Verification of Immediate Observation Petri Nets

Chana Weil-Kennedy, *Technical University of Munich*

joint work with Mikhail Raskin, Javier Esparza

European Research Council
Established by the European Commission

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Petri nets & reachability

Petri nets are a classic formal model for the representation of concurrent systems.

Reachability problem: Given a Petri net $\mathcal{N}$, and markings M_0 and M can marking M_0 reach marking M in $\mathcal{N}$?

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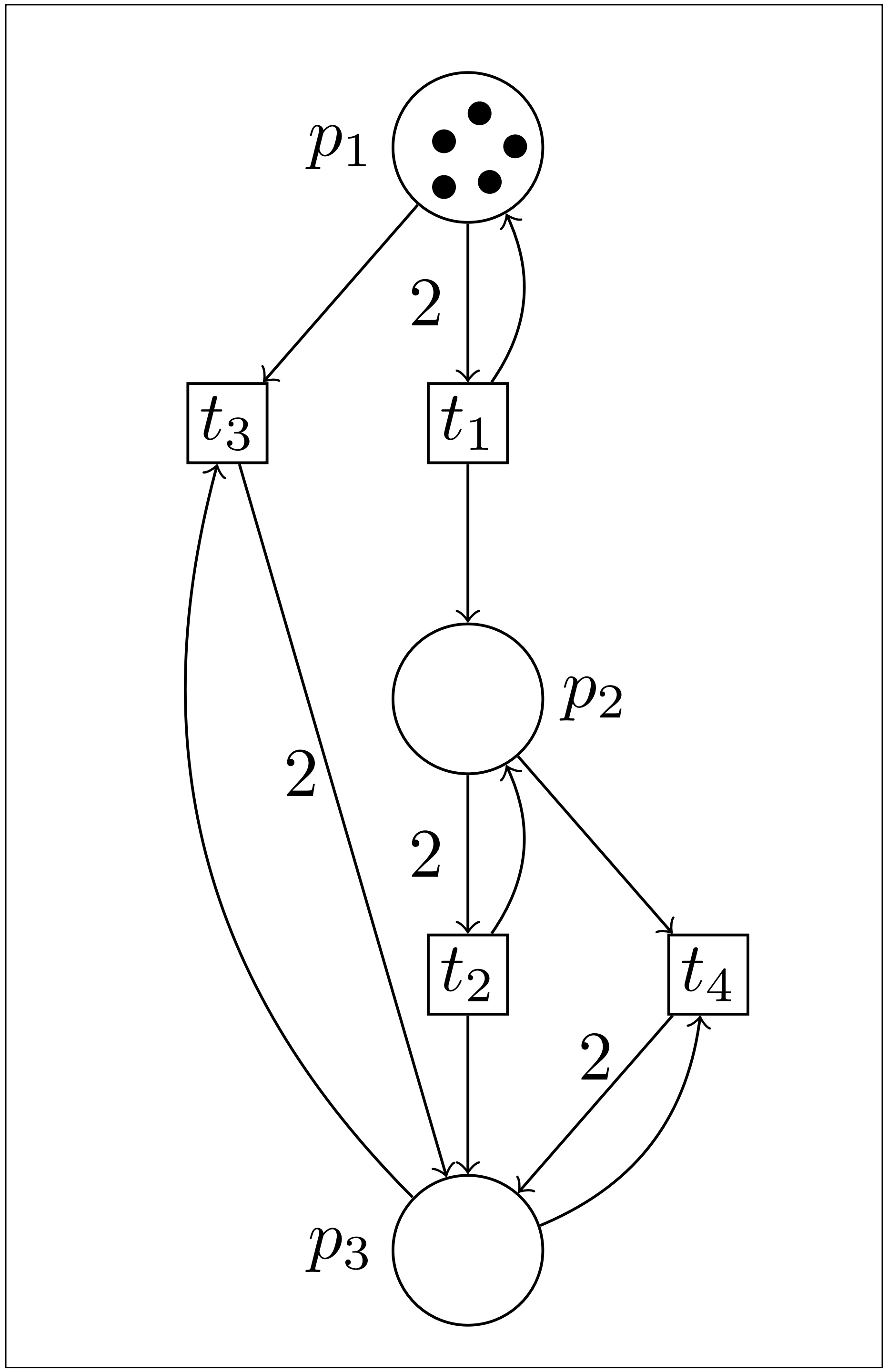
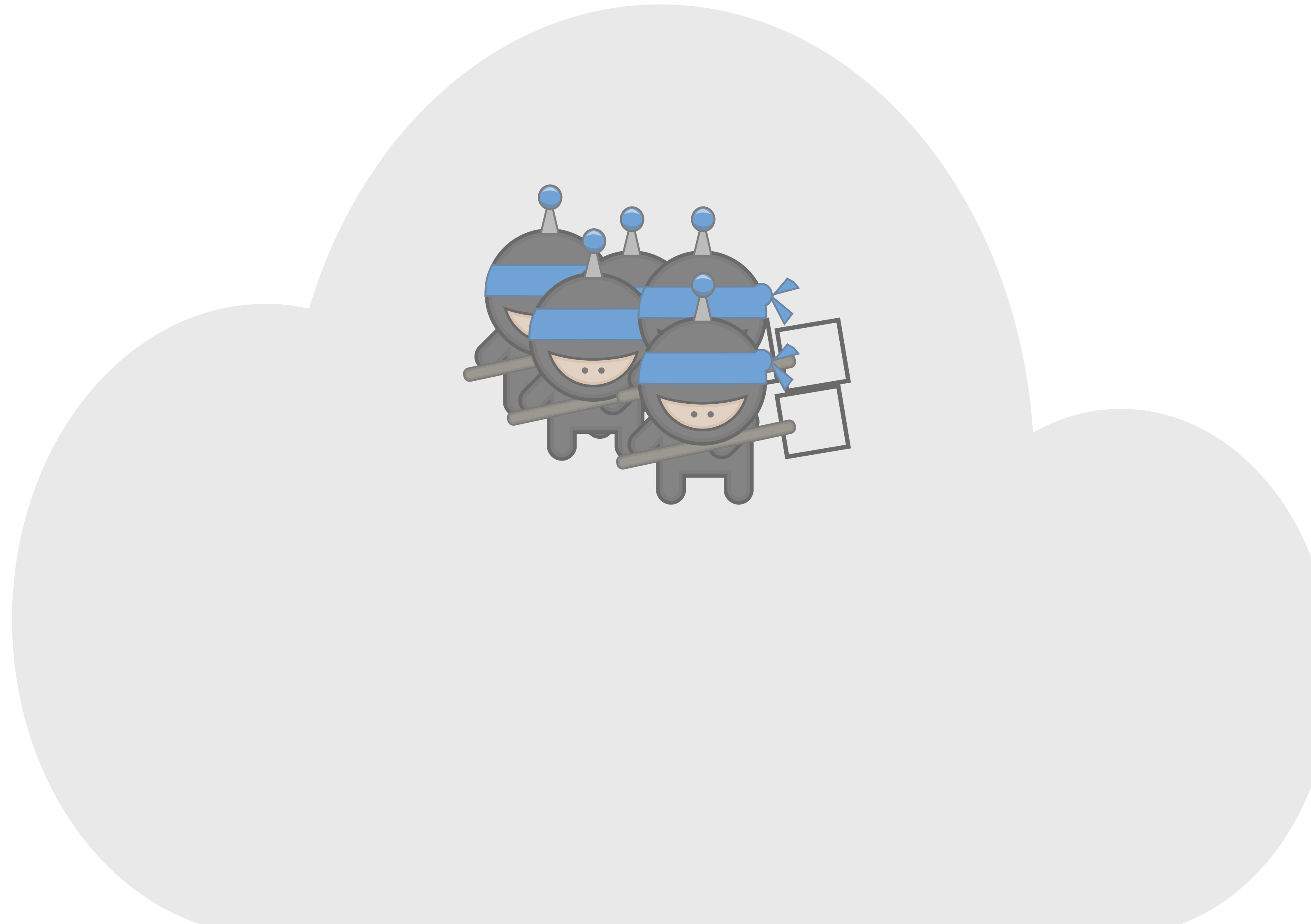
Study subclasses of Petri nets

Example



[The computational power of population protocols, Angluin et al.,'06]



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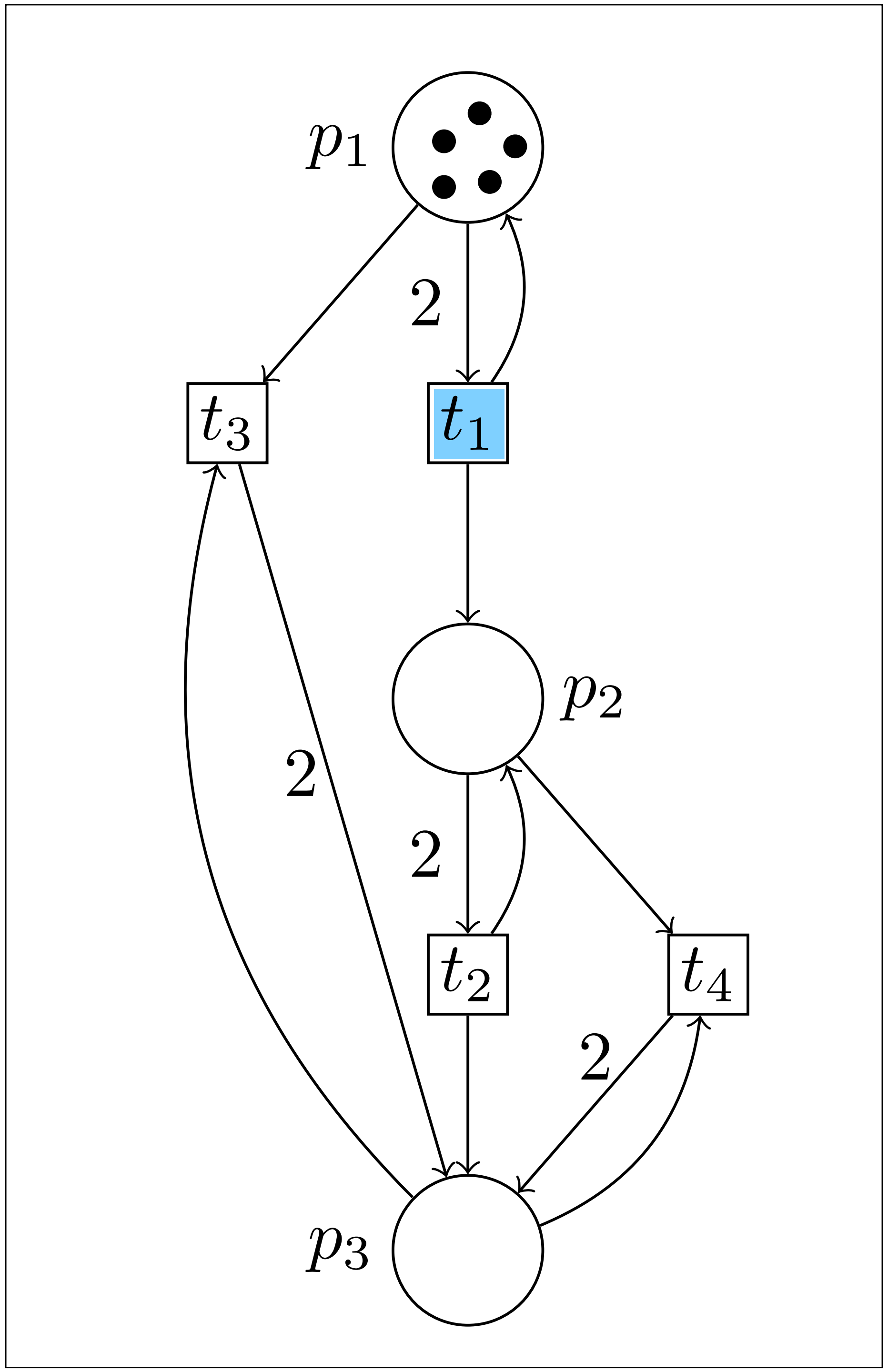

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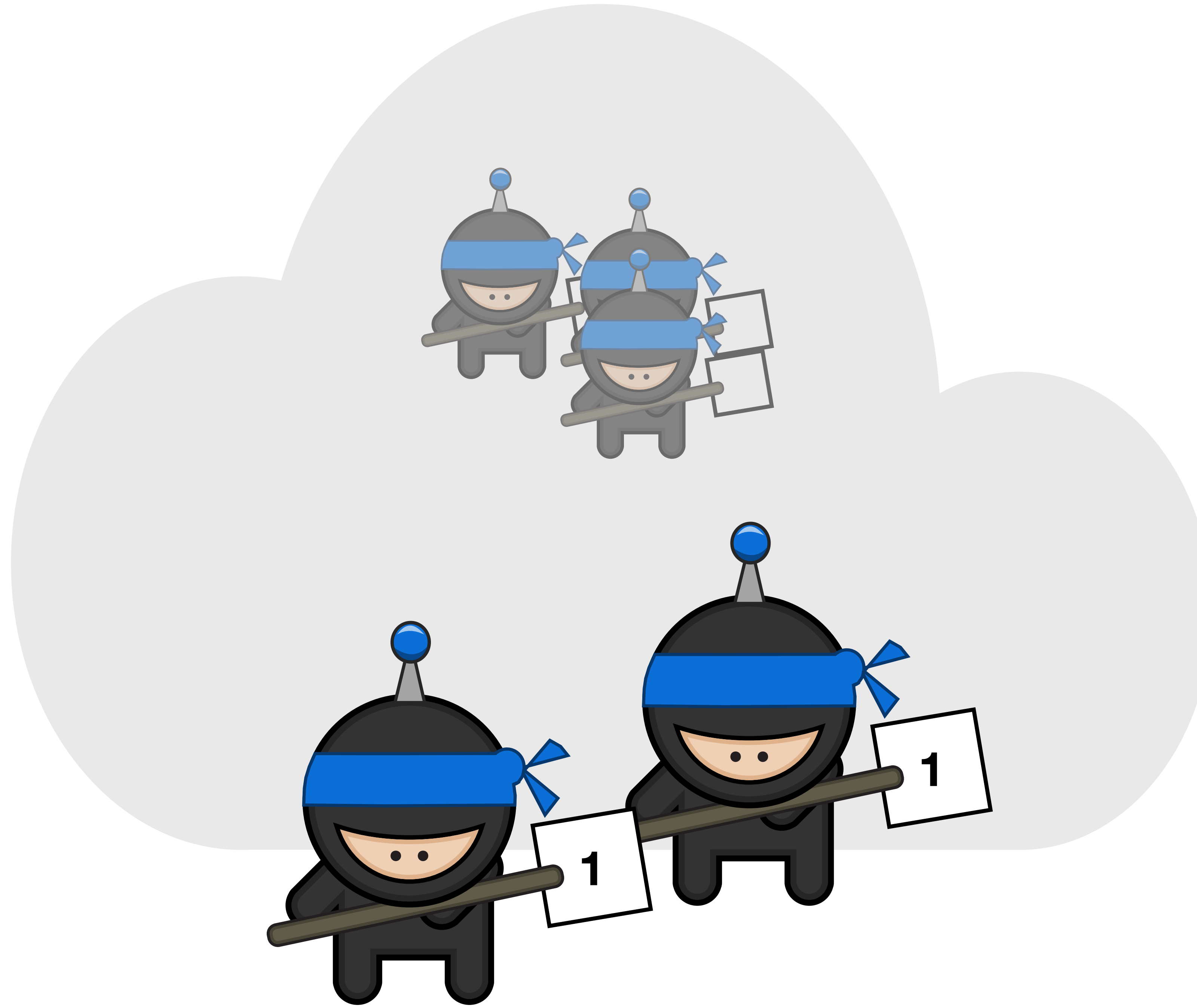

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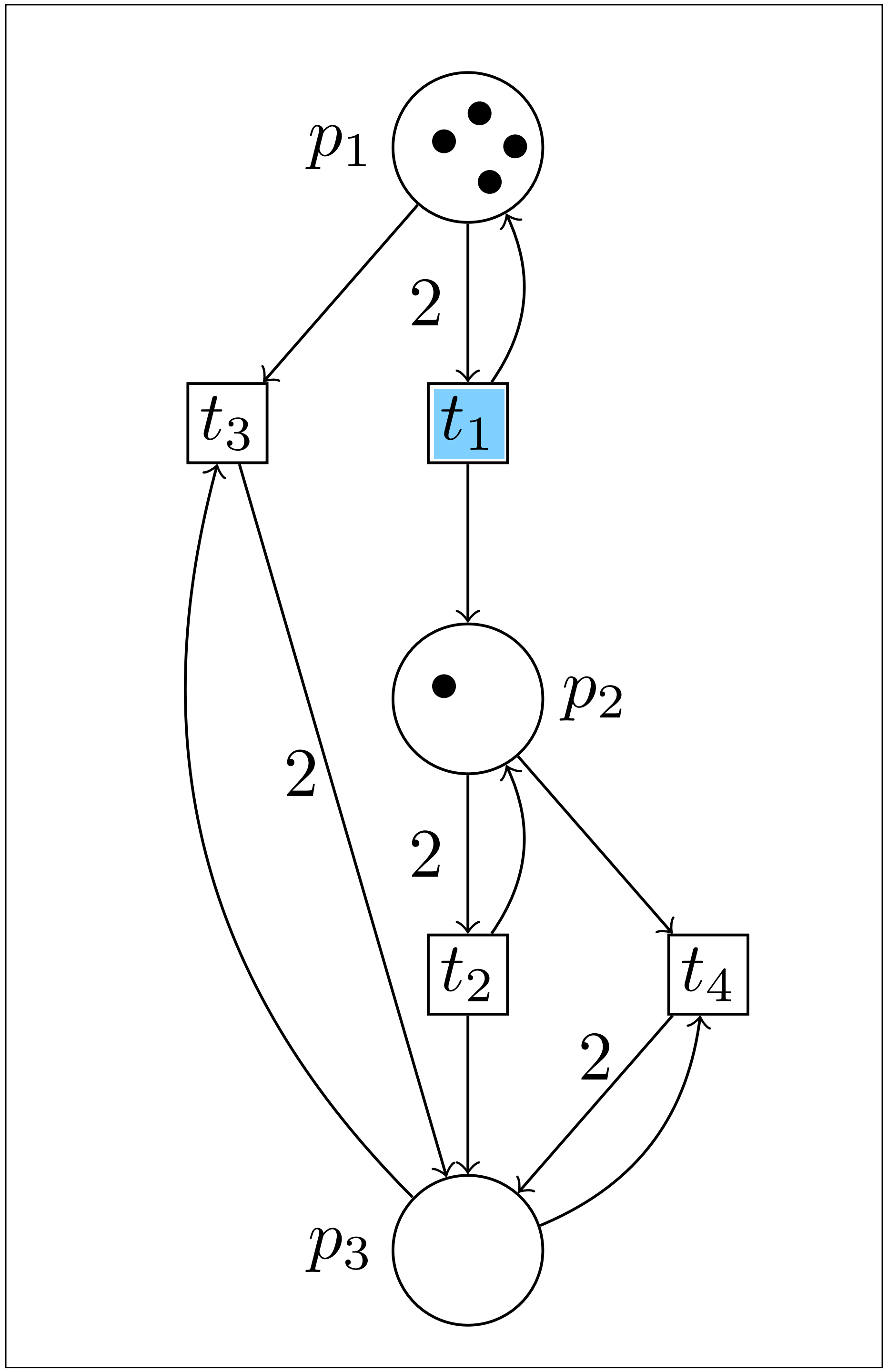

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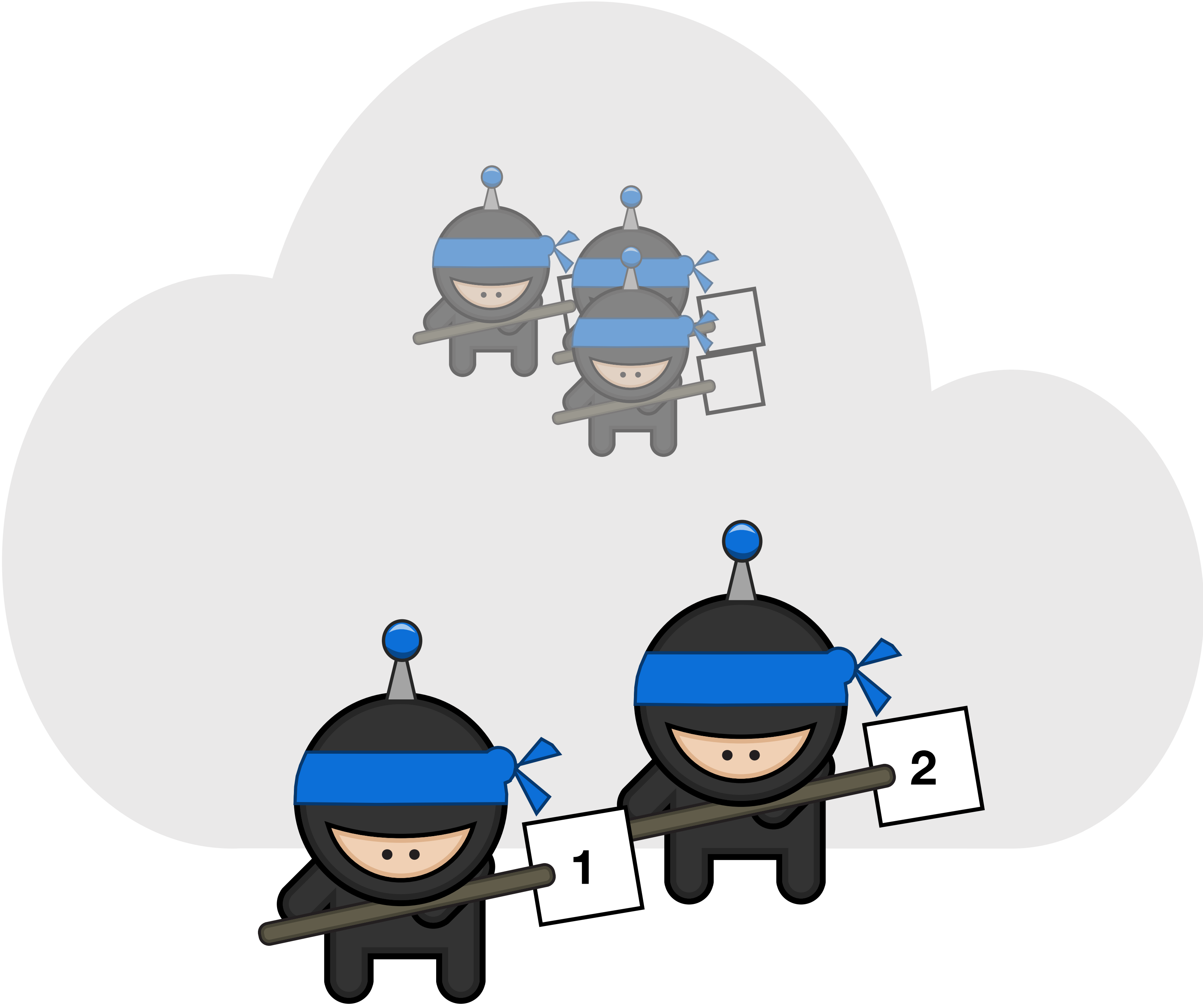

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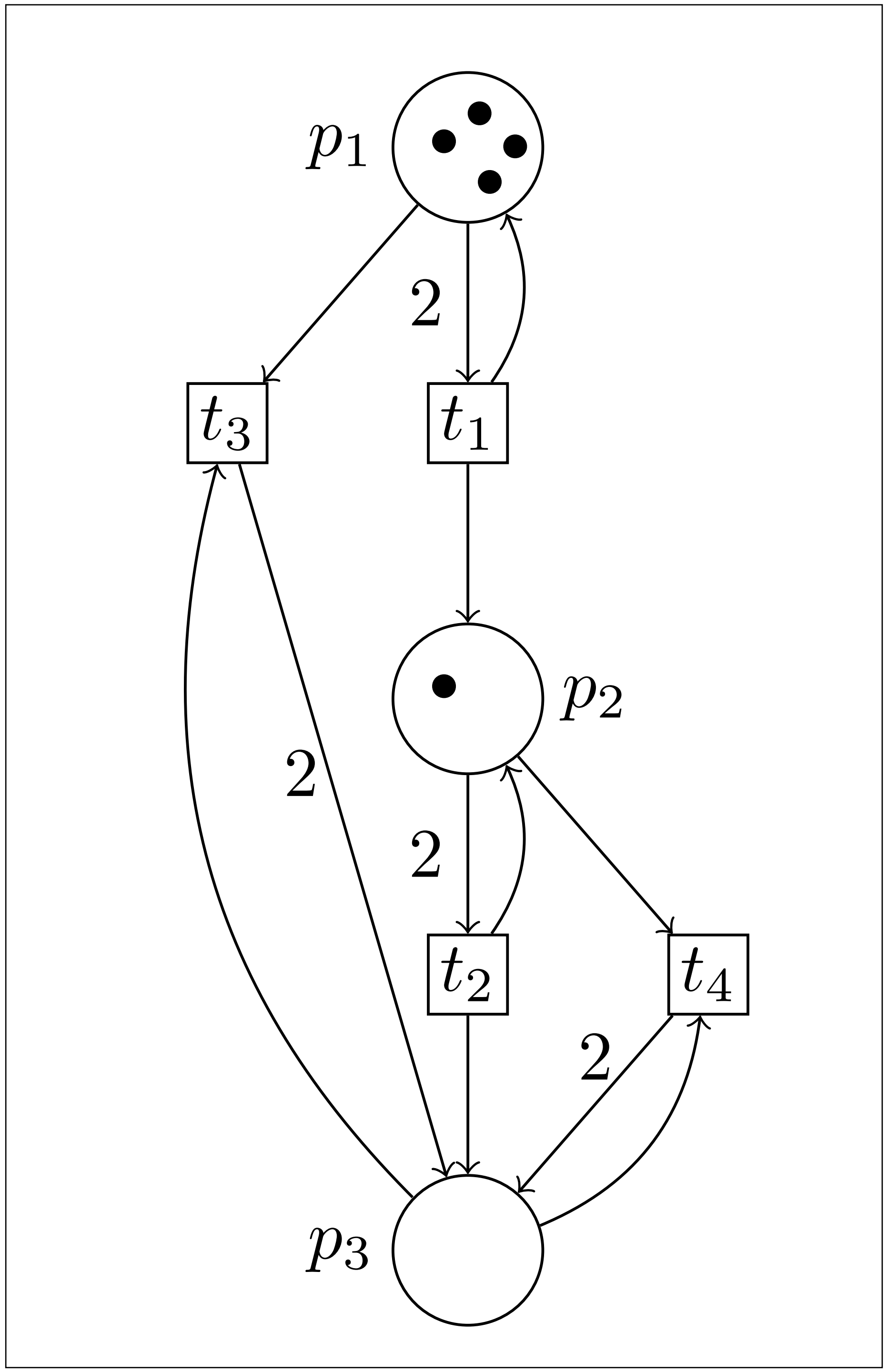

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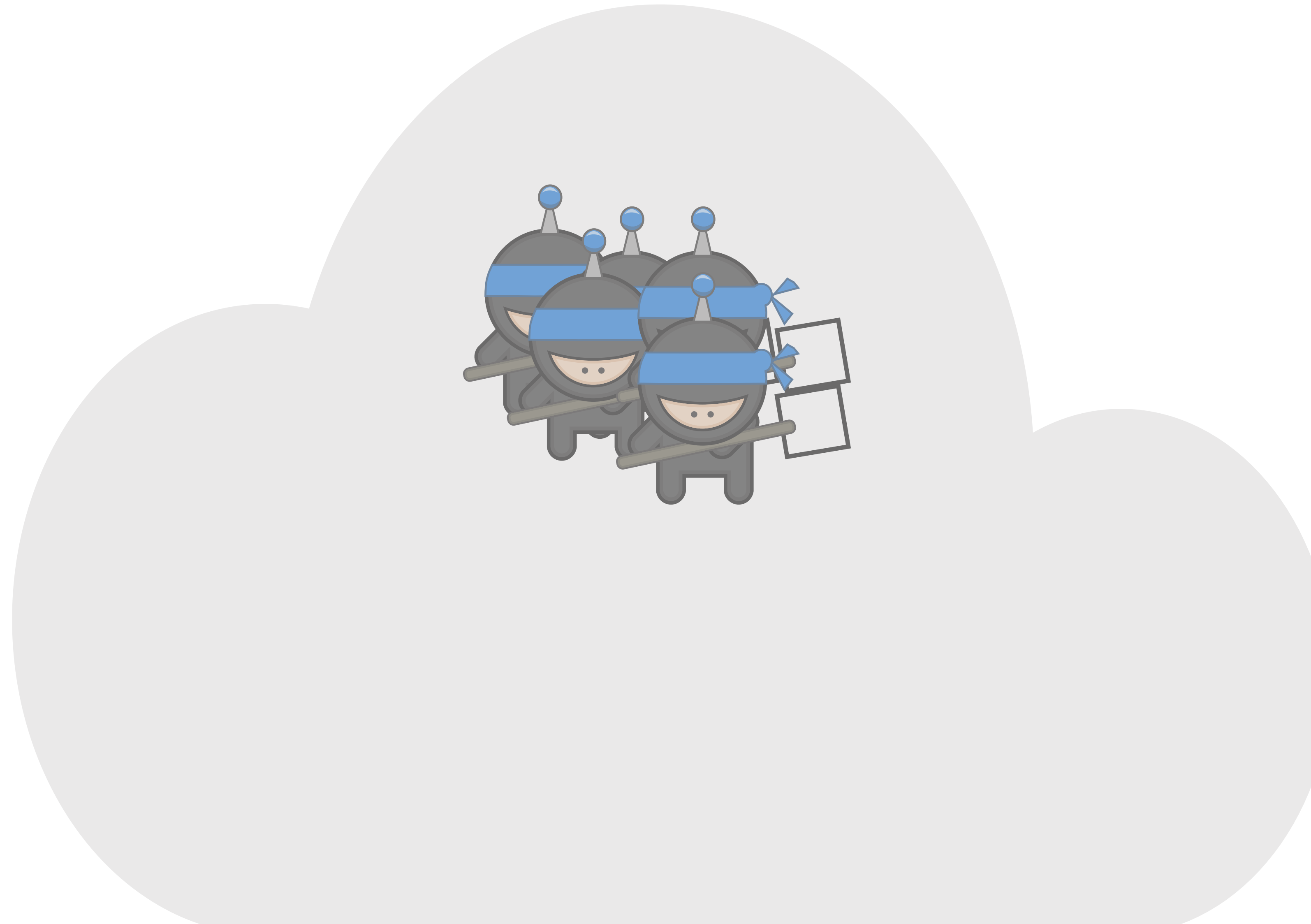

