VI. (5)

Alternating Currents

The magnitude of alternating current changes continuously with time and its direction reverses periodically. It is represented by

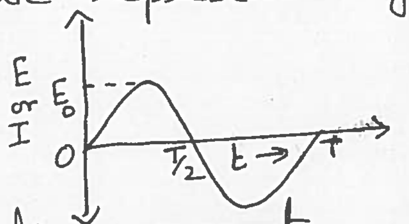
$$I = I_0 \sin\omega t$$
or
$$I = I_0 \cos\omega t$$

where I is the instantaneous value of current (the magnitude of current at any instant of time)

I_0 is the peak or maximum value of alternating current

ω is the angular frequency.

Similarly alternating emf may be represented by



$$E = E_0 \sin\omega t$$
or
$$E = E_0 \cos\omega t.$$

Mean or average value of alternating current

Mean or average value of alternating current over half cycle is that value of steady current which would send the same amount of charge through a circuit in the time of half cycle as is sent by the ac through the same circuit in the same time.

Let the alternating current be represented by

$$I = I_0 \sin\omega t$$

If the strength of current I is assumed to remain constant for a small time dt then the small amount of charge sent in a small time dt is

$$dq = I\,dt$$

If q is the total charge sent by the ac in the first half cycle then

$$q = \int dq = \int_0^{T/2} I_0 \sin\omega t \, dt.$$

$$= \frac{I_0}{\omega}\left[-\cos\omega t\right]_0^{T/2}$$

$$= -\frac{I_0}{\omega}\left[\cos\omega \frac{T}{2} - \cos 0\right]$$

$$= -\frac{I_0}{\omega}\left[\cos \frac{2\pi}{T}\cdot\frac{T}{2} - \cos 0\right]$$

$$\therefore \quad q = \frac{2I_0}{\omega}$$

If I_m represents the mean value of ac over the first half cycle then

$$q = I_m \times \frac{T}{2}.$$

$$I_m = \frac{2I_0}{\omega} \times \frac{2}{T}$$

$$= \frac{4 I_0 T}{2\pi T}$$

$$I_m = \frac{2I_0}{\pi}$$

$$\text{or} \quad \boxed{I_m = .637\, I_0}$$

The mean or average value of ac over the positive half cycle is 63.7% of the peak value of ac.

Mean or average value of alternating emf: is that value of constant emf which would send the same amount of charge through a circuit in the time of half cycle as is sent by alternating emf through the same circuit in the same time.

$$E = E_0 \sin\omega t.$$

If current remains constant for a small time dt then the small amount of charge sent by alternating emf in time dt is

$$dq = I\, dt$$

$$dq = \frac{E_0}{R} \sin\omega t\, dt.$$

Total charge sent by alternating emf through half cycle

$$q = \int_0^{T/2} \frac{E_0}{R} \sin\omega t\, dt.$$

$$= -\frac{E_0}{\omega R}\left[\cos\omega t\right]_0^{T/2}$$

$$q = \frac{2E_0}{\omega R}.$$

If E_m is the mean value of alternating emf over the first half cycle then.

$$q = \frac{E_m}{R}\left(\frac{T}{2}\right)$$

$$E_m = \frac{2E_0 R}{\omega R} \times \frac{2}{T}.$$

$$E_m = \frac{2E_0}{\pi}$$

$\therefore$ Mean value of alternating emf over a half cycle is 63.7% of E_0. or $\boxed{E_m = \cdot 637 E_0}$

Note: The mean or average value of ac over a complete cycle is zero.

Root mean square value (I_{rms}) or virtual value (I_v) or effective value (I_{eff}) of alternating current

It is that value of steady current which would generate the same amount of heat in a given resistance in a given time as is generated by the ac when passed through the same resistance for the same time.

Let alternating current be represented by

$$I = I_0 \sin\omega t.$$

If this current flows through resistance R for time dt then the amount of heat produced is

$$dH = I^2 R\, dt.$$

Total amount of heat produced over a complete cycle is

$$H = \int_0^T I^2 R\, dt$$

$$= \int_0^T I_0^2 \sin^2\omega t\, R\, dt.$$

$$= I_0^2 R \int_0^T \sin^2 \omega t \, dt$$

$$= I_0^2 R \int_0^T \left[\frac{1 - \cos 2\omega t}{2}\right] dt$$

$$= \frac{I_0^2 R}{2} \left[\int_0^T dt - \int_0^T \cos 2\omega t \, dt\right]$$

$$= \frac{I_0^2 R}{2} \left[(T) - \left(\frac{\sin 2\omega t}{2\omega}\right)_0^T\right]$$

$$= \frac{I_0^2 R}{2} \left[(T) - \left(\frac{\sin 2 \cdot 2\pi}{2\omega T} \cdot T - \sin 0\right)\right]$$

$$H = \frac{I_0^2 R}{2}[T - 0] \qquad [\because \sin 4\pi = 0]$$

If rms value of ac is I_v then the amount of heat produced in the same time T through the same resistance R is

$$H = I_v^2 R T$$

$$I_v^2 R T = \frac{I_0^2 R T}{2}$$

$$\boxed{I_v = \frac{I_0}{\sqrt{2}}}$$

I_v is 70.7% of the peak value of alternating current

<u>Root mean square value of alternating emf</u>

It is that value of steady emf which would generate the same amount of heat in a given time as that of alternating emf through the same resistance and for the same time.

