The Complexity of Verifying Observation Population Protocols

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joint work with Javier Esparza and Mikhail Raskin

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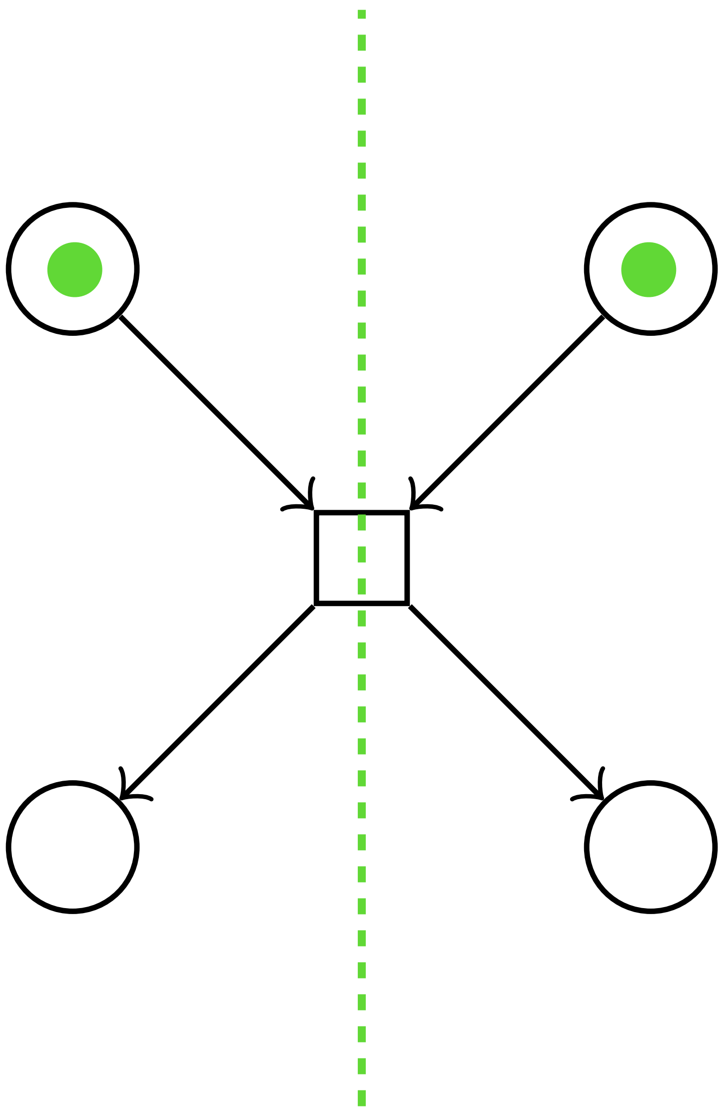
Population protocols

[Angluin, Aspnes, Diamadi, Fischer, Peralta, ’04]

- Distributed computing model where anonymous finite-state mobile agents jointly compute a function.

- Agents communicate through *rendez-vous*.

- Motivating scenarios : networks of passively mobile sensors, propagation of trust, chemical reactions networks



