Verification of Immediate Observation Population Protocols

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joint work with Javier Esparza, Pierre Ganty, Rupak Majumdar

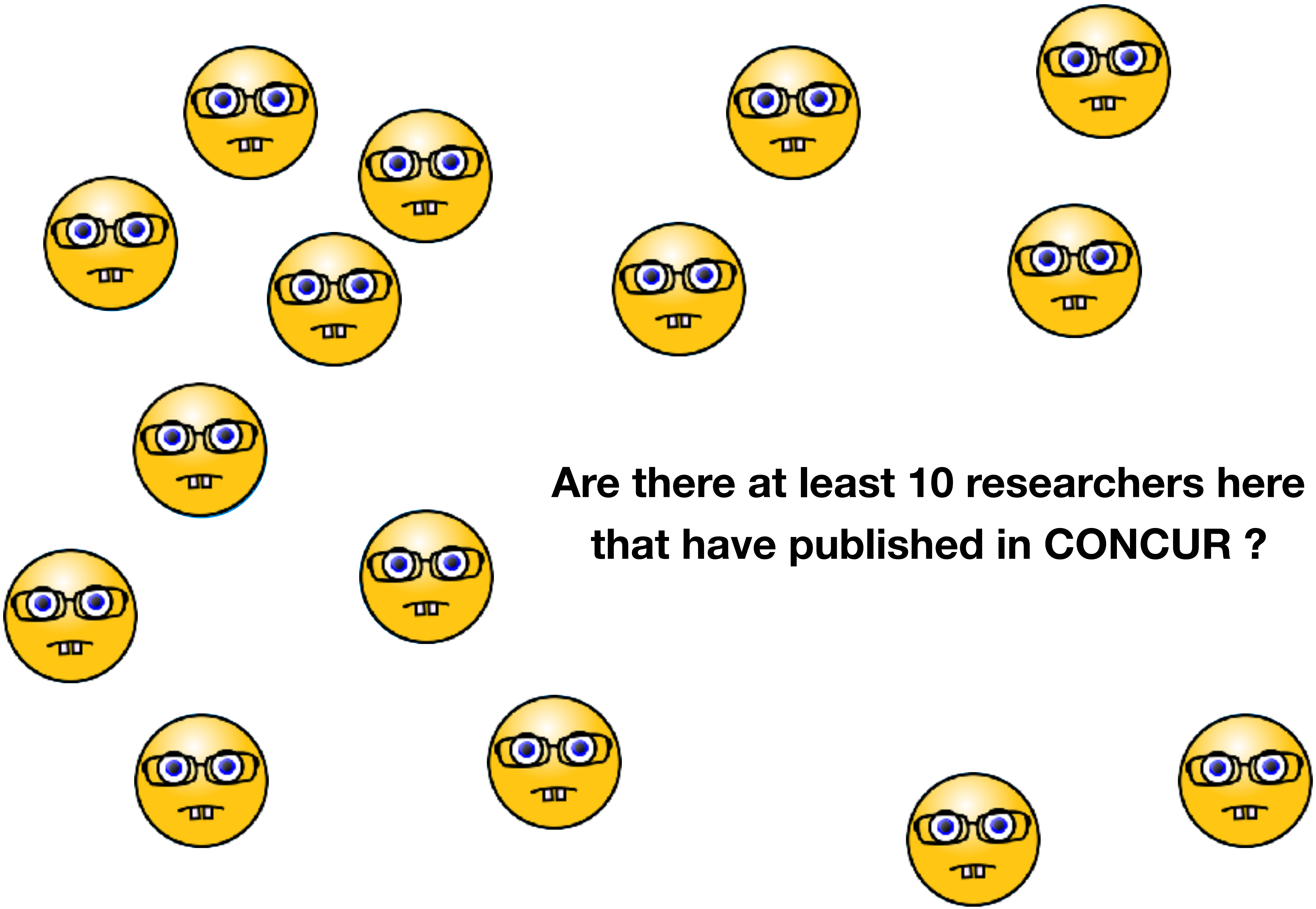
European Research Council

Established by the European Commission

Introduction

- Population protocols were introduced in 2004 by Angluin et al.
- They are a model of distributed computation by anonymous, identical, finite-state mobile agents with no global knowledge
- Motivating scenarios : networks of passively mobile sensors, propagation of trust, distributed computation in chemical reactions

Researchers at a Reception



Are there at least 10 researchers here that have published in CONCUR ?

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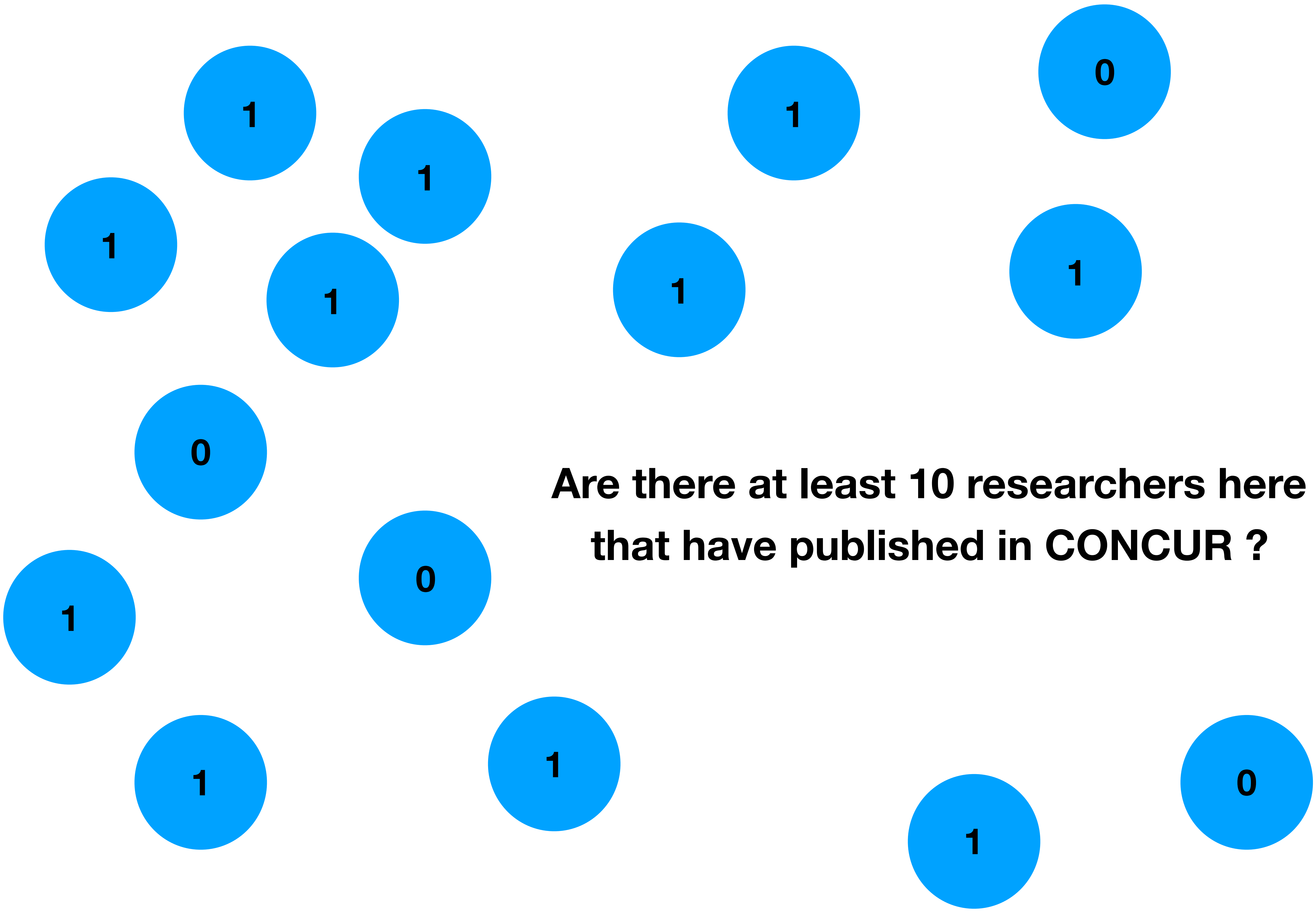

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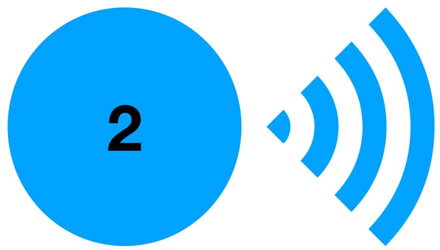

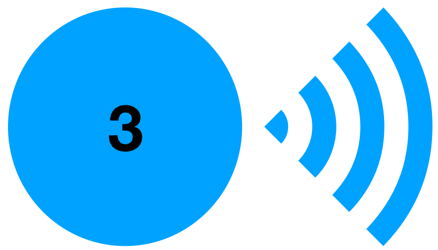

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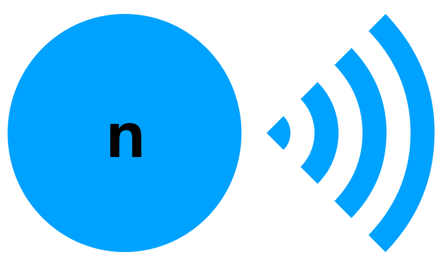