$$E = E_0 \sin \omega t.$$

If the current is assumed to remain constant for a small time dt then the amount of heat generated by the alternating emf in the time dt is

$$dH = I^2 R \, dt$$

$\therefore$ Total heat generated over a complete cycle

$$H = \int_0^T I^2 R \, dt.$$

$$H = \frac{E_0^2}{R}\int_0^T \sin^2 \omega t \, dt$$

$$= \frac{E_0^2}{R}\int_0^T \left[\frac{1-\cos 2\omega t}{2}\right] dt$$

$$= \frac{E_0^2}{2R}\left[\int_0^T dt - \int_0^T \cos 2\omega t \, dt\right]$$

$$H = \frac{E_0^2 T}{2R} \qquad \left[\because \int_0^T \cos 2\omega t \, dt = 0\right]$$

Suppose $H = I_v^2 R T$

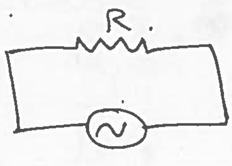
$$= \frac{E_v^2 T}{R}$$

$$E_v^2 = \frac{E_0^2 T R}{2 R T}$$

$$\boxed{E_v = \frac{E_0}{\sqrt{2}}}$$

<u>A C circuit containing resistance only</u>

Suppose a resistance is connected to a source of emf then



$$E = E_0 \sin \omega t \quad — (1)$$

If I is the current at any instant of time t then the potential drop across R will be equal to the applied emf E

$$\therefore IR = E = E_0 \sin \omega t$$

$$I = \frac{E_0 \sin \omega t}{R} \quad — (2)$$

The current through the circuit is maximum when $\sin \omega t = 1$ ie $I = I_0$ when $\sin \omega t = 1$

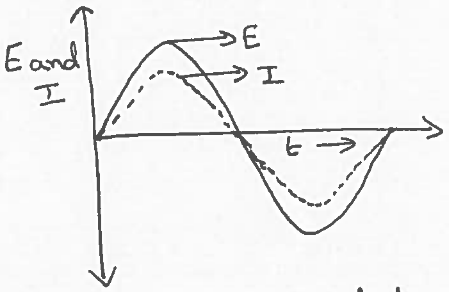
$$I_0 = \frac{E_0}{R}. \quad — (3)$$

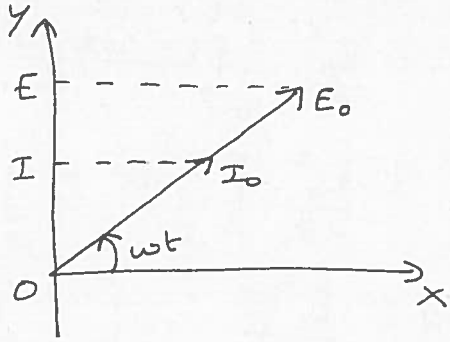
Substituting (3) in (2)

$$I = I_0 \sin \omega t \quad — (4).$$

From (1) and (4) it is clear that alternating voltage and current are in phase.

∴ The behaviour of R is the same in ac and dc circuits.





Graphical representation of alternating voltage and current with respect to t for an ac circuit containing resistance only.

Phasor diagram

Phasor diagram : The phasor diagram or vector diagram represents the phase relationship between alternating emf and alternating current.

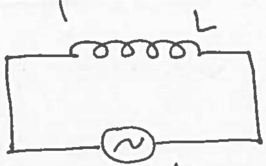
E_0 and I_0 are represented by means of phasors rotating in the anticlockwise direction and the length of the arrow indicates the maximum value

In a circuit containing only resistance both phasors are in the same direction making an angle ωt with the X-axis.

The projection on any axis gives the instantaneous value of E and I.

