FoSSaCS 2022

Parameterized Analysis of Reconfigurable Broadcast Networks

A. R. Balasubramanian, Lucie Guillou, <u>Chana Weil-Kennedy</u>

Technical University of Munich and *ENS Rennes*

European Research Council
Established by the European Commission

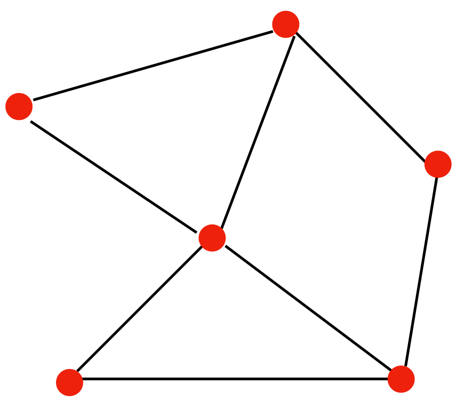
The project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 787367

Broadcast Networks

Broadcast communication used in networks of

identical, **finite-state** nodes that run the same protocol

as in cache coherence protocols, or communication protocols on ad-hoc networks

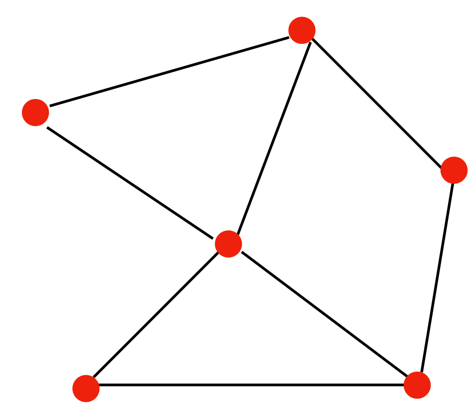


Broadcast Networks

Broadcast communication used in networks of

identical, **finite-state** nodes that run the same protocol

as in cache coherence protocols, or communication protocols on ad-hoc networks



Broadcast network = shared broadcast protocol + communication topology

finite automaton with transitions labeled
by sends or receives

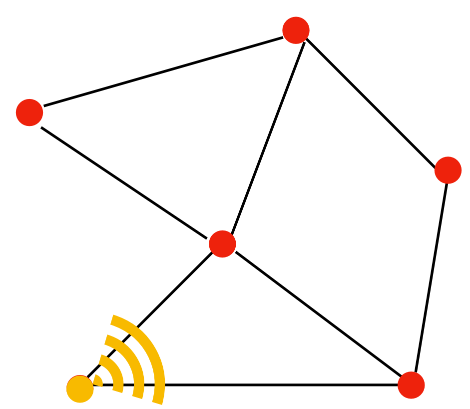
agents send to their neighbors

Broadcast Networks

Broadcast communication used in networks of

identical, **finite-state** nodes that run the same protocol

as in cache coherence protocols, or communication protocols on ad-hoc networks



Broadcast network = shared broadcast protocol + communication topology

finite automaton with transitions labeled by sends or receives

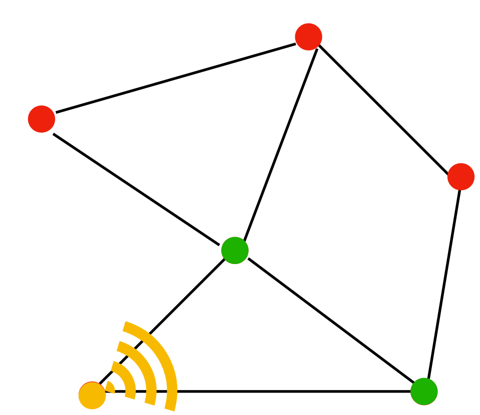
agents send to their neighbors

Broadcast Networks

Broadcast communication used in networks of

identical, **finite-state** nodes that run the same protocol

as in cache coherence protocols, or communication protocols on ad-hoc networks



Broadcast network = shared broadcast protocol + communication topology

finite automaton with transitions labeled by sends or receives

agents send to their neighbors

Broadcast Networks

- **Parameterized** systems: properties should hold independently of number of agents

  E.g.: Control state reachability - does there exist an initial configuration which can reach a configuration putting an agent in the control state?

Broadcast Networks

- **Parameterized** systems: properties should hold independently of number of agents

  E.g.: Control state reachability - does there exist an initial configuration which can reach a configuration putting an agent in the control state? **Undecidable**

  [Delzanno,Sangnier,Zavattaro,CONCUR'10]

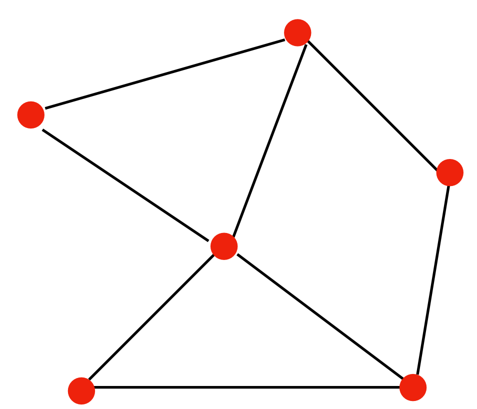
Broadcast Networks

- **Parameterized** systems: properties should hold independently of number of agents

E.g.: Control state reachability - does there exist an initial configuration which can reach a configuration putting an agent in the control state? **Undecidable**

[Delzanno,Sangnier,Zavattaro,CONCUR'10]

introduce **reconfigurable** broadcast networks



communication topology can change

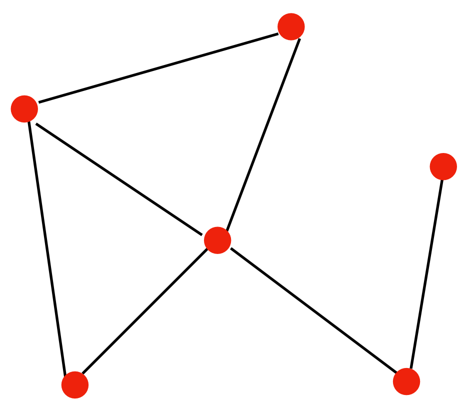
Broadcast Networks

- **Parameterized** systems: properties should hold independently of number of agents

E.g.: Control state reachability - does there exist an initial configuration which can reach a configuration putting an agent in the control state? **Undecidable**

[Delzanno,Sangnier,Zavattaro,CONCUR'10]

introduce **reconfigurable** broadcast networks



communication topology can change

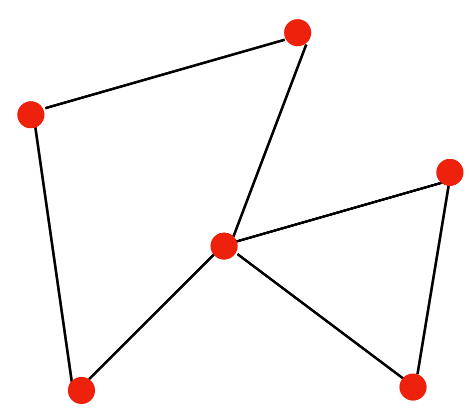
Broadcast Networks

- **Parameterized** systems: properties should hold independently of number of agents

E.g.: Control state reachability - does there exist an initial configuration which can reach a configuration putting an agent in the control state? **Undecidable**

[Delzanno,Sangnier,Zavattaro,CONCUR'10]

introduce **reconfigurable** broadcast networks



communication topology can change

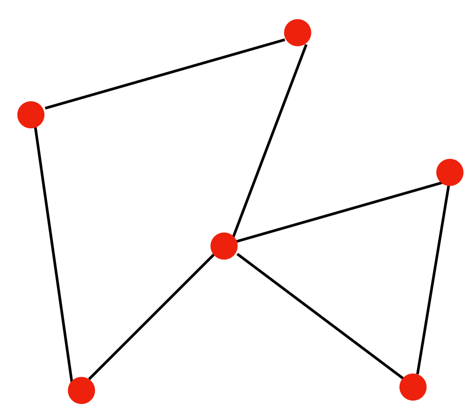
Broadcast Networks

- **Parameterized** systems: properties should hold independently of number of agents

E.g.: Control state reachability - does there exist an initial configuration which can reach a configuration putting an agent in the control state? **Undecidable**

[Delzanno,Sangnier,Zavattaro,CONCUR'10]

introduce **reconfigurable** broadcast networks



communication topology can change

- Models channel reconfiguration / message loss / adversary picking the receiving nodes

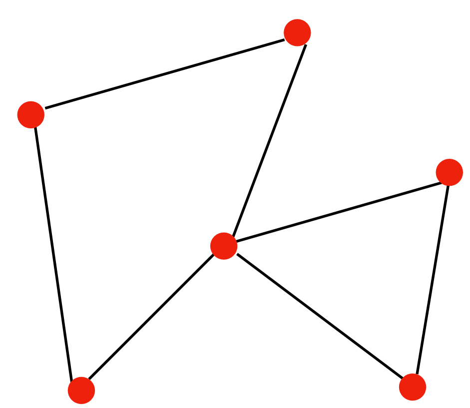
Broadcast Networks

- **Parameterized** systems: properties should hold independently of number of agents

E.g.: Control state reachability - does there exist an initial configuration which can reach a configuration putting an agent in the control state? **Undecidable**

[Delzanno,Sangnier,Zavattaro,CONCUR'10]

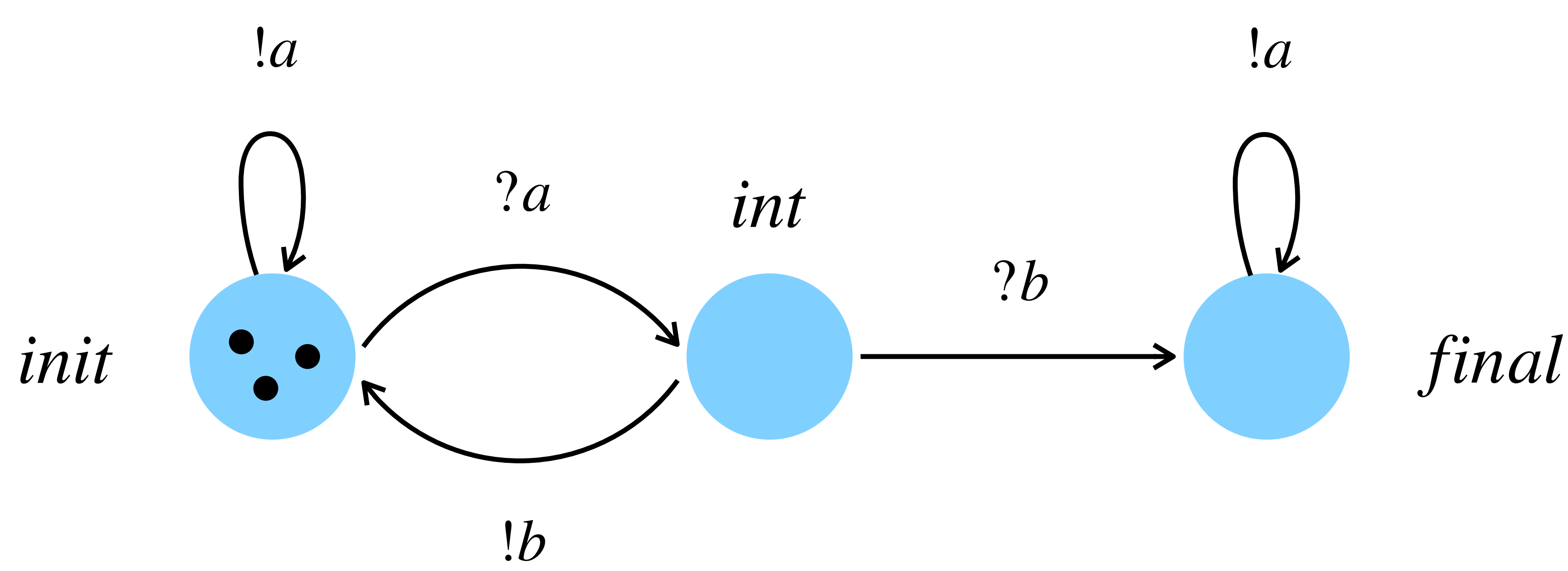
introduce **reconfigurable** broadcast networks



communication topology can change

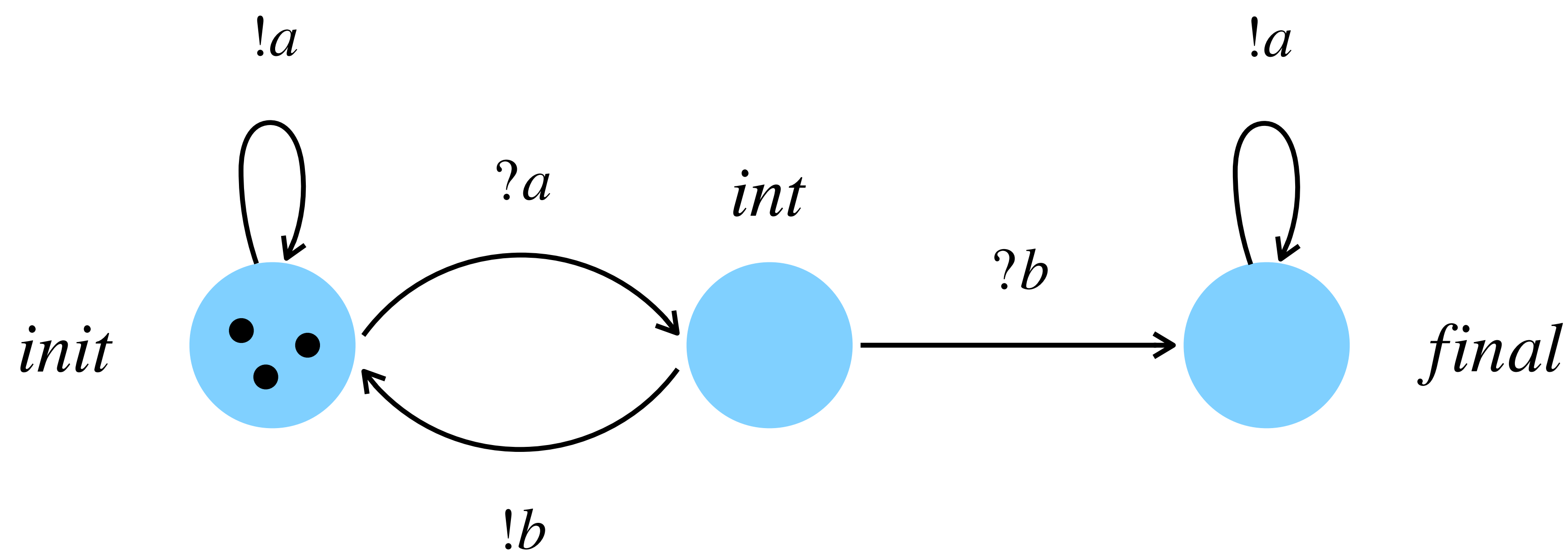
- Models channel reconfiguration / message loss / adversary picking the receiving nodes
- Control state reachability solvable in **P-TIME** [Delzanno et al, FSTTCS'12]