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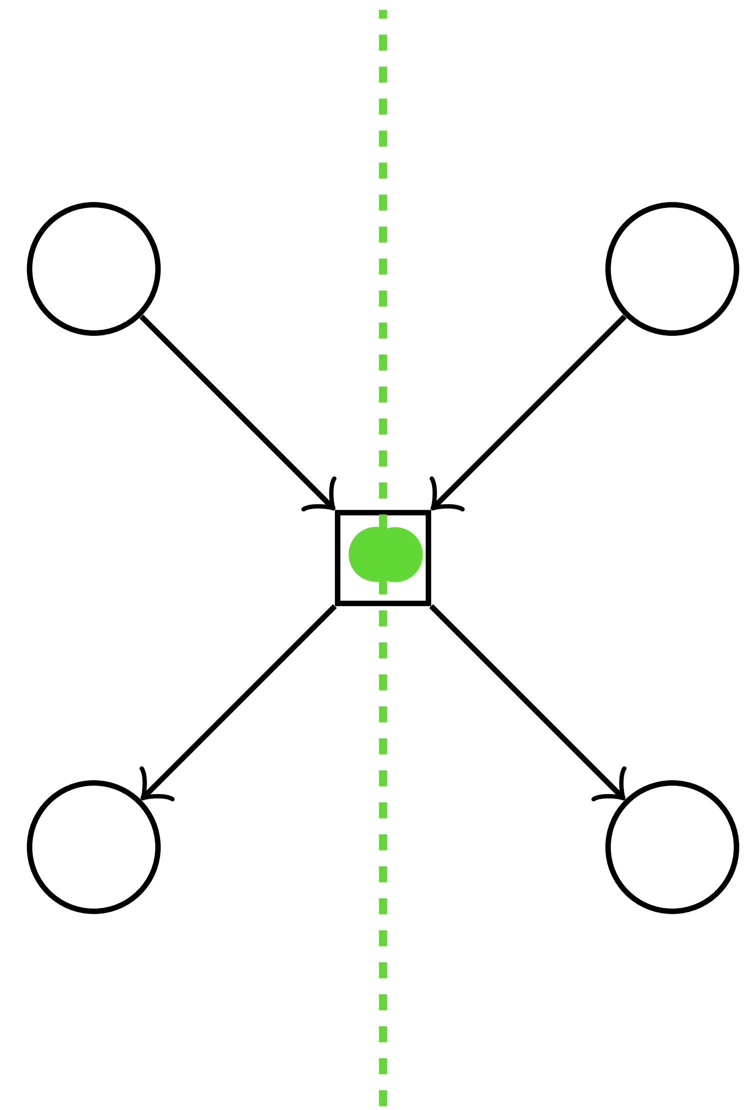

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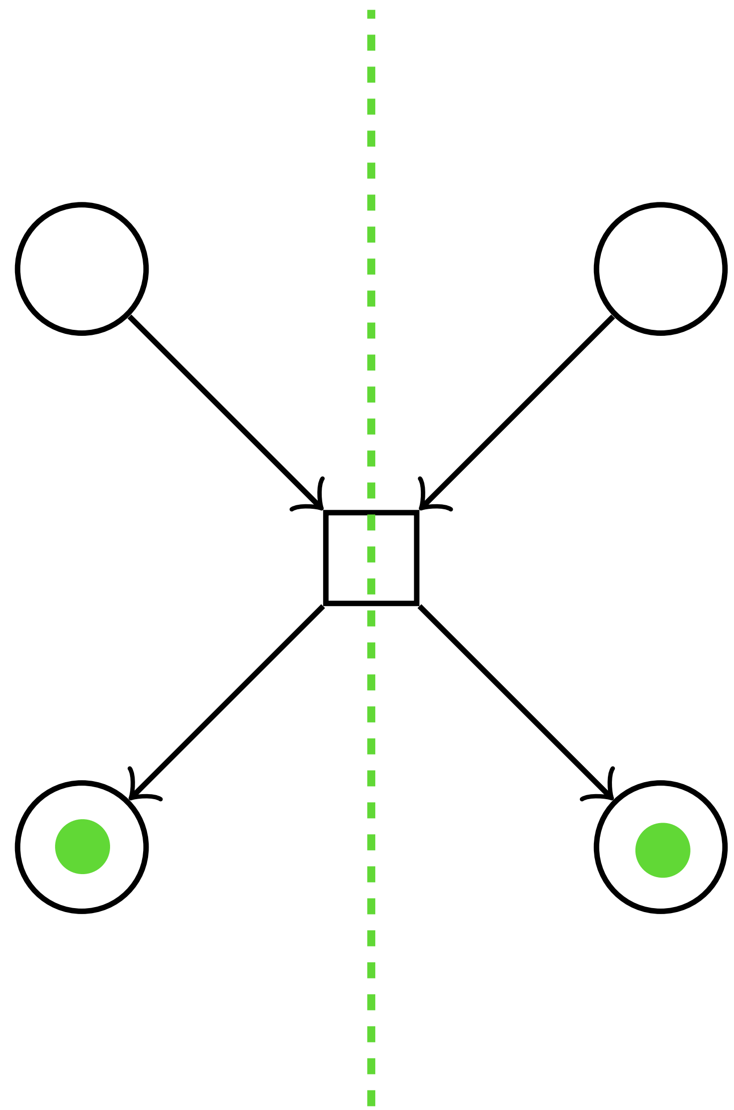

Verifying correctness - results

- Verifying whether a protocol is correct is TOWER-hard for general population protocols. *[Esparza, Ganty, Leroux, Majumdar, '15]*

- We investigate the correctness problem for two subclasses: **immediate observation** and **delayed observation** population protocols.