AC circuit containing inductance only

Let the source of alternating emf be represented by $E = E_0 \sin \omega t$ —(1)



When the coil is connected to a source of alternating current then the emf induced in the coil is

$$e = -L\frac{dI}{dt}.$$

Now, total instantaneous emf in the circuit is

$$E - L\frac{dI}{dt} = 0$$

[It is equal to 0 because there is no circuit element across which the potential drop can occur]

$$\therefore E = L\frac{dI}{dt}$$

$$E_0 \sin\omega t = L\frac{dI}{dt}.$$

$$dI = \frac{E_0}{L}\sin\omega t\, dt$$

Integrating the above expression

$$I = -\frac{E_0}{L\omega}\cos\omega t.$$

$$I = \frac{E_0}{L\omega}\sin(\omega t - \pi/2) \quad -(2)$$

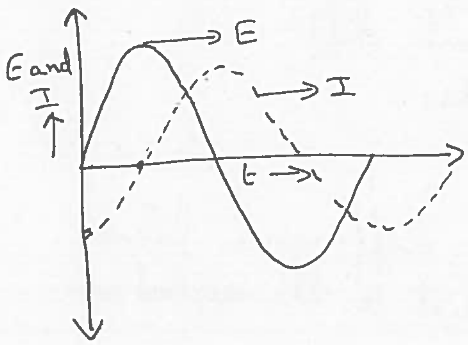
The current is maximum when $\sin(\omega t - \pi/2) = 1$

$$\therefore I_0 = \frac{E_0}{L\omega} \quad —— (3)$$

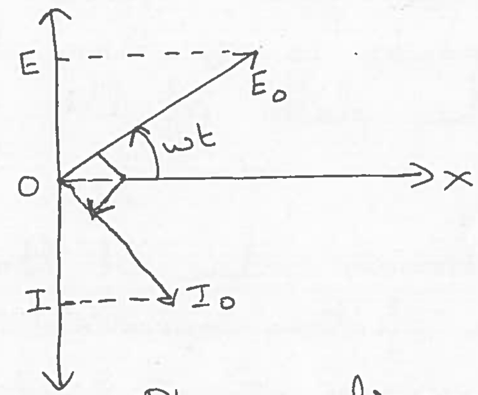
Substituting (3) in (2)

$$I = I_0 \sin(\omega t - \pi/2) \quad -(4)$$

Comparing (1) and (4) it is clear that in a circuit containing only L the current lags the voltage by $\pi/2$.



Graphical representation



Phasor diagram.

It is clear from the phasor diagram that the phasor E_0 makes an angle ωt with OX. and that the current lags the voltage by $\pi/2$.

Inductive reactance (X_L):

Considering the relation $I_0 = \frac{E_0}{L\omega}$

and comparing with Ohm's law we can infer that ωL represents the resistance offered by the inductor L. This is the inductive reactance denoted by X_L.

<u>Note</u> : (1) $X_L = \omega L = 2\pi \nu L$

where ν is the frequency of ac.

(2) The unit of X_L is ohm.

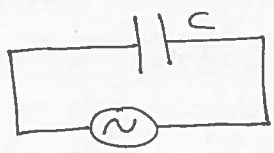
(3) In dc circuits, $\nu = 0$ $\therefore X_L = 0$.

$\therefore$ A pure inductor offers zero resistance to dc.

(4) Higher the frequency of ac more is the inductive reactance and at very high frequencies an inductor in a circuit nearly amounts to an open circuit.

<u>AC circuit containing only capacitance</u>

Suppose the alternating emf is



$$E = E_0 \sin \omega t \rightarrow (1)$$

As the current flows, suppose the charge on the capacitor is q, then the potential difference between the plates of the capacitor is

$$V = q/C.$$

The instantaneous value of the potential difference between the plates of the capacitor must be equal to the applied emf

$$\therefore E = E_0 \sin \omega t = q/C$$

$$q = C E_0 \sin \omega t.$$

If I is the instantaneous value of current then,

$$I = \frac{dq}{dt} = \frac{d}{dt}[C E_0 \sin \omega t]$$

$$I = C E_0 \omega \cos \omega t$$

$$I = \frac{E_0}{1/C\omega} \sin(\omega t + \pi/2) \rightarrow (2)$$

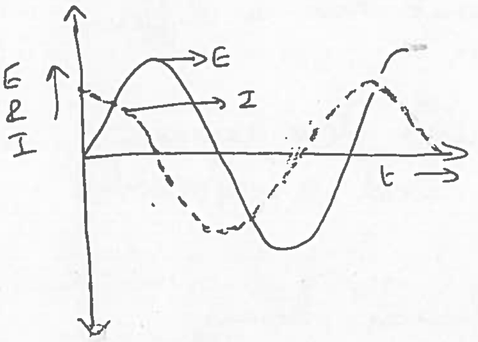
The current is maximum when $\sin(\omega t + \pi/2) = 1$

$$\Rightarrow I_0 = \frac{E_0}{1/C\omega} \longrightarrow (3)$$

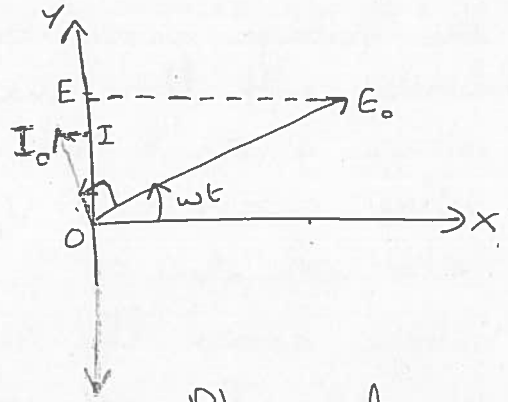
Substituting (3) in (2)

$$I = I_0 \sin(\omega t + \pi/2) \rightarrow (4)$$

Comparing (1) and (4) it is clear that in a circuit containing only C the current leads the voltage by $\pi/2$.



