Mathematics 2017 (Outside Delhi) Term II

SET I

Time allowed : 3 Hours

Maximum marks : 90

SECTION — A

1. What is the common difference of an A.P. in which $a_{21} - a_7 = 84$? **[1]**

Solution : Given, $a_{21} - a_7 = 84$

$\Rightarrow \quad (a + 20d) - (a + 6d) = 84$

$\Rightarrow \quad a + 20d - a - 6d = 84$

$\Rightarrow \quad 20d - 6d = 84$

$\Rightarrow \quad 14d = 84$

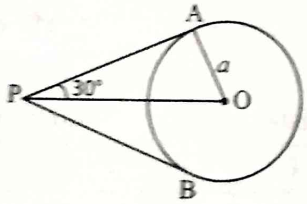
$\Rightarrow \quad d = \dfrac{84}{14} = 6$

Hence common difference = 6. **Ans.**

2. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O, is $60°$, then find the length of OP. **[1]**

Solution : Given, $\angle APB = 60°$

$\Rightarrow \quad \angle APO = 30°$



In right angle ΔOAP,

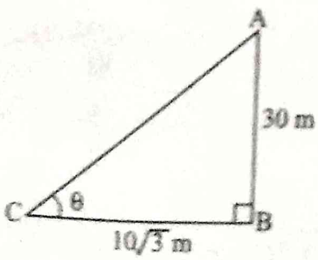
$$\frac{OP}{OA} = \text{cosec } 30°$$

$$\frac{OP}{a} = 2 \Rightarrow OP = 2a$$ **Ans.**

3. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun ? **[1]**

Solution : In ΔABC,

$$\tan\theta = \frac{AB}{BC}$$



$$\tan\theta = \frac{30}{10\sqrt{3}} = \sqrt{3}$$

$\tan\theta = \tan 60° \quad \Rightarrow \quad \theta = 60°$

Hence angle of elevation is $60°$. **Ans.**

4. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0·18. What is the number of rotten apples in the heap ? **[1]**

Solution : Total apples = 900

$P(E) = 0{\cdot}18$

$$\frac{\text{No. of rotten apples}}{\text{Total no. of apples}} = 0{\cdot}18$$

$$\frac{\text{No. of rotten apples}}{900} = 0{\cdot}18$$

No. of rotten apples $= 900 \times 0{\cdot}18$

$= 162$ **Ans.**

SECTION — B

5. Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other. **[2]**

Solution : Given, equation is $px^2 - 14x + 8 = 0$

Let one root $= \alpha$,

then other root $= 6\alpha$

Sum of roots $= -\dfrac{b}{a}$;

$$\alpha + 6\alpha = \frac{-(-14)}{p}$$

$$7\alpha = \frac{14}{p};$$

$$\alpha = \frac{14}{p \times 7}$$

or $$\alpha = \frac{2}{p} \quad ...(i)$$

Product of roots $= \dfrac{c}{a}$

$$(\alpha)(6\alpha) = \frac{8}{p}$$

$$6\alpha^2 = \frac{8}{p} \quad ...(ii)$$

Putting value of α from eq. (i)

$$6\left(\frac{2}{p}\right)^2 = \frac{8}{p}$$

$$6 \times \frac{4}{p^2} = \frac{8}{p}$$

$\Rightarrow$ $24p = 8p^2$

$\Rightarrow$ $8p^2 - 24p = 0$

$\Rightarrow$ $8p\,(p-3) = 0$

$\Rightarrow$ Either $8p = 0 \Rightarrow p = 0$

$\Rightarrow$ $p - 3 = 0 \Rightarrow p = 3$

or

For $p = 0$, given condition is not satisfied

$\therefore$ $p = 3$ **Ans.**

Which term of the progression 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$, ... is the first negative term ? **[2]**

