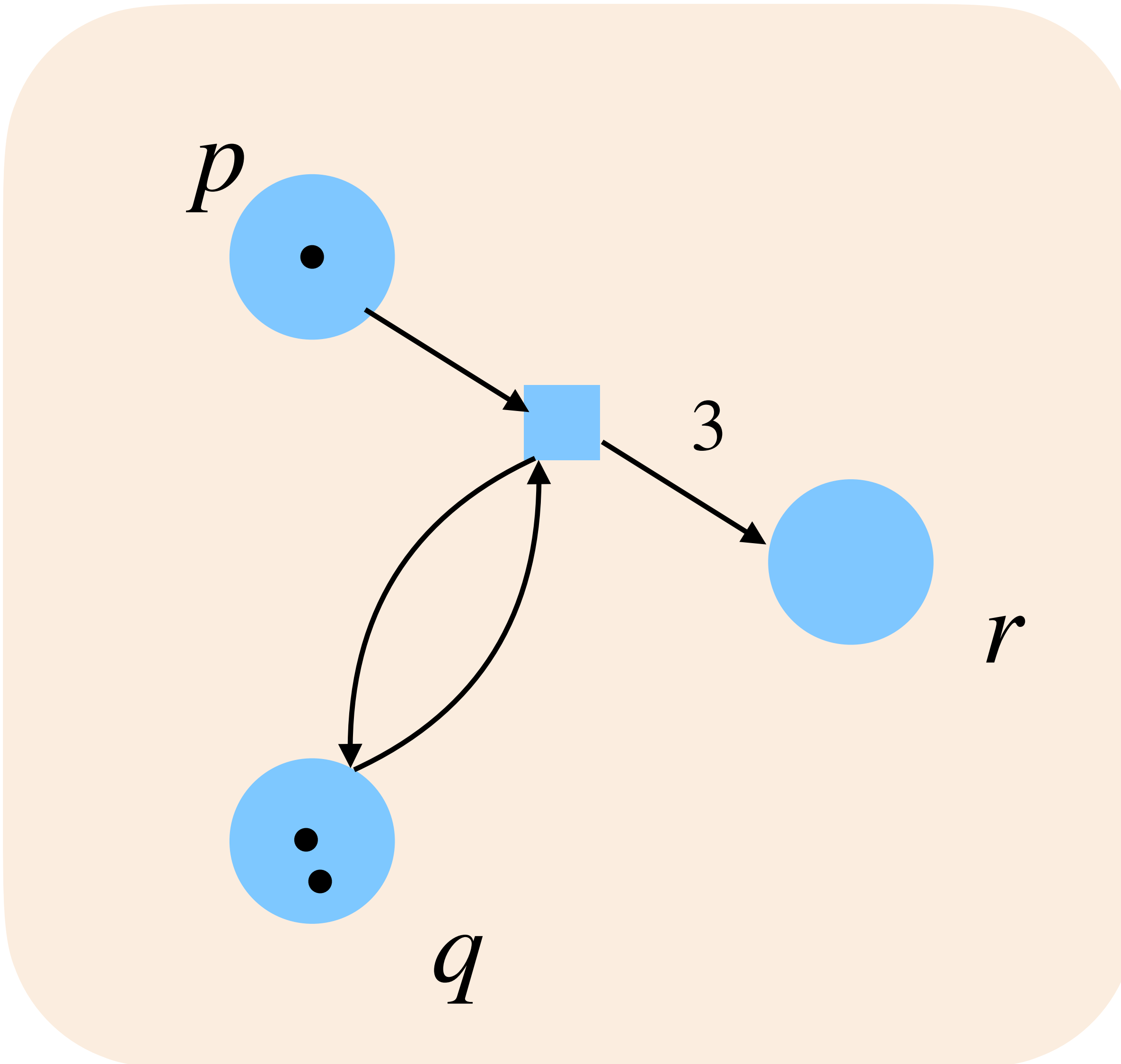

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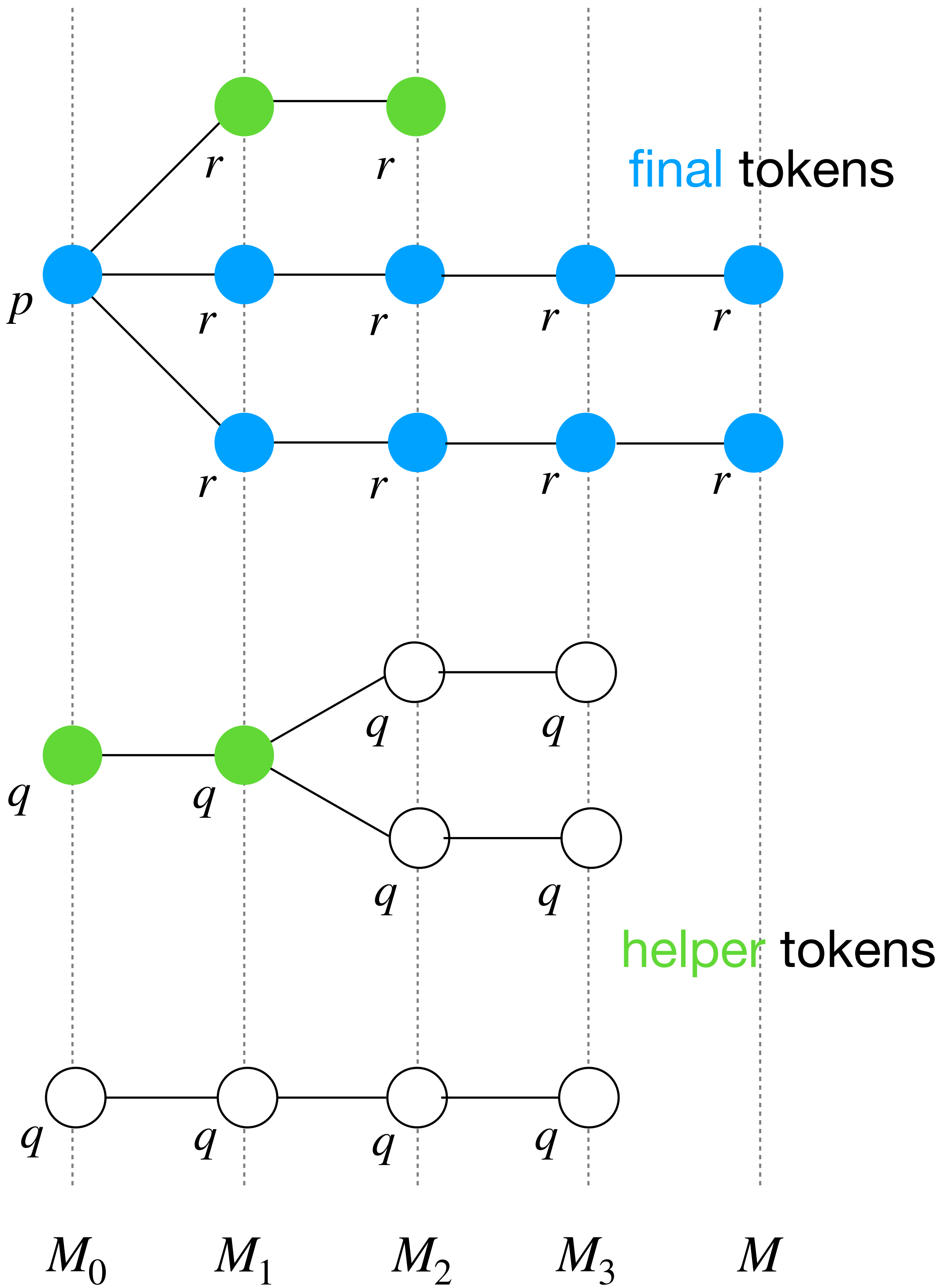

