I [A]

Electrostatics

The term electrostatics refers to frictional electricity or charges at rest.

Example: When a glass rod is rubbed with silk, the glass rod becomes positively charged and silk negatively charged. Similarly when ebonite is rubbed with wool, ebonite acquires a negative charge and wool becomes positively charged.

Origin of electric charge: When two substances are rubbed together energy is being provided to overcome the friction between them. It is this energy that is used to transfer the electrons from a substance having a lower work function to that which has a higher work function.

Since the +vely charged body loses electrons its mass reduces by a negligible amount and the -vely charged body increases in mass by a small amount.

* Properties of electric charge

1. Quantization of electric charge: It is the property of electric charge by virtue of which it exists only in discrete packets of a certain minimum amount of charge ie $q = ne$

   where $n = \pm 1, \pm 2, \pm 3 \ldots\ldots$

   and $e = 1.6 \times 10^{-19}$ C

   The cause of quantization is because when two substances are rubbed against each other only integral number of electrons can be transferred from one body to another.

2. Conservation of electric charge: It states that in an isolated system, the net charge always

remains constant which implies that charges can neither be created nor destroyed.

example: (a) A γ-ray photon materialises into an electron and positron.

$$\gamma \rightarrow e^- + e^+$$

(b) In all nuclear transformation the proton number remains unchanged.

$$U_{92}^{238} \rightarrow Th_{90}^{234} + He_2^4$$

3. Electric charges are additive in nature
The total charge on a body is the algebraic sum of the charges located at the different parts of a body

4. Electric charge is not affected by the motion of the charged particle.

5. Like charges repel each other and unlike charges repel each other.

COULOMB'S LAW: It states that two point charges attract or repel each other with a force which is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them.

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = k\frac{q_1 q_2}{r^2}$$

where q_1 and q_2 are the two charges placed at a distance r from each other.

The value of k depends on the nature of the medium and the system of units. In the CGS system $k = 1$ and in the SI system $k = 1/4\pi\epsilon_0$ where ϵ_0 is the absolute electrical permittivity of free space or air. The numerical value of $1/4\pi\epsilon_0 = 9 \times 10^9 \ Nm^2C^{-2}$

and $\epsilon_0 = 8.85 \times 10^{-12} \ C^2 N^{-1} m^{-2}$.

Unit of charge:

In the SI system

$$F_{vacuum} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

If $q_1 = q_2 = q$ and $r = 1m$ and $F_{vac} = 9 \times 10^9 N$

then,
$$9 \times 10^9 = \frac{9 \times 10^9 q^2}{1^2}$$

$$q^2 = 1 \Rightarrow q = \pm 1C.$$

Therefore, one coulomb is that charge which will repel an equal and similar charge with a force of 9×10^9 N when placed in vacuum at a distance of 1m from it.

In the CGS system the unit of charge is stat coulomb where $1C = 3 \times 10^9$ stat C

Relative permittivity or Dielectric constant.

When the charges are placed in some other medium other than air, the force between the charges gets greatly affected.

We know $F_{vacuum} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$

The force between the same two charges

at the same distance r in a medium is

$$F_{med} = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2}$$

$$\frac{F_{vac}}{F_{med}} = \frac{\epsilon}{\epsilon_0} = \epsilon_r \text{ or } K.$$

ϵ_r is called the relative permittivity of the medium with respect to vacuum and is denoted by K called the dielectric constant of the medium.

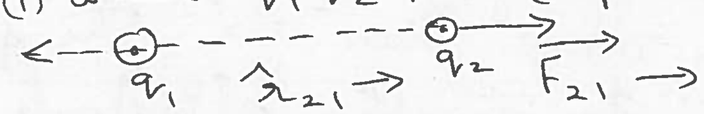
Hence, dielectric constant of the medium is defined as the ratio of the force between two charges placed at a certain distance apart in air to the force between the same two charges placed the same distance apart in medium

Note: The force between two charges placed at the same distance apart in water becomes 1/80 times the force between the same two charges kept same distance apart in air because the dielectric constant of water is about 80.

$$\therefore F_{med} = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0 K} \frac{q_1 q_2}{r^2}$$

Coulomb's law in vector form

(1) when $q_1 q_2 > 0$ (repulsive



$\vec{F}_{21}$ and $\hat{r}_{21}$ are along the same direction

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}$$

Similarly $\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$

Since $\hat{r}_{12} = -\hat{r}_{21}$

$$\vec{F}_{21} = -\vec{F}_{12}$$

This proves that force between the charges is equal and opposite. Coulomb's law also proves that electrostatic forces are repulsive forces.

<u>Superposition principle</u>: When a number of charges are interacting, the total force on a given charge is the vector sum of the forces exerted on it by all other charges.

