Time allowed : 3 Hours | Maximum marks : 90

SECTION — A

1. In ΔDEW, AB $\parallel EW$. If $AD = 4$ cm, $DE = 12$ cm and $DW = 24$ cm, then find the value of DB. [1]

Solution : Let $BD = x$ cm,

$DW = 24$ cm,

Then, $BW = (24 - x)$ cm, $AE = 12 - 4 = 8$ cm

In ΔDEW, $AB \parallel EW$

$\therefore \quad \frac{AD}{AE} = \frac{BD}{BW}$ [Thales' Theorem]

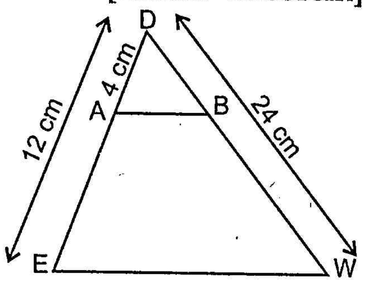
$\Rightarrow \quad \frac{4}{8} = \frac{x}{24 - x}$

$\Rightarrow \quad 8x = 96 - 4x$

$\Rightarrow \quad 12x = 96$

$\Rightarrow \quad x = \frac{96}{12} = 8$ cm

$\therefore \quad DB = 8$ cm **Ans.**



2. If ΔABC is right angled at B, what is the value of $\sin(A + C)$. [1]

Solution : $\angle B = 90°$ [Given]

We know that in Δ ABC,

$\angle A + \angle B + \angle C = 180°$

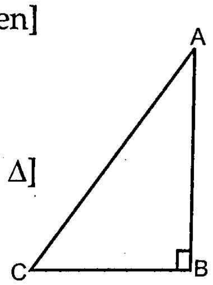
[Angle sum property of a Δ]

$\Rightarrow \angle A + \angle C + 90° = 180°$

$\Rightarrow \quad \angle A + \angle C = 180° - 90°$

$= 90°$

$\therefore \quad \sin(A + C) = \sin 90° = 1$ **Ans.**



3. If $\sqrt{3}\sin\theta = \cos\theta$, find the value of $\frac{3\cos^2\theta + 2\cos\theta}{3\cos\theta + 2}$. [1]

Solution : $\sqrt{3}\sin\theta = \cos\theta$ [Given]

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$ or $\tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan\theta = \tan 30° \Rightarrow \theta = 30°$

Now,

$$\frac{3\cos^2\theta + 2\cos\theta}{3\cos\theta + 2} = \frac{\cos\theta(3\cos\theta + 2)}{(3\cos\theta + 2)}$$

$= \cos\theta$

Put $\theta = 30°$

$\Rightarrow \quad \cos 30° = \frac{\sqrt{3}}{2}$ **Ans.**

4. From the following frequency distribution, find the median class :

Cost of living index	1400-1550	1550-1700	1700-1850	1850-2000
Number of weeks	8	15	21	8

[1]

Solution :

Cost of living index	No. of weeks (f)	c.f.
1400-1550	8	8
1550-1700	15	23
1700-1850	21	44
1850-2000	8	52
	$\Sigma f = 52$	

Here, $n = 52$

$\Rightarrow \quad \frac{n}{2} = \frac{52}{2} = 26,$

26 will lie in the class interval 1700-1850.

$\therefore$ Median class is 1700-1850. **Ans.**

SECTION — B

5. Show that $3\sqrt{7}$ is an irrational number. [2]

Solution : Let us assume, to the contrary, that $3\sqrt{7}$ is rational.

That is, we can find co-prime a and b ($b \neq 0$) such that $3\sqrt{7} = \frac{a}{b}$

Rearranging, we get $\sqrt{7} = \frac{a}{3b}$

Since 3, a and b are integers, $\frac{a}{3b}$ can be written in the form of $\frac{p}{q}$, so $\frac{a}{3b}$ is rational, and so $\sqrt{7}$ is rational.

But this contradicts the fact that $\sqrt{7}$ is irrational. So, we conclude that $3\sqrt{7}$ is irrational.

Hence Proved.