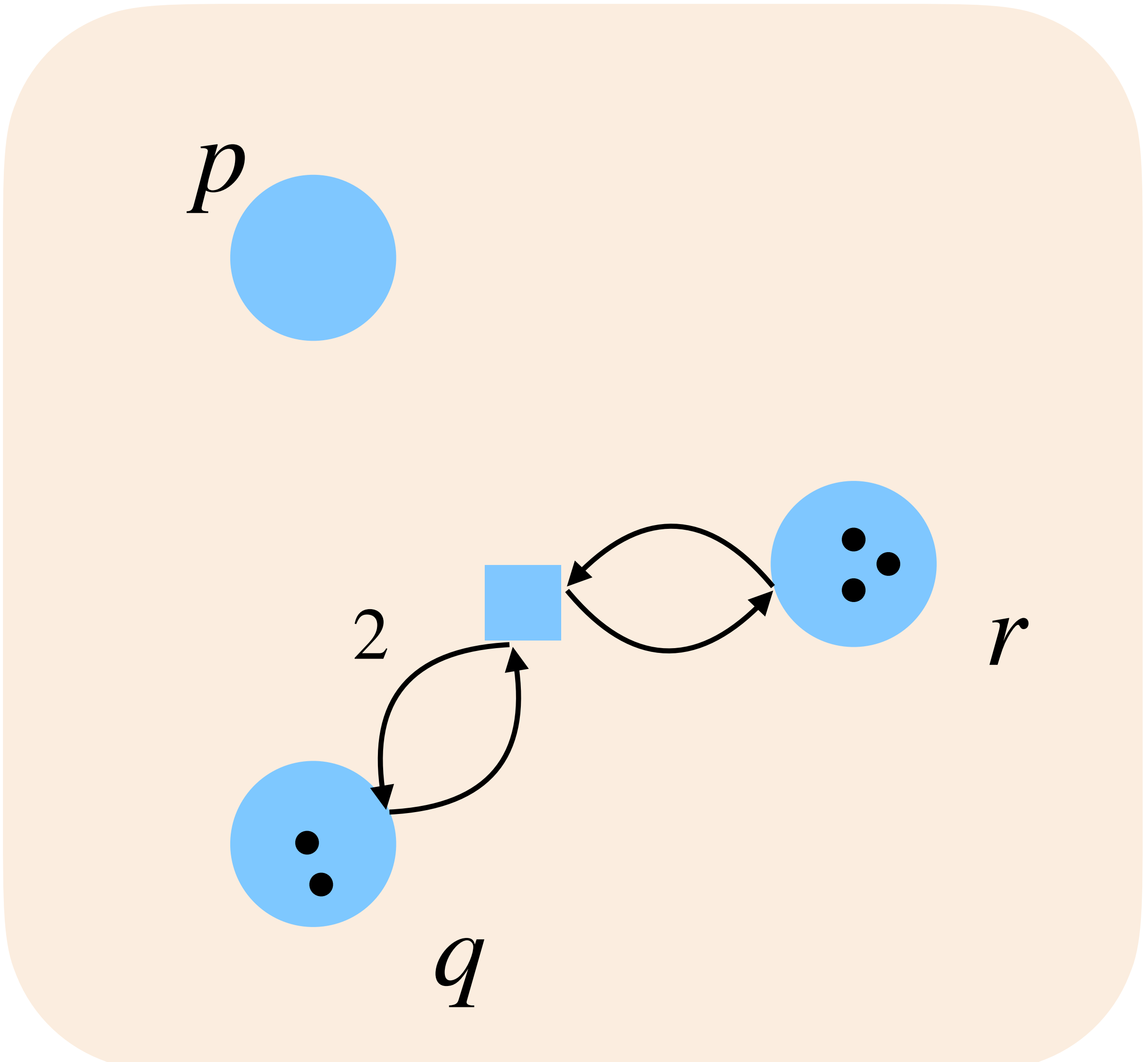

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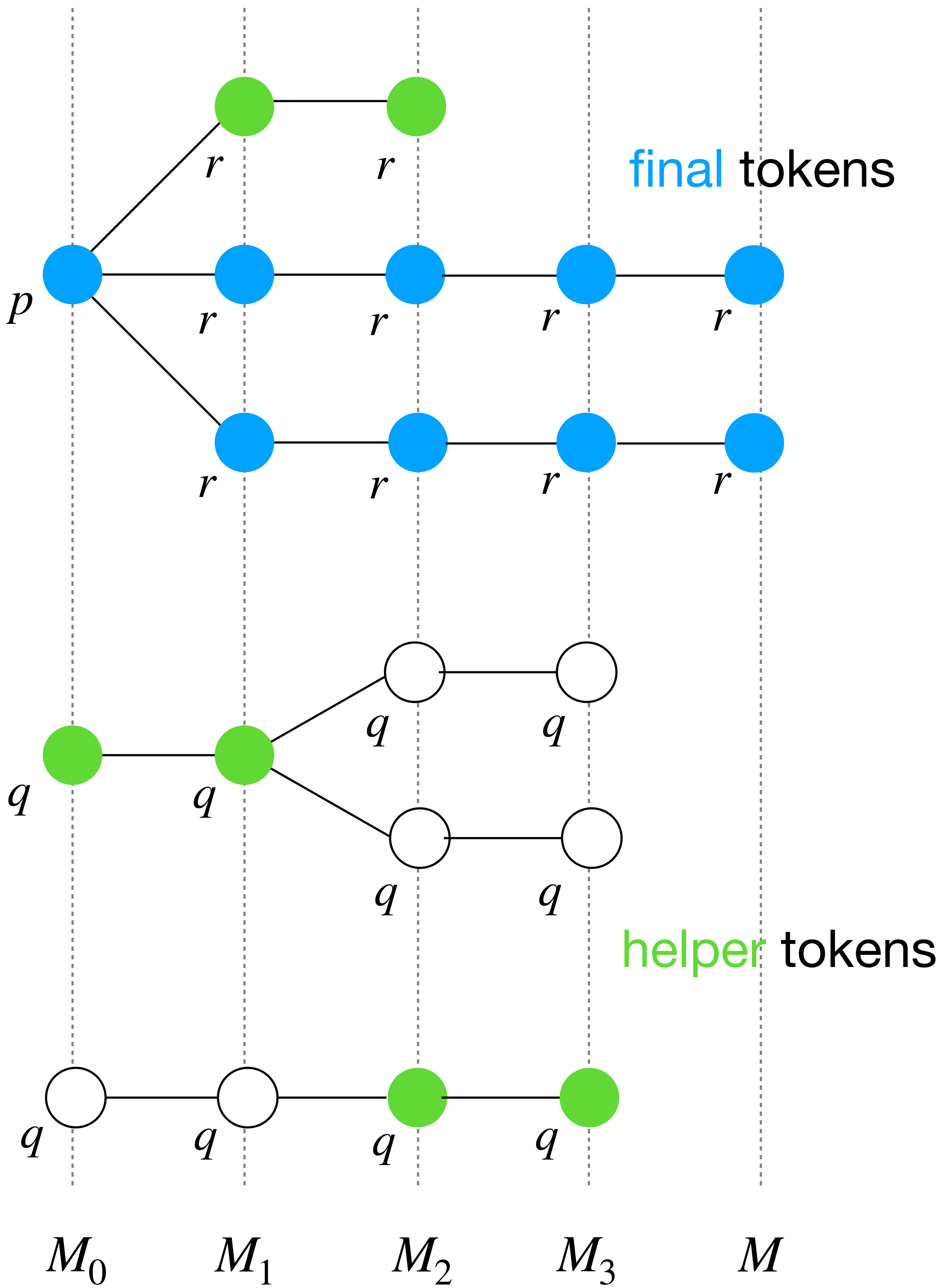