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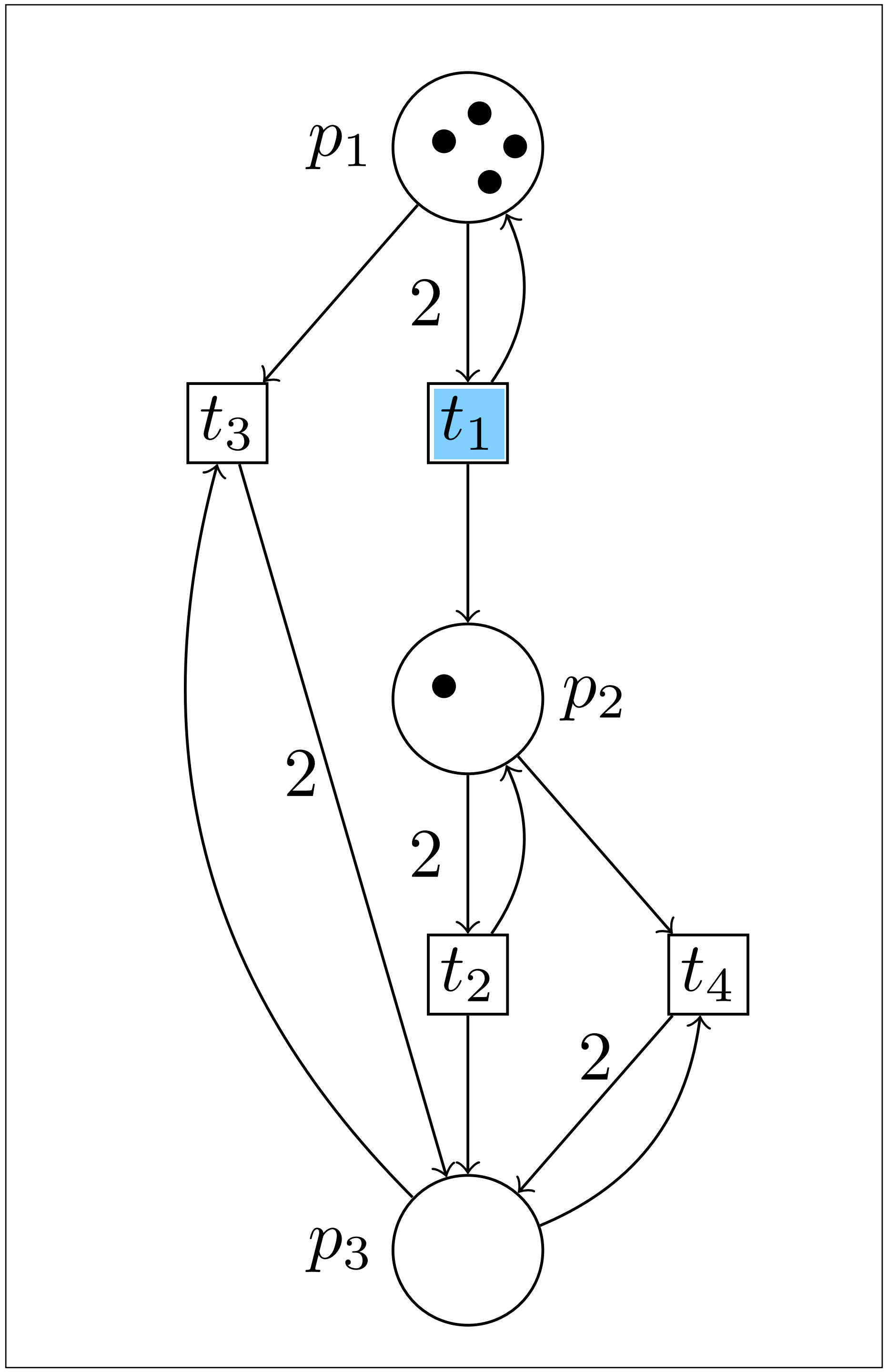

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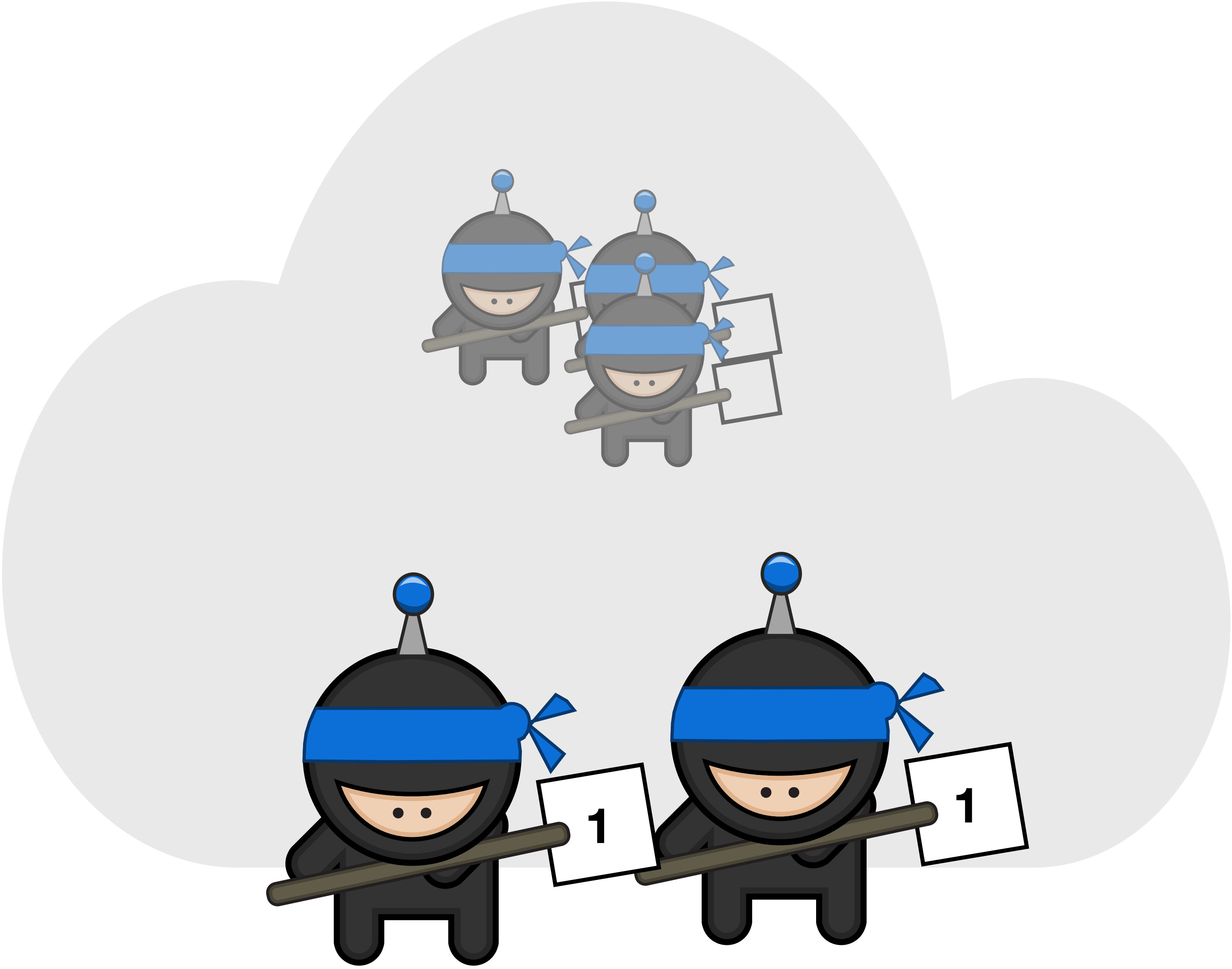

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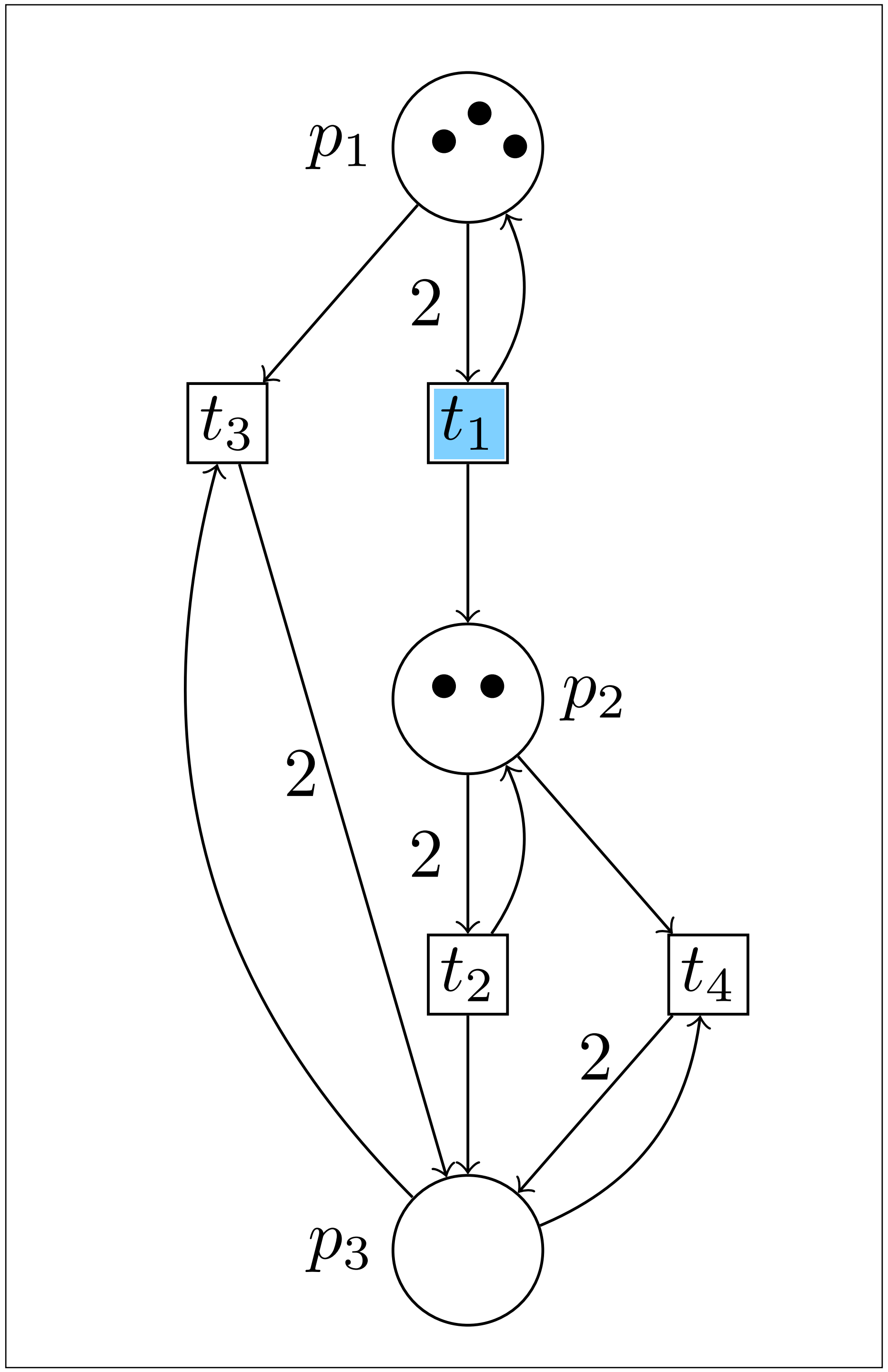

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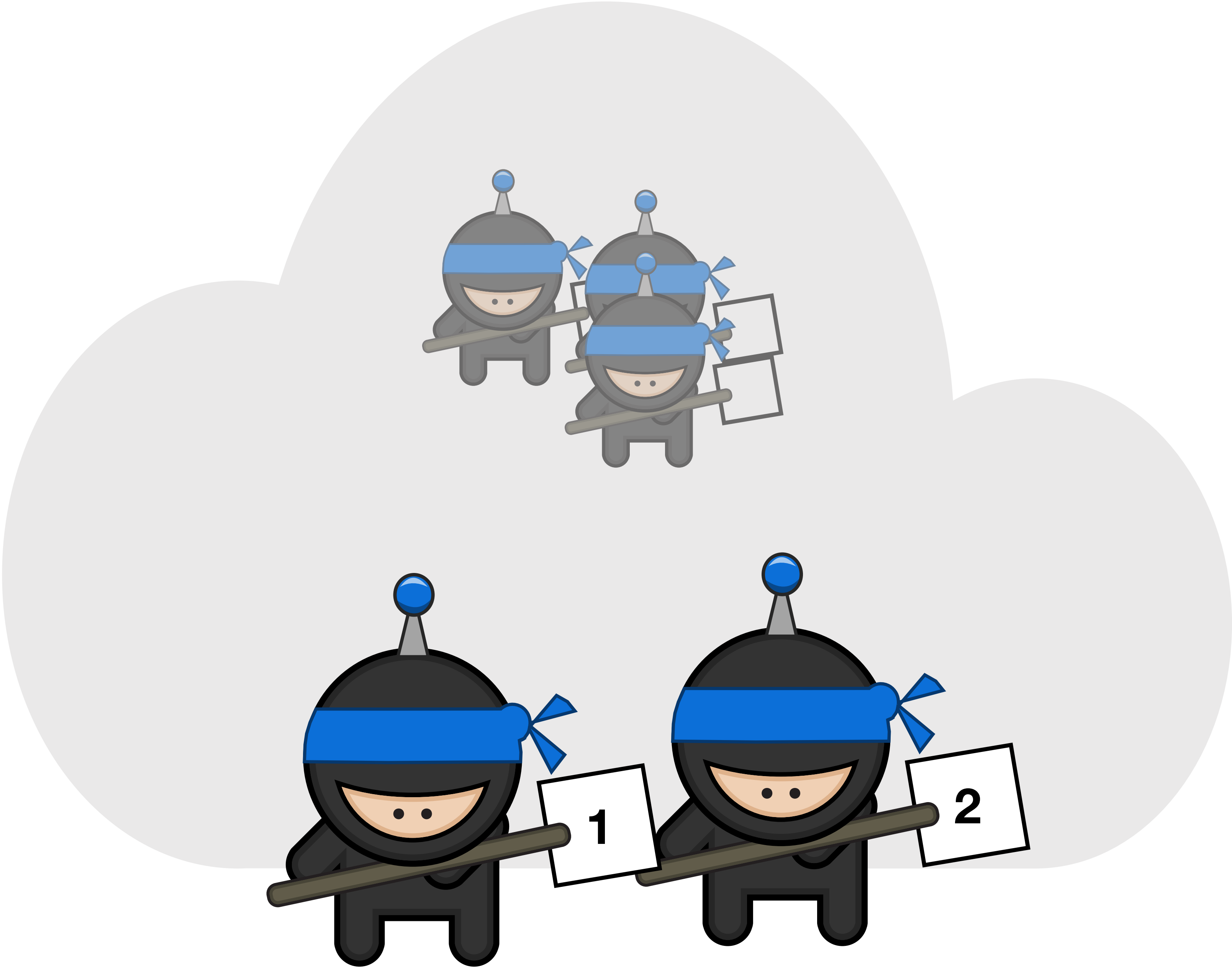

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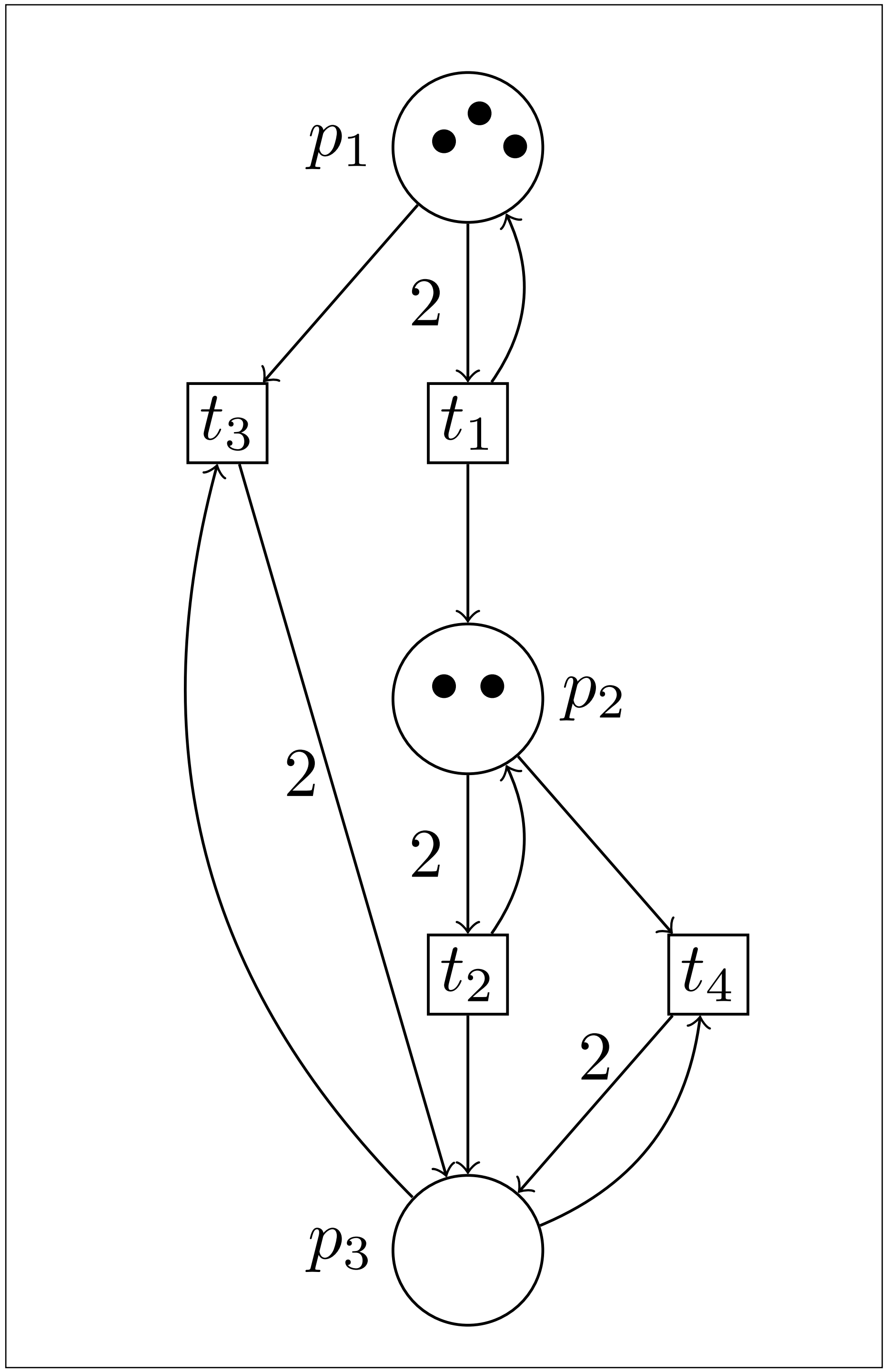

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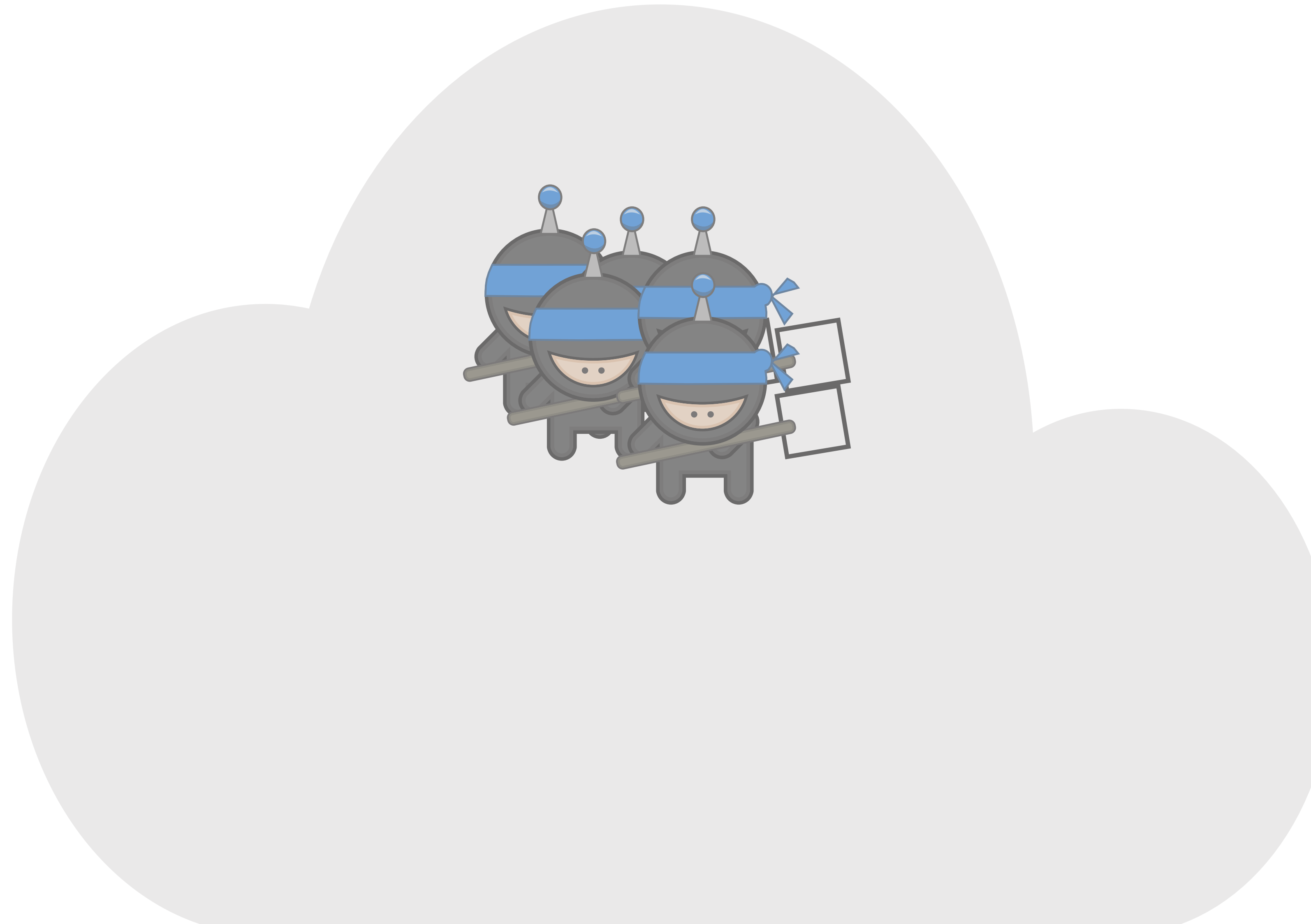

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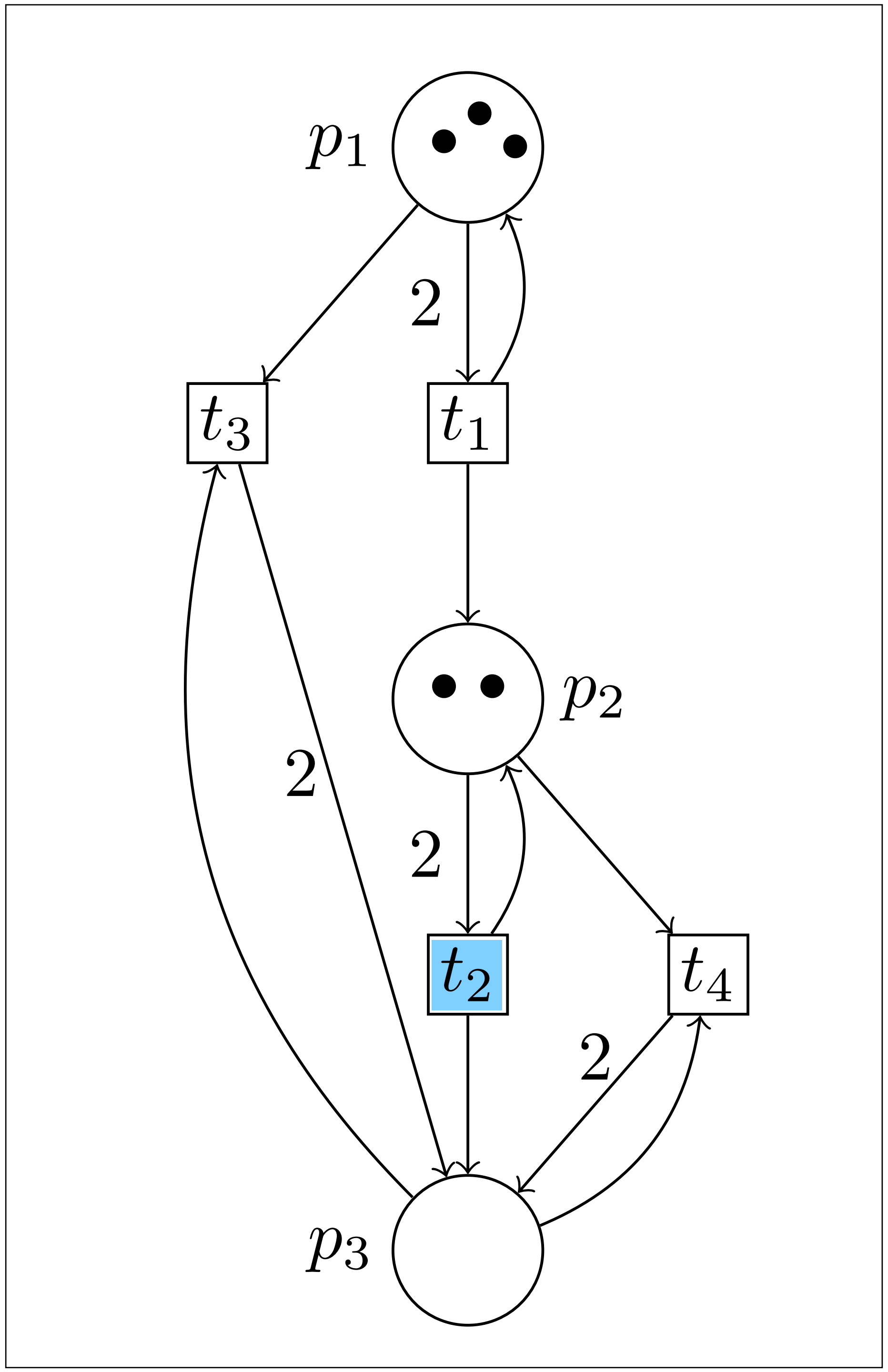

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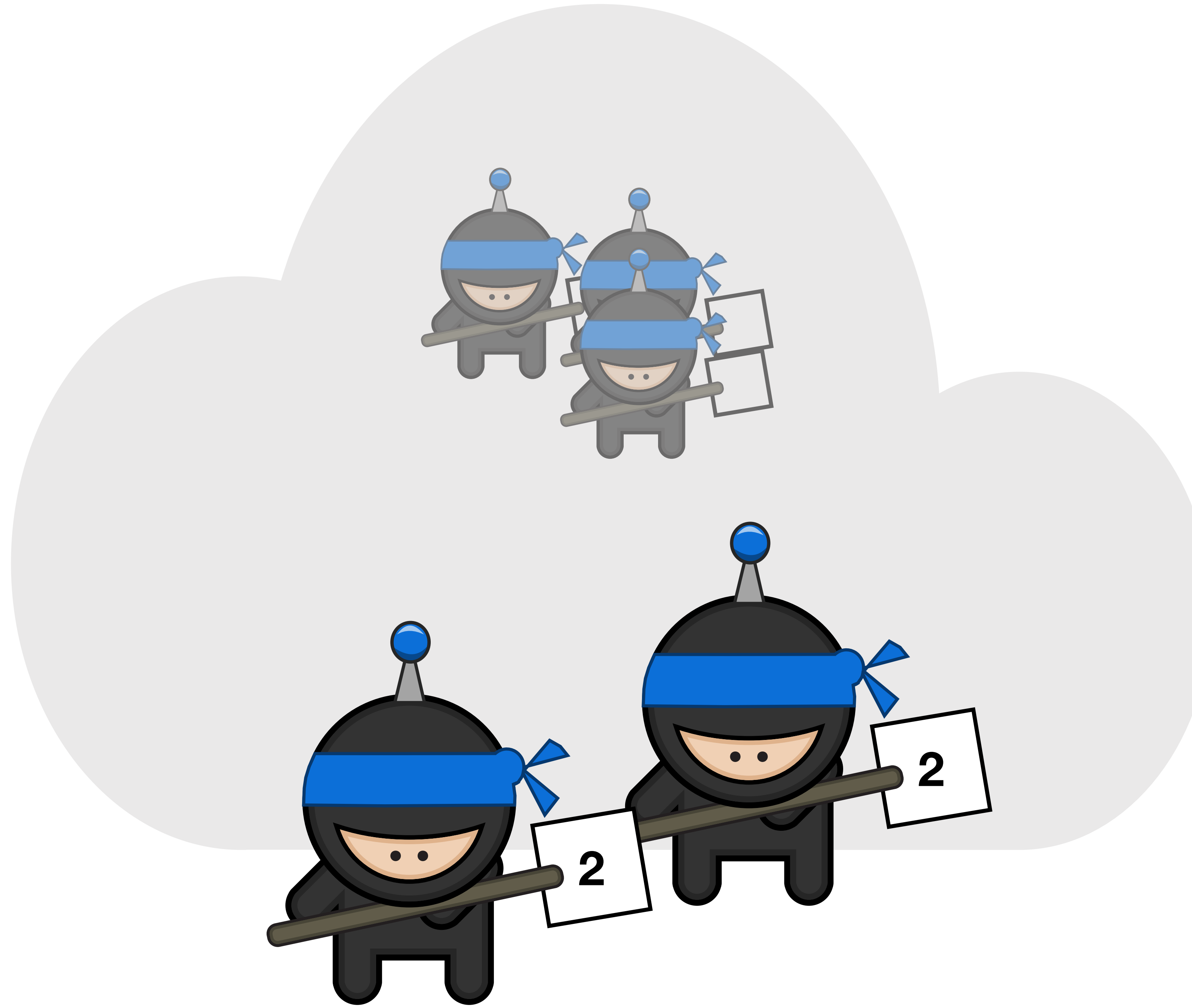