6. Explain why $(17 \times 5 \times 11 \times 3 \times 2 + 2 \times 11)$ is a composite number ? [2]

Solution : $17 \times 5 \times 11 \times 3 \times 2 + 2 \times 11 = 17 \times 5 \times 3 \times 22 + 22$ ...(i)

$= 22(17 \times 5 \times 3 + 1)$

$= 22(255 + 1) = 2 \times 11 \times 256$

Equation (i) is divisible by 2, 11 and 256, which means it has more than 2 prime factors.

$\therefore$ $(17 \times 5 \times 11 \times 3 \times 2 + 2 \times 11)$ is a composite number. **Ans.**

7. Find whether the following pair of linear equations is consistent or inconsistent :

$$3x + 2y = 8$$
$$6x - 4y = 9 \qquad [2]$$

Solution : Here, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{2}{-4} = \frac{-1}{2}$.

$$\frac{1}{2} \neq \frac{-1}{2}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, which will give a unique solution.

Hence, given pair of linear equations is consistent. **Ans.**

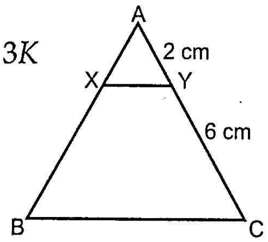
8. ***X* and *Y* are points on the sides *AB* and *AC* respectively of a triangle ABC such that $\frac{AX}{AB}$, *AY* = 2 cm and *YC* = 6 cm. Find whether *XY* || *BC* or not.** [2]

Solution : $\frac{AX}{AB} = \frac{1}{4}$

i.e., $AX = 1K, AB = 4K$ (K–constant)

$\therefore$ $BX = AB - AX$

$= 4K - 1K = 3K$



Now, $\frac{AX}{XB} = \frac{1K}{3K} = \frac{1}{3}$

And, $\frac{AY}{YC} = \frac{2}{6} = \frac{1}{3}$

$\frac{AX}{XB} = \frac{AY}{YC}$

$\therefore$ $XY \parallel BC$ (By converse of Thales' theorem) **Ans.**

9. Prove the following identity :

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = 1 - \sin\theta.\cos\theta. \qquad [2]$$

Solution : L.H.S. $= \frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta}$

$$= \frac{(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta.\cos\theta)}{(\sin\theta + \cos\theta)}$$

$[a^3 + b^3 = (a + b)(a^2 + b^2 - ab)]$

$= 1 - \sin\theta.\cos\theta =$ R.H.S.

$[\because \sin^2\theta + \cos^2\theta = 1]$ **Hence Proved.**

10. Show that the mode of the series obtained by combining the two series S_1 and S_2 given below is different from that of S_1 and S_2 taken separately:

S_1 : 3, 5, 8, 8, 9, 12, 13, 9, 9

S_2 : 7, 4, 7, 8, 7, 8, 13 [2]

Solution : Mode of S_1 series = 9

Mode of S_2 series = 7

After combining S_1 and S_2, the new series will be = 3, 5, 8, 8, 9, 12, 13, 9, 9, 7, 4, 7, 8, 7, 8, 13.

Mode of combined series = 8 (maximum times)

Mode of (S_1, S_2) is different from mode of S_1 and mode of S_2 separately. **Hence Proved.**

SECTION — C

11. The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly. [3]

Solution : To find the length of the longest rod that can measure the dimensions of the room exactly, we have to find HCF.

Length, L = 8 m 50 cm = 850 cm = $2^1 \times 5^2 \times 17$

Breadth, B = 6 m 25 cm = 625 cm = 5^4

Height, H = 4 m 75 cm = 475 cm = $5^2 \times 19$

$\therefore$ HCF of L, B and H is 5^2 = 25 cm

$\therefore$ Length of the longest rod = 25 cm **Ans.**

12. Solve by elimination :

$$3x - y = 7$$
$$2x + 5y + 1 = 0 \qquad [3]$$

Solution :
$$3x - y = 7 \qquad ...(i)$$
$$2x + 5y = -1 \qquad ...(ii)$$

Multiplying equation (i) by 5 and solving it with equation (ii), we get

$$\begin{aligned} 2x + 5y &= -1 \\ 15x - 5y &= 35 \\ \hline 17x \quad &= 34 \end{aligned}$$