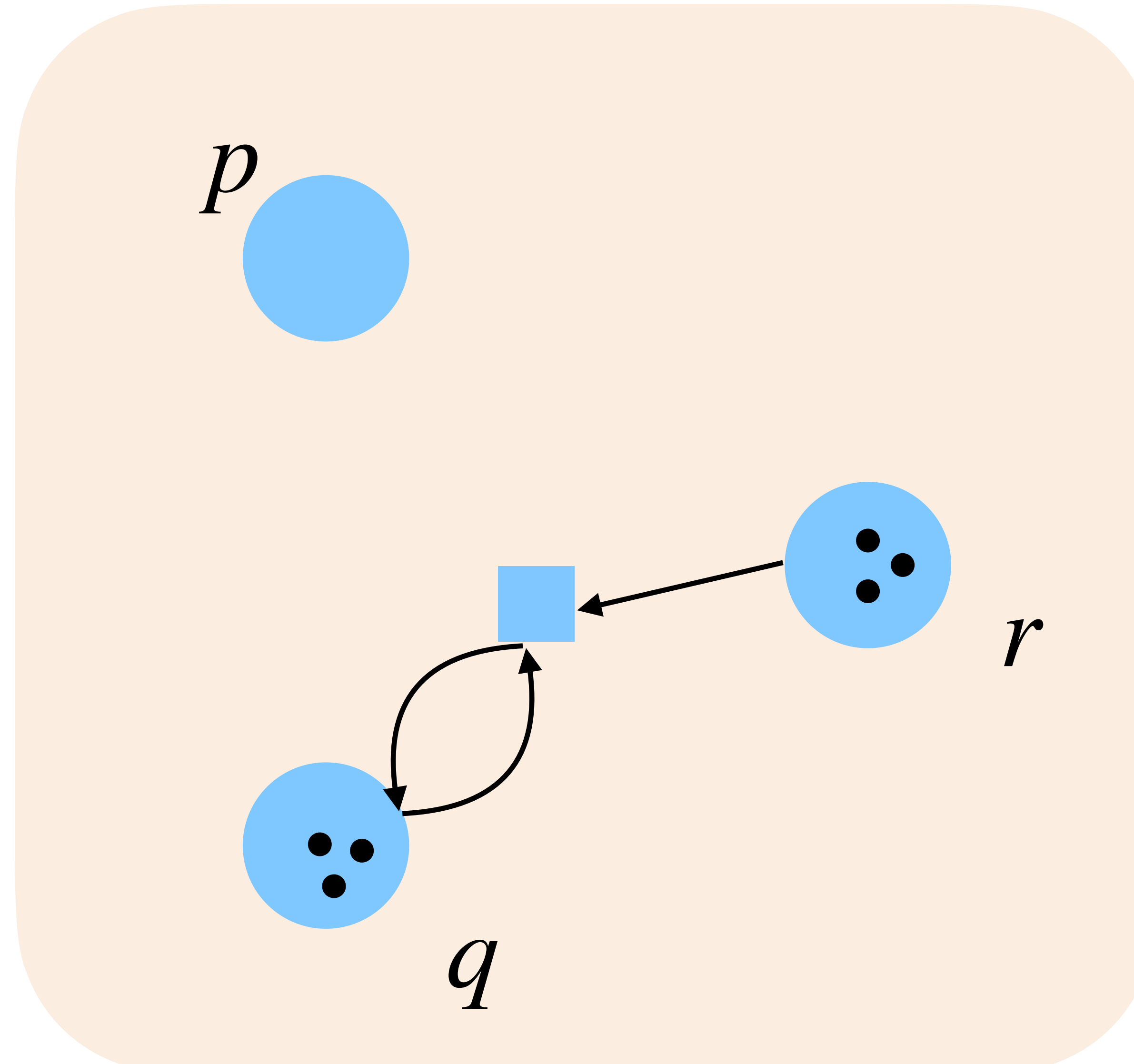

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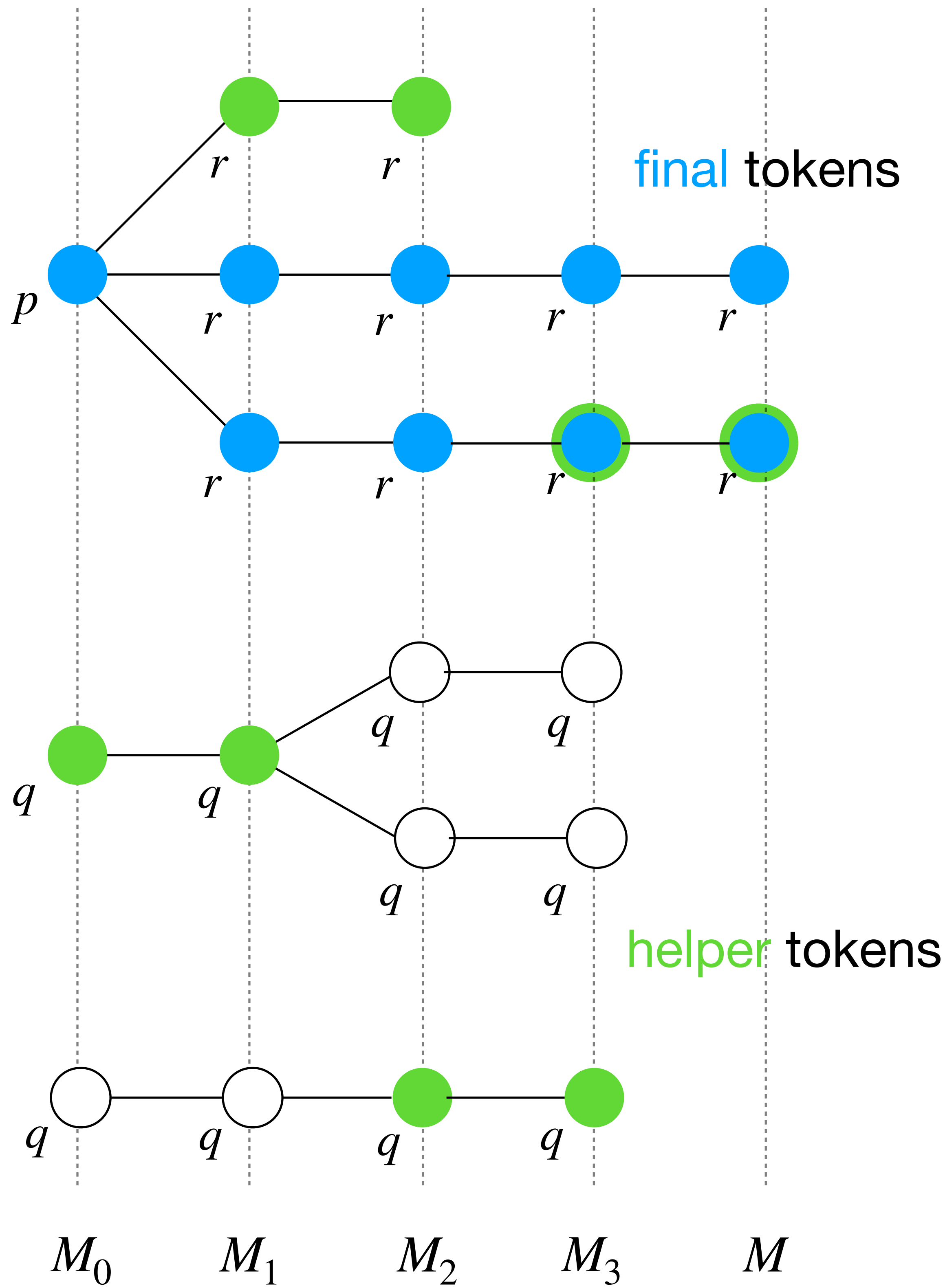

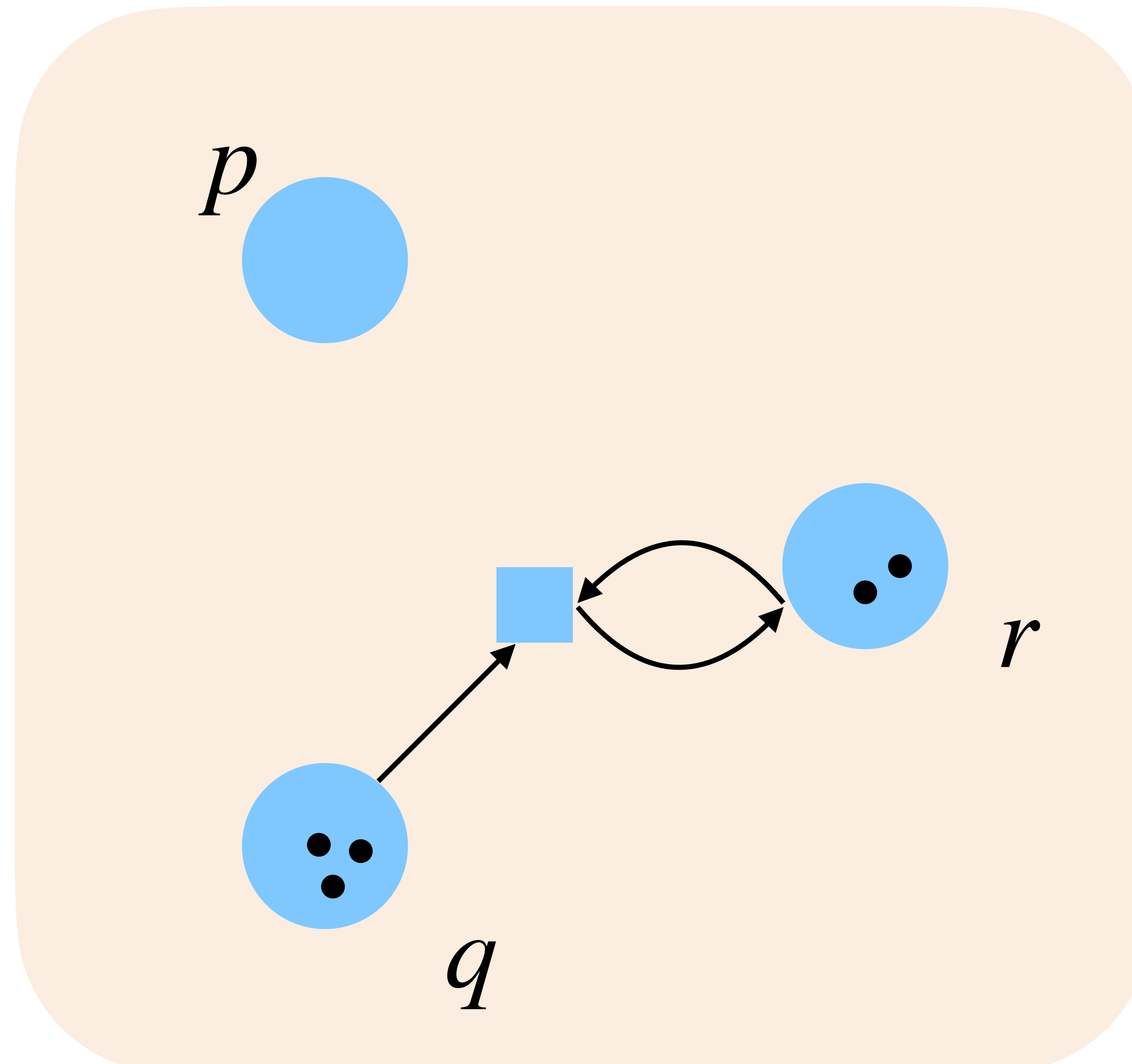

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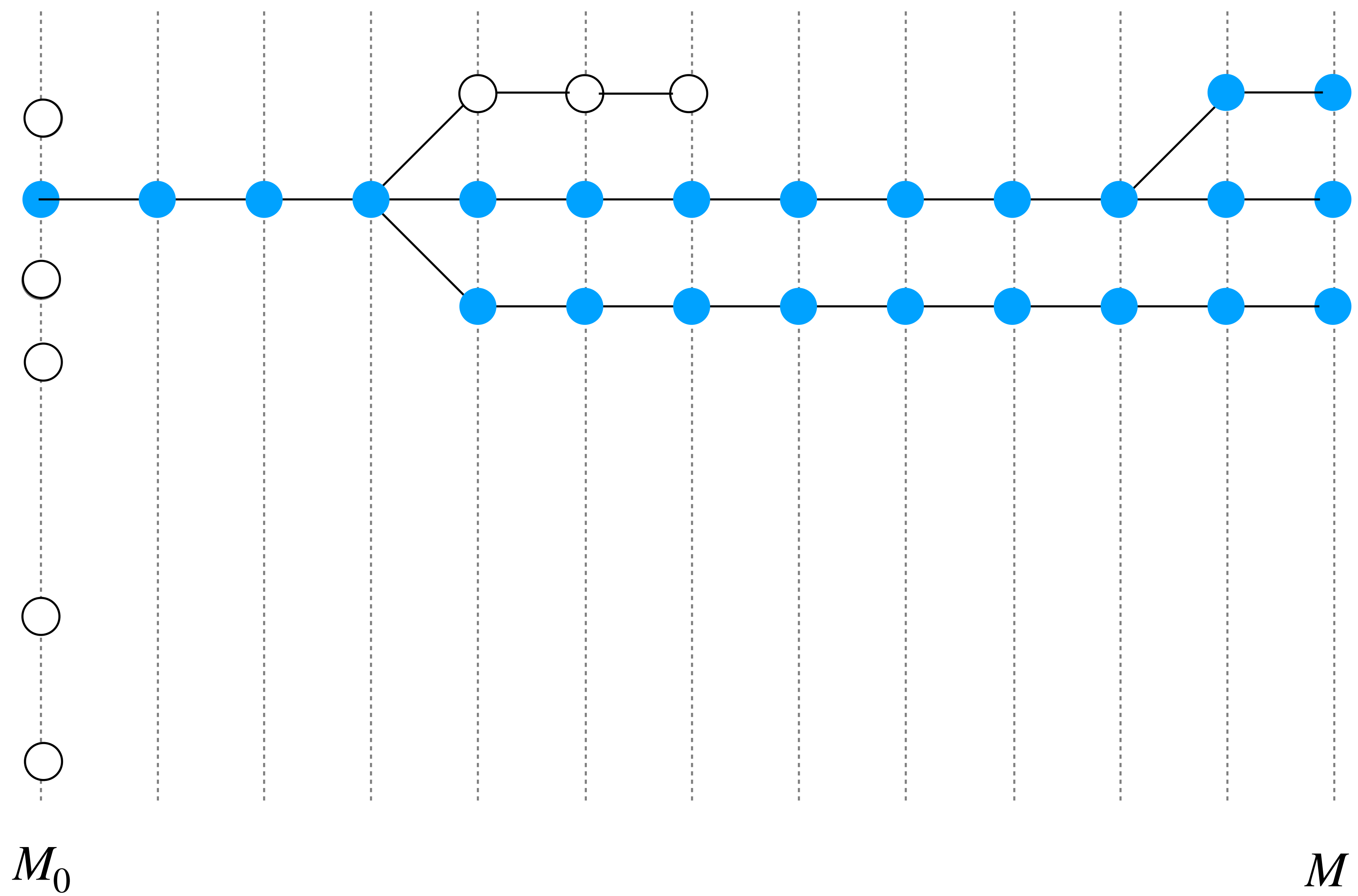
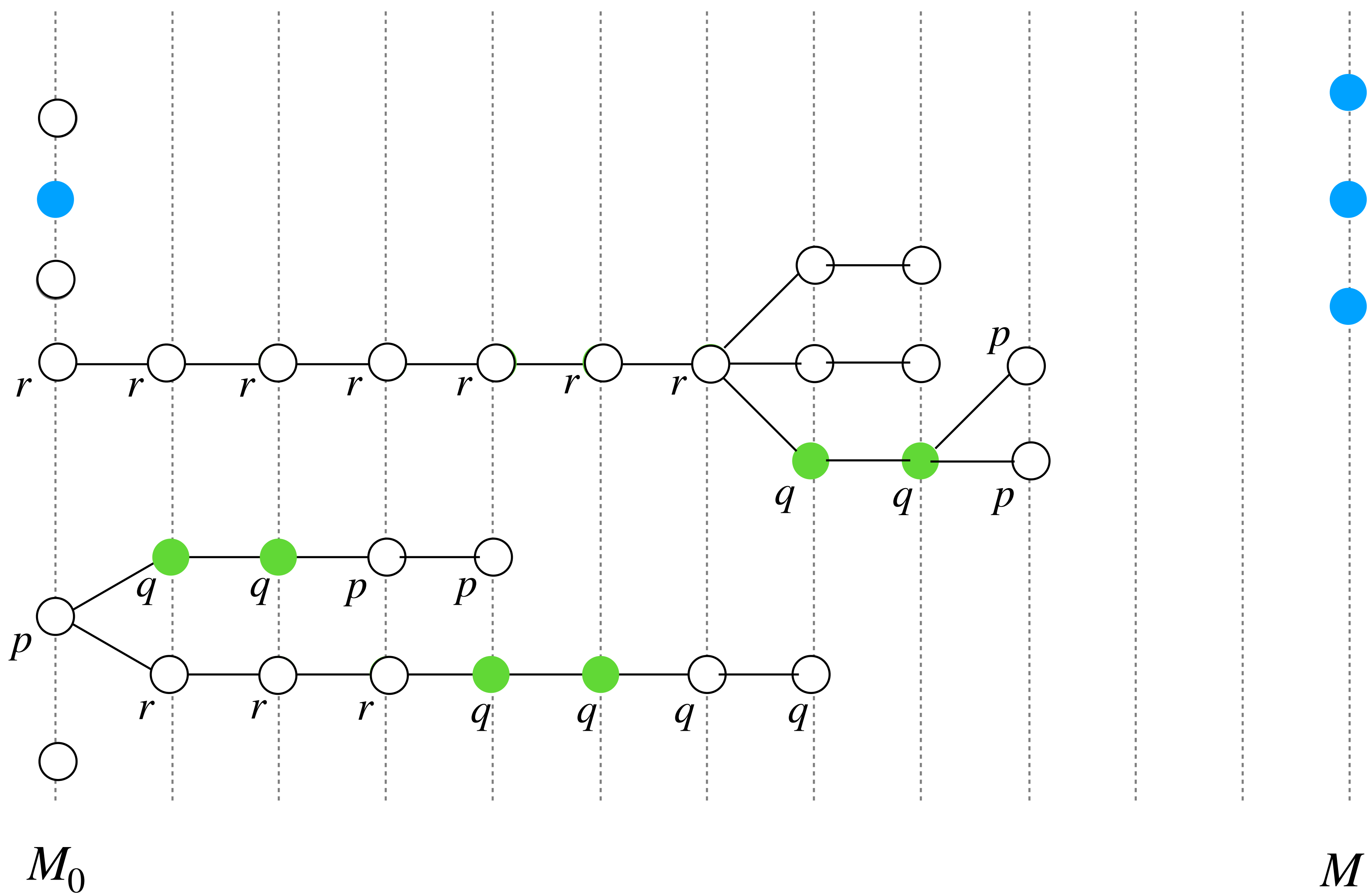