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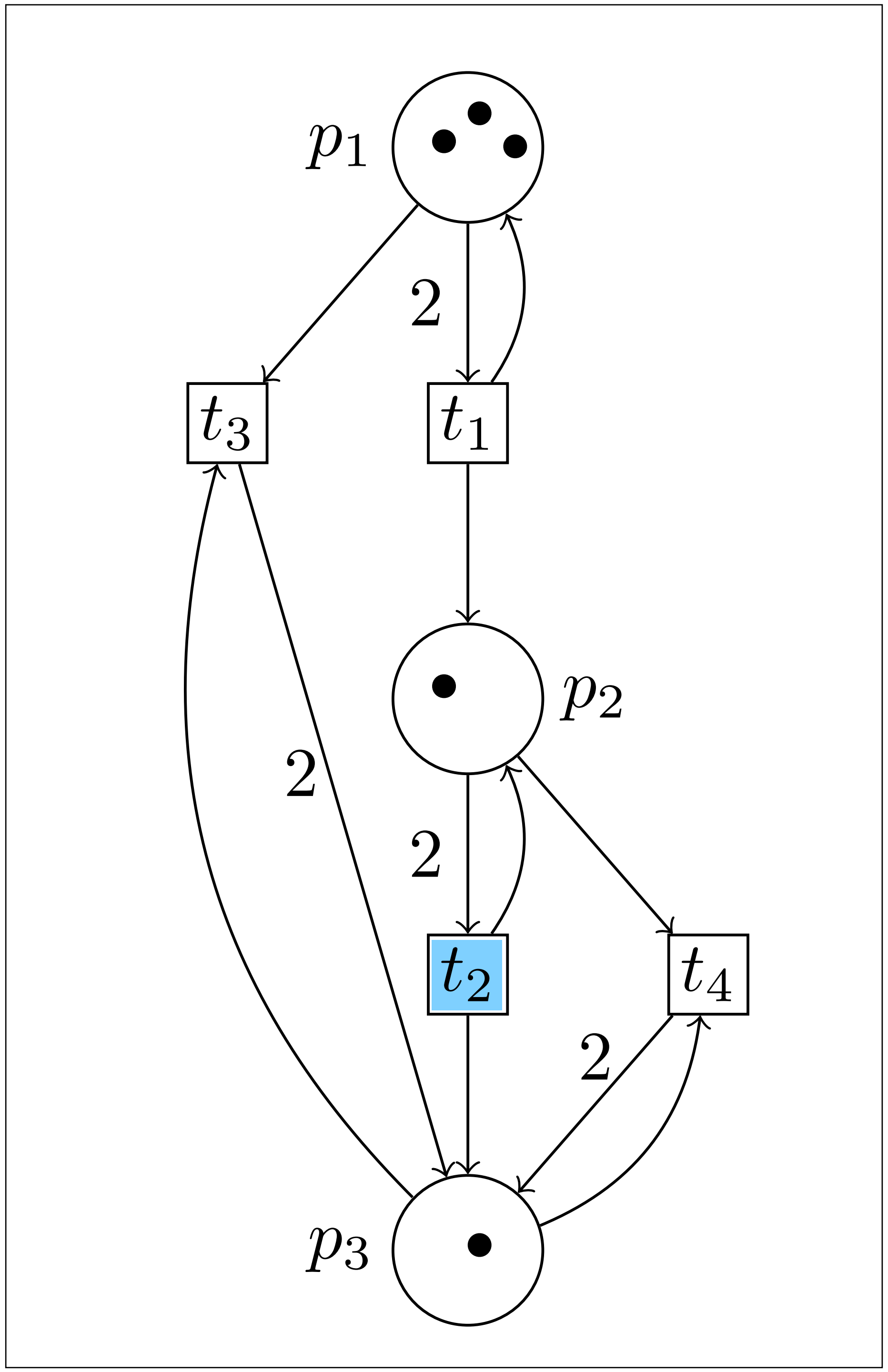

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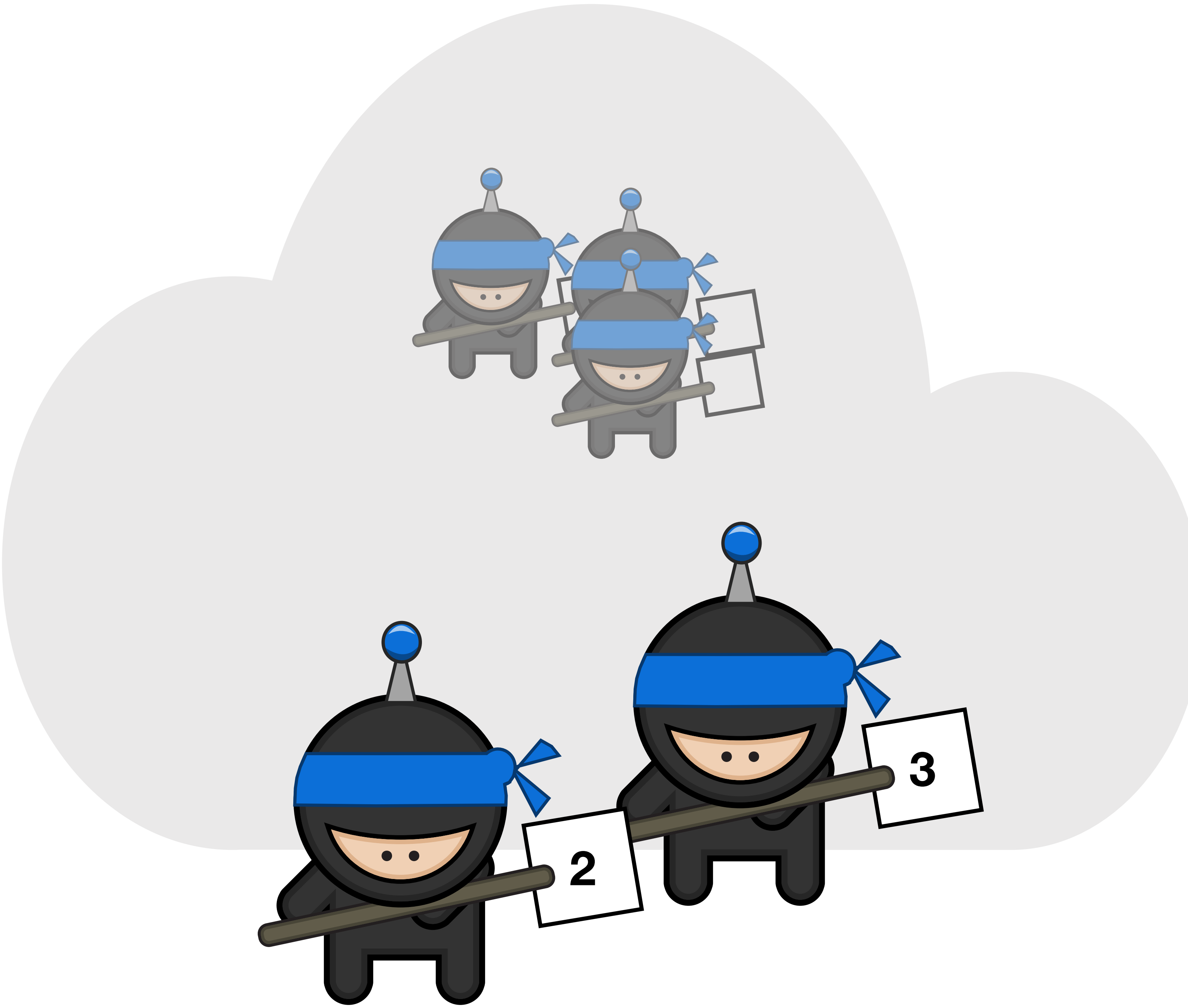

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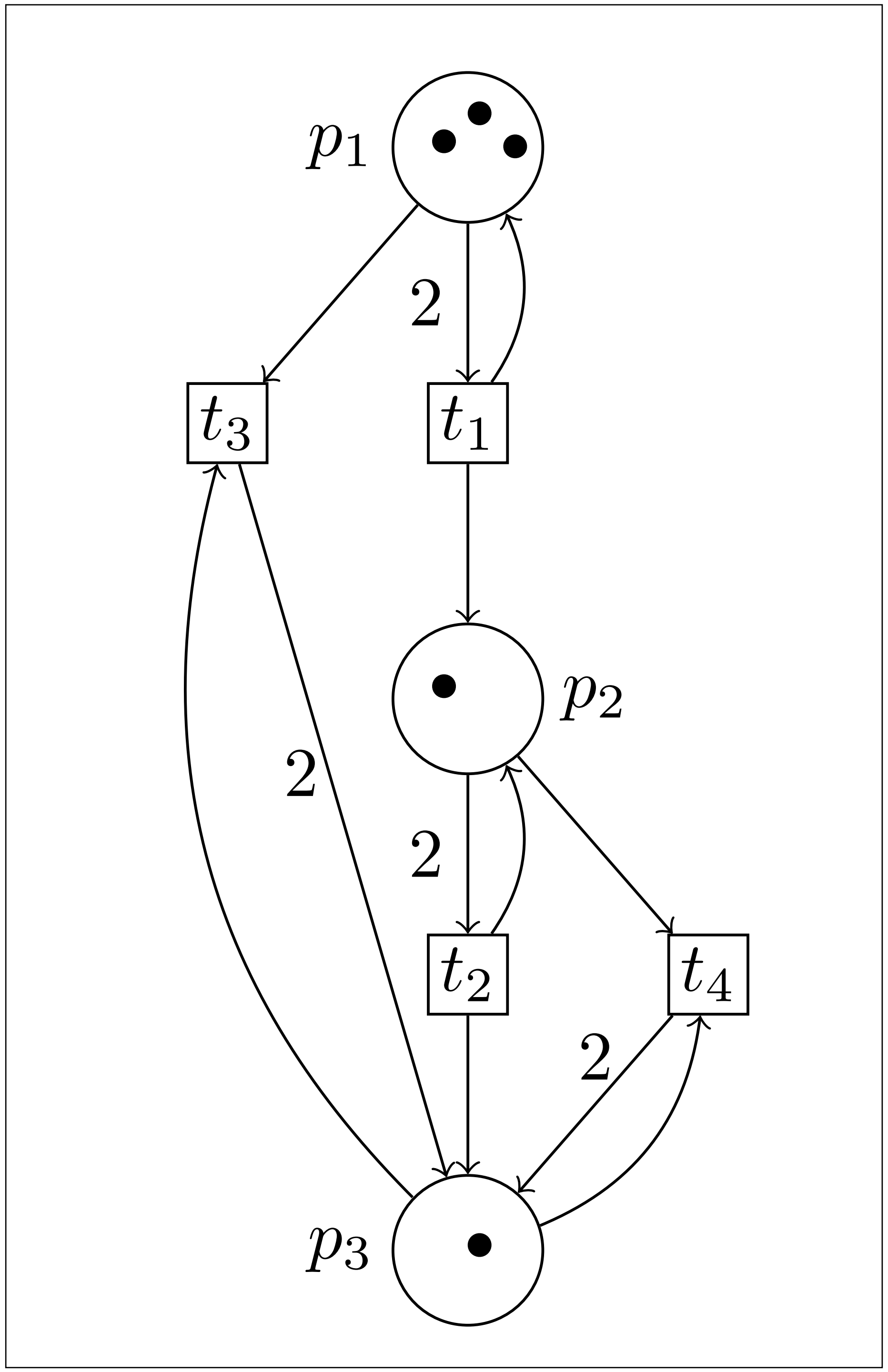

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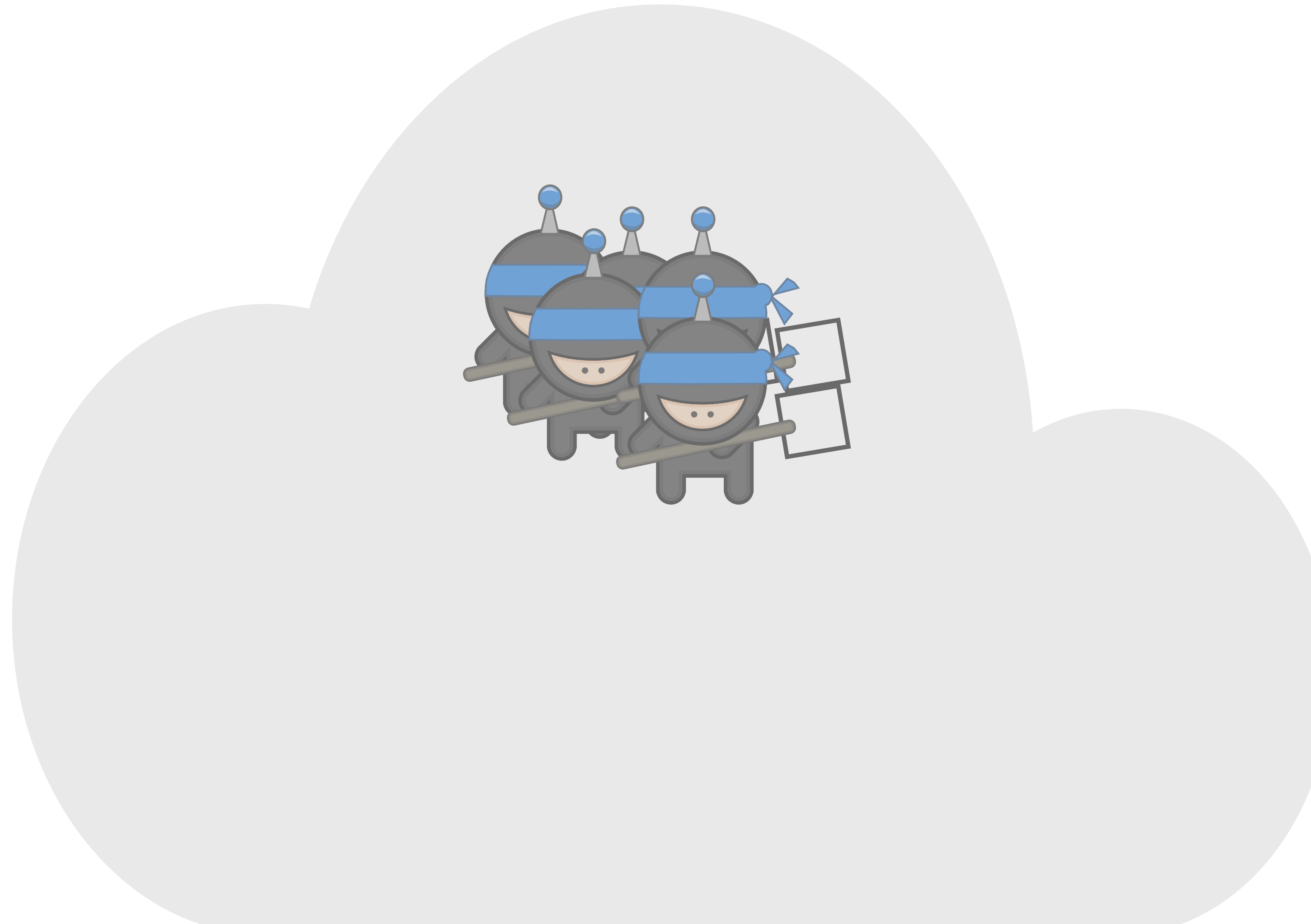

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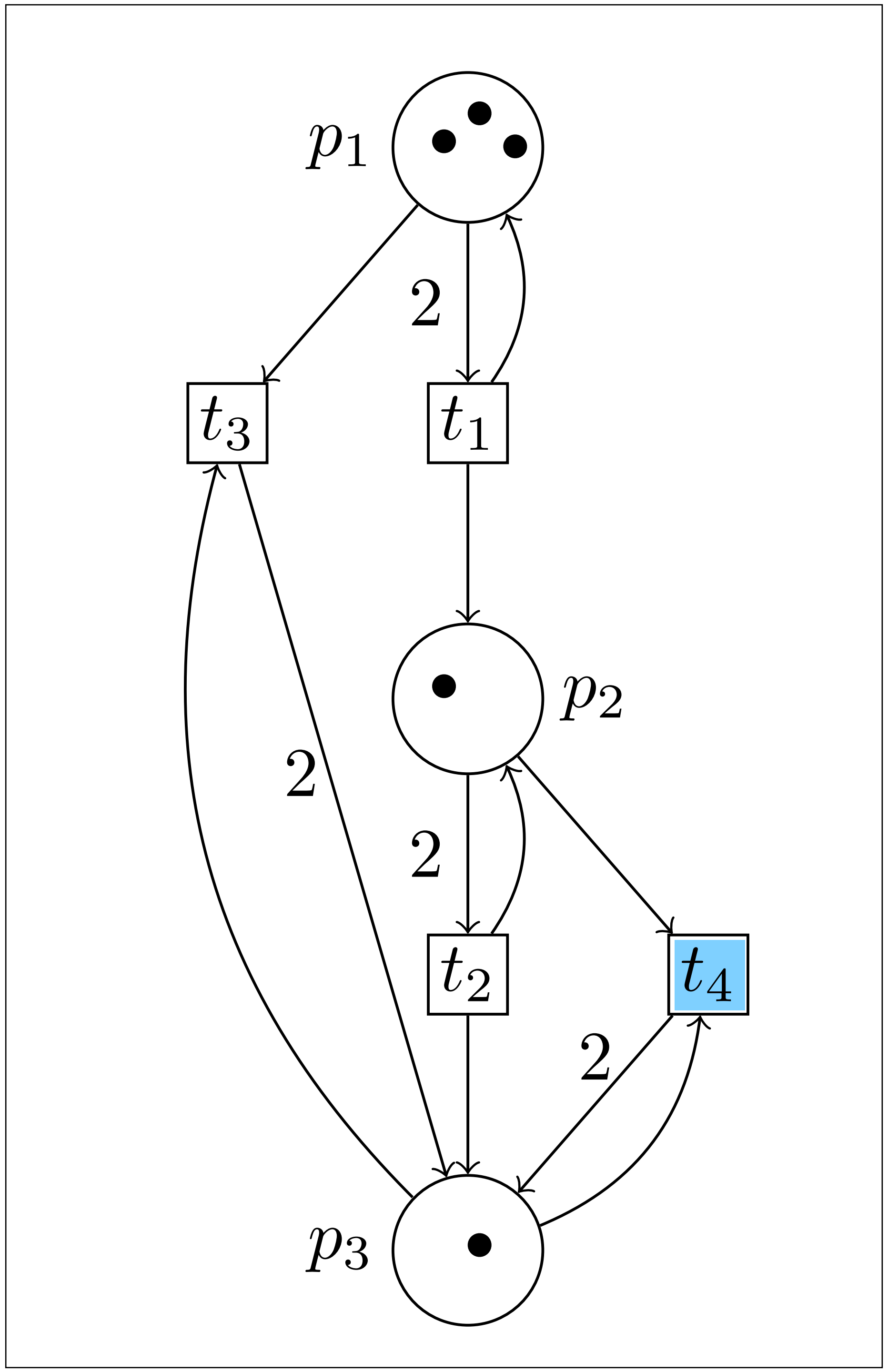

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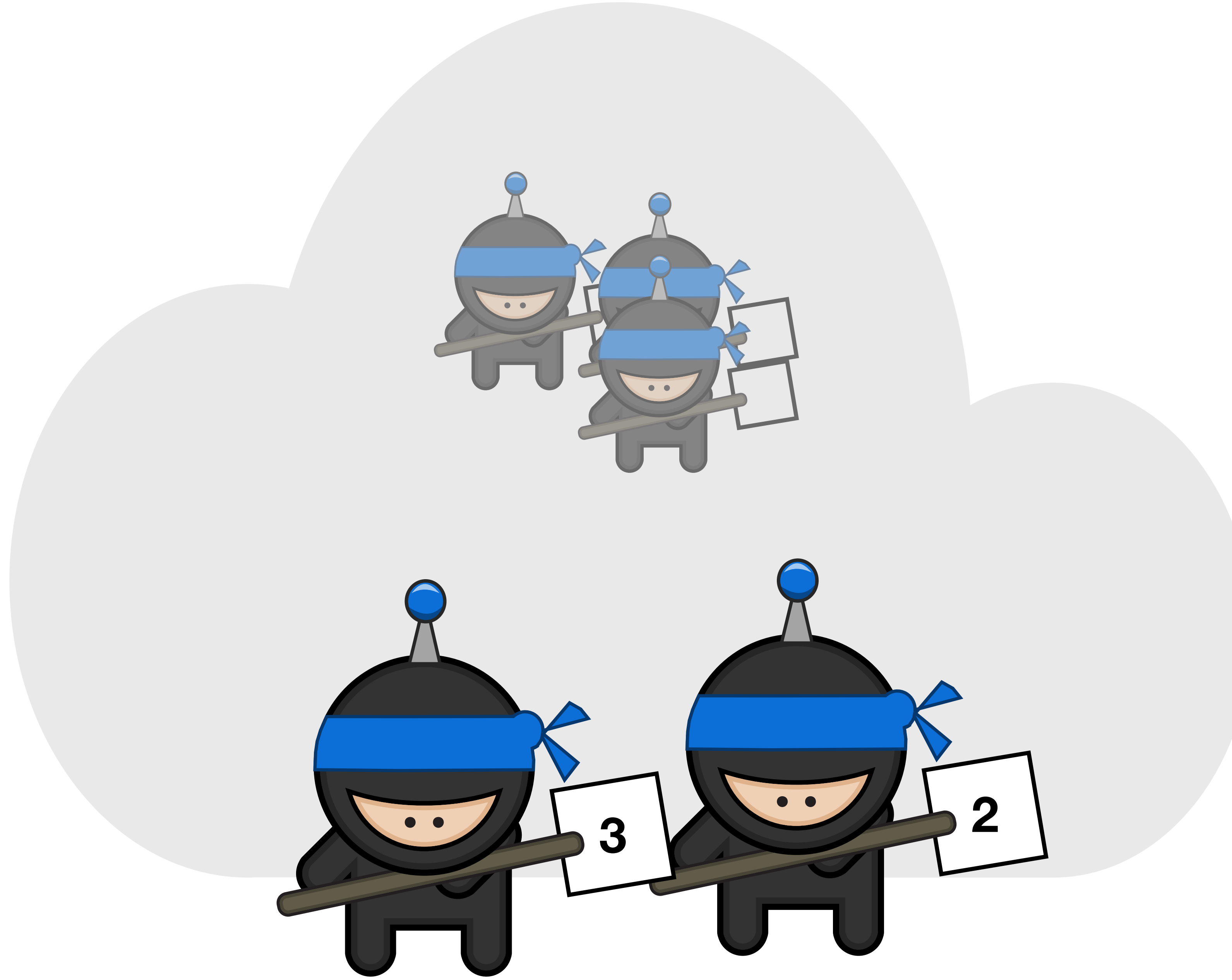

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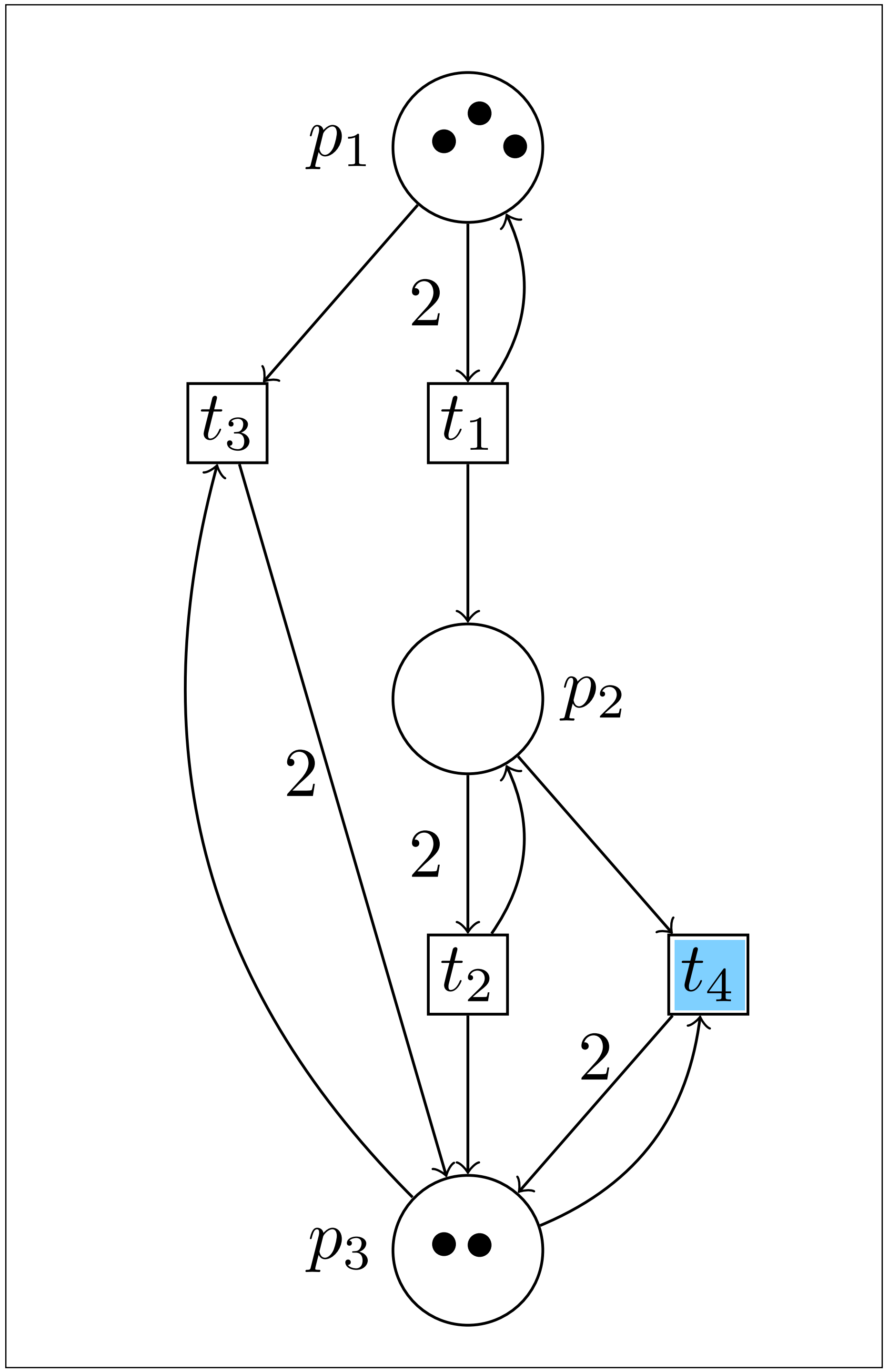

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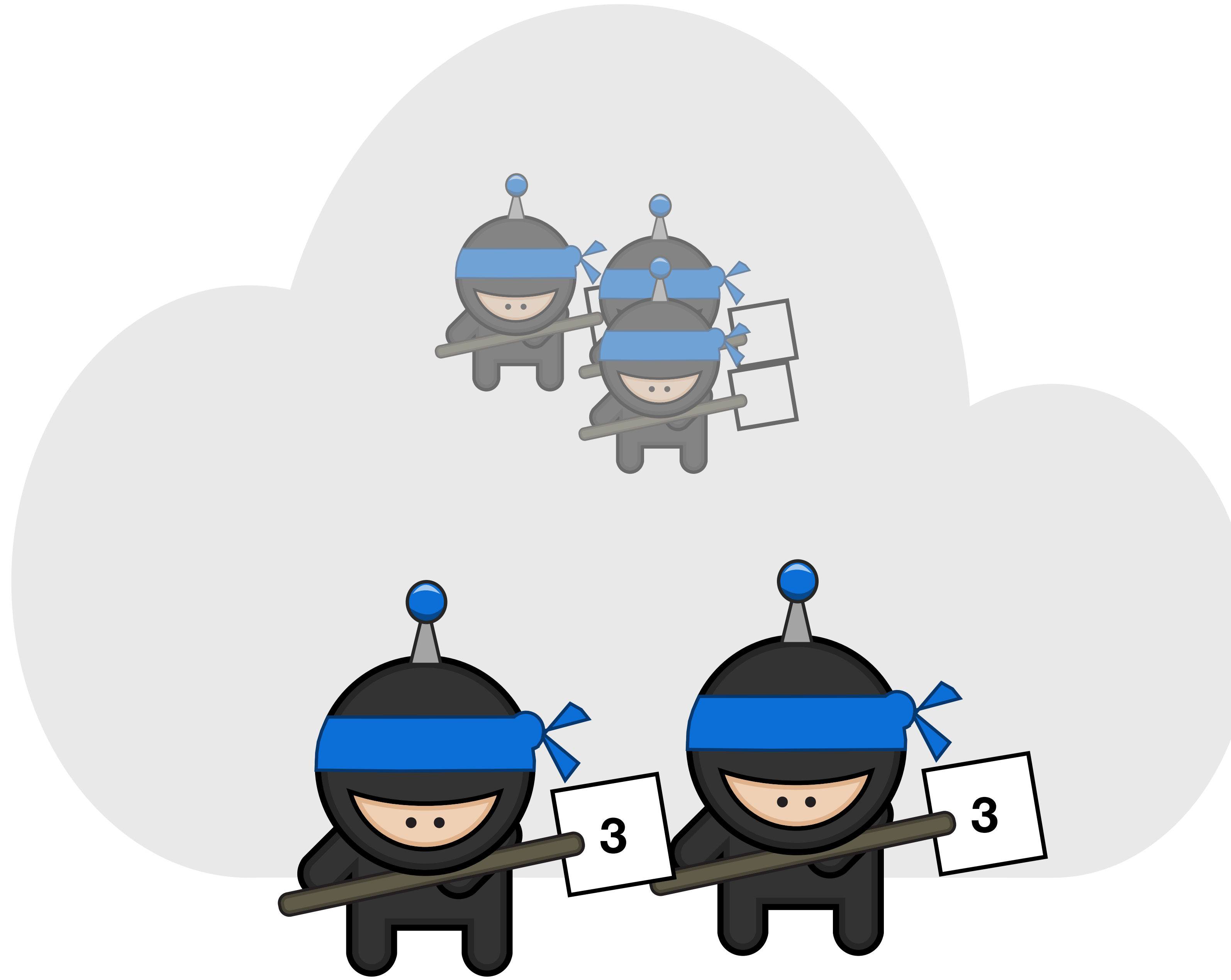