Graphical representation



Phasor diagram.
Current phasor leads voltage by $\pi/2$

Capacitative reactance (X_c)

Considering the relation $I_0 = \frac{E_0}{1/C\omega}$ and comparing with Ohm's law we can infer that $1/C\omega$ represents the . . resistance of the circuit. It is the capacitative reactance denoted by X_c.

Note (1) $X_c = \frac{1}{C\omega} = \frac{1}{C\,2\pi\nu}$ where ν is the frequency of ac

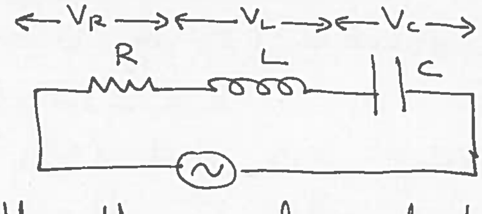
(2) The unit of X_c is ohm

(3) In a dc circuit, $\nu = 0$, $\therefore X_c$ is infinite. In other words a capacitor blocks dc.

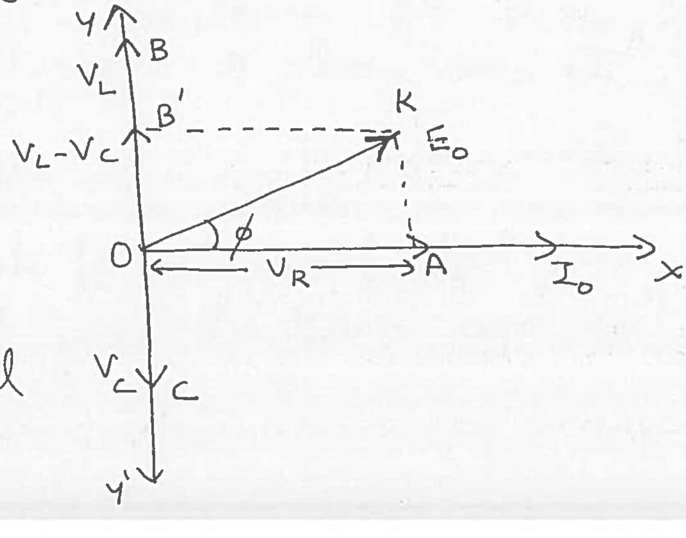
(4) Since $X_c = \frac{1}{2\pi\nu C}$, a capacitor is a conductor at very high frequencies of ac and nearly amounts to an open circuit at very low frequencies

VX.

AC circuit containing resistance, inductance and capacitance



1. Since resistance, inductance and capacitance are in series the current at any instant through the three elements has the same amplitude and phase.
2. However, the voltage across each element has a different phase relationship with the current.
   (a) The voltage across R is in phase with the current. The max: voltage across R is $V_R = I_0 R$. and is represented by the vector $\vec{OA}$ along OX.
   (b) The voltage across the inductor leads the current by 90°. The maximum voltage across L is $V_L = I_0 X_L$. and is represented by the $\vec{OB}$ along OY.
   (c) The voltage across the capacitor lags the current by 90°. The maximum voltage across C is $V_C = X_C I_0$. and is represented by $\vec{OC}$ along OY'.
3. Since V_L and V_C have a phase difference of 180° the net reactive voltage is $V_L - V_C$ (assuming $V_L > V_C$) and is represented by OB'.
4. Hence, the vector sum of V_R and $(V_L - V_C)$ is the phasor E_0 making an angle ϕ with the current phasor I_0.



$$E_0 = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(I_0 R)^2 + (X_L I_0 - X_C I_0)^2}$$

$$E_0 = I_0 \sqrt{R^2 + (X_L - X_C)^2}$$

$\frac{E_0}{I_0} = Z$, where Z is the total effective resistance of a LCR circuit known as the impedance of the circuit.