1. Keep the final tokens

PSPACE reachability



1. Keep the final tokens

2. Reduce the number of helper tokens

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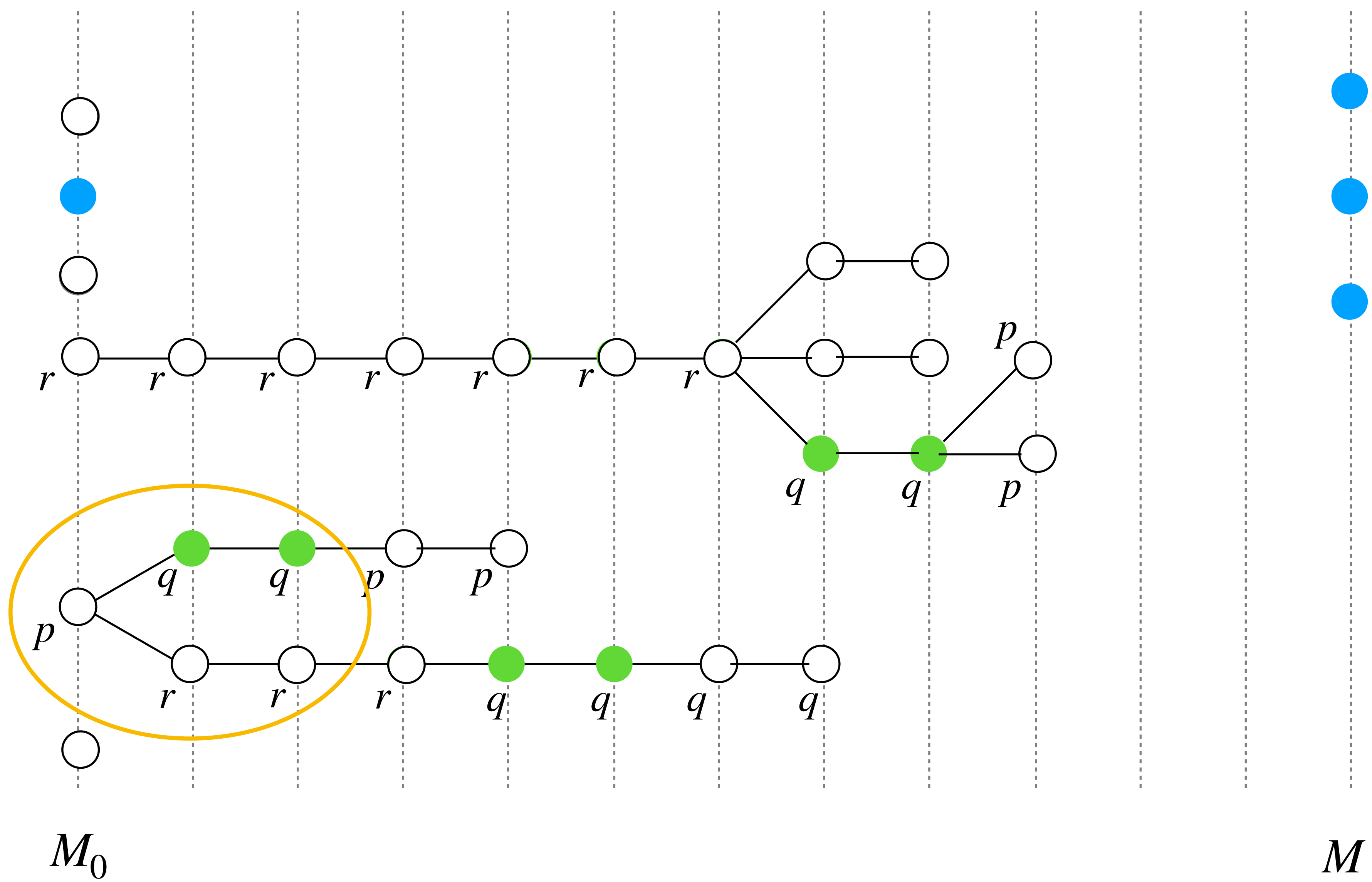

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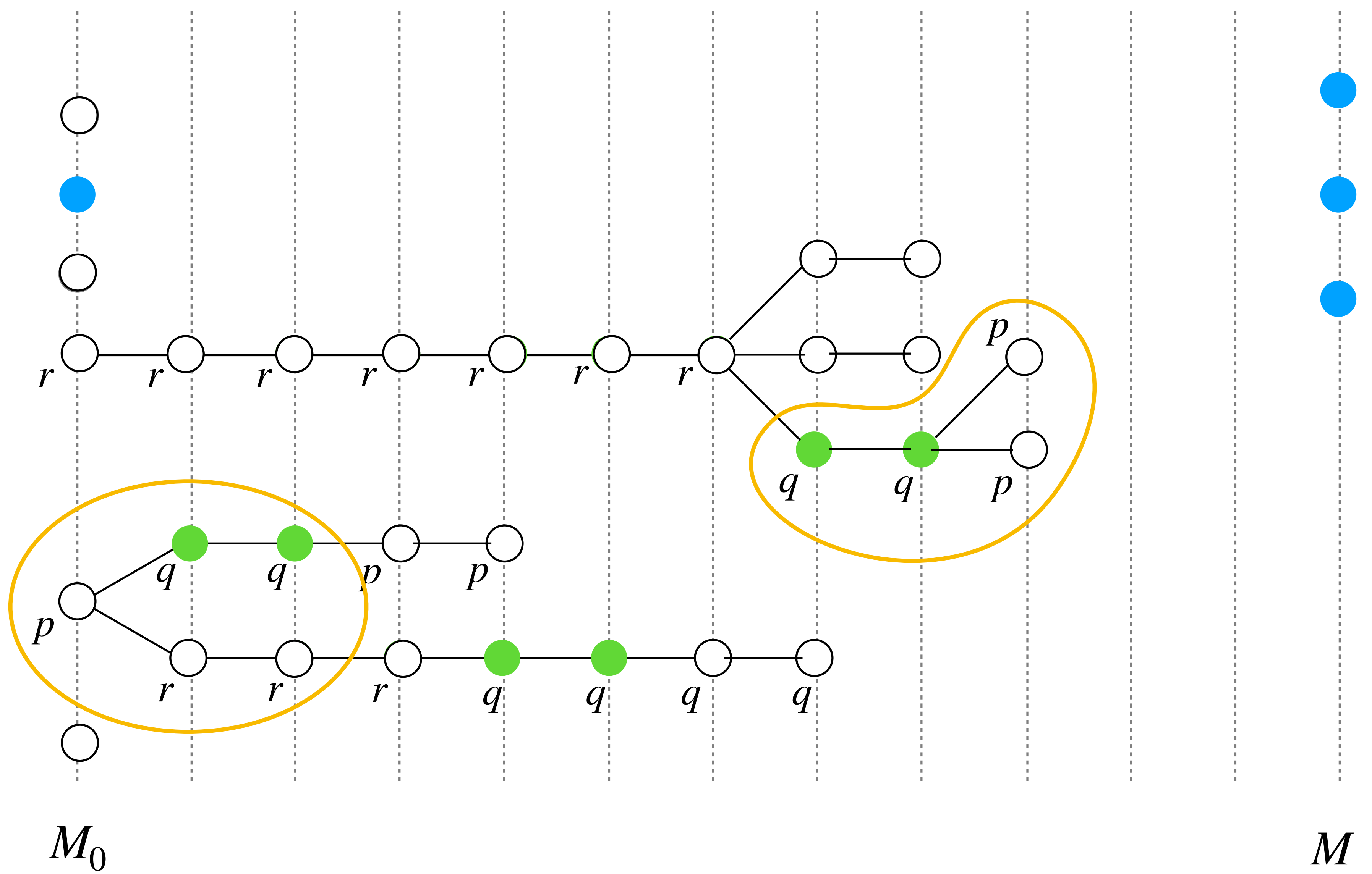

