SECTION — A

1. If the quadratic equation $px^2 - 2\sqrt{5}px + 15 = 0$, has two equal roots then find the value of p. **[1]**

Solution : The given quadratic equation is,

$$px^2 - 2\sqrt{5}px + 15 = 0$$

This is of the form

$$ax^2 + bx + c = 0$$

Where, $a = p, b = -2\sqrt{5}p, c = 15$

We have,
$$\begin{aligned} D &= b^2 - 4ac \\ &= (-2\sqrt{5}p)^2 - 4 \times p \times 15 \\ &= 20p^2 - 60p \\ &= 20p(p-3) \end{aligned}$$

For real and equal roots, we must have :

$$D = 0, \Rightarrow 20p(p-3) = 0$$
$$\Rightarrow p = 0, p = 3$$

$p = 0$, is not possible as whole equation will be zero.

Hence, 3 is the required value of p. **Ans.**

2. In figure 1, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. Find the sun's altitude. **[1]**

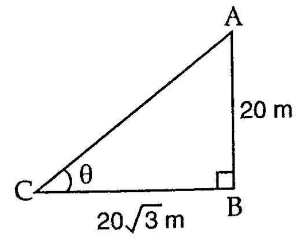


Figure 1

Solution : Given, AB is the tower and BC is its shadow.

$\therefore$ $$\tan\theta = \frac{AB}{BC}$$

$$\left[\because \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right]$$

$\Rightarrow$ $$\tan\theta = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $$\tan\theta = \tan 30^\circ \left[\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right]$$

$\Rightarrow$ $$\theta = 30^\circ$$ **Ans.**

3. Two different dice are tossed together. Find the probability that the product of two numbers on the top of the dice is 6. **[1]**

Solution : When two dice are thrown simultaneously, all possible outcomes are :

$$S = \begin{bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{bmatrix}$$

Total number of all outcomes $= 6 \times 6 = 36$

Favourable outcomes of getting the product as 6 are :

(2, 3), (3, 2), (1, 6), (6, 1)

Hence, number of favourable outcomes getting product as 6 is 4.

Probability that the product of the two numbers on the top of the die is 6

$$= \frac{4}{36}$$

$$= \frac{1}{9}$$ **Ans.**

4. In figure 2, PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT = 60^\circ$, find $\angle PRQ$. **[1]**

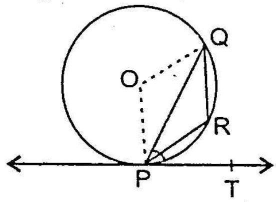


Figure 2

Solution : Given, O is the centre of the given circle

$\therefore$ OQ and OP are the radius of circle.

$\because$ PT is a tangent

$\therefore$ $$OP \perp PT$$

So, $$\angle OPT = 90^\circ$$

$\therefore$ $$\angle OPQ = 90^\circ - \angle QPT$$
$$\angle OPQ = 90^\circ - 60^\circ$$
[Given, $\angle QPT = 60^\circ$]
$$\angle OPQ = 30^\circ$$

$\therefore$ $$\angle OQP = 30^\circ$$ [$\because$ ΔOPQ is isosceles triangle]

Now, in ΔOPQ

$$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$$
$$\angle POQ + 30^\circ + 30^\circ = 180^\circ$$
$$\angle POQ = 120^\circ$$

reflex $\angle POQ = 360^\circ - 120^\circ = 240^\circ$

$\therefore \qquad \angle PRQ = \frac{1}{2}$ reflex $\angle POQ$

[$\because$ The angle substended by an arc of a circle at the centre is double the angle substended by it at any point on the remaining part of the circle]

$$\angle PRQ = \frac{1}{2} \times 240^\circ$$

Hence, $\qquad \angle PRQ = 120^\circ$ **Ans.**

SECTION — B

5. In figure 3, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120^\circ$, then prove that, $OR = PR + RQ$. **[2]**

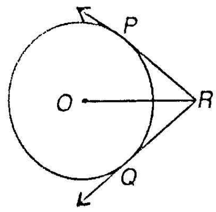
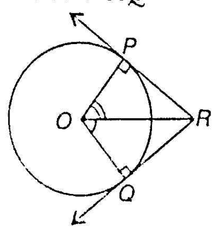


Figure 3

Solution : O is the centre of the circle and $\angle PRQ = 120^\circ$

Construction : Join OP, OQ

To prove : $OP = PR + RQ$



Proof : We know that,

Tangent to a circle is perpendicular to the radius at the point of tangent *i.e.*, $OP \perp RP$ and $OQ \perp RQ$.