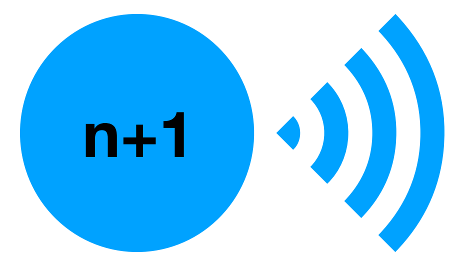

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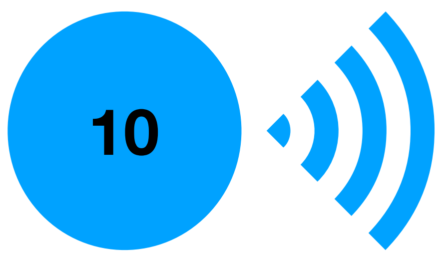
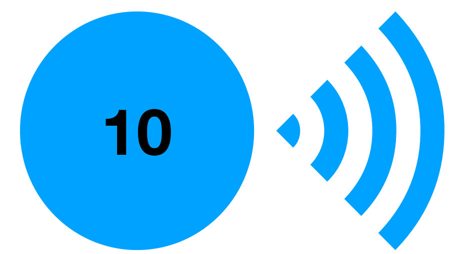
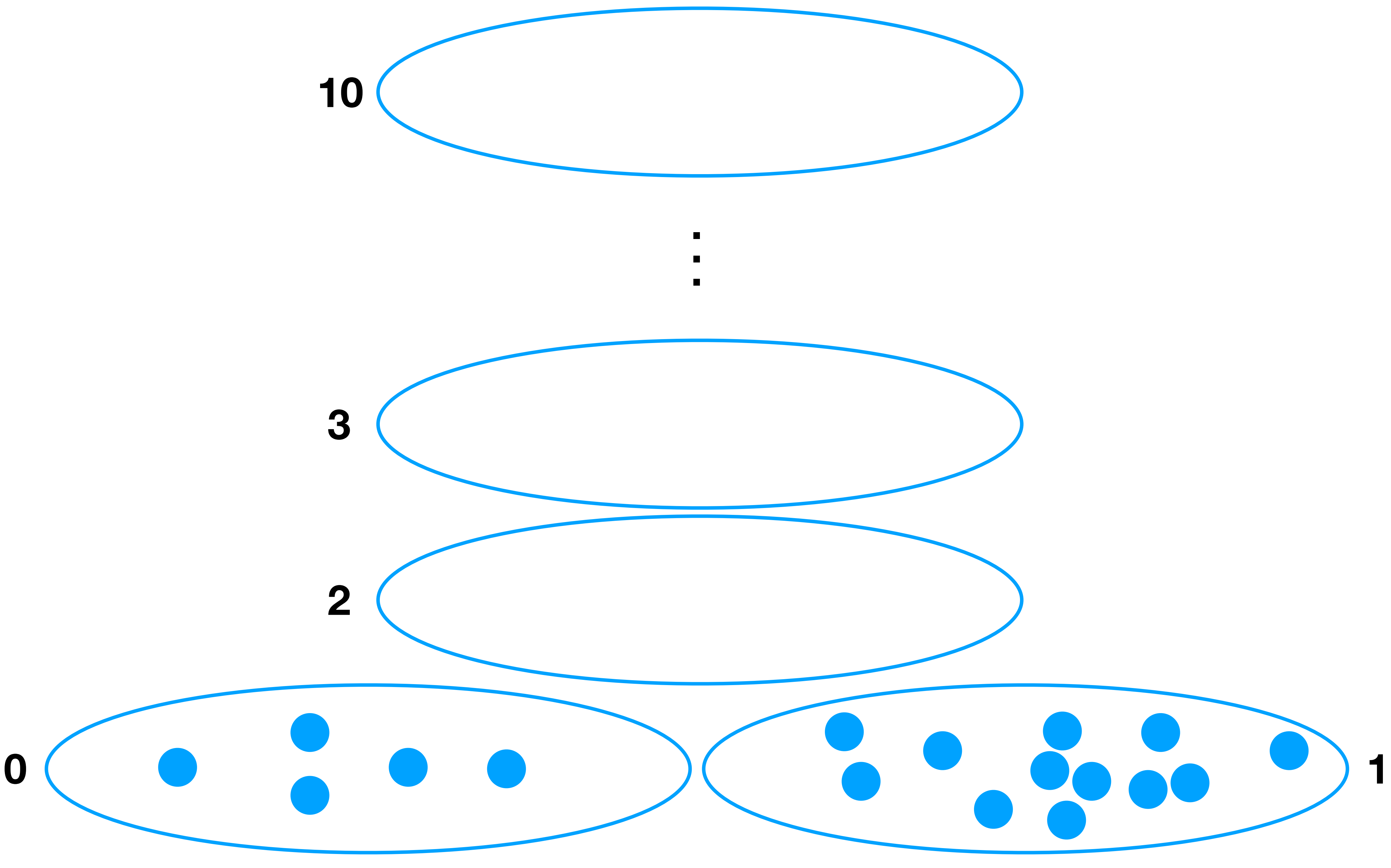




Counting Researchers Protocol



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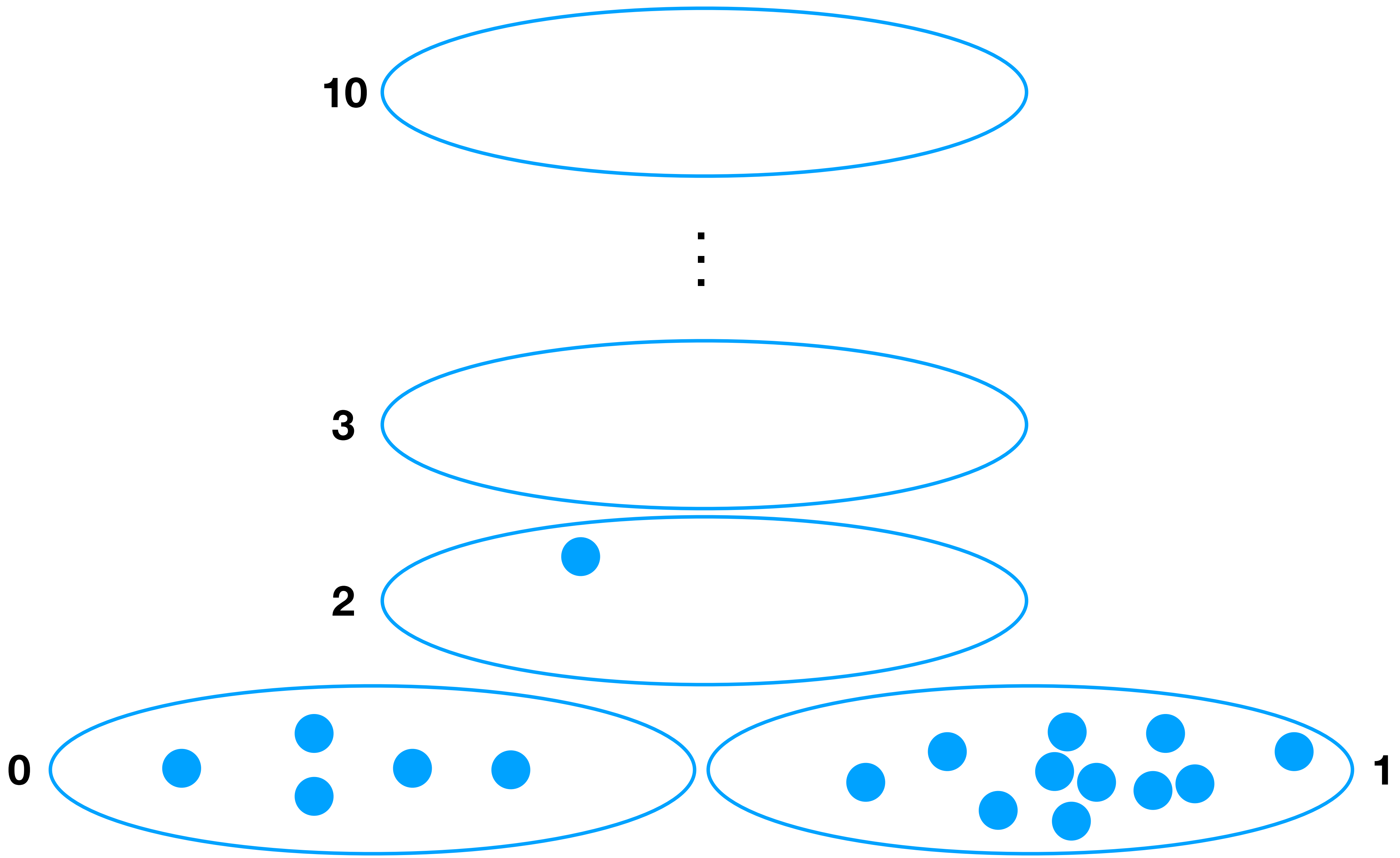

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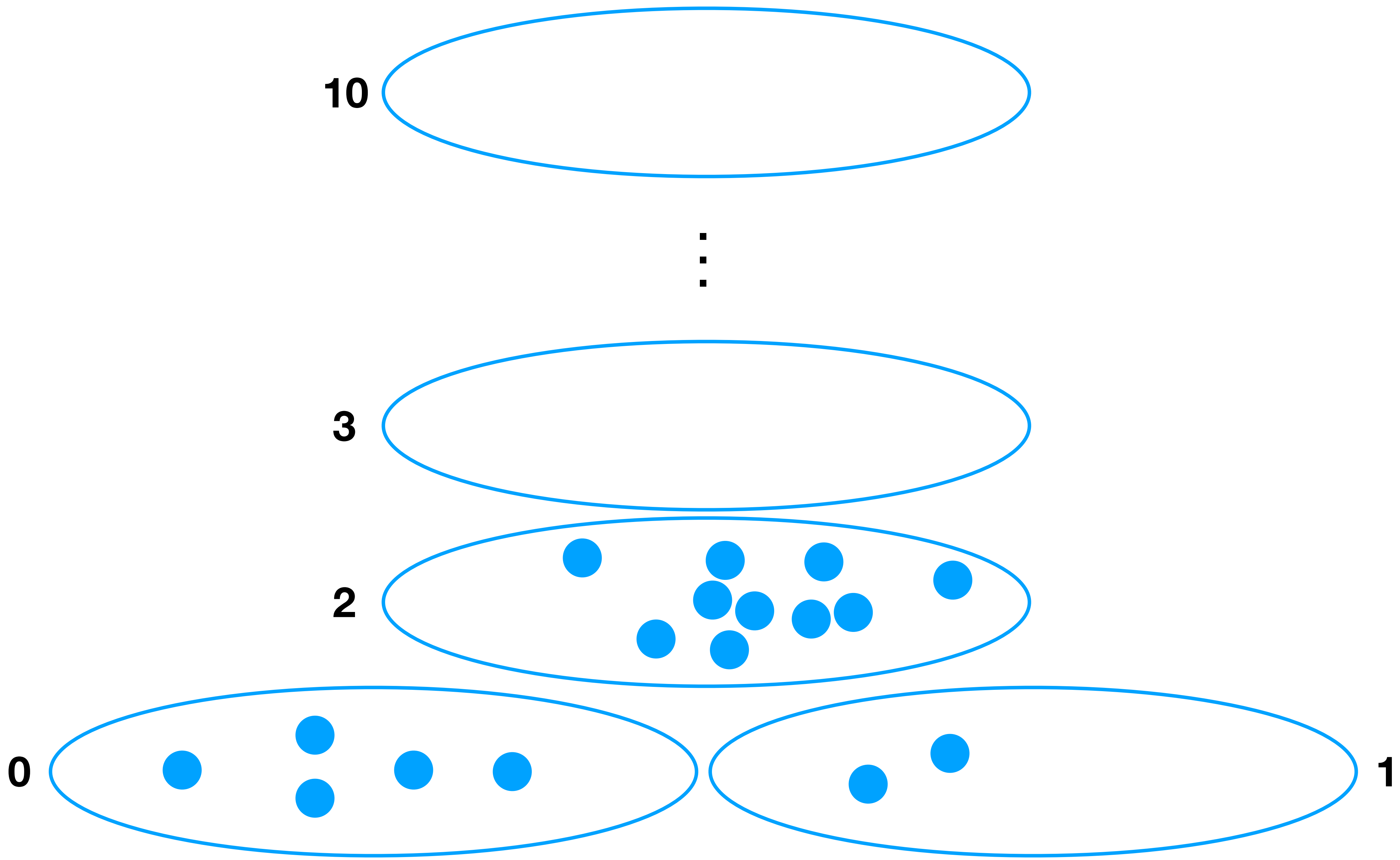