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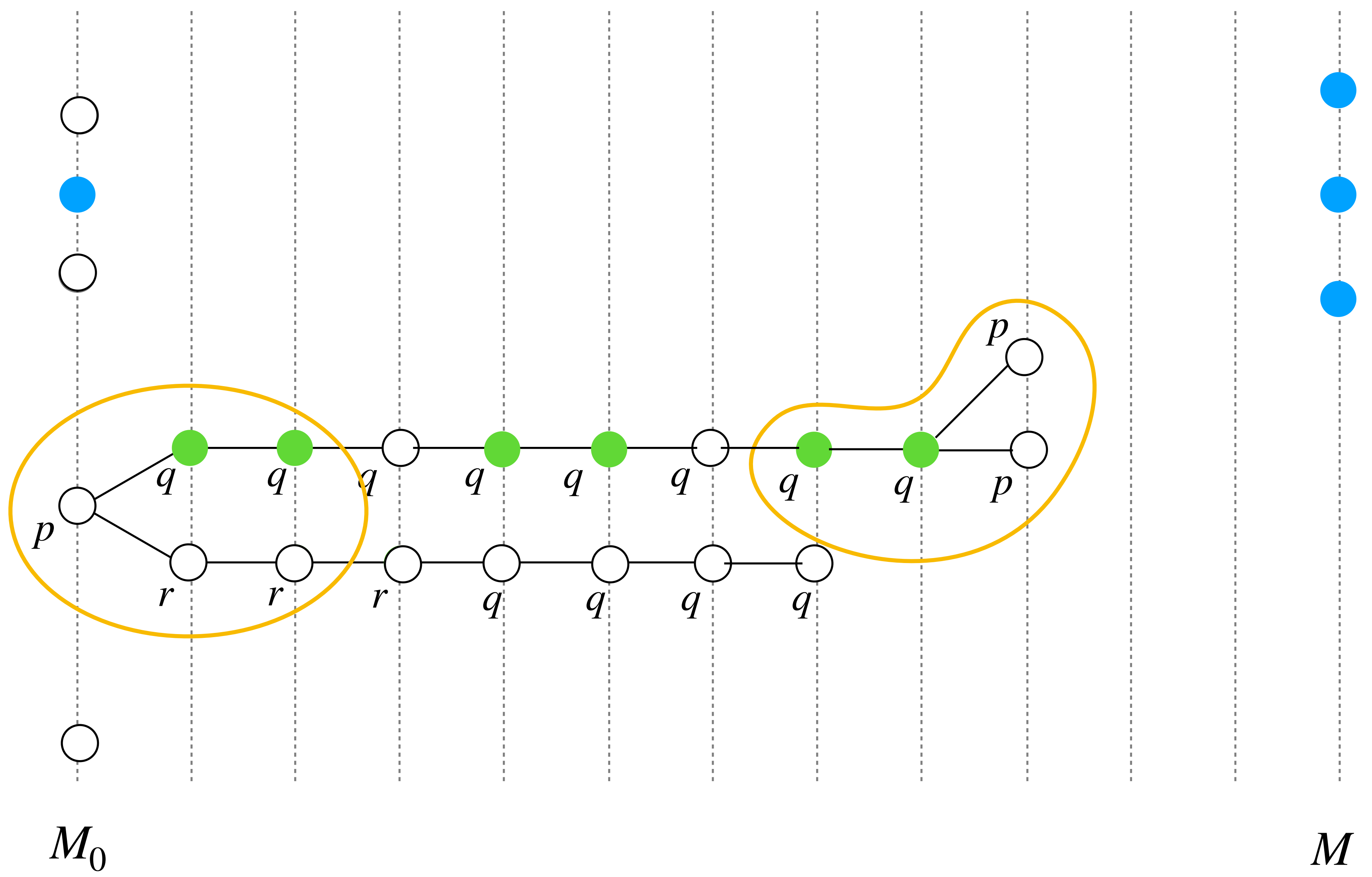

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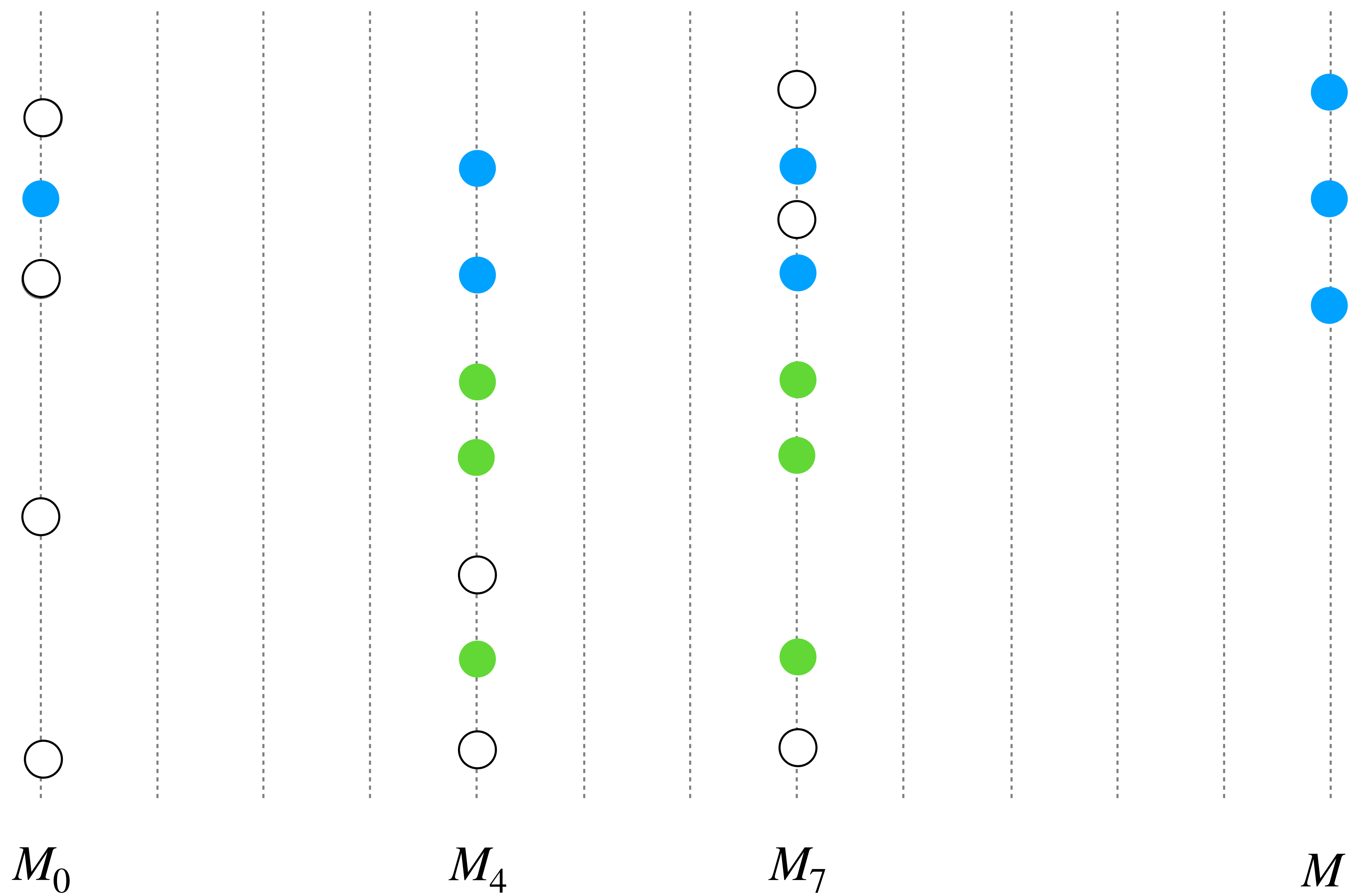

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C. Weil-Kennedy, TUM

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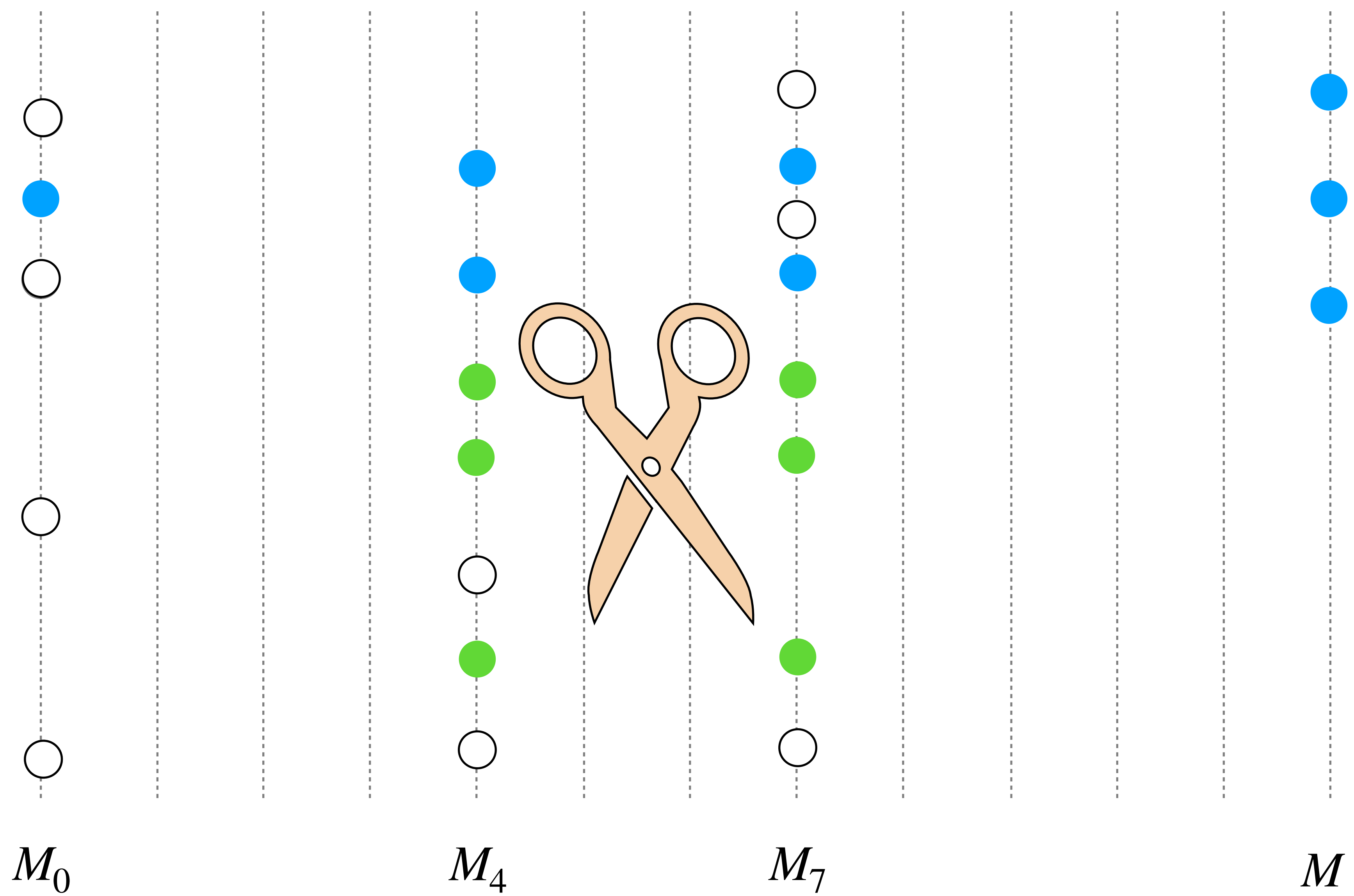