$\therefore \qquad \angle OPR = \angle OQR = 90^\circ$

Now, in ΔOPR and ΔOQR,

$OP = OQ$ [Radius of circle]

$OR = OR$ [Common]

$\angle OPR = \angle OQR = 90^\circ$ [Each 90°]

$\therefore \qquad \Delta OPR \cong \Delta OQR$ [By SSA congruence]

So, $\qquad PR = QR$ [By c.p.c.t.]

and $\qquad \angle ORP = \angle ORQ = \frac{120^\circ}{2} = 60^\circ$

Now, in ΔOPR

$$\cos 60^\circ = \frac{PR}{OR} \qquad \left[\because \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}\right]$$

$$\frac{1}{2} = \frac{PR}{OR}$$

$OR = 2PR$

$OR = PR + PR$

$OR = PR + RQ$ [$\because PR = RQ$]

Hence, $\qquad OR = PR + RQ.$ **Hence Proved**

6. In figure 4, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm^2, then find the lengths of sides AB and AC. **[2]**

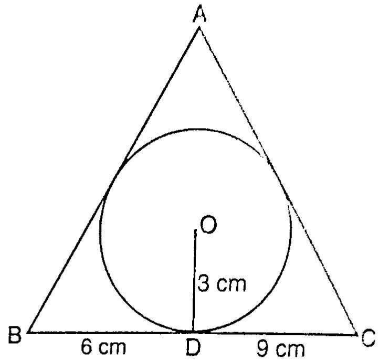
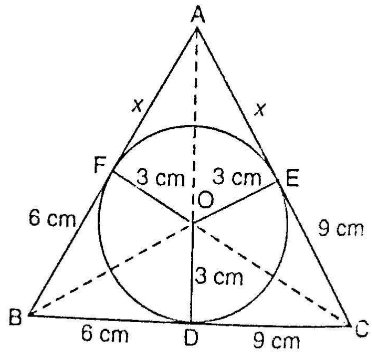


Figure 4

Solution : Given, in ΔABC, circle touch the triangle at point D, F and E respectively and let the lengths of the segment AF be x.



So, $\qquad BF = BD = 6$ cm [Tangent from point B]

$CE = CD = 9$ cm [Tangent from point C]

and $\qquad AE = AF = x$ cm [Tangent from point A]

Now, $\qquad$ Area of $\Delta OBC = \frac{1}{2} \times BC \times OD$

$$= \frac{1}{2} \times (6 + 9) \times 3$$

$$= \frac{45}{2} \text{ cm}^2$$

Area of $\Delta OCA = \frac{1}{2} \times AC \times OE$

$$= \frac{1}{2} \times (9 + x) \times 3$$

$$= \frac{3}{2}(9 + x) \text{ cm}^2$$

$$\text{Area of } \Delta BOA = \frac{1}{2} \times AB \times OF$$

$$= \frac{1}{2} \times (6 + x) \times 3$$

$$= \frac{3}{2} (6 + x) \text{ cm}^2$$

Area of $\Delta ABC = 54 \text{ cm}^2$ [Given]

$\because$ Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔBOA

$$54 = \frac{45}{2} + \frac{3}{2}(9+x) + \frac{3}{2}(6+x)$$

$$\Rightarrow \quad 54 \times 2 = 45 + 27 + 3x + 18 + 3x$$

$$\Rightarrow \quad 108 - 45 - 27 - 18 = 6x$$

$$\Rightarrow \quad 6x = 18$$

$$\Rightarrow \quad x = 3$$

So, $AB = AF + FB = x + 6 = 3 + 6 = 9$ cm

and $AC = AE + EC = x + 9 = 3 + 9 = 12$ cm

Hence, lengths of AB and AC are 9 cm and 12 cm respectively. **Ans.**

7. Solve the following quadratic equation for x:

$$4x^2 + 4bx - (a^2 - b^2) = 0$$ **[2]**

Solution : The given equation is

$$4x^2 + 4bx - (a^2 - b^2) = 0 \quad \ldots(i)$$

Comparing equation (i) with quadratic equation $Ax^2 + Bx + C = 0$, we get

$A = 4, B = 4b, C = -(a^2 - b^2)$

By quadratic formula

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x = \frac{-4b \pm \sqrt{16b^2 + 4 \times 4 \times (a^2 - b^2)}}{2 \times 4}$$

$$x = \frac{-4b \pm \sqrt{16b^2 + 16a^2 - 16b^2}}{8}$$

$$x = \frac{-4b \pm 4a}{8}$$

$$x = \frac{-b \pm a}{2}$$

Therefore, $x = \frac{-b - a}{2} \Rightarrow -\left(\frac{a + b}{2}\right)$

or $x = \frac{-b + a}{2} \Rightarrow \frac{a - b}{2}$

Hence, $x = -\left(\frac{a + b}{2}\right)$ and $x = \frac{a - b}{2}$. **Ans.**

8. In an A.P., if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the A.P., where S_n denotes the sum of its first n terms. **[2]**

Solution : Given, $S_5 + S_7 = 167$

$$\Rightarrow \quad \frac{5}{2}(2a + 4d) + \frac{7}{2}(2a + 6d) = 167$$

$$\Rightarrow \quad \frac{5}{2} \times 2(a + 2d) + \frac{7}{2} \times 2(a + 3d) = 167$$

$$\Rightarrow \quad 5a + 10d + 7a + 21d = 167$$

$$\Rightarrow \quad 12a + 31d = 167 \quad \ldots(i)$$

$$\Rightarrow \quad \frac{10}{2}(2a + 9d) = 235$$

$$\Rightarrow \quad 10a + 45d = 235$$

$$\Rightarrow \quad 2a + 9d = 47 \quad \ldots(ii)$$

On multiplying equation (ii) by 6, we get :

$$12a + 54d = 282 \quad \ldots(iii)$$

On subtracting equation (i) from (iii), we get :

$$\begin{array}{r} 12a + 54d = 282 \\ 12a + 31d = 167 \\ - \quad\; - \quad\;\; - \\ \hline 23d = 115 \end{array}$$

$$\Rightarrow \quad d = 5$$

Substituting value of d in equation (i), we get

$$12a + 31 \times 5 = 167$$

$$12a + 155 = 167$$

$$\Rightarrow \quad 12a = 12$$

$$\Rightarrow \quad a = 1$$

Hence A.P. is 1, 6, 11.... **Ans.**

9. The points A(4, 7), B(p, 3) and C(7, 3) are the vertices of a right triangle, right-angled at B. Find the value of p. **[2]**

Solution : The given points are $A(4, 7)$, $B(p, 3)$ and $C(7, 3)$.