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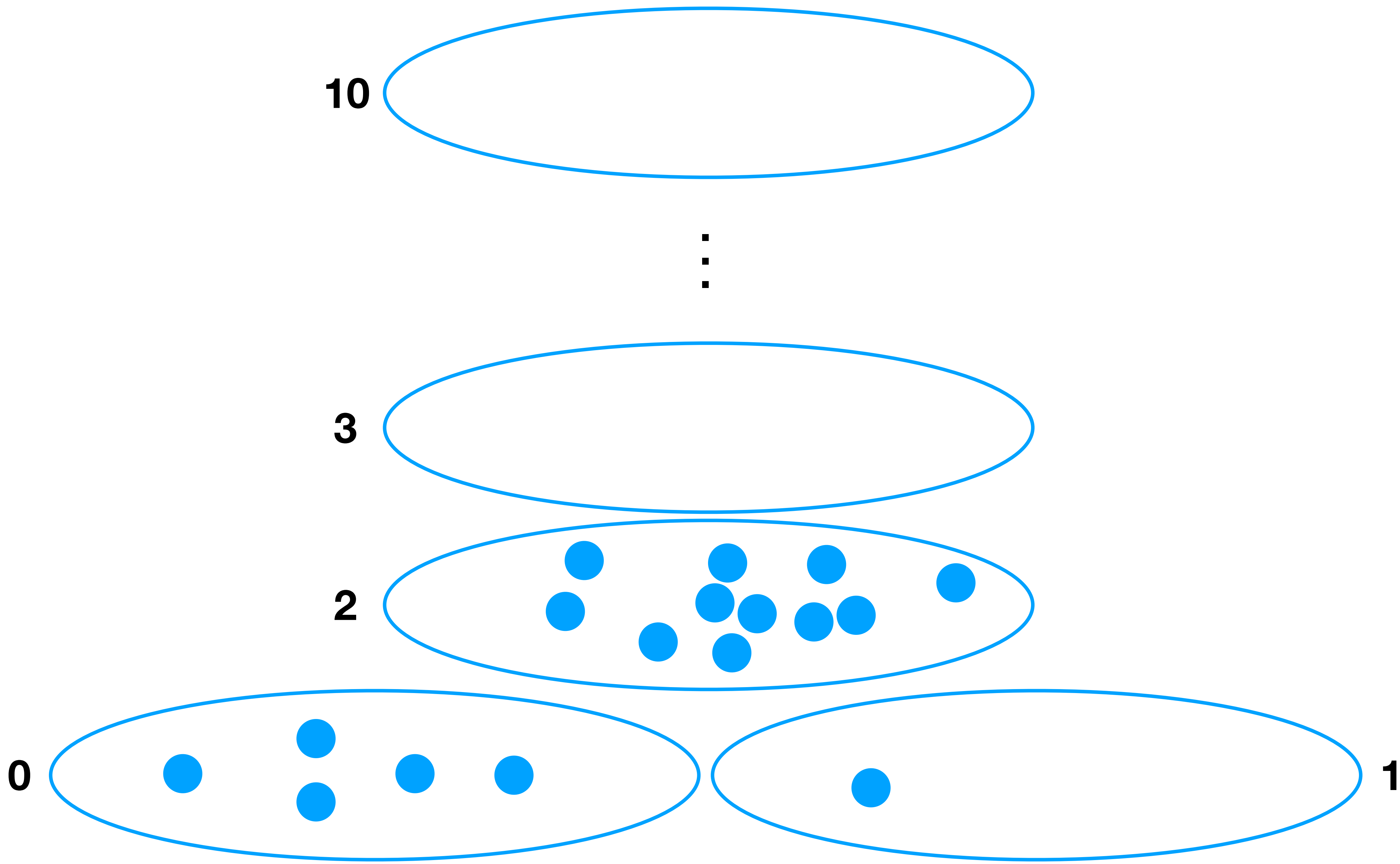

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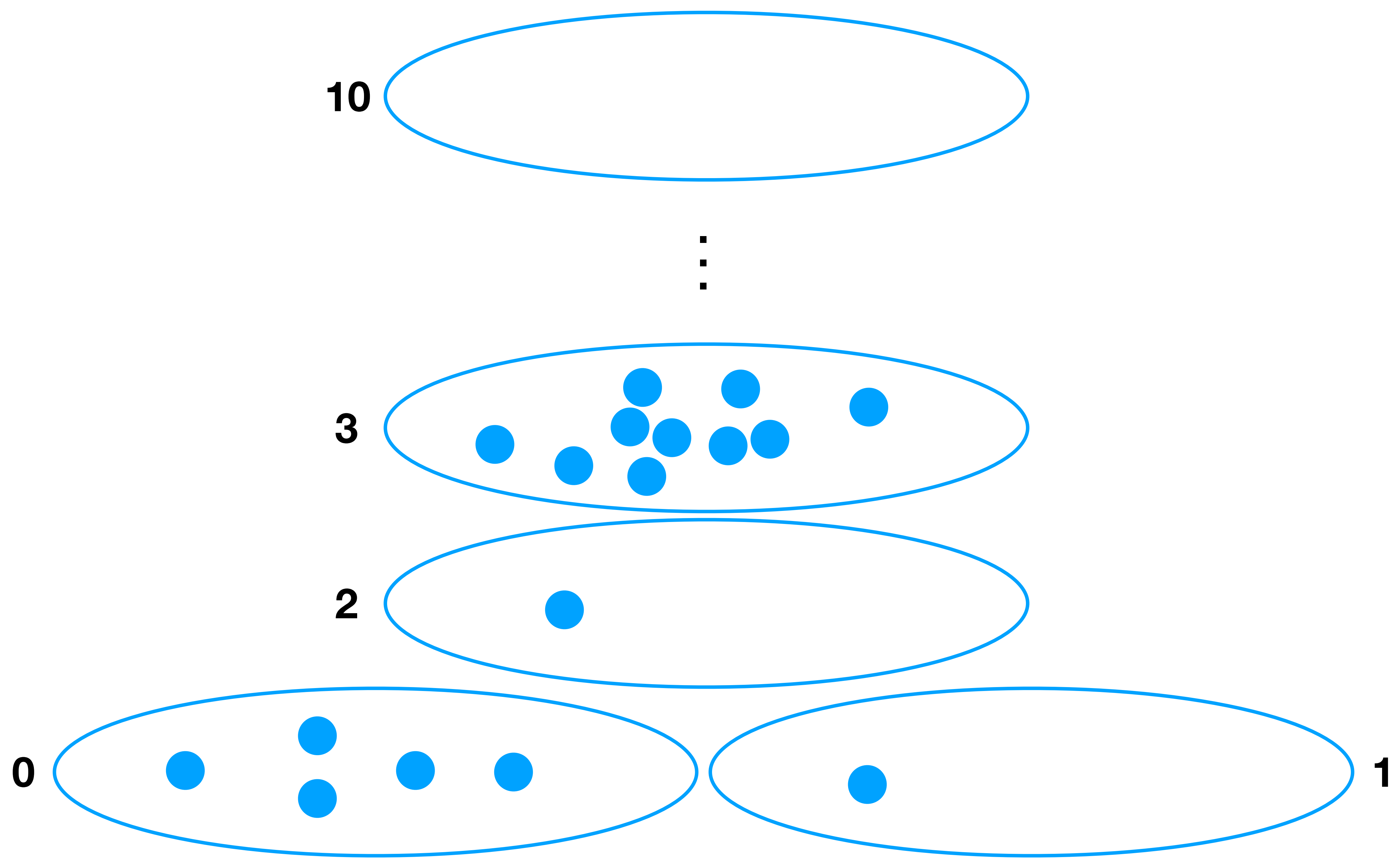

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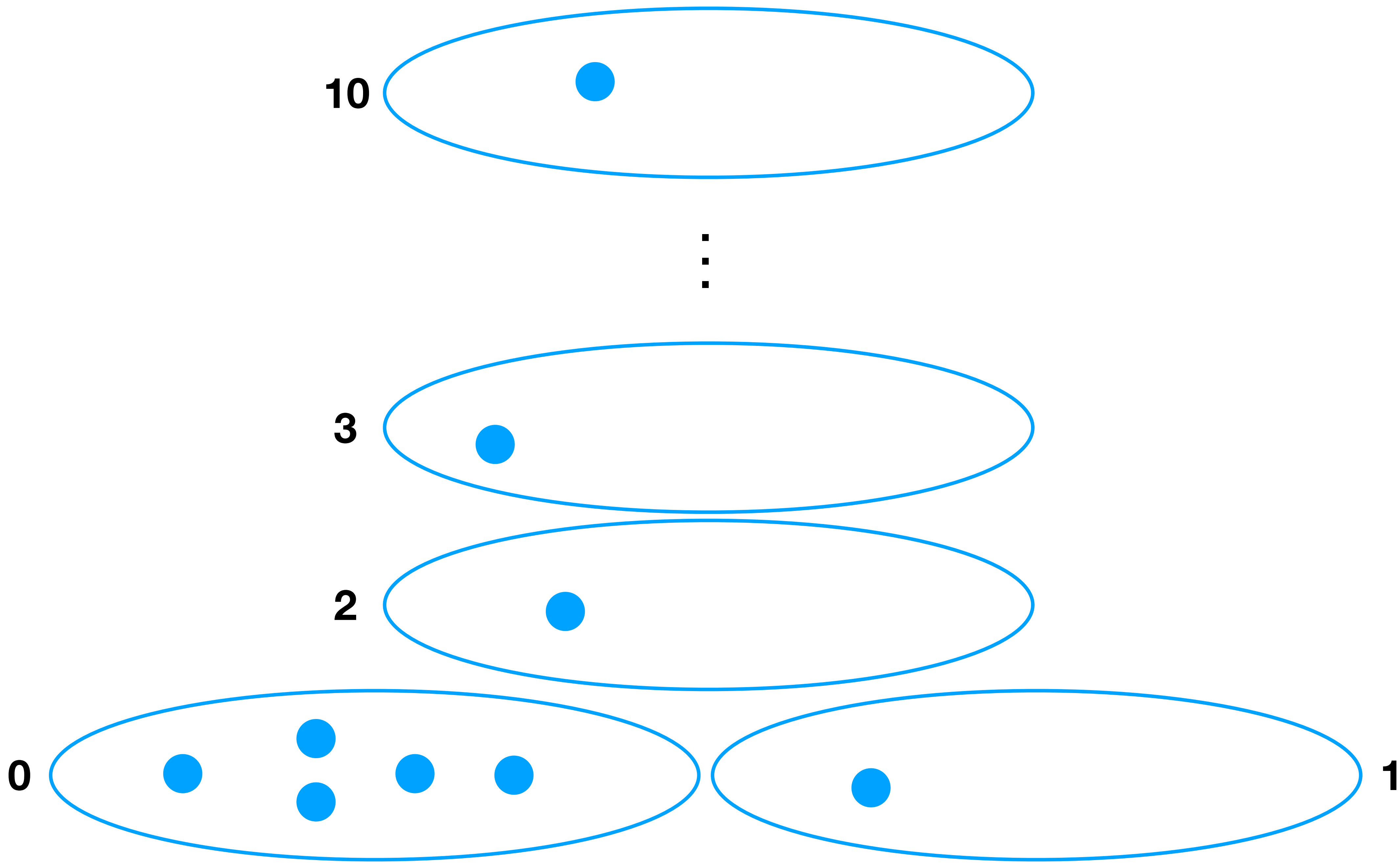