5. Also, $\text{Tan}\,\phi = \dfrac{AK}{OA} = \dfrac{V_L - V_C}{V_R} = \dfrac{I_0(X_L - X_C)}{I_0 R}$

$$\therefore \text{Tan}\,\phi = \frac{X_L - X_C}{R}$$

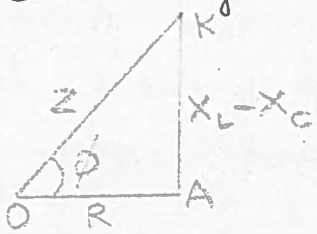
When $X_L = X_C$, $\text{Tan}\,\phi = 0$ and $\phi = 0$. Now, the current and voltage are in phase with each other and it is a non-inductive circuit. (a circuit with only resistance because $X_L = X_C$). graph (page 19)

Impedance triangle

(a) The opposition offered to the flow of current due to a resistor in a circuit is known as resistance. It is denoted by R and represents the base OA of the triangle

(b) The reactance is the resistance offered by an inductor X_L or the resistance offered by a capacitor X_C to the flow of current in an ac circuit or the resistance offered by an inductor and capacitor in an ac circuit (Assuming $X_L > X_C$, $X_L - X_C$ can be represented by the perpendicular AK of the triangle)

(c) The effective resistance offered by a resistor, inductor and capacitor to the flow of current in an ac circuit is called the impedance Z. It is represented by the diagnol OK of the triangle. $\angle AOK$ is called the phase angle by which the voltage leads the current.



Series resonance circuit

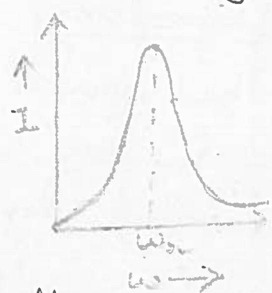
A circuit in which an inductor, capacitor and resistor are connected in series and admits maximum current corresponding to a given frequency of ac is called series resonance circuit.

In a LCR circuit, $Z = \sqrt{R^2 + (L\omega - 1/C\omega)^2}$

(a) At very low frequencies X_L is negligible but capacitive reactance is very high.

(b) However as the frequency of alternating emf increases X_L increases and X_C decreases.

(c) For a particular value of ω called resonant frequency (ω_r), $X_L = X_C$.



$$\therefore \omega_r L = \frac{1}{\omega_r C}$$

$$\therefore \boxed{\omega_r = \frac{1}{\sqrt{LC}}} \quad \text{or} \quad \boxed{\nu = \frac{1}{2\pi\sqrt{LC}}}$$

(d) At the resonant frequency, $Z = R$, that is the impedance is minimum and current through the LCR circuit is maximum.

(e) It is known as an acceptor circuit and is used in TV and radio receiver sets. To hear a particular radio station, we tune the radio. In tuning, we vary the capacitance of a capacitor such that the resonant frequency of the circuit becomes nearly equal to the frequency of the radio signal received by the antenna.

(f) Resonance phenomenon is exhibited by a circuit only if both L and C are present in a circuit [$\because$ voltages across L and C cancel each other]. There can be no resonance in a RL or RC circuit.

Quality factor or Sharpness of resonance: It is the ratio of voltage across L or C to the applied voltage at resonance [applied voltage at resonance is equal to the voltage across R]

$$\therefore Q = \frac{I(\omega_r L)}{IR} = \frac{\omega_r L}{R}$$

$$\Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}} \quad \left(\because \omega_r = \frac{1}{\sqrt{LC}}\right)$$

Expression for bandwidth of the circuit

Consider two values of ω, say ω_1 and ω_2 such that

$$\omega_1 = \omega_r + \Delta\omega$$
$$\omega_2 = \omega_r - \Delta\omega$$

where ω_r is the resonant frequency.

The difference $\omega_1 - \omega_2 = 2\Delta\omega$ called the bandwidth of the circuit. The smaller the value of $\Delta\omega$ sharper or narrower is the resonance

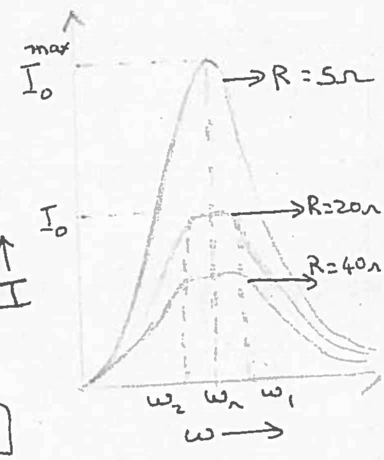
$$\text{At } \omega_1, \quad I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}} \rightarrow (1)$$

Let us choose ω such that the max: current corresponding to it is I_0 and the assumption is $I_0 = \frac{1}{\sqrt{2}} I_0^{max}$

$$\therefore I_0 = \frac{I_0^{max}}{\sqrt{2}} = \frac{E_0}{\sqrt{2}R} \quad — (2)$$

(I_0^{max} is the max: current corresponding to ω_r)