Solution : Given, A.P. is 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$

$$= 20, \frac{77}{4}, \frac{37}{2}, \frac{71}{4}, \ldots$$

Here, $a = 20$, $d = \frac{77}{4} - 20 = \frac{77-80}{4} = \frac{-3}{4}$

Let a_n is first negative term

$\Rightarrow$ $a_n + (n-1)d < 0$

$\Rightarrow$ $20 + (n-1)\left(-\frac{3}{4}\right) < 0$

$\Rightarrow$ $20 - \frac{3}{4}n + \frac{3}{4} < 0$

$\Rightarrow$ $20 + \frac{3}{4} < \frac{3}{4}n$

$\Rightarrow$ $\frac{83}{4} < \frac{3}{4}n$

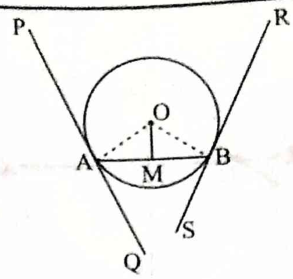
$\Rightarrow$ $n > \frac{83}{4} \times \frac{4}{3}$

$\Rightarrow$ $n > \frac{83}{3} = 27.66$

28th term will be first negative term of given A.P. **Ans.**

7. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord. **[2]**

Solution : Given, a circle of radius OA and centred at O with chord AB and tangents PQ & RS are drawn from point A and B respectively.



Draw $OM \perp AB$, and join OA and OB.

In $\Delta\, OAM$ and $\Delta\, OMB$,

$OA = OB$ (Radii)

$OM = OM$ (Common)

$\angle OMA = \angle OMB$ (Each 90°)

$\therefore$ $\Delta\, OAM \cong \Delta\, OMB$ (R.H.S. cong.)

$\therefore$ $\angle OAM = \angle OBM$ (CPCT)

Also, $\angle OAP = \angle OBR = 90°$ (Line joining point of contact of tangent to centre is perpendicular on it)

On addition,

$\angle OAM + \angle OAP = \angle OBM + \angle OBR$

$\Rightarrow$ $\angle PAB = \angle RBA$

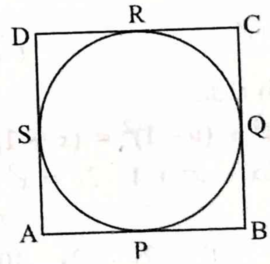
$\Rightarrow$ $\angle PAQ - \angle PAB = \angle RBS - \angle RBA$

$\Rightarrow$ $\angle QAB = \angle SBA$ **Hence Proved.**

8. A circle touches all the four sides of a quadrilateral $ABCD$. Prove that

$$AB + CD = BC + DA$$ **[2]**

Solution : Given, a quad. $ABCD$ and a circle touches its all four sides at P, Q, R, and S respectively.



To prove : $AB + CD = BC + DA$

L.H.S. $= AB + CD$

$= AP + PB + CR + RD$

$= AS + BQ + CQ + DS$

(Tangents from same external point are always equal)

$= (AS + SD) + (BQ + QC)$

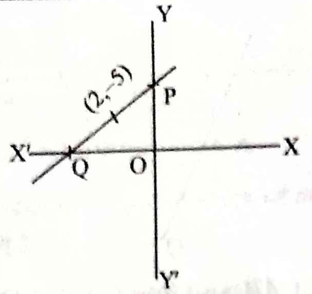
$= AD + BC$

$=$ R.H.S. **Hence Proved.**

9. A line intersects the y-axis and x-axis at the points P and Q respectively. If $(2, -5)$ is the mid-point of PQ, then find the co-ordinates of P and Q. **[2]**

Solution : Let co-ordinate of $P\ (0, y)$

Co-ordinate of $Q\ (x, 0)$



Mid-point is (2, – 5)

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (2, -5)$$

$$\Rightarrow \quad \left(\frac{x+0}{2}, \frac{0+y}{2}\right) = (2, -5)$$

$$\Rightarrow \quad \frac{x}{2} = 2; \quad \frac{y}{2} = -5$$

$$\Rightarrow \quad x = 4; \quad y = -10$$

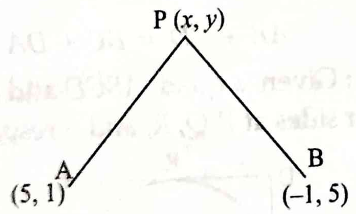
Co-ordinate of P (0, – 10)