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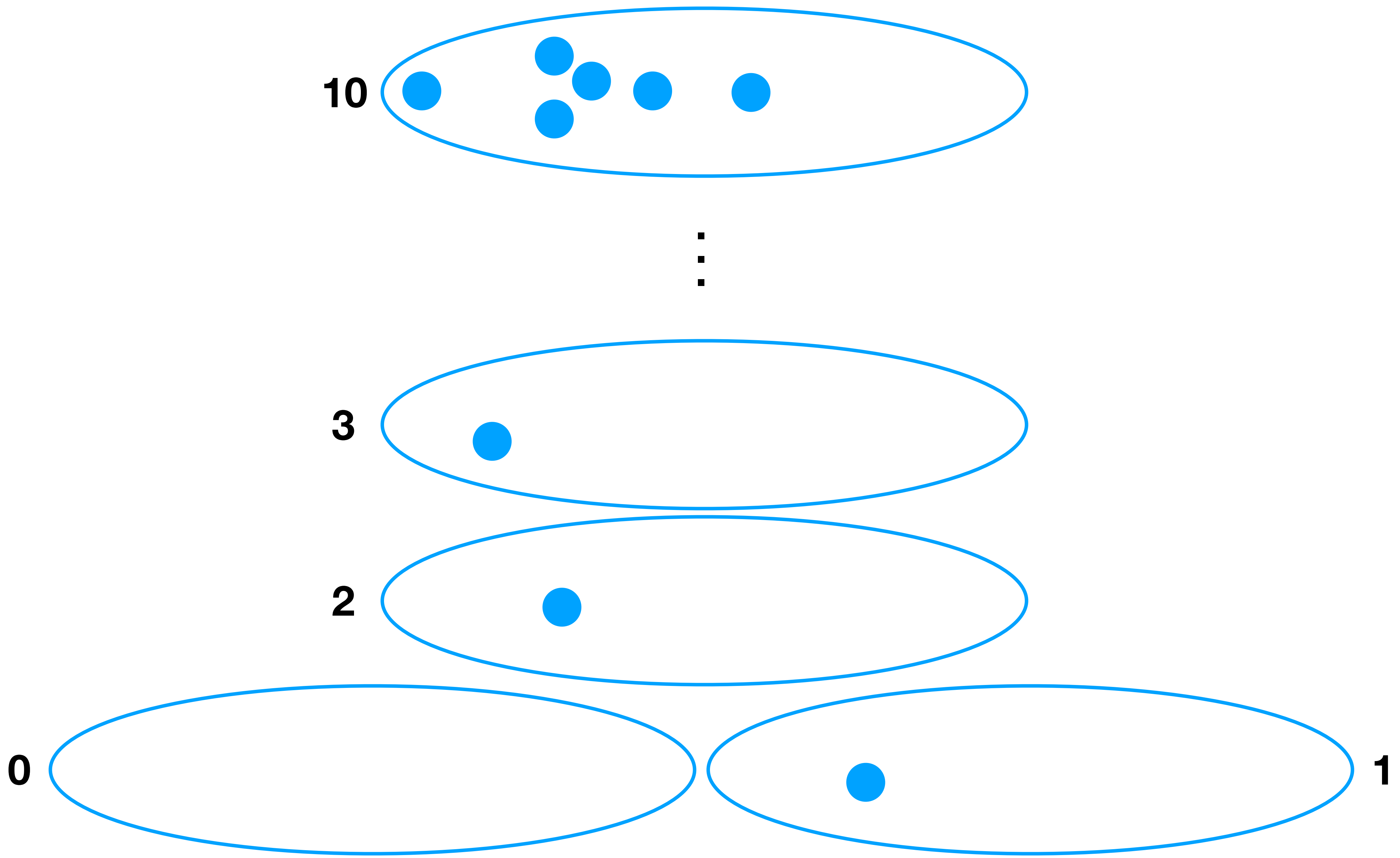

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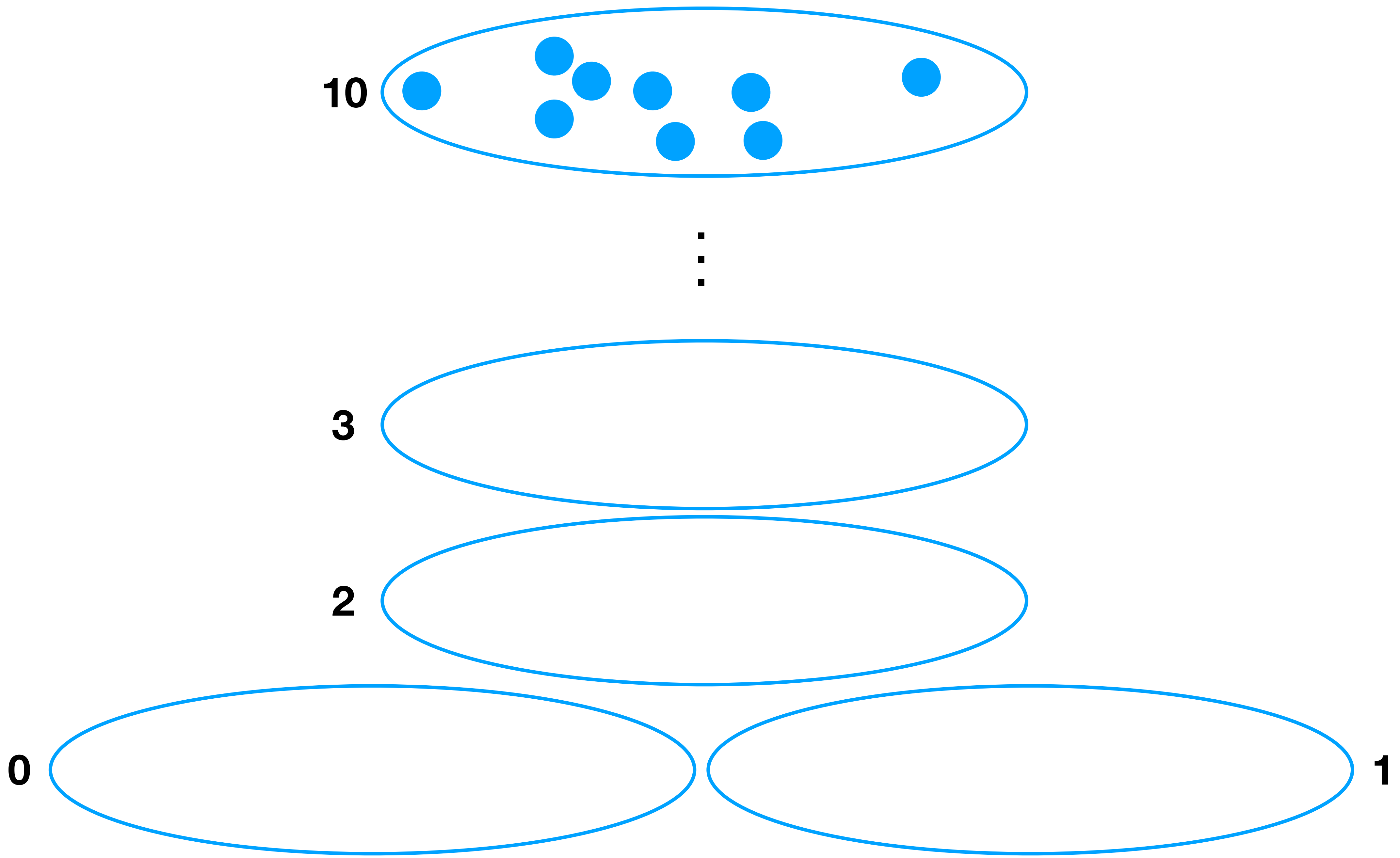
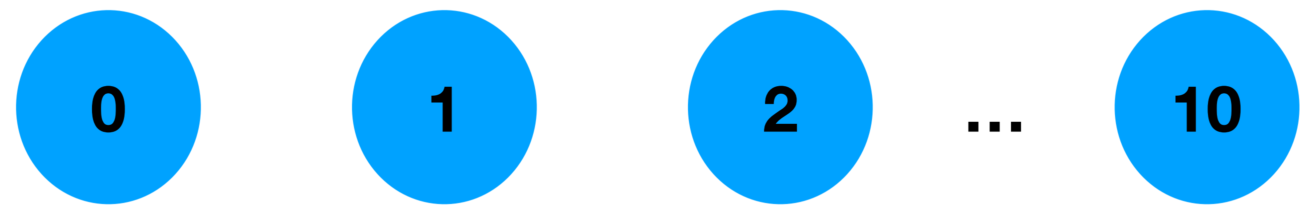


Population Protocol

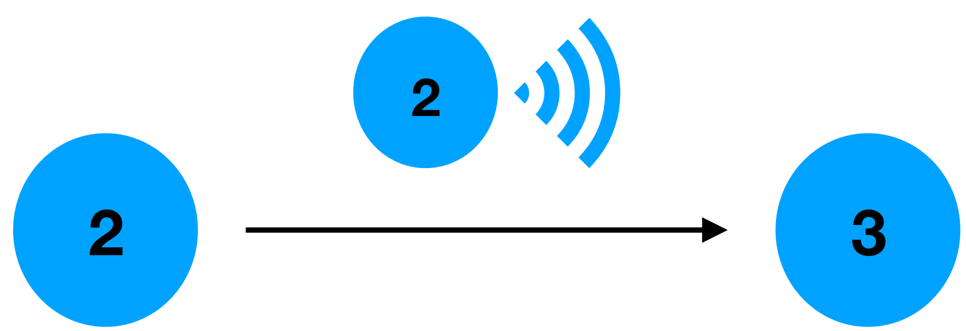
States 0 1 2 ... 10

Population Protocol

States



Transitions



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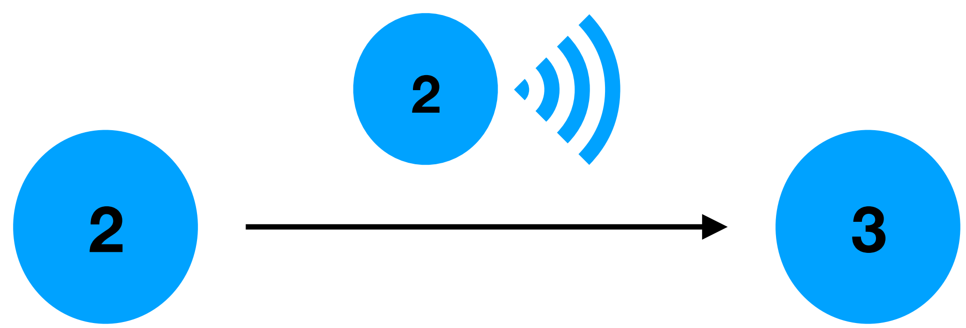

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Only agents in 0

and in 1

Population Protocol

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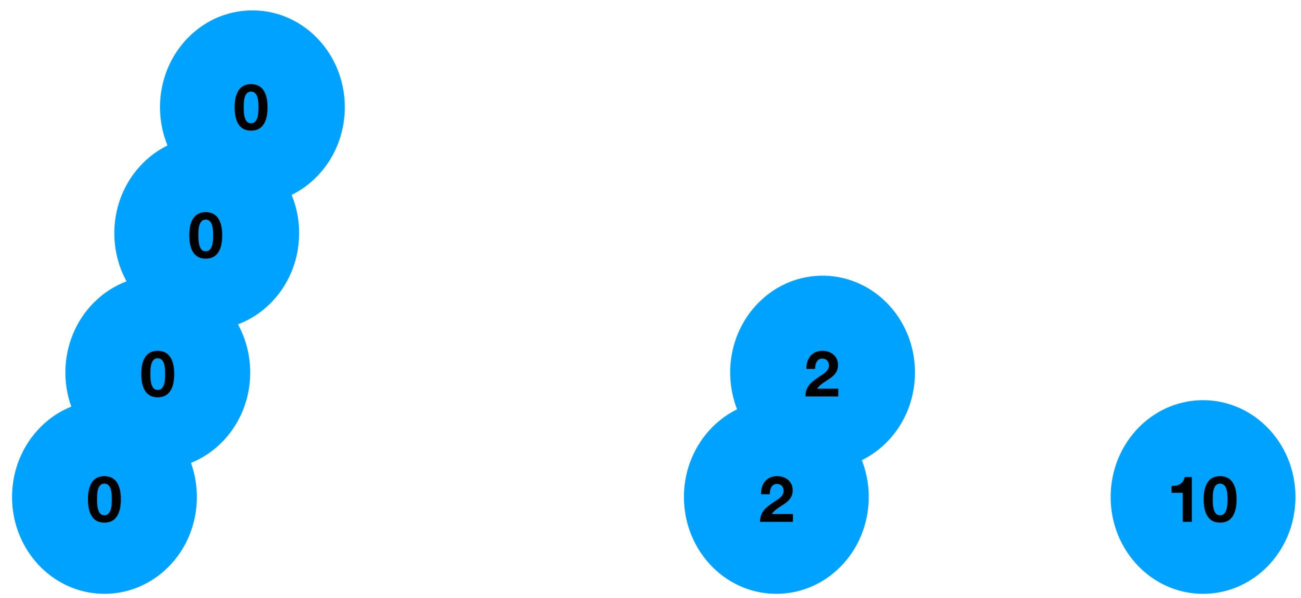
Output function

10 → **true**

n → **false** **0 ≤ n < 10**

Configuration

States = { 0 1 2 ... 10 }



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This protocol **computes** the predicate “at least 10 researchers have published at CONCUR”

Predicate Computation

Definition :

A population protocol **computes a predicate**

if and only if

every run starting in an initial configuration eventually reaches a configuration in which everyone agrees on the same output and does so forever

The Problem

Given $\mathcal{P}$ a population protocol, does $\mathcal{P}$ compute a predicate ?