Reconfigurable Broadcast Networks



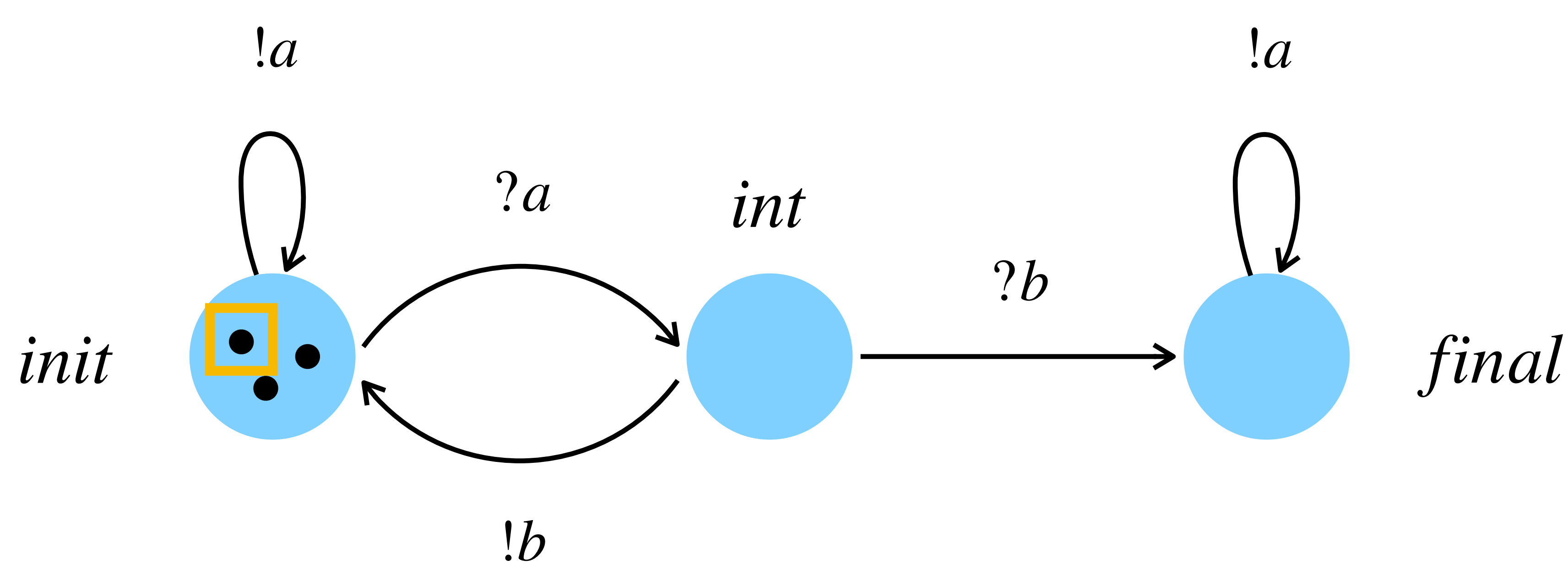
Reconfigurable Broadcast Networks

Goal: put an agent in $final$



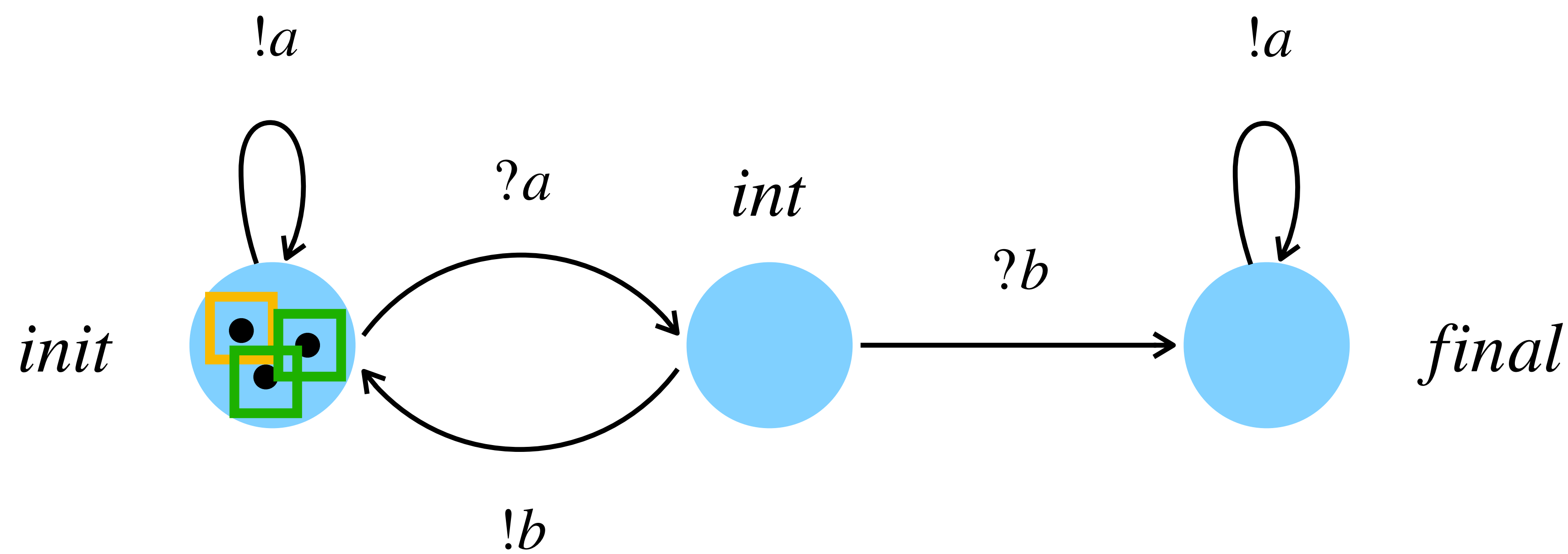
Reconfigurable Broadcast Networks

Goal: put an agent in *final*



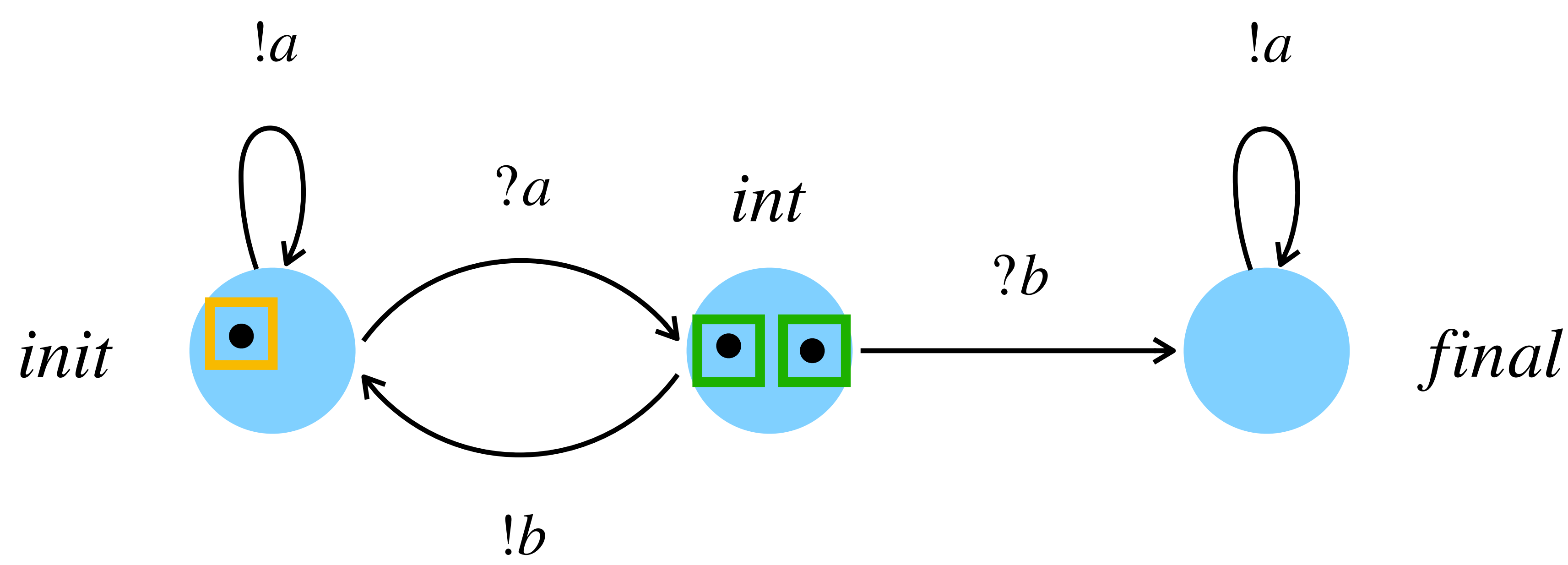
Reconfigurable Broadcast Networks

Goal: put an agent in *final*



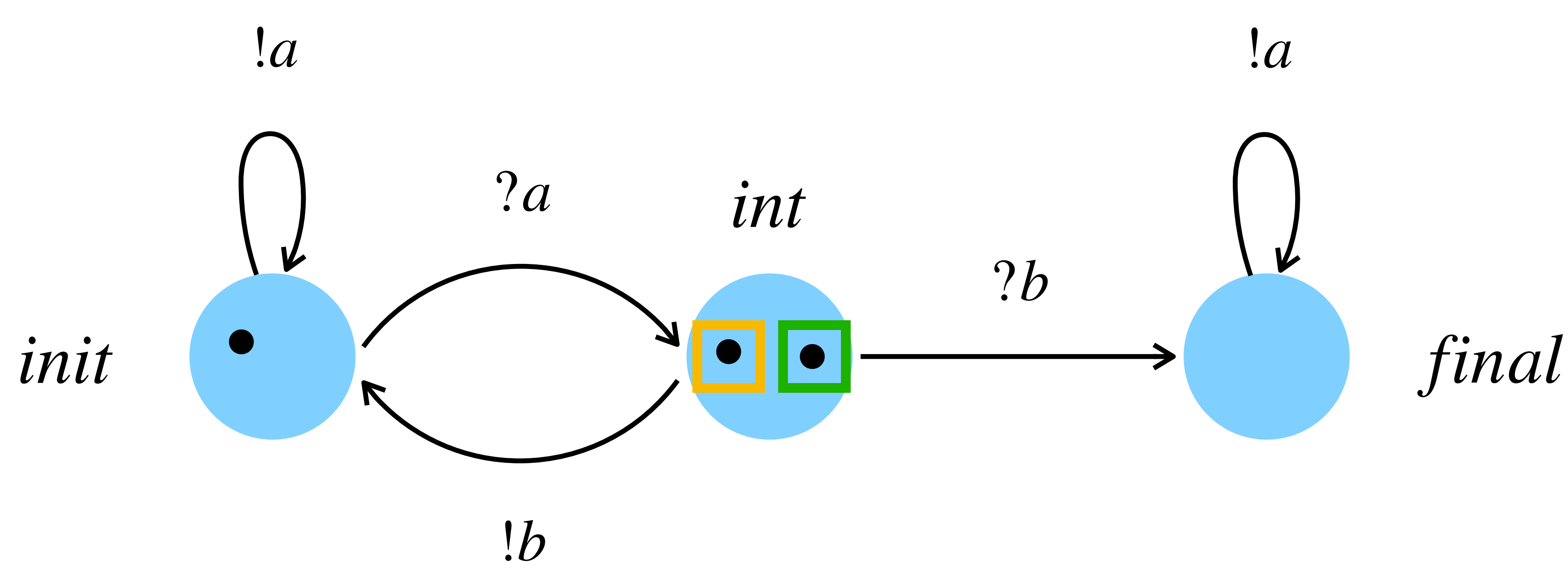
Reconfigurable Broadcast Networks

Goal: put an agent in *final*



