Mathematics 2015 (Outside Delhi) Term II — SET II

Note : Except for the following questions, all the remaining questions have been asked in previous set.

SECTION — B

10. If $A(4, 3)$, $B(-1, y)$ and $C(3, 4)$ are the vertices of a right triangle ABC, right-angled at A, then find the value of y. [2]

Solution : Given the triangle ABC, right angled at A.

Now,
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$AB = \sqrt{(-1-4)^2 + (y-3)^2}$$
$$AB = \sqrt{(-5)^2 + (y-3)^2}$$
$$AB = \sqrt{25 + (y-3)^2}$$
$$AB = \sqrt{25 + y^2 + 9 - 6y}$$
$$AB = \sqrt{34 + y^2 - 6y}$$

$$BC = \sqrt{(3-(-1))^2 + (4-y)^2}$$
$$BC = \sqrt{(4)^2 + (4-y)^2}$$
$$BC = \sqrt{16 + 16 + y^2 - 8y}$$
$$BC = \sqrt{32 + y^2 - 8y}$$

And
$$AC = \sqrt{(3-4)^2 + (4-3)^2}$$
$$AC = \sqrt{(-1)^2 + (1)^2}$$
$$AC = \sqrt{1+1}$$
$$AC = \sqrt{2} \text{ units}$$

Given, ΔABC is a right angled triangle

So, by Pythagoras theorem
$$BC^2 = AC^2 + AB^2$$
$$(\sqrt{32 + y^2 - 8y})^2 = (\sqrt{2})^2 + (\sqrt{34 + y^2 - 6y})^2$$
$$32 + y^2 - 8y = 2 + 34 + y^2 - 6y$$

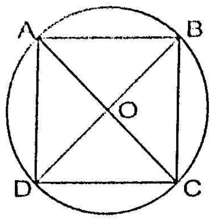
$$-2y = 4$$
$$y = -2$$

Hence, the value of y is -2. **Ans.**

SECTION — C

18. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if the area of the circle is 1256 cm². [Use $\pi = 3.14$] **[3]**

Solution :



Given that the area of the circle is 1256 cm².

$\because$ Area of the circle $= \pi r^2$

$$1256 = \frac{3.14}{100} \times r^2$$
$$r^2 = \frac{1256 \times 100}{314}$$
$$r = \sqrt{400}$$
$$r = 20 \text{ cm}$$

Now, $ABCD$ are the vertices of a rhombus.

$\therefore$ $\angle A = \angle C$ ...(i)

[opposite angles of rhombus]

But $ABCD$ lie on the circle.

So, $ABCD$ is called cyclic quadrilateral

$\therefore$ $\angle A + \angle C = 180°$ ...(ii)

On using equation (i), we get

$$\angle A + \angle A = 180°$$
$$2\angle A = 180°$$
$$\angle A = 90°$$

so, $\angle C = 90°$ [From eq. (i)]

$\therefore$ $ABCD$ is square.

So, BD is a diameter of circle.

[$\because$ The angle in a semicircle is a right angle triangle]

Now, Area of rhombus $= \frac{1}{2} \times$ product of diagonals

$$= \frac{1}{2} \times 40 \times 40$$
$$= 800 \text{ cm}^2$$

Hence, Area of rhombus is 800 cm². **Ans.**

19. Solve for x :

$$2x^2 + 6\sqrt{3}x - 60 = 0$$ **[3]**

Solution : Consider the given equation

$$2x^2 + 6\sqrt{3}x - 60 = 0$$
$$\Rightarrow \quad x^2 + 3\sqrt{3}x - 30 = 0 \quad ...(i)$$

Comparing equation (i) by

$$ax^2 + bx + c = 0$$

We get $a = 1, b = 3\sqrt{3}, c = -30.$

By quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-3\sqrt{3} \pm \sqrt{27 + 120}}{2}$$
$$x = \frac{-3\sqrt{3} \pm \sqrt{147}}{2}$$

Hence value for $x = \dfrac{-3\sqrt{3} \pm \sqrt{147}}{2}$ **Ans.**

20. The 16th term of an AP is five times its third term. If its 10th term is 41, then find the sum of its first fifteen terms. **[3]**

Solution : Given that 16th term of an A.P. is five time its 3rd term.

i.e., $a + (16-1)d = 5[a + (3-1)d]$

$\Rightarrow$ $a + 15d = 5[a + 2d]$

$\Rightarrow$ $a + 15d = 5a + 10d$

$\Rightarrow$ $4a - 5d = 0$ ...(i)

Also given that,

$$a_{10} = 41$$

$\Rightarrow$ $a + (10-1)d = 41$

$\Rightarrow$ $a + 9d = 41$ ...(ii)

On multiplying equation (ii) by 4, we get

$$4a + 36d = 164 \quad ...(iii)$$

Subtracting equation (iii) from (i), we get

$$\begin{array}{r} 4a - 5d = 0 \\ 4a + 36d = 164 \\ - \quad - \quad\quad - \\ \hline -41d = -164 \end{array}$$
$$d = 4$$

