Mathematics 2016 (Delhi) Term II

SET II

> **Note : Except for the following questions, all the remaining questions have been asked in previous set.**

SECTION — B

10. How many terms of the A.P. 27, 24, 21, ... should be taken so that their sum is zero? **[2]**

Solution : Given, A.P. is 27, 24, 21, ...

We have, $a = 27$, $d = 24 - 27 = 21 - 24 = -3$

Now, $S_n = 0$

Therefore, $$S_n = \frac{n}{2}[2a + (n-1)d] = 0$$

$$\Rightarrow \frac{n}{2}[2(27) + (n-1)(-3)] = 0$$

$$\Rightarrow 54 - 3n + 3 = 0$$

$$\Rightarrow 57 - 3n = 0$$

$$\Rightarrow 3n = 57$$

$$\therefore n = 19$$

Hence, the no. of terms are 19 **Ans.**

SECTION — C

18. Solve for x : **[3]**

$$\frac{x+1}{x+1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}; x \neq 1, -2, 2$$

Solution : We have, $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$; $x \neq 1, -2, 2$

$$\frac{(x+1)(x+2)+(x-2)(x-1)}{(x-1)(x+2)} = \frac{4(x-2)-(2x+3)}{x-2}$$

$(x-2)\,[x^2 + x + 2x + 2 + x^2 - 2x - x + 2] = [4x - 8 - 2x - 3]\,(x^2 + x - 2)$

$(x-2)\,(2x^2 + 4) = (2x - 11)\,(x^2 + x - 2)$

$2x^3 + 4x - 4x^2 - 8 = 2x^3 + 2x^2 - 4x - 11x^2 - 11x + 22$

$4x - 4x^2 - 8 = -9x^2 - 15x + 22$

$5x^2 + 19x - 30 = 0$

$5x^2 + 25x - 6x - 30 = 0$

$5x\,(x + 5) - 6\,(x + 5) = 0$

$(5x - 6)\,(x + 5) = 0$

$$x = -5, \frac{6}{5}$$

$\therefore \quad x = -5 \text{ or } x = \frac{6}{5}$ **Ans.**

19. Two different dice are thrown together. Find the probability of :
(i) getting a number greater than 3 on each die
(ii) getting a total of 6 or 7 of the numbers on two dice **[3]**

Solution : Total outcomes = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3) (6, 4), (6, 5), (6, 6)}

$\Rightarrow$ Total no. of outcomes = 36

(i) Let E_1 be the event of getting a number greater than 3 on each die.
Favourable outcomes = {(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}
No. of favourable outcomes = 9

$\therefore \quad P(E_1) = \frac{9}{96} = \frac{1}{4}$ **Ans.**

(ii) Let E_2 be the event of getting a total of 6 or 7 of the numbers on two dice.
Favourable outcomes = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}
$\Rightarrow$ No. of favourable outcomes = 11

$\therefore \quad P(E_2) = \frac{11}{36}$ **Ans.**

20. A right circular cone of radius 3 cm, has a curved surface area of 47.1 cm². Find the volume of the cone. (use π = 3.14) **[3]**

Solution : Given, radius of right circular cone = 3 cm
and, curved surface area = 47·1 cm²

$\therefore \quad \pi r l = 47{\cdot}1$

$$l = \frac{47{\cdot}1}{3{\cdot}14 \times 3} = 5 \text{ cm}$$

$\therefore \quad h = \sqrt{l^2 - r^2}$

$= \sqrt{(5)^2 - (3)^2}$

$= \sqrt{25 - 9} = 4 \text{ cm}$

Now, Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3{\cdot}14 \times 3 \times 3 \times 4$

$= 37{\cdot}68 \text{ cm}^3$ **Ans.**

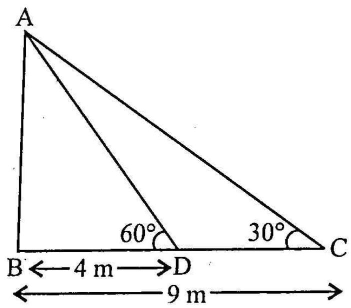
SECTION — D

28. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are 60° and 30° respectively. Find the height of the tower. **[4]**

Solution : Let length of tower is h

In ΔABD

$$\tan 60° = \frac{h}{4} \quad \text{...(i)}$$



In ΔABC

$$\tan 30° = \frac{h}{9}$$

$$\cot(90° - 30°) = \frac{h}{9}$$

$$\cot 60° = \frac{h}{9} \quad \text{...(ii)}$$

Multiplying eq. (i) and (ii), we get

$$\tan 60° . \cot 60° = \frac{h}{4} \times \frac{h}{9}$$

$$1 = \frac{h^2}{36}$$

$h = 6$ m **Ans.**

Note : In this question, it has not been specified whether two points from tower are taken in same or opposite side we have taken these points on the same side of tower.