(Adding)

$\Rightarrow$ $x = \frac{34}{17} = 2$

Putting the value of x in (i), we have

$3(2) - y = 7$

$\Rightarrow$ $6 - y = 7 \Rightarrow -y = 7 - 6$

$\Rightarrow$ $y = -1$

$\therefore$ $x = 2, y = -1$ **Ans.**

13. Find a quadratic polynomial, the sum and product of whose zeroes are 0 and $-\frac{3}{5}$ respectively. Hence find the zeroes. **[3]**

Solution : Quadratic polynomial

$$= x^2 - (\text{Sum of zeroes})\, x + \text{Product of zeroes}$$

$$= x^2 - (0)\, x + \left(\frac{-3}{5}\right) = x^2 - \frac{3}{5}$$

$$= (x)^2 - \left(\sqrt{\frac{3}{5}}\right)^2$$

$$= \left(x - \sqrt{\frac{3}{5}}\right)\left(x + \sqrt{\frac{3}{5}}\right) \quad \left[\begin{array}{l}\text{By applying} \\ (a^2 - b^2) = (a+b)(a-b)\end{array}\right]$$

Zeroes are, $x - \sqrt{\frac{3}{5}} = 0$ or $x + \sqrt{\frac{3}{5}} = 0$

$$\Rightarrow \quad x = \sqrt{\frac{3}{5}} \quad \text{or} \quad x = -\sqrt{\frac{3}{5}}$$

$$x = \sqrt{\frac{3}{5} \times \frac{5}{5}} \quad \text{or} \quad x = -\sqrt{\frac{3}{5} \times \frac{5}{5}}$$

$$x = \frac{\sqrt{15}}{5} \quad \text{or} \quad x = \frac{-\sqrt{15}}{5}$$

Ans.

14. The sum of the digits of a two digit number is 8 and the difference between the number and that formed by reversing the digits is 18. Find the number. **[3]**

Solution : Let unit digit $= x$

Tens digit $= y$

So, original number = unit digit + 10 × tens digit

$$1 = x + 10y$$

According to question,

$$\text{Sum of digits} = 8$$

so, $\quad x + y = 8 \quad$ ...(i)

On reversing the digits, unit digit $= y$

Tens digit $= x$

so, $\quad$ New number $= 10x + y$

According to question,

$$\text{Difference} = 18$$

$$\Rightarrow x + 10y - (10x + y) = 18$$

$$\Rightarrow \quad x + 10y - 10x - y = 18$$

$$\Rightarrow \quad 9y - 9x = 18$$

$$\Rightarrow \quad y - x = 2 \quad \text{...(ii)}$$

By adding eq. (i) and (ii)

$$2y = 10$$

$$y = \frac{10}{2} \Rightarrow y = 5$$

Put the value of y in eq. (i)

$$x + y = 8$$

$$\Rightarrow \quad x + 5 = 8$$

$$\Rightarrow \quad x = 8 - 5$$

$$\Rightarrow \quad x = 3$$

$\therefore$ Original number $= 10y + x$

$$= 10 \times 5 + 3$$

$$= 50 + 3$$

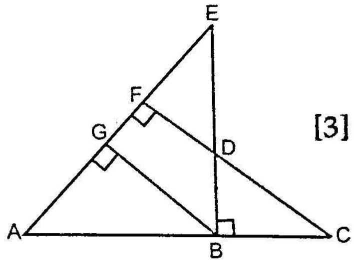
$$= 53$$

Ans.

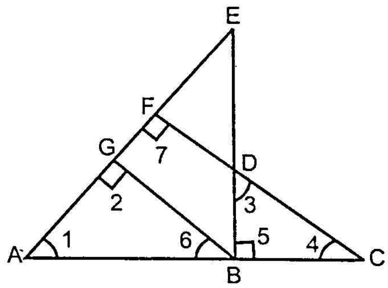
15. In given figure, EB ⊥ AC, BG ⊥ AE and CF ⊥ AE
Prove that :

(i) ΔABG ~ ΔDCB

(ii) $\frac{BC}{BD} = \frac{BE}{BA}$ **[3]**



Solution :



Given : EB ⊥ AC, BG ⊥ AE and CF ⊥ AE

To prove : (i) ΔABG ~ ΔDCB

(ii) $\frac{BC}{BD} = \frac{BE}{BA}$

Proof : (i) In Δ ABG and Δ DCB, BG || CF as corresponding angles are equal.