On putting the value of d in eq. (i), we get

$$4a - 5 \times 4 = 0$$
$$4a = 20$$
$$a = 5$$

Now, $S_{15} = \frac{15}{2}[2a + (15-1)d]$

$$S_{15} = \frac{15}{2}(2 \times 5 + 14 \times 4)$$
$$= \frac{15}{2} 2(5 + 14 \times 2)$$
$$= 15(5 + 28)$$
$$= 15 \times 33$$
$$S_{15} = 495$$

Hence, sum of first fifteen terms is 495. **Ans.**

SECTION — D

28. A bus travels at a certain average speed for a distance of 75 km and then travels a distance of 90

km at an average speed of 10 km/h more than the first speed. If it takes 3 hours to complete the total journey, find its first speed. **[4]**

Solution : Let x be the initial speed of the bus we know that

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

or

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Thus, we have

$$3 = \frac{75}{x} + \frac{90}{x+10}$$

$$\Rightarrow \quad 3 = \frac{75(x+10)+90x}{x(x+10)}$$

$$\Rightarrow \quad 3(x)(x+10) = 75x + 750 + 90x$$

$$\Rightarrow \quad 3x^2 + 30x = 75x + 750 + 90x$$

$$\Rightarrow \quad 3x^2 - 135x - 750 = 0$$

$$\Rightarrow \quad x^2 - 45x - 250 = 0$$

$$\Rightarrow \quad x^2 - 50x + 5x - 250 = 0$$

$$\Rightarrow \quad x(x-50) + 5(x-50) = 0$$

$$\Rightarrow \quad (x+5)(x-50) = 0$$

$$\Rightarrow \quad x = -5 \text{ or } x = 50$$

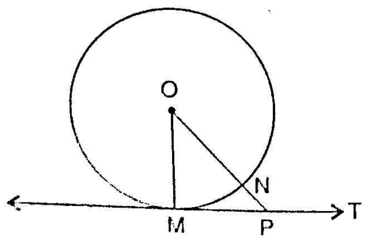
Since, speed cannot be negative

So, $\quad x = 50$

Hence, the initial speed of bus is 50 km/hr. **Ans.**

29. **Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.** **[4]**

Solution :



Given,

A circle with centre O and a tangent T at a point M of the circle.

To prove : $OM \perp T$

Construction : Take a point P, other than M on T. Join OP.

Proof : P is a point on the tangent T, other than the point of contact M.

$\therefore$ P lies outside the circle.

Let OP intersect the circle at N.

Then, $\quad ON < OP \quad$ ...(i)

[$\because$ a part is less than whole]

But $\quad OM = ON \quad$ ...(ii)

[Radii of the same circle]

$\therefore \quad OM < OP \quad$ [Using (ii)]

Thus, OM is shorter than any other line segment joining O to any point T, other than M.

But a shortest distance between a point and a line is the perpendicular distance.

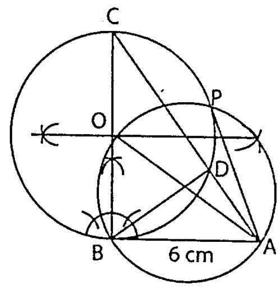
$\therefore \quad OM \perp T$

Hence, OM is perpendicular on T. **Hence Proved.**

30. **Construct a right triangle ABC with $AB = 6$ cm, $BC =$ 8 cm and $\angle B = 90°$. Draw BD, the perpendicular from B on AC. Draw the circle through B, C and D and construct the tangents from A to this circle.** [4]

Solution : Steps of construction :

(i) Draw a line segment AB = 6 cm.



(ii) Make a right angle at point B and draw BC = 8 cm.

(iii) Draw a perpendicular BD to AC.

(iv) Taking BC as diameter, draw a circle which passes through points B, C and D.

(v) Join A to O and taking AO as diameter, draw second circle.

(vi) From point A, draw tangents AB and AP.

31. **Find the values of k so that the area of the triangle with vertices $(k+1, 1)$, $(4, -3)$ and $(7, -k)$ is 6 sq. units.** [4]

Solution : Given, the vertices are $(k+1, 1)$, $(4, -3)$ and $(7, -k)$ and the area of the triangle is 6 square units.

Therefore,

$$\text{Area} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$6 = \frac{1}{2}[(k+1)(-3+k) + 4(-k-1) + 7(1+3)]$$

$$12 = (k+1)(k-3) + 4(-k-1) + 28$$

$$12 = k^2 - 3k + k - 3 - 4k - 4 + 28$$

$$k^2 - 6k + 9 = 0$$

$$k^2 - 3k - 3k + 9 = 0$$

$$k(k-3) - 3(k-3) = 0$$

$$(k-3)(k-3) = 0$$

$$\therefore \quad k = 3, 3$$

Hence, value of k is 3. **Ans.**