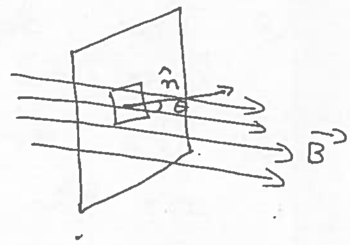
V

Electromagnetic Induction

The phenomenon of generating current/emf due to change in the magnetic flux linked with the coil is electro magnetic induction.

Magnetic flux (ϕ): Magnetic flux $\Delta\phi$ through an area ΔS is defined as the product of the magnitude of the area element and the component of $\vec{B}$ along normal to the area element

$$\Delta\phi = (B\cos\theta)\,\Delta S$$
$$= \vec{B}\cdot\Delta\vec{S}$$



Magnetic flux over the entire surface is

$$\phi = \int \vec{B}\cdot d\vec{s}$$
or
$$\phi = \vec{B}\cdot\vec{A} = BA\cos\theta.$$

[θ is the smallest angle that the normal to the surface makes with the magnetic field].

Note (a) For N turns, $\phi = NBA\cos\theta$.

(b) S.I unit of magnetic flux is weber (Wb)

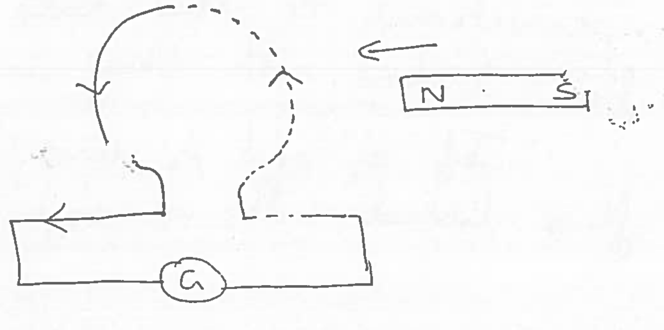
(c) It is a scalar quantity

(d) Its dimensions $[ML^2T^{-2}A^{-1}]$

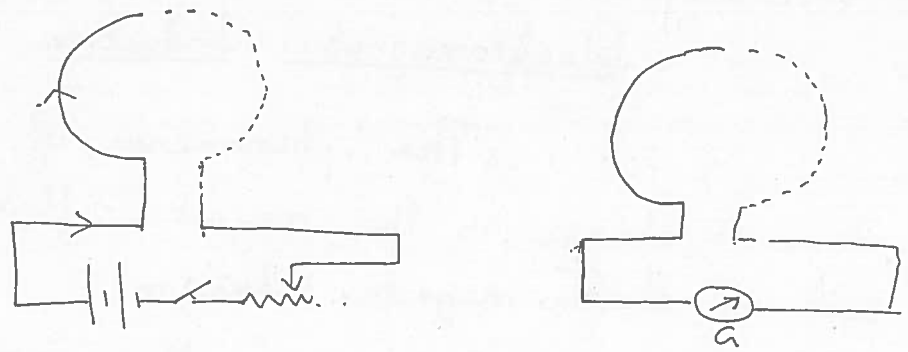
(e) From dimensional formula it can be shown that the unit of flux is volt - sec.

FARADAY'S EXPERIMENTS

EXPT - I.



EXPT - II



Observations

(1) In the first experiment the galvanometer showed deflection when there was relative motion between the coil and the magnet.

(2) The deflection was temporary and lasted as long as there was relative motion.

(3) The extent of deflection was found to increase as the magnet moved faster.

(4) On reversing the polarity the deflection in the galvanometer was in the opposite direction.

(5) In the second experiment the galvanometer showed deflection while pressing or releasing the key.

Conclusion

It is clear from the above observations that in order to produce an induced emf in the coil the magnetic flux linked with the coil must change.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

1. Whenever the amount of magnetic flux linked with a circuit changes, an emf is induced in the circuit. The induced emf lasts so long as the change in magnetic flux continues.

2. The magnitude of induced emf in a circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.

If ϕ_1 and ϕ_2 are the amount of magnetic flux linked with a coil during a time 't' then according to the second law

$$e \propto \frac{\phi_2 - \phi_1}{t}$$

$$e = k\frac{(\phi_2 - \phi_1)}{t}$$

or

$$\boxed{e = -\frac{d\phi}{dt}} \quad [\because k = 1]$$

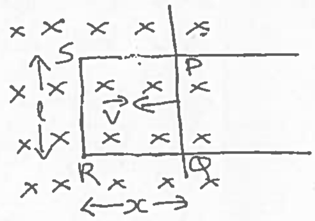
[-ve sign is because the induced emf always opposes any change in magnetic flux linked with the circuit]

Lenz's law: [This law gives the direction of the induced current in the circuit]

According to this law the direction of induced current will appear in such a direction that it opposes the change in magnetic flux responsible for its production.