Suppose there are n charges $q_1, q_2, q_3, \ldots q_n$ distributed in space and interacting with each other then according to the superposition principle the total force on charge q_1 is given by

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \cdots + \vec{F}_{1n}$$

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} + \cdots + \frac{q_1 q_n}{r_{1n}^2}\hat{r}_{1n}\right]$$

<u>Continuous charge distribution</u>

We know that electric charge is quantized but in most practical situations, the magnitude of the charge is so small as compared to the charges on the bodies that quantization of charge may be ignored. On charged bodies of reasonable size the charge can have any continuous value.

Continuous charge distribution ca be one, two or three dimensional.

Linear charge density: When the charge is distributed along a line (straight or curved) it is called linear charge density.

$$\lambda = q/l$$

unit Cm^{-1}

For a circular ring $\lambda = q/2\pi r$

Surface charge density: When the charge is distributed over a surface (plane or curved) it is called surface charge density

$$\sigma = q/A$$

unit Cm^{-2}.

For a spherical conductor $\sigma = q/4\pi r^2$

Volume charge density: When the charge is distributed over a volume of the object then it is called volume charge distribution.

$$\rho = q/V$$

unit Cm^{-3}

For a charged sphere where the charges are distributed throughout the volume of the conductor

$$\rho = \frac{q}{\frac{4}{3}\pi r^3}$$

Electrostatic field

Electric field due to a point charge may be defined as the space around the point charge in which electrostatic force of attraction or repulsion due to the charge can be experienced by any other charge.

Relation between electric field intensity and force

Electric field intensity is defined as the force experienced by an unit positive charge placed at that point.

If $\vec{F}$ is the force acting on a test charge q_0 at any point then electric field intensity at that point is

$$\vec{E} = \lim_{q_0 \to 0} \vec{F}/q_0 .$$

SI unit is N/C.

The direction of electric field intensity is the same as the direction of $\vec{F}$ [ie the direction along which the test charge q_0 would move if free to do so].

Electric field intensity due to point charge

To calculate the electric field intensity at any point P due to a point charge q at O, such that OP = r, imagine a small +ve test charge q_0 at P.

According to Coulomb's law, force at P is

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2} \hat{r}$$

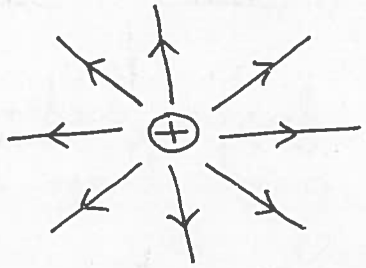
$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} q/r^2 \, \hat{r} .$$

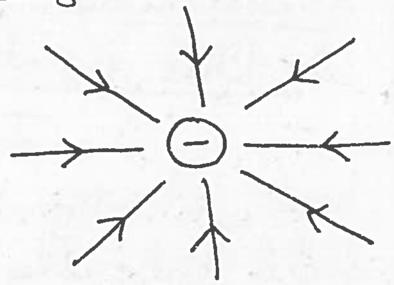
$$\therefore \boxed{E = \frac{q}{4\pi\epsilon_0 r^2}}$$

Electric field lines :: It may be defined as a path straight or curved such that the tangent to it at any point gives the direction of field intensity at that point. [drawn in the direction in which a +ve test charge would move].

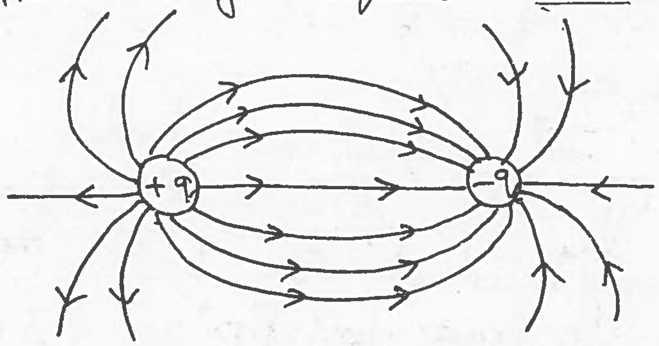
Electric field lines due to

(a) a single +ve point charge $(q > 0)$ (b) a single −ve point charge $(q < 0)$.



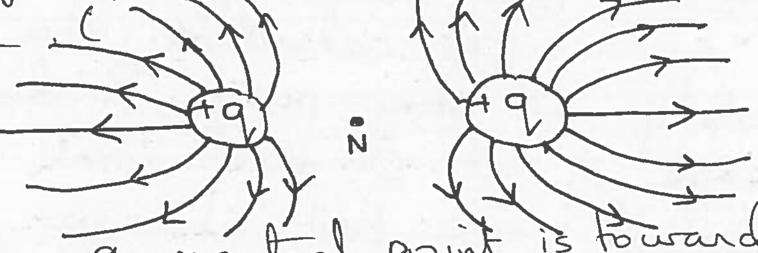


(c) a pair of equal and opposite charges.