PSPACE-complete $\qquad$ Π_2^p-complete

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Immediate Observation Population Protocols

[Angluin, Aspnes, Eisenstat, Ruppert, '07]

- Subclass introduced to model one-way communication.

- An agent *observes* another agent's state and *immediately* updates its own based on this information.

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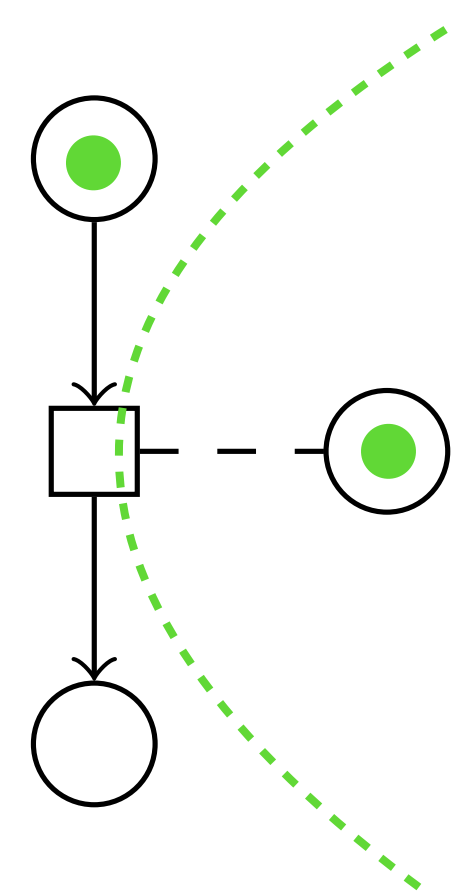

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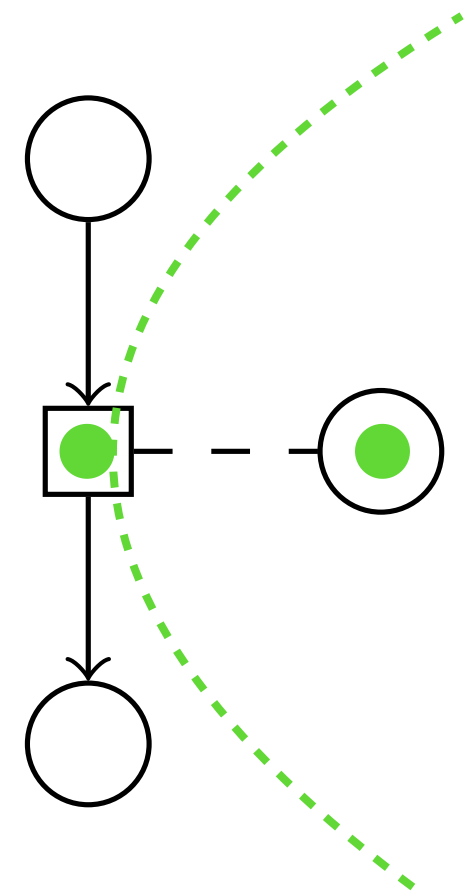