Lenz's law and conservation of energy

Let 'r' be the resistance of movable arm PQ of the rectangular conductor. The remaining arms have negligible resistance



Therefore the current in the loop is $I = e/r$

$$\therefore I = \frac{Blv}{r}$$

Because of the magnetic field there will be a force on the arm PQ

$$\therefore F = BIl = \frac{B^2l^2v}{r}$$ directed outwards opposite to the velocity of the rod.

If the arm PQ is moved with a constant speed 'v' the power

$$P = Fv = \frac{B^2l^2v^2}{r} \text{ --- (1)}$$

The agent that does this work is mechanical and this mechanical energy is dissipated as Joule heat.

$$P_J = I^2r = \left[\frac{Blv}{r}\right]^2 r = \frac{B^2l^2v^2}{r} \text{ --- (2)}$$

Thus mechanical energy needed to move the arm PQ is converted into electrical energy (induced emf) and then to thermal energy.

Qualitative study of Lenz's law and energy conservation: When the north pole of the barmagnet is brought towards the coil a north polarity is induced in the coil and work has to be done against the force of repulsion between them. It is this mechanical work done in moving the magnet with respect to the coil that changes into electrical energy producing induced current.

When the magnet is not moved, no work is done and hence no induced current.

Therefore Lenz's law obeys the principle of conservation of energy.

Fleming's right hand rule: The direction of induced current is given by this rule.

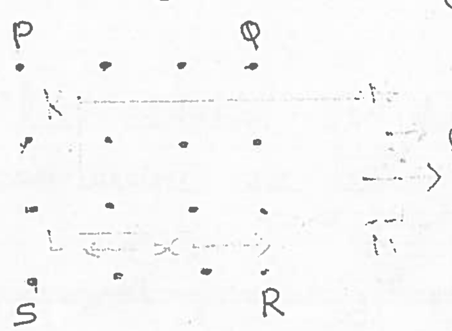
If the fore finger, middle finger and thumb are stretched in three mutually ⊥ directions then the fore finger represents the direction of magnetic field, the thumb represents the direction of motion of the conductor and the direction of induced current is given by the middle finger.

Various methods of producing induced emf

Since $\phi = BA\cos\theta$, induced emf can be produced by

(1) Changing magnetic field: The magnetic field can be changed by moving a magnet towards or away from the coil or by pressing and releasing the key.

(2) Changing area:



(a) Consider a uniform magnetic field PQRS directed out of the plane of the paper.

(b) A rectangular loop KLMN such that $KL = l$ is held partially in the field.

(c) At any instant 't' suppose x be the position of the loop in the magnetic field and the loop is moved with a velocity v then the area of the loop associated with the magnetic field changes and an emf is induced in it.

(d) <u>To calculate</u> <u>the</u> <u>induced</u> <u>emf</u> : Suppose in time Δt the loop is moved out of the magnetic field through a small distance Δx then the decrease in area of the loop is $-l\Delta x$.

Change in magnetic flux $d\phi = -Bl\Delta x$

$$e = -\frac{d\phi}{dt} = -\left[-Bl\frac{\Delta x}{\Delta t}\right]$$

$$\boxed{e = Blv}$$

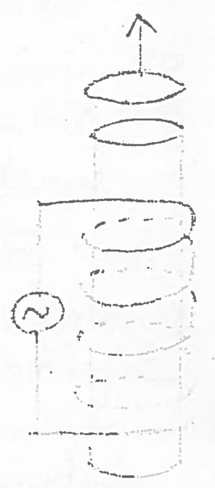
where v is the velocity of the loop. If R is the resistance of the loop then $i = \frac{e}{R} = \frac{Blv}{R}$.

<u>Note</u> : (a) If the loop is moved in such a manner that the entire area remains in the magnetic field no induced current appears. (b) The relation between the charge flow through a circuit and change in magnetic flux is $\Delta Q = \frac{\Delta \phi_B}{R}$

(3) <u>Induced</u> <u>emf</u> <u>by</u> <u>changing</u> <u>relative</u> <u>orientation</u> <u>of</u> <u>coil</u> <u>and</u> <u>magnetic</u> <u>field</u> : If the coil is rotated in a magnetic field then the angle θ between the normal to the coil and B changes $\therefore$ emf is induced.