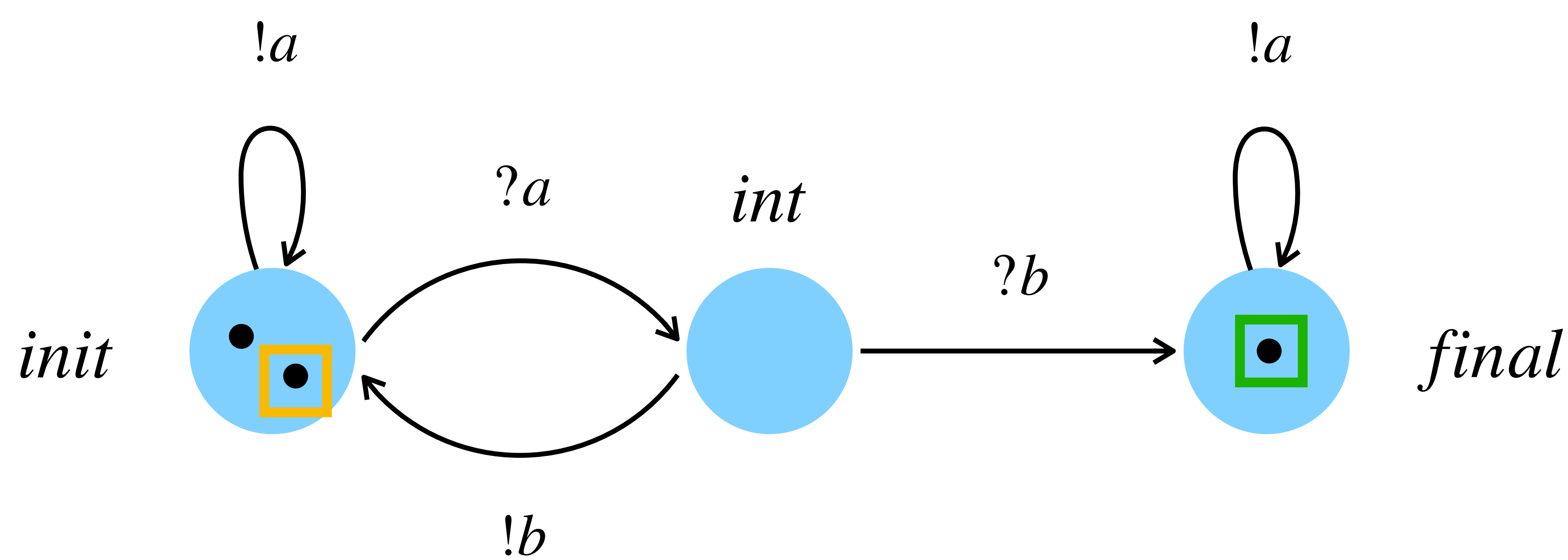
Reconfigurable Broadcast Networks

Goal: put an agent in $final$



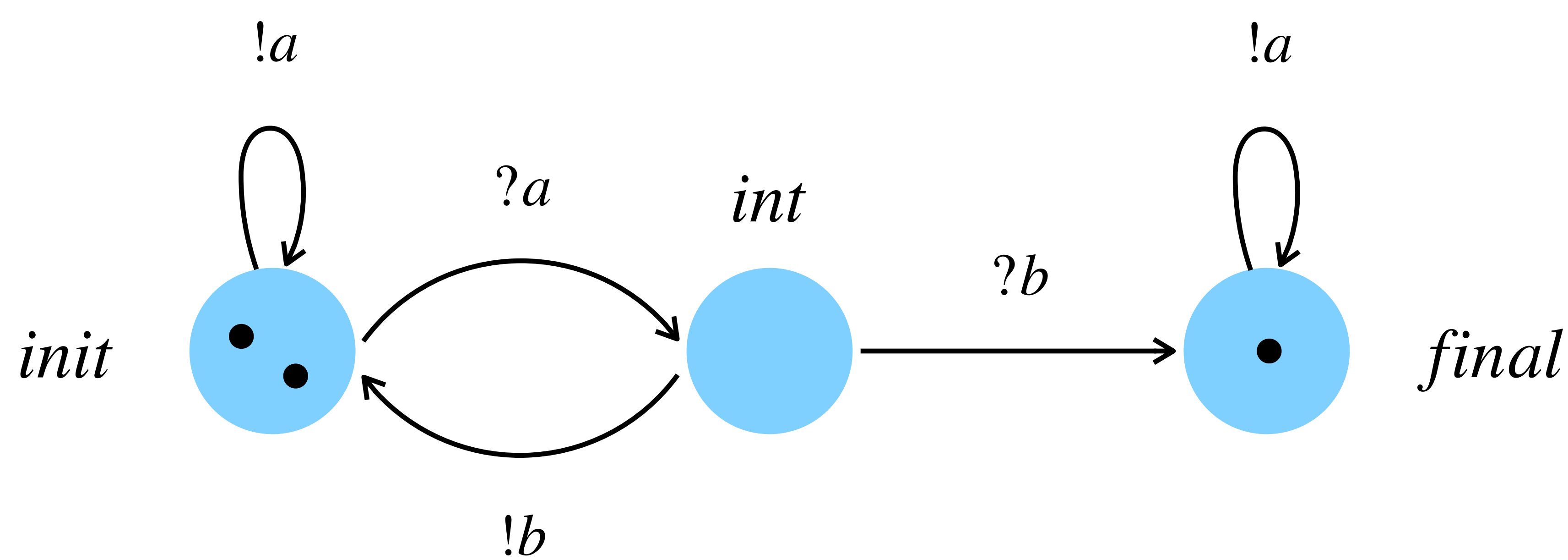
Reconfigurable Broadcast Networks

Goal: put an agent in final ✓



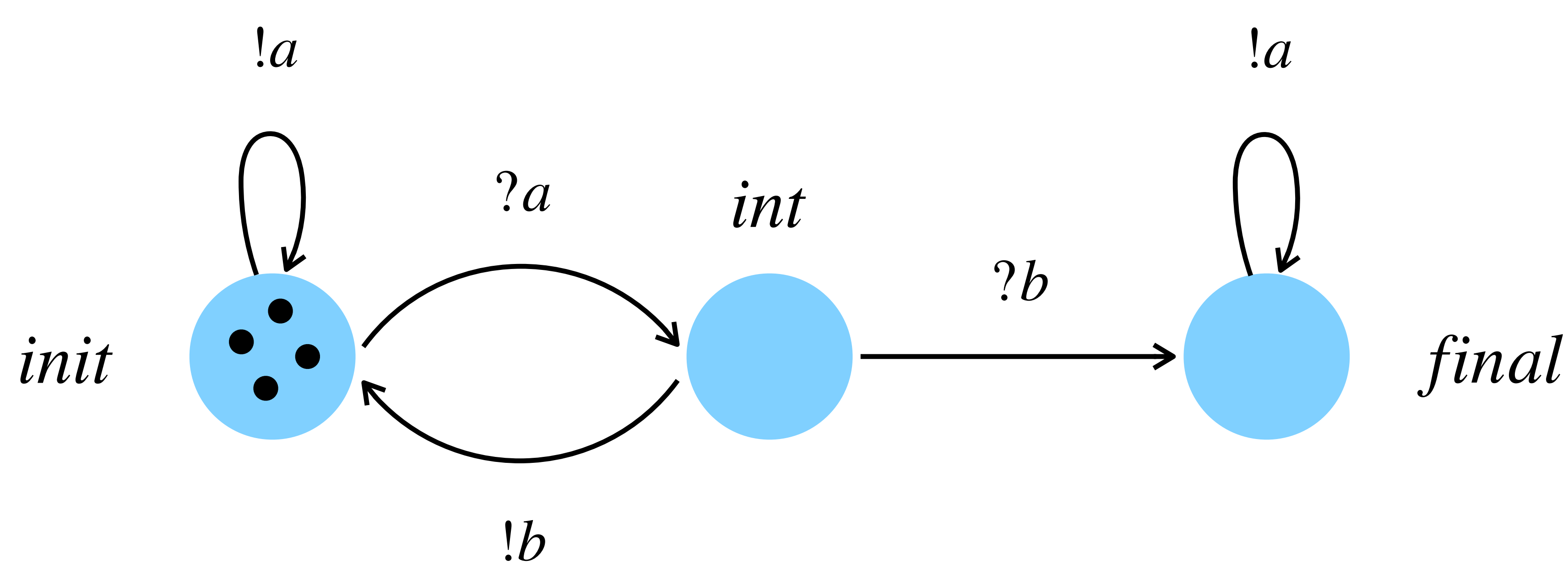
Reconfigurable Broadcast Networks

Goal: put an agent in final ✓

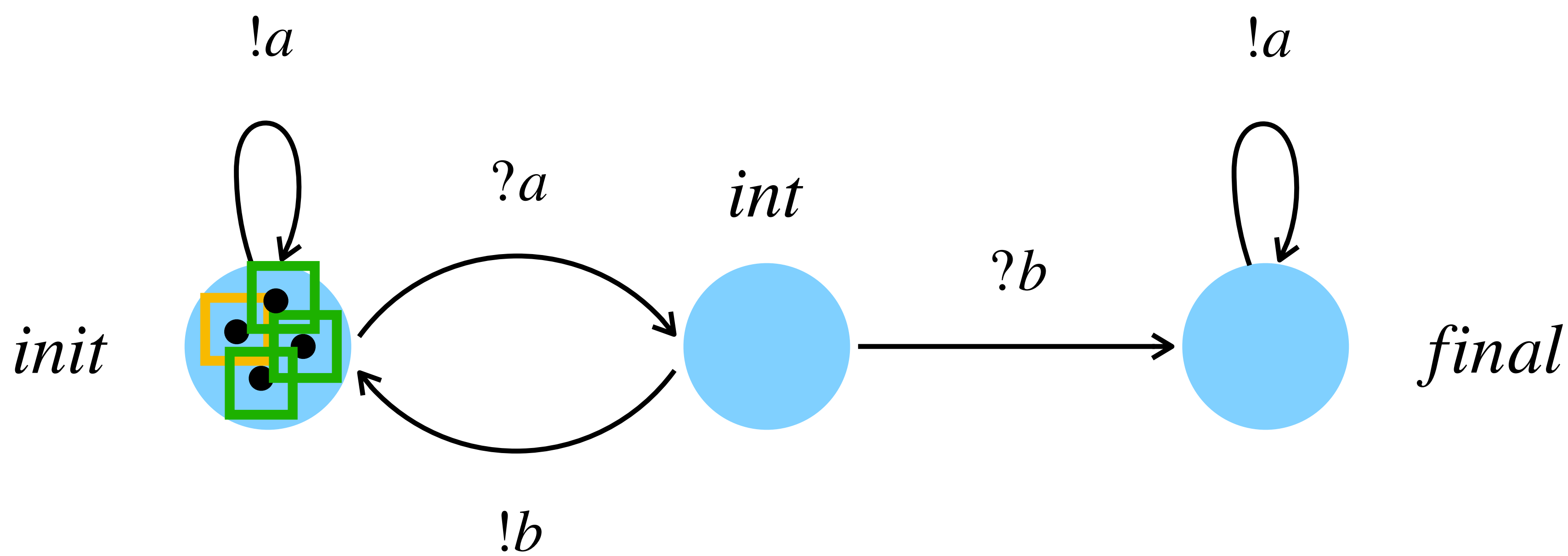


- agents communicate by selective broadcast
- broadcast and receives happen at the same time
- multiple receives happen simultaneously