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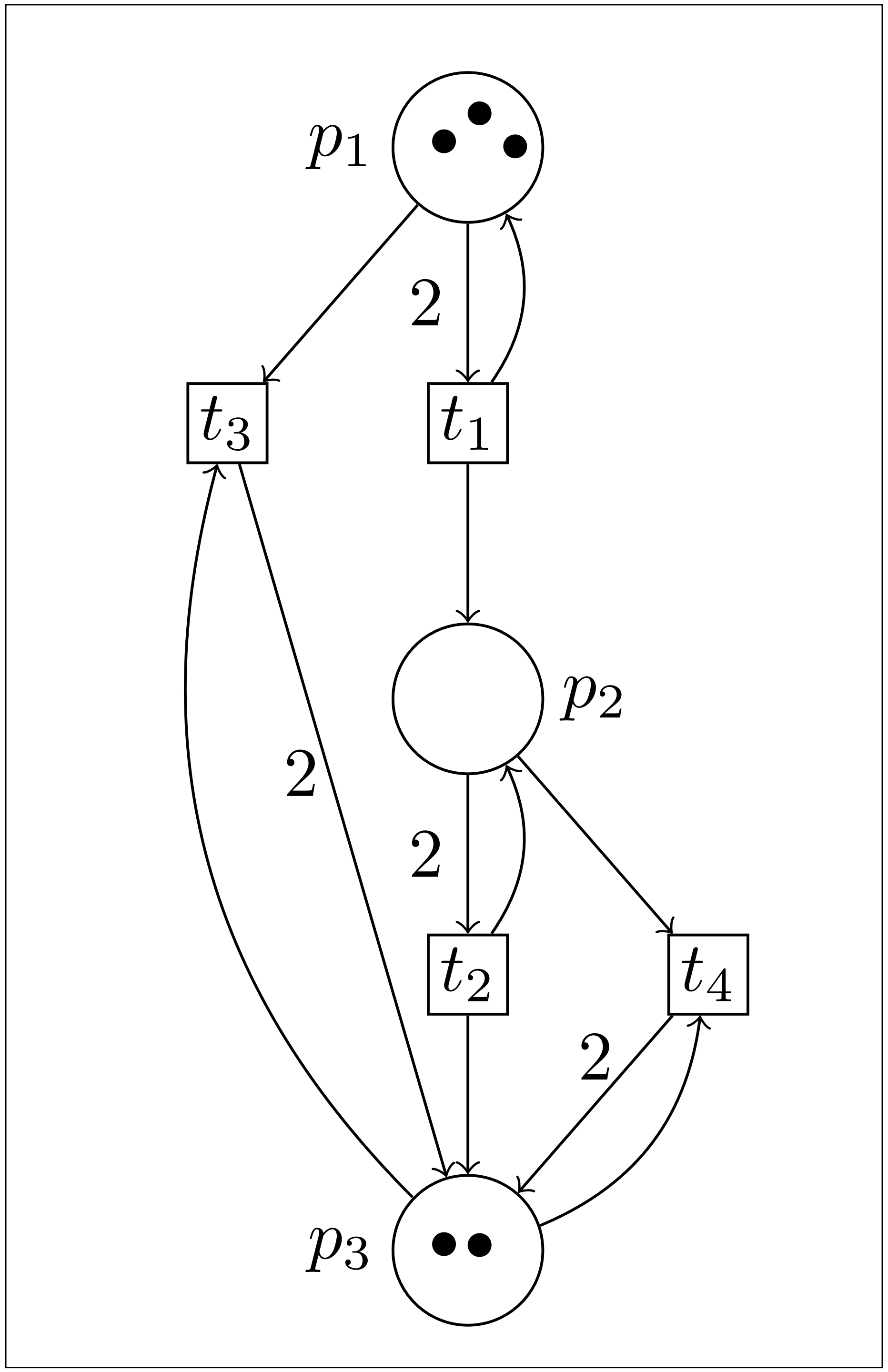

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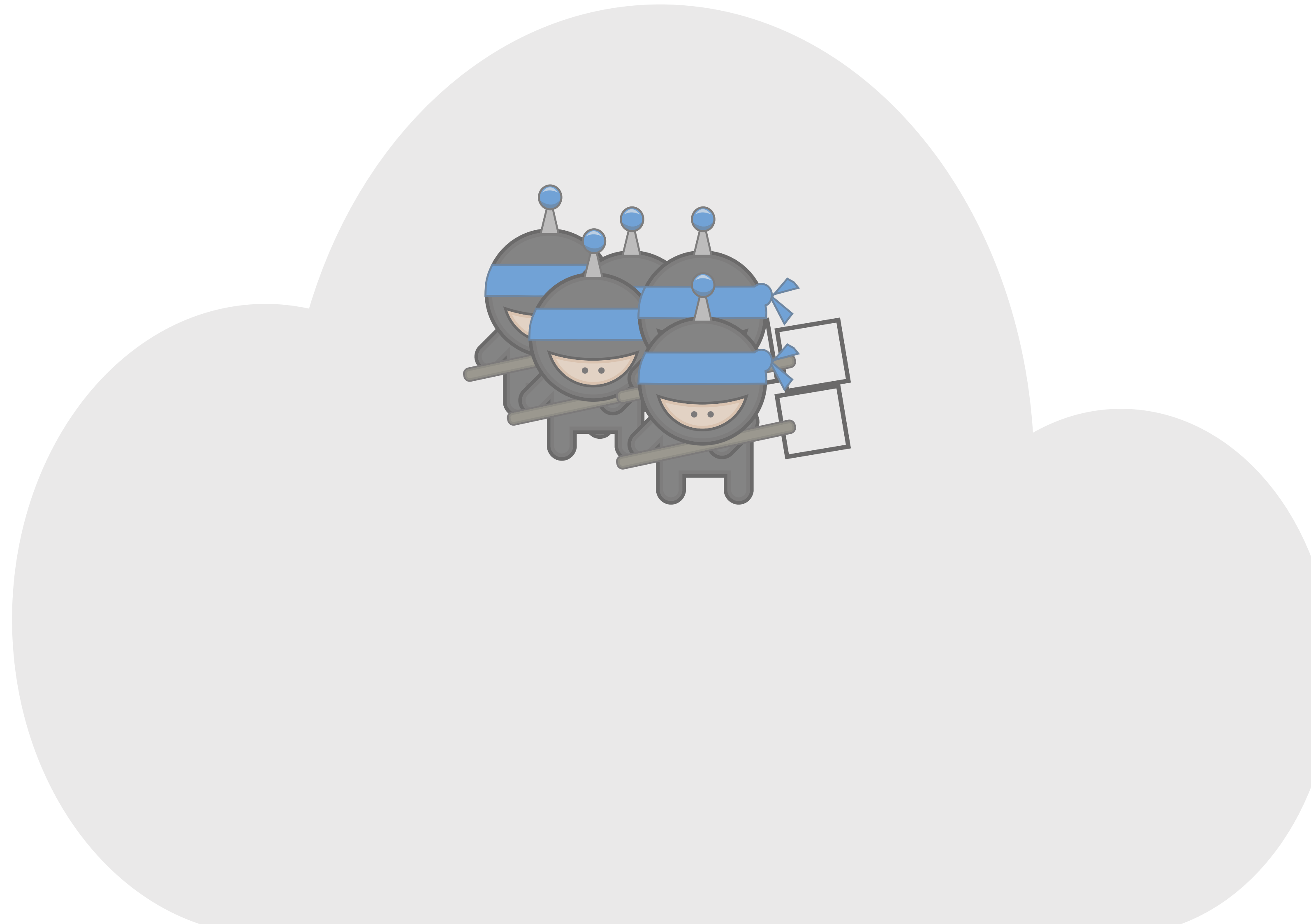

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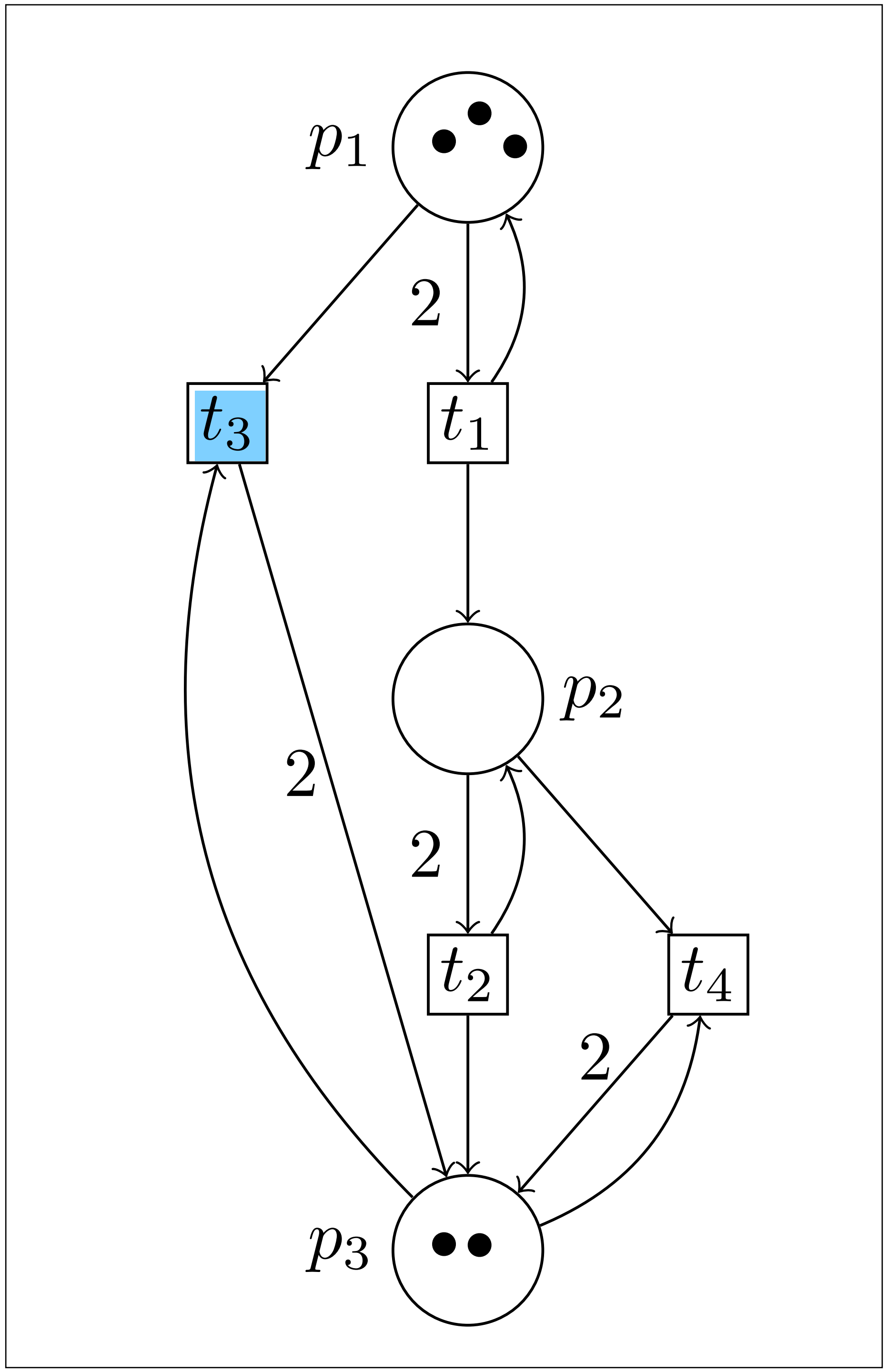

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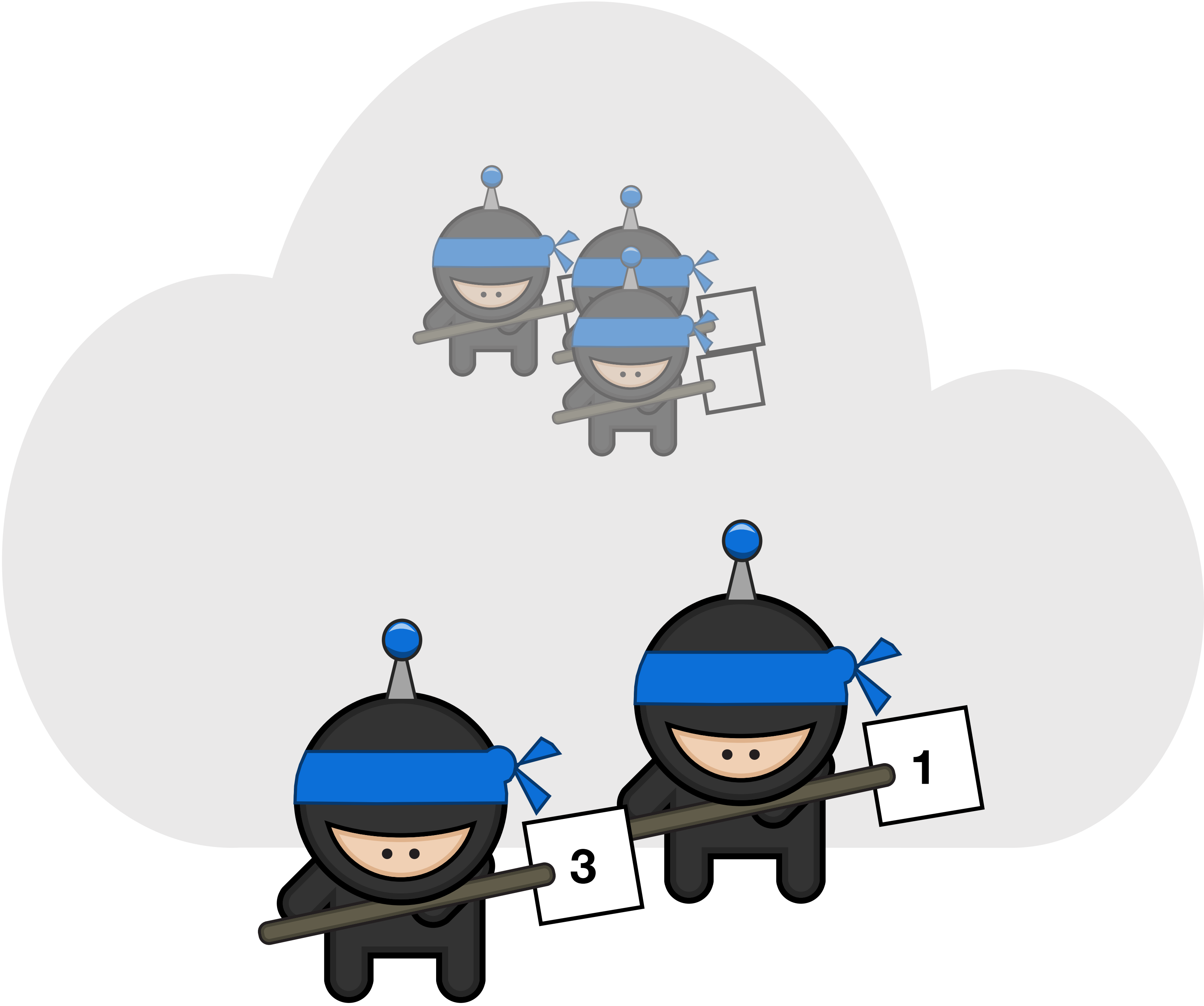

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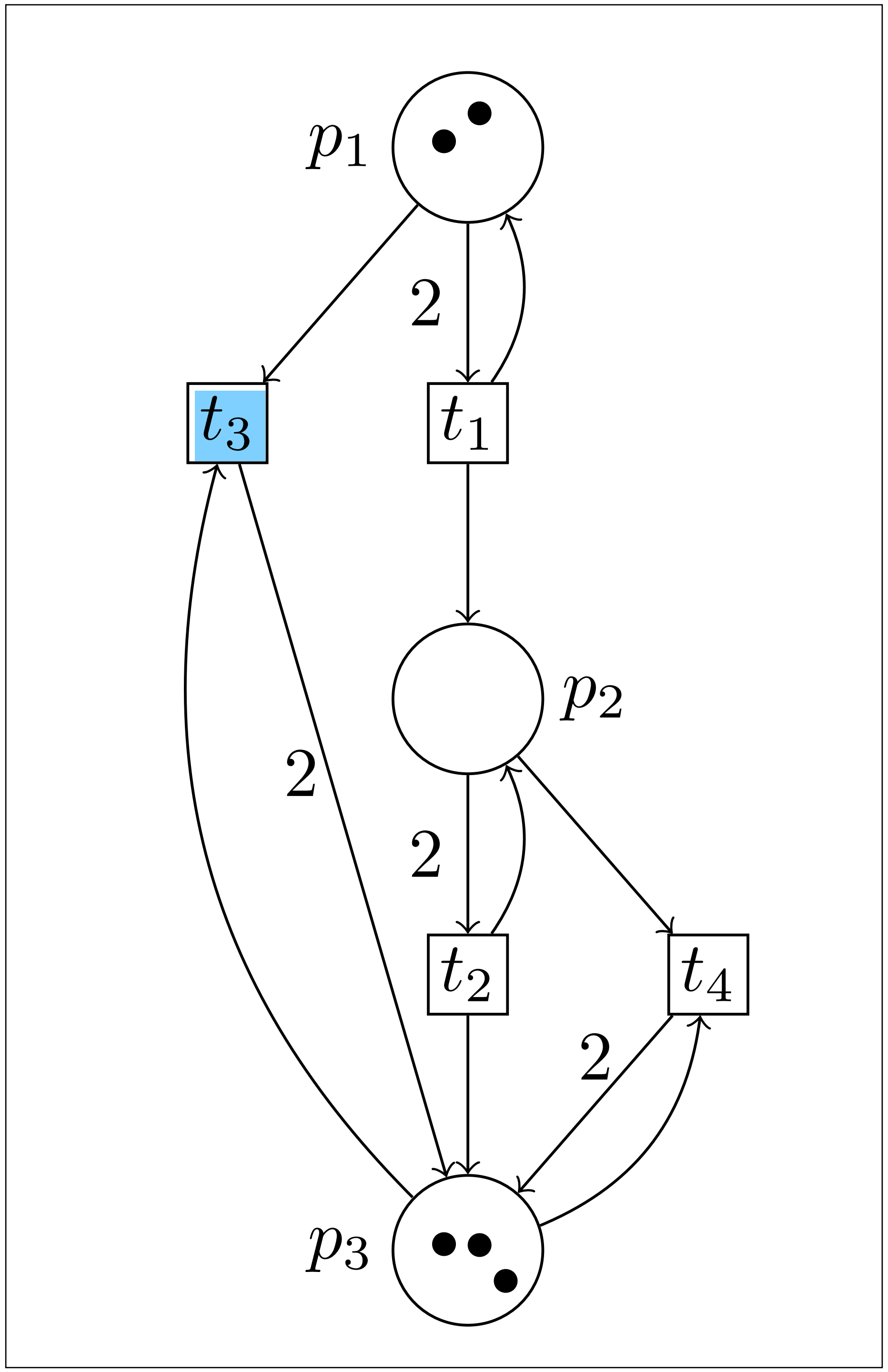

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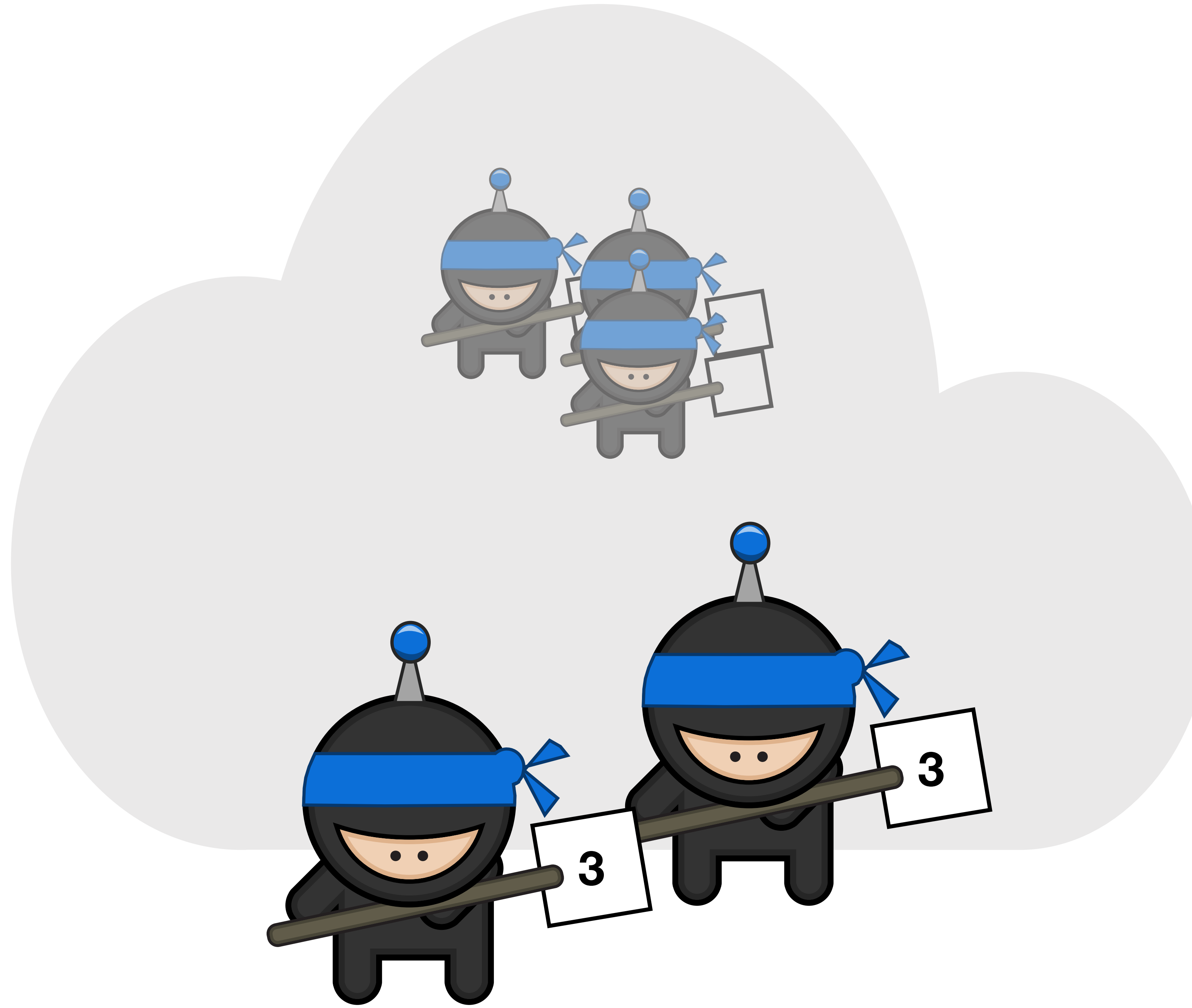

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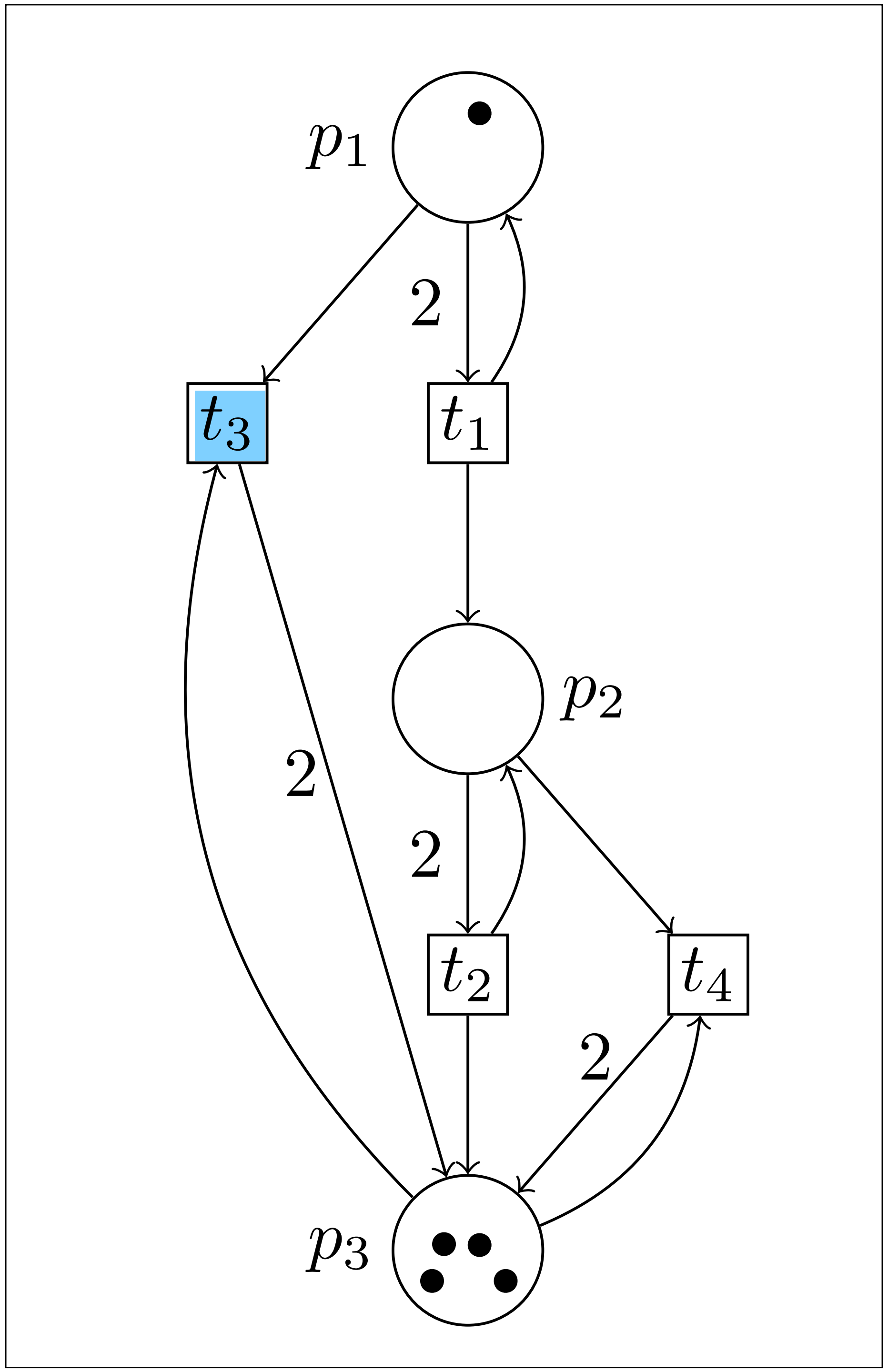

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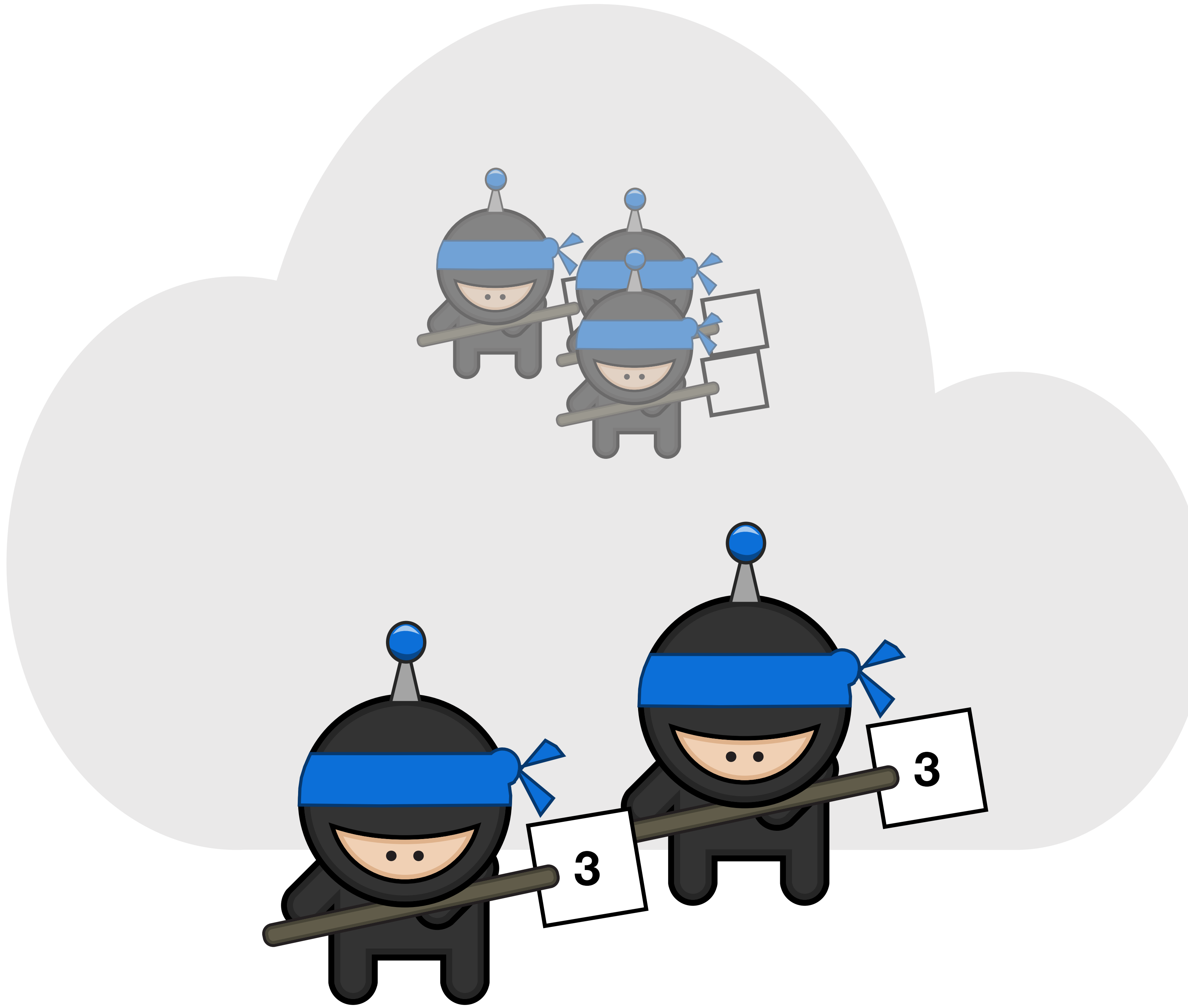

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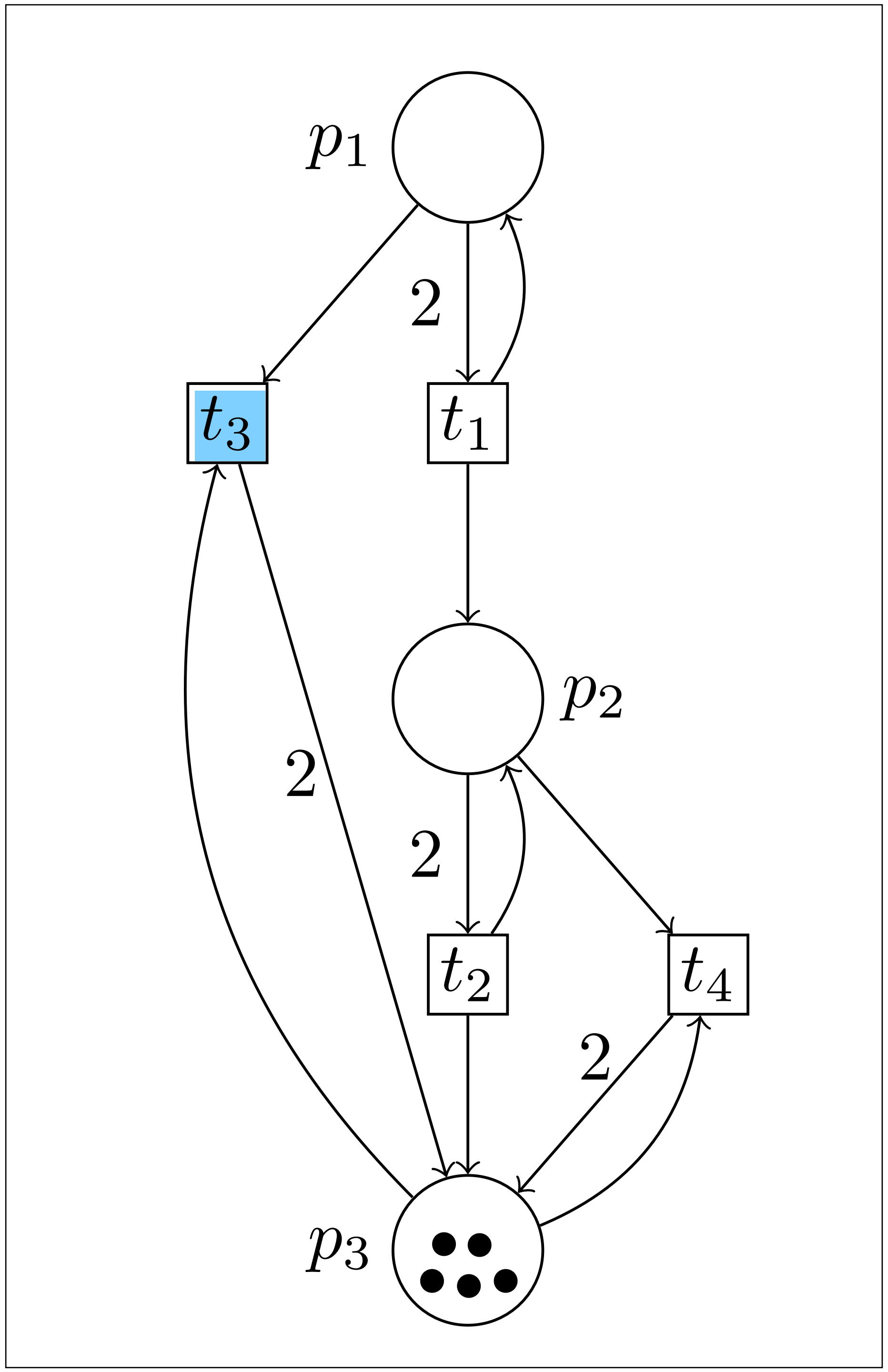

