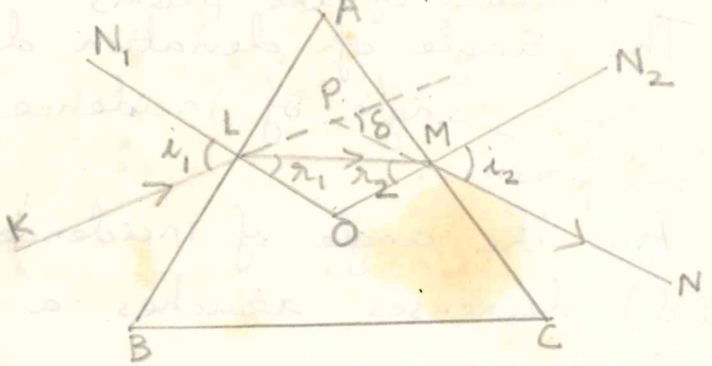
VIII [B]

Dispersion of light

Prism: A prism is a transparent medium bound by two plane faces inclined to each other at a certain angle. These plane faces are known as the refracting faces and the angle formed between them is known as the angle of the prism. The line along which the two refracting faces meet is the refracting edge of the prism.

Refraction through a prism: and prism formula – To determine the refractive index of the prism.



KL is the incident ray, incident on the face AB of the prism. It is refracted along LM and on being incident on the face AC emerges along MN. The ray of light undergoes two refractions on passing through the prism and hence deviates through a certain angle from the original path.

To calculate the angle of deviation

Consider $\triangle PLM$,

$$\delta = \angle PLM + \angle PML$$
$$= (i_1 - r_1) + (i_2 - r_2)$$
$$= (i_1 + i_2) - (r_1 + r_2) \quad \text{— (1)}$$

Considering Δ OLM,

$$\angle O + r_1 + r_2 = 180^\circ \quad — (2)$$

In quadilateral ALOM

$$\angle L + \angle M = 180^\circ$$ [each angle = 90°, normal to the surface]

$$\Rightarrow \angle A + \angle O = 180^\circ \quad — (3)$$

Combining (2) and (3)

$$\angle O + r_1 + r_2 = \angle A + \angle O$$

$$\boxed{A = r_1 + r_2} \quad — (4)$$

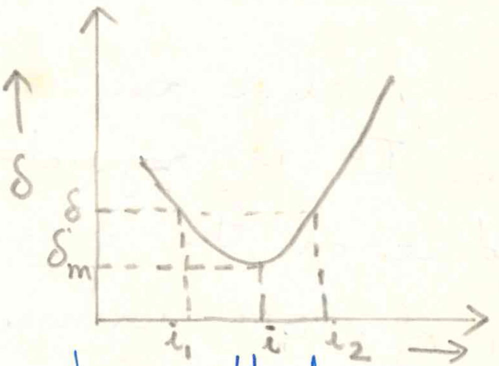
Substituting (4) in (1)

$$\boxed{\delta = (i_1 + i_2) - A} \quad — (5)$$

This is the deviation for a large angle prism.

The angle of deviation depends on the angle of the prism, angle of incidence and the material of the prism.

As the angle of incidence (i) increases the deviation (δ) decreases reaches a minimum and again increases.



From the graph it is clear that corresponding to one angle of δ there are two angles of incidence (i_1 & i_2). However at minimum deviation $\delta = \delta_m$ (a) $i_1 = i_2$ (b) the incident ray and emergent ray are symmetrical with respect to the refracting faces (c) $\therefore$ refracted ray is parallel to the base of the prism.

Thus at minimum deviation position

$$i_1 = i_2 = i$$

$$\therefore r_1 = r_2 = r$$

Since $r_1 + r_2 = A$.

$$r = A/2 \quad \text{— (1)}$$

We know $\delta = (i_1 + i_2) - A$

$$\therefore \delta_m = 2i - A$$

$$i = \frac{A + \delta_m}{2} \quad \text{— (2)}$$

According to Snell's law, $\mu = \dfrac{\sin i}{\sin r}$ — (3)

Substituting (1) and (2) in (3)

$$\boxed{\mu = \frac{\sin \frac{A+\delta_m}{2}}{\sin A/2}}$$

→ This is the Prism formula.