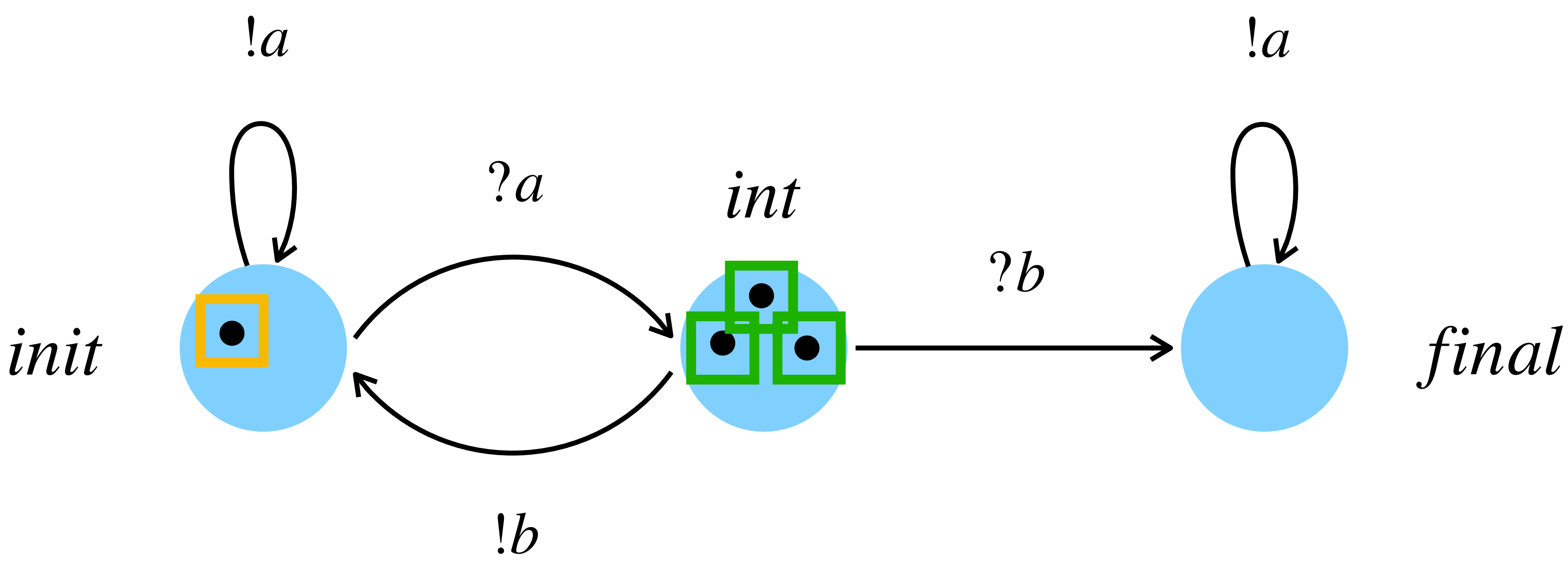
Reconfigurable Broadcast Networks



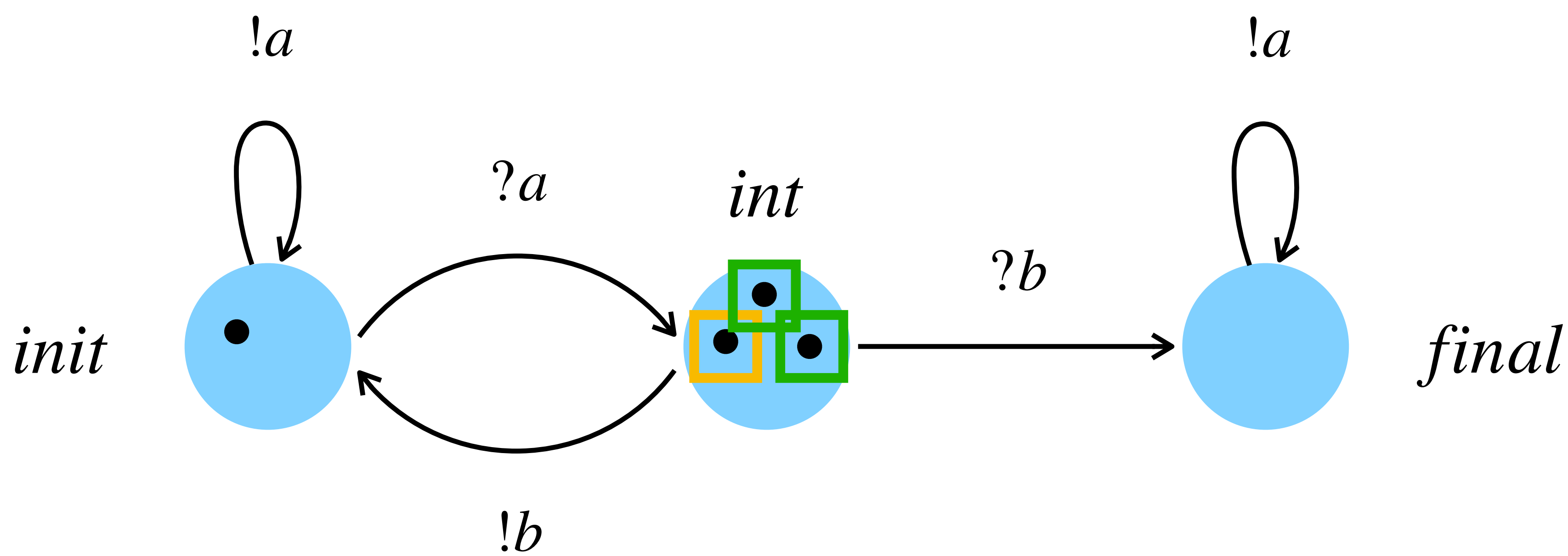
Reconfigurable Broadcast Networks



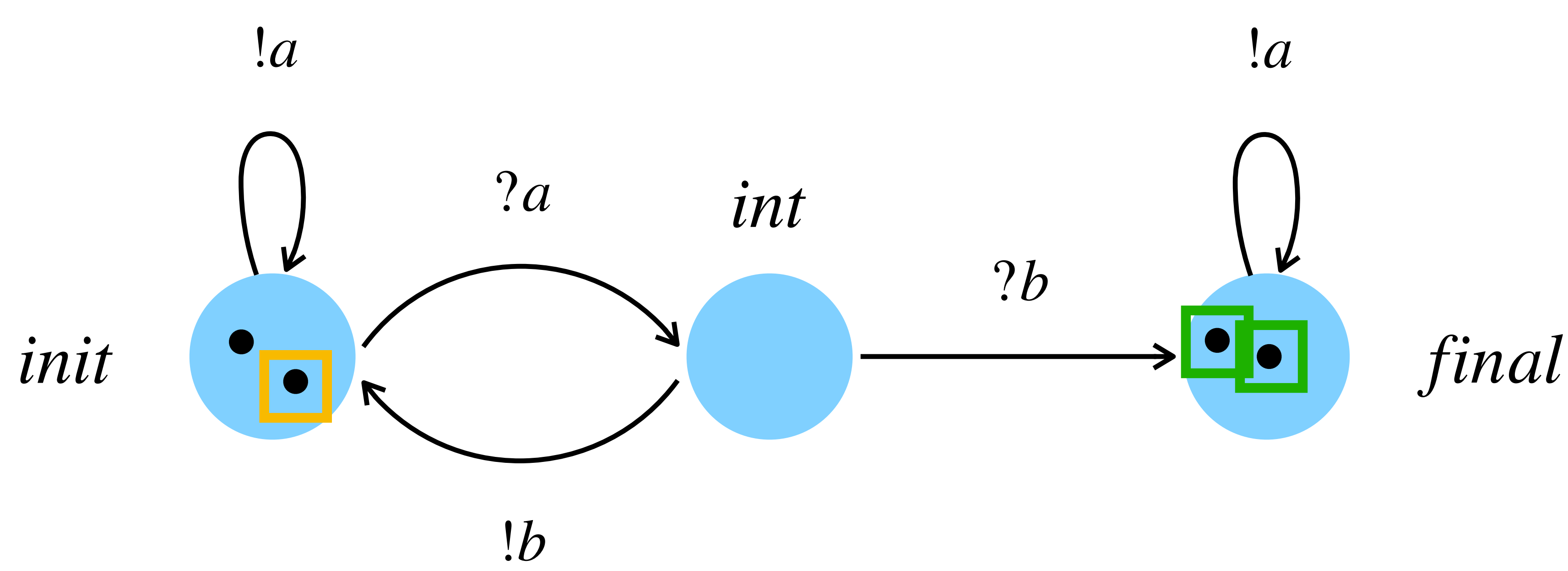
Reconfigurable Broadcast Networks



Reconfigurable Broadcast Networks



Reconfigurable Broadcast Networks



Parameterized problems

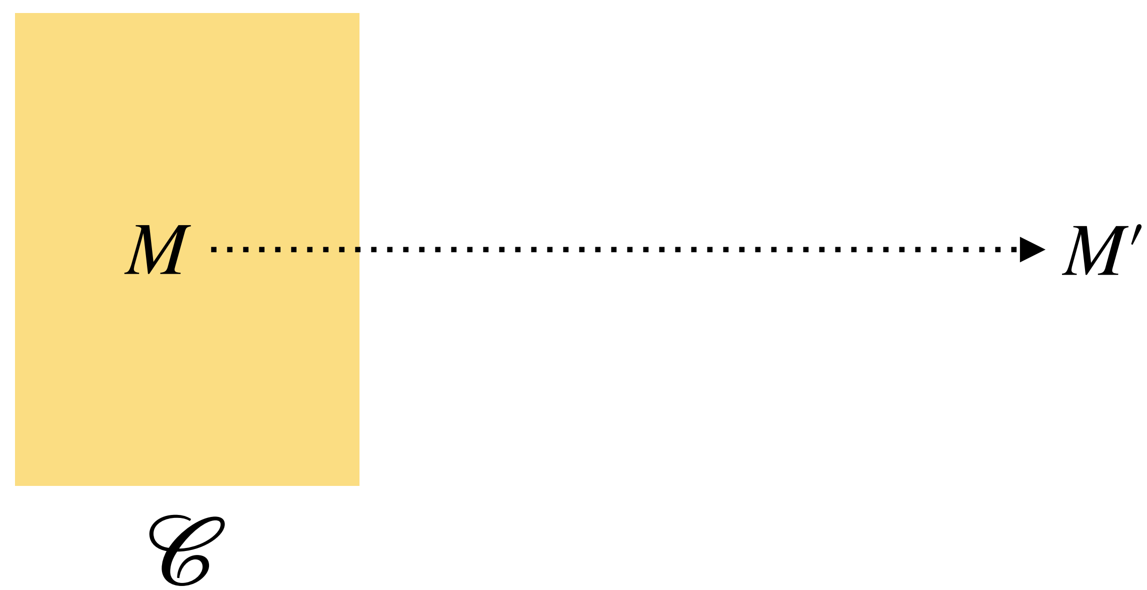
number of agents in q

A **cube** is a boolean combination of constraints $\underset{\in \mathbb{N}}{a} \leq \#q \leq \underset{\in \mathbb{N} \cup \infty}{b}$

Parameterized problems

number of agents in q

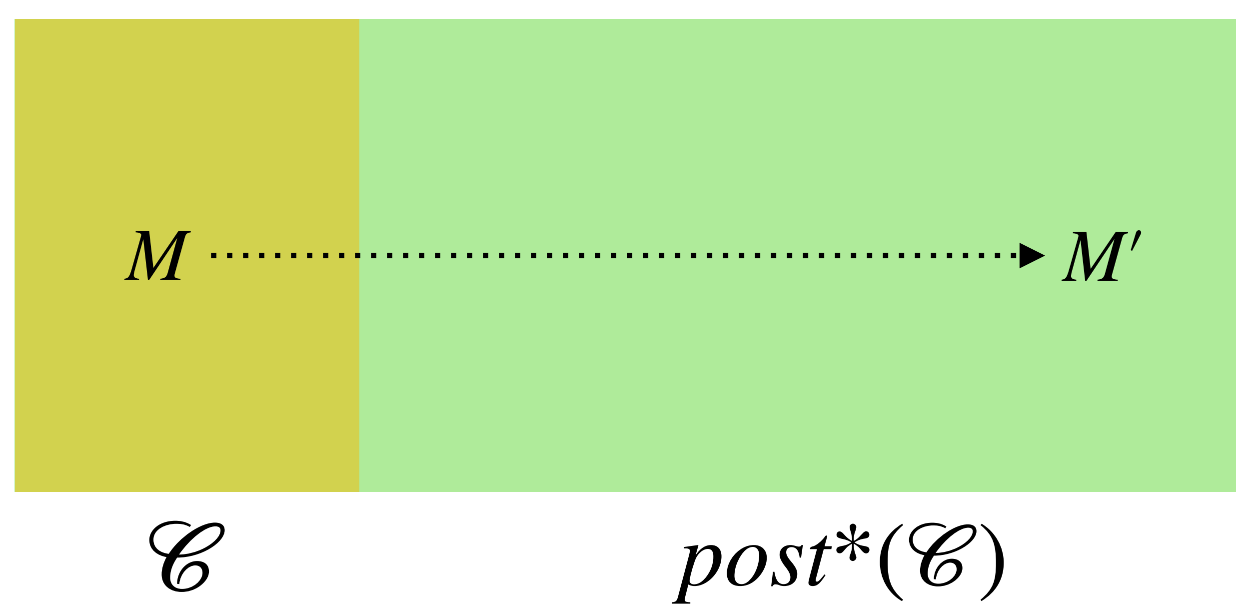
A **cube** is a boolean combination of constraints $\underset{\in \mathbb{N}}{a} \leq \#q \leq \underset{\in \mathbb{N} \cup \infty}{b}$



Parameterized problems

number of agents in q

A **cube** is a boolean combination of constraints $\underset{\in \mathbb{N}}{a} \leq \#q \leq \underset{\in \mathbb{N} \cup \infty}{b}$

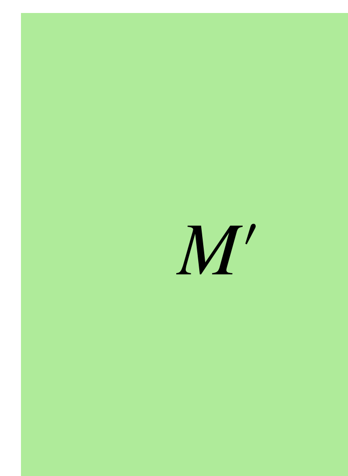


Parameterized problems

number of agents in q

A **cube** is a boolean combination of constraints $\quad a \leq \#q \leq b$

$\in \mathbb{N} \qquad \in \mathbb{N} \cup \infty$



$\mathscr{C}'$

Parameterized problems

A **cube** is a boolean combination of constraints $\underset{\in \mathbb{N}}{a} \leq \overset{\text{number of agents in } q}{\#q} \leq \underset{\in \mathbb{N} \cup \infty}{b}$

$pre^*(\mathscr{C}')$ $\mathscr{C}'$

Parameterized problems

A **cube** is a boolean combination of constraints $\underset{\in \mathbb{N}}{a} \leq \#q \leq \underset{\in \mathbb{N} \cup \infty}{b}$

parameterized problems: emptiness and membership of sets described by boolean operators, pre^* and $post^*$ over cubes

Parameterized problems

A **cube** is a boolean combination of constraints $\underset{\in \mathbb{N}}{a} \leq \#q \leq \underset{\in \mathbb{N} \cup \infty}{b}$

parameterized problems: emptiness and membership of sets described by boolean operators, pre^* and $post^*$ over cubes

- control state reachability from cube $\mathscr{C}_{init}$ to state $final$:

Parameterized problems

A **cube** is a boolean combination of constraints $\underset{\in \mathbb{N}}{a} \leq \#q \leq \underset{\in \mathbb{N} \cup \infty}{b}$

parameterized problems: emptiness and membership of sets described by boolean operators, pre^* and $post^*$ over cubes

- control state reachability from cube $\mathscr{C}_{init}$ to state $final$:

$$post^*(\mathscr{C}_{init}) \cap (1 \leq \#final) \neq \emptyset$$

Parameterized problems

A **cube** is a boolean combination of constraints $\underset{\in \mathbb{N}}{a} \leq \#q \leq \underset{\in \mathbb{N} \cup \infty}{b}$

parameterized problems: emptiness and membership of sets described by boolean operators, pre^* and $post^*$ over cubes

- control state reachability from cube $\mathscr{C}_{init}$ to state $final$:

$$post^*(\mathscr{C}_{init}) \cap (1 \leq \#final) \neq \emptyset$$

- reachability from cube $\mathscr{C}$ to cube $\mathscr{C}'$: $post^*(\mathscr{C}) \cap \mathscr{C}' \neq \emptyset$

Parameterized problems

A **cube** is a boolean combination of constraints $\underset{\in \mathbb{N}}{a} \leq \#q \leq \underset{\in \mathbb{N} \cup \infty}{b}$

parameterized problems: emptiness and membership of sets described by boolean operators, $pre*$ and $post*$ over cubes

- control state reachability from cube $\mathscr{C}_{init}$ to state $final$:

$$post*(\mathscr{C}_{init}) \cap (1 \leq \#final) \neq \varnothing$$

- reachability from cube $\mathscr{C}$ to cube $\mathscr{C}'$: $post*(\mathscr{C}) \cap \mathscr{C}' \neq \varnothing$

- almost-sure reachability from cube $\mathscr{C}_{init}$ to cube $\mathscr{C}_{final}$: $post*(\mathscr{C}_{init}) \subseteq pre*(\mathscr{C}_{final})$

Result

parameterized problems: emptiness and membership of sets described by boolean operators, pre^* and $post^*$ over cubes

PSPACE

Result

parameterized problems: emptiness and membership of sets described by boolean operators, pre^* and $post^*$ over cubes

PSPACE

Previous results:

[Esparza, Raskin, *W.-K.*, PN'19]

- PSPACE-hardness result for RBN reachability and configuration-coverability

Result

parameterized problems: emptiness and membership of sets described by boolean operators, pre^* and $post^*$ over cubes

PSPACE

Previous results:

[Esparza, Raskin, *W.-K.*, PN'19]

- PSPACE-hardness result for RBN reachability and configuration-coverability

- [Delzanno et al., FSTTCS'12]
  - Control-state reachability in PTIME
  - Reachability from the initial cube to any cube is PSPACE-complete

Result

parameterized problems: emptiness and membership of sets described by boolean operators, pre^* and $post^*$ over cubes

PSPACE

Previous results:

[Esparza, Raskin, *W.-K.*, PN'19]

- PSPACE-hardness result for RBN reachability and configuration-coverability

- [Delzanno et al., FSTTCS'12]
  - Control-state reachability in PTIME
  - Reachability from the initial cube to any cube is PSPACE-complete

- Almost-sure reachability is in EXPSPACE and is PSPACE-hard [Bouyer et al., ICALP'16]

PSPACE algorithm

parameterized problems: emptiness and membership of sets described by boolean operators, pre^* and $post^*$ over cubes

PSPACE

$post^*$ (or pre^*) of a cube is a "small" cube

PSPACE algorithm

parameterized problems: emptiness and membership of sets described by boolean operators, pre^* and $post^*$ over cubes

PSPACE

$post^*$ (or pre^*) of a cube is a “small” cube

➡ $post^*(\mathscr{C}) \cap \mathscr{C}'$ is small cube

PSPACE algorithm

parameterized problems: emptiness and membership of sets described by boolean operators, pre^* and $post^*$ over cubes

PSPACE

$post^*$ (or pre^*) of a cube is a "small" cube

→ $post^*(\mathscr{C}) \cap \mathscr{C}'$ is small cube

→ if non-empty, $post^*(\mathscr{C}) \cap \mathscr{C}'$ contains a small configuration

PSPACE algorithm

parameterized problems: emptiness and membership of sets described by boolean operators, pre^* and $post^*$ over cubes

PSPACE

$post^*$ (or pre^*) of a cube is a “small” cube

➡ $post^*(\mathscr{C}) \cap \mathscr{C}'$ is small cube

➡ if non-empty, $post^*(\mathscr{C}) \cap \mathscr{C}'$ contains a small configuration

Algorithm for checking if $\mathscr{C}'$ is reachable from $\mathscr{C}$

- guess two small configurations M in $\mathscr{C}$ and M’ in $\mathscr{C}'$ of same size
- guess a path from M to M’

PSPACE algorithm

parameterized problems: emptiness and membership of sets described by boolean operators, pre^* and $post^*$ over cubes

PSPACE

$post^*$ (or pre^*) of a cube is a "small" cube

$post^*(\mathscr{C}) \cap \mathscr{C}'$ is small cube

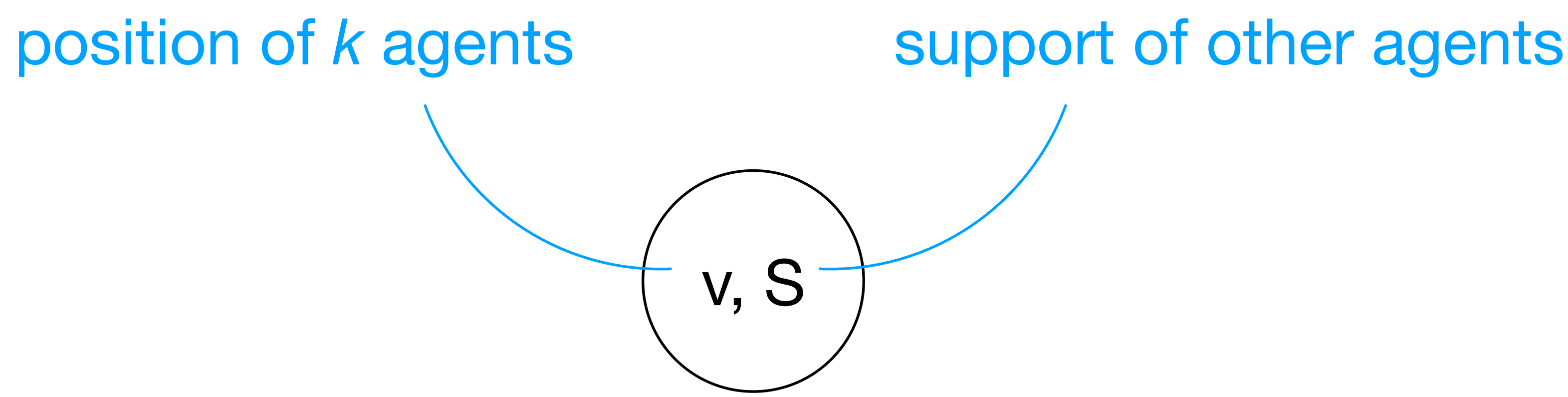
if non-empty, $post^*(\mathscr{C}) \cap \mathscr{C}'$ contains a small configuration

Algorithm for checking if $\mathscr{C}'$ is reachable from $\mathscr{C}$

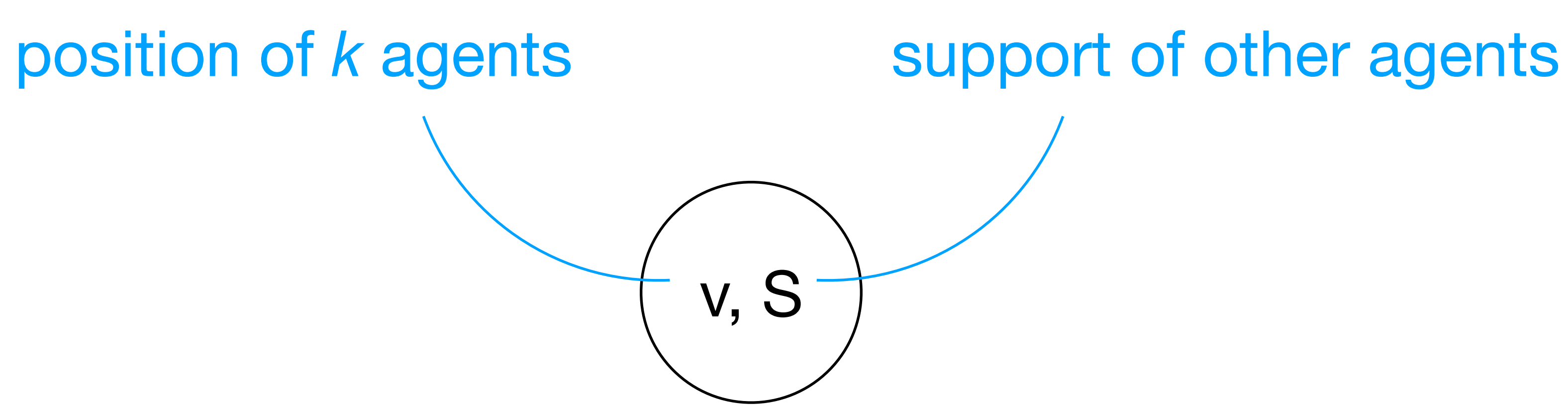
- guess two small configurations M in $\mathscr{C}$ and M' in $\mathscr{C}'$ of same size
- guess a path from M to M'