$\angle 2 = \angle 5$ [Each 90°]

$\angle 6 = \angle 4$ [Corresponding angles]

$\therefore$ Δ ABG ~ Δ DCB **Hence Proved.**

[By AA similarity]

$\angle 1 = \angle 3$ [CPCT]

(ii) In Δ ABE and Δ DBC

$\angle 1 = \angle 3$ [Proved above]

$\angle ABE = \angle 5$

[Each is 90°, EB ⊥ AC (Given)]

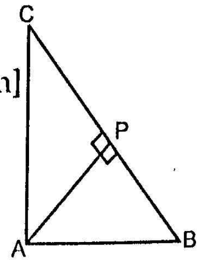
Δ ABE ~ Δ DBC [By AA similarity]

In similar triangles, corresponding sides are proportional

$\therefore \quad \frac{BC}{BD} = \frac{BE}{BA}$ **Hence Proved.**

16. In triangle ABC, if AP ⊥ BC and $AC^2 = BC^2 - AB^2$, then prove that $PA^2 = PB \times CP$. **[3]**

Solution : $AC^2 = BC^2 - AB^2$ [Given]



$AC^2 + AB^2 = BC^2$

$\therefore \quad \angle BAC = 90°$

[By converse of Pythagoras' theorem]

$\Delta APB \sim \Delta CPA$

[If a perpendicular is drawn from the vertex of the right angle of a triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other]

$\Rightarrow \quad \frac{AP}{CP} = \frac{PB}{PA}$

[In similar triangle, corresponding sides are proportional]

$\Rightarrow \quad PA^2 = PB \cdot CP$ **Hence Proved.**

17. If $\sin\theta = \frac{12}{13}$, $0° < \theta < 90°$, find the value of :

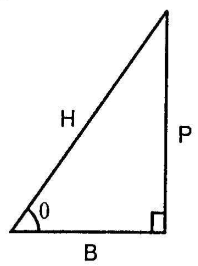
$$\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta.\cos\theta} \times \frac{1}{\tan^2\theta}$$ **[3]**

Solution : Given, $\sin\theta = \frac{12}{13}$

$\Rightarrow \quad \frac{P}{H} = \frac{12}{13}$

Let, P = 12K, H = 13K

$P^2 + B^2 = H^2$ [Pythagoras theorem]



$(12K)^2 + B^2 = (13K)^2$

$144K^2 + B^2 = 169K^2$

$B^2 = 169K^2 - 144K^2$

$= 25K^2$

$B = 5K$

$\therefore \quad \cos\theta = \frac{B}{H} = \frac{5K}{13K} = \frac{5}{13}$

$\tan\theta = \frac{P}{B} = \frac{12K}{5K} = \frac{12}{5}$

Now, $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta.\cos\theta} \times \frac{1}{\tan^2\theta}$

On solving,

$$= \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

$$= \frac{\frac{144-25}{169}}{\frac{120}{169}} \times \frac{25}{144}$$

$$= \frac{119}{120} \times \frac{25}{144} = \frac{595}{3456}$$ Ans.

18. If $\sec\theta + \tan\theta = p$, prove that $\sin\theta = \frac{p^2-1}{p^2+1}$ **[3]**

Solution : R.H.S. $= \frac{p^2-1}{p^2+1}$

$$= \frac{(\sec\theta+\tan\theta)^2 - 1}{(\sec\theta+\tan\theta)^2+1}$$

$$= \frac{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1}{\sec^2\theta + \tan^2 + 2\sec\theta\tan\theta + 1}$$

[By $(a+b)^2 = a^2 + b^2 + 2ab$]

$$= \frac{(\sec^2\theta - 1) + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + (1+\tan^2\theta) + 2\sec\theta\tan\theta}$$

$$= \frac{\tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + \sec^2\theta + 2\sec\theta\tan\theta}$$