Since A, B and C are the vertices of a right angled triangle

then, $(AB)^2 + (BC)^2 = (AC)^2$ [By Pythagoras theorem]

$$[(p - 4)^2 + (3 - 7)^2] + [(7 - p)^2 + (3 - 3)^2] = [(7 - 4)^2 + (3 - 7)^2]$$

$$(p - 4)^2 + (-4)^2 + (7 - p)^2 = (3)^2 + (-4)^2$$

$$p^2 + 16 - 8p + 16 + 49 + p^2 - 14p = 9 + 16$$

$$2p^2 - 22p + 56 = 0$$

$$p^2 - 11p + 28 = 0$$

$$p^2 - 7p - 4p + 28 = 0$$

$$p(p - 7) - 4(p - 7) = 0$$

$p = 4$ or 7

$p \neq 7$ (As B and C will coincide)

So, $p = 4$. **Ans.**

10. Find the relation between x and y if the points $A(x, y)$, $B(-5, 7)$ and $C(-4, 5)$ are collinear. **[2]**

Solution : Given that the points $A(x, y)$, $B(-5, 7)$ and $C(-4, 5)$ are collinear.

So, the area formed by the vertices are 0.

Therefore,

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[x(7-5) - 5(5-y) - 4(y-7)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[x(2) - 5(5-y) - 4(y-7)] = 0$$

$$\Rightarrow \quad 2x - 25 + 5y - 4y + 28 = 0$$

$$\Rightarrow \quad 2x + y + 3 = 0$$

$$-2x - 3 = y$$

which is the required, relation between x and y *i.e.*, $y = -2x - 3$. **Ans.**

SECTION — C

11. The 14th term of an AP is twice its 8th term. If its 6th term is -8, then find the sum of its first 20 terms. **[3]**

Solution : In the given AP, let first term $= a$ and common difference $= d$

Then, $T_n = a + (n-1)d$

$\Rightarrow \quad T_{14} = a + (14-1)d = a + 13d$

and $\quad T_8 = a + (8-1)d = a + 7d$

Now, $\quad T_{14} = 2T_8$ (Given)

$$a + 13d = 2(a + 7d)$$

$$a + 13d = 2a + 14d$$

$$a = -d \quad \text{...(i)}$$

Also, $\quad T_6 = a + (6-1)d$

$\Rightarrow \quad a + 5d = -8$ ...(ii)

Putting the value of a from eq. (i), we get

$$-d + 5d = -8$$

$$4d = -8$$

$$d = -2$$

Substituting $d = -2$ in eq. (ii), we get

$$a + 5(-2) = -8$$

$$a = 10 - 8$$

$$a = 2$$

$\therefore$ Sum of first 20 terms is

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{20}{2}[2 \times 2 + (20-1)(-2)]$$

$$= 10[4 - 38]$$

$$= -340$$ **Ans.**

12. Solve for x :

$$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$$ **[3]**

Solution : We have, $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$$\Rightarrow \quad \sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$$

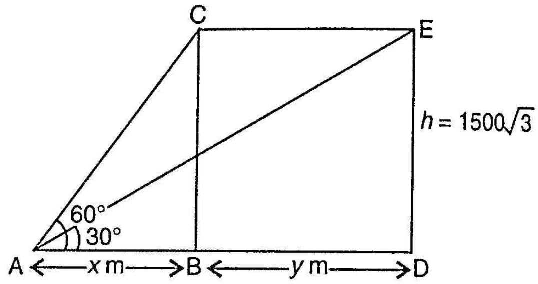
$$\Rightarrow \quad \sqrt{3}x(x - \sqrt{6}) + \sqrt{2}(x - \sqrt{6}) = 0$$

$$\Rightarrow \quad (\sqrt{3}x + \sqrt{2})(x - \sqrt{6}) = 0$$

$$\Rightarrow \quad x = -\sqrt{\frac{2}{3}} \text{ or } \sqrt{6}$$ **Ans.**

13. The angle of elevation of an aeroplane from point A on the ground is 60°. After flight of 15 seconds, the angle of elevation change to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the plane in km/hr. **[3]**

Solution :



Let BC be the height at which the aeroplane flying.

Then, $\quad BC = 1500\sqrt{3}$ m

In 15 seconds, the aeroplane moves from C to E and makes angle of elevation 30°.