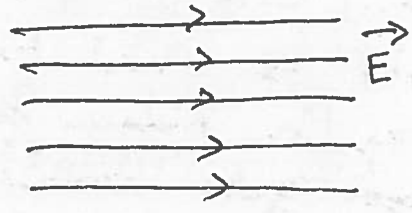
(d) a pair of equal and similar charges

$q_1 = q_2$,



Note : If $q_1 > q_2$ neutral point is towards q_2.

e. Uniform electric field



Properties of electric field lines

1. Electric field lines start from positive charges and end at negative charges. For a single charge they may start or end at infinity.
2. In a charge free region, electric field lines can be taken to be continuous curves without any breaks.
3. Two field lines can never cross each other. [If it did so, the field at the point of intersection will not have a unique direction, which is not possible.]
4. Electrostatic field lines do not form closed loops. This follows from the conservative nature of electric field.
5. Electrostatic field lines around two like charges show mutual repulsion and that between equal and opposite charges show mutual attraction.

Electric dipole:

It is a pair of equal and opposite charges separated by a very small distance.

* Electric dipole moment ($\vec{p}$) : It is equal to the product of magnitude of either charge and the distance between them.

$$\boxed{\vec{p} = q(2\vec{a})}$$

It is a vector quantity and is directed from the $-ve$ to the $+ve$ charge.

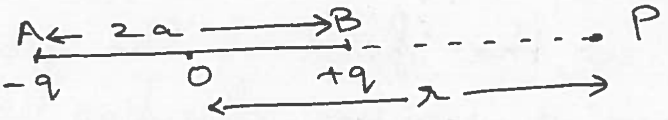
[2a is the distance between the two charges]

SI unit is Cm.

A dipole is said to be ideal when the charge q is large and the distance between the charges gets smaller and smaller.

Electric field intensity along axial line of a dipole:

Consider two point charges $-q$ and $+q$ at A and B separated by a small distance $2a$. The point P at which the electric field intensity due to the dipole is to be determined is at a distance 'r' from the centre of the dipole.



Electric field intensity at P due to $-q$ at A

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2} \text{ along PA}$$

Electric field intensity at P due to $+q$ at B

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2} \text{ along BP.}$$

Resultant field intensity $\vec{E} = \vec{E_2} - \vec{E_1}$

$$E = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] \text{ along BP.}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{4ar}{(r^2-a^2)^2}\right]$$

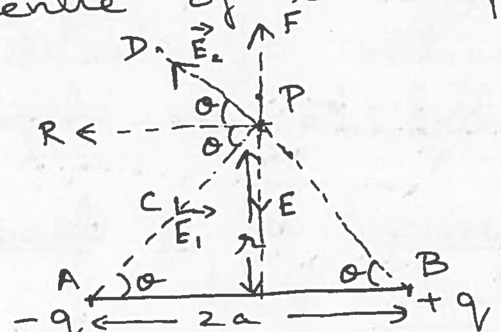
$$E = \frac{2pr}{4\pi\epsilon_0 (r^2-a^2)^2} \text{ along BP} \quad [\because p = 2aq]$$

In case of a short dipole $[a << r]$

$$\boxed{\vec{E} = \frac{2\vec{p}}{4\pi\epsilon_0 r^3}}.$$

Electric field intensity along equatorial line of a dipole.

Consider two charges $-q$ and $+q$ at A and B and separated by a small distance $2a$. The point P at which the electric field intensity is to be determined is at a distance r from the centre of the dipole.



Electric field intensity at P due to $-q$ at A is

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{(a^2+r^2)} \text{ along PC}$$

Electric field intensity at P due to $+q$ at B is

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{(a^2+r^2)} \text{ along PD}$$

$\vec{E_1}$ can be resolved into rectangular components $E_1 \cos\theta$ along PR and $E_1 \sin\theta$ along PE

Similarly $\vec{E_2}$ can be resolved into rectangular components $E_2 \cos\theta$ along PR and $E_2 \sin\theta$ along PF.