$\left[\begin{array}{l}\sec^2\theta - 1 = \tan^2\theta \\ \sec^2\theta = 1 + \tan^2\theta\end{array}\right]$

$$= \frac{2\tan^2\theta + 2\sec\theta\tan\theta}{2\sec^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{2\tan\theta(\tan\theta + \sec\theta)}{2\sec\theta(\sec\theta + \tan\theta)} = \frac{\tan\theta}{\sec\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$= \sin\theta =$ L.H.S. **Hence Proved.**

19. Find the mean of the following distribution by Assumed Mean Method :

Class interval	Frequency
10-20	8
20-30	7
30-40	12
40-50	23
50-60	11

60-70	13
70-80	8
80-90	6
90-100	12

[3]

Solution :

Class interval	Frequency (f_i)	X_i	$d_i = X_i - 55$	$f_i d_i$
10-20	8	15	-40	-320
20-30	7	25	-30	-210
30-40	12	35	-20	-240
40-50	23	45	-10	-230
50-60	11	55	0	0
60-70	13	65	10	130
70-80	8	75	20	160
80-90	6	85	30	180
90-100	12	95	40	480
	$\Sigma f_i = 100$			$\Sigma f_i d_i = -50$

Let A = 55

$$\text{Mean} = A + \frac{\sum f_i d_i}{\sum f_i} = 55 + \left(\frac{-50}{100}\right)$$

$$= 55 - \frac{50}{100} = 55 - 0.5 = 54.5 \textbf{ Ans.}$$

20. The average score of boys in the examination of a school is 71 and that of the girls is 73. The average score of the school in the examination is 71.8. Find the ratio of number of boys in the number of girls who appeared in the examination. **[3]**

Solution : Let the number of boys = n_1

and number of girls = n_2

Average boys' score = $71 = \overline{X_1}$ (Let)

Average girls' score = $73 = \overline{X_2}$ (Let)

$$\text{Combined mean} = \frac{n_1\overline{X}_1 + n_2\overline{X}_2}{n_1 + n_2}$$

$$71.8 = \frac{n_1(71) + n_2(73)}{n_1 + n_2}$$

$$71n_1 + 73n_2 = 71.8n_1 + 71.8n_2$$

$$71n_1 - 71.8n_1 = 71.8n_2 - 73n_2$$

$$-0.8n_1 = -1.2n_2$$

$$\frac{n_1}{n_2} = \frac{1.2}{0.8} \Rightarrow \frac{n_1}{n_2} = \frac{3}{2}$$

$$\Rightarrow \quad n_1 : n_2 = 3 : 2$$

$\therefore$ No. of boys : No. of girls = 3 : 2. **Ans.**

SECTION — D

21. Find HCF of numbers 134791, 6341 and 6339 by Euclid's division algorithm. **[4]**

Solution : First we find HCF of 6339 and 6341 by Euclid's division method

```
6339) 6341 (1
      6339
      ----
         2) 6339 (3169
            6
            -
             3
             2
            --
             13
             12
            --
              19
              18
             ---
              1) 2 (2
                 2
                 -
                 0
```

$$6341 > 6339$$

$$\Rightarrow \quad 6341 = 6339 \times 1 + 2$$

Also, $\quad 6339 = 2 \times 3169 + 1$

$$2 = 1 \times 2 + 0$$

$\therefore$ HCF of 6341 and 6339 is 1.

Now, we find the HCF of 134791 and 1

$$134791 = 1 \times 134791 + 0$$

$\therefore$ HCF of 134791 and 1 is 1.

Hence, HCF of given three numbers is 1. **Ans.**

22. Draw the graph of the following pair of linear equations :

$x + 3y = 6$ and $2x - 3y = 2$

Find the ratio of the areas of the two triangles formed by first line, $x = 0$, $y = 0$ and second line, $x = 0$, $y = 0$. **[4]**