<u>Calculation of angle of deviation for a small angle prism.</u>

From Snell's law, $\mu = \dfrac{\sin i}{\sin r}$.

If angles are small, $i_1 = \mu r_1$ — (1)

and $i_2 = \mu r_2$ — (2)

Adding (1) and (2)

$$i_1 + i_2 = \mu (r_1 + r_2) \quad \text{— (3)}$$

Substituting (3) in (5)

$$\delta = \mu (r_1 + r_2) - A$$

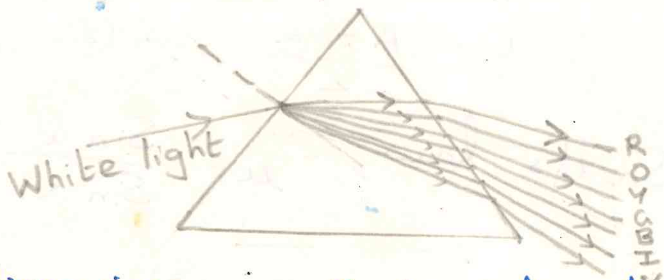
$$= \mu A - A$$

$$\boxed{\delta = A(\mu - 1)}$$

<u>Note</u>: The above expression gives the deviation produced by a prism of small angle provided the angle of incidence is small.

<u>Dispersion of light</u>: The phenomenon of the splitting of a beam of white light into its constituent colours on passing through a prism is dispersion.

The band of colours obtained is called as spectrum [VIBGYOR]



<u>Cause of dispersion</u> (a) Each colour has its characteristic wavelength (λ). The wavelength and velocity of violet colour is smaller than red colour.

(b) The refractive index (μ) of the material depends on wavelength as given by Cauchy's formula.

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \dots$$

A, B, C are constants.

$\therefore$ μ is different for different colours.

(c) Since $\delta = (\mu - 1)A$.

It is clear that different colours deviate through different angles on passing through the prism. Hence they are seen as separate.

-X- Note: Which colour deviates more, violet or red?

We know $\lambda_v < \lambda_r$

Since $\lambda \propto \frac{1}{\mu}$. (Cauchy's formula)

$\mu_v > \mu_r$.

Since $\mu \propto \delta$ [$\because \delta = (\mu - 1)A$]

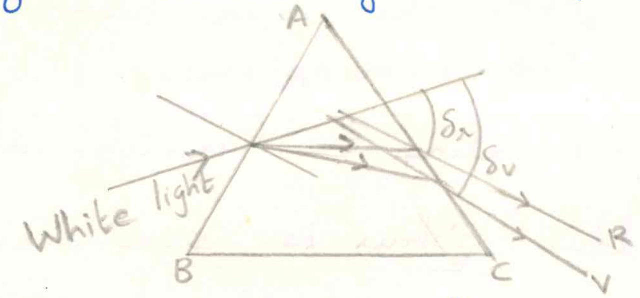
$\delta_v > \delta_r$.

<u>Angular dispersion</u>: Angular dispersion produced by a prism for white light is the difference in the angles of deviation of the two extreme colours.

Angular dispersion = $\delta_v - \delta_r$

$$\delta_v - \delta_r = (\mu_v - 1)A - (\mu_r - 1)A$$
$$= (\mu_v - \mu_r)A$$

$\therefore$ Angular dispersion depends on the angle of the prism and nature of material of the prism.



Dispersive power (ω): It is the ratio of angular dispersion to the mean deviation produced by the prism.

$$\omega = \frac{\delta_v - \delta_r}{\delta}$$ (δ is the deviation produced by yellow colour)

$$= \frac{(\mu_v - \mu_r)A}{(\mu - 1)A}$$

$$\therefore \omega = \frac{(\mu_v - \mu_r)}{\mu - 1}$$

Some important concepts

(a) Colour of the sky: The blue colour of the sky is on account of scattering of sunlight by the large number of molecules in the atmosphere.

According to Rayleigh, the intensity of scattering $(I) \propto \frac{1}{\lambda^4}$

where λ is the wavelength of light.

As $\lambda_b < \lambda_r$, maximum scattering is of blue colour. $\therefore$ The sky appears blue.

(b) Clouds appear white: Clouds being in the lower layers of the atmosphere contain large dust particles and water droplets. These scatterers do not obey Rayleigh scattering law ($\because$ size of particles is not comparable to the wavelength of light).