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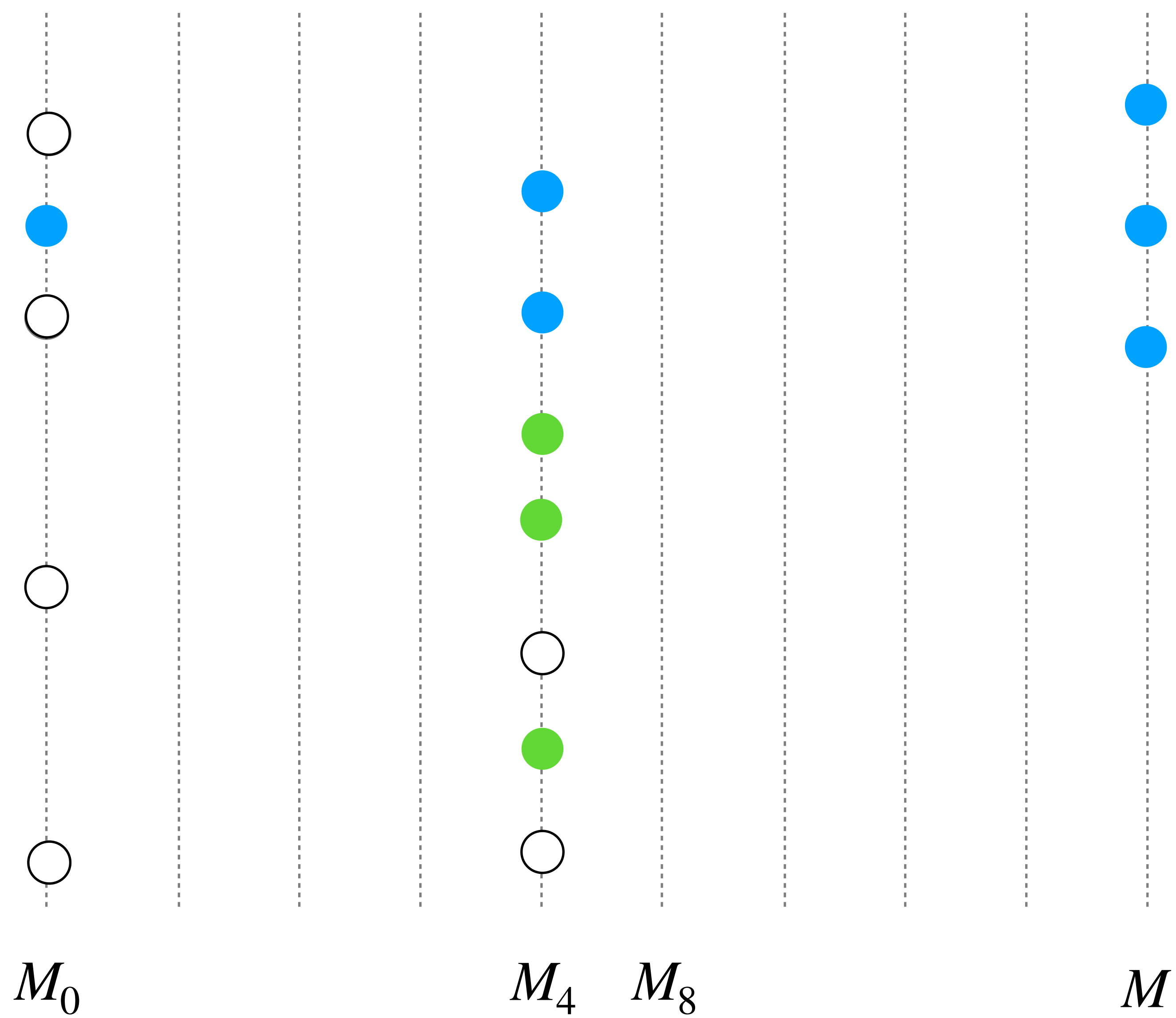

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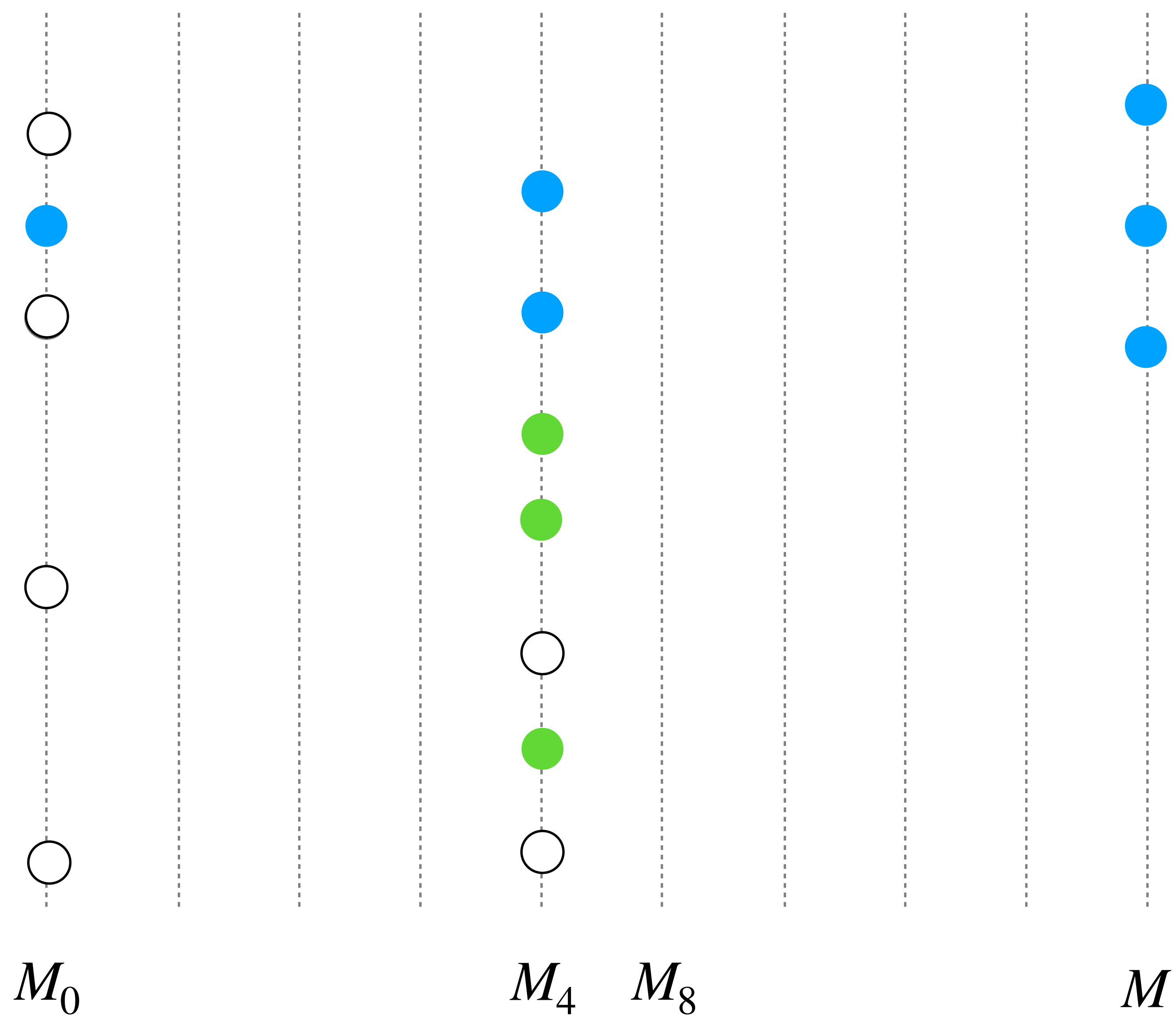