EXPSPACE-hard

non primitive-recursive

[J. Esparza, P. Ganty, J. Leroux, R. Majumdar, ’16]

Immediate Observation Population Protocols

[D. Angluin, J. Aspnes, D. Eisenstat, E. Ruppert, ’07]

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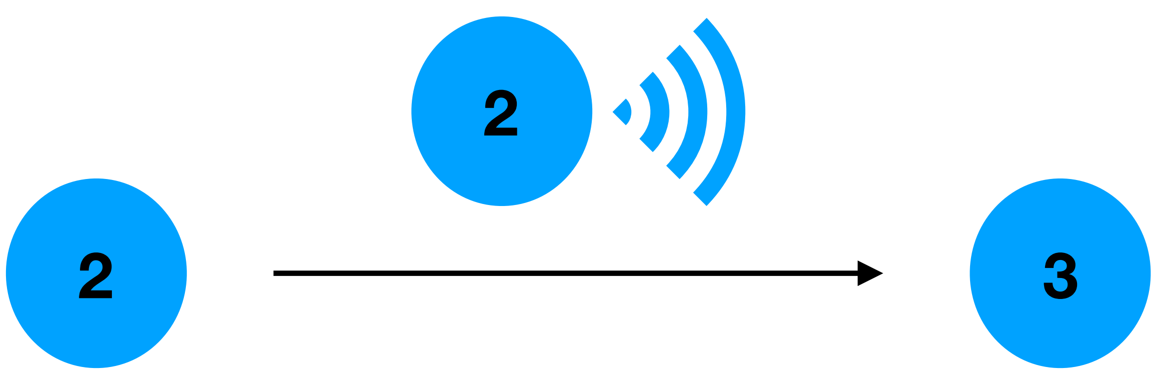

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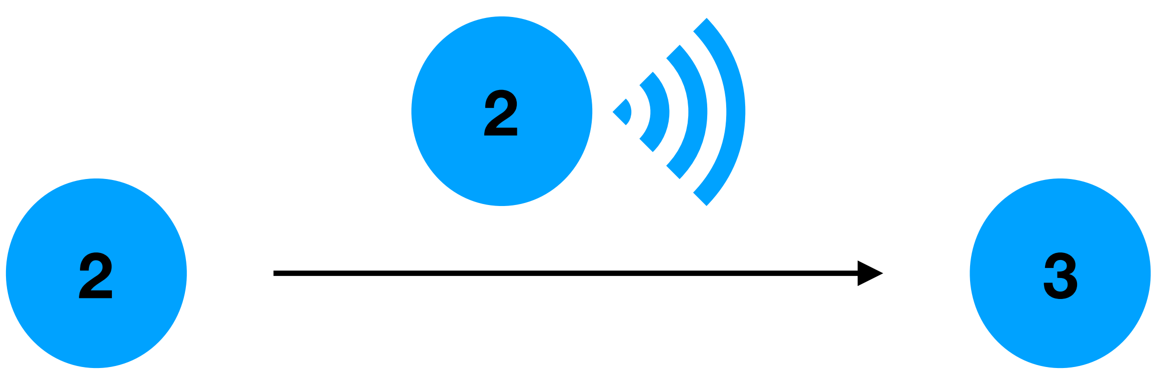

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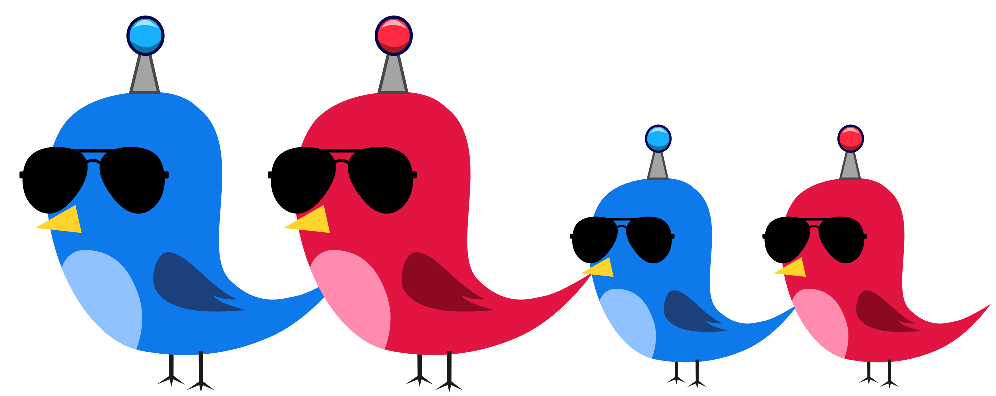

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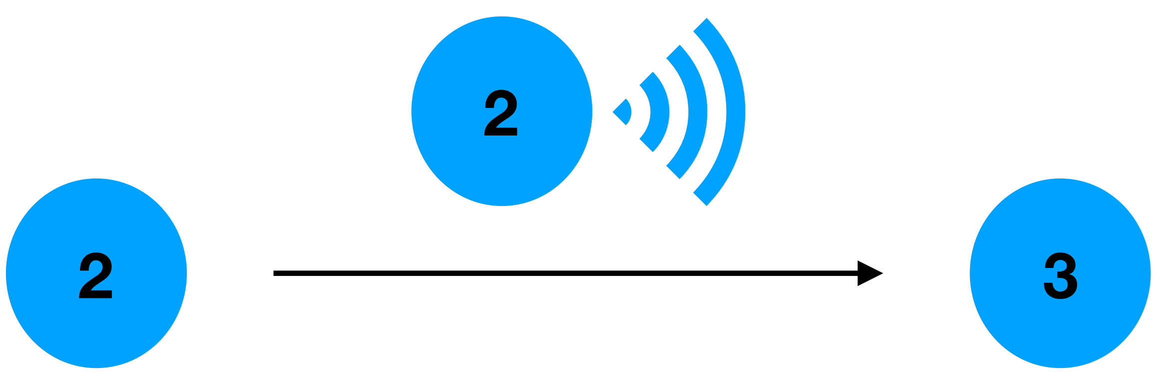

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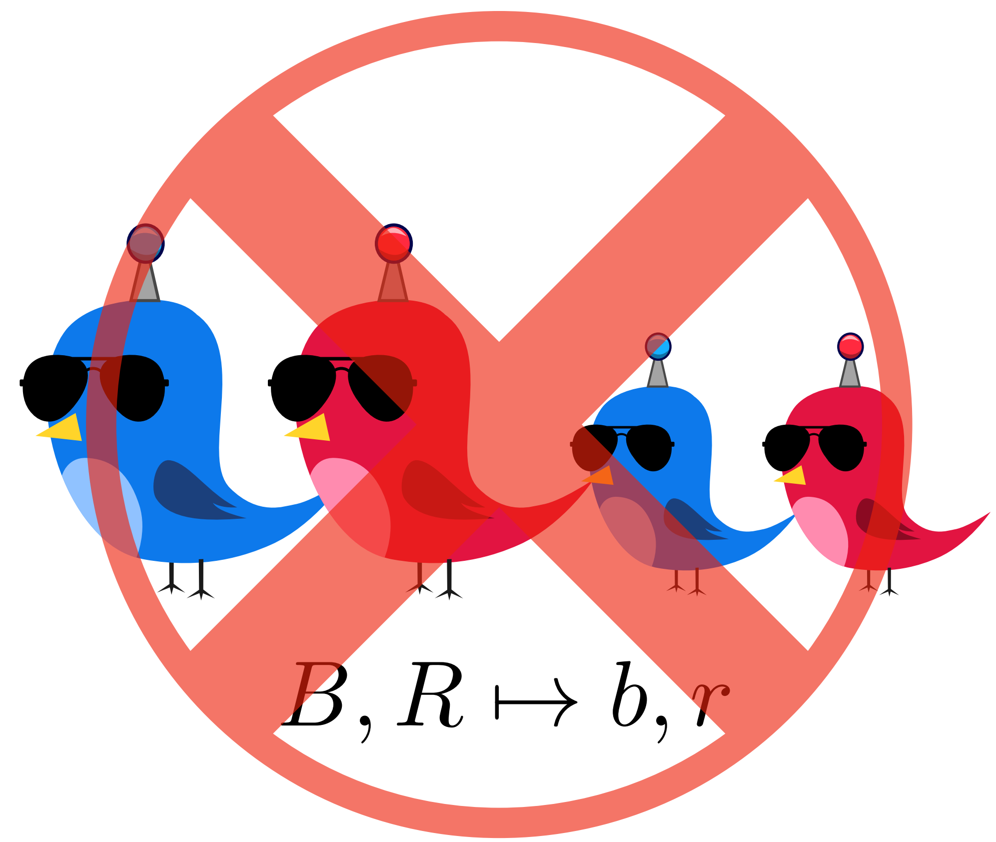

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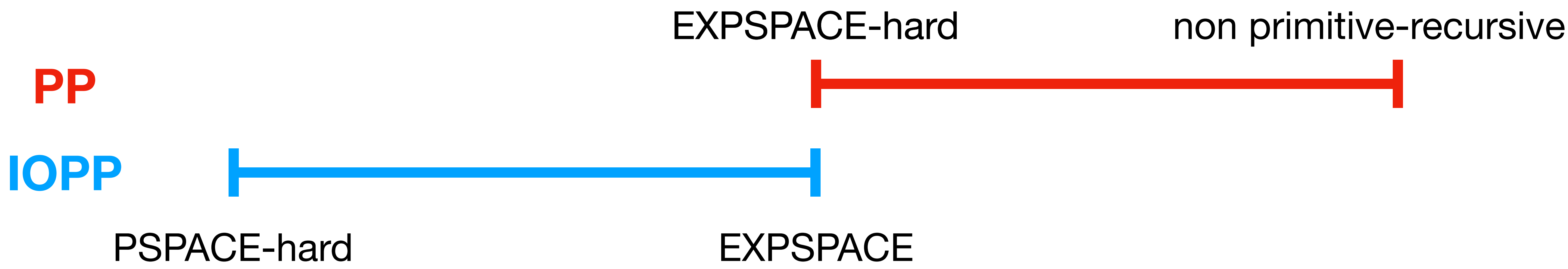
- Disadvantage of IOPP : less expressivity

- Advantage of IOPP : can be implemented on top of one-way communication models
  - e.g. sensor networks
  - e.g. networks with unidirectional communication channels

Result

Given $\mathcal{P}$ a population protocol, does $\mathcal{P}$ compute a predicate ?