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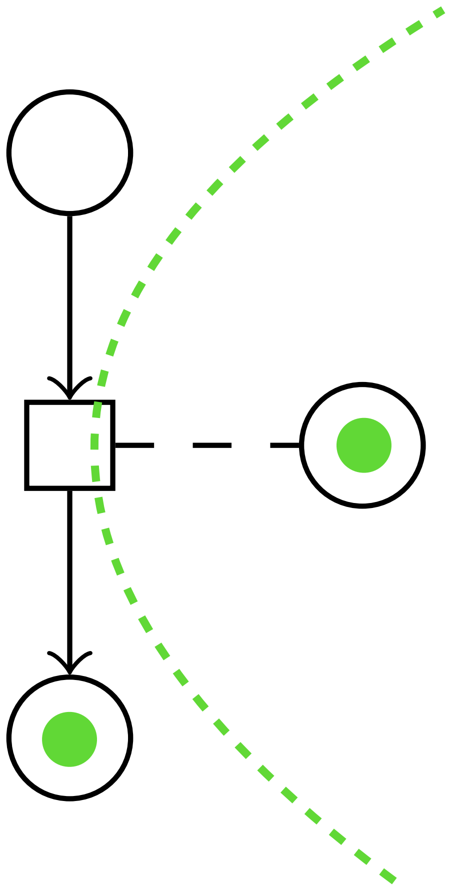
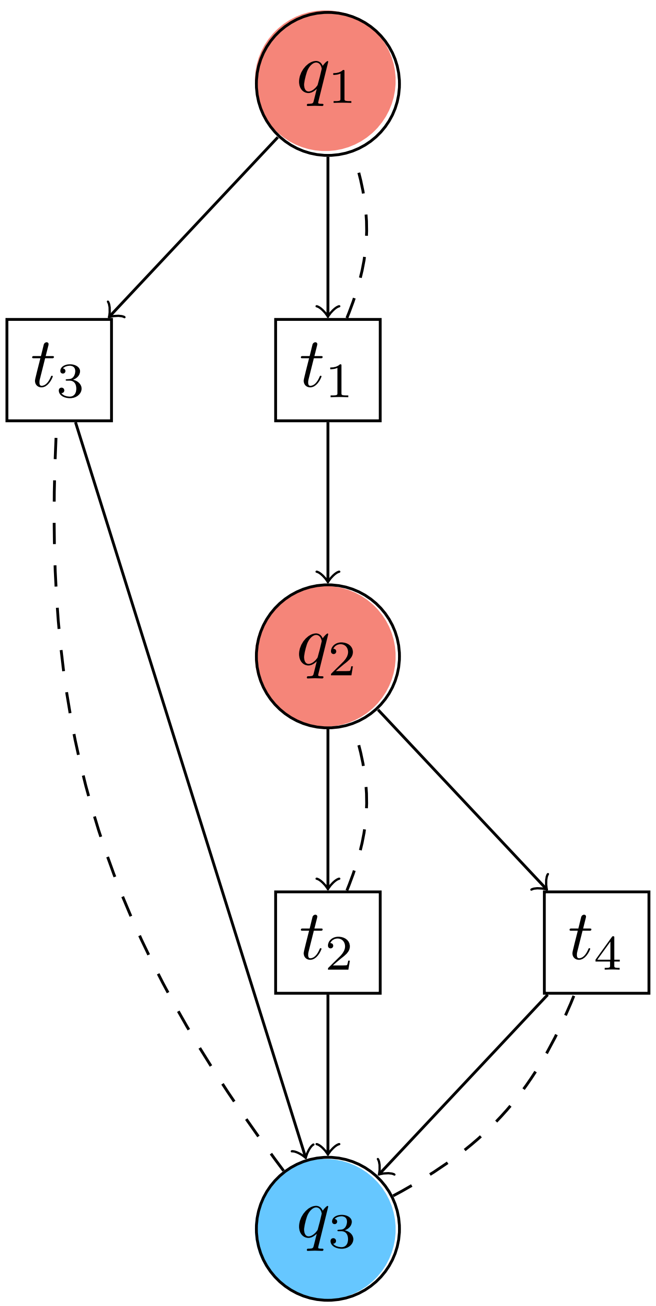


Approach to the PSPACE algorithm

- **Express correctness** as a boolean formula over sets of agent configurations with *pre** and *post**
- Find a **good representation** for sets of configurations
- Show that we only need to verify the formula for **small configurations**