Let $AB = x$ m, $BD = y$ m

So, $\quad AD = (x + y)$ m

In ΔABC,

$$\tan 60° = \frac{BC}{AB}$$

$$\sqrt{3} = \frac{1500\sqrt{3}}{x} \quad [\because \tan 60° = \sqrt{3}]$$

$$x = 1500 \text{ m} \quad \text{...(i)}$$

In ΔEAD

$$\tan 30° = \frac{ED}{AD} \quad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$\frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{x + y}$$

$$x + y = 1500 \times 3$$

$$y = 4500 - 1500 = 3000 \text{ m}$$

[Using equation (i)]

$$\text{Speed of aeroplane} = \frac{\text{Distance}}{\text{Time}}$$

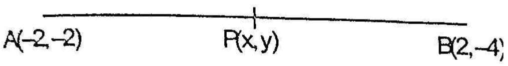
$$= \frac{3000}{15}$$

$$= 200 \text{ m/s or } 720 \text{ km/hr} \quad \textbf{Ans.}$$

14. If the coordinates of points A and B are (– 2, – 2) and (2, – 4) respectively find the coordinates of P such that AP = $\frac{3}{7}$ AB, where P lies on the line segment AB. [3]

Solution : Here P (x, y) divides line segment AB

such that AP = $\frac{3}{7}$AB



$$\Rightarrow \quad \frac{AP}{AB} = \frac{3}{7}$$

$$\Rightarrow \quad \frac{AB}{AP} = \frac{7}{3}$$

$$\Rightarrow \quad \frac{AB}{AP} - 1 = \frac{7}{3} - 1$$

$$\Rightarrow \quad \frac{AB - AP}{AP} = \frac{4}{3}$$

$$\Rightarrow \quad \frac{BP}{AP} = \frac{4}{3}$$

$$\Rightarrow \quad \frac{AP}{BP} = \frac{3}{4}$$

$\therefore$ P divides AB in the ratio 3 : 4 $(m : n)$

The coordinates of P are (x, y)

Therefore,

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

$$x = \frac{3 \times 2 + 4(-2)}{3+4}, y = \frac{3(-4) + 4(-2)}{3+4}$$

$$x = \frac{6-8}{7}, y = \frac{-12-8}{7}$$

$$x = \frac{-2}{7}, y = \frac{-20}{7}$$

Therefore, co-ordinates of $P(x, y)$ are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$

Ans.

15. A probability of selecting a red ball at random from a jar that contains only red, blue and orange is $\frac{1}{4}$. The probability of selecting a blue ball at random from the same jar is $\frac{1}{3}$. If the jar contains 10 orange balls, find the total number of balls in the jar. [3]

Solution : Given, a jar contains red, blue and orange balls.

Let the number of red balls $= x$

and the number of blue balls $= y$

Number of orange balls = 10

Then, total number of balls $= x + y + 10$

Let P be the probability of selecting a red ball from the jar

$$P = \frac{x}{x+y+10}$$

But P(a red balls) $= \frac{1}{4}$ (Given)

$$\therefore \quad \frac{1}{4} = \frac{x}{x+y+10}$$

$$x + y + 10 = 4x$$

$$3x - y = 10 \quad \ldots\text{(i)}$$

Similarly, P(a blue ball) $= \dfrac{y}{x+y+10}$

But P(a blue ball) $= \dfrac{1}{3}$

$$\therefore \quad \frac{1}{3} = \frac{y}{x+y+10}$$

$$x + y + 10 = 3y$$

$$x - 2y = -10 \quad \ldots\text{(ii)}$$

On multiplying equation (ii) by 3, we get

$$3x - 6y = -30 \quad \ldots\text{(iii)}$$

On subtracting equation (iii) from (i)

$$\begin{array}{r} 3x - y = 10 \\ 3x - 6y = -30 \\ - \quad + \qquad + \\ \hline 5y = 40 \\ y = 8 \end{array}$$

On putting the value of y in (iii), we get

$$3x - 6 \times 8 = -30$$

$$3x = -30 + 48$$

$$x = \frac{18}{3}$$

$$x = 6$$

Total number of balls $= x + y + 10$

$= 6 + 8 + 10$

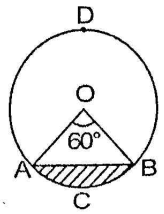
$= 24$

Hence, total number of balls in the jar is 24. **Ans.**

16. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is 60°. Also find the area of the corresponding major segment. [Use $\pi = \frac{22}{7}$] [3]

Solution : Let ACB be the given arc subtending an angle of 60° at the centre.

Here, $r = 14$ cm and $\theta = 60°$.



Area of the minor segment ACBA

= (Area of the sector $OACBO$) – (Area of ΔOAB)

$$= \frac{\pi r^2\theta}{360°} - \frac{1}{2}r^2\sin\theta$$

$$= \frac{22}{7}\times 14\times 14\times\frac{60°}{360°} - \frac{1}{2}\times 14\times 14\times\sin 60°$$

$$= \frac{308}{3} - 7\times 14\times\frac{\sqrt{3}}{2}$$

$$= \frac{308}{3} - 49\sqrt{3}$$

$= 17.89\text{ cm}^2$

Area of the major segment $BDAB$

= Area of circle – Area of minor segment $ACBA$

$= \pi r^2 - 17.89$

$$= \frac{22}{7}\times 14\times 14 - 17.89$$

$= 616 - 17.89$

$= 598.11 \approx 598\text{ cm}^2$ **Ans.**

17. Due to sudden floods, some welfare associations jointly requested the government to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4 m with the conical upper part of same diameter but of height 2.8 m and the canvas to be used costs ₹ 100 per sq. m. Find amount the associations will have to pay. What values are shown by these associations ? **[3]**