* <u>EDDY</u> <u>CURRENTS</u> : It is the current induced in the body of the conductor when the amount of magnetic flux linked with the conductor changes.

<u>Example - 1</u> : A light metallic disc placed on a current carrying solenoid is thrown up in air.



<u>Reason</u> : As current through the solenoid increases the magnetic flux along the axis of the solenoid increases. The eddy currents magnetise the disc such that if the upper end of solenoid acquires north polarity the lower face of the disc also acquires a north polarity. Hence the metallic disc is thrown up in air.

Example - 2 : The damping effect of a flat metallic plate between the pole pieces of an electromagnet

Reason : When the electromagnet is switched on and the metallic plate is at the mean position then the magnetic flux linked with the plate is maximum. As it goes to the extreme position the magnetic flux linked with the plate decreases and eddy currents are induced in it. The direction of eddy currents is so as to oppose the change taking place and coil quickly returns to the mean position. This damping effect is called as electromagnetic damping.

Applications of eddy currents :

(a) Electromagnetic damping : Used in designing dead beat galvanometers. (concept explained above).

(b) Induction furnace : The large eddy currents developed produces so much heat that the substance melts.

(c) Induction motor

(d) Speedometer

(e) Diathermy

(f) Electromagnetic brakes

Disadvantages of eddy currents

(1) Eddy currents oppose relative motion.

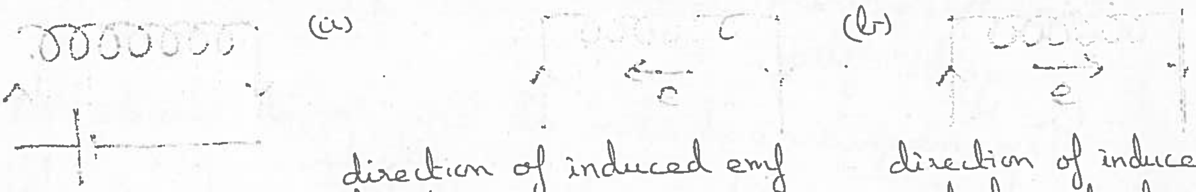
(2) Loss of energy takes place in the form of heat

(3) Excessive heat produced due to eddy currents may break the insulation of various appliances.

Note: Eddy currents can be minimised by using laminated cores as in the case of transformers and dynamos.

The iron core is in the form of thin sheets electrically insulated from each other. (Since resistance is largely increased eddy currents can be reduced)

Self induction: It is the property of a coil by virtue of which the coil opposes any change in the strength of current flowing through it by inducing an emf in itself.



(a) direction of induced emf due to increasing current (key pressed)

(b) direction of induced emf due to decreasing current (key released)

Coefficient of self induction (L) or Self inductance

If I is the current flowing through a coil at any time and ϕ is the magnetic flux linked with the coil at that time.

$$\phi \propto I$$

$$\phi = LI.$$

L is the coefficient of self induction or self inductance of the coil.

(i) If $I = 1\ A$, $\phi = L$.

$\therefore$ Coefficient of self induction of a coil is numerically equal to the amount of magnetic flux linked with the coil when unit current passes through it.

(ii) Also, $e = -\dfrac{d\phi}{dt}$

$$\boxed{e = -L\frac{dI}{dt}} \quad [\because \phi = LI]$$

If $\dfrac{dI}{dt} = 1\ A/sec$, then $e = -L$.

$\therefore$ Coefficient of self induction of a coil is equal to the emf induced in the coil when the rate of change of current through the coil is unity.

The unit of self inductance L is henry (H).

Definition of 1 henry: Self inductance of a coil is 1 henry when a current change at the rate of 1 A/sec through the coil induces an emf of 1V in the coil.

* Self inductance of a long solenoid

Consider a solenoid of length l having N number of turns in it. A time varying current is set up through the solenoid which gives rise to a time varying magnetic flux.