Co-ordinate of Q (4, 0) **Ans.**

10. If the distances of $P(x, y)$, from $A(5, 1)$ and $B(-1, 5)$ are equal, then prove that $3x = 2y$. **[2]**

Solution : Given, $PA = PB$

$$\Rightarrow \quad \sqrt{(x-5)^2+(y-1)^2} = \sqrt{(x+1)^2+(y-5)^2}$$



Squaring both sides

$$(x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2$$

$$\Rightarrow x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 1 + 2x + y^2 + 25 - 10y$$

$$\Rightarrow \quad -10x - 2y = 2x - 10y$$

$$\Rightarrow \quad -10x - 2x = -10y + 2y$$

$$\Rightarrow \quad 12x = 8y$$

$$\Rightarrow \quad 3x = 2y$$ **Hence Proved.**

SECTION — C

11. If $ad \neq bc$, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots. **[3]**

Solution : Given, $ad \neq bc$

$$(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$$

$$D = b^2 - 4ac$$

$$= [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)$$

$$= 4[a^2c^2 + b^2d^2 + 2abcd] - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$$

$$= 4[a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2]$$

$$= 4[-a^2d^2 - b^2c^2 + 2abcd]$$

$$= -4[a^2d^2 + b^2c^2 - 2abcd]$$

$$= -4[ad - bc]^2$$

D is negative

Hence given equation has no real roots. **Hence Proved.**

12. The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P. **[3]**

Solution : Given, $a = 5, a_n = 45, S_n = 400$

We have, $$S_n = \frac{n}{2}[a + a_n]$$

$$\Rightarrow \quad 400 = \frac{n}{2}[5 + 45]$$

$$\Rightarrow \quad 400 = \frac{n}{2}[50]$$

$$\Rightarrow \quad 25n = 400 \Rightarrow n = \frac{400}{25}$$

$$\Rightarrow \quad n = 16$$

Now, $$a_n = a + (n-1)d$$

$$\Rightarrow \quad 45 = 5 + (16-1)d$$

$$\Rightarrow \quad 45 - 5 = 15d$$

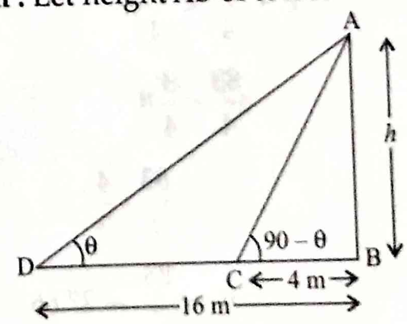
$$\Rightarrow \quad 15d = 40$$

$$\Rightarrow \quad d = \frac{8}{3}$$

So, $$n = 16 \text{ and } d = \frac{8}{3}$$ **Ans.**

13. On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower. **[3]**

Solution : Let height AB of tower $= h$.



In ΔABC,

$$\frac{AB}{BC} = \tan(90 - \theta)$$

$$\frac{h}{4} = \cot\theta$$

In ΔABD,

$$\frac{AB}{BC} = \tan\theta$$

$$\frac{h}{16} = \tan\theta \qquad \text{...(ii)}$$

Multiply eq. (i) and (ii)

$$\frac{h}{4}\times\frac{h}{16} = \cot\theta\times\tan\theta$$

$$\frac{h^2}{64} = 1$$

$$\left[\because \cot\theta\times\tan\theta = \frac{1}{\tan\theta}\times\tan\theta = 1\right]$$

$\Rightarrow h^2 = 64 \Rightarrow h = 8$ m

Height of tower = 8 m.

Ans.