1. Keep the final tokens

$\leq |M|$ per intermediate marking

2. Reduce the number of helper tokens

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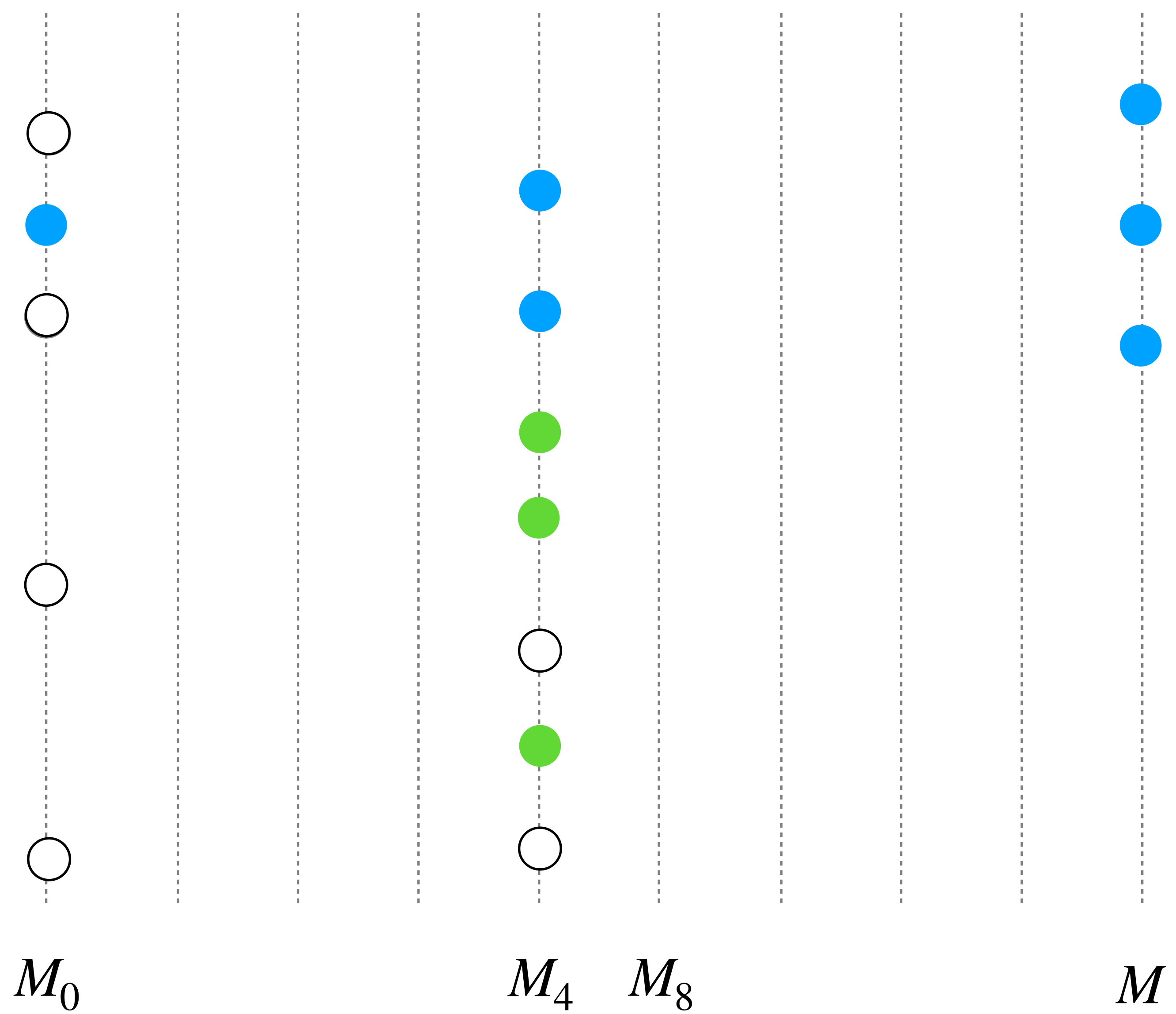
C. Weil-Kennedy, TUM

PSPACE reachability



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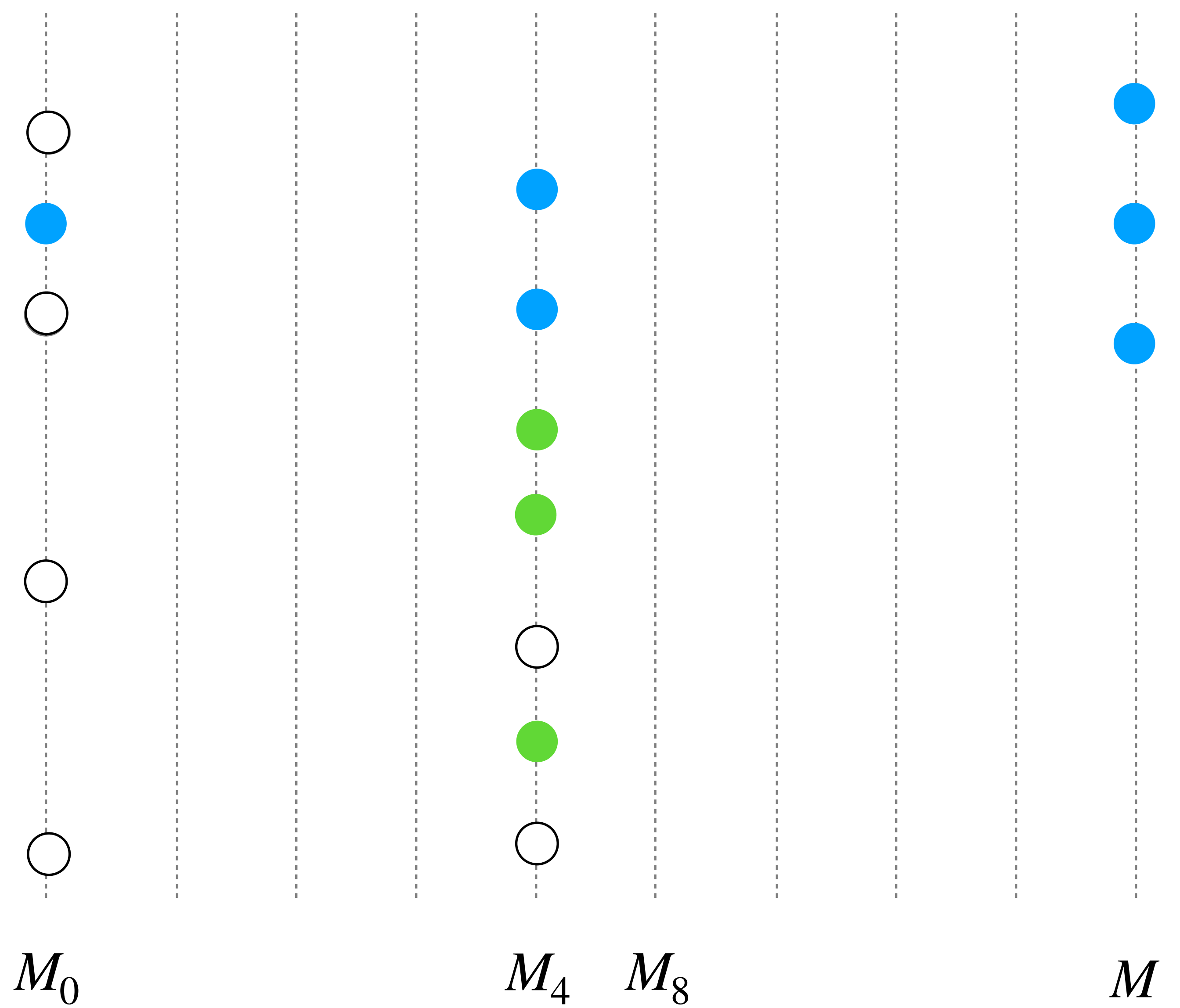

1. Keep the final tokens $\boxed{\leq |M| \text{ per intermediate marking}}$

2. Reduce the number of helper tokens $\boxed{\leq n^2 \text{ per intermediate marking}}$

3. Cut out repetitions of final + helper $\boxed{\text{accelerated length} \leq O(\text{\# such configurations})}$

Flatness

[Leroux, Sutre, '05]

flat $\exists$ sequence $t_1^* t_2^* \dots t_l^*$ such that

$$M_0 \xrightarrow{*} M \text{ iff } M_0 \xrightarrow{t_1^{k_1} t_2^{k_2} \dots t_l^{k_l}} M$$

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BPP, IO nets

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BPP, IO nets

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such that $l \in O(|M|n)^n$ *and* $\forall i,\ M_i \in O(|M_o||M|n\gamma)^n$

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BIO nets

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$$M_0 \xrightarrow{t_1^{k_1}} M_1 \xrightarrow{t_2^{k_2}} M_2 \rightarrow \dots \xrightarrow{t_l^{k_l}} M_l = M$$

such that $l \in O(|M|n)^n$ *and* $\forall i,\ M_i \in O(|M_o||M|n\gamma)^n$

Flatness

[Leroux, Sutre, '05]

flat $\exists$ sequence $t_1^* t_2^* \ldots t_l^*$ such that

$$M_0 \xrightarrow{*} M \text{ iff } M_0 \xrightarrow{t_1^{k_1} t_2^{k_2} \ldots t_l^{k_l}} M$$

$\downarrow$

BPP, IO nets

pre*-flat $\forall M, \exists$ sequence $t_1^* t_2^* \ldots t_l^*$ such that

$$M_0 \xrightarrow{*} M \text{ iff } M_0 \xrightarrow{t_1^{k_1} t_2^{k_2} \ldots t_l^{k_l}} M$$

$\downarrow$

BIO nets

Main Theorem

In a BIO net with n places, and transitions producing $\leq \gamma$ *tokens*

$$\textit{If } M_0 \xrightarrow{*} M$$

then $\exists$ *markings* $M_1, M_2, \ldots, M_l$ $\exists$ *transitions* $t_1, t_2, \ldots, t_l$ $\exists$ *constants* $k_1, k_2, \ldots k_l \geq 0$

$$M_0 \xrightarrow{t_1^{k_1}} M_1 \xrightarrow{t_2^{k_2}} M_2 \rightarrow \ldots \xrightarrow{t_l^{k_l}} M_l = M$$

such that $l \in O(|M|n)^n$ *and* $\forall i,\ M_i \in O(|M_o||M|n\gamma)^n$

model checking tools with acceleration techniques
e.g. FAST [Bardin, Finkel, Leroux, Petrucci, '03]

Conclusion

non-elementary
[Czerwinzki, Lasota, Lazic, Leroux, Mazowiecki, ’19]

PSPACE-complete
[Esparza, Raskin, *W.-K.*, ’20]