Since $E_1 = E_2$ the $\sin\theta$ components cancel each other and the resultant intensity is

$$E = E_1\cos\theta + E_2\cos\theta$$

$$= 2E_1\cos\theta$$

$$= \frac{2}{4\pi\epsilon_0}\frac{q}{(a^2+r^2)}\frac{a}{(a^2+r^2)^{1/2}} \quad \left[\because \cos\theta = \frac{a}{(a^2+r^2)^{1/2}}\right.$$

$$E = \frac{p}{4\pi\epsilon_0 (a^2+r^2)^{3/2}} \text{ along PR}$$

If $a \ll r$ (short dipole)

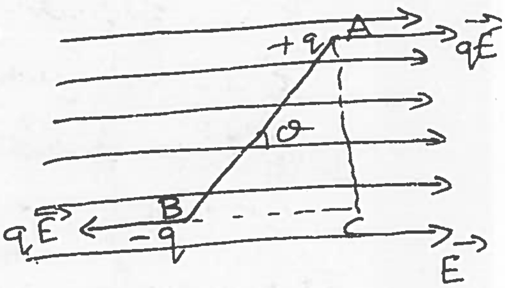
$$\boxed{\vec{E} = \frac{\vec{p}}{4\pi\epsilon_0 r^3}}$$

Note (i) for a short dipole, electric field intensity due to a dipole along the axial line is twice that along the equatorial line.

(ii) The direction of electric field intensity along the axial line is parallel to dipole moment and for any point along the equatorial line the direction of electric field intensity is antiparallel to the dipole moment.

Torque experienced by a dipole in an uniform electric field

Consider a dipole of length 2a placed in an uniform electric field $\vec{E}$.



Force acting on the charge q at A = $q\vec{E}$ along $\vec{E}$

force acting on the charge -q at B = $q\vec{E}$ opposite to $\vec{E}$.

Net force = 0.

Since the two forces are equal, unlike and parallel it consitutes a couple which rotates the dipole in the clockwise direction tending to align it in the direction of the electric field.

$$\tau = F \text{ (arm of the couple)}$$
$$= qE[AC]$$
$$= qE(2a\sin\theta)$$
$$= pE\sin\theta$$
$$\therefore \boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

The direction of the (τ) torque is given by the right hand screw rule and is $\perp$ to $\vec{p}$ and $\vec{E}$

Note

(i) The net translatory force on the dipole is 0.

(ii) When $\vec{p}$ is along $\vec{E}$, $\theta = 0 \Rightarrow \tau = 0$.
The dipole is said to be in stable equilibrium.

(iii) When $\vec{p}$ is directed opposite to $\vec{E}$ $\theta = 180^\circ$
$\Rightarrow \tau = 0$ and the dipole is in unstable equilibrium

(iv) τ is maximum when $\theta = 90^\circ$.

$$\tau = pE$$

(v) unit of torque is Nm & dimensions $[ML^2T^{-2}]$

(vi) When the dipole is in a non-uniform electric field the net force and torque $\neq 0$.. As the dipole sets itself parallel to the field the torque becomes zero but net force still persists which tends to displace the dipole in the direction of the field.

Potential energy of a dipole in an uniform electric field

Potential energy of a dipole is the energy possessed by a dipole by virtue of its particular position in the electric field.
Small amount of work done in rotating the dipole through an angle $d\theta$ against the torque is $dW = \tau \, d\theta$

$$= pE \sin\theta \, d\theta$$

Total work done $W = pE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$

$$W = -pE \left[\cos\theta_2 - \cos\theta_1 \right]$$

If $\theta_1 = 90^\circ$ and $\theta_2 = \theta$

$$W = -pE \left[\cos\theta - \cos 90 \right]$$

Since the work done is stored in the dipole as its potential energy

$$\boxed{U = -\vec{p} \cdot \vec{E}}$$

It is a scalar quantity and measured in joule.

- -

Area vector: Although area is a scalar quantity it is represented as a vector to deal with specific problems.

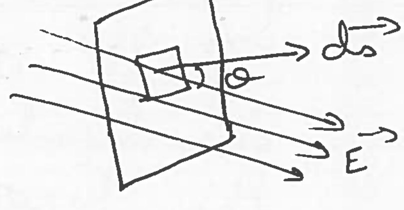
If an area element ds is represented as $\vec{ds}$ then the direction of the area vector is drawn perpendicular to the area element and is along the outward drawn normal to the area element

$$\vec{ds} = ds\,\hat{n}$$

Electric flux [ϕ_E] : Electric flux over an area in an electric field represents the total number of electric field lines crossing that area.

OR

Electric flux is the product of surface area and the component of electric field intensity normal to the area



$$d\phi_E = ds[E\cos\theta]$$

$$= E\,ds\cos\theta$$

$$d\phi_E = \vec{E} \cdot \vec{ds}$$

Note : (i) Electric flux is a scalar and its unit is Nm^2C^{-1}

(ii) It may be positive or negative depending on the angle between $\vec{E}$ and $\vec{ds}$.

(iii) In general, $\phi_E = \oint \vec{E} \cdot \vec{ds}$

VX GAUSS's THEOREM AND APPLICATIONS