Solution : Diameter of the tent $(d) = 4.2$ m

∴ Radius of the tent $(r) = 2.1$ m $\left[\because r = \frac{d}{2}\right]$

Height of the cylindrical part of tent $(h) = 4$ m

Height of conical part $(H) = 2.8$ m

Slant height of conical part $(l) = \sqrt{H^2 + r^2}$

$$l = \sqrt{(2.8)^2 + (2.1)^2}$$

$$l = \sqrt{7.84 + 4.41}$$

$$l = \sqrt{12.25}$$

$l = 3.5$ m

Curved surface area of the cylinder $= 2\pi rh$

$$= 2\times\frac{22}{7}\times 2.1\times 4 \quad \left[\because \pi = \frac{22}{7}\right]$$

$= 2\times 22\times 0{\cdot}3\times 4$

$= 52.8\text{ m}^2$

Curved surface area of conical tent $= \pi rl$

$$= \frac{22}{7}\times 2.1\times 3.5$$

$= 22\times 0.3\times 3.5$

$= 23.1\text{ m}^2$

Total area of cloth required for building one tent

= C.S.A. of cylinder + C.S.A. of conical tent

$= (52.8 + 23.1)\text{m}^2$

$= 75.9\text{ m}^2$

Cost of building one tent $= 75.9\times 100$

= ₹ 7590

Total cost of 100 tents = ₹ (7590 × 100)

= ₹ 7,59,000

Cost to be borne by the associations (50% of the cost)

$$= \frac{759000\times 50}{100}$$

= ₹ 379500

Hence, the association will have to pay ₹ 379500.

Values shown by associations are helping the flood victims and showing concern for humanity. **Ans.**

18. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of each bottle, if 10% liquid is wasted in this transfer. **[3]**

Solution : Internal diameter of hemispherical bowl = 36 m

∴ Radius of hemispherical bowl $(r) = 18$ cm

Volume of liquid $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3}\times\pi\times 18^3$$

∵ Diameter of bottle = 6 cm

∴ Radius of bottle = 3 cm

Now, volume of a cylindrical bottle $= \pi R^2 h$

$$= \pi\, 3^2\, h$$
$$= 9\pi h$$

Volume of liquid to be transfer = volume of liquid −10% volume of liquid

$$= \frac{2}{3}\pi 18^3 - \frac{10}{100}\left(\frac{2}{3}\pi 18^3\right)$$
$$= \frac{2}{3}\pi 18^3\left(1-\frac{10}{100}\right)$$
$$= \frac{2}{3}\pi 18^3 \times \frac{9}{10}$$
$$= \pi \times 18^3 \times \frac{3}{5}$$

Number of cylindrical bottles

$$= \frac{\text{Volume of liquid to be transfered}}{\text{Volume of a bottle}}$$

$$72 = \frac{\pi \times 18 \times 18 \times 18 \times \frac{3}{5}}{9\pi h}$$

$$h = \frac{27}{5} = 5.4 \text{ cm}$$

Hence, height of each bottle will be 5.4 cm. **Ans.**

19. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have ? Find the cost of painting the total surface area of the solid so formed, at the rate of ₹ 5 per 100 sq. cm [Use $\pi = 3.14$] [3]

Solution : Side of the cubical block (a) = 10 cm

Longest diagonal of the cubical block $= a\sqrt{3}$

$$= 10\sqrt{3} \text{ cm}$$

Since the cube is surmounted by a hemisphere, therefore the side of the cube should be equal to the diameter of the hemisphere.

$\because$ Diameter of the sphere = 10 cm

$\therefore$ Radius of the sphere (r) = 5 cm

$$\left[\because \text{Radius} = \frac{\text{Diameter}}{2}\right]$$

Total surface area of solid = T.S.A. of the cube + C.S.A. of hemisphere – Inner cross-section area of hemisphere

$$= 6a^2 + 2\pi r^2 - \pi r^2$$
$$= 6a^2 + \pi r^2$$
$$= 6(10)^2 + 3.14(5)^2$$
$$[\because \pi = 3.14]$$
$$= 600 + 25 \times 3.14$$
$$= 600 + 78.5$$
$$= 678.5 \text{ cm}^2$$

Cost of painting per square metre is ₹ 5

$$\text{Total cost for painting} = \frac{₹\ 678.5}{100} \times 5$$
$$= ₹\ 33.92$$

Hence, total cost for painting will be ₹ 33.92 **Ans.**

20. 504 cones each of diameter 3.5 cm and height 3 cm are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. $\left[\pi = \frac{22}{7}\right]$ [3]

Solution : Diameter of each cone (d) = 3.5 cm

$$\text{Radius of each cone } (r) = \frac{3.5}{2} = \frac{7}{4} \text{cm} \quad \left[\because r = \frac{d}{2}\right]$$

Height of each cone (h) = 3 cm

Volume of 504 cones = 504 × Volume of one cone

$$= 504 \times \frac{1}{3}\pi r^2 h$$
$$= 504 \times \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 3$$