A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag. **[3]**

Solution : Given, no. of white balls = 15

Let no. of black balls $= x$

$\therefore$ Total balls $= (15 + x)$

According to the question,

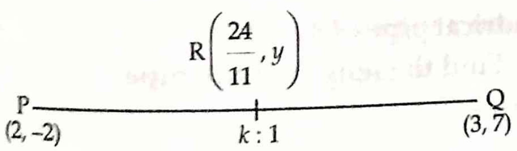
$$P\text{ (Black ball)} = 3\times P\text{ (White ball)}$$

$$\Rightarrow \frac{x}{(15+x)} = 3\times\frac{15}{(15+x)}$$

$$\Rightarrow x = 45$$

$\therefore$ No. of black balls in bag = 45 **Ans.**

5. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points $P(2, -2)$ and $Q(3, 7)$? Also find the value of y. **[3]**



Solution : Let point R divides PQ in the ratio $k : 1$

$$R = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$\Rightarrow \left(\frac{24}{11}, y\right) = \left(\frac{k(3)+1(2)}{k+1}, \frac{k(7)+1(-2)}{k+1}\right)$$

$$= \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$$

$$\Rightarrow \frac{3k+2}{k+1} = \frac{24}{11}$$

$$\Rightarrow 11(3k+2) = 24(k+1)$$

$$\Rightarrow 33k + 22 = 24k + 24$$

$$\Rightarrow 33k - 24k = 24 - 22$$

$$\Rightarrow 9k = 2 \Rightarrow k = 2/9$$

$$\therefore k : 1 = 2 : 9$$

Now,

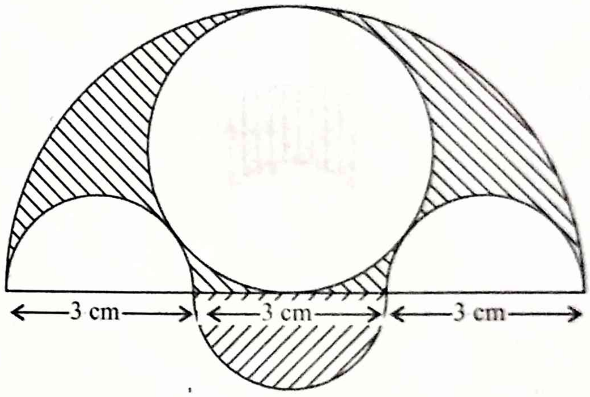
$$y = \frac{7k-2}{k+1} = \frac{7\left(\frac{2}{9}\right)-2}{\frac{2}{9}+1}$$

$$= \frac{\frac{14}{9}-2}{\frac{2}{9}+1} = \frac{\frac{14-18}{9}}{\frac{2+9}{9}} = \frac{-4}{11}$$

Line PQ divides in the ratio 2 : 9 and value of $y = \frac{-4}{11}$

Ans.

16. Three semicircles each of diameter 3 cm, a circle of diameter 4·5 cm and a semicircle of radius 4·5 cm are drawn in the given figure. Find the area of the shaded region. **[3]**



Solution : Given, radius of large semi-circle = 4·5 cm

$$\text{Area of large semi-circle} = \frac{1}{2}\pi R^2$$

$$= \frac{1}{2}\times\frac{22}{7}\times 4{\cdot}5\times 4{\cdot}5$$

Diameter of inner circle = 4·5 cm

$$\Rightarrow r = \frac{4{\cdot}5}{2}\text{ cm}$$

$$\text{Area of inner circle} = \pi r^2$$

$$= \frac{22}{7}\times\frac{4{\cdot}5}{2}\times\frac{4{\cdot}5}{2}$$

Diameter of small semi-circle = 3 cm

$$r = \frac{3}{2}\text{ cm}$$

$$\text{Area of small semi-circle} = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2}\times\frac{22}{7}\times\frac{3}{2}\times\frac{3}{2}$$

Area of shaded region

= Area of large semi circle + Area of 1 small semi-circle – Area of inner circle – Area of 2 small semi circle

$$= \frac{1}{2} \times \frac{22}{7} \times 4{\cdot}5 \times 4{\cdot}5 + \frac{1}{2} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}$$

$$- \frac{22}{7} \times \frac{4{\cdot}5}{2} \times \frac{4{\cdot}5}{2} - 2 \times \frac{1}{2} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}$$