Solution : **First Line** $\qquad$ **Second Line**

$x + 3y = 6 \qquad 2x - 3y = 12$

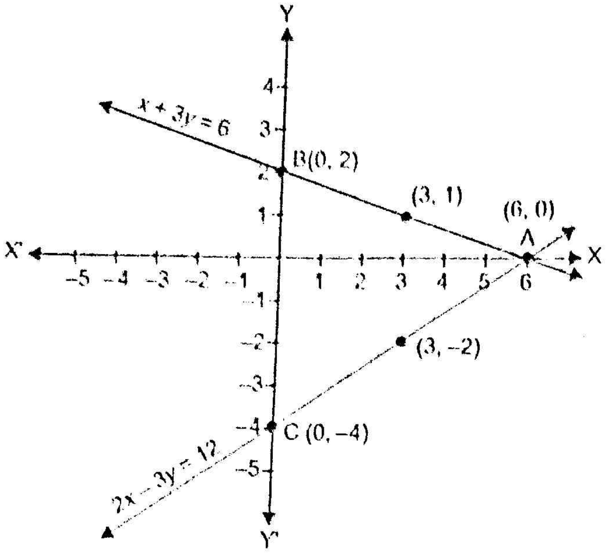
$\Rightarrow \quad x = 6 - 3y \qquad \Rightarrow 2x = 12 + 3y$

$$\Rightarrow x = \frac{12 + 3y}{2}$$

x	6	3	0
y	0	1	2

x	6	3	0
y	0	−2	−4

(6, 0), (3, 1), (0, 2) $\qquad$ (6, 0), (3, −2), (0, −4)



Area of triangle

$= \frac{1}{2} \times \text{base} \times \text{corresponding altitude}$

$$\therefore \quad \frac{\text{Area of } \Delta AOB}{\text{Area of } \Delta AOC} = \frac{1/2 \times OA \times OB}{1/2 \times OA \times OC}$$

$$\Rightarrow \quad \frac{OB}{OC} = \frac{2}{4} = \frac{1}{2}$$

$\therefore$ Required ratio = 1 : 2 **Ans.**

23. If the polynomial $(x^4 + 2x^3 + 8x^2 + 12x + 18)$ is divided by another polynomial $(x^2 + 5)$, the remainder comes out to be $(px + q)$, find the values of p and q. **[4]**

Solution :

$$\begin{array}{r|l} & x^2 + 2x + 3 \\ \hline x^2 + 5 & x^4 + 2x^3 + 8x^2 + 12x + 18 \\ & +x^4 \qquad\quad +5x^2 \\ & - \qquad\qquad - \\ \hline & 2x^3 + 3x^2 + 12x + 18 \\ & +2x^3 \qquad\quad + 10x \\ & - \qquad\qquad - \\ \hline & 3x^2 + 2x + 18 \\ & +3x^2 \qquad\quad + 15 \\ & - \qquad\qquad - \\ \hline & 2x + 3 \end{array}$$

Remainder = $2x + 3$

i.e., $\quad px + q = 2x + 3$

$\therefore \quad p = 2, q = 3$ **Ans.**

24. What must be subtracted from $p(x) = 8x^4 + 14x^3 - 2x^2 + 8x - 12$ so that $4x^2 + 3x - 2$ is factor of $p(x)$? This quesiton was given to group of students for working together. Do you think teacher should promote group work? **[4]**

Solution : For this,

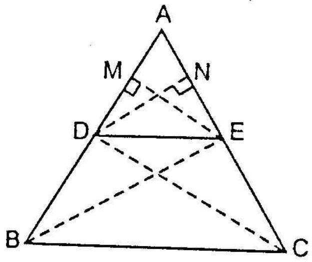
$$\begin{array}{r|l} & 2x^2 + 2x - 1 \\ \hline 4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 8x - 12 \\ & +8x^4 + 6x^3 - 4x^2 \\ & - \quad - \quad + \\ \hline & 8x^3 + 2x^2 + 8x - 12 \\ & +8x^3 + 6x^2 - 4x \\ & - \quad - \quad + \\ \hline & -4x^2 + 12x - 12 \\ & -4x^2 - 3x + 2 \\ & + \quad + \quad - \\ \hline & 15x - 14 \end{array}$$

Polynomial to be subtracted is $(15x - 14)$.

Value : Yes, as it increases confidence and team spirit among students. **Ans.**

25. Prove "If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio". **[4]**

Solution : Given, In ΔABC, $DE \parallel BC$



To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Draw $EM \perp AB$ and $DN \perp AC$. Join B to E and C to D.