Let radius of sphere be R cm

$\therefore$ Volume of sphere = Volume of 504 cones

$$\frac{4}{3} \times \frac{22}{7} \times R^3 = 504 \times \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 3$$

$$R = \sqrt[3]{\frac{3 \times 3 \times 7 \times 7 \times 7 \times 3}{2 \times 2 \times 2}}$$

$$R = \frac{21}{2} \text{cm}$$

Hence, diameter of sphere = 2R = 21 cm. **Ans.**

Now, surface area of sphere $= 4\pi R^2$

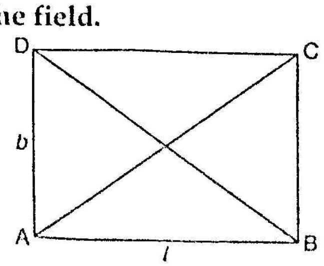
$$= 4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$$
$$= 63 \times 22$$
$$= 1386 \text{ cm}^2$$

Hence, surface area of sphere is 1386 cm². **Ans.**

SECTION — D

21. The diagonal of a rectangular field is 16 m more than the shorter side. If the longer side is 14 m more than the shorter side, then find the lengths of the sides of the field. [4]

Solution :



Let l be the length of the longer side and b be the length of the shorter side.

Given that the length of the diagonal of the rectangular field is 16 m more than shorter side.

Thus, Diagonal $= 16 + b$

Since longer side is 14 m more than shorter side,

$\therefore l = 14 + b.$

We know,

$$(\text{Diagonal})^2 = (\text{Length})^2 + (\text{Breadth})^2$$

[By Pythagoras theorem]

$$\therefore \quad (16 + b)^2 = (14 + b)^2 + b^2$$

$$256 + b^2 + 32b = 196 + b^2 + 28b + b^2$$

$$b^2 - 4b - 60 = 0$$

$$b^2 - 10b + 6b - 60 = 0$$

$$b(b - 10) + 6(b - 10) = 0$$

$$(b + 6)(b - 10) = 0$$

$$\Rightarrow \quad b = -6 \text{ or } +10$$

As breadth cannot be negative

$\therefore$ Breadth $(b) = 10$ m.

Now, length of rectangular field $= (14 + b)$ m

$= (14 + 10)$ m

$= 24$ m

Thus, length of rectangular field is 24 cm and breadth is 10 m. **Ans.**

22. Find the 60[th] term of the A.P. 8, 10, 12.... if it has a total of 60 terms and hence find the sum of its last 10 terms. [4]

Solution : Consider the given A.P. 8, 10, 12, ...

Hence the first term is 8

And the common difference

$$d = 10 - 8 = 2$$

or

$$12 - 10 = 2$$

Therefore, 60[th] term is

$$a_{60} = 8 + (60 - 1)\,2$$

$$\Rightarrow \quad a_{60} = 8 + 59 \times 2$$

$$\Rightarrow \quad a_{60} = 126$$

We need to find the sum of last 10 terms

Since, sum of last 10 terms = sum of first 60 terms – sum of first 50 terms.

$$S_{10} = \frac{60}{2}[2 \times 8 + (60 - 1)2] - \frac{50}{2}[2 \times 8 + (50 - 1)2]$$

$$= \frac{60}{2} \times 2[8 + 59] - \frac{50}{2} \times 2[8 + 49]$$

$$= 60 \times 67 - 50 \times 57$$

$$= 4020 - 2850$$

$$= 1170$$

Hence, the sum of last 10 terms is 1170. **Ans.**

23. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed ? [4]

Solution : Let the average speed of the train be x km/hr.

Then, new average speed of the train $= (x + 6)$ km/hr

Time taken by train to cover 54 km $= \dfrac{54}{x}$ hrs

And time taken by train to cover 63 km

$$= \frac{63}{(x+6)} \text{ hrs}$$

According to the question,

$$\frac{54}{x} + \frac{63}{x+6} = 3$$

$$\frac{54(x+6)+63x}{x(x+6)} = 3$$

$$54x + 324 + 63x = 3x(x + 6)$$

$$324 + 117x = 3x^2 + 18x$$

$$3x^2 - 99x - 324 = 0$$

$$x^2 - 33x - 108 = 0$$

$$x^2 - 36x + 3x - 108 = 0$$

$$x(x - 36) + 3(x - 36) = 0$$

$$(x + 3)(x - 36) = 0$$

$$x = -3 \text{ or } 36$$

Since, speed cannot be negative

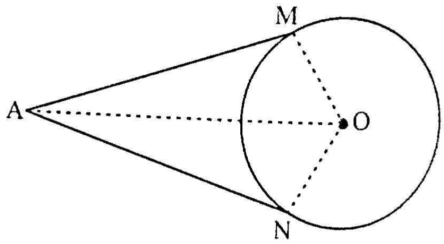
$$\therefore \quad x = 36$$

so, First speed of train = 36 km/hr **Ans.**

24. Prove that the lengths of the tangents drawn from an external point to a circle are equal. [4]

Solution : Given, Tangents AM and AN are drawn from point A to a circle with centre O.

To prove : $AM = AQ$



Construction : Join OM, ON and OA

Proof : Since AM is a tangent at M and OM is radius

$$\therefore \quad OM \perp AM$$

Similarly, $\quad ON \perp AN$

Now, in ΔOMA and ΔONA.

$OM = ON$ (radii of same circle)

$OA = OA$ (common)

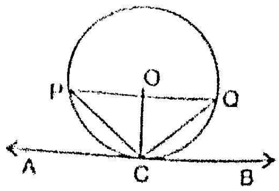
$\angle OMA = \angle ONA = 90°$

$\therefore \quad \Delta OMA \cong \Delta ONA$ (By RHS congruence)

Hence, $\quad AM = AN$ (By cpct) **Hence Proved.**

25. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc. [4]

Solution :



Given, C is the mid-point of the minor arc PQ and O is the centre of the circle and AB is tangent to the circle through point C.