Substituting (2) in (1)



$$R\sqrt{2} = \sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2} \quad \left[\because I_0 = \frac{E_0}{\sqrt{2}R}\right]$$

$$2R^2 = R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2 \quad \text{(Squaring both sides)}$$

$$R = \omega_1 L - \frac{1}{\omega_1 C} \quad \text{(Taking root)}$$

$$R = (\omega_r + \Delta\omega)L - \frac{1}{(\omega_r + \Delta\omega)C} \quad [\because \omega_1 = \omega_r + \Delta\omega]$$

$$R = \omega_r L\left(1 + \frac{\Delta\omega}{\omega_r}\right) - \frac{1}{\omega_r C\left(1 + \frac{\Delta\omega}{\omega_r}\right)}$$

$$R = \omega_r L\left[1 + \frac{\Delta\omega}{\omega_r}\right] - \frac{\omega_r L}{\left[1 + \frac{\Delta\omega}{\omega_r}\right]} \quad \left[\because \omega_r^2 = \frac{1}{LC}\right]$$

$$R = \omega_r L\left[1 + \frac{\Delta\omega}{\omega_r}\right] - \omega_r L\left[1 - \frac{\Delta\omega}{\omega_r}\right] \quad \left[\because \frac{\Delta\omega}{\omega_r} << 1 \text{ and } \left(1 + \frac{\Delta\omega}{\omega_r}\right)^{-1} \text{ is equal to } \left[1 - \frac{\Delta\omega}{\omega_r}\right]\right]$$

$$\therefore R = 2\omega_r L \frac{\Delta\omega}{\omega_r}$$

$$\frac{\omega_r L}{R} = \frac{\omega_r}{2\Delta\omega}$$

The ratio $\frac{\omega_r L}{R}$ is called quality factor $\therefore \boxed{Q = \frac{\omega_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}}$

Also the bandwidth $\boxed{2\Delta\omega = R/L}$

Lower the resistance greater is the value of Q or sharpness of ——— (refer graph)

Average power associated with a resistance in an ac circuit.

Since the magnitude of current and voltage keeps changing at every instant of time in an ac circuit the instantaneous power is the product of instantaneous values of current and voltage.

Instantaneous power $= EI$

$$= E_0 I_0 \sin^2 \omega t.$$

If the power is assumed to remain constant for a small time dt then the work done is

$$dW = E_0 I_0 \sin^2 \omega t \, dt$$

Work done in maintaining current for a full cycle is

$$W = \int_0^T E_0 I_0 \sin^2 \omega t \, dt$$

$$= E_0 I_0 \left[\int_0^T \left(\frac{1 - \cos 2\omega t}{2} \right) dt \right]$$

$$\therefore W = \frac{E_0 I_0 T}{2} \quad \left[\because \int_0^T \cos 2\omega t \, dt = 0 \right]$$

Average power supplied to R over a complete cycle is

$$P_{ave} = \frac{W}{T} = \frac{E_0 I_0 T}{2T}$$

$$= \frac{E_0 I_0}{2}$$

$$\boxed{P_{ave} = \frac{E_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} = E_v I_v}$$

Average power associated with an inductor in an ac circuit

In an inductor linked ac circuit the instantaneous voltage and current are

$$E = E_0 \sin \omega t$$

$$I = I_0 \sin (\omega t - \pi/2)$$

$$= - I_0 \cos \omega t.$$

Work done in maintaining current for a full cycle is

$$\therefore W = -E_0 I_0 \int_0^T \sin\omega t \cos\omega t \, dt .$$

$$= -\frac{E_0 I_0}{2} \int_0^T \sin 2\omega t \, dt .$$

$$= -\frac{E_0 I_0}{2} \left[-\frac{\cos 2\omega t}{2\omega} \right]_0^T = \frac{E_0 I_0}{4\omega} \left[\cos 2 \cdot \frac{2\pi}{\cancel{T}} \cancel{T} - \cos 0 \right]$$

$$W = \frac{E_0 I_0}{4\omega} [1 - 1] = 0 . \quad (\cos 4\pi = 1)$$

$\therefore$ The average power associated with an inductor in an ac circuit over a complete cycle is 0. [The energy stored in the inductor during the building up of current in L is returned to the source during the decay of current].

Average power associated with a capacitor in an ac circuit

In a capacitor, $E = E_0 \sin\omega t$

$I = I_0 \sin(\omega t + \pi/2)$.

$= I_0 \cos\omega t$

. Work done over a complete cycle in ac circuit with a capacitor is

$$W = E_0 I_0 \int_0^T \sin\omega t \cos\omega t \, dt$$

$$= \frac{E_0 I_0}{2} \int_0^T \sin 2\omega t \, dt$$

$$= -\frac{E_0 I_0}{2} \left[\frac{\cos 2\omega t}{2\omega} \right]_0^T = -\frac{E_0 I_0}{4\omega} \left[\cos 2 \cdot \frac{2\pi}{\cancel{T}} . \cancel{T} - \cos 0 \right]$$

$$= -\frac{E_0 I_0}{4\omega} [1 - 1] \quad [\because \cos 4\pi = 1]$$

$$\therefore W = 0 .$$

$\therefore$ Average power associated with a capacitor in an ac circuit over a complete cycle is 0. [The energy stored in the capacitor during charging is returned to the source during the discharging of the capacitor]

Average power associated with a LCR circuit

Let the applied emf be $E = E_0 \sin\omega t$.