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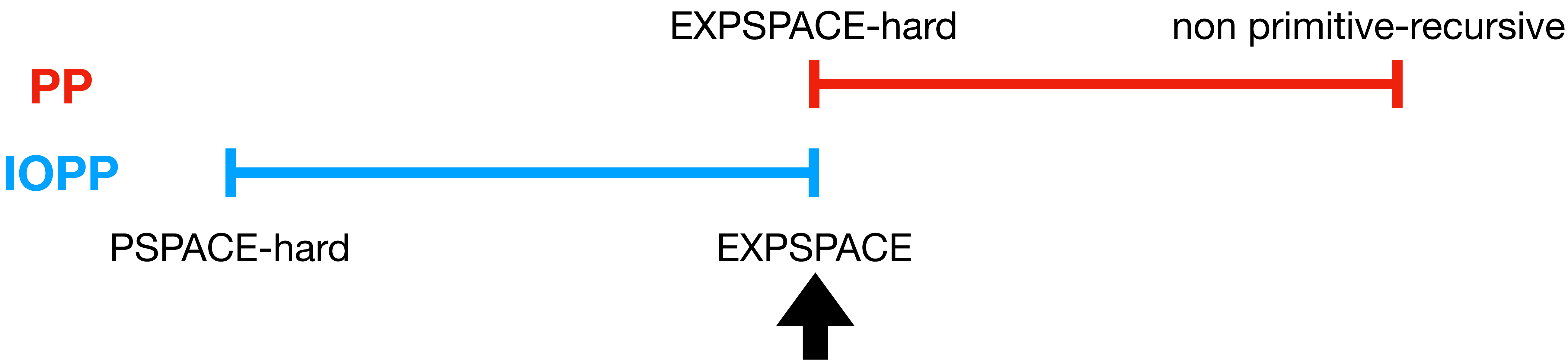

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Key Idea

We will :

- **Reformulate** the problem of “computing a predicate”
- Find a **good representation** for sets of configurations
- **Bound the number of iterations** needed to calculate *pre** and *post**

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$$\wedge$$

$$pre^*(\mathcal{ST}_0) \cap pre^*(\mathcal{ST}_1) \cap \mathcal{I} = \emptyset$$

$\mathcal{I}$ the initial configurations

$\mathcal{ST}_b$ the stable *b*-consensus configurations for *b* $\in \{0, 1\}$

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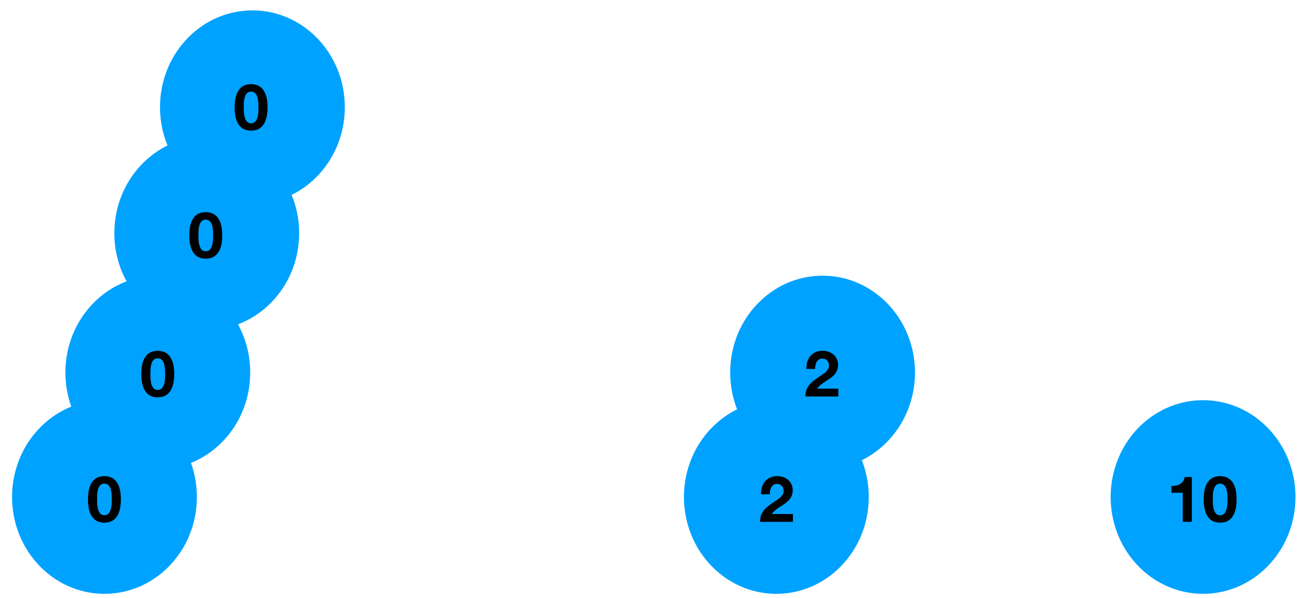

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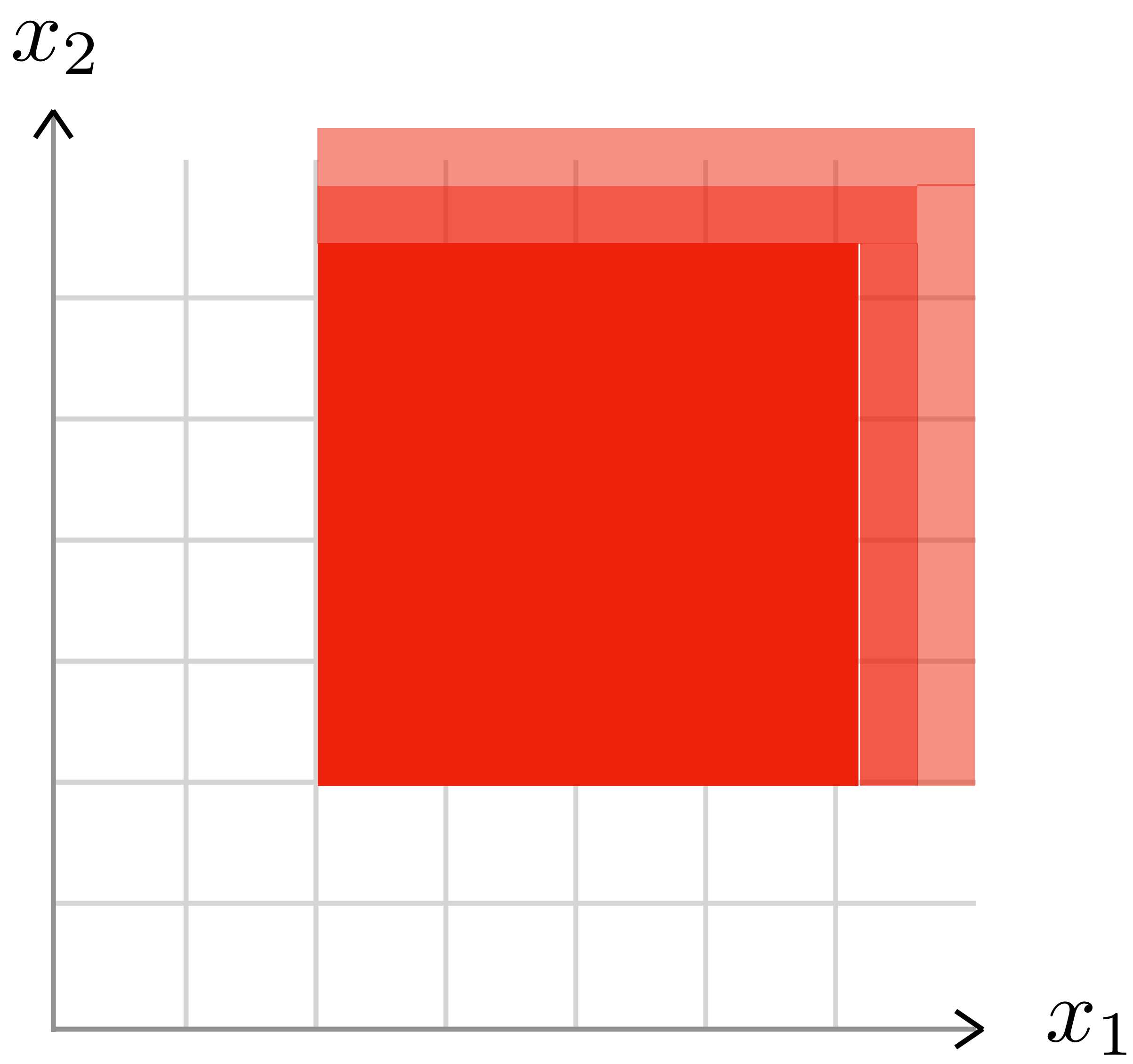
Counting Constraints



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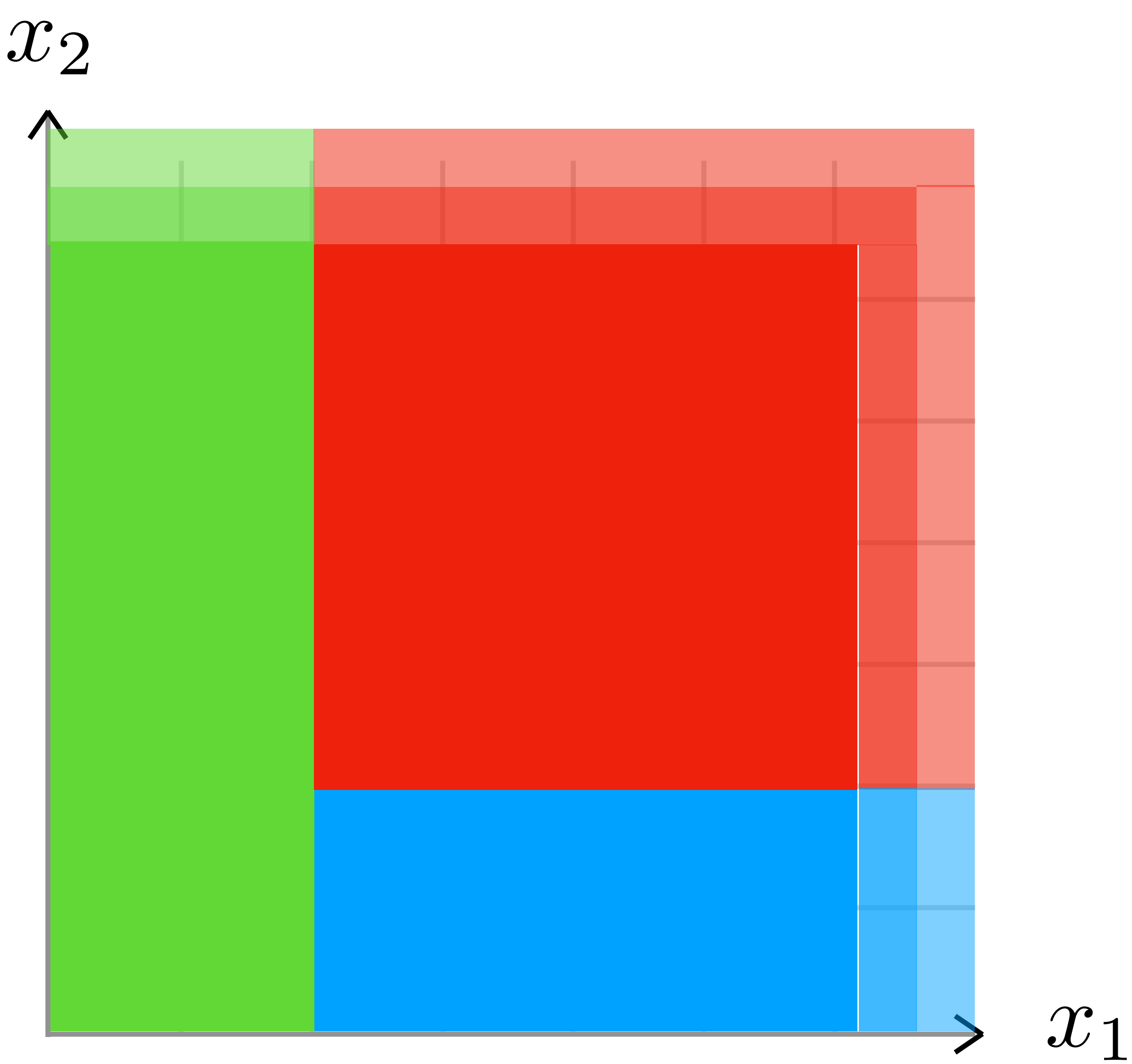

$$\neg \begin{array}{l} 2 \leq x_1 \leq \infty \\ 2 \leq x_2 \leq \infty \end{array}$$

$$\Downarrow$$

$$\begin{array}{l} 2 \leq x_1 \leq \infty \\ 0 \leq x_2 \leq 2 \end{array} \vee \begin{array}{l} 0 \leq x_1 \leq 2 \\ 0 \leq x_2 \leq \infty \end{array}$$