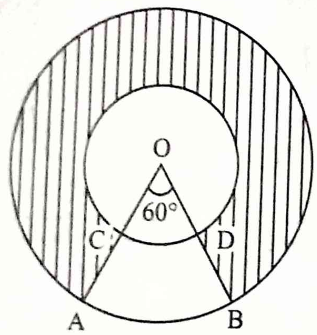
$$= \frac{1}{2} \times \left[20{\cdot}25 + \frac{9}{4}\right] - \frac{22}{7}\left[\frac{20{\cdot}25}{4} + \frac{9}{4}\right]$$

$$= \frac{11}{7} \times \frac{90}{4} - \frac{22}{7} \times \frac{29{\cdot}25}{4}$$

$$= \frac{990 - 643{\cdot}5}{28} = \frac{346{\cdot}5}{28}$$

$= 12{\cdot}37$ cm^2 (approx.) **Ans.**

17. In the given figure, two concentric circles with centre O have radii 21 cm and 42 cm. If $\angle AOB$ = 60°, find the area of the shaded region. [Use $\pi = \frac{22}{7}$] **[3]**



Solution : Angle for shaded region = 360° – 60°

= 300°

Area of shaded region

$$= \frac{\pi\theta}{360}(R^2 - r^2)$$

$$= \frac{22}{7} \times \frac{300}{360}[42^2 - 21^2]$$

$$= \frac{22}{7} \times \frac{5}{6} \times 63 \times 21$$

= 3465 cm^2 **Ans.**

18. Water in a canal, 5·4 m wide and 1·8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation ? **[3]**

Solution : Width of canal = 5·4 m

Depth of canal = 1·8 m

Length of water in canal for 1 hr = 25 km

= 25000 m

Volume of water flown out from canal in 1 hr

$= l \times b \times h$

$= 5{\cdot}4 \times 1{\cdot}8 \times 25000$

$= 243000$ m^3

Volume of water for 40 min $= 243000 \times \frac{40}{60} = 162000$ m^3

Area to be irrigated with 10 cm standing water in field

$$= \frac{\text{Volume}}{\text{Height}} = \frac{162000 \times 100}{10} \text{ m}^2$$

= 1620000 m^2

= 162 hectare **Ans.**

19. The slant height of a frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum. **[3]**

Solution : Slant height of frustum 'l' = 4 cm

Perimeter of upper top = 18 cm

$\Rightarrow$ $2\pi R = 18$ cm $\Rightarrow R = \frac{9}{\pi}$ cm

Perimeter of lower bottom = 6 cm

$\Rightarrow$ $2\pi r = 6 \Rightarrow r = \frac{3}{\pi}$ cm

Curved S.A. of frustum $= \pi\, l\, [R + r]$

$$= \pi \times 4 \times \left[\frac{9}{\pi} + \frac{3}{\pi}\right]$$

$$= \pi \times 4 \times \frac{12}{\pi} = 48 \text{ cm}^2$$ **Ans.**

20. The dimensions of a solid iron cuboid are 4·4 m × 2·6 m × 1·0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe. **[3]**

Solution : Inner radius of pipe 'r' = 30 cm

Thickness of pipe = 5 cm

$\therefore$ Outer radius = 30 + 5

$\Rightarrow$ R = 35 cm

Now, Vol. of hollow pipe = Vol. of cuboid

$\pi h (R^2 - r^2) = l \times b \times h$

$$\frac{22}{7} \times h\,[35^2 - 30^2] = 4{\cdot}4 \times 2{\cdot}6 \times 1 \times 100 \times 100 \times 100$$

$$\frac{22}{7} \times h \times 65 \times 5 = 44 \times 26 \times 1 \times 100 \times 100$$

$$h = \frac{44 \times 26 \times 100 \times 100 \times 7}{22 \times 65 \times 5}$$

= 11200 cm

= 112 m **Ans.**

SECTION — D

21. Solve for x :

$$\frac{1}{x+1}+\frac{3}{5x+1}=\frac{5}{x+4}, x \neq -1, -\frac{1}{5}, -4$$ **[4]**