If alternating current lags behind the applied emf by ϕ then,

$$I = I_0 \sin(\omega t - \phi).$$

Assuming power to remain constant for a small time dt, the work done is

$$dW = E_0 I_0 \sin\omega t \sin(\omega t - \phi)\, dt$$

$$dW = E_0 I_0 \sin\omega t \left[\sin\omega t \cos\phi - \cos\omega t \sin\phi\right] dt.$$

$$W = E_0 I_0 \left[\cos\phi \int_0^T \sin^2\omega t\, dt - \sin\phi \int_0^T \sin\omega t \cos\omega t\, dt\right]$$

$$= \frac{E_0 I_0}{2}\left[\cos\phi \int_0^T dt - \cos\phi \int_0^T \cos 2\omega t\, dt - \sin\phi \int_0^T \sin 2\omega t\, dt\right]$$

$$= \frac{E_0 I_0}{2} \cos\phi\, [T] \qquad \left[\because \int_0^T \cos 2\omega t = \int_0^T \sin 2\omega t = 0\right]$$

$\therefore$ Average power over a complete cycle is

$$P = \frac{W}{T} = \frac{E_0 I_0 T \cos\phi}{2T}$$

$$P = \frac{E_0 I_0 \cos\phi}{2}$$

$$\therefore \boxed{P_{ave} = E_v I_v \cos\phi}$$

Note: (a) In a LCR circuit $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

(b) In a LC circuit $\cos\phi = R/Z = 0.$ $[\because R = 0]$

(c) In a RC circuit $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_C^2}}$

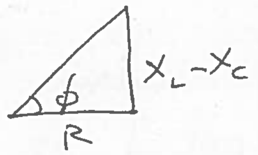
(d) In a RL circuit $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_L^2}}$

Power factor: It is the ratio of true power to apparent power or virtual power.

$$\text{Power factor} = \cos\phi = \frac{P}{E_v I_v}$$

Case (i) In a non inductive circuit, $X_L = X_C$

$$\boxed{\cos\phi = \frac{R}{\sqrt{R^2}} = 1}$$



or $\phi = 0$.

This is the maximum value of power factor

Case (ii) In the case of pure inductor or capacitor.

$$\cos\phi = 0.$$
$$\text{or } \phi = 90^\circ$$

$\therefore$ Current through a pure inductor or pure capacitor which consumes no power for its maintenance in the circuit is called wattless current or idle current

It is for this reason that inductors and capacitors are most suitable for controlling ac.

Advantages of ac.

(1) It can be easily converted to dc using rectifiers

(2) ac voltages can be varied using transformers.

(3) It can be transmitted over long distances without much loss in power using step up transformers.

(4) It is easy to generate ac.

Disadvantages of ac.

(1) Phenomena like electroplating, electrotyping etc require dc.

(2) Since ac is transmitted more from the surface of the wire than from inside, several fine insulated wires are used than a single thick wire

(3) ac is more dangerous than dc for the same voltage. [$\because$ peak value of ac $= \sqrt{2}\, E_v$].

Energy stored in an inductor.

When ac is applied to a circuit containing an inductor the current in it grows from $0 \to I_0$ and the emf induced is given by $e = -L\frac{dI}{dt}$.

To maintain the growth of current, power has to be supplied from an external source

$$\therefore E = -e = L\frac{dI}{dt}.$$

Power supplied is,

$$\frac{dW}{dt} = EI = LI\frac{dI}{dt}.$$

$$\therefore dW = LI\,dI.$$

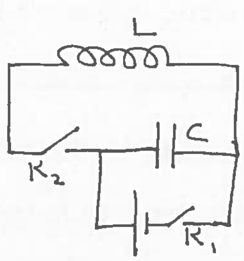
$\therefore$ Total work done by the external source in building the current from $0 \to I_0$ is.

$$W = L\int_0^{I_0} I\,dI.$$

$$\therefore \boxed{\text{Energy stored} = W = \frac{LI_0^2}{2}}$$

LC oscillations

1) Key K_1 is closed and the capacitor gets charged. The energy is stored in the form of electrostatic energy



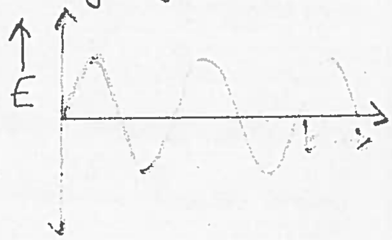
$$U_C = \frac{1}{2}CV^2$$

(2) K_1 is kept open and K_2 closed, then the capacitor starts discharging through L. Due to change in magnetic flux linked with L, an emf is induced in the opposite direction. Energy stored appears in the form of magnetic energy $U_L = \frac{1}{2}LI^2$.