The magnetic induction at any point inside a solenoid carrying current is $B = \mu_0 \frac{NI}{l}$ $[\because n = N/l]$

$\therefore$ Magnetic flux linked with each turn of the solenoid

$$\phi_{\text{each turn}} = BA$$

$$= \frac{\mu_0 NIA}{l}$$

where A is the area of each turn of the solenoid

Total magnetic flux linked with all the turns of the solenoid

$$\phi_{\text{all turns}} = \frac{\mu_0 NIA}{l} \times N$$

$$\phi_{\text{all turn}} = \frac{\mu_0 N^2 IA}{l} \quad \text{— (1)}$$

Since $\phi = LI$ — (2)

Equating (1) & (2), $\boxed{L = \frac{\mu_0 N^2 A}{l}}$

L depends on the number of turns, area of each turn and the nature of the material of the core on which the coil is wound. (Self inductance plays the role of inertia)

Mutual induction: It is the property of two coils by virtue of which each coil opposes any change in the strength of current flowing through the other coil by developing an induced emf.

When the key is pressed magnetic flux linked with P and thereby S increases and an emf is induced in S. The direction is so as to oppose the change.

When the key is released (break) the magnetic flux decreases and direction of induced emf is in the same direction of cell current so as to prolong the decay of current.

Coefficient of mutual induction: (M) or Mutual Inductance

If I is the current in one coil then ϕ is the amount of magnetic flux linked with the neighbouring coil.

$$\phi \propto I$$

$$\phi = MI$$

(a) If $I = 1A$, then $\phi = M$.

$\therefore$ Coefficient of mutual induction or mutual inductance of two coils is numerically equal to the magnetic flux linked with one coil when unit current flows through the neighbouring coil.

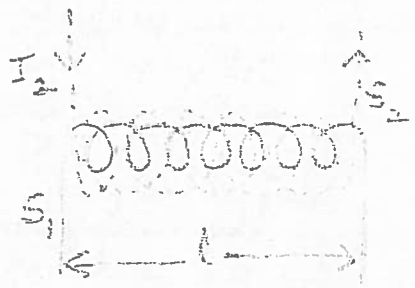
(b) Also, $\boxed{e = -\frac{d\phi}{dt} = -M\frac{dI}{dt}}$ $[\because \phi = MI]$

If $\frac{dI}{dt} = 1 A/sec$ then $M = e$.

Thus coefficient of mutual induction of two coils is equal to the emf induced in one coil when the rate of change of current through the other coil is unity.

SI unit of M is henry (H)

Mutual inductance of two long solenoids



Let N_1 be the number of turns of a long air cored inner solenoid S_1 of radius r_1. Another outer solenoid S_2, of radius r_2 having N_2 turns is wound over S_1 and a time varying current I_2 passes through S_2. l is the length of each solenoid.

Due to time varying current the magnetic flux linked with each turn of S_1 is

$$\phi_{1\,(\text{each turn})} = B_2 A_1$$
$$= \frac{\mu_0 N_2 I_2 A_1}{l} \quad \left[\because B_2 = \frac{\mu_0 N_2 I}{l}\right.$$

$\therefore$ Magnetic flux linked with all turns of S_1 is

$$\phi_{1\,(\text{all turns})} = \frac{\mu_0 N_2 I_2 A_1}{l} \times N_1 \quad — (1)$$

Also, $\phi_1 = M_{12} I_2 \quad —(2)$

Equating (1) and (2)

$$\boxed{M_{12} = \frac{\mu_0 N_1 N_2 A_1}{l}}$$

M_{12} is the mutual inductance of coil 1 with respect to coil 2. M depends on the geometry of coils (that is the size, shape, no: of turns etc), and also on the nature of the material on which it is wound [then $\mu = \mu_0 \mu_r$]

<u>Note</u>: Suppose the time varying current passes through S_1; the flux due to S_1 is confined inside S_1 only because there is no magnetic field outside S_1. Thus $\phi_2 = B_1 A_1$

Also $\phi_2 = M_{21} I_1$

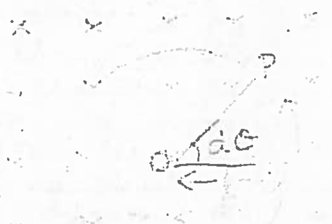
Hence $M_{21} = \dfrac{\mu_0 N_1 N_2 A_1}{l}$

$\therefore \boxed{M_{12} = M_{21}}$

* <u>An important conceptual question</u>

1. A copper rod of length l rotates with an angular speed ω in a uniform magnetic field B. Find the emf developed between the two ends of the rod. The field is $\perp$ to the motion of the rod.

To calculate the emf imagine a closed loop by connecting the centre with any point on the circumference. The induced emf is equal to B× (rate of change of area of loop).