Correctness - an example



Protocol for $(n,0,0)$
such that $n \geq 3$

- Goal of a protocol: compute a function $f : \mathbb{N}^k \rightarrow \{$ true, false $\}$
- Configurations are number of agents in each state
- Correctness: for every initial configuration C_0, the protocol "computes" $f(C_0)$

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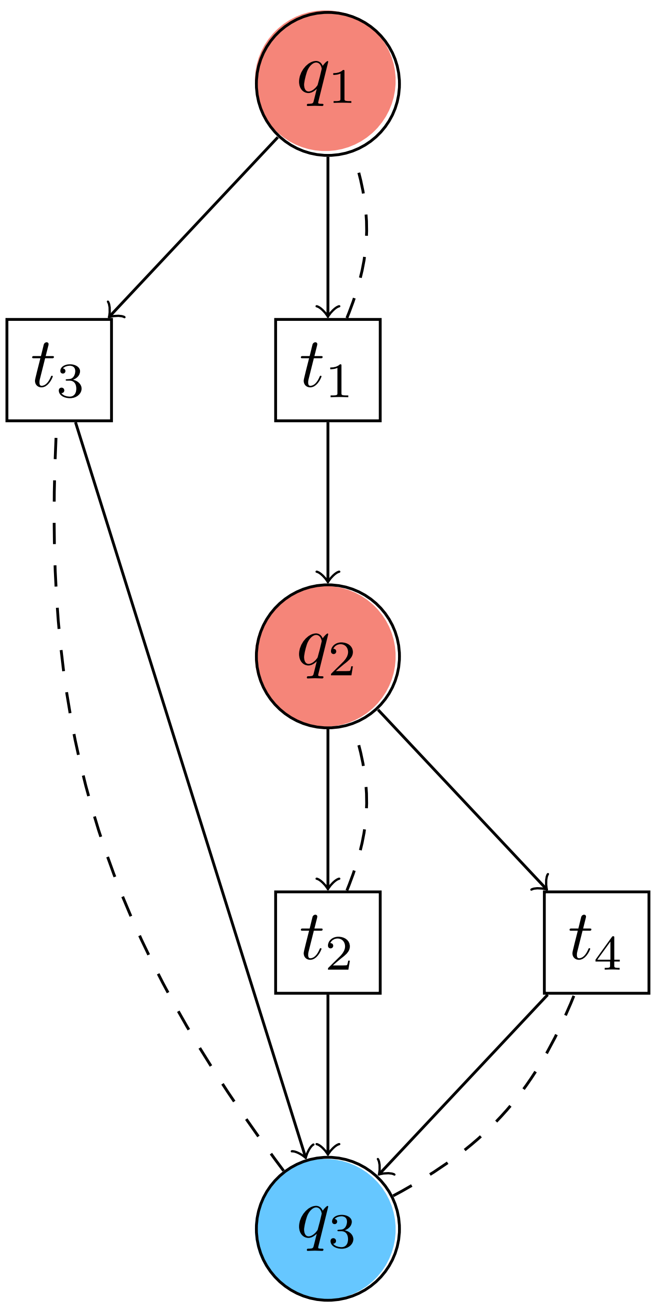

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Initial configurations: $C_0^{(n)} = (\ n,\ 0,\ 0\ )$

This protocol is correct if and only if for every initial configuration $C_0^{(n)}$:

- $n \geq 3 \Rightarrow$ all configurations reachable from $C_0^{(n)}$ can reach the configuration with all agents in q_3.
- $n < 3 \Rightarrow$ there is no reachable configuration with an agent in q_3.