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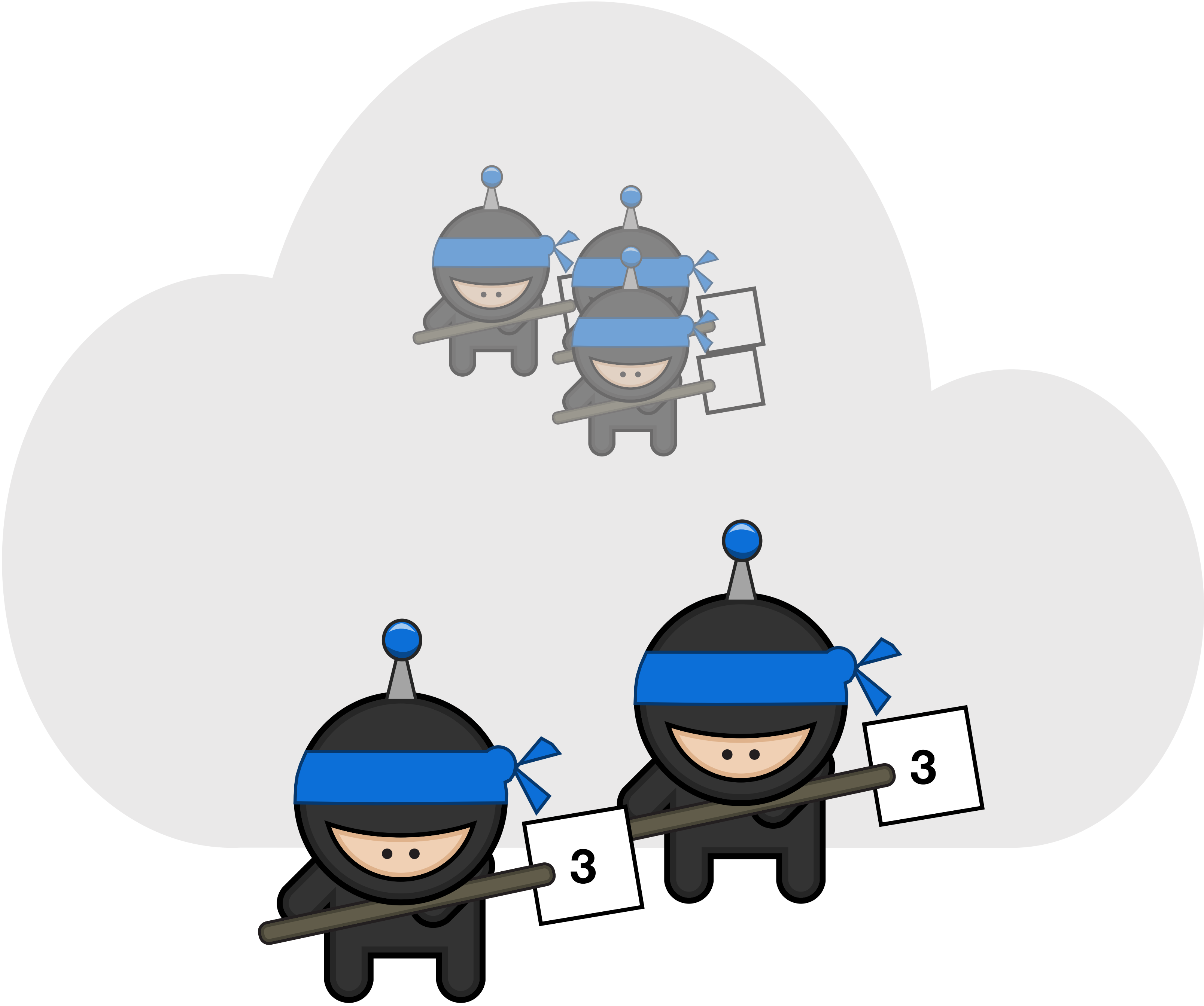


In this talk

Part 1: Immediate observation nets

Parameterized reachability is easy

+ an intuition of why

Part 2: Branching immediate observation nets

Parameterized reachability is still easy

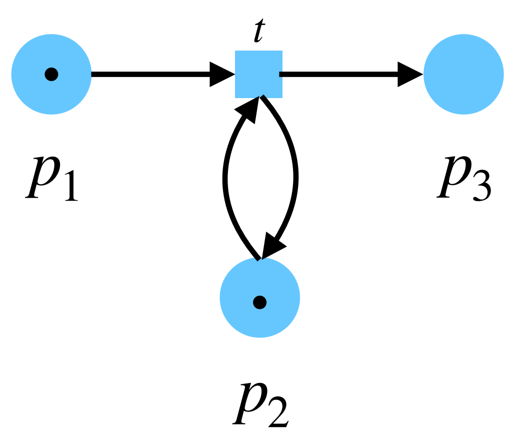
and BIO nets are expressive

Part 1:
Immediate observation nets

Immediate Observation nets

[Esparza, Raskin, *W.-K.*, ’19]

Immediate Observation nets (IO)



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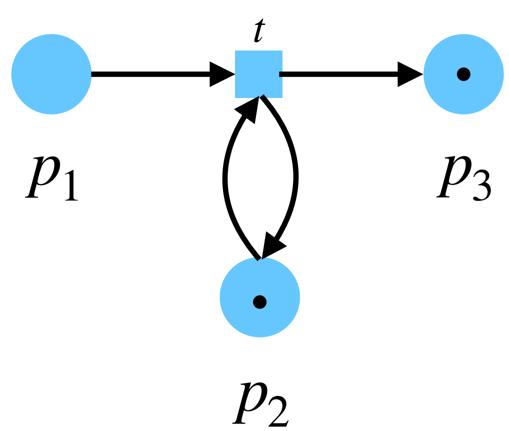

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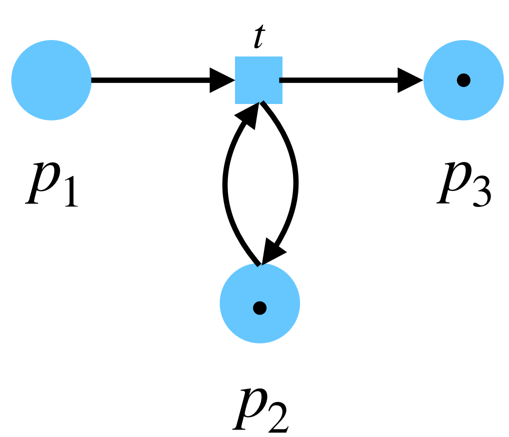

- introduced to study **immediate observation** population protocols (distributed computing model).

  [Angluin, Aspnes, Eisenstat, Ruppert, ’07]

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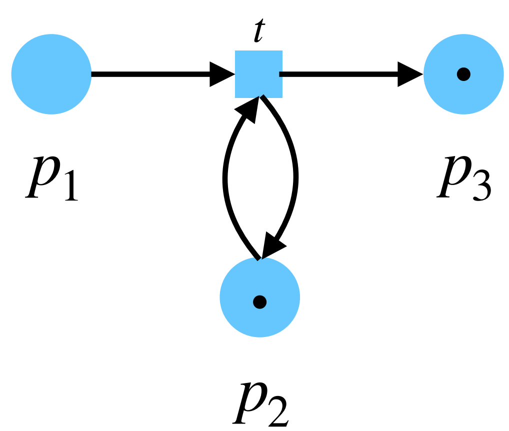

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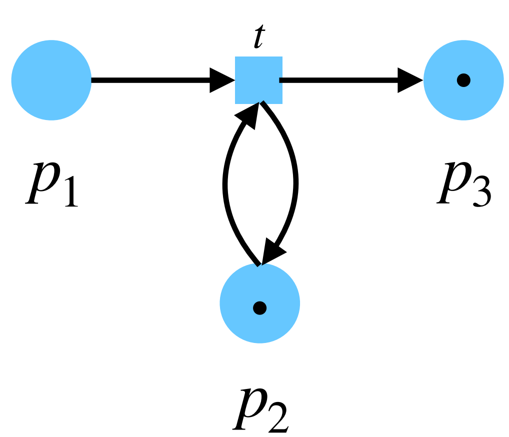

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➡ In these application domains we are interested in *parameterized* problems.

Cube-reachability

number of tokens in q

A **cube** is a boolean combination of constraints $a \leq \#q \leq b$

$\in \mathbb{N}$ $\in \mathbb{N} \cup \infty$

cube-reachability: given cubes $\mathscr{C}$ and $\mathscr{C}'$, does there exist $M \in \mathscr{C}$ and $M' \in \mathscr{C}'$ such that M reaches M' ?

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PSPACE-complete for IO nets

➡ correctness of IO population protocols is in PSPACE

[Esparza, Raskin, *W.-K.*, '19]

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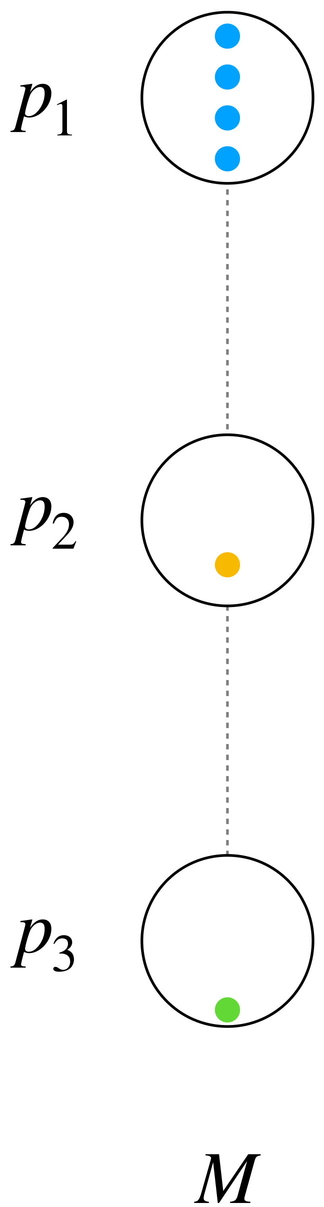

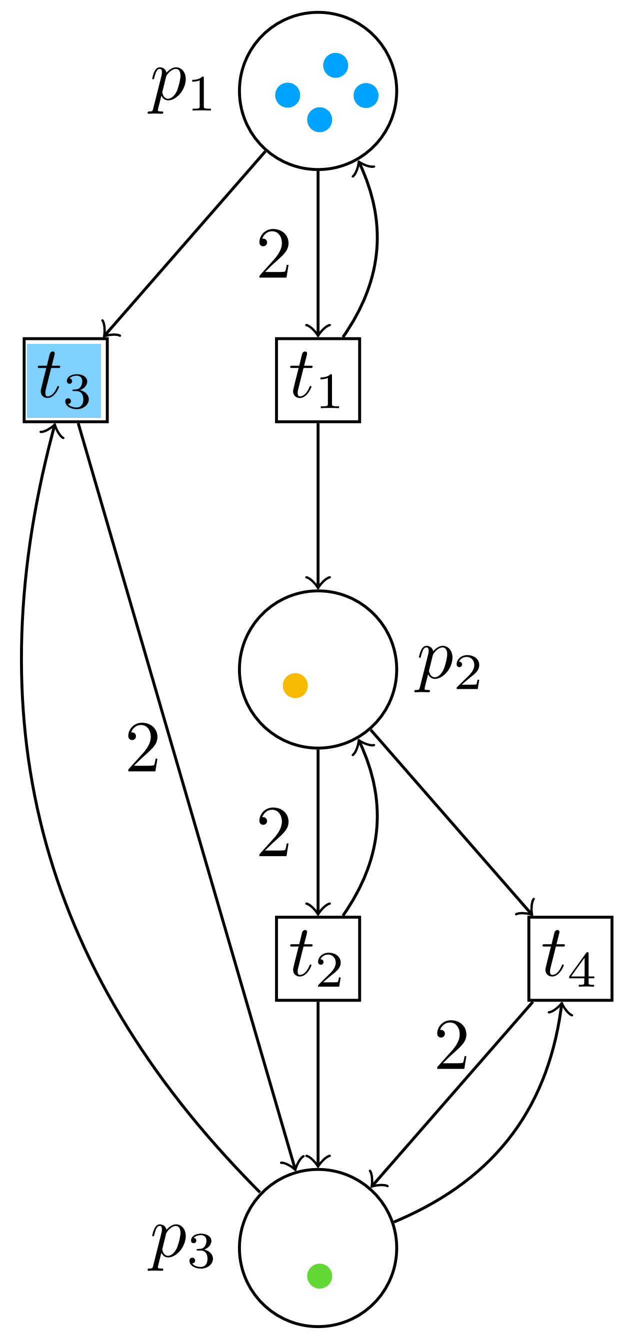

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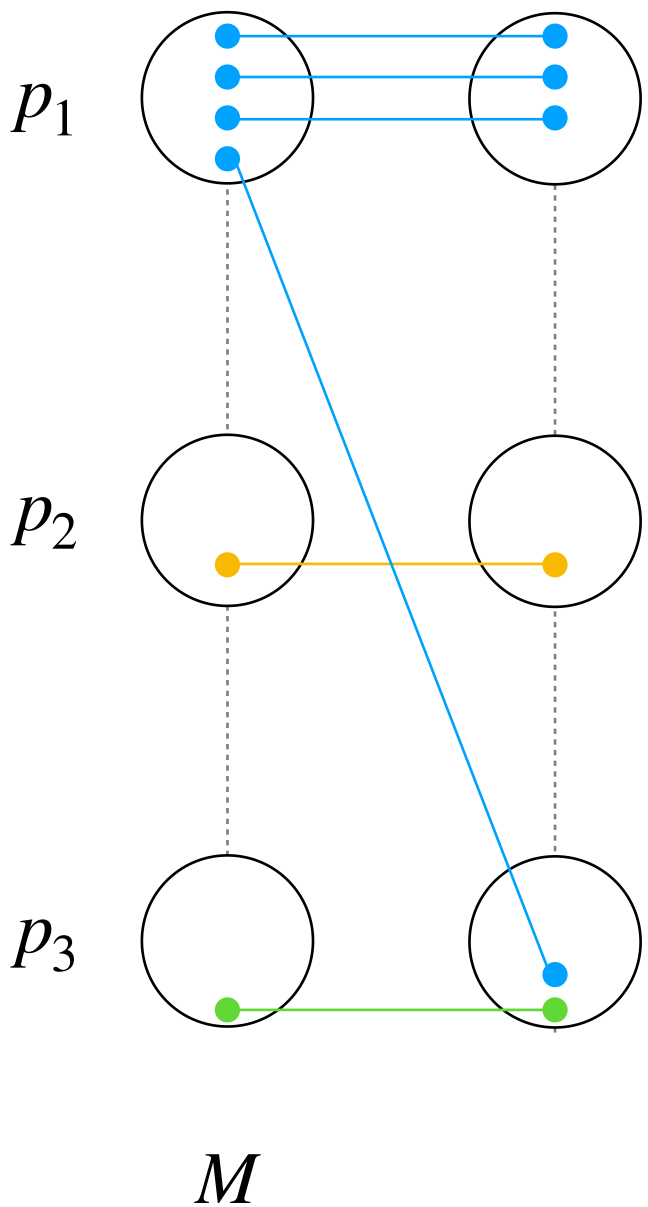

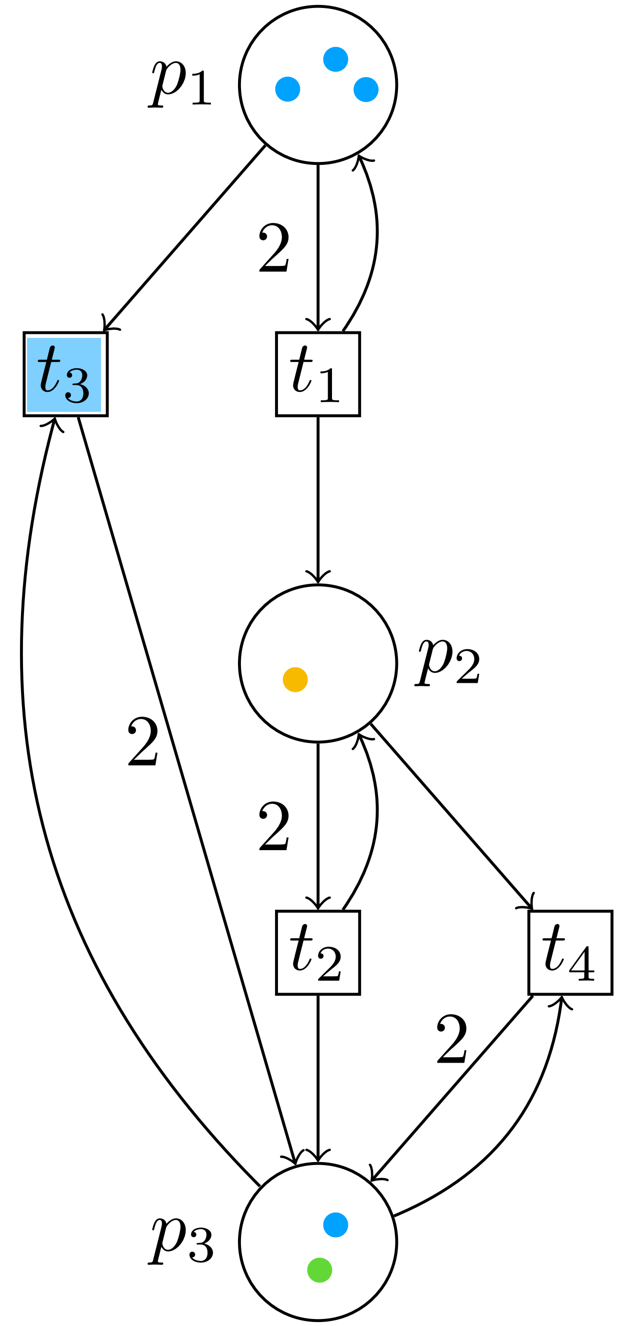

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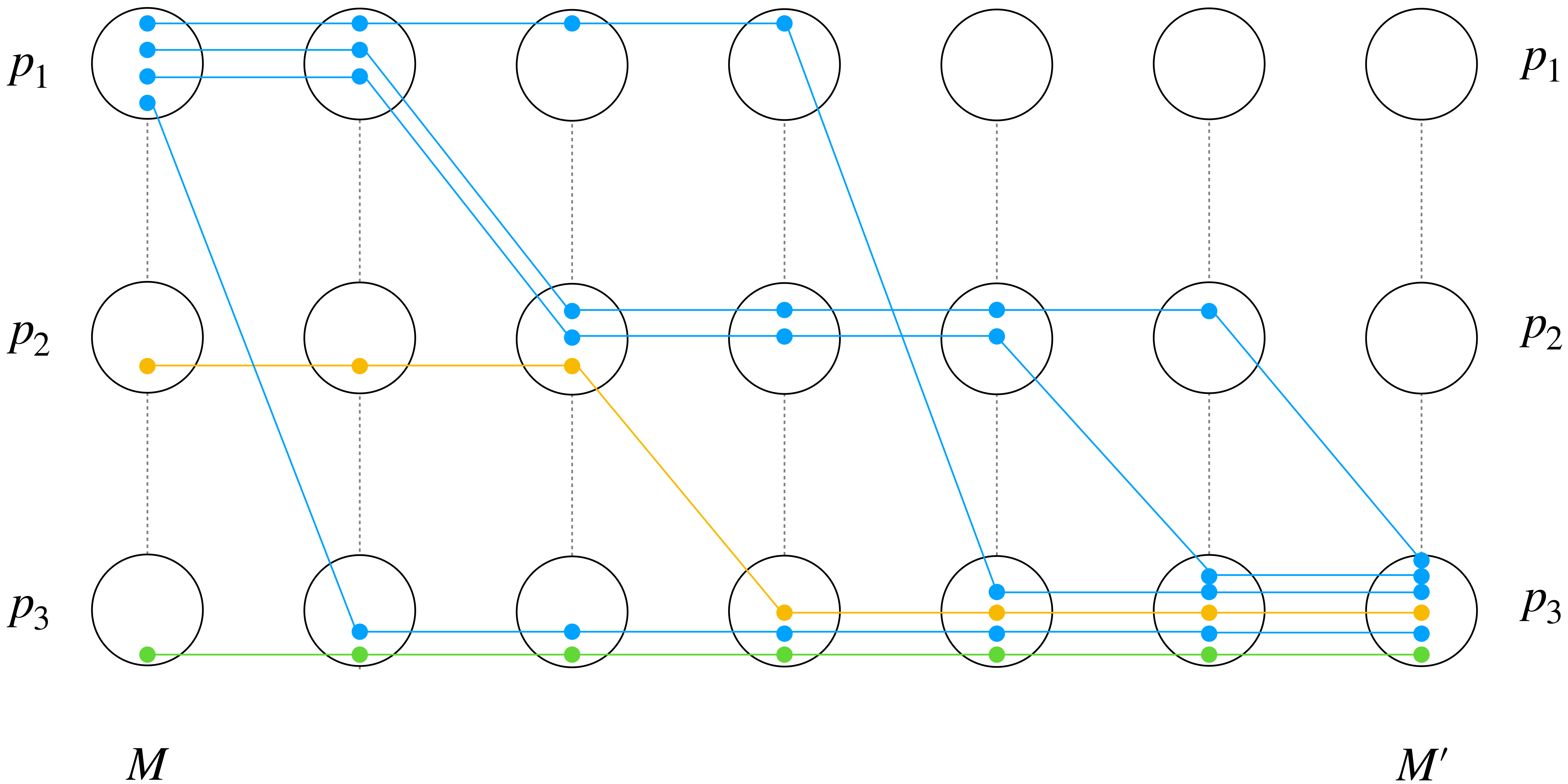

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Pruning

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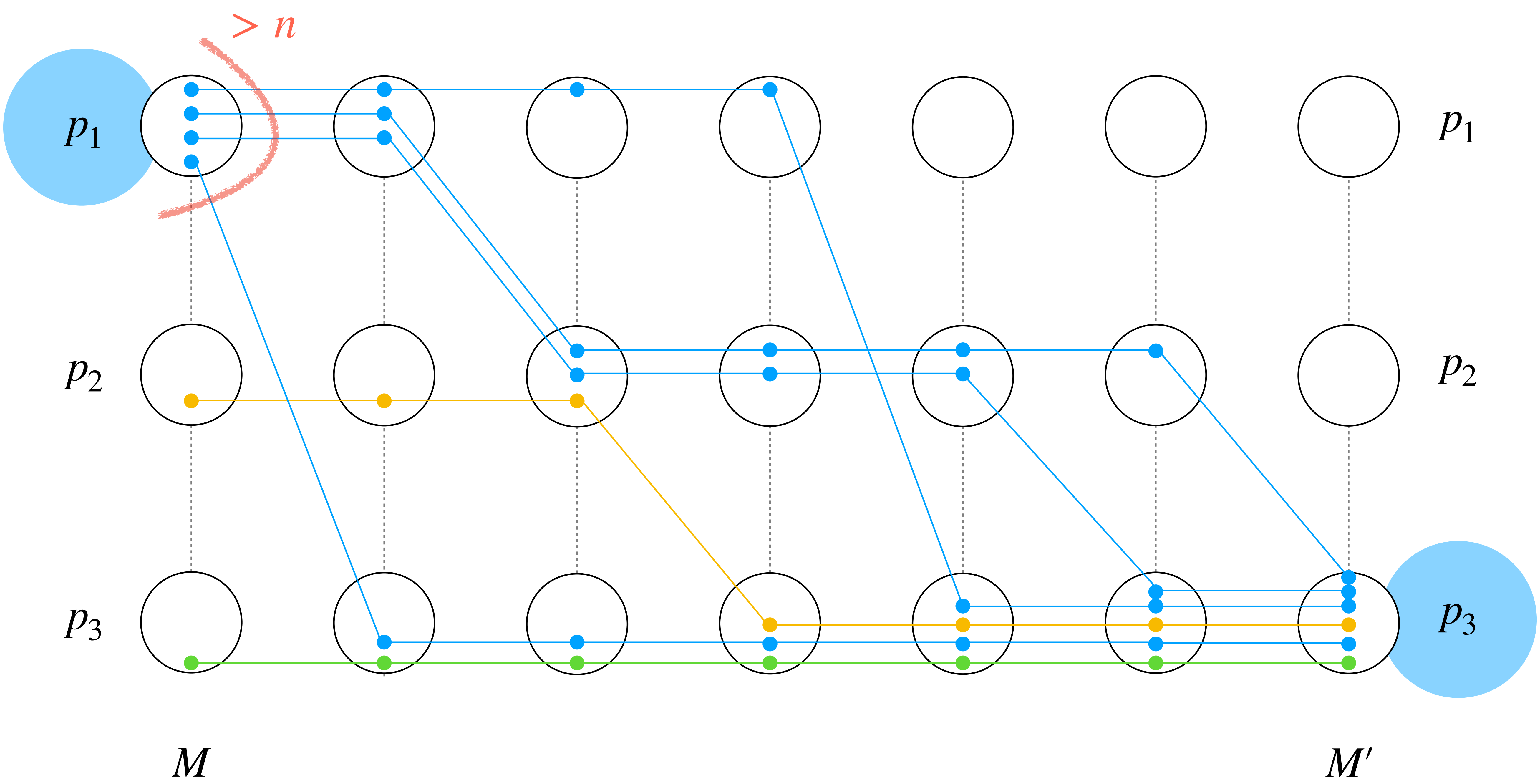

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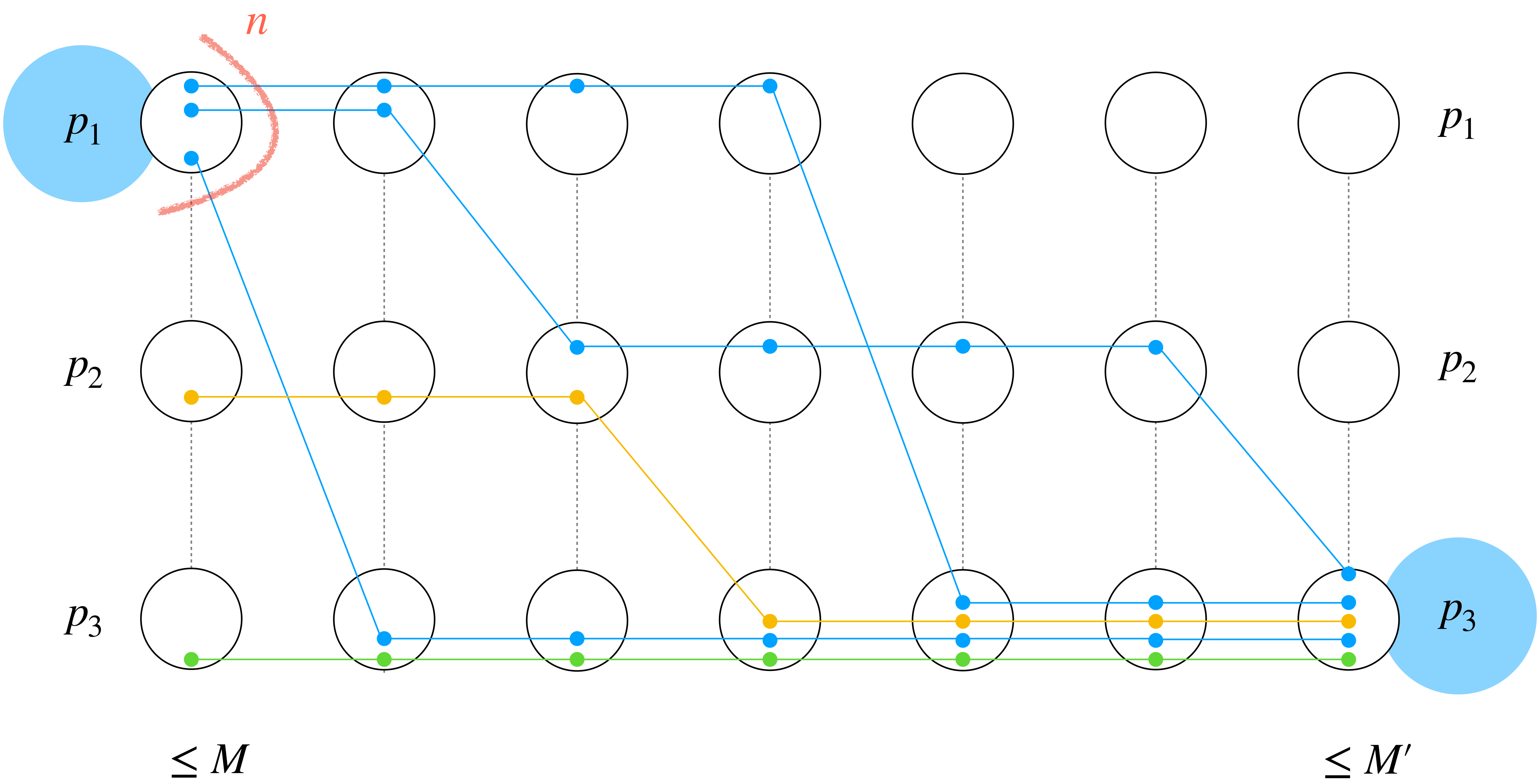