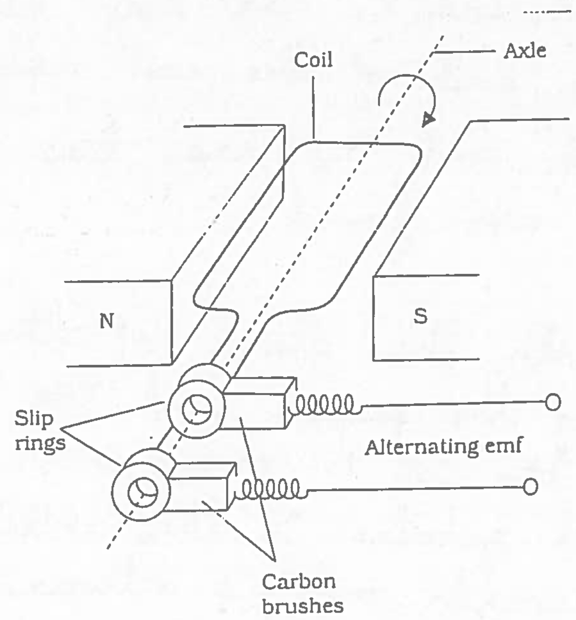


If $d\theta$ is the angle between the rod and the radius of the circle at P at time t, then the area of arc formed by the rod and radius $\therefore \pi l^2 \dfrac{d\theta}{2\pi} = dA$

$\therefore e = \dfrac{d\phi}{dt} \Rightarrow e = B\dfrac{dA}{dt} \;[\because \phi = BA] \Rightarrow e = \dfrac{B l^2 d\theta}{2\,dt} \;\left[\because dA = \dfrac{l^2 d\theta}{2}\right.$

$\therefore \boxed{e = \dfrac{B l^2 \omega}{2}}$

AC Generator



During the first half rotation of the coil.
[AB goes into the plane of the paper,
Magnetic field from North pole to South pole,
and using fleming's right hand rule induced current from
A → B]

It is a device which produces electrical energy from mechanical energy

Principle : An ac generator is based on the phenomenon of electromagnetic induction. ie whenever the amount of magnetic flux linked with a coil changes an emf is induced in the coil.

Construction

(a) Armature : ABCD is a rectangular coil consisting of a number of turns of insulated copper wire wound on a laminated soft iron core.

(b) Field magnets : N and S are the pole pieces of a strong electromagnet [illegible] which the coil is rotated. The axis of rotation is perpendicular to magnetic field lines

(c) <u>Slip rings</u> : R_1 and R_2 are two hollow metallic rings to which the ends of the coil are attached.

(d) <u>Brushes</u> : B_1 and B_2 are two flexible carbon rods that are fixed.

<u>Working</u>

(1) To start with, the plane of the coil is ⊥ to the plane of the paper and the magnetic flux linked with the coil is maximum.

(2) As the coil rotates in the anticlockwise direction AB moves inwards and CD outwards. According to Fleming's right hand rule current induced is from $A \rightarrow B$ and $C \rightarrow D$. ∴ In the external circuit current flows from $B_2 \rightarrow B_1$.

(3) After half a rotation, AB moves outwards and CD inwards. The induced current is from $B \rightarrow A$ and $D \rightarrow C$. ∴ In the external circuit current flows from $B_1 \rightarrow B_2$.

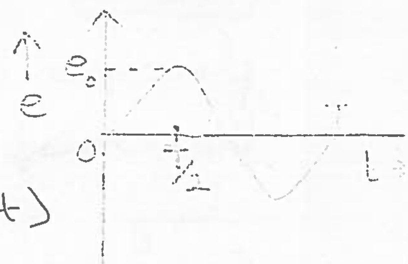
The induced current is alternating in nature and is repeated several times.

(4) Magnetic flux linked with the coil is

$$\phi = N(\vec{B} \cdot \vec{A})$$

$$= NBA \cos\theta$$

$$= NBA \cos\omega t$$



$$e = -\frac{d\phi}{dt} = -NBA\omega(-\sin\omega t)$$

$$\boxed{e = NAB\omega \sin\omega t}$$

N → number of turns of the coil.

A → area of cross section of each turn of the coil.

θ → is the angle that the normal to the plane of the coil makes with the magnetic field.

Induced emf e is maximum when $\sin\omega t = 1$ ∴ $\boxed{e = e_0 \sin\omega t}$ $[e_0 = NAB\omega]$