A good representation - counting constraints

We consider infinite sets of configurations defined by counting constraints.

number of agents in q_2

lower bound

upper bound

- An expression $2 \leq x_2 \leq 5$ is an *atomic bound.*

A good representation - counting constraints

We consider infinite sets of configurations defined by counting constraints.

e.g. in a protocol with two states q_1 and q_2

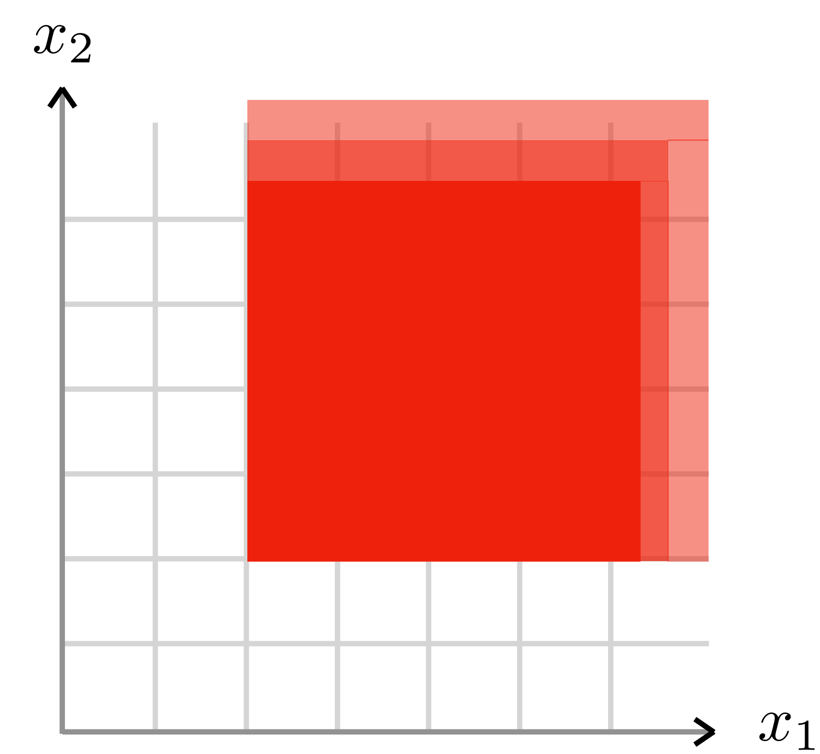
number of agents in q_2

lower bound

upper bound

- An expression $2 \leq x_2 \leq 5$ is an *atomic bound.*

- *Counting constraints* are boolean combinations of atomic bounds.



$$2 \leq x_1 \leq \infty \wedge$$
$$2 \leq x_2 \leq \infty$$

Main Theorem

Theorem

For *P* an IO protocol with *n* states, for Γ a counting constraint describing a set *S*,

1. there exist counting constraints for *pre*(S)* and *post*(S)*
2. the size of these counting constraints is $\leq size(\Gamma) + n^3$

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For P an IO protocol with n states, for Γ a counting constraint describing a set S,

1. there exist counting constraints for *pre*(S)* and *post*(S)*
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essentially the largest number of agents in a minimal configuration

Applying the Main Theorem

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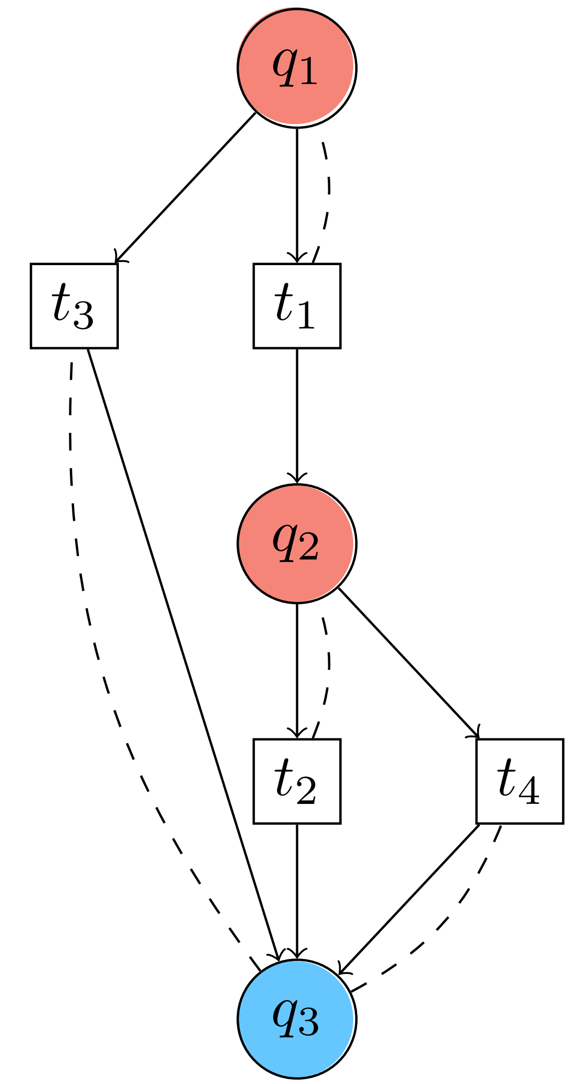