Savitch: NPSPACE = PSPACE

Symbolic graph

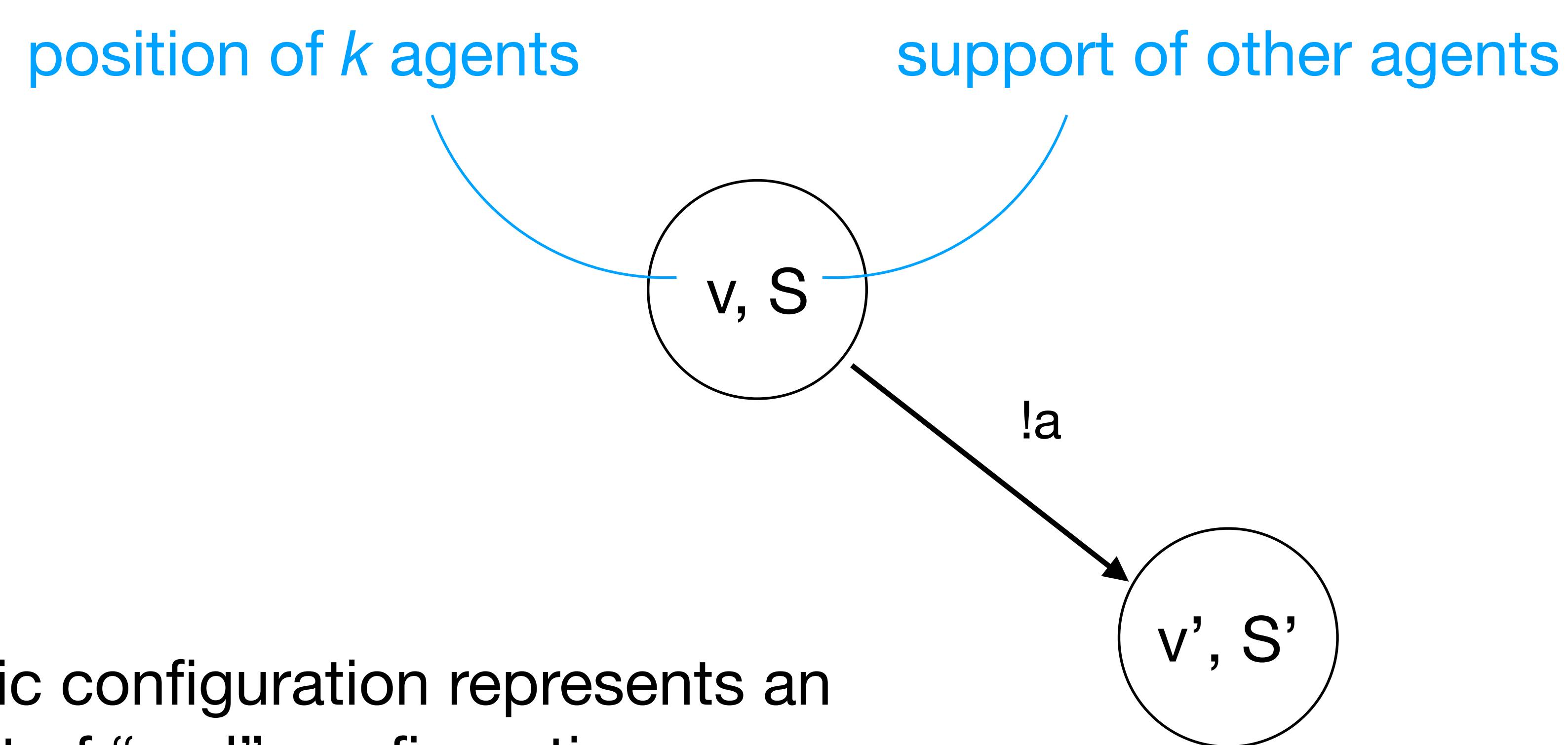


Symbolic graph



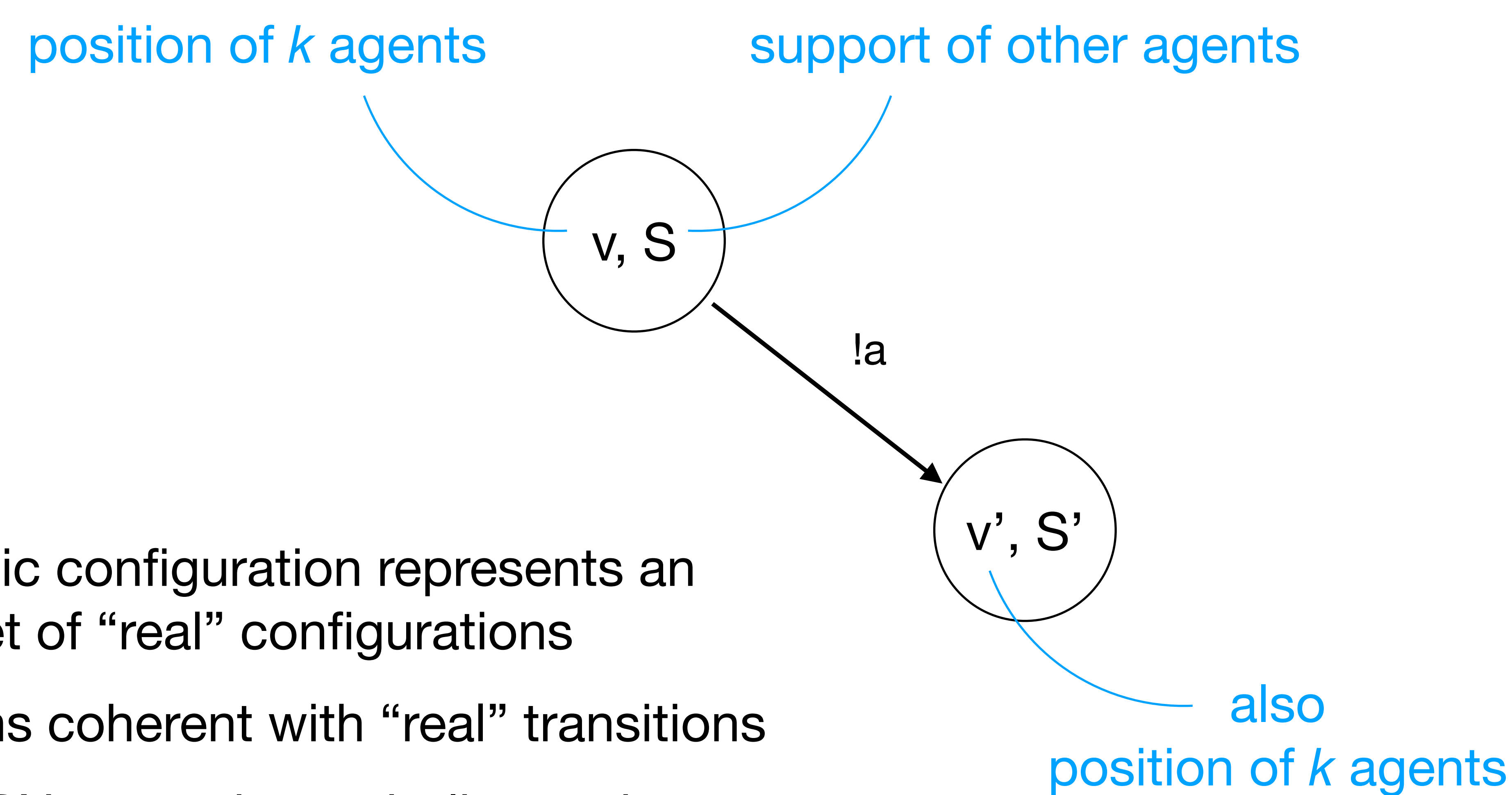
- A symbolic configuration represents an infinite set of "real" configurations

Symbolic graph



- A symbolic configuration represents an infinite set of "real" configurations
- Transitions coherent with "real" transitions
- Run in RBN ⇒ run in symbolic graph

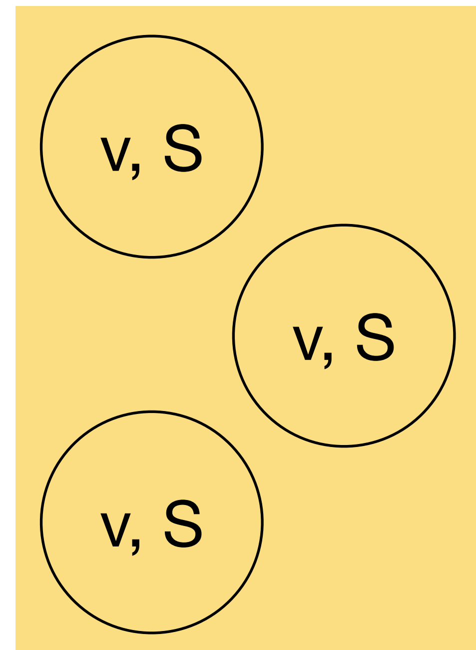
Symbolic graph



- A symbolic configuration represents an infinite set of "real" configurations
- Transitions coherent with "real" transitions
- Run in RBN ⇒ run in symbolic graph

Symbolic graph

$\mathscr{C} \longrightarrow$



$\mathscr{C}_{SG}$

$a \leq \#q \leq b$

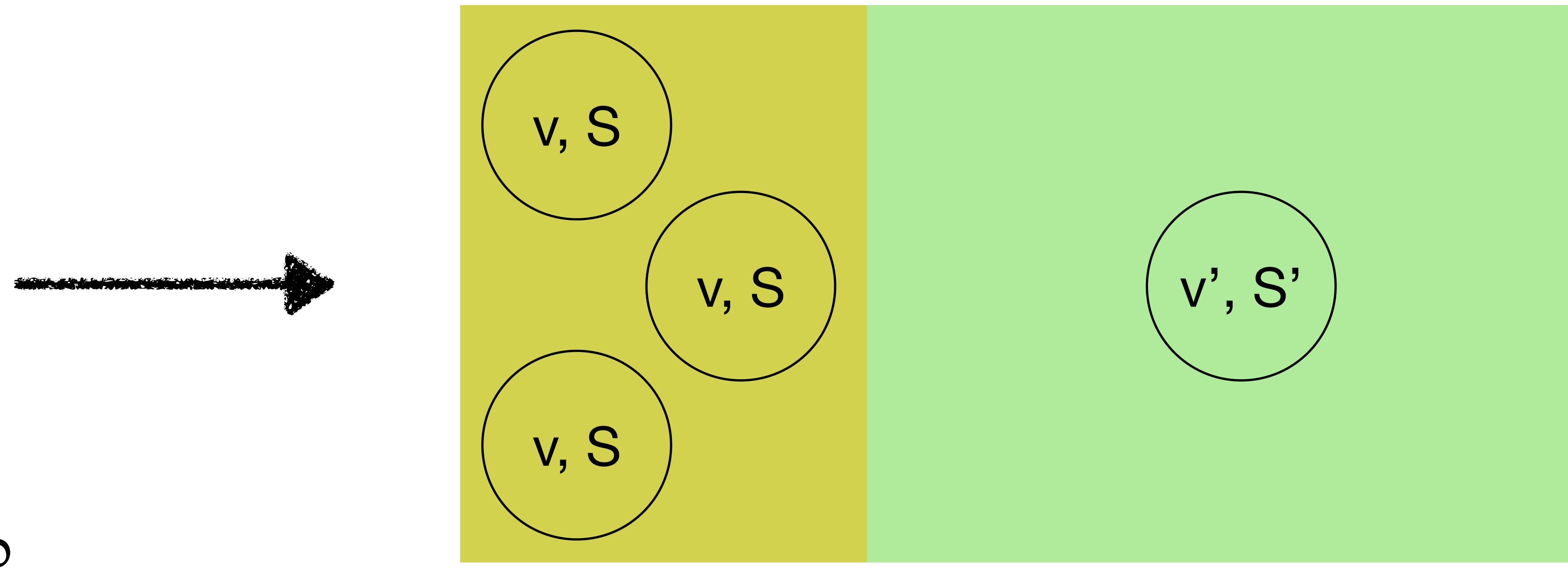
$a \leq \#q < \infty$

Symbolic graph

$\mathscr{C}$

$a \leq \#q \leq b$

$a \leq \#q < \infty$



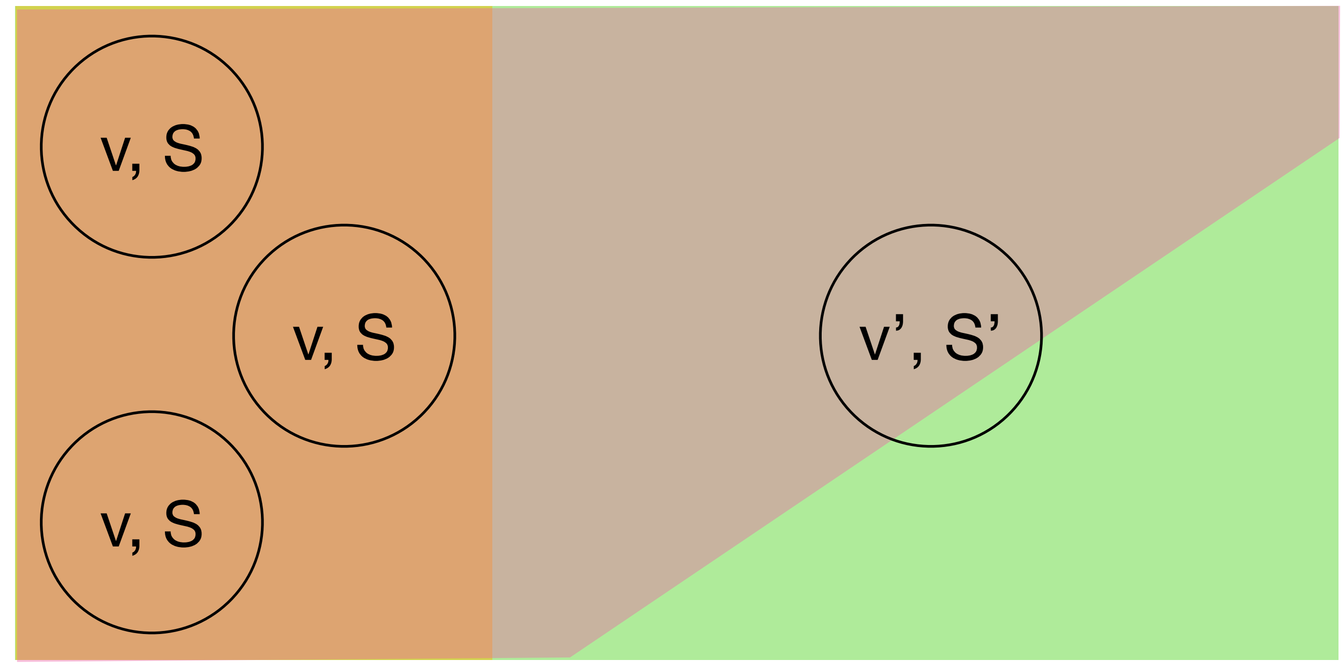
$\mathscr{C}_{SG}$ $post^*(\mathscr{C}_{SG})$

Symbolic graph

$\mathscr{C}$

$a \leq \#q \leq b$

$a \leq \#q < \infty$



$\mathscr{C}_{SG}$

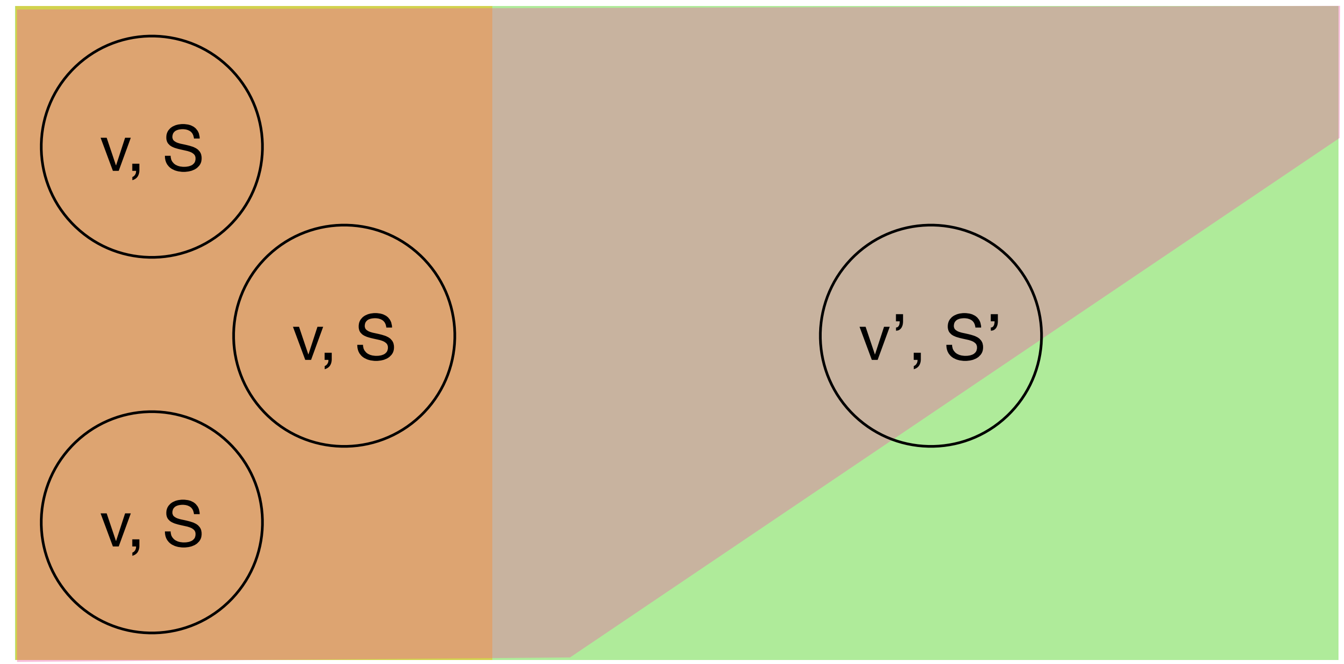
$post^*(\mathscr{C}) \subseteq post^*(\mathscr{C}_{SG}) \subsetneq post^*(\mathscr{C})$