Proof : In ΔADE and ΔBDE

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times DB \times EM} = \frac{AD}{DB} \quad \text{...(i)}$$

[Area of $\Delta = \frac{1}{2} \times$ base $\times$ corresponding altitude]

In ΔADE and ΔCDE

$$\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN} = \frac{AE}{EC} \quad \text{...(ii)}$$

Since, $\quad DE \parallel BC$ [Given]

$\therefore \quad ar(\Delta BDE) = ar(\Delta CDE)$ ...(iii)

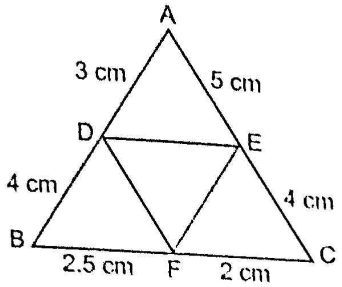
[As on the same base and between the same parallel sides are equal in area]

From eq. (i), (ii) and (iii)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence Proved.

26. In the given figure, $AD = 3$ cm, $AE = 5$ cm, $BD = 4$ cm, $CE = 4$ cm, $CF = 2$ cm, $BF = 2.5$ cm, then find the pair of parallel lines and hence their lengths. [4]



Solution : $\frac{EC}{EA} = \frac{CF}{FB}$ and $\frac{CF}{FB} = \frac{2}{2.5} = \frac{4}{5}$

$\Rightarrow \quad \frac{EC}{EA} = \frac{CF}{FB}$

In ΔABC, $EF \parallel AB$

[Converse of Thales' theorem]

Also, $\quad \frac{CE}{CA} = \frac{4}{4+5} = \frac{4}{9}$ ...(i)

$\frac{CF}{CB} = \frac{2}{2+2.5} = \frac{2}{4.5} = \frac{4}{9}$

$\frac{EC}{EA} = \frac{CF}{CB}$

$\angle ECF = \angle ACB$ [Common]

$\Delta CFE \sim \Delta CBA$ [SAS similarity]

$\Rightarrow \quad \frac{EF}{AB} = \frac{CE}{CA}$

[In similar Δ's, corresponding sides are proportional]

$\Rightarrow \quad \frac{EF}{7} = \frac{4}{9}$

[$\because AB = 3 + 4 = 7$ cm]

$\therefore \quad EF = \frac{28}{9}$ cm and $AB = 7$ cm **Ans.**

27. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, where $0 < A + B < 90°$, $A > B$, find A and B. Also calculate $\tan A . \sin(A + B) + \cos A . \tan(A - B)$. [4]

Solution : Given, $\tan(A + B) = \sqrt{3}$, $\tan(A - B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan(A + B) = \tan 60°$

$(A + B) = 60°$ ...(i)

And, $\tan(A - B) = \tan 30°$

$(A - B) = 30°$ ...(ii)

On adding eq. (i) & (ii)

$$\begin{aligned} A + B &= 60° \\ \underline{A - B} &\underline{= 30°} \\ \underline{2A} &\underline{= 90°} \end{aligned}$$ [By adding]

$\Rightarrow \quad A = \frac{90°}{2} = 45°$

From eq. (i), $A + B = 60°$

$45° + B = 60°$

$B = 15°$

$\therefore \quad A = 45°, B = 15°$

Now, $\tan A . \sin(A + B) + \cos A . \tan(A - B)$

$= \tan 45° . \sin(60°) + \cos 45° . \tan(30°)$

$= 1 \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{3}}{2} + \frac{\sqrt{6}}{6}$

$= \frac{3\sqrt{3} + \sqrt{6}}{6}$ **Ans.**

28. Prove that :

$(1 + \cot A + \tan A).(\sin A - \cos A)$

$$= \frac{\sec^3 A - \operatorname{cosec}^3 A}{\sec^2 A . \operatorname{cosec}^2 A}$$ [4]

Solution : L.H.S. $= (1 + \cot A + \tan A)(\sin A - \cos A)$

$= \left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A)$

$= \left(\frac{\sin A \cos A + \cos^2 A + \sin^2 A}{\sin A . \cos A}\right)(\sin A - \cos A)$