Solution : Given, $\frac{1}{x+1}+\frac{3}{5x+1}=\frac{5}{x+4}$

$\Rightarrow \quad \frac{1}{x+1}-\frac{5}{x+4}=\frac{-3}{5x+1}$

$\Rightarrow \quad \frac{(x+4)-5(x+1)}{(x+1)(x+4)}=\frac{-3}{5x+1}$

$\Rightarrow \quad \frac{x+4-5x-5}{x^2+5x+4}=\frac{-3}{5x+1}$

$\Rightarrow \quad \frac{(-4x-1)}{x^2+5x+4}=\frac{-3}{5x+1}$

$\Rightarrow \quad (4x+1)(5x+1)=3(x^2+5x+4)$

$\Rightarrow \quad 20x^2+4x+5x+1=3x^2+15x+12$

$\Rightarrow \quad 17x^2-6x-11=0$

$\Rightarrow \quad 17x^2-17x+11x-11=0$

$\Rightarrow \quad 17x(x-1)+11(x-1)=0$

$\Rightarrow \quad (x-1)(17x+11)=0$

$\Rightarrow$ Either $x=1$ or $x=\frac{-11}{17}$ **Ans.**

22. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank ? **[4]**

Solution : Let tank fill by one tap = x hrs

Other tap = $(x+3)$ hrs

Together they fill by $3\frac{1}{13}=\frac{40}{13}$ hrs

Now,

$\Rightarrow \quad \frac{1}{x}+\frac{1}{x+3}=\frac{13}{40}$

$\Rightarrow \quad \frac{x+3+x}{(x)(x+3)}=\frac{13}{40}$

$\Rightarrow \quad \frac{2x+3}{x^2+3x}=\frac{13}{40}$

$\Rightarrow \quad 13x^2+39x=80x+120$

$\Rightarrow \quad 13x^2-41x-120=0$

$\Rightarrow \quad 13x^2-65x+24x-120=0$

$\Rightarrow \quad 13x(x-5)+24(x-5)=0$

$\Rightarrow \quad (x-5)(13x+24)=0$

Either $x-5=0$ or $13x+24=0$

$x=5, x=-24/13$ (Rejected)

One tap fill the tank in 5 hrs

So other tap fill the tank in $5+3=8$ hrs **Ans.**

23. If the ratio of the sum of the first n terms of two A.P.s is $(7n+1):(4n+27)$, then find the ratio of their 9^{th} terms. **[4]**

Solution : Ratio of sum of first n terms of two A.P.s are

$$\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]}=\frac{7n+1}{4n+27}$$

Put $n=17$

$\Rightarrow \quad \frac{2a+(16)d}{2A+(16)D}=\frac{120}{95}$

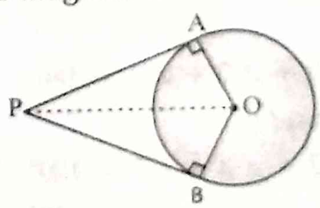
$\frac{2a+(16)d}{2A+(16)D}=\frac{120}{95}=\frac{24}{19}$

$\frac{a+8d}{A+8D}=\frac{24}{19}$

Hence ratio of 9^{th} terms of two A.P.s is 24 : 19 **Ans.**

24. Prove that the lengths of two tangents drawn from an external point to a circle are equal. **[4]**

Solution : Given, a circle with centre O and external point P. Two tangents PA and PB are drawn.



To prove : $PA = PB$

Const. : Join radius OA and OB also join O to P.

Proof : In ΔOAP and ΔOBP

$OA = OB$ (Radii)

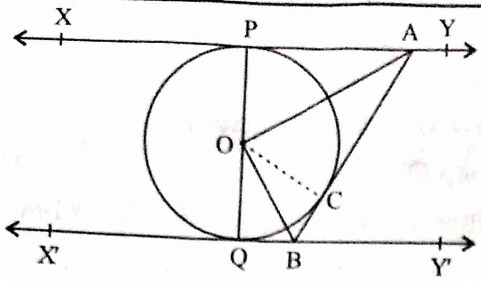
$\angle A = \angle B$ (Each 90°)

$OP = OP$ (Common)

$\therefore \quad \Delta AOP \cong \Delta BOP$ (RHS cong.)