Symbolic graph

$\mathscr{C} \longrightarrow$

$a \leq \#q \leq b$

$a \leq \#q < \infty$



$\mathscr{C}_{SG}$

$post^*(\mathscr{C}) \subseteq post^*(\mathscr{C}_{SG}) \subsetneq post^*(\mathscr{C})$

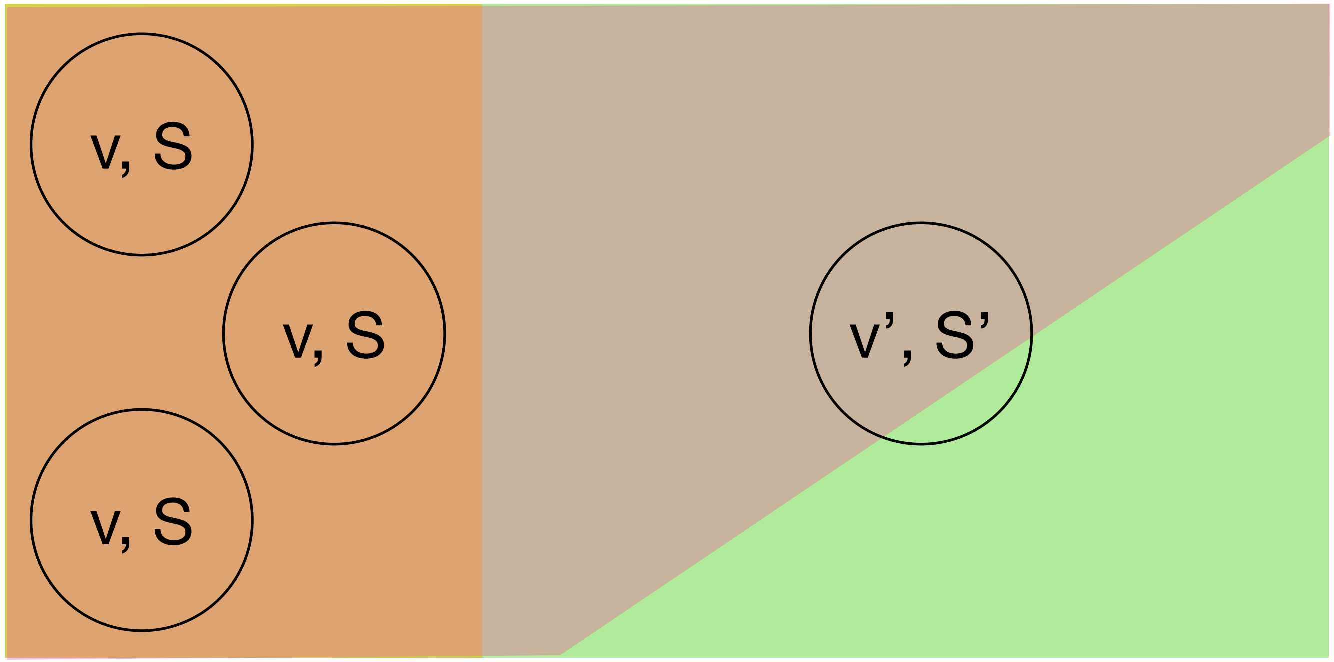
$\exists N$ such that $\forall$ v', S' $\forall C'$ with $\begin{cases} C'(q) \geq v'(q) + N & \text{if } q \in S' \\ C'(q) = v'(q) & \text{if } q \notin S' \end{cases}$ $C' \in post^*(\mathscr{C})$

Symbolic graph

$\mathscr{C} \longrightarrow$

$a \leq \#q \leq b$

$a \leq \#q < \infty$



$\mathscr{C}_{SG}$

$post^*(\mathscr{C}) \subseteq post^*(\mathscr{C}_{SG}) \subsetneq post^*(\mathscr{C})$

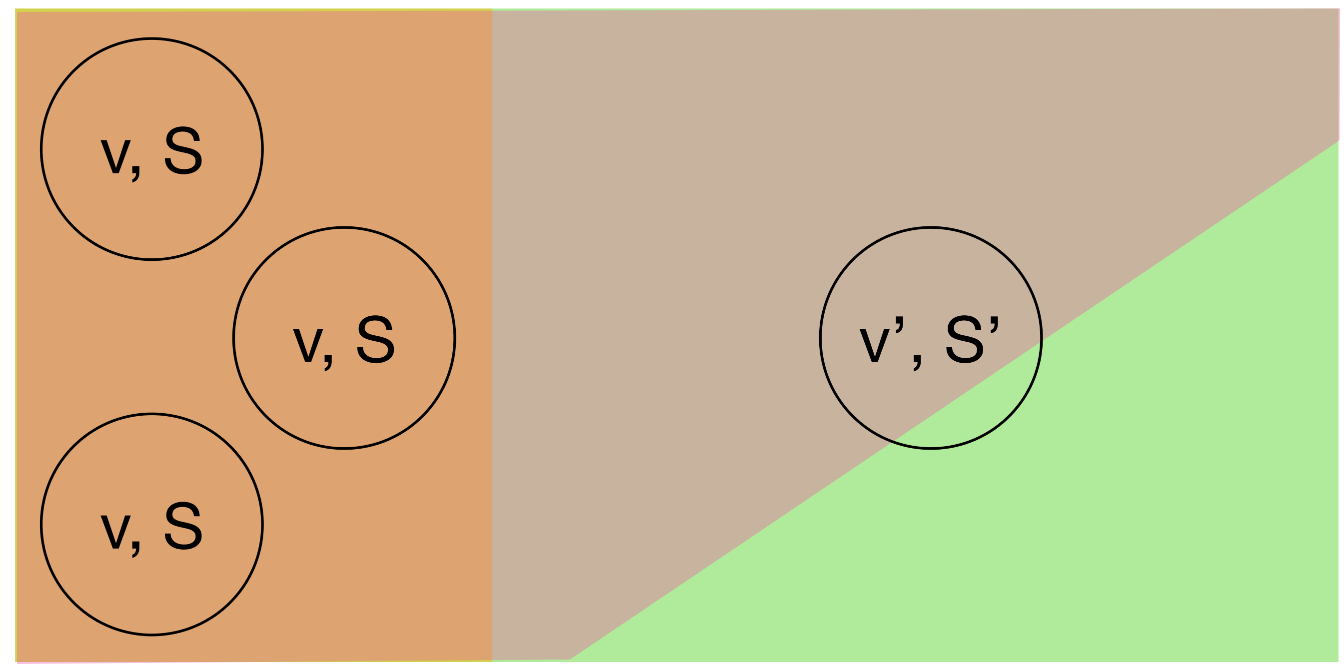
$\exists N$ such that $\forall$ v', S' $\forall C'$ with $\begin{cases} C'(q) \geq v'(q) + N & \text{if} \quad q \in S' \\ C'(q) = v'(q) & \text{if} \quad q \notin S' \end{cases}$ $C' \in post^*(\mathscr{C})$

Symbolic graph

$\mathscr{C} \longrightarrow$

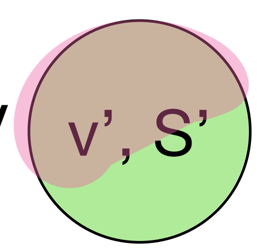
$a \leq \#q \leq b$

$a \leq \#q < \infty$

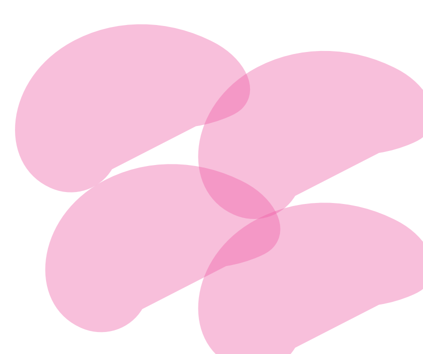


$\mathscr{C}_{SG}$

$post^*(\mathscr{C}) \subseteq post^*(\mathscr{C}_{SG}) \subsetneq post^*(\mathscr{C})$

$\exists N$ such that $\forall$ 

$\forall C'$ with $\begin{cases} C'(q) \geq v'(q) + N & \text{if } q \in S' \\ C'(q) = v'(q) & \text{if } q \notin S' \end{cases}$ $C' \in post^*(\mathscr{C})$

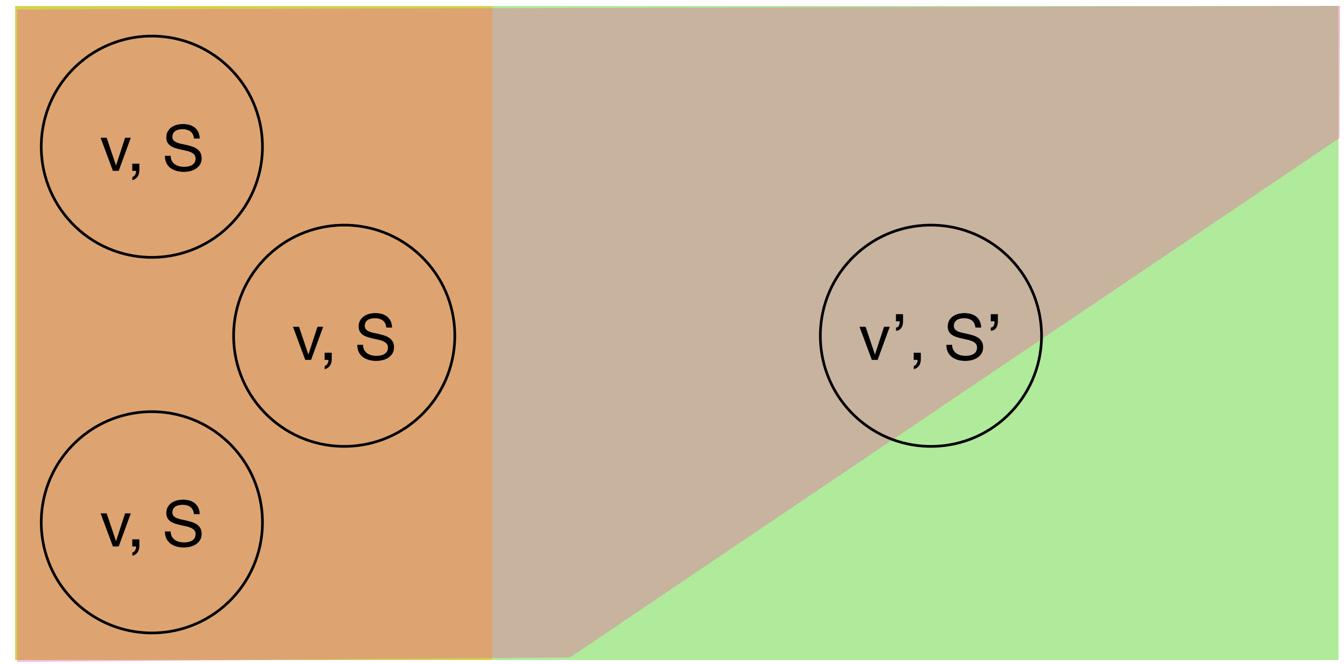
$post^*(\mathscr{C}) =$  is a finite union of cubes

Symbolic graph

$\mathscr{C}$

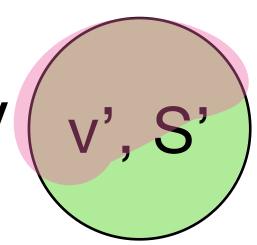
$a \leq \#q \leq b$

$a \leq \#q < \infty$



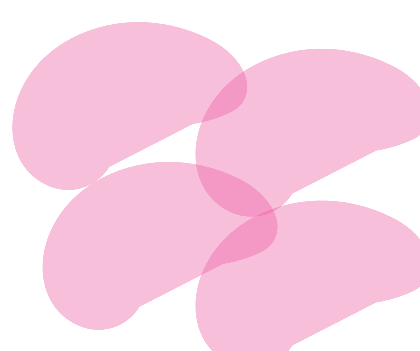
$\mathscr{C}_{SG}$

$post^*(\mathscr{C}) \subseteq post^*(\mathscr{C}_{SG}) \subsetneq post^*(\mathscr{C})$