$$post^*\begin{pmatrix} 3 \leq q_1 \leq \infty \wedge \\ 0 \leq q_2 \leq 0 \wedge \\ 0 \leq q_3 \leq 0 \end{pmatrix} \subseteq pre^*\begin{pmatrix} 0 \leq q_1 \leq 0 \wedge \\ 0 \leq q_2 \leq 0 \wedge \\ 3 \leq q_3 \leq \infty \end{pmatrix}$$

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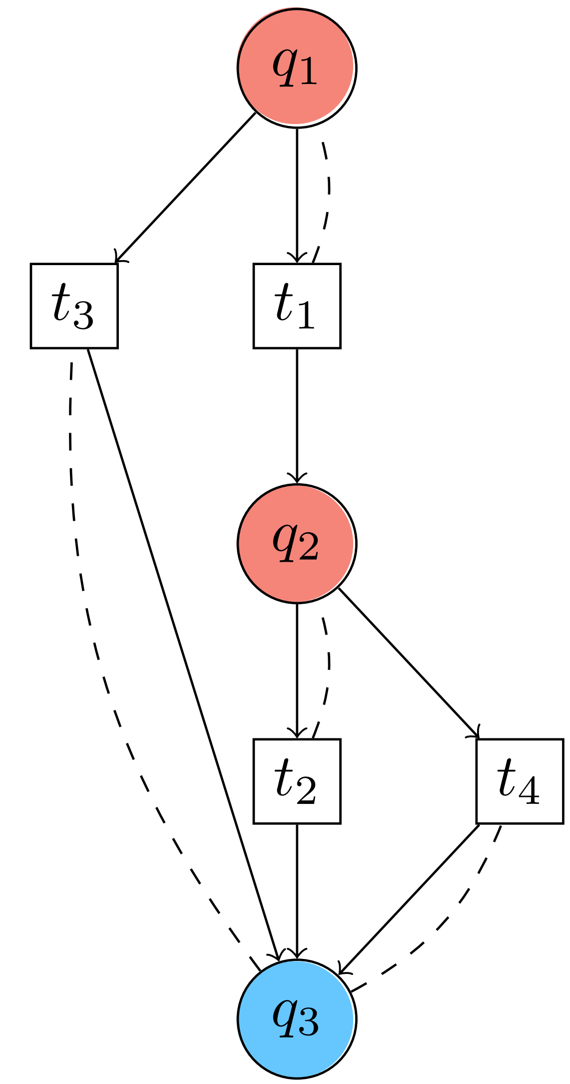

$$post^*\begin{pmatrix} 3 \leq q_1 \leq \infty \wedge \\ 0 \leq q_2 \leq 0 \wedge \\ 0 \leq q_3 \leq 0 \end{pmatrix} \quad \cap \quad \overline{pre^*\begin{pmatrix} 0 \leq q_1 \leq 0 \wedge \\ 0 \leq q_2 \leq 0 \wedge \\ 3 \leq q_3 \leq \infty \end{pmatrix}} \quad = \quad \emptyset$$

By the Main Theorem, if the intersection is not empty then it contains a "small" configuration

Conclusion

- We solved the **correctness** problem for subclasses of population protocols: **immediate observation** and **delayed observation**.

PSPACE-complete $\qquad\qquad$ Π_2^p-complete

- **Future work**: solve the correctness problem for the remaining three subclasses introduced in seminal paper of Angluin et al.

[The Computational Power of Population Protocols, Angluin et al.*, '07]*

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Thank you !