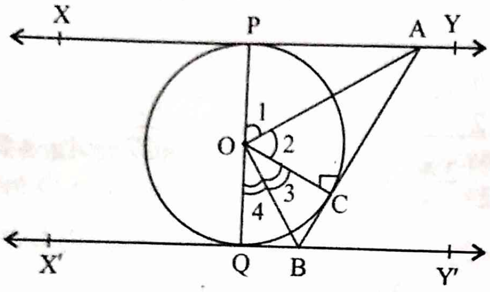
$\therefore \quad PA = PB$ (cpct) **Hence Proved.**

25. In the given figure, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, is intersecting XY at A and $X'Y'$ at B. Prove that $\angle AOB = 90°$. **[4]**



Solution : Given, XY & $X'Y'$ are parallel

Tangent AB is another tangent which touches the circle at C.



To prove : $\angle AOB = 90°$

Const. : Join OC.

Proof : In ΔOPA and ΔOCA

$OP = OC$ (Radii)

$\angle OPA = \angle OCA$ (Radius $\perp$ tangent)

$OA = OA$ (Common)

$\therefore$ $\Delta OPA \cong \Delta OCA$ (CPCT)

$\therefore$ $\angle 1 = \angle 2$ ...(i)

Simila[illegible] $\Delta OQB \cong \Delta OCB$

$\therefore$ $\angle 3 = \angle 4$ ...(ii)

Also, POQ is a diameter of circle

$\therefore$ $\angle POQ = 180°$ (Straight angle)

$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$

From eq. (i) and (ii)

$$\angle 2 + \angle 2 + \angle 3 + \angle 3 = 180°$$

$$2(\angle 2 + \angle 3) = 180°$$

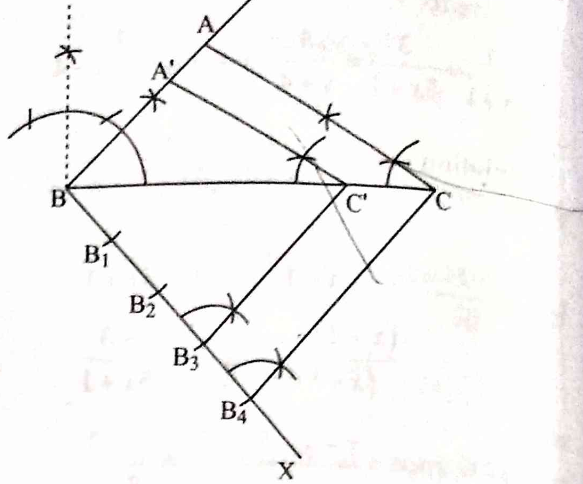
$$\angle 2 + \angle 3 = 90°$$

Hence, $\angle AOB = 90°$ **Hence Proved.**

26. Construct a triangle ABC with side $BC = 7$ cm, $\angle B = 45°$, $\angle A = 105°$. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the ΔABC. **[4]**

Solution : $BC = 7$ cm, $\angle B = 45°$, $\angle A = 105°$

$$\angle C = 180° - (\angle B + \angle A)$$
$$= 180° - (45° + 105°)$$
$$= 180° - 150°$$
$$= 30°$$



Steps of construction—

(i) Draw a line segment $BC = 7$ cm.

(ii) Draw an angle 45° at B and 30° at C. They intersect at A.

(iii) Draw an acute angle at B.

(iv) Divide angle ray in 4 equal parts as B_1, B_2, B_3 and B_4.

(v) Join B_4 to C.

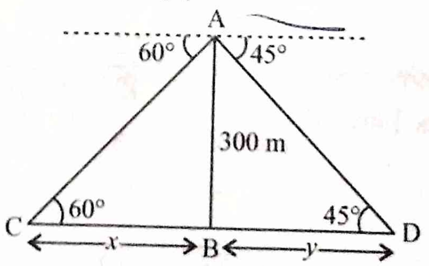
(vi) From B_3, draw a line parallel to B_4C intersecting BC at C'.

(vii) Draw another line parallel to CA from C' intersecting AB ray at A'.

(viii) $\Delta A'BC'$ is required triangle such that $\Delta A'BC' \sim \Delta ABC$ with $A'B = \frac{3}{4} AB$.