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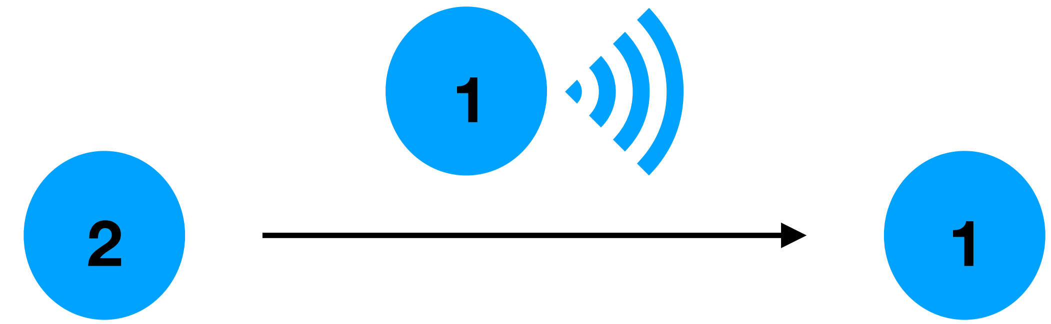

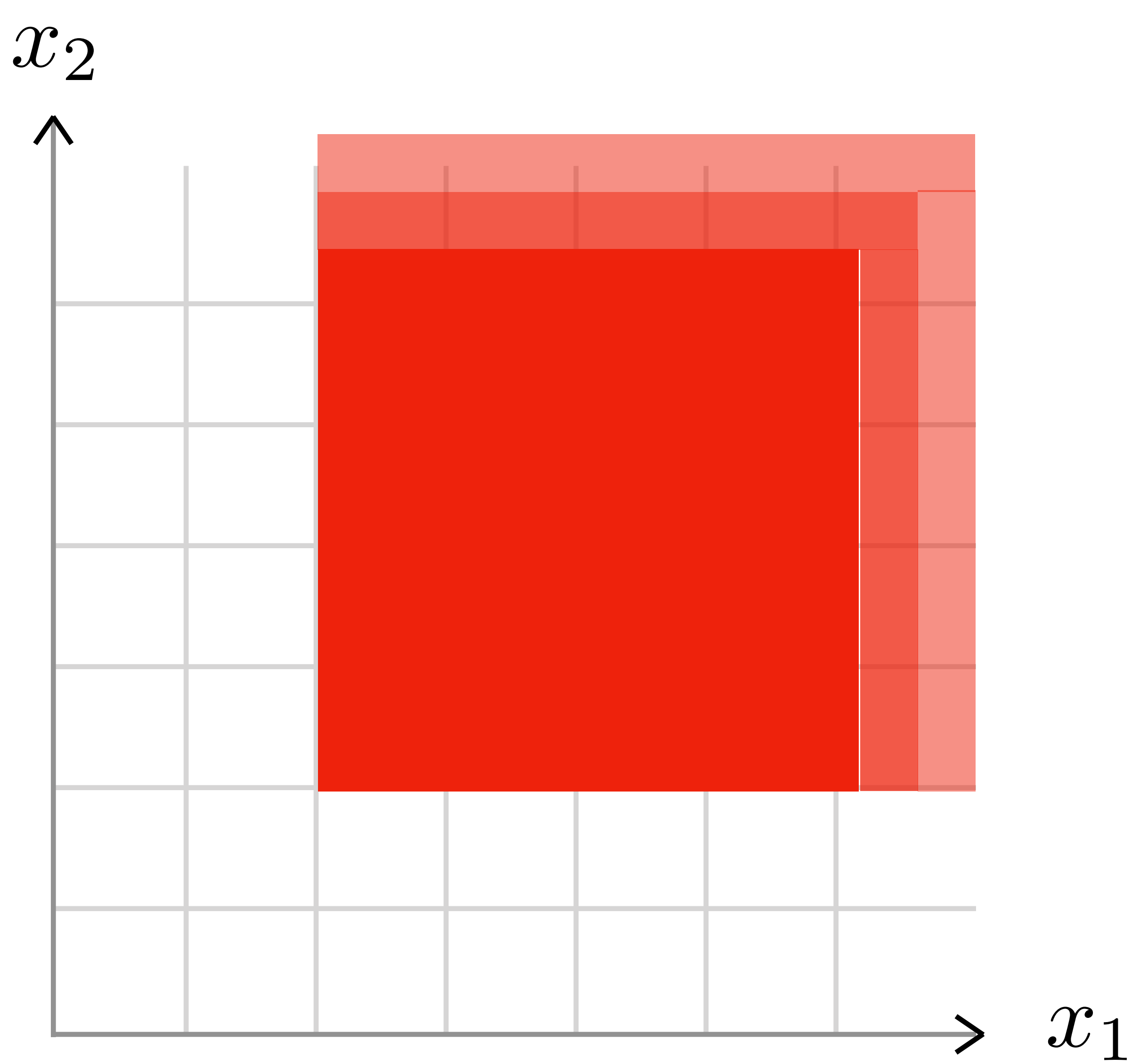

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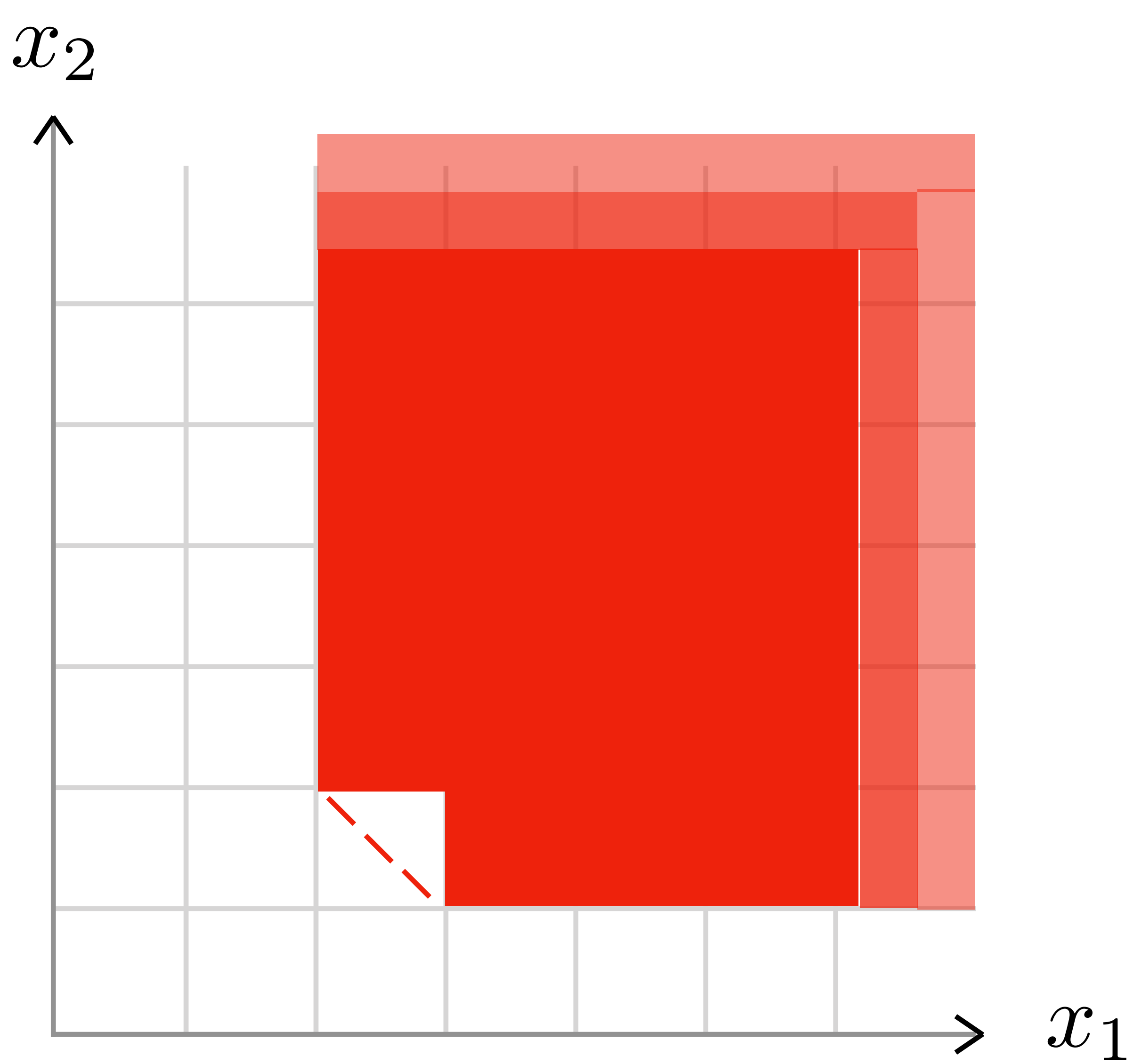