$\exists N$ such that $\forall$ 

$\forall C'$ with $\begin{cases} C'(q) \geq v'(q) + N & \text{if} \quad q \in S' \\ C'(q) = v'(q) & \text{if} \quad q \notin S' \end{cases}$ $C' \in post^*(\mathscr{C})$

exponential in $\mathscr{C}$ and RBN

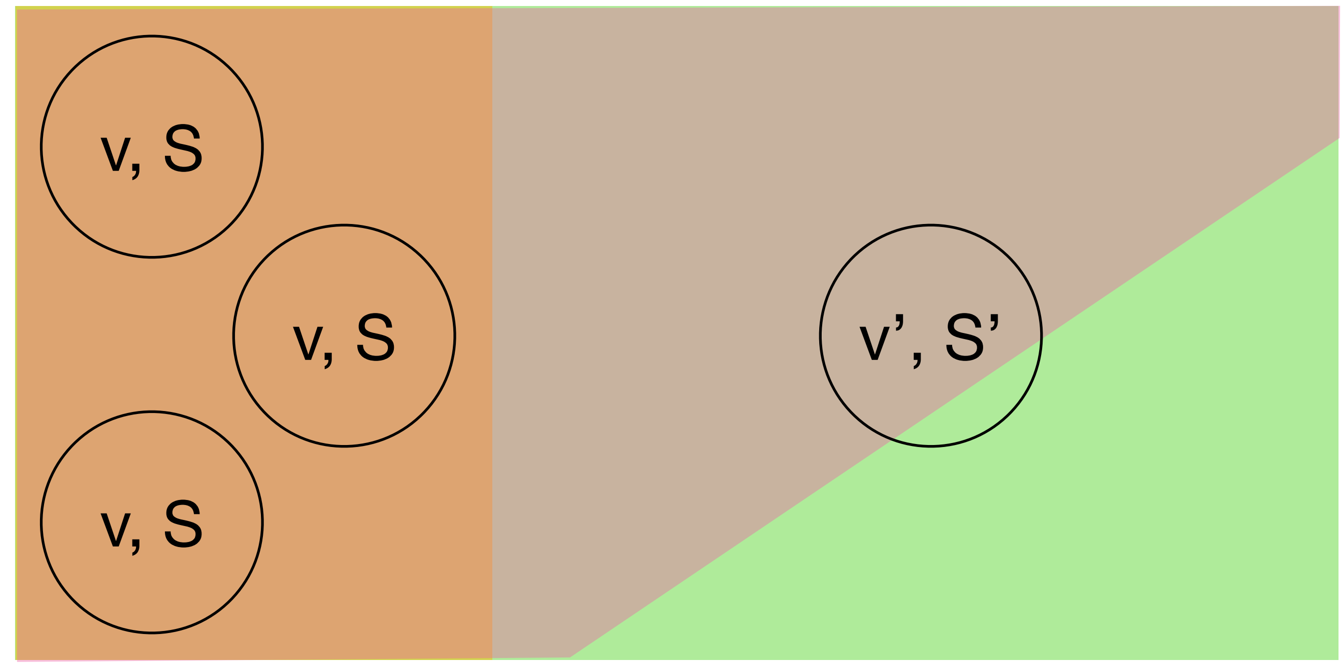
$post^*(\mathscr{C}) =$  is a finite union of cubes

Symbolic graph

$\mathscr{C}$

$a \leq \#q \leq b$

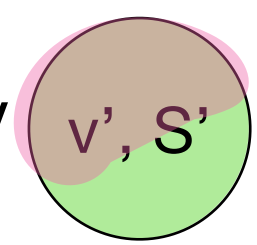
$a \leq \#q < \infty$



$\mathscr{C}_{SG}$

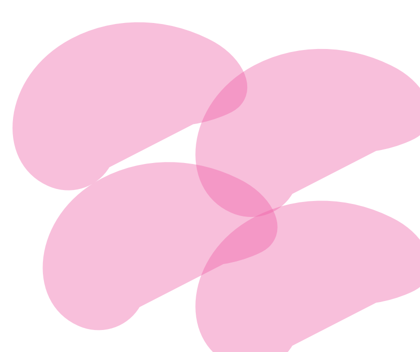
$post^*(\mathscr{C}_{SG})$

$post^*(\mathscr{C}) \subseteq$ $\subsetneq post^*(\mathscr{C})$

$\exists N$ such that $\forall$ 

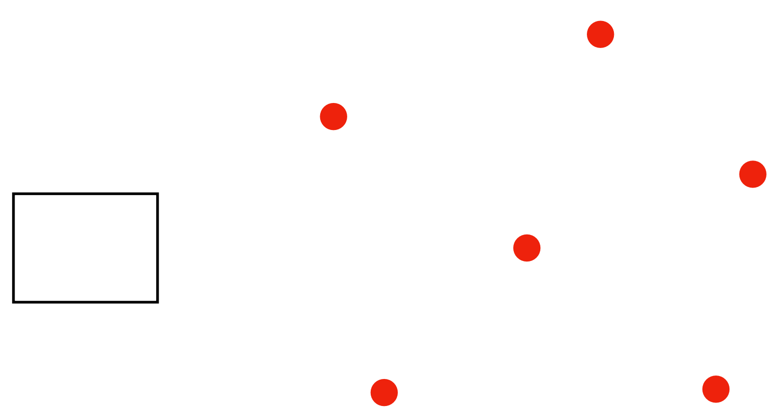
$\forall C'$ with $\begin{cases} C'(q) \geq v'(q) + N & \text{if} \quad q \in S' \\ C'(q) = v'(q) & \text{if} \quad q \notin S' \end{cases}$ $C' \in post^*(\mathscr{C})$

exponential in $\mathscr{C}$ and RBN

$post^*(\mathscr{C}) =$  is a finite union of exponential cubes

Asynchronous shared-memory systems

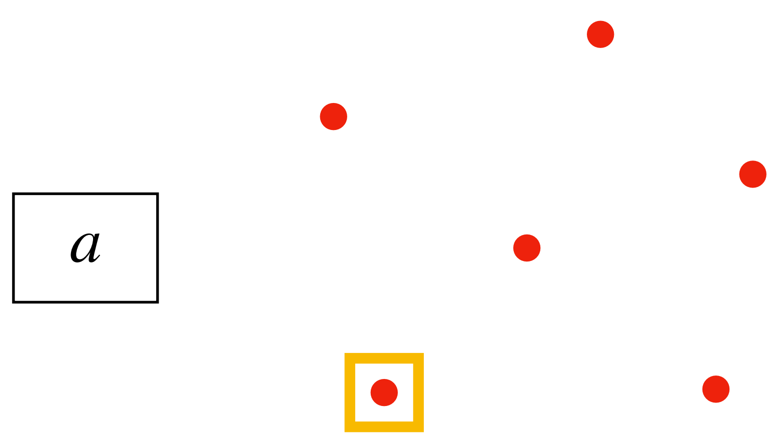
[Esparza, Ganty & Majumdar, CAV '13]



communication by writing
to a shared register

Asynchronous shared-memory systems

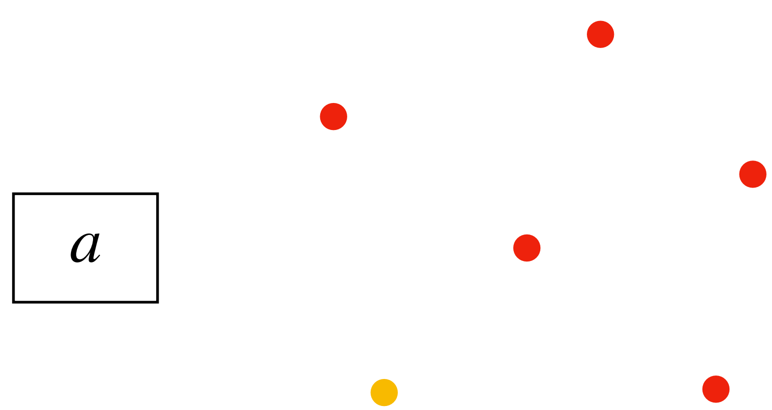
[Esparza, Ganty & Majumdar, CAV ’13]



communication by writing
to a shared register

Asynchronous shared-memory systems

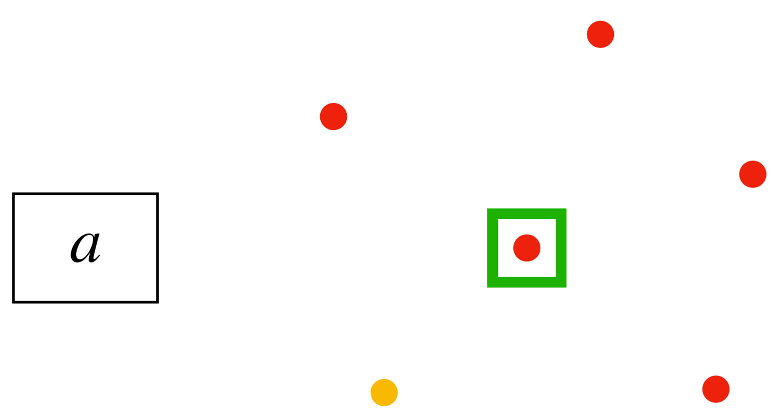
[Esparza, Ganty & Majumdar, CAV ’13]



communication by writing
to a shared register

Asynchronous shared-memory systems

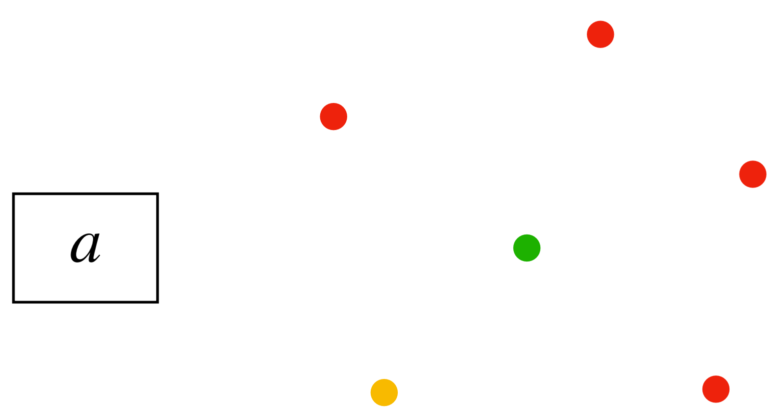
[Esparza, Ganty & Majumdar, CAV '13]



communication by writing
to a shared register

Asynchronous shared-memory systems

[Esparza, Ganty & Majumdar, CAV ’13]

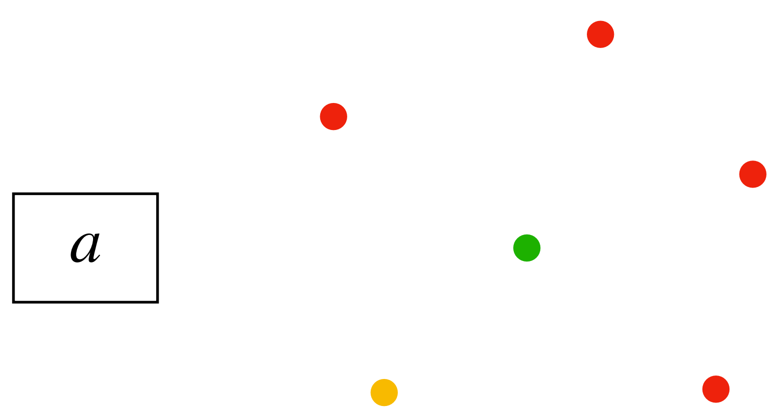


communication by writing
to a shared register

Asynchronous shared-memory systems

[Esparza, Ganty & Majumdar, CAV ’13]

Asynchronous shared-memory systems are equivalent to RBN for these parameterized problems [*A. R. B.*, *W.-K.*, Gandalf’21]

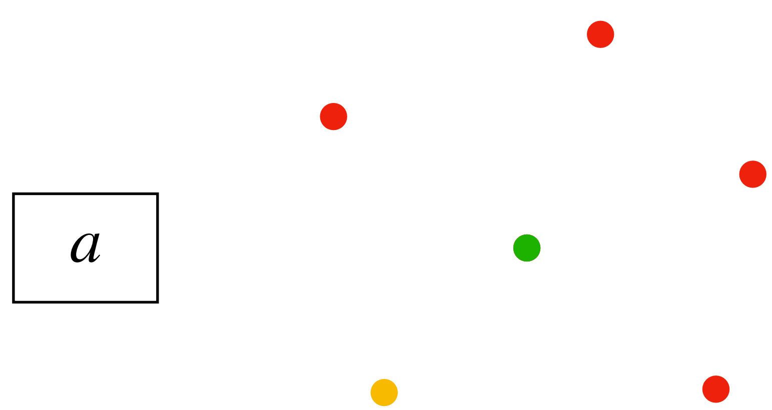


communication by writing
to a shared register

Asynchronous shared-memory systems

[Esparza, Ganty & Majumdar, CAV ’13]

Asynchronous shared-memory systems are equivalent to RBN for these parameterized problems [*A. R. B.*, *W.-K.*, Gandalf’21]



communication by writing
to a shared register

→ close the [Bouyer et al., ICALP’16] PSPACE-EXPSPACE complexity gap for almost-sure reachability

Conclusion

- Host of parameterized problems for RBN can be solved in PSPACE

Conclusion

- Host of parameterized problems for RBN can be solved in PSPACE

- Tight result: reachability and coverability are already PSPACE-hard

Conclusion

- Host of parameterized problems for RBN can be solved in PSPACE
- Tight result: reachability and coverability are already PSPACE-hard
- Complexity jump: in static broadcast network, these parameterized problems are undecidable

Conclusion

- Host of parameterized problems for RBN can be solved in PSPACE

- Tight result: reachability and coverability are already PSPACE-hard

- Complexity jump: in static broadcast network, these parameterized problems are undecidable

- Results spread: to asynchronous shared-memory systems

Conclusion

- Host of parameterized problems for RBN can be solved in PSPACE
- Tight result: reachability and coverability are already PSPACE-hard
- Complexity jump: in static broadcast network, these parameterized problems are undecidable
- Results spread: to asynchronous shared-memory systems

thank you!