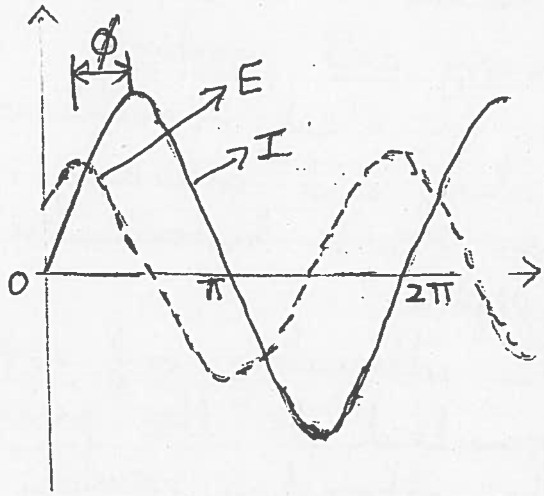
(3) When the capacitor has fully discharged the magnetic flux linked with L decreases and the induced emf charges the capacitor in the opposite direction.

(4) The capacitor discharges again and the process is repeated several times.

Thus energy obtained once from the source keeps oscillating between the capacitor and the inductor. If the circuit has no resistance then there will be no loss of energy and undamped oscillations are obtained. The frequency of oscillation is $\boxed{\nu = \frac{1}{2\pi\sqrt{LC}}}$

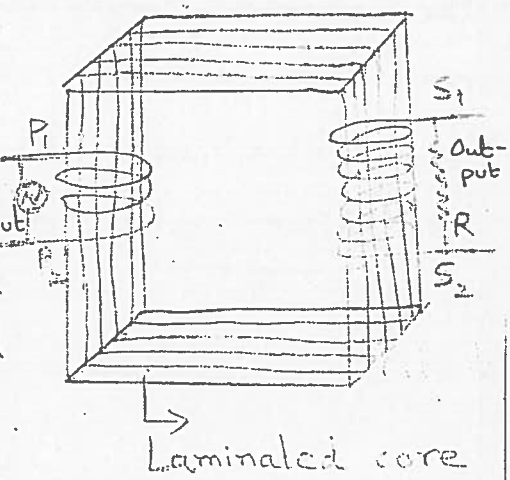


graph of series LCR circuit



TRANSFORMER : It is a device used for increasing or decreasing ac voltage.

Principle : It is based on the principle of mutual induction. ie. Whenever the amount of magnetic flux linked with a coil changes, an emf is induced in the neighbouring coil.



Construction:

1. It consists of a soft iron core made of laminated sheets well insulated from each other.
2. Two coils P_1P_2 and S_1S_2 are wound on the same core that are insulated from each other and from the core.
3. The alternating source is connected to the primary coil P_1P_2 and a load resistance R is connected to the secondary coil S_1S_2.

Theory and working

1. For an ideal transformer the resistances in the primary and secondary is negligible. The energy losses due to hysteresis in the iron core is also negligible.
2. The alternating emf supplied by the ac source connected to the primary is $E = E_0 \sin\omega t$.

The alternating primary current induces an alternating magnetic flux ϕ_B in the core. As the core extends through the secondary winding, the induced flux extends through the turns of the secondary.

3. The emf induced per turn is the same in the primary and secondary and according to faraday's law of electromagnetic induction

$$E_{per\ turn} = \frac{d\phi_B}{dt} = \frac{E_p}{N_p} = \frac{E_s}{N_s}.$$

where N_p and N_s are the total number of turns in the primary and the secondary.

$$\boxed{E_s = E_p \frac{N_s}{N_p}}$$

$\frac{N_s}{N_p}$ is K called the transformation ratio.

If $N_s > N_p$, $E_s > E_p$ and it is a step-up transformer and if $N_s < N_p$, $E_s < E_p$ and it is a step down transformer.

4. Since there is no loss of energy in the process

$$I_p E_p = E_s I_s$$

$$I_s = I_p \frac{E_p}{E_s}$$

$$\boxed{I_s = I_p \frac{N_p}{N_s}} \Rightarrow I_s = \frac{I_p}{K}$$

For a step up transformer, $K > 1$ $\therefore I_s < I_p$.

5. <u>Efficiency of a transformer</u>

It is the ratio of output power to input power.

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{E_s I_s}{E_p I_p}$$

Efficiency is always less than 1 because of many energy losses.

(a) Copper <u>loss</u>: It is the loss of energy in the form of heat in the copper coils of the transformer. In high current, low voltage windings these are minimised by using thick wire.

(b) Eddy currents: The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by using a laminated iron core

(c) Flux leakage: Not all of the flux due to primary passes through the secondary due to the air gaps in the core. Flux leakage can be minimised by winding the primary and secondary coils one over the other.

(d) Hysteresis: The magnetisation of the core is repeatedly reversed by the alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material that has low hysteresis loss.

Note: The large scale transmission of electrical energy over long distances is done with the use of transformers. The voltage output is stepped up so that current is reduced and consequently the I^2R loss is reduced considerably.

The voltage is stepped down in stages before a power supply of 240 V reaches our homes