29. Construct a triangle ABC in which $BC = 6$ cm, $AB = 5$ cm and $\angle ABC = 60°$.

Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of ΔABC. **[4]**

Solution : Steps of Construction—

(i) Draw a line segment $BC = 6$ cm.

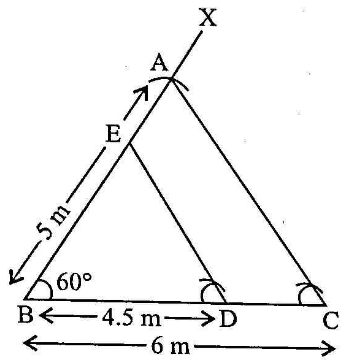
(ii) Construct $\angle XBC = 60°$

(iii) With B as centre and radius equal to 5 cm draw an arc which intersect XB at A.

(iv) Join AC. Thus, ΔABC is obtained.

(v) Draw D on BC such that $BD = \frac{3}{4}\ BC = \left(\frac{3}{4}\times 6\right)$ cm $= \frac{9}{2}$ cm $= 4{\cdot}5$ cm

(vi) Draw $DE \parallel CA$, cutting BA at E.

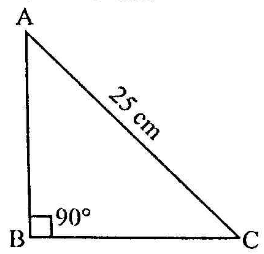


Then, $\Delta\ BDE$ is the required triangle similar to $\Delta\ ABC$ such that each side of $\Delta\ BDE$ is $\frac{3}{4}$ times the corresponding side of $\Delta\ ABC$.

30. The perimeter of a right triangle is 60 cm. If its hypotenuse is 25 cm. Find the area of the triangle. **[4]**

Solution : Given, the perimeter of right triangle = 60 cm

and hypotenuse = 25 cm



$$\therefore \quad AB + BC + CA = 60 \text{ cm}$$

$$AB + BC + 25 = 60$$

$$\therefore \quad AB + BC = 35 \qquad ...(i)$$

Now, by pythagoras theorem,

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(25)^2 = (AB)^2 + (BC)^2$$

$$\therefore \quad AB^2 + BC^2 = 625 \qquad ...(ii)$$

we, know that, $(a + b)^2 = a^2 + b^2 + 2ab$

then, $\quad (AB + BC)^2 = (AB)^2 + (BC)^2 + 2AB{\cdot}BC$

$$(35)^2 = 625 + 2\,AB{\cdot}BC$$

$$\therefore \quad 2AB{\cdot}BC = 1225 - 625$$

$$2AB{\cdot}BC = 600$$

$$\therefore \quad AB{\cdot}BC = 300$$

$$\therefore \quad \text{Area of } \Delta\,ABC = \frac{1}{2} \times AB \times BC$$

$$= \frac{1}{2} \times 300 = 150 \text{ cm}^2 \quad \text{Ans.}$$

31. A thief, after committing a theft, runs at a uniform speed of 50 m/ minute. After 2 minutes, a policeman runs to catch him. He goes 60 m in first minute and increases his speed by 5 m/minute every succeeding minute. After how many minutes, the policeman will catch the thief? **[4]**

Solution : Let total time be n minutes

Since policeman runs after two minutes he will catch the thief in $(n - 2)$ minutes.

Total distance covered by thief = 50 m /min × n min = $(50\,n)$ m

Now, total distance covered by the policeman = (60) + (60 + 5) + (60 + 5 + 5) + ... + $(n - 2)$ terms

i.e., 60 + 65 + 70 + ... + $(n - 2)$ terms

$$\therefore\ S_{n-2} = \frac{n-2}{2}\,[2 \times 60 + (n-3)\,5]$$

$$\Rightarrow \quad \frac{n-2}{2}\,[120 + (n-3)\,5] = 50n$$

$$\Rightarrow \quad n - 2\,(120 + 5n - 15) = 100n$$

$$\Rightarrow \quad 120n - 240 + 5n^2 - 10n - 15n + 30 = 100n$$

$$\Rightarrow \quad 5n^2 - 5n - 210 = 0$$

$$\Rightarrow \quad n^2 - n - 42 = 0$$

$$\Rightarrow \quad n^2 - (7 - 6)\,n - 42 = 0$$

$$\Rightarrow \quad n^2 - 7n + 6n - 42 = 0$$

$$\Rightarrow \quad n(n - 7) + 6\,(n - 7) = 0$$

$$\Rightarrow \quad (n + 6)\,(n - 7) = 0$$

$$n = 7 \text{ or } n = -6 \text{ (neglect)}$$

Hence, policeman will catch the thief in $(7 - 2)$ *i.e.*, 5 minutes.

Ans.