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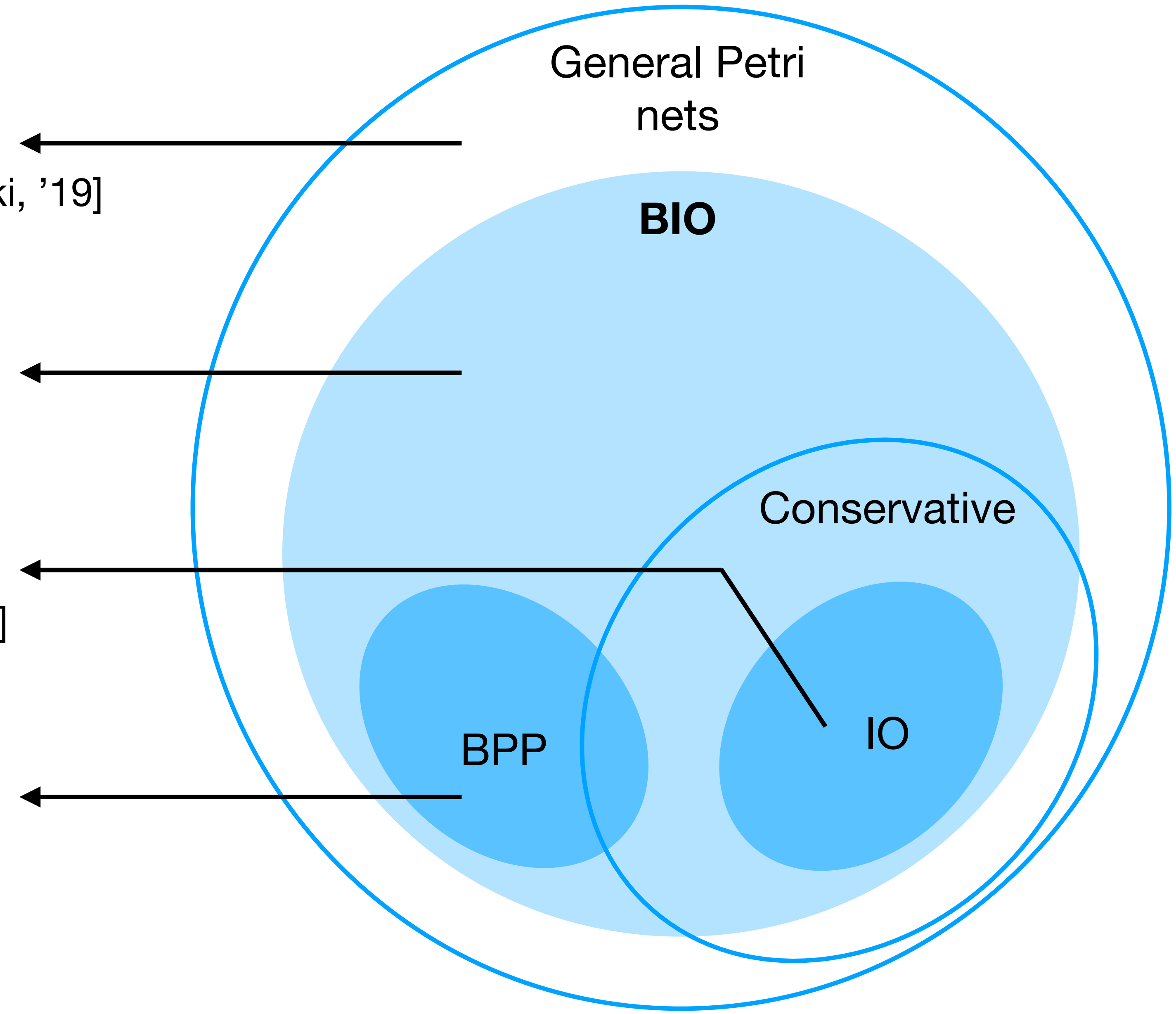
NP-complete
[Esparza, ’97]



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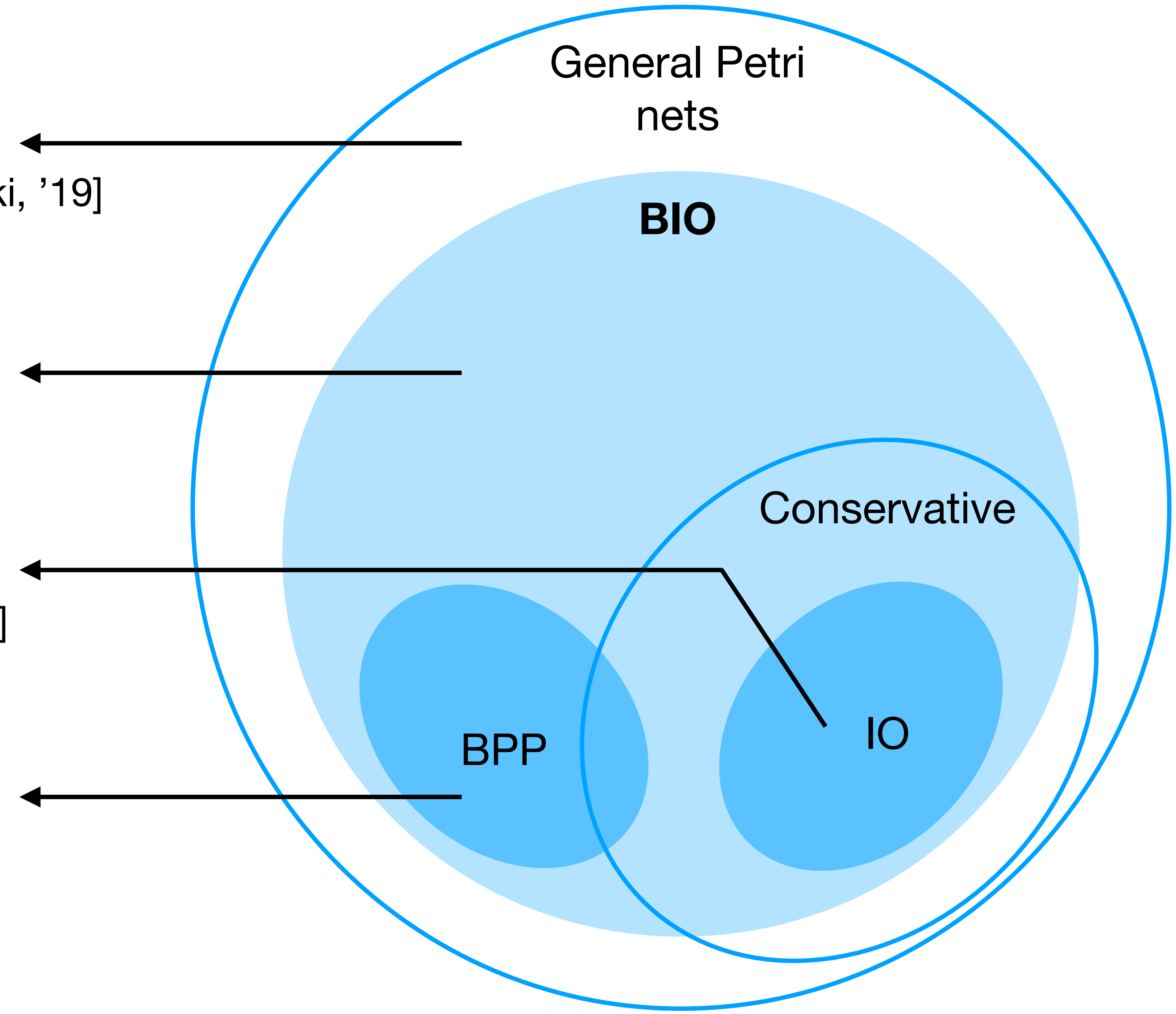

Other results:

- reachability between possibly infinite sets of markings (*cubes)* is also PSPACE-complete
- this also holds for coverability, liveness, and more

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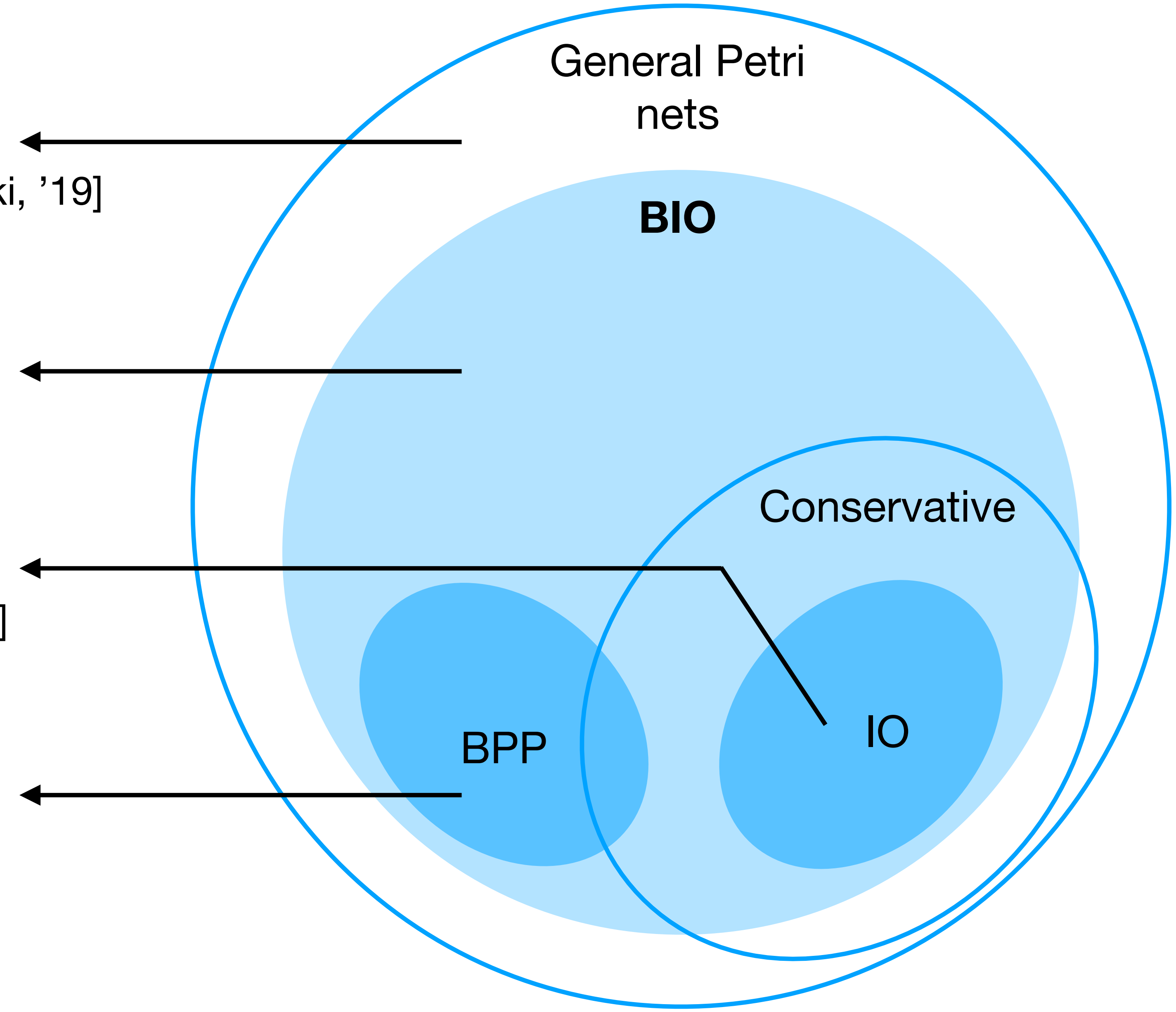

Other results:

- reachability between possibly infinite sets of markings (*cubes)* is also PSPACE-complete
- this also holds for coverability, liveness, and more

Future: Investigate consequences in chemical reaction networks, formal languages, etc.