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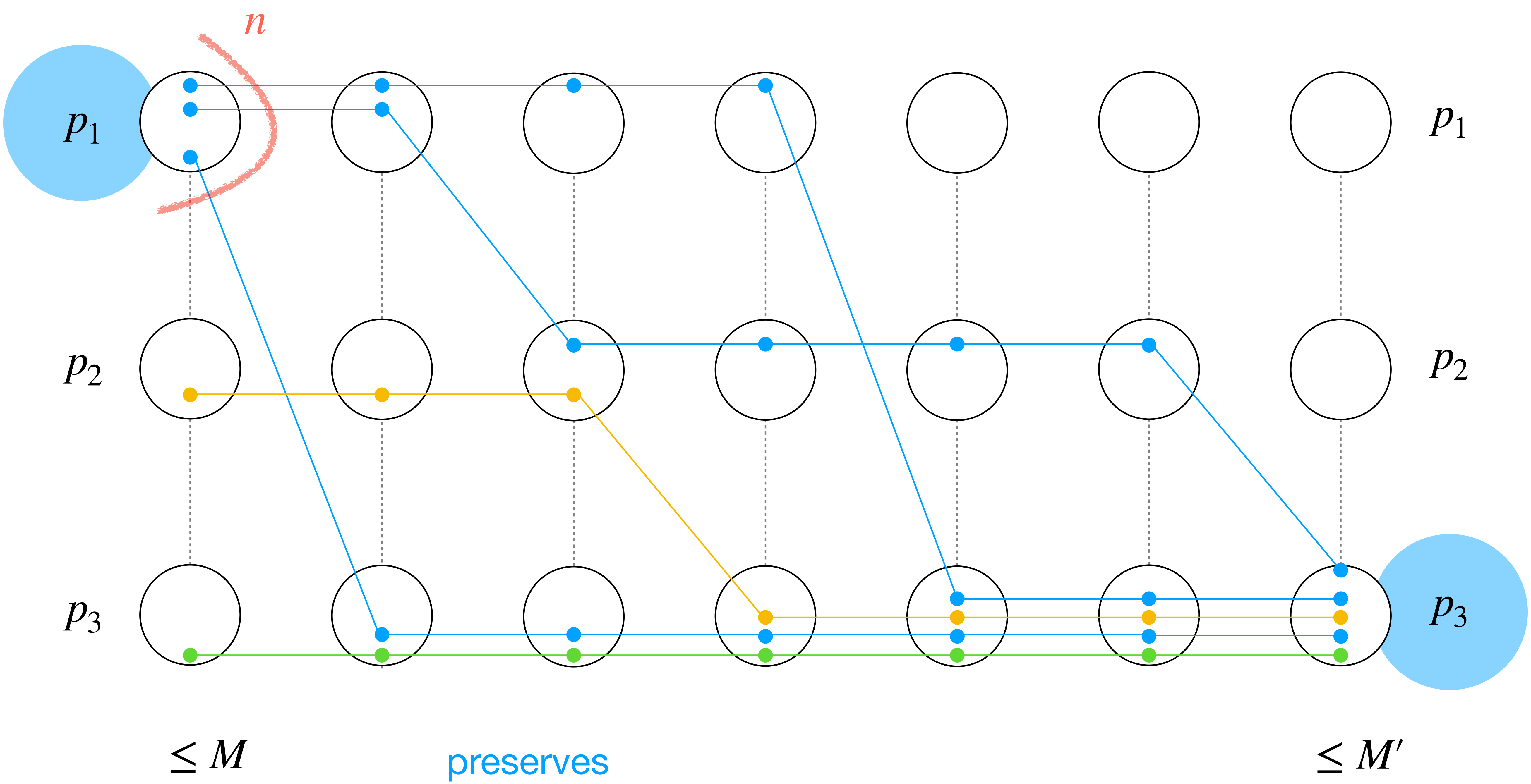
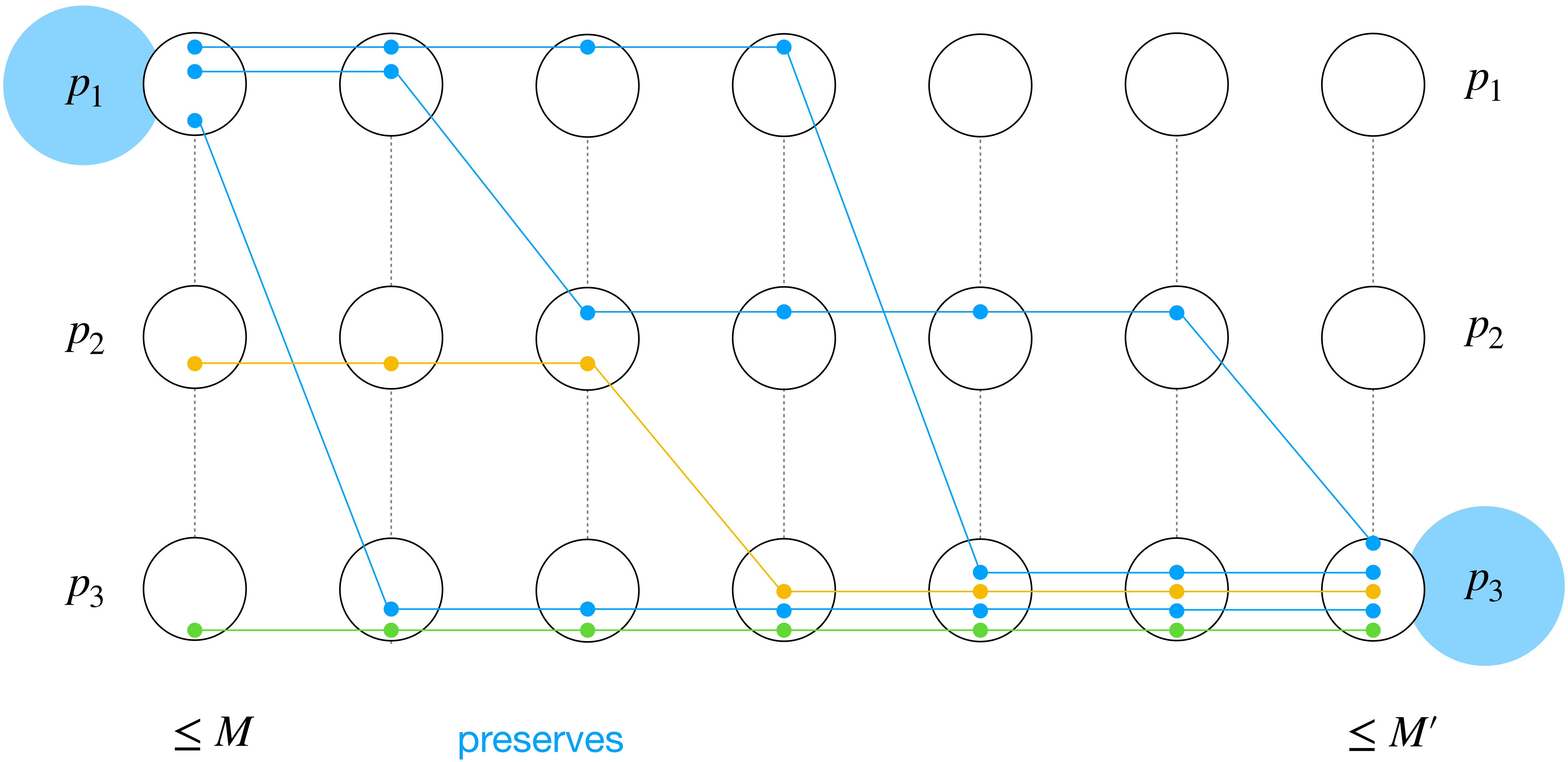


preserves

- support of initial and final markings
- validity of firing sequence

Boosting



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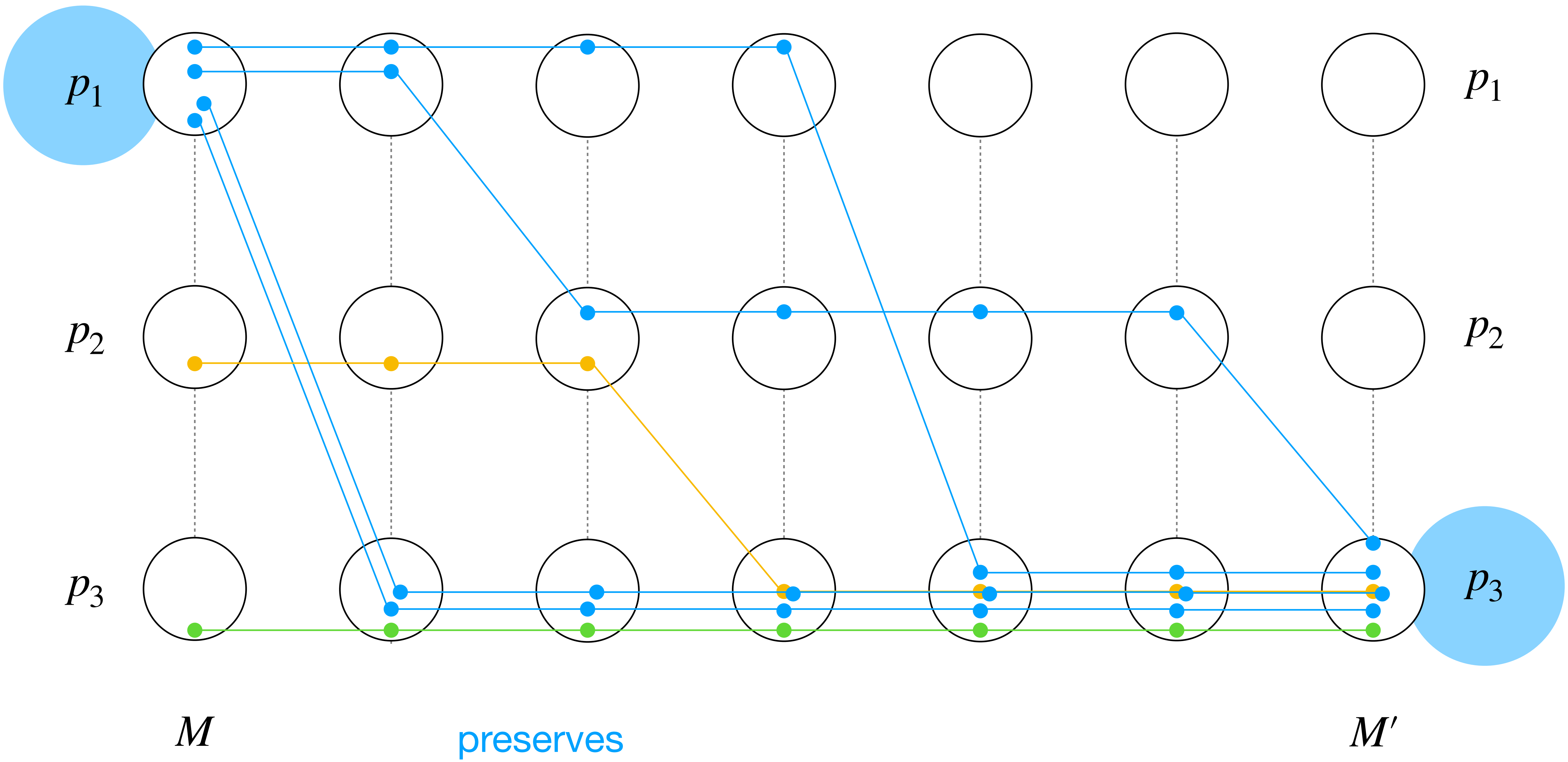

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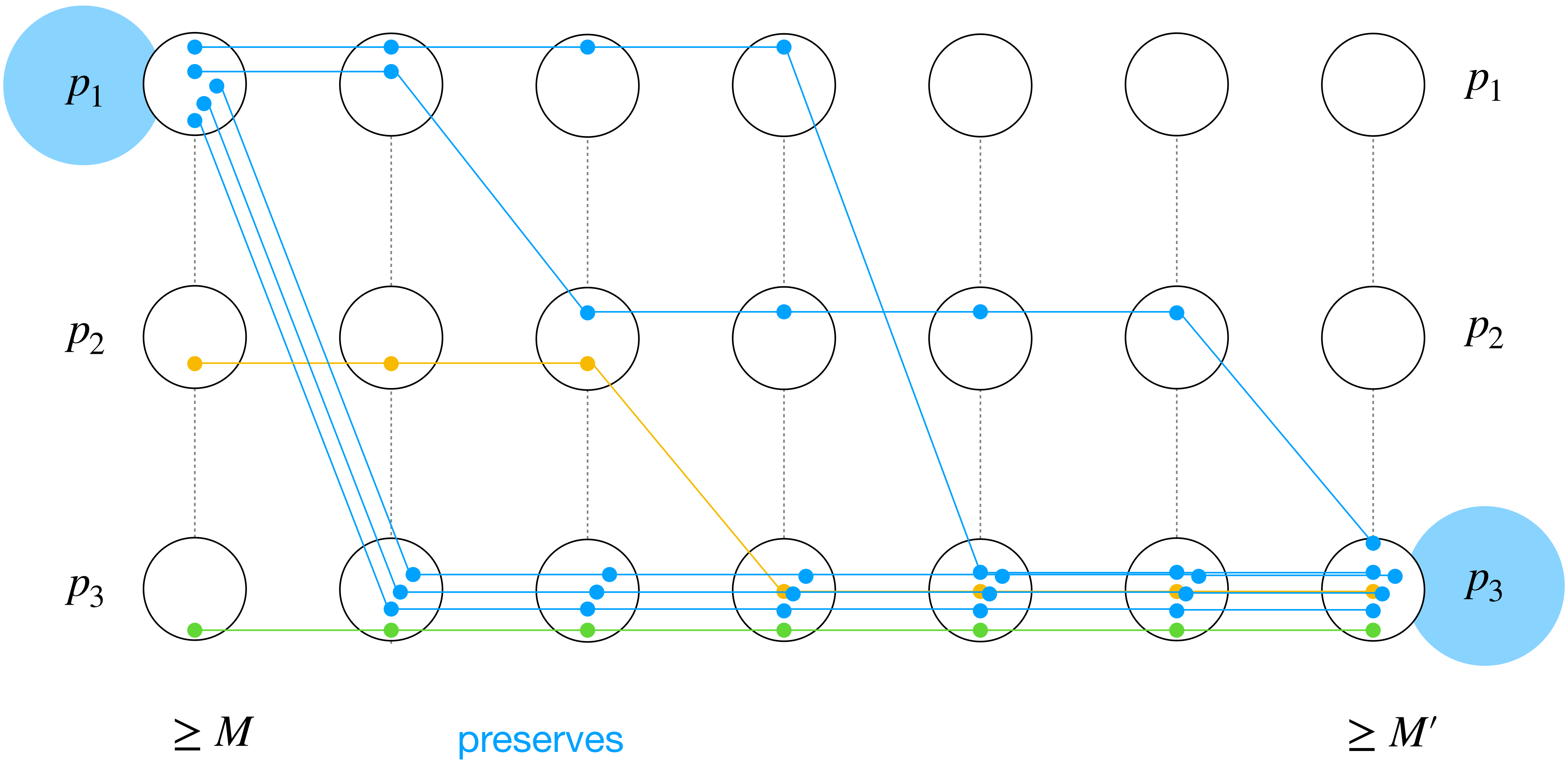

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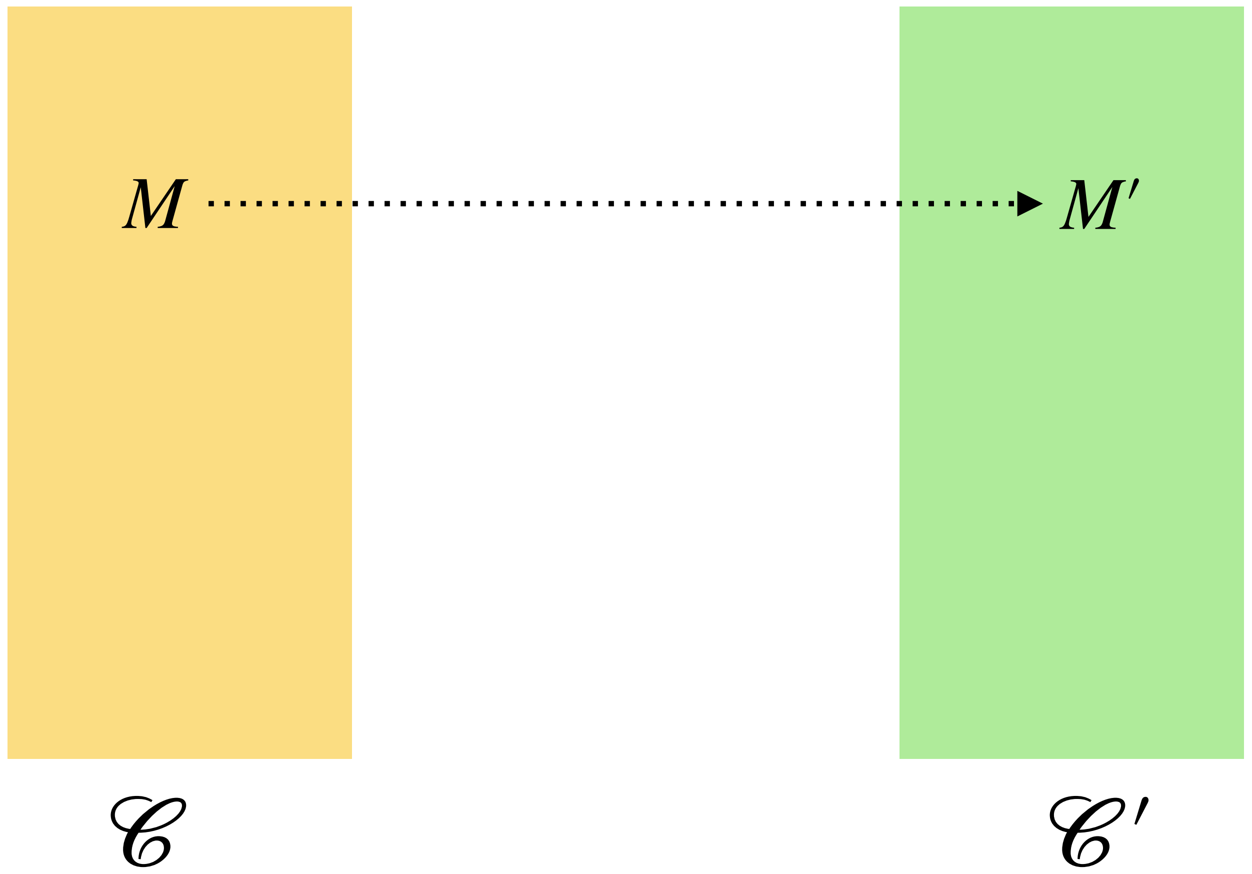

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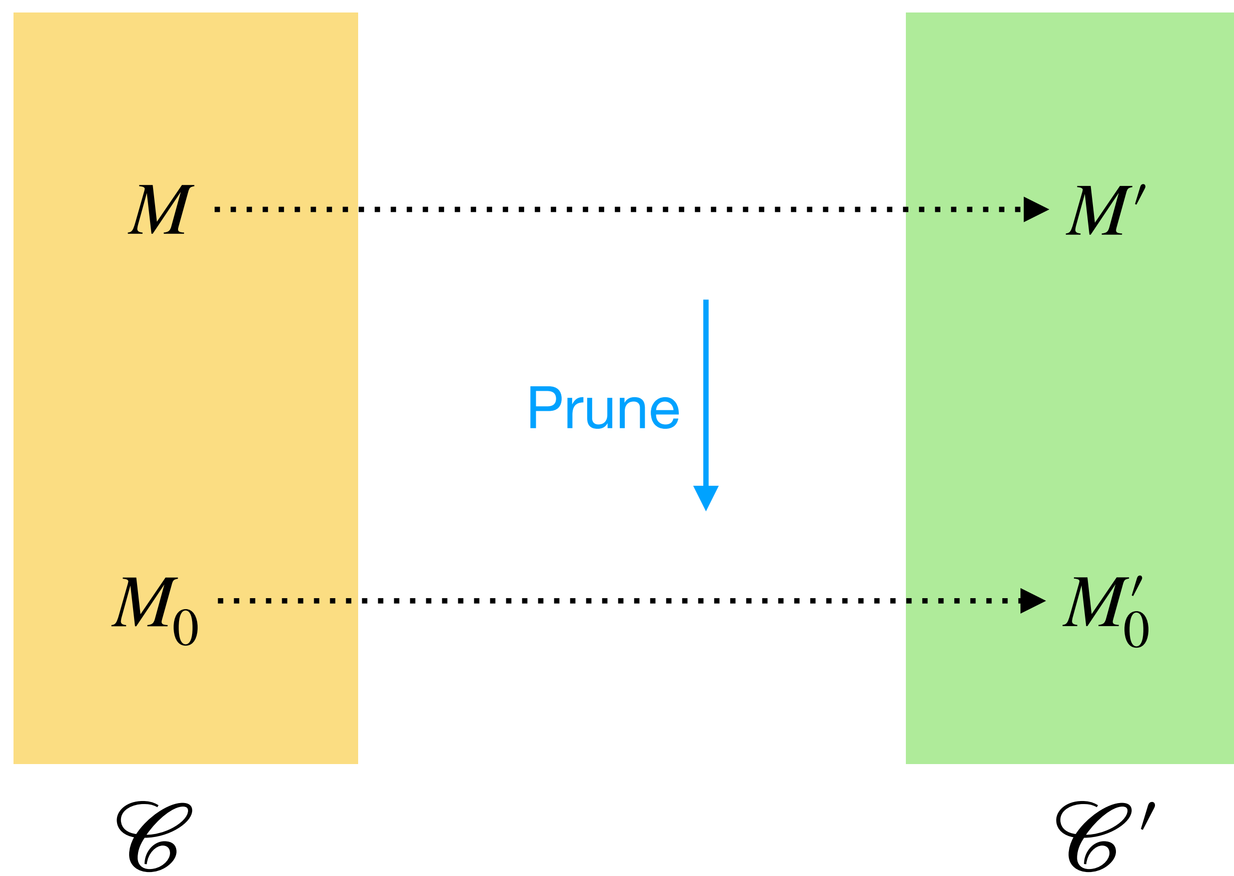

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+ lower bound $\mathscr{C}$
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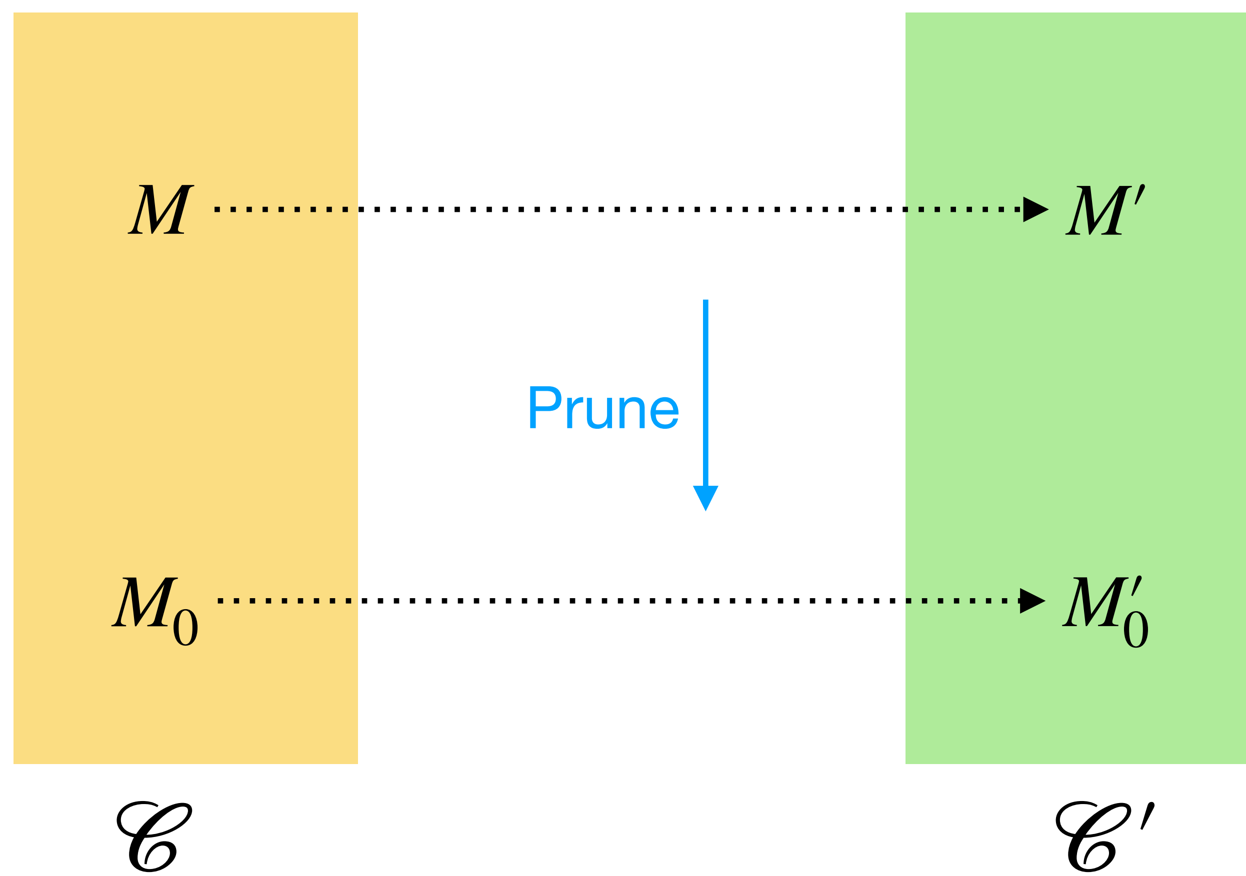

- NPSPACE = PSPACE
- non-deterministically pick small markings M_0 and M'_0
- check if M_0 reaches M'_0

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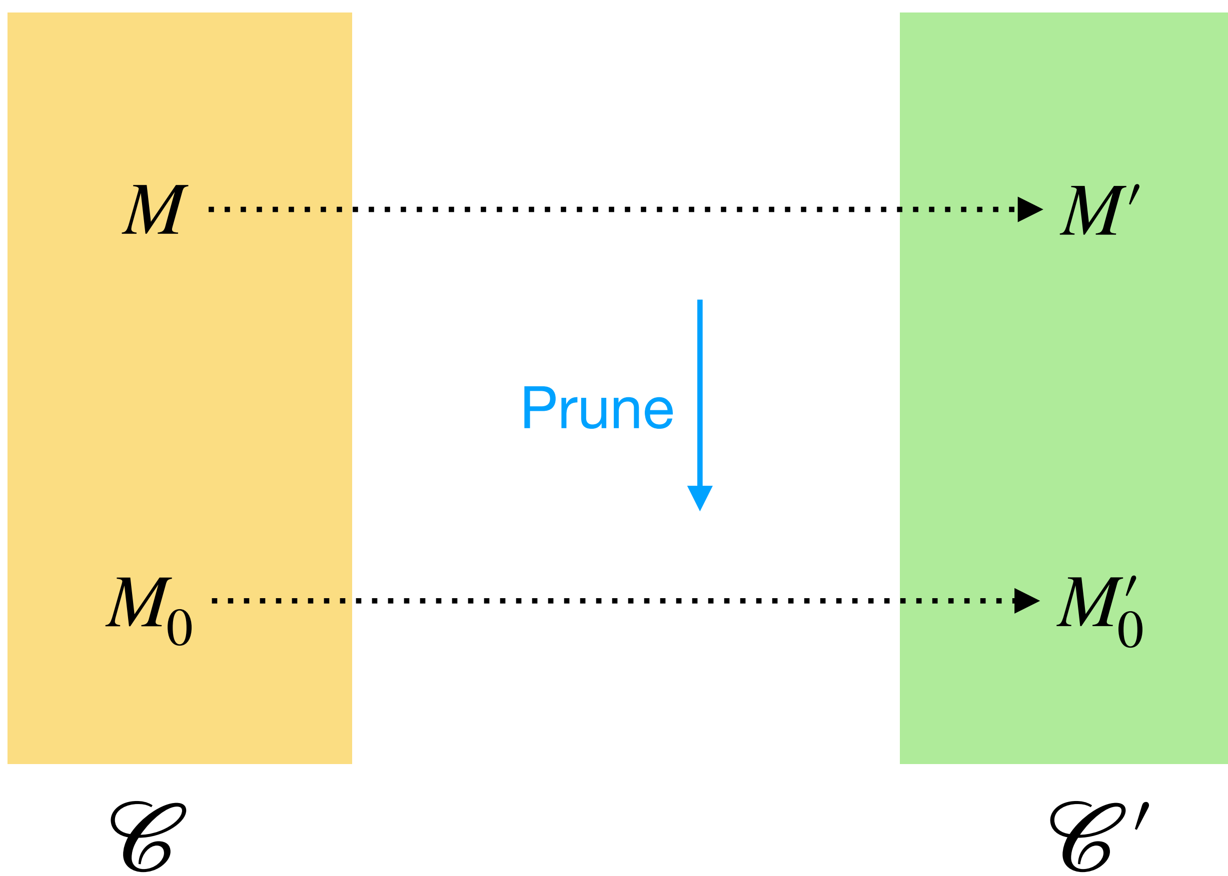

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- check if M_0 reaches M'_0

PSPACE

Parameterized problems

A **cube** is a boolean combination of constraints $\underset{\in \mathbb{N}}{a} \leq \#q \leq \underset{\in \mathbb{N} \cup \infty}{b}$

cube-reachability: given cubes $\mathscr{C}$ and $\mathscr{C}'$, does there exist $M \in \mathscr{C}$ and $M' \in \mathscr{C}'$ such that M reaches M'?