Construction : Draw PC and QC.

To prove : $PQ \parallel AB$

Proof : It is given that C is the mid-point of the arc PQ.

So, Minor arc PC = Minor arc QC

$\Rightarrow$ $PC = QC$

Hence ΔPQC is an isosceles triangle.

Thus the perpendicular bisector of the side PQ of ΔPQC passes through vertex C.

But we know that the perpendicular bisector of a chord passes through centre of the circle.

So, the perpendicular bisector of PQ passes through the center O of the circle.

Thus, the perpendicular bisector of PQ passes through the points O and C.

$\Rightarrow$ $PQ \perp OC$ ...(i)

AB is perpendicular to the circle through the point C on the circle

$\Rightarrow$ $AB \perp OC$ ...(ii)

From equations (i) and (ii), the chord PQ and tangent AB of the circle are perpendicular to the same line OC.

Hence, $AB \parallel PQ$

or $PQ \parallel AB$ **Hence Proved.**

26. Construct a $\Delta\, ABC$ in which $AB = 6$ cm, $\angle A = 30°$ and $\angle B = 60°$. Construct another $\Delta\, AB'C'$ similar to $\Delta\, ABC$ with base $AB' = 8$ cm. [4]

Solution : Steps of construction :

(i) Draw a line segment AB= 6 cm.

(ii) Construct $\angle ABP$= 60° and $\angle QAB$ = 30°

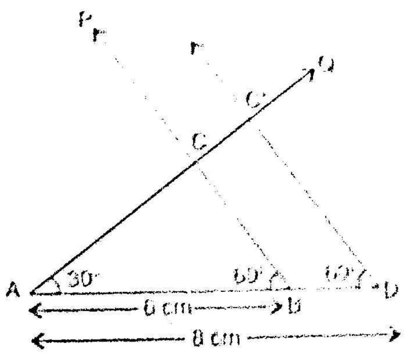
(iii) Join AC and BC such that C is the intersection point of BP and AQ.

Thus, ΔABC is the required triangle.

(iv) Extend AB to B', such that AB' = 8 cm.

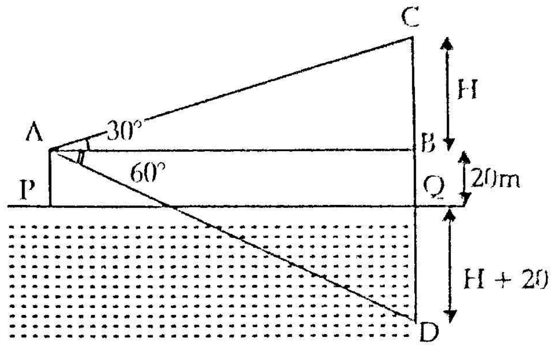
(v) Draw $B'C' \parallel BC$ cutting AC produced at C'.

Then, $\Delta AB'C'$ is the required triangle similar to ΔABC.



27. At a point A, 20 m above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A. [4]

Solution :



Let PQ be the surface of the lake. A is the point vertically above P such that AP = 20 m.

Let C be the position of the cloud and D be its reflection in the lake.

Let $BC = H$ metres

Now, In $\Delta\, ABD$

$$\tan 60° = \frac{BD}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{H + 20 + 20}{AB}$$

$$\Rightarrow \quad \sqrt{3}.\, AB = H + 40$$

$$\Rightarrow \quad AB = \frac{H + 40}{\sqrt{3}} \quad ...(i)$$

And, in $\Delta\, ABC$

$$\tan 30° = \frac{BC}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{H}{AB}$$

$$AB = \sqrt{3}H \quad ...(ii)$$

From eq. (i) and (ii)

$$\frac{H + 40}{\sqrt{3}} = \sqrt{3}\, H$$

$$\Rightarrow \quad 3H = H + 40$$

$$\Rightarrow \quad 2H = 40 \Rightarrow H = 20$$

Putting the value of H in eq. (ii), we get

$$AB = 20\sqrt{3}$$

Again, in $\Delta\, ABC$

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$= \left(20\sqrt{3}\right)^2 + (20)^2$$

$$= 1200 + 400$$

$$= 1600$$

$$AC = \sqrt{1600} = 40$$

Hence, the distance of cloud from A is 40 m.

Ans.

28. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is

(i) A card of spade or an ace

(ii) A black king

(iii) Neither a jack nor a king

(iv) Either a king or a queen [4]

Solution : (i) Let S be the sample space of drawing a card from a well-shuffled deck

Then, $S = 52$

There are 13 spade cards and 4 aces in a deck.

As a ace of spade is included in 13 spade cards, so, there are 13 spade cards and 3 aces.

A card of spade or an ace can be drawn in

$$13 + 4 - 1 = 16 \text{ (ways)}$$

Probability of drawing a card of spade or an ace.

$$P = \frac{16}{52} = \frac{4}{13}.$$

(ii) There are 2 black king cards in a deck.

Probability of drawing a black king

$$P = \frac{2}{52}$$

$$P = \frac{1}{26}$$

(iii) There are 4 jack and 4 king cards in a deck.