$= \frac{\sin^3 A - \cos^3 A}{\sin A . \cos A}$

[Using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$$= \frac{\frac{\sin^3 A}{\sin^3 A . \cos^3 A} - \frac{\cos^3 A}{\sin^3 A . \cos^3 A}}{\frac{\sin A \cos A}{\sin^3 A . \cos^3 A}}$$

[Dividing Num. & Denom. by $\sin^3 A . \cos^3 A$]

$= \frac{\sec^3 A - \operatorname{cosec}^3 A}{\sec^2 A . \operatorname{cosec}^2 A}$ = R.H.S. **Hence Proved.**

29. Prove the identity :

$$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{1 - 2\cos^2 A}$$ [4]

Solution : L.H.S. $= \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$

$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}$

$$= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A + \sin^2 A + \cos^2 A - 2\sin A \cos A}{\sin^2 A - \cos^2 A}$$

$$= \frac{1+1}{1-\cos^2 A - \cos^2 A} \qquad [\because \sin^2 A + \cos^2 A = 1, \ \sin^2 A = 1 - \cos^2 A]$$

$$= \frac{2}{1-2\cos^2 A} = \text{R.H.S.}$$ **Hence Proved.**

30. The following table gives the daily income of 50 workers of a factory. Draw both types ("less than type" and " greater than type") ogives.

Daily income (in ₹)	No. of workers
100 – 120	12
120 – 140	14
140 – 160	8
160 – 180	6
180 – 200	10

[4]

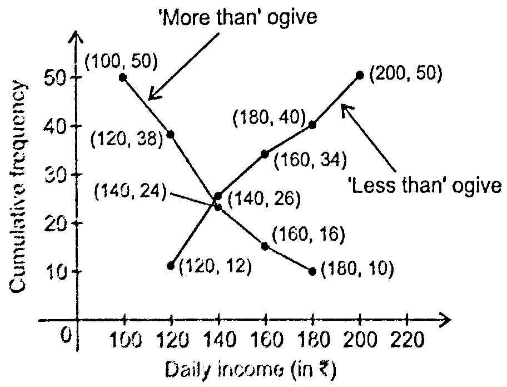
Solution :

Less than ogive

Daily Income (in ₹)	No. of workers (c.f.)
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

More than ogive

Daily Income (in ₹)	No. of workers (c.f.)
More than 100	50
More than 120	38
More than 140	24
More than 160	16
More than 180	10



31. In a class test, marks obtained by 120 students are given in the following frequency distribution. If it is given that mean is 59, find the missing frequencies x and y.

Marks	No. of students
0 – 10	1
10 – 20	3
20 – 30	7
30 – 40	10
40 – 50	15
50 – 60	x
60 – 70	9
70 – 80	27
80 – 90	18
90 – 100	y

[4]

Solution :

Marks	No. of Students f_i	X_i	$d_i = \frac{X_i - 55}{10}$	$f_i d_i$	
0-10	1	5	–5	–5	
10-20	3	15	–4	–12	
20-30	7	25	–3	–21	–73
30-40	10	35	–2	–20	
40-50	15	45	–1	–15	
50-60	x	A= 55	0	0	
60-70	9	65	1	9	
70-80	27	75	2	54	117 + 4y
80-90	18	85	3	54	
90-100	y	95	4	$4y$	
	$\Sigma f_i = 90 + x + y$			$\Sigma f_i d_i = 44 + 4y$	

$$\Sigma f_i = 90 + x + y$$

But $\qquad \Sigma f_i = 120$ [Given]

$\therefore \qquad 90 + x + y = 120$

$$x = 120 - 90 - y = 30 - y \qquad ...(i)$$

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} \times h$$

$$\Rightarrow \qquad 59 = 55 + \left(\frac{44+4y}{120} \times 10\right)$$

[A = 55, h = 10, Σf_i = 120]

$$\Rightarrow \qquad 59 - 55 = \frac{4(11+y)}{12}$$

$$\Rightarrow \qquad 4 \times 3 = 11 + y$$

$$\Rightarrow \qquad y = 12 - 11 = 1$$

From eq. (i), $\quad x = 30 - 1 = 29$

$\therefore \qquad x = 29, y = 1$ **Ans.**