PSPACE

parameterized problems: verifying predicates using boolean operators and reachability operators pre^* and $post^*$ over cubes

PSPACE

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PSPACE

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PSPACE

$pre^*(\mathscr{C})$ is the set of markings that can reach $\mathscr{C}$

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parameterized problems: verifying predicates using boolean operators and reachability operators pre^* and $post^*$ over cubes

PSPACE

$pre^*(\mathscr{C})$ is the set of markings that can reach $\mathscr{C}$

$post^*(\mathscr{C})$ is the set of markings that $\mathscr{C}$ can reach

e.g. reachability from cube $\mathscr{C}$ to cube $\mathscr{C}'$: $post^*(\mathscr{C}) \cap \mathscr{C}' \neq \emptyset$

Parameterized problems

A **cube** is a boolean combination of constraints $\underset{\in \mathbb{N}}{a} \leq \#q \leq \underset{\in \mathbb{N} \cup \infty}{b}$

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PSPACE

parameterized problems: verifying predicates using boolean operators and reachability operators pre^* and $post^*$ over cubes

PSPACE

e.g. almost-sure reachability from cube $\mathscr{C}_{init}$ to cube $\mathscr{C}_{final}$

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parameterized problems: verifying predicates using boolean operators and reachability operators pre^* and $post^*$ over cubes

PSPACE

e.g. almost-sure reachability from cube $\mathscr{C}_{init}$ to cube $\mathscr{C}_{final}$

$$post^*(\mathscr{C}_{init}) \subseteq pre^*(\mathscr{C}_{final})$$

IO nets are flat

[Leroux, Sutre, '05]

Flat

$\exists$ sequence $t_1^* t_2^* \ldots t_\ell^*$ such that $\forall M_0 \forall M$, $M_0 \xrightarrow{*} M$ iff $M_0 \xrightarrow{t_1^{k_1} t_2^{k_2} \ldots t_\ell^{k_\ell}} M$

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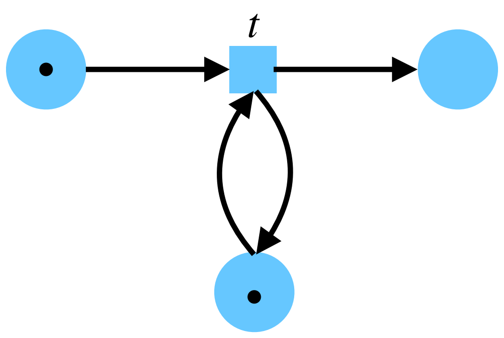
$\longrightarrow$ check **reachability properties** with model checking **tools** that use acceleration techniques **e.g. FAST** [Bardin, Finkel, Leroux, Petrucci, ’03]

Part 2:
Branching immediate observation nets

Branching immediate observation nets

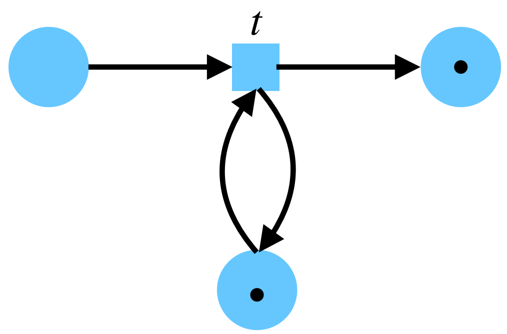
Immediate Observation nets (IO)



- Conservative
- Communication

Branching immediate observation nets

Immediate Observation nets (IO)



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Branching immediate observation nets

[Christensen et al., ’93] [Yen, ’97] [Lasota, ’09] [Mayr, Weihmann, ’15]

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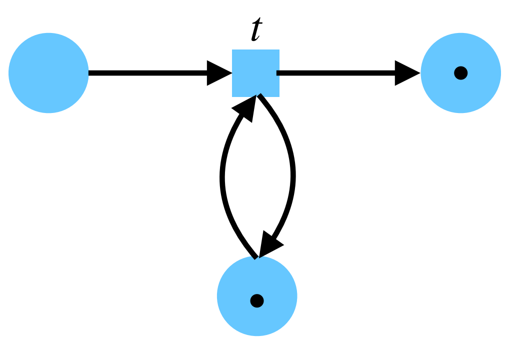

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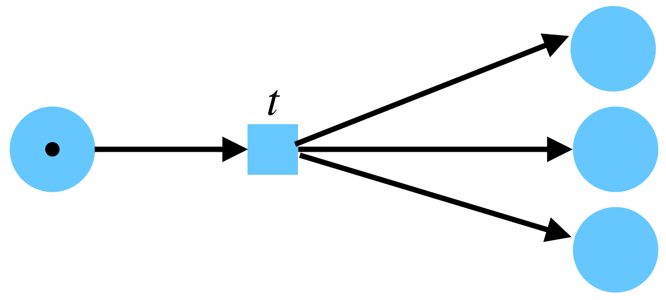
Branching Parallel Processes (BPP)



- Token creation and destruction
- Communication-free

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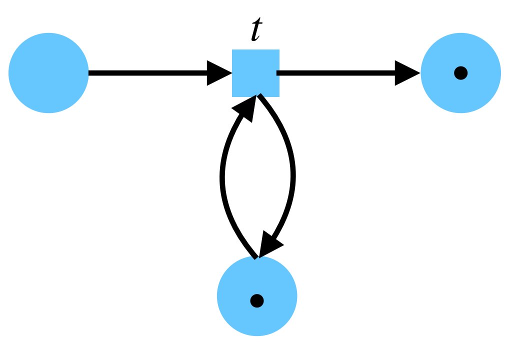

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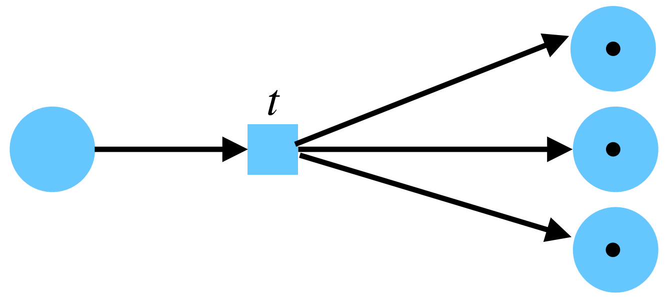

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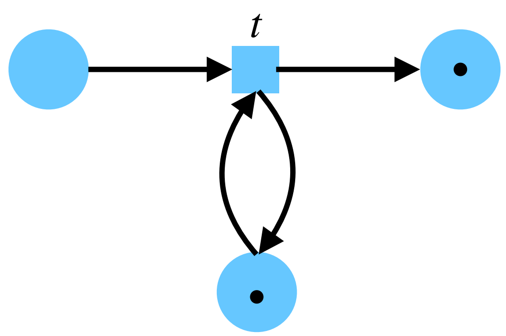

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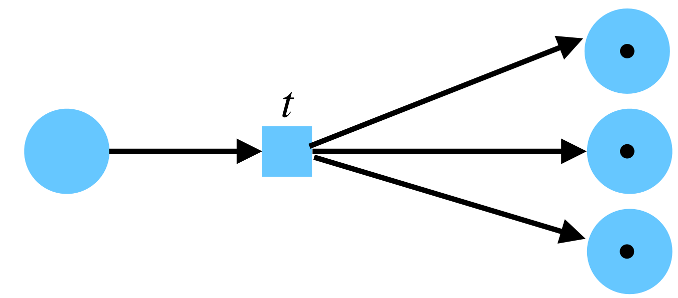

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Branching Immediate Observation nets (BIO)

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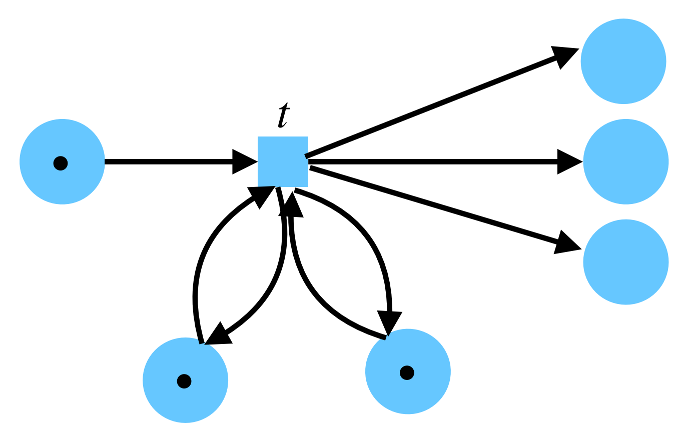

[Esparza, Raskin, *W.-K.*, ’20]

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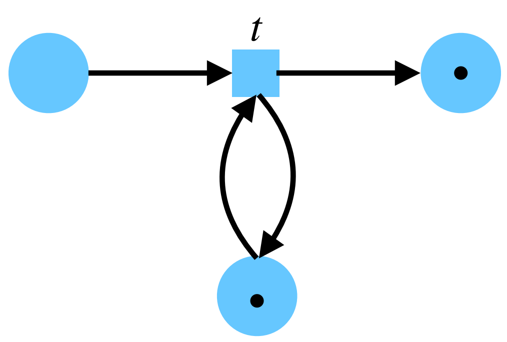

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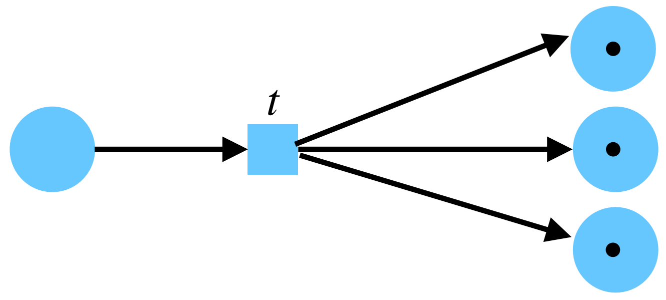

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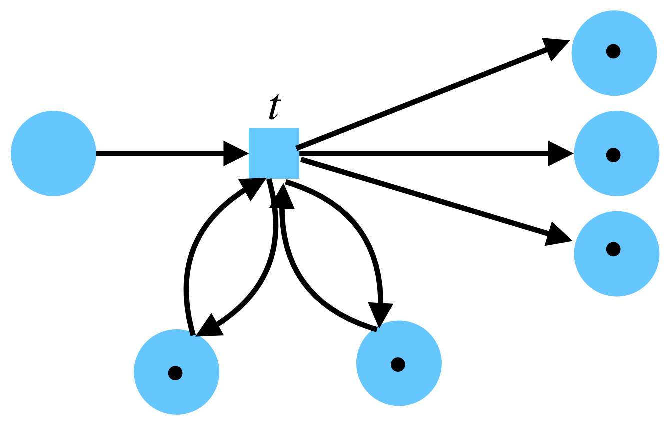

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C. Weil-Kennedy, TUM

Cube-reachability

number of tokens in q

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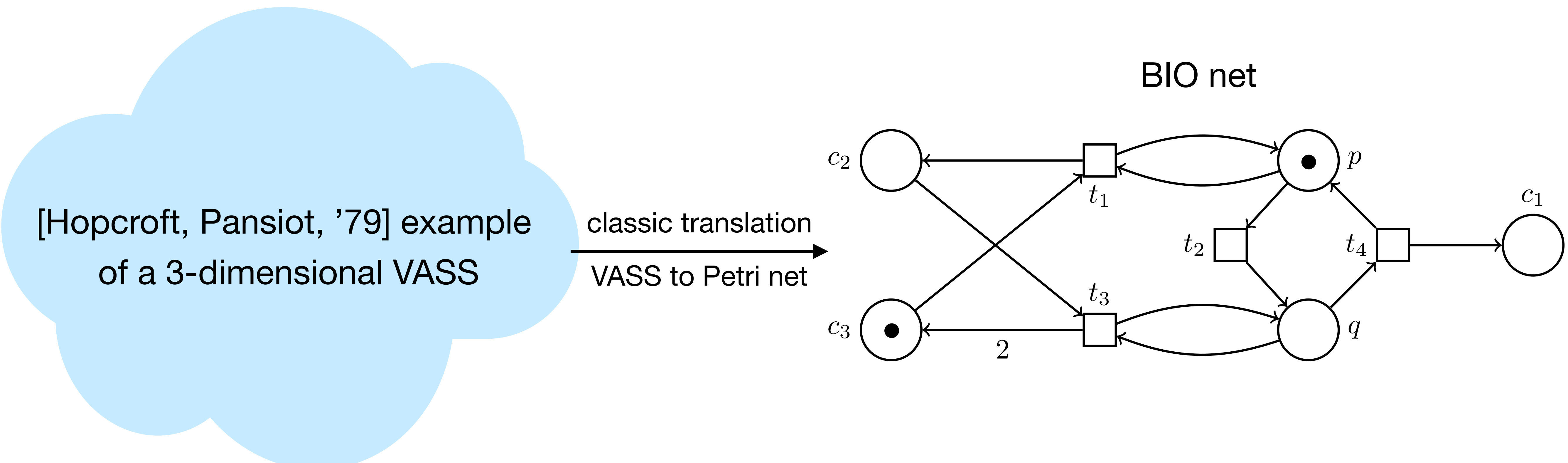
parameterized problems: verifying predicates using boolean operators and reachability operators pre^* and $post^*$ over cubes

Non-semilinear reachability

BIO nets can have **non-semilinear** reachability set



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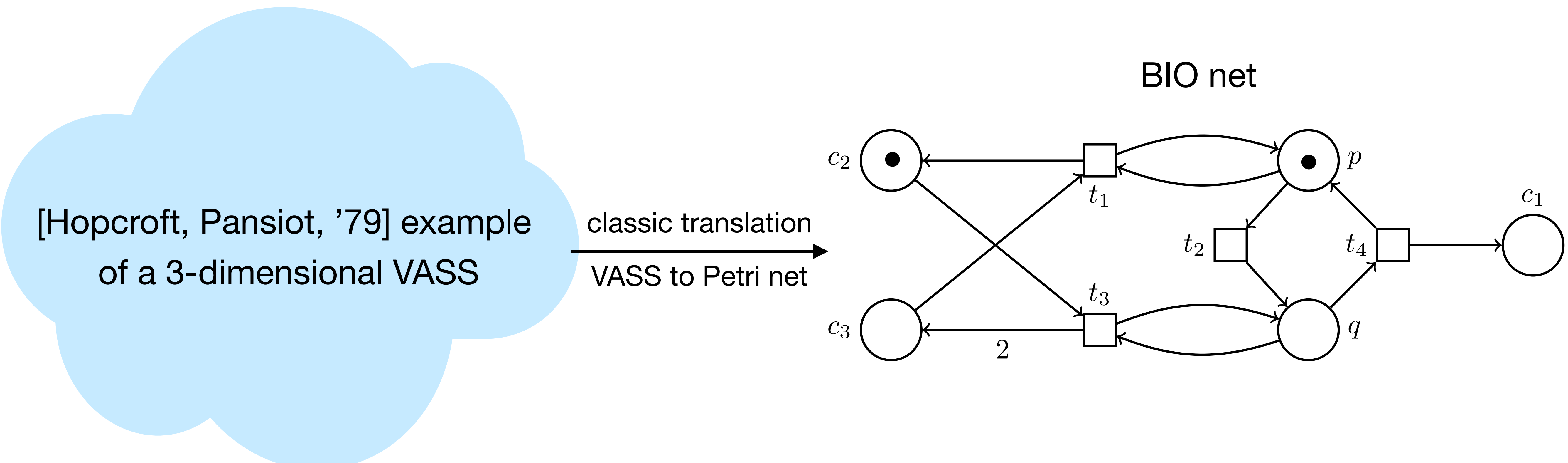

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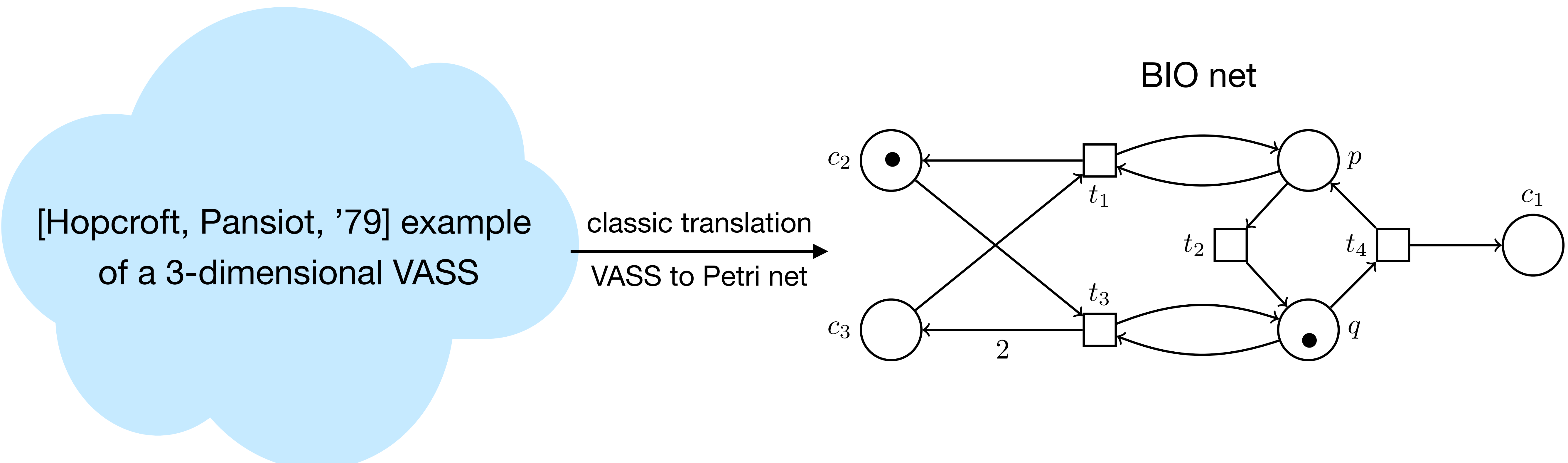

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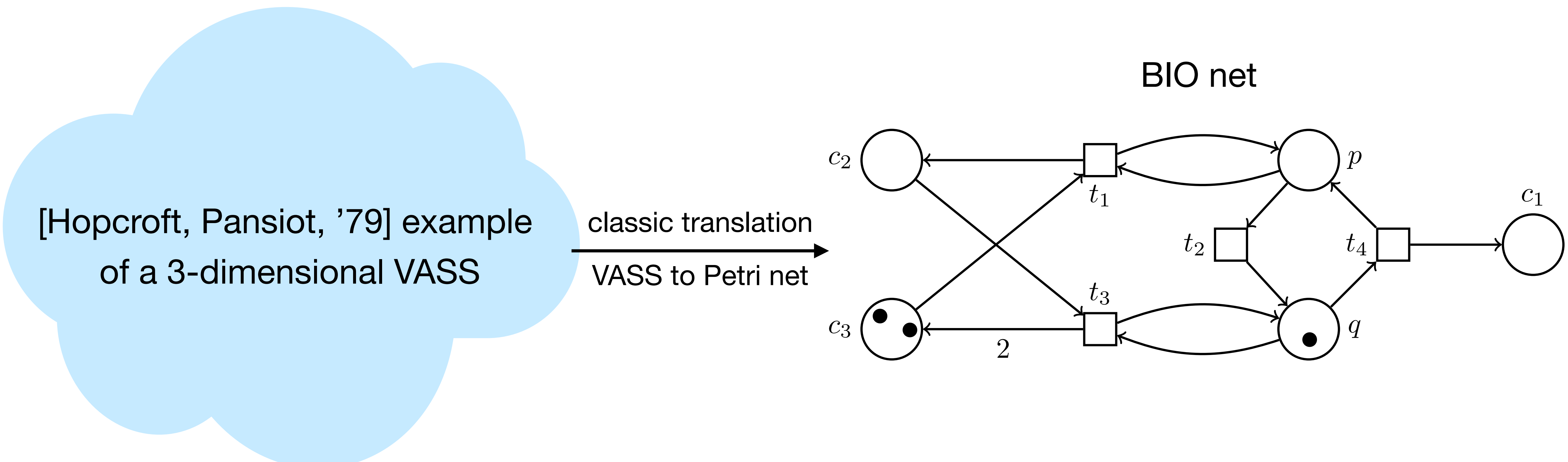

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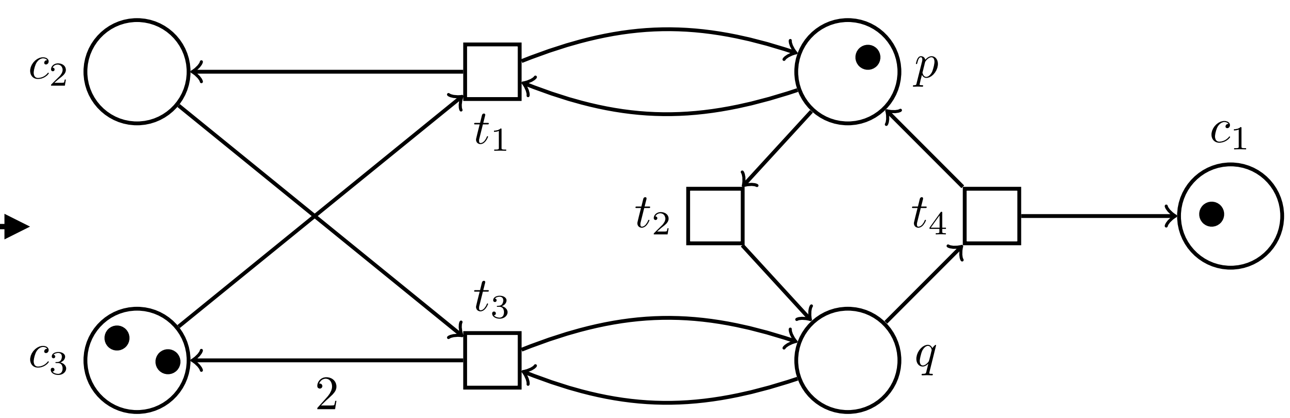
[Hopcroft, Pansiot, '79] example of a 3-dimensional VASS

classic translation

VASS to Petri net

BIO net



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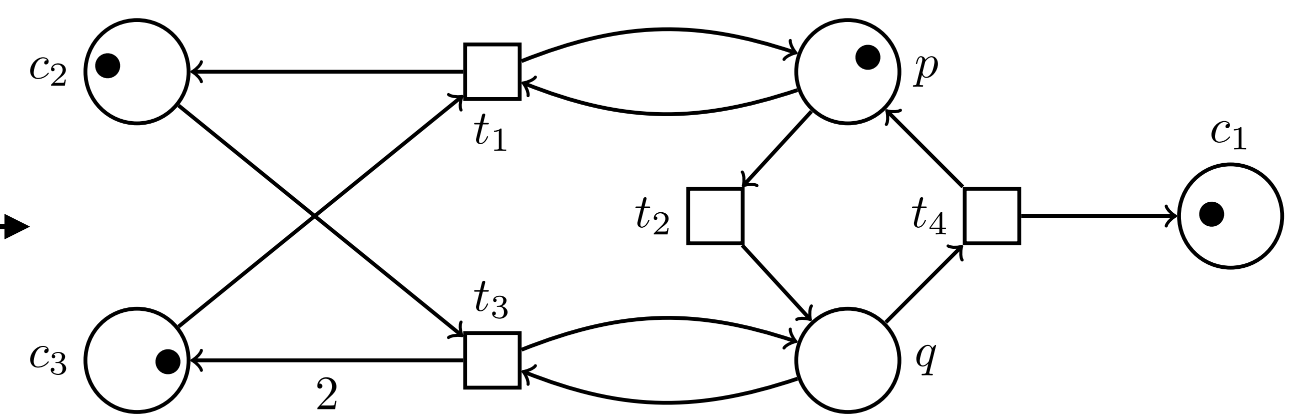

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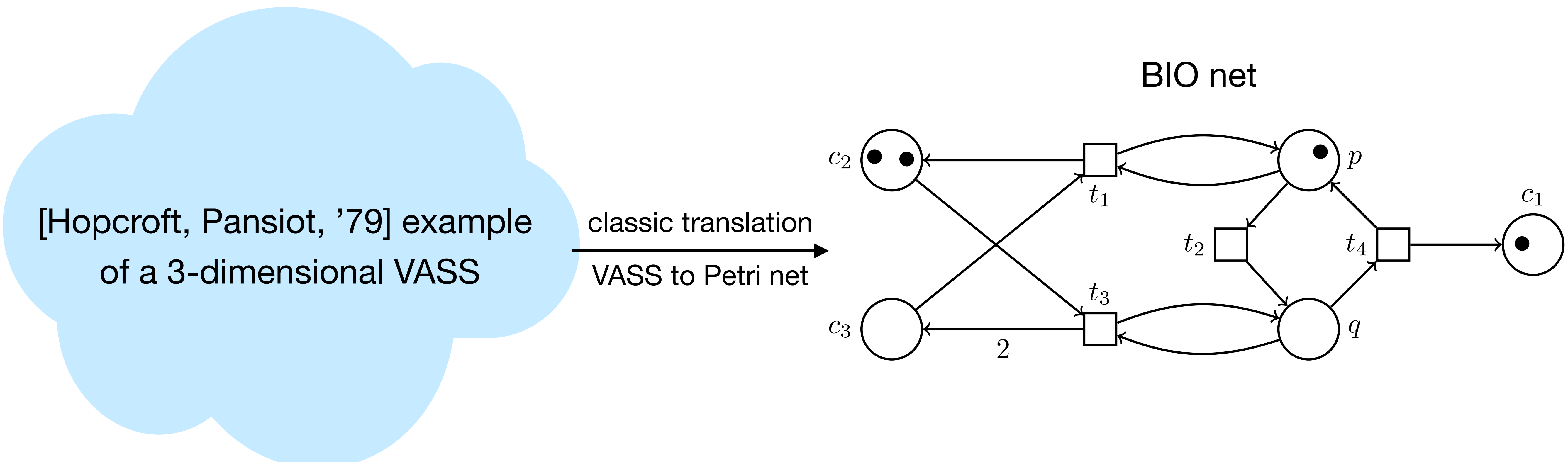

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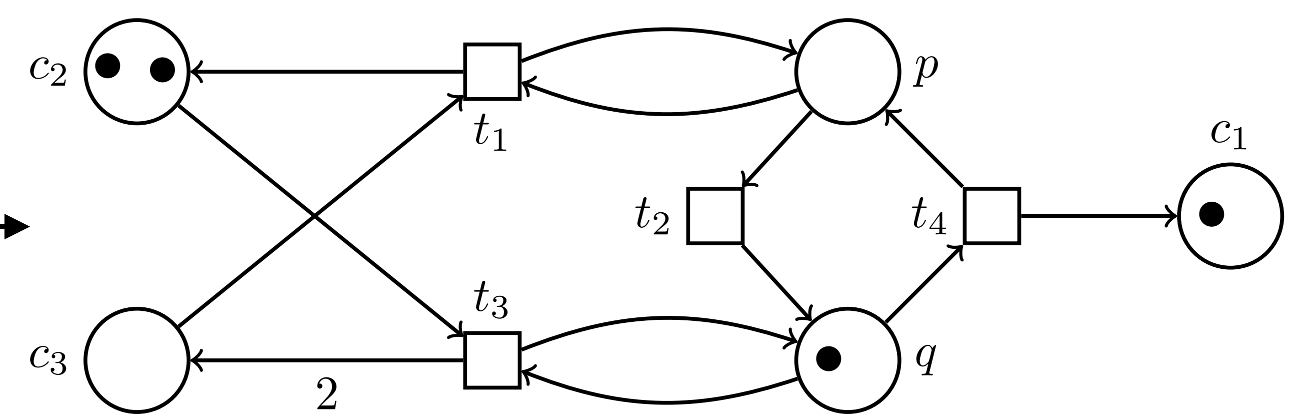

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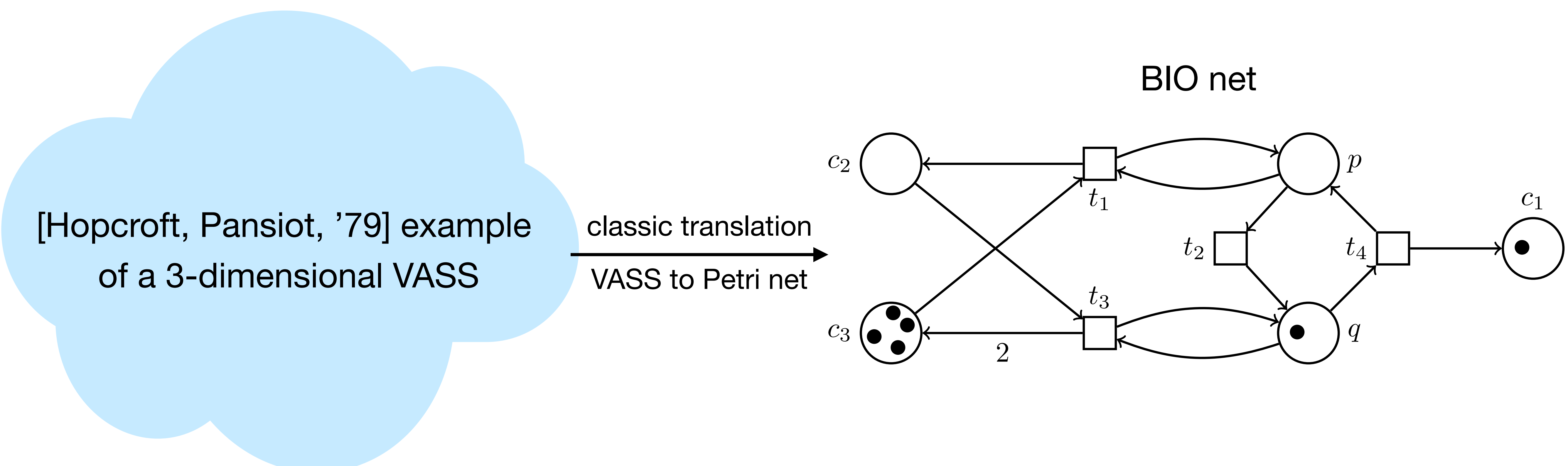

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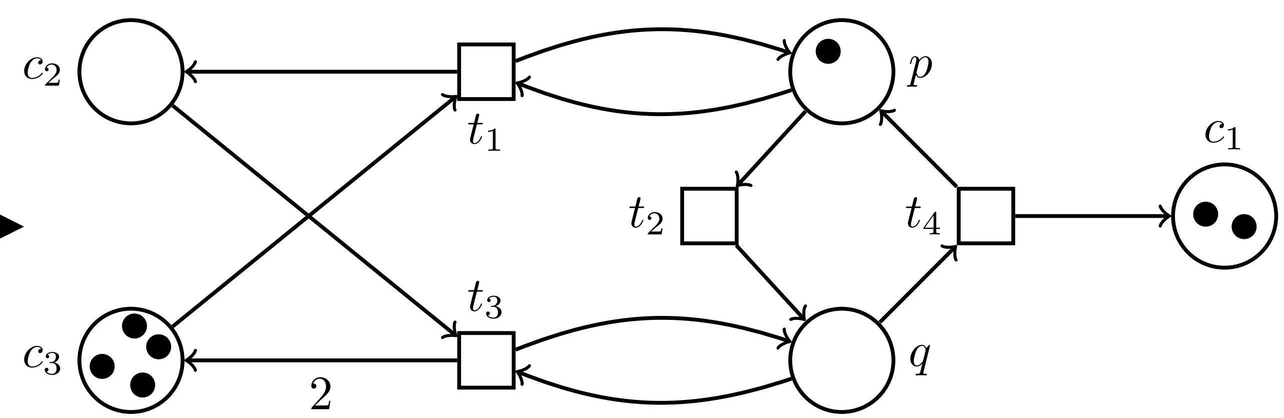