So, there are 52 – 8 = 44 cards which are neither jack nor king

Probability of drawing a card which is neither a jack nor a king

$$P = \frac{44}{52}$$

$$P = \frac{11}{13}$$

(iv) There are 4 queen and 4 king cards in a deck.

So, there are 8 cards which are either king or queen.

Probability of drawing a card which is either king or a queen

$$P = \frac{8}{52}$$

$$P = \frac{2}{13}$$ Ans.

29. Find the values of k so that the area of the triangle with vertices $(1, -1)$, $(-4, 2k)$ and $(-k, -5)$ is 24 sq. units. [4]

Solution : The vertices of the given ΔABC are $A(1, -1)$, $B(-4, 2k)$ and $C(-k, -5)$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[1(2k+5) + (-4)(-5+1) + (-k)(-1-2k)]$$

$$= \frac{1}{2}[2k + 5 + 16 + k + 2k^2]$$

$$= \frac{1}{2}[2k^2 + 3k + 21]$$

Area of ΔABC = 24 sq. units (Given)

$$\therefore \quad \frac{1}{2}[2k^2 + 3k + 21] = 24$$

$$[2k^2 + 3k + 21] = 48$$

$$\Rightarrow \quad 2k^2 + 3k + 21 = 48$$

$$\Rightarrow \quad 2k^2 + 3k - 27 = 0$$

$$\Rightarrow \quad 2k^2 + 9k - 6k - 27 = 0$$

$$\Rightarrow \quad k(2k+9) - 3(2k+9) = 0$$

$$\Rightarrow \quad (k-3)(2k+9) = 0$$

$$k = 3 \text{ or } -\frac{9}{2}$$

Hence, $k = 3$ or $k = -\frac{9}{2}$. Ans.

30. In figure 5, $PQRS$ is a square lawn with side PQ = 42 m Two circular flower beds are there on the sides PS and QR with center at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts). [4]

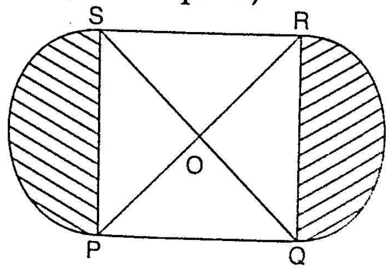
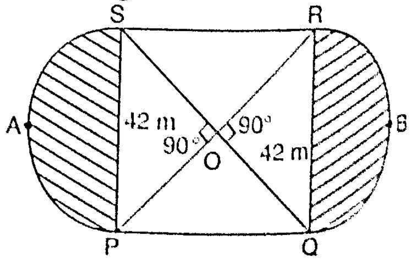


Figure 5

Solution :



Given, $PQRS$ is a square with side 42 m.

Let its diagonals intersect at O.

Then, $OP = OQ = OR = OS$

and $\angle POS = \angle QOR = 90°$

$$PR^2 = PQ^2 + QR^2$$

$$PR = (\sqrt{2} \times 42) \text{m}$$

Now, $OP = \frac{1}{2}\times(\text{diagonal}) = 21\sqrt{2}$ m

$\because$ Area of flower bed PAS = Area of flower bed QBR

$\therefore$ Total area of the two flower beds = Area of flower bed PAS + Area of flower bed QBR

$= 2\times[\text{Area of sector } OPAS - \text{Area of } \Delta POS]$

$$= 2\times\left[\pi r^2 \frac{\theta}{360^\circ} - \frac{1}{2}r^2 \sin\theta\right]$$

[Where, $\theta = 90^\circ$]

$$= 2\times\left[\frac{22}{7}\times(21\sqrt{2})^2 \frac{90^\circ}{360^\circ} - \frac{1}{2}\times 21\sqrt{2}\times 21\sqrt{2}\right]$$

[$\because \sin 90^\circ = 1$]

$$= 2\times\left[\frac{22}{7}\times 21\times 21\times 2\times\frac{1}{4} - \frac{1}{2}\times 21\times 21\times 2\right]$$

$= 2[33 \times 21 - 441]$

$= 2[693 - 441]$

$= 504 \text{ m}^2$

Hence area of flower beds is 504 m^2. **Ans.**

31. **From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire. [4]**

Solution : Height of the cylinder (h) = 10 cm

Radius of base of cylinder (r) = 4.2 cm

Now,

Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 4.2 \times 4.2 \times 10$

$= 554.4 \text{ cm}^3$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\times\frac{22}{7} \times 4{\cdot}2 \times 4{\cdot}2 \times 4{\cdot}2$

$= 155.232 \text{ cm}^3$

Volume of the rest of the cylinder after scooping out the hemisphere from each end = Volume of cylinder – 2 × Volume of hemisphere

$= 554.4 - 2 \times 155.232$

$= 554.4 - 310.464$

$= 243.936 \text{ cm}^3$.

The remaining cylinder is melted and converted into a new cylindrical wire of 1.4 cm thickness.

So, radius of cylindrical wire = 0.7 cm

Volume of remaining cylinder = Volume of new cylindrical wire

$243.936 = \pi R^2 H$

$243{\cdot}936 = \frac{22}{7} \times 0{\cdot}7 \times 0{\cdot}7 \times H$

$\Rightarrow$ $H = 158.4$ cm **Ans.**