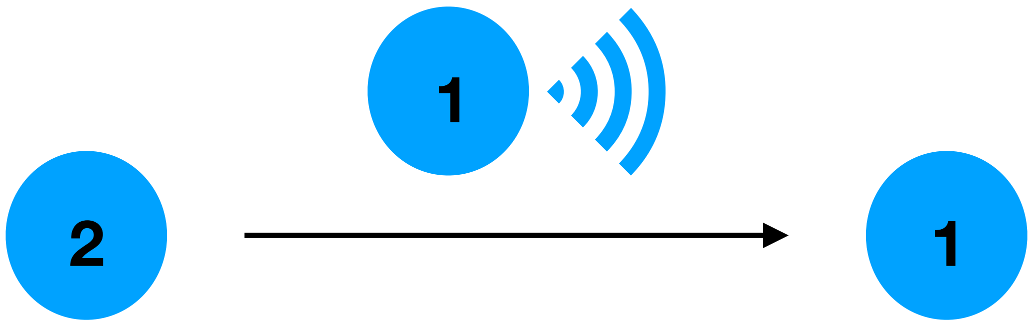

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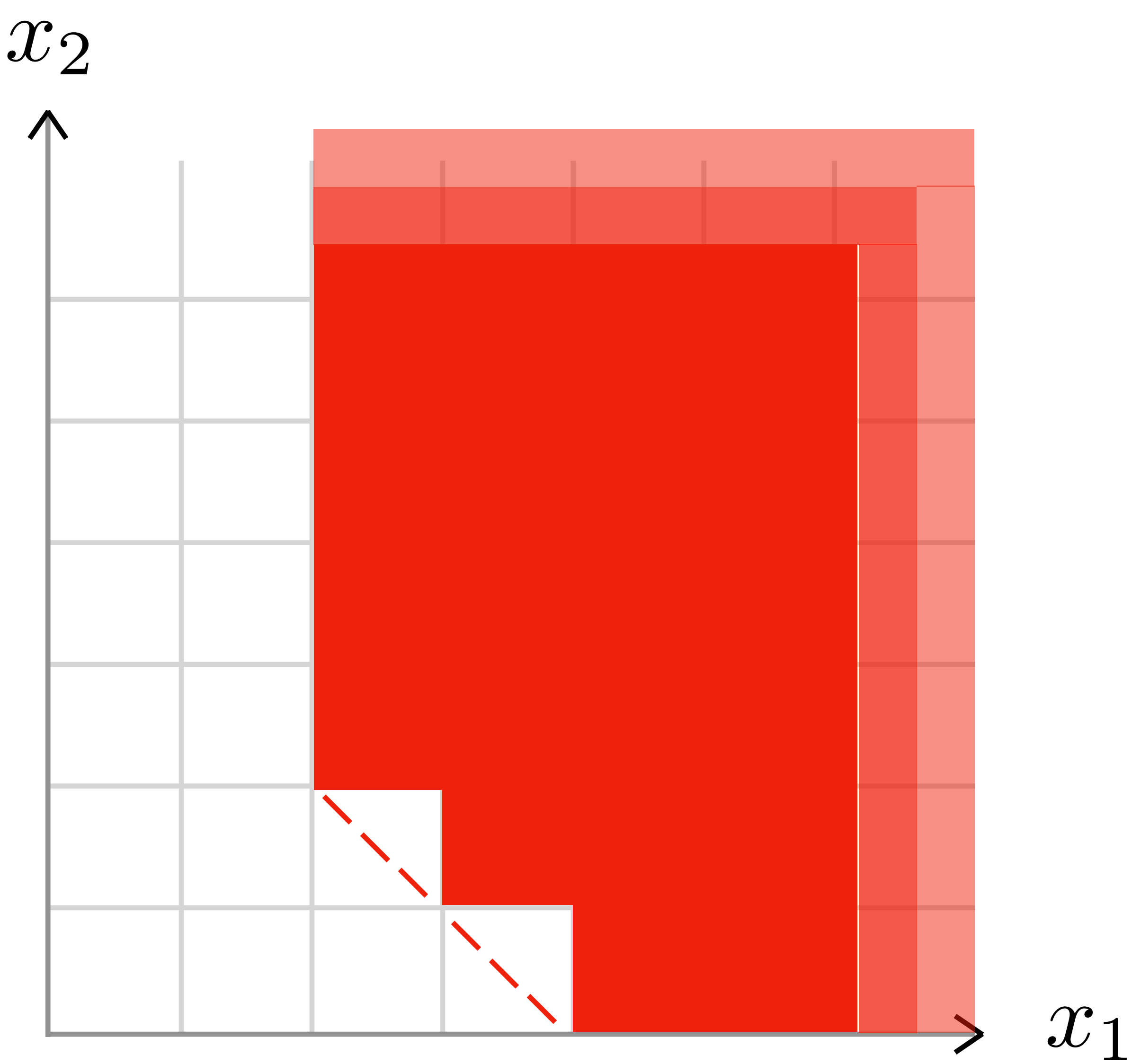

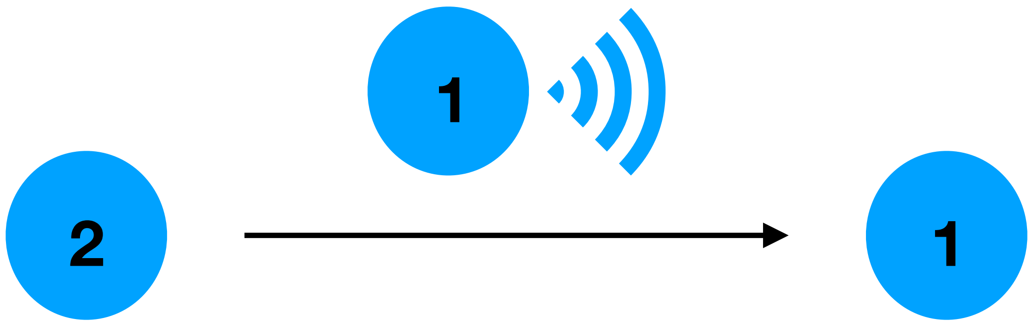

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$$\Downarrow$$
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Counting Constraints

- Counting constraints represent possibly **infinite** sets of configurations
- Counting constraints are closed under Boolean combinations
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This is the **fundamental** property for counting constraints
and it is **not true in the general population protocol framework**

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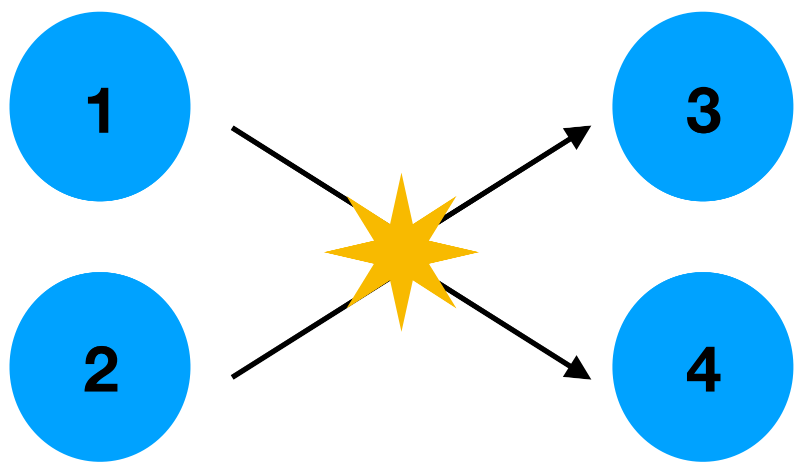

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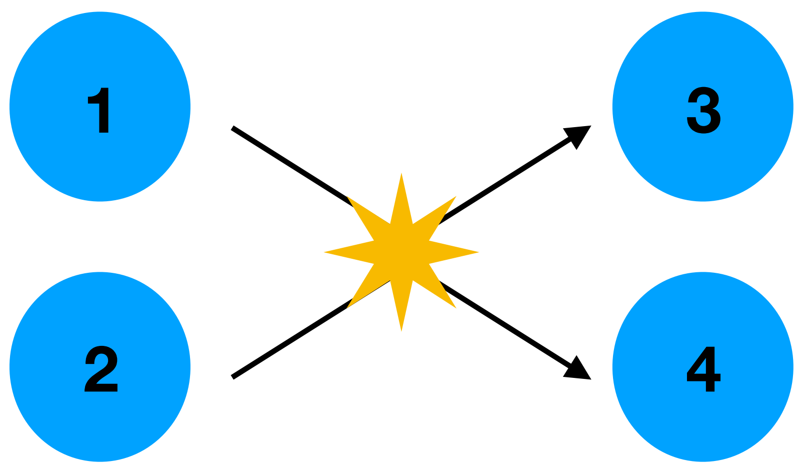

Consider a protocol with a unique transition and with agents only in state 1 and 2 in the initial configurations

$\mathcal{I} =$ configurations of the form $(n_1, n_2, 0, 0)$

$post^*(\mathcal{I}) =$ configurations of the form (n'_1, n'_2, n, n)

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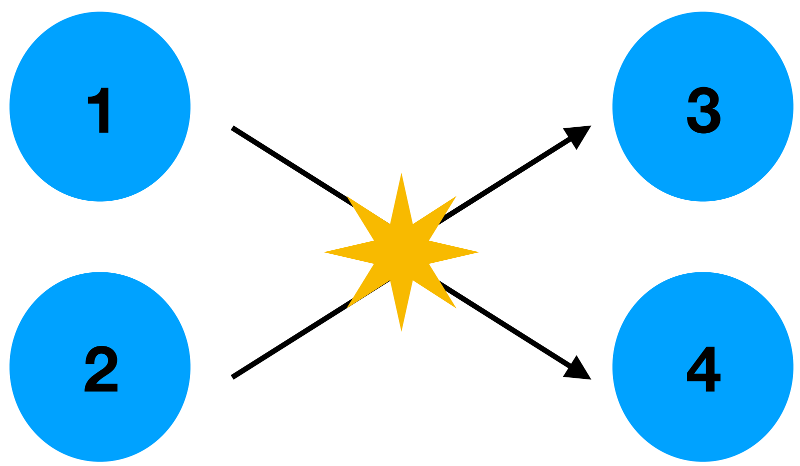

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This cannot be expressed with counting constraints

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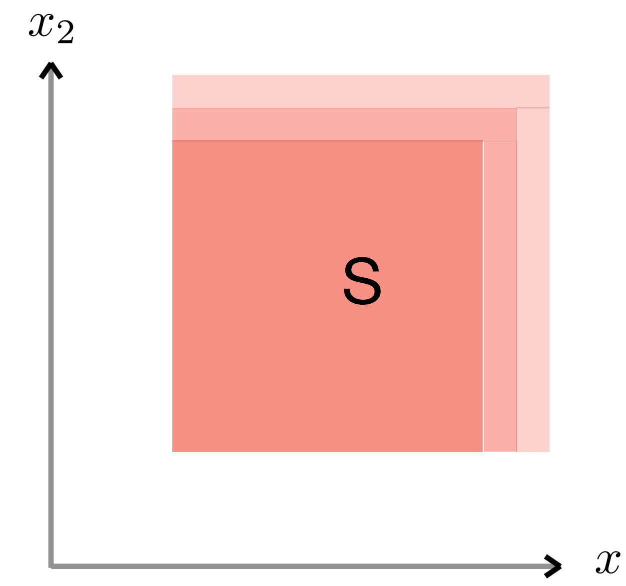
Idea

[C. Rackoff, '78]

- A theorem by Rackoff gives K such that

$$post^*(S) = \bigcup_{i \geq 0}^{K} post^i(S)$$

but only for S an **upward closed set**



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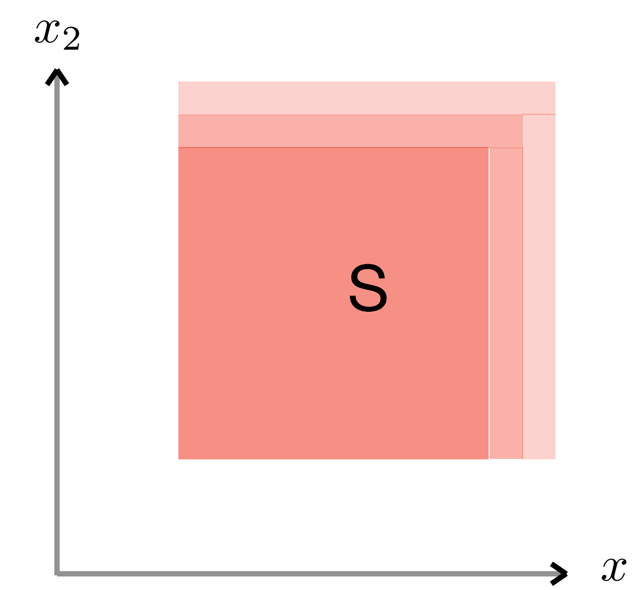
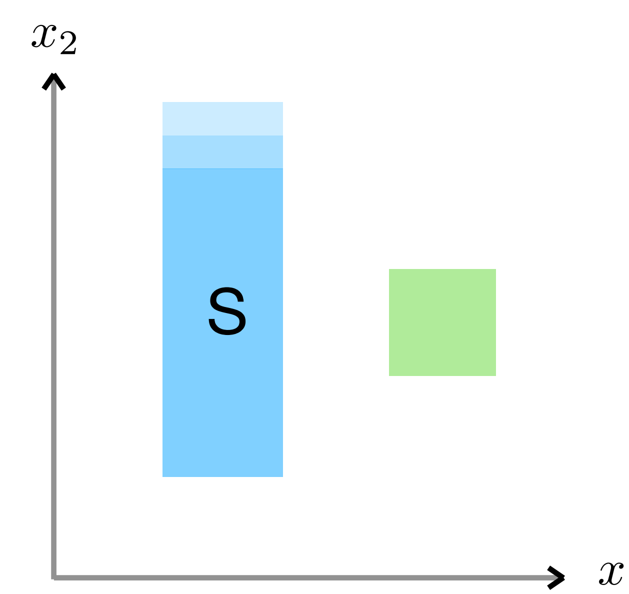


- We **generalize** this result by applying Rackoff a finite number of times — we split runs starting in S and apply Rackoff to each segment



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