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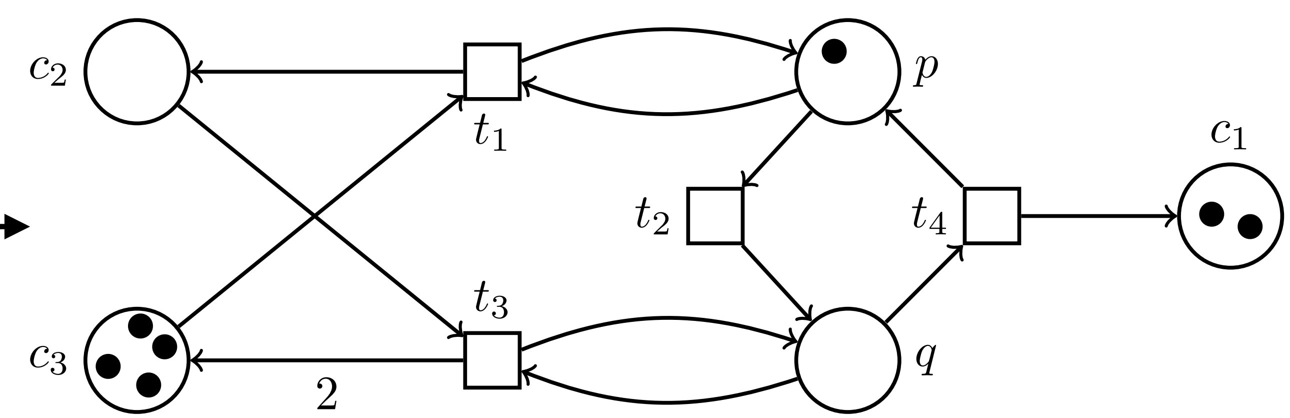

$$c_2 + c_3 \le 2^{c_1}$$

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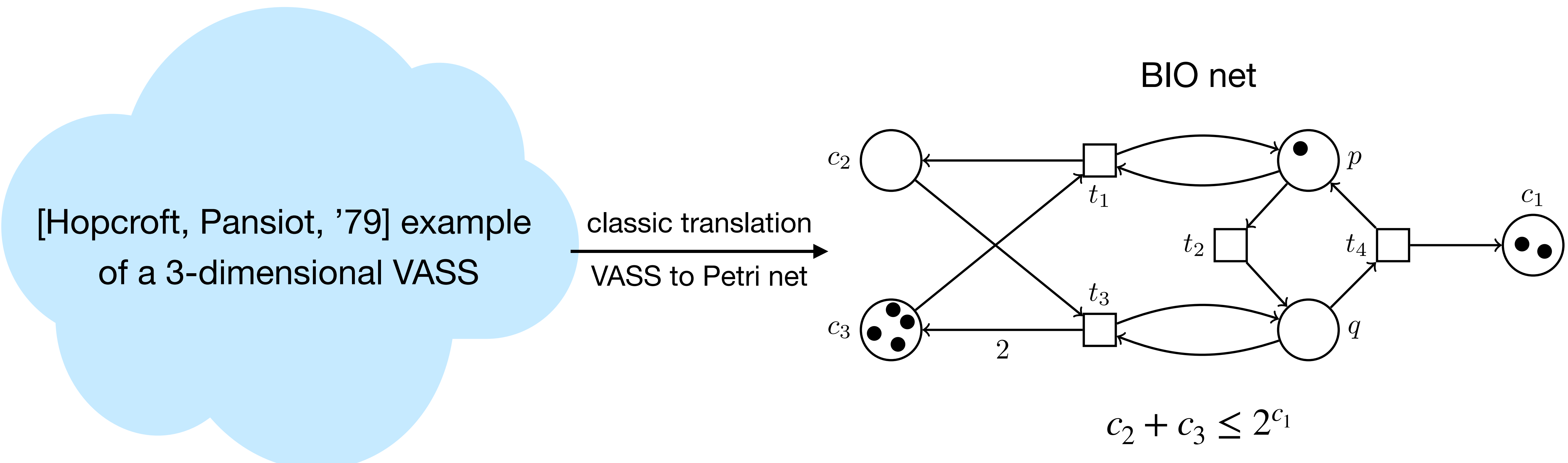

$$c_2 + c_3 \leq 2^{c_1}$$

Until now, unbounded Petri net classes with provably simpler reachability than the general case have semilinear reachability sets

BIO nets are locally flat

[Leroux, Sutre, '05]

Flat

$\exists$ sequence $t_1^* t_2^* \ldots t_\ell^*$ such that $\forall M_0 \forall M$, $M_0 \xrightarrow{*} M$ iff $M_0 \xrightarrow{t_1^{k_1} t_2^{k_2} \ldots t_\ell^{k_\ell}} M$

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Locally flat

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$\longrightarrow$ check **reachability properties** with
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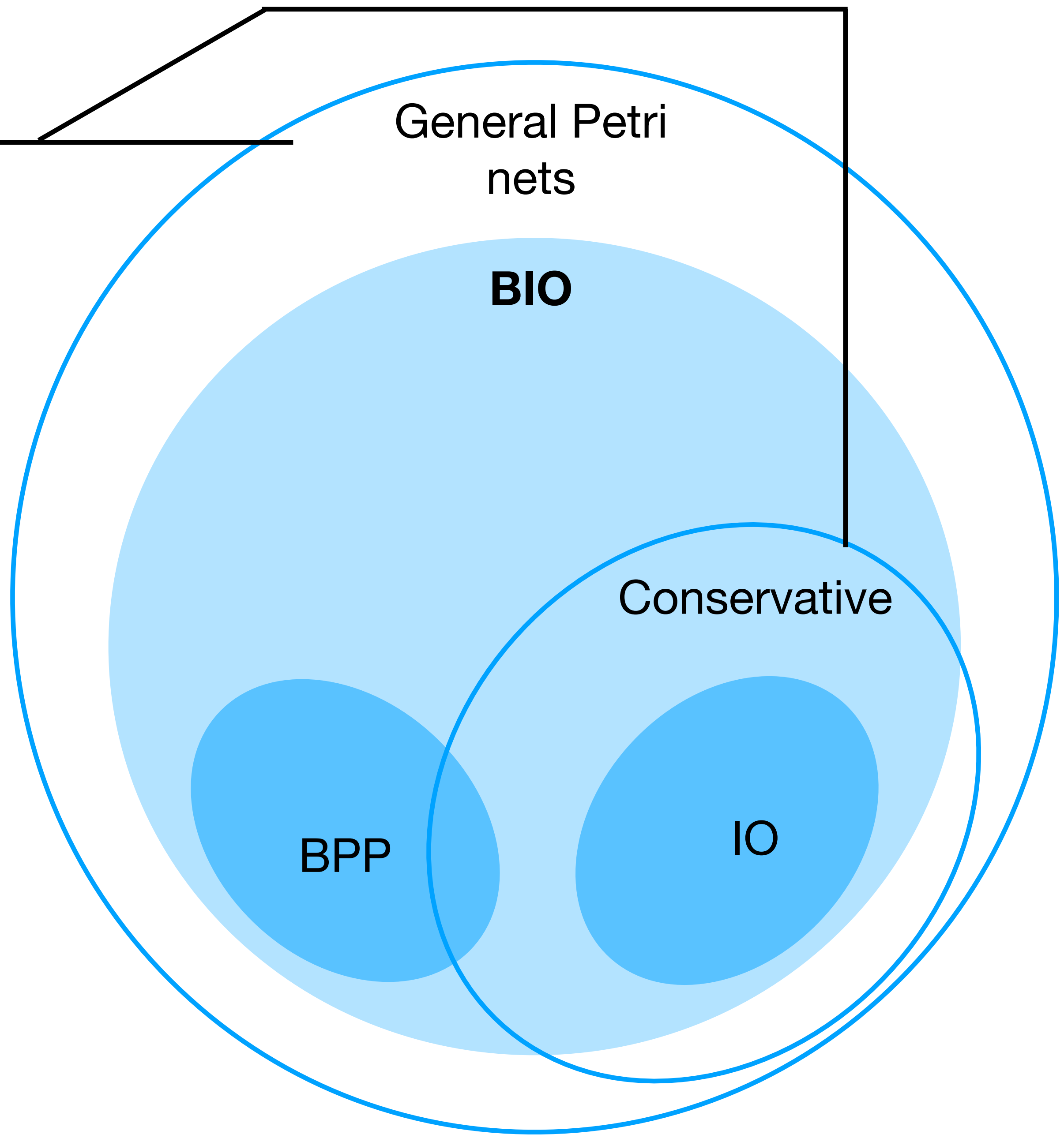
e.g. FAST [Bardin, Finkel, Leroux, Petrucci, '03]

Cube-reachability summary

non-elementary
[Czerwinzki, Lasota, Lazic, Leroux, Mazowiecki, '19]



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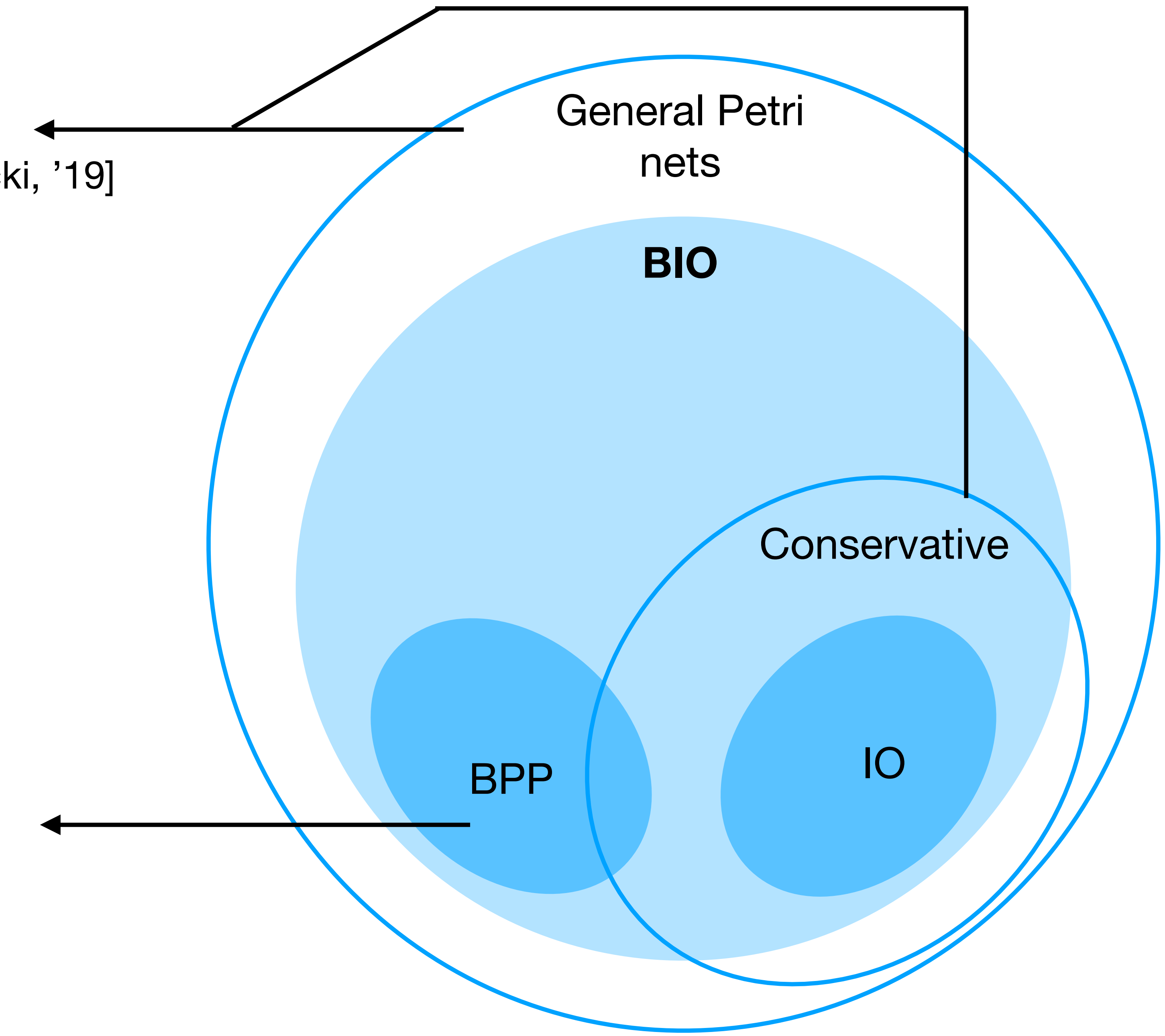

NP-complete
[Esparza, ’97]

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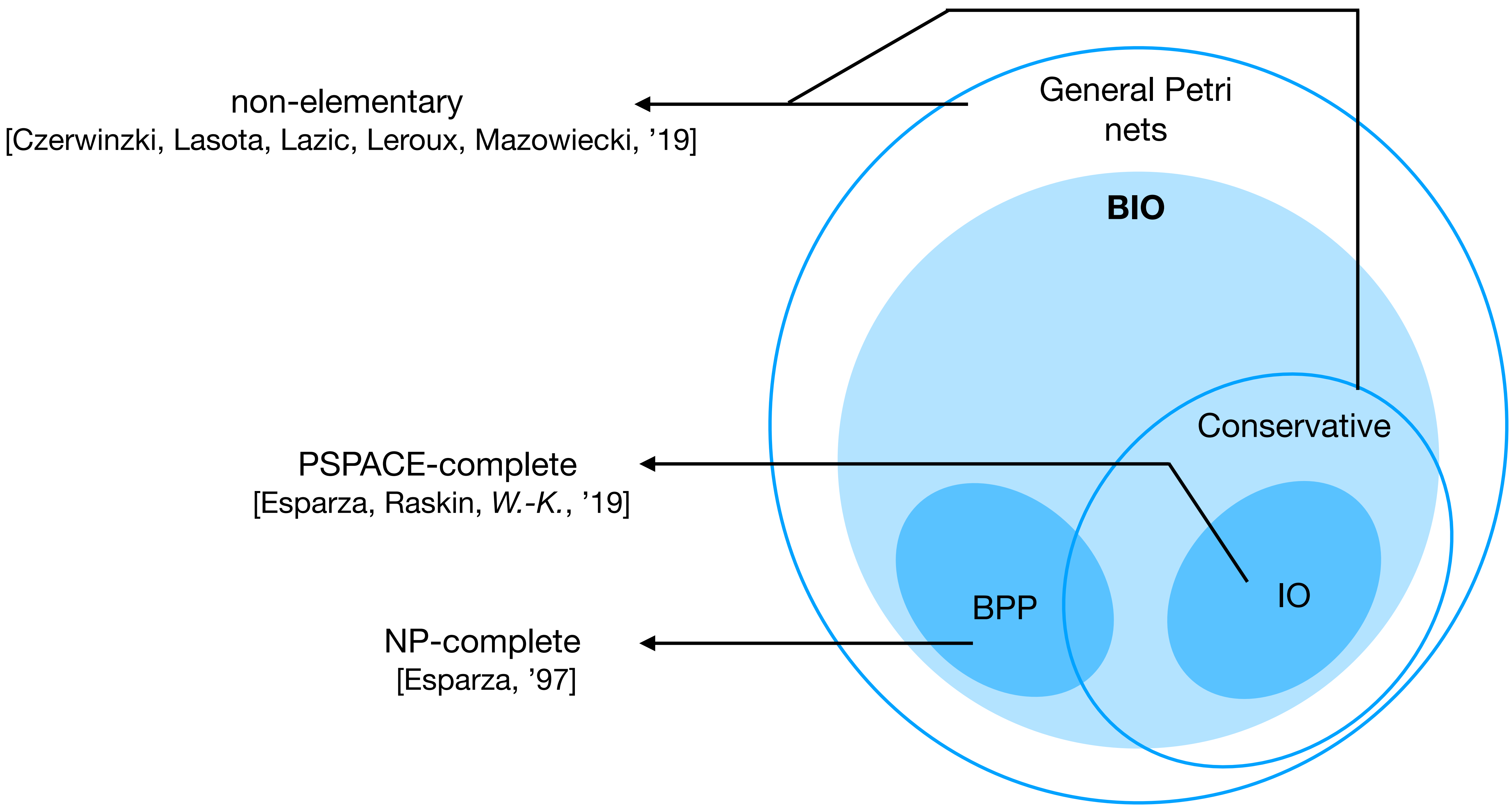
Cube-reachability summary



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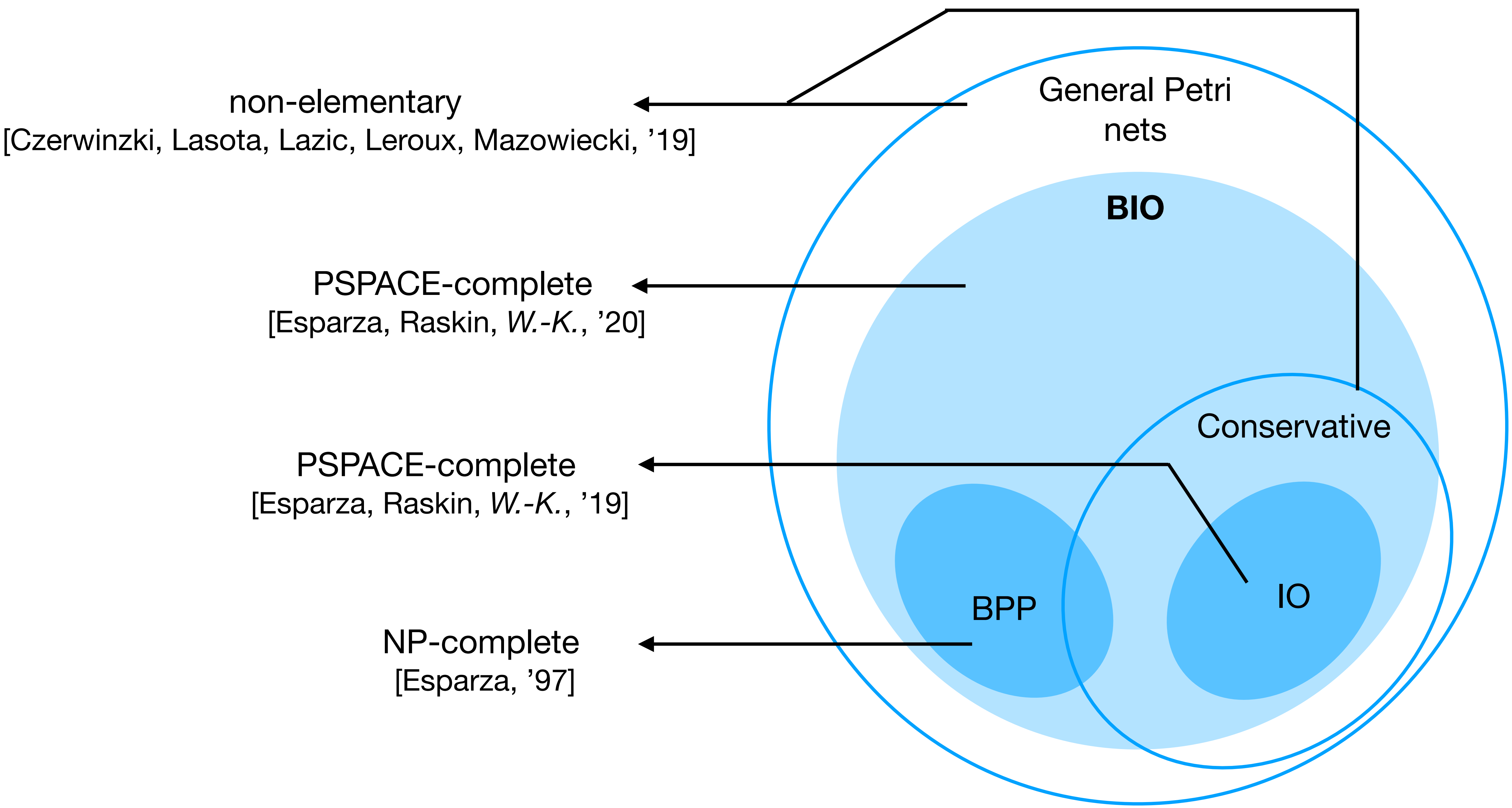
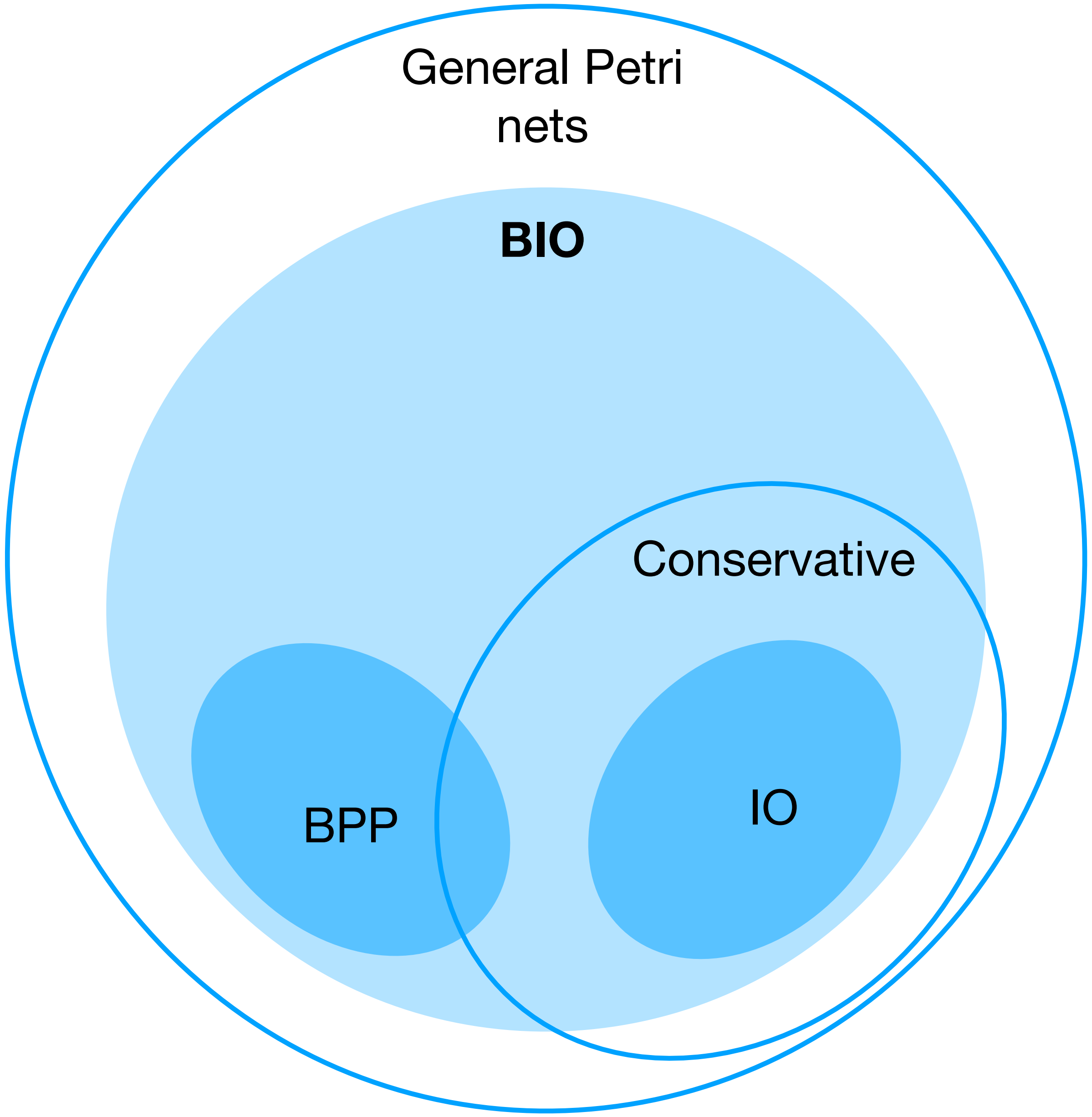


Conclusion

- IO nets introduced to model population protocols: allowed to solve correctness
- cube-parameterized problems are in PSPACE
- BIO nets generalize BPP and IO nets, still have PSPACE cube-reachability
- BIO nets are first class of Petri nets with non-semilinear reachability set and elementary reachability problem
- IO nets are flat & BIO nets are locally flat, allowing efficient model checking



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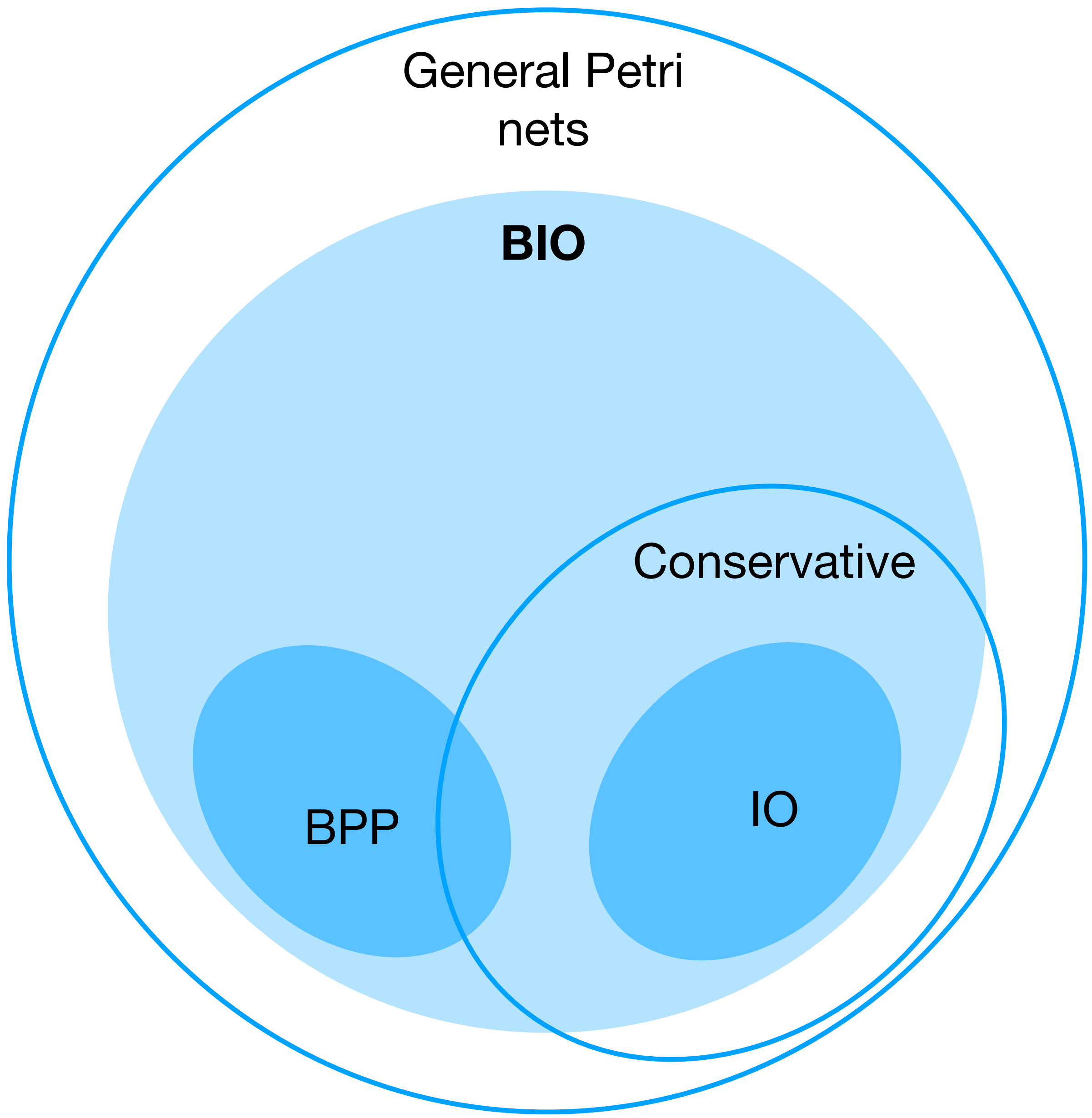

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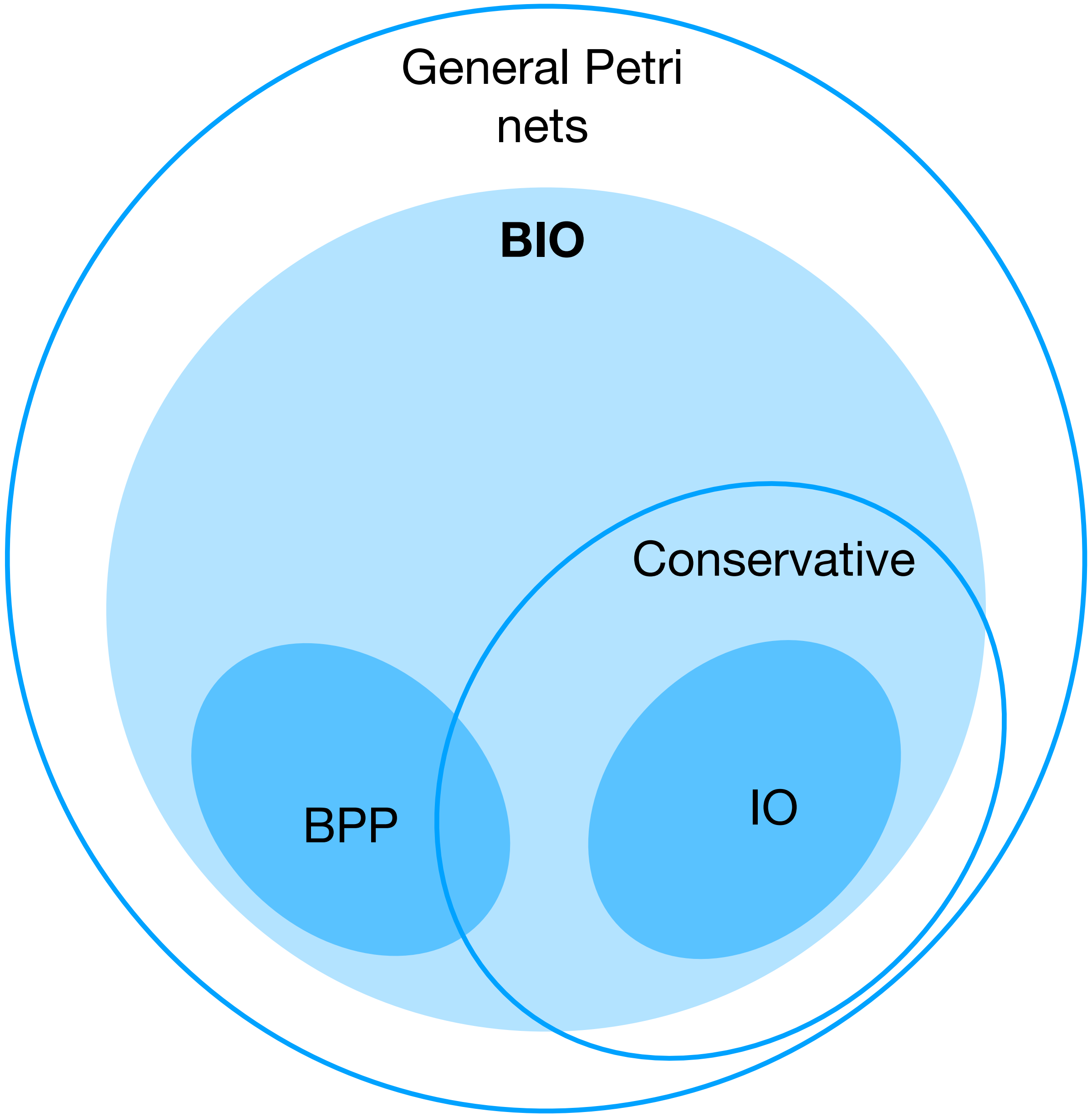

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Thank you!