27. An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45° and 60° respectively. Find the width of the river. [Use $\sqrt{3} = 1{\cdot}732$] **[4]**

Solution : Let aeroplane is at A, 300 m high from a river. C and D are opposite banks of river.



In right ΔABC,

$$\frac{BC}{AB} = \cot 60°$$

$$\Rightarrow \quad \frac{x}{300} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{300}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$= 100\sqrt{3}$ m

$= 100 \times 1{\cdot}732 = 173{\cdot}2$ m

In right ΔABD,

$$\frac{BD}{AB} = \cot 45^\circ$$

$$\frac{y}{300} = 1 \Rightarrow y = 300$$

Width of river $= x + y$

$= 173{\cdot}2 + 300$

$= 473{\cdot}2$ m **Ans.**

If the points $A(k + 1, 2k)$, $B(3k, 2k + 3)$ and $C(5k - 1, 5k)$ are collinear, then find the value of k. **[4]**

Solution : Since $A(k + 1, 2k)$, $B(3k, 2k + 3)$ and $C(5k - 1, 5k)$ are collinear points, so area of triangle $= 0$.

$$\begin{vmatrix} k+1 & 2k \\ 3k & 2k+3 \\ 5k-1 & 5k \\ k+1 & 2k \end{vmatrix}$$

$$\Delta = \frac{1}{2}\,[(k+1)(2k+3) - 6k^2 + 15k^2 - (5k-1)(2k+3) + 2k(5k-1) - (k+1)(5k)]$$

$$0 = \frac{1}{2}\,[2k^2 + 5k + 3 - 6k^2 + 15k^2 - 10k^2 - 13k + 3 + 10k^2 - 2k - 5k^2 - 5k]$$

$$0 = \frac{1}{2}\,[6k^2 - 15k + 6]$$

$\Rightarrow \quad 6k^2 - 15k + 6 = 0$

$\Rightarrow \quad 6k^2 - 12k - 3k + 6 = 0$

$\Rightarrow \quad 6k(k-2) - 3(k-2) = 0$

$\Rightarrow \quad (k-2)(6k-3) = 0$

$k = 2$ or $k = \frac{1}{2}$ **Ans.**

Two different dice are thrown together. Find the probability that the numbers obtained have

(i) even sum, and

(ii) even product. **[4]**

Solution : When two different dice are thrown together

Total outcomes $= 6 \times 6 = 36$

(i) For even sum—Favourable outcomes are

(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)

No. of favourable outcomes = 18

$$P\text{(even sum)} = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{18}{36} = \frac{1}{2}$$ **Ans.**

(ii) For even product—Favourable outcomes are

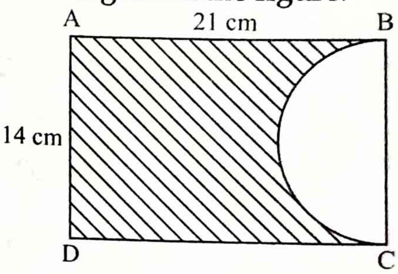
(1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6).

No. of favourable outcomes = 27

$$P\text{(even product)} = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{27}{36} = \frac{3}{4}$$ **Ans.**

30. In the given figure, $ABCD$ is a rectangle of dimensions 21 cm × 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure. **[4]**



Solution : Area of shaded region

= Area of rectangle – Area of semi circle

$= l \times b - \frac{1}{2}\pi r^2$

$= 21 \times 14 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$

$= 294 - 77$

$= 217$ cm^2

Perimeter of shaded region $= 2l + b + \pi r$

$= 2 \times 21 + 14 + \frac{22}{7} \times 7$

$= 42 + 14 + 22$

$= 78$ cm **Ans.**

31. In a rain-water harvesting system, the rain-water from a roof of 22 m × 20 m drains into a cylindrical tank having diameter of base 2 m and heigth 3·5 m. If the tank is full, find the rainfall in cm. Write your views on water conservation. **[4]**

Solution : Volume of water collected in system = Volume of cylindrical tank

$$L \times B \times H = \pi r^2 h$$

$$22 \times 20 \times H = \frac{22}{7} \times 1 \times 1 \times 3{\cdot}5$$

$$22 \times 20 \times H = 11$$