Mathematics 2018

Time allowed : 3 hours **Maximum marks : 80**

General Instructions :

(i) All questions are compulsory.
(ii) This question paper consists of 30 questions divided into four sections – A, B, C and D.
(iii) Section A contains 6 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each. Section D contains 8 questions of 4 marks each.
(iv) There is no overall choice. However, an internal choice has been provided in four questions of 3 marks each and 3 questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of calculator is not permitted.

SECTION – A

1. If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k. [1]

Solution : Given quadratic equation is,

$$x^2 - 2kx - 6 = 0$$

$x = 3$ is a root of above equation, then

$$(3)^2 - 2k(3) - 6 = 0$$
$$9 - 6k - 6 = 0$$
$$3 - 6k = 0$$
$$3 = 6k$$
$$\therefore \quad k = \frac{3}{6} = \frac{1}{2}$$
$$k = \frac{1}{2}$$

Ans.

2. What is the HCF of smallest prime number and the smallest composite number ? [1]

Solution : Smallest prime number = 2

Smallest composite number = 4

Prime factorisation of 2 is 1×2

Prime factorisation of 4 is 1×2^2

$\therefore$ HCF (2, 4) = 2 **Ans.**

3. Find the distance of a point $P(x,y)$ from the origin. [1]

Solution : The given point is $P(x, y)$.

The origin is O (0, 0)

Distance of point P from origin,

$$PO = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(x-0)^2 + (y-0)^2}$$
$$= \sqrt{x^2 + y^2} \text{ unit}$$

Ans.

4. In an AP, if the common difference (d) = –4 and the seventh term (a_7) is 4, then find the first term. [1]

Solution : Given,

$$d = -4$$
$$a_7 = 4$$
$$a + 6d = 4$$
$$a + 6(-4) = 4$$
$$a - 24 = 4$$
$$a = 4 + 24$$
$$a = 28$$

Ans.

5. What is the value of $(\cos^2 67° - \sin^2 23°)$? [1]

Solution : We have, $\cos^2 67° - \sin^2 23°$

$$= \cos^2 67° - \cos^2 (90° - 23°)$$
$$[\because \sin(90° - \theta) = \cos\theta]$$
$$= \cos^2 67° - \cos^2 67°$$
$$= 0$$

Ans.

6. Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then find $\frac{ar\,\Delta ABC}{ar\,\Delta PQR}$. [1]

Solution : Given, $\Delta ABC \sim \Delta PQR$

And $\frac{AB}{PQ} = \frac{1}{3}$

Now, $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2}$$
$$= \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

Ans.

SECTION – B

7. Given that $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number. [2]

Solution : Given, $\sqrt{2}$ is irrational number.

Let $\sqrt{2} = m$

Suppose, $5 + 3\sqrt{2}$ is a rational number.

So, $$5 + 3\sqrt{2} = \frac{a}{b} \quad (a \neq b, b \neq 0)$$
$$3\sqrt{2} = \frac{a}{b} - 5$$
$$3\sqrt{2} = \frac{a-5b}{b}$$
or $$\sqrt{2} = \frac{a-5b}{3b}$$
So, $$\frac{a-5b}{3b} = m$$

But $\frac{a-5b}{3b}$ is rational number, so m is rational number which contradicts the fact that $m = \sqrt{2}$ is irrational number.

So, our supposition is wrong.

Hence, $5 + 3\sqrt{2}$ is also irrational. **Hence Proved.**

8. In fig. 1, $ABCD$ is a rectangle. Find the values of x and y. [2]

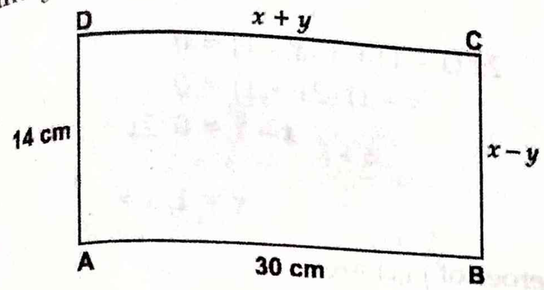


Figure 1

Solution : Given, $ABCD$ is a rectangle.

$\therefore \quad AB = CD$

$30 = x + y$

$\Rightarrow$ or $\quad x + y = 30 \quad$ ...(i)

Similarly, $\quad AD = BC$

$14 = x - y$

$\Rightarrow$ or $\quad x - y = 14 \quad$ ...(ii)

On adding eq. (i) and (ii), we get

$2x = 44$

$\Rightarrow \quad x = 22$

Putting the value of x in eq. (i), we get

$22 + y = 30$

$\Rightarrow \quad y = 30 - 22$

$\Rightarrow \quad y = 8$

So, $x = 22$, $y = 8$. **Ans.**

9. **Find the sum of first 8 multiples of 3.** [2]

Solution : First 8 multiples of 3 are

3, 6, 9............upto 8 terms

We can observe that the above series is an AP with $a = 3$, $d = 6 - 3 = 3$, $n = 8$

Sum of n terms of an A.P. is given by,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_8 = \frac{8}{2}[2 \times 3 + (8-1)(3)]$$

$$= 4[6 + 7 \times 3]$$

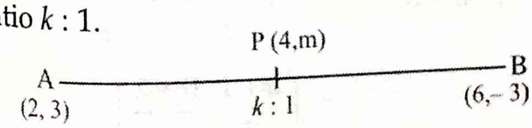
$$= 4[6 + 21]$$

$$= 4 \times 27$$

$$\Rightarrow \quad S_8 = 108$$ **Ans.**

10. **Find the ratio in which $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence find m.** [2]

Solution : Let P divides line segment AB in the ratio $k : 1$.



Coordinates of P

$$P = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$(4, m) = \left(\frac{k \times 6 + 1 \times 2}{k+1}, \frac{k \times (-3) + 1 \times 3}{k+1}\right)$$

$$(4, m) = \left(\frac{6k+2}{k+1}, \frac{-3k+3}{k+1}\right)$$

On comparing, we get

$$\left(\frac{6k+2}{k+1}\right) = 4$$

$$\Rightarrow \quad 6k + 2 = 4 + 4k$$

$$\Rightarrow \quad 6k - 4k = 4 - 2$$

$$\Rightarrow \quad 2k = 2$$

$$\Rightarrow \quad k = 1$$

Hence, P divides AB in the ratio $1 : 1$. **Ans.**

From (i), $\dfrac{-3(1)+3}{1+1} = m$

$$\Rightarrow \quad \frac{-3+3}{2} = m$$

$$\Rightarrow \quad m = 0$$ **Ans.**

11. **Two different dice are tossed together. Find the probability :**

(i) of getting a doublet.

(ii) of getting a sum 10, of the numbers on the two dice. [2]

Solution : Total outcomes on tossing two different dice = 36

(i) A : getting a doublet

$A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$\therefore$ Number of favourable outcomes of $A = 6$

$$\therefore \quad P(A) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{6}{36} = \frac{1}{6}$$ **Ans.**

(ii) B : getting a sum 10.

$B = \{(4, 6), (5, 5), (6, 4)\}$

$\therefore$ Number of favourable outcomes of B = 3

$$\therefore \quad P(B) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$\therefore \quad = \frac{3}{36} = \frac{1}{12}$$ **Ans.**

12. **An integer is chosen at random between 1 and 100. Find the probability that it is :**

(i) divisible by 8.

(ii) not divisible by 8. [2]

Solution : Total numbers are 2, 3, 4, ..., 99

(i) Let E be the event of getting a number divisible by 8.

$E = \{8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96\}$

$= 12$

$$P(E) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{12}{98} = 0.1224$$

(ii) Let E' be the event of getting a number not divisible by 8.

Then, $P(E') = 1 - P(E)$

$= 1 - 0.1224$

$= 0.8756$ **Ans.**

SECTION – C

13. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers. [3]

Solution :

2	404
2	202
101	101
	1

2	96
2	48
2	24
2	112
2	6
3	3
	1

Prime factorization of 404 = 2 × 2 × 101

Prime factorization of 96 = 2 × 2 × 2 × 2 × 2 × 3

$\therefore$ HCF = 2 × 2 = 4

And LCM = 2 × 2 × 2 × 2 × 2 × 3 × 101

= 9696

$\therefore$ HCF = 4, LCM = 9696 **Ans.**

Verification

HCF × LCM = Product of two numbers

4 × 9696 = 404 × 96

38784 = 38784 **Hence Verified.**

14. Find all zeroes of the polynomial $(2x^4 - 9x^3 + 5x^2 + 3x - 1)$ if two of its zeroes are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$. [3]

Solution : Here, $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

And two of its zeroes are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$.

Quadratic polynomial with zeroes is given by,

$\{x - (2 + \sqrt{3})\}.\{x - (2 - \sqrt{3})\}$

$\Rightarrow$ $(x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$

$\Rightarrow$ $(x - 2)^2 - (\sqrt{3})^2$

$\Rightarrow$ $x^2 - 4x + 4 - 3$

$\Rightarrow$ $x^2 - 4x + 1 = g(x)$ (say)

Now, $g(x)$ will be a factor of $p(x)$ so $g(x)$ will be divisible by $p(x)$

$$
\begin{array}{r|l}
 & 2x^2 - x - 1 \\
x^2 - 4x + 1 & 2x^4 - 9x^3 + 5x^2 + 3x - 1 \\
 & 2x^4 \mp 8x^3 \pm 2x^2 \\
\hline
 & -x^3 + 3x^2 + 3x \\
 & \mp x^3 \pm 4x^2 \mp x \\
\hline
 & -x^2 + 4x - 1 \\
 & \mp x^2 \pm 4x \mp 1 \\
\hline
 & \times
\end{array}
$$

For other zeroes,

$2x^2 - x - 1 = 0$

$2x^2 - 2x + x - 1 = 0$

$2x(x - 1) + 1(x - 1) = 0$

$(x - 1)(2x + 1) = 0$

or $x - 1 = 0$ $2x + 1 = 0$

$x = 1, x = \frac{-1}{2}$

Zeroes of $p(x)$ are

$1, \frac{-1}{2}, 2 + \sqrt{3}$ and $2 - \sqrt{3}$. **Ans.**

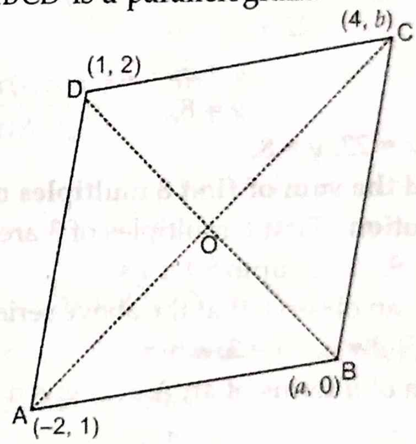
15. If A (–2, 1), B (a, 0), C (4, b) and D (1, 2) are the vertices of a parallelogram $ABCD$, find the values of a and b. Hence find the lengths of its sides. [3]

OR

If A (–5, 7), B (–4, –5), C (–1, –6) and D (4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral $ABCD$.

Solution :

Given, $ABCD$ is a parallelogram.



Midpoint of $AC = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-2 + 4}{2}, \frac{1 + b}{2}\right)$

$= \left(\frac{2}{2}, \frac{1 + b}{2}\right)$

$= \left(1, \frac{1 + b}{2}\right)$

Midpoint of $BD = \left(\frac{x'_1 + x'_2}{2}, \frac{y'_1 + y'_2}{2}\right)$

$= \left(\frac{a + 1}{2}, \frac{0 + 2}{2}\right)$

$= \left(\frac{a + 1}{2}, \frac{2}{2}\right)$

$= \left(\frac{a + 1}{2}, 1\right)$

Since, diagonals of a parallelogram bisect each other,

$$\therefore \quad \left(1, \frac{1+b}{2}\right) = \left(\frac{a+1}{2}, 1\right)$$

On comparing, we get

$$\therefore \quad \frac{a+1}{2} = 1 \qquad \frac{1+b}{2} = 1$$

$$\Rightarrow \quad a+1 = 2 \qquad \Rightarrow 1+b = 2$$

$$\Rightarrow \quad a = 1 \qquad \Rightarrow b = 1$$

Ans.

Therefore, the coordinates of vertices of parallelogram $ABCD$ are $A(-2, 1)$, $B(1, 0)$, $C(4, 1)$ and $D(1, 2)$

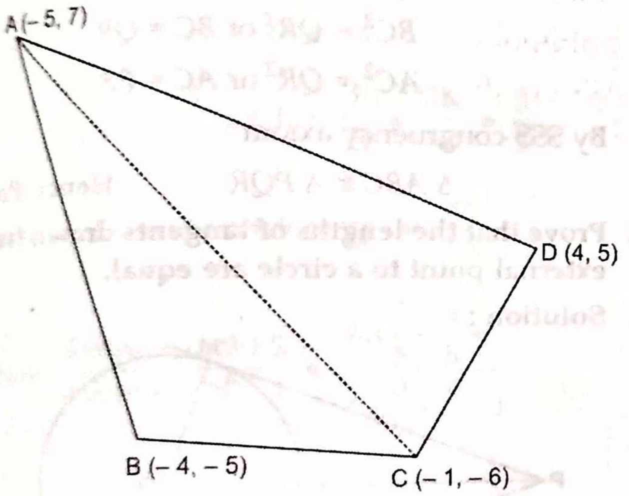
Length of side $AB = DC = \sqrt{(1+2)^2 + (0-1)^2}$

$$= \sqrt{9+1} = \sqrt{10} \text{ units}$$

And, $AD = BC = \sqrt{(1+2)^2 + (2-1)^2}$

$$= \sqrt{9+1} = \sqrt{10} \text{ units}$$ **Ans.**

OR



Given $ABCD$ is quadrilateral.

By joining points A and C, the quadrilateral is divided into two triangles.

Now, Area of quad. $ABCD$ = Area of ΔABC + Area of ΔACD

Area of ΔABC

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[-5(-5+6) - 4(-6-7) - 1(7+5)]$$

$$= \frac{1}{2}[-5(1) - 4(-13) - 1(12)]$$

$$= \frac{1}{2}(-5 + 52 - 12)$$

$$= \frac{1}{2}(35) = \frac{35}{2} \text{ sq. units.}$$

Area of ΔADC

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[-5(5+6) + 4(-6-7) + (-1)(7-5)]$$

$$= \frac{1}{2}[-5(11) + 4(-13) - 1(2)]$$

$$= \frac{1}{2}(-55 + 52 - 12)$$

$$= \frac{1}{2}|-109| = \frac{109}{2} \text{ sq. units.}$$

Area of quadrilateral $ABCD$

$$= \frac{35}{2} + \frac{109}{2}$$

$$= \frac{144}{2} = 72 \text{ sq. units.}$$ **Ans.**

16. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed. **[3]**

Solution : Let the usual speed of plane be x km/h.

Increased speed = $(x + 100)$ km/h.

$\therefore$ Distance to cover = 1500 km.

Time taken by plane with usual speed = $\frac{1500}{x}$ hr.

Time taken by plane with increased speed

$$= \frac{1500}{(100+x)} \text{ hrs.}$$

According to the question,

$$\frac{1500}{x} - \frac{1500}{(100+x)} = \frac{30}{60} = \frac{1}{2}$$

$$1500\left[\frac{1}{x} - \frac{1}{x+100}\right] = \frac{1}{2}$$

$$1500\left[\frac{x+100-x}{(x)(x+100)}\right] = \frac{1}{2}$$

$$\frac{1500 \times 100}{x^2 + 100x} = \frac{1}{2}$$

$$x^2 + 100x = 300000$$

$$x^2 + 100x - 300000 = 0$$

$$x^2 + 600x - 500x - 300000 = 0$$

$$x(x + 600) - 500(x + 600) = 0$$

$$(x + 600)(x - 500) = 0$$

Either $x + 600 = 0$

$x = -600$ (Rejected)

or $x - 500 = 0$

$x = 500$

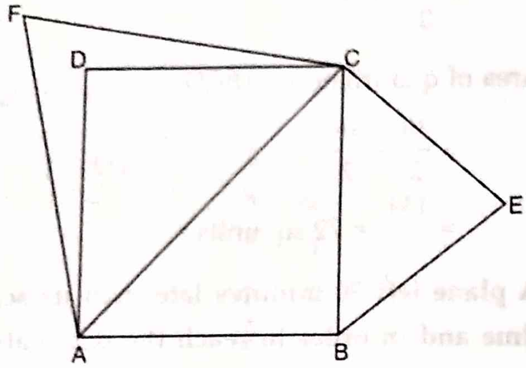
$\therefore$ Usual speed of plane = 500 km/hr. **Ans.**

17. Prove that the area of an equilateral triangle described on one side of the square is equal to half the area of the equilateral triangle described on one of its diagonal. **[3]**

OR

If the area of two similar triangles are equal, prove that they are congruent.

Solution : Let $ABCD$ be a square with side 'a'.



In ΔABC,

$$AC^2 = AB^2 + BC^2$$
$$= a^2 + a^2$$
$$= 2a^2$$
$$AC = \sqrt{2a^2} = \sqrt{2}\,a.$$

Area of equilateral ΔBEC (formed on side BC of square $ABCD$)

$$= \frac{\sqrt{3}}{4} \times (\text{side})^2$$
$$= \frac{\sqrt{3}}{4} a^2 \quad \text{....(i)}$$

Area of equilateral ΔACF (formed on diagonal AC of square $ABCD$)

$$= \frac{\sqrt{3}}{4} (\sqrt{2}\,a)^2$$
$$= \frac{\sqrt{3}}{4} (2a^2)$$
$$= 2\frac{\sqrt{3}}{4} a^2 \quad \text{....(ii)}$$

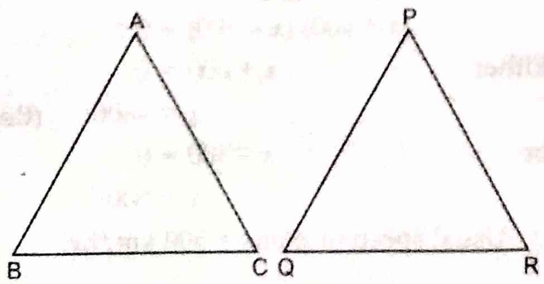
From eq. (i) and (ii),

$$\text{ar } \Delta ACF = 2 \times \text{ar } \Delta BCE$$

or $\quad \text{ar } (\Delta BCE) = \frac{1}{2} \text{ar } (\Delta ACF)$

i.e., area of triangle described on one side of square is half the area of triangle described on its diagonal. **Hence Proved.**

OR



Given, $\quad \Delta ABC \sim \Delta PQR$

And $\quad \text{ar } (\Delta ABC) = \text{ar } (\Delta PQR)$

To prove :

$$\Delta ABC \cong \Delta PQR$$

Proof :

Given, $\Delta ABC \sim \Delta PQR$

$$\therefore \quad \frac{\text{ar } (\Delta ABC)}{\text{ar } (\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

(Ratio of area of similar triangles is equal to the square of corresponding sides)

But $\quad \frac{\text{ar } (\Delta ABC)}{\text{ar } (\Delta PQR)} = 1 \quad$ (Given)

$$\therefore \quad \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} = 1$$

So, $\quad AB^2 = PQ^2 \text{ or } AB = PQ$

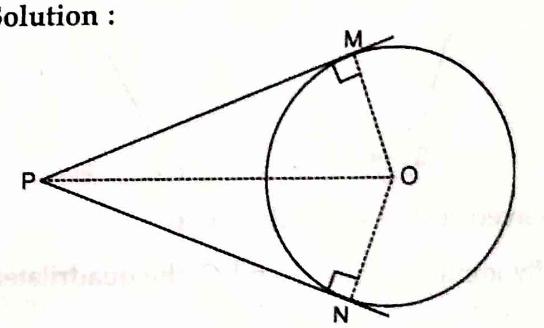
$$BC^2 = QR^2 \text{ or } BC = QR$$
$$AC^2 = QR^2 \text{ or } AC = PR$$

By SSS congruency axiom

$\Delta ABC \cong \Delta PQR$ **Hence Proved.**

18. Prove that the lengths of tangents drawn from an external point to a circle are equal. **[3]**

Solution :



Given : a circle with centre O on which two tangents PM and PN are drawn from an external point P.

To prove :

$$PM = PN$$

Construction : Join OM, ON and OP.

Proof : Since tangent and radius are perpendicular at point of contact,

$$\therefore \quad \angle OMP = \angle ONP = 90°$$

In ΔPOM and ΔPON,

$OM = ON$ (Radii)

$\angle OMP = \angle ONP$

$PO = OP$ (Common)

$\therefore \quad \Delta OMP \cong \Delta ONP$ (RHS cong.)

$\therefore \quad PM = PN$ (CPCT)

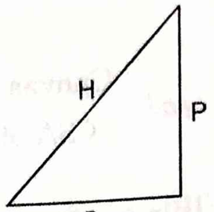
Hence Proved.

19. If $4 \tan \theta = 3$, evaluate $\left(\dfrac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$ **[3]**

OR

If $\tan 2A = \cot (A - 18°)$, where 2A is an acute angle, find the value of A.

Solution : Given, $4 \tan \theta = 3$,



$\Rightarrow$ $$\tan \theta = \frac{3}{4}\left(=\frac{P}{B}\right)$$

$P = 3K, B = 4K,$

Now, $$H = \sqrt{P^2 + B^2}$$

$$= \sqrt{(3K)^2 + (4K)^2}$$

$$= \sqrt{9K^2 + 16K^2}$$

$$= \sqrt{25K^2}$$

$\Rightarrow$ $$H = 5K$$

$\therefore$ $$\sin \theta = \frac{P}{H} = \frac{3K}{5K} = \frac{3}{5}$$

and $$\cos \theta = \frac{B}{H} = \frac{4K}{5K} = \frac{4}{5}$$

Now, $$\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \frac{4 \times \frac{3}{5} - \frac{4}{5} + 1}{4 \times \frac{3}{5} + \frac{4}{5} - 1}$$

$$= \frac{\left(\frac{12}{5} - \frac{4}{5} + 1\right)}{\left(\frac{12}{5} + \frac{4}{5} - 1\right)}$$

$$= \frac{\left(\frac{12 - 4 + 5}{5}\right)}{\left(\frac{12 + 4 - 5}{5}\right)}$$

$$= \frac{13/5}{11/5}$$

$$= \frac{13}{11}$$ **Ans.**

OR

Given, $$\tan 2A = \cot (A - 18°)$$

$\Rightarrow$ $$\cot (90° - 2A) = \cot (A - 18°)$$
$[\because \tan\theta = \cot (90° - \theta)]$

$\Rightarrow$ $$90° - 2A = A - 18°$$

$\Rightarrow$ $$90° + 18° = A + 2A$$

$\Rightarrow$ $$108° = 3A$$

$\Rightarrow$ $$A = \frac{108°}{3}$$

$\Rightarrow$ $$A = 36°$$ **Ans.**

20. **Find the area of the shaded region in Fig. 2, where arcs drawn with centres *A*, *B*, *C* and *D* intersect in pairs at mid-points *P*, *Q*, *R* and *S* of the sides *AB*, *BC*, *CD* and *DA* respectively of a square *ABCD* of side 12 cm. [Use $\pi = 3.14$]** **[3]**

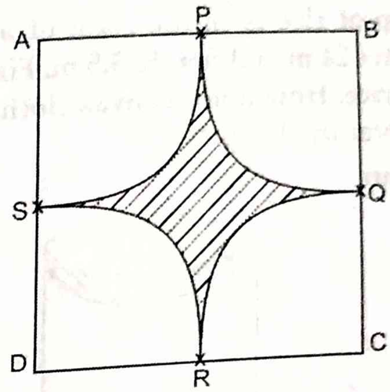
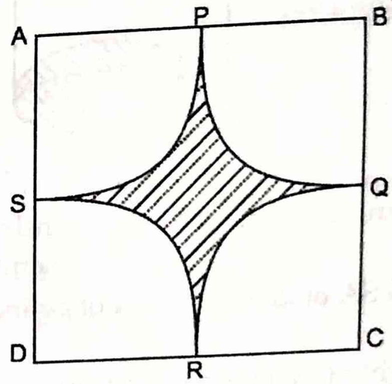


Figure 2

Solution :



Given, *ABCD* is a square of side 12 cm.

P, *Q*, *R* and *S* are the mid points of sides *AB*, *BC*, *CD* and *AD* respectively.

Area of shaded region

$$= \text{Area of square} - 4 \times \text{Area of quadrant}$$

$$= a^2 - 4 \times \frac{1}{4}\pi r^2$$

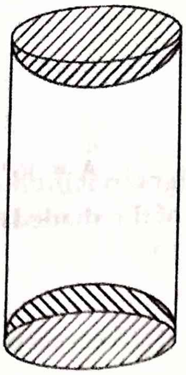
$$= (12)^2 - 3.14 \times (6)^2$$

$$= 144 - 3.14 \times 36$$

$$= 144 - 113.04$$

$$= 30.96 \text{ cm}^2$$ **Ans.**

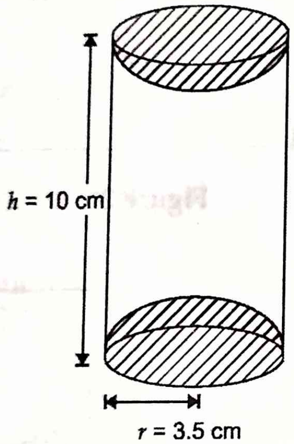
21. **A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 3. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article.** **[3]**



OR

A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap ?

Solution :



Given, Radius (r) of cylinder = Radius of hemisphere = 3.5 cm.

Total SA of article = CSA of cylinder + 2 × CSA of hemisphere

Height of cylinder, h = 10 cm

$$\begin{aligned}\text{TSA} &= 2\pi rh + 2 \times 2\pi r^2 \\ &= 2\pi rh + 4\pi r^2 \\ &= 2\pi r\,(h + 2r) \\ &= 2 \times \frac{22}{7} \times 3.5\,(10 + 2 \times 3.5) \\ &= 2 \times 22 \times 0.5 \times (10 + 7) \\ &= 2 \times 11 \times 17 \\ &= 374 \text{ cm}^2\end{aligned}$$

OR

Base diameter of cone = 24 m.

$\therefore$ Radius r = 12 m

Height of cone, h = 3.5 m

Volume of rice in conical heap

$$\begin{aligned}&= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5 \\ &= 528 \text{ m}^3\end{aligned}$$

Now, slant height, $l = \sqrt{h^2 + r^2}$

$$\begin{aligned}&= \sqrt{(3.5)^2 + (12)^2} \\ &= \sqrt{12.25 + 144} \\ &= \sqrt{156.25} \\ &= 12.5 \text{ m}\end{aligned}$$

Canvas cloth required to just cover the heap = CSA of conical heap = πrl

$$\begin{aligned}&= \frac{22}{7} \times 12 \times 12.5 \\ &= \frac{3300}{7} \text{ m}^2 \\ &= 471.43 \text{ m}^2.\end{aligned}$$

Ans.

22. The table below shows the salaries of 280 persons : **[3]**

Salary (In thousand ₹)	No. of Persons
5 – 10	49
10 – 15	133
15 – 20	63
20 – 25	15
25 – 30	6
30 – 35	7
35 – 40	4
40 – 45	2
45 – 50	1

Calculate the median salary of the data.

Solution :

Salary	No. of Persons	Cumulative frequency ($c.f.$)
5 – 10	49	49
l 10 – 15	133 f	182
15 – 20	63	245
20 – 25	15	260
25 – 30	6	266
30 – 35	7	273
35 – 40	4	277
40 – 45	2	279
45 – 50	1	280
Total	280	

$$\frac{N}{2} = \frac{280}{2} = 140$$

The cumulative frequency just greater than 140 is 182.

$\therefore$ Median class is 10 – 15.

$\Rightarrow l = 10, h = 5, N = 280, c.f. = 49$ and $f = 133$

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h$$

$$= 10 + \left(\frac{140-49}{133}\right) \times 5$$

$$= 10 + \frac{91 \times 5}{133}$$

$$= 10 + \frac{455}{133}$$

$$= 10 + 3.42$$

$$= 13.42$$ **Ans.**

SECTION – D

A motor boat whose speed is 18 km/hr in still water takes 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. [4]

OR

A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed ?

Solution : Given, speed of motor boat in still water = 18 km/hr.

Let speed of stream = x km/hr.

$\therefore$ Speed of boat downstream = $(18 + x)$ km/hr.

And speed of boat upstream = $(18 - x)$ km/hr.

Time of the upstream journey = $\frac{24}{(18-x)}$

Time of the downstream journey = $\frac{24}{(18+x)}$

According to the question,

$$\frac{24}{(18-x)} - \frac{24}{(18+x)} = 1$$

$$\frac{24(18+x)-24(18-x)}{(18-x)(18+x)} = 1$$

$$\frac{24 \times 18 + 24x - 24 \times 18 + 24x}{324 - x^2} = 1$$

$$\frac{48x}{324 - x^2} = 1$$

$$48x = 324 - x^2$$

$\Rightarrow$ $x^2 + 48x - 324 = 0$

$\Rightarrow$ $x^2 + 54x - 6x - 324 = 0$

$\Rightarrow$ $x(x + 54) - 6(x + 54) = 0$

$\Rightarrow$ $(x + 54)(x - 6) = 0$

Either $x + 54 = 0$

$x = -54$

Rejected, as speed cannot be negative

or $x - 6 = 0$

$x = 6$

Thus, the speed of the stream is 6 km/hr. **Ans.**

OR

Let original average speed of train be x km/hr.

$\therefore$ Increased speed of train = $(x + 6)$ km/hr.

Time taken to cover 63 km with average speed $= \frac{63}{x}$ hr.

Time taken to cover 72 km with increased speed $= \frac{72}{(x+6)}$ hr.

According to the question,

$$\frac{63}{x} + \frac{72}{x+6} = 3$$

$\Rightarrow$ $\frac{63(x+6)+72(x)}{(x)(x+6)} = 3$

$\Rightarrow$ $\frac{63x+378+72x}{x^2+6x} = 3$

$\Rightarrow$ $135x + 378 = 3(x^2 + 6x)$

$\Rightarrow$ $135x + 378 = 3x^2 + 18x$

$\Rightarrow$ $3x^2 + 18x - 135x - 378 = 0$

$\Rightarrow$ $3x^2 - 117x - 378 = 0$

$\Rightarrow$ $3(x^2 - 39x - 126) = 0$

$\Rightarrow$ $x^2 - 39x - 126 = 0$

$\Rightarrow$ $x^2 - 42x + 3x - 126 = 0$

$\Rightarrow$ $x(x - 42) + 3(x - 42) = 0$

$\Rightarrow$ $(x - 42)(x + 3) = 0$

Either $x - 42 = 0$

$x = 42$

or $x + 3 = 0$

$x = -3$

Rejected (as speed cannot be negative)

Thus, average speed of train is 42 km/hr. **Ans.**

24. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15. Find the numbers. [4]

Solution: Let the first term of AP be a and d be the common difference.

Let four consecutive terms of an AP be $a-3d$, $a-d$, $a+d$ and $a+3d$

According to the question,

$$a - 3d + a - d + a + d + a + 3d = 32$$

$\Rightarrow$ $4a = 32$

$\Rightarrow$ $a = 8$ ...(i)

Also,

$$(a - 3d)(a + 3d) : (a - d)(a + d) = 7 : 15$$

$$\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$$

$$\frac{64 - 9d^2}{64 - d^2} = \frac{7}{15}$$

[From (i) put $a = 8$]

$$15(64 - 9d^2) = 7(64 - d^2)$$

$$960 - 135d^2 = 448 - 7d^2$$

$$960 - 448 = 135d^2 - 7d^2$$

$$512 = 128d^2$$

$$d^2 = \frac{512}{128}$$

$$d^2 = 4$$

$$\Rightarrow \quad d = \pm 2$$

For $d = 2$, four terms of AP are,

$$a - 3d = 8 - 3(2) = 2$$
$$a - d = 8 - 2 = 6$$
$$a + d = 8 + 2 = 10$$
$$a + 3d = 8 + 3(2) = 14$$

For $d = -2$, four terms are

$$a - 3d = 8 - 3(-2) = 14$$
$$a - d = 8 - (-2) = 10$$
$$a + d = 8 + (-2) = 6$$
$$a + 3d = 8 + 3(-2) = 2$$

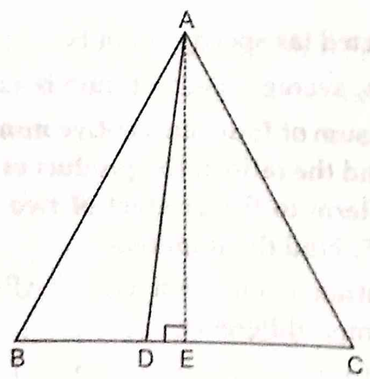
Thus, the four terms of AP series are 2, 6, 10, 14 or 14, 10, 6, 2. **Ans.**

25. In an equilateral ΔABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove that $9(AD)^2 = 7(AB)^2$. **[4]**

OR

Prove that, in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Solution :



Given, ABC is an equilateral triangle and D is a point on BC such that $BD = \frac{1}{3}BC$.

To prove :

$$9AD^2 = 7AB^2$$

Construction : Draw $AE \perp BC$

Proof : $BD = \frac{1}{3}BC$ ...(i)(Given)

$AE \perp BC$

We know that perpendicular from a vertex of equilateral triangle to the base divides base in two equal parts.

$$\therefore \quad BE = EC = \frac{1}{2}BC \quad \text{...(ii)}$$

In ΔAEB,

$$AD^2 = AE^2 + DE^2$$

(Pythagoras theorem)

or $\quad AE^2 = AD^2 - DE^2$ ...(iii)

Similarly, In ΔAEB,

$$AB^2 = AE^2 + BE^2$$

$$= AD^2 - DE^2 + \left(\frac{1}{2}BC\right)^2 \quad \text{[From (ii) and (iii)]}$$

$$= AD^2 - (BE - BD)^2 + \frac{1}{4}BC^2$$

$$= AD^2 - BE^2 - BD^2 + 2.BE.BD + \frac{1}{4}BC^2$$

$$= AD^2 - \left(\frac{1}{2}BC\right)^2 - \left(\frac{1}{3}BC\right)^2 + 2.\frac{1}{2}BC.\frac{1}{3}BC + \frac{1}{4}BC^2$$

$$AB^2 = AD^2 - \frac{1}{9}BC^2 + \frac{1}{3}BC^2$$

$$\Rightarrow \quad AB^2 = AD^2 + \frac{2}{9}BC^2$$

$$\Rightarrow \quad AB^2 = AD^2 + \frac{2}{9}AB^2 \quad (\because BC = AB)$$

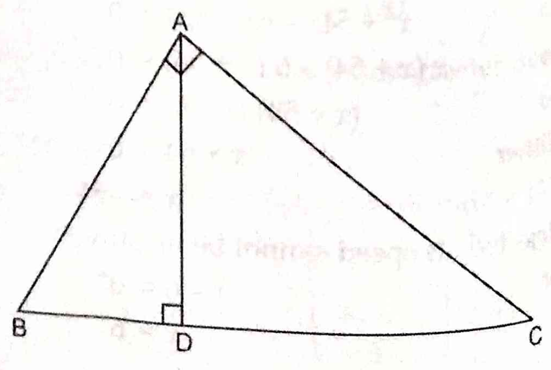
$$\Rightarrow \quad AB^2 - \frac{2}{9}AB^2 = AD^2$$

$$\Rightarrow \quad \frac{7}{9}AB^2 = AD^2$$

$$\Rightarrow \quad 7AB^2 = 9AD^2$$

Or $\quad 9(AD)^2 = 7(AB)^2$ **Hence Proved.**

OR



Given : ΔABC is a right angle triangle, right angled at A.

To prove : $BC^2 = AB^2 + AC^2$

Construction : Draw $AD \perp BC$.

Proof : In ΔADB and ΔBAC,

$\angle B = \angle B$ (Common)

$\angle ADB = \angle BAC$ (Each 90°)

$\therefore$ $\Delta ADB \sim \Delta BAC$

(By AA similarity axiom)

$\frac{AB}{BC} = \frac{BD}{AB}$ (CPCT)

$\therefore$ $AB^2 = BC \times BD$ ...(i)

Similarly,

$\Delta ADC \sim \Delta CAB$

$\frac{AC}{BC} = \frac{DC}{AC}$

$AC^2 = BC \times DC$ ...(ii)

On adding eq. (i) and (ii)

$AB^2 + AC^2 = BC \times BD + BC \times CD$

$= BC(BD + CD)$

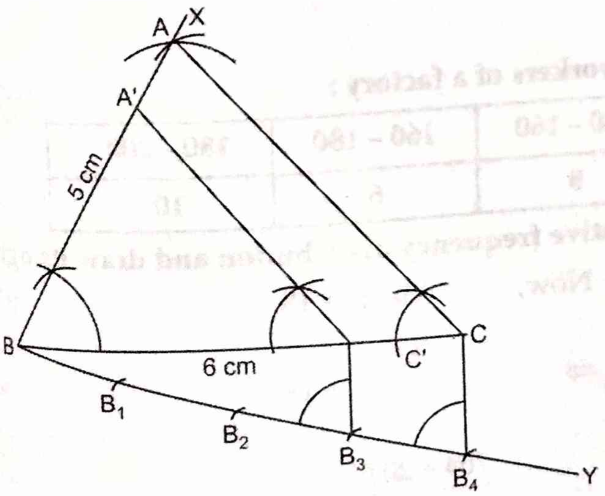
$= BC \times BC$

$AB^2 + AC^2 = BC^2$

$\Rightarrow$ $BC^2 = AB^2 + AC^2$ **Hence Proved.**

6. Draw a triangle ABC with $BC = 6$ cm, $AB = 5$ cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the ΔABC. **[4]**

Solution :



Steps of construction –

(i) Draw a line segment $BC = 6$ cm.

(ii) Construct $\angle XBC = 60°$.

(iii) With B as centre and radius equal to 5 cm, draw an arc intersecting XB at A.

(iv) Join AC. Thus, ΔABC is obtained.

(v) Draw an acute angle $\angle CBY$ below of B.

(vi) Mark 4-equal parts on BY as B_1, B_2, B_3 and B_4.

(vii) Join B_4 to C.

(viii) From B_3, draw a line parallel to B_4C intersecting BC at C'.

(ix) Draw another line parallel to CA from C', intersecting AB at A.'

(x) $\Delta A'BC'$ is required triangle which is similar to ΔABC such that $BC' = \frac{3}{4}BC$.

27. Prove that : $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$. **[4]**

Solution : L.H.S. $= \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$$= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$$

$$= \frac{\sin A}{\cos A}\frac{(1 - 2\sin^2 A)}{[2(1 - \sin^2 A) - 1]}$$

$[\because \cos^2 A = 1 - \sin^2 A]$

$$= \frac{\sin A}{\cos A}\frac{(1-2\sin^2 A)}{(2-2\sin^2 A-1)}$$

$$= \frac{\sin A}{\cos A}\frac{(1 - 2\sin^2 A)}{(1 - 2\sin^2 A)}$$

$= \tan A =$ R.H.S. **Hence Proved.**

28. The diameters of the lower and upper ends of a bucket in the form of a frustum of a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find :

(i) The area of the metal sheet used to make the bucket.

(ii) Why we should avoid the bucket made by ordinary plastic ? [Use $\pi = 3.14$] **[4]**

Solution : Given, Height of frustum, $h = 24$ cm.

Diameter of lower end = 10 cm.

$\therefore$ Radius of lower end, $r = 5$ cm.

Diameter of upper end = 30 cm.

$\therefore$ Radius of upper end, R = 15 cm.

Slant height, $l = \sqrt{h^2 + (R - r)^2}$

$= \sqrt{(24)^2 + (15 - 5)^2}$

$= \sqrt{576 + 100}$

$= \sqrt{676}$

$= 26$ cm

(i) Area of metal sheet used to make the bucket

$=$ CSA of frustum + Area of base

$= \pi l(R + r) + \pi r^2$

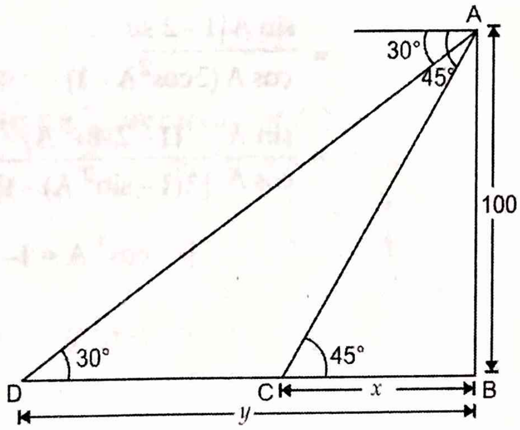
$= \pi[26(15 + 5) + (5)^2]$

$= 3.14 (26 \times 20 + 25)$
$= 3.14 (520 + 25)$
$= 3.14 \times 545$
$= 1711.3 \text{ cm}^2$ **Ans.**

(ii) We should avoid the bucket made by ordinary plastic because plastic is harmful to the environment and to protect the environment its use should be avoided.

29. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$] **[4]**

Solution :



Let AB be the light house and two ships be at C and D.

In ΔABC,

$$\frac{BC}{AB} = \cot 45°$$

$$\Rightarrow \quad \frac{x}{100} = 1$$

$$\Rightarrow \quad x = 100 \quad \text{...(i)}$$

Similarly, in ΔABD,

$$\frac{BD}{AB} = \cot 30°$$

$$\Rightarrow \quad \frac{y}{100} = \sqrt{3}$$

$$\Rightarrow \quad y = 100\sqrt{3} \quad \text{...(ii)}$$

Distance between two ships $= y - x$

$= 100\sqrt{3} - 100$ [from (i) and (ii)]

$= 100(\sqrt{3} - 1)$

$= 100 (1.732 - 1)$

$= 100 (0.732)$

$= 73.2$ m **Ans.**

30. The mean of the following distribution is 18. Find the frequency f of the class 19–21.

Class	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
Frequency	3	6	9	13	f	5	4

[4]

OR

The following distribution gives the daily income of 50 workers of a factory :

Daily Income (in ₹)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

Solution :

C.I.	Mid value x_i	f_i	f_ix_i
11–13	12	3	36
13–15	14	6	84
15–17	16	9	144
17–19	18	13	234
19–21	20	f	20f
21–23	22	5	110
23–25	24	4	96
Total		$\Sigma f_i = 40 + f$	$\Sigma f_ix_i = 704 + 20f$

Now, Mean = 18 (Given)

$$\Rightarrow \quad \frac{\Sigma f_i x_i}{\Sigma f_i} = 18$$

$$\therefore \quad \frac{704 + 20f}{40 + f} = 18$$

$$\Rightarrow \quad 704 + 20f = 18(40 + f)$$

$$\Rightarrow \quad 704 + 20f = 720 + 18f$$

$$\Rightarrow \quad 20f - 18f = 720 - 704$$

$$\Rightarrow \quad 2f = 16$$

$$\Rightarrow \quad f = 8$$ **Ans.**

OR

Less than type cumulative frequency distribution :

Daily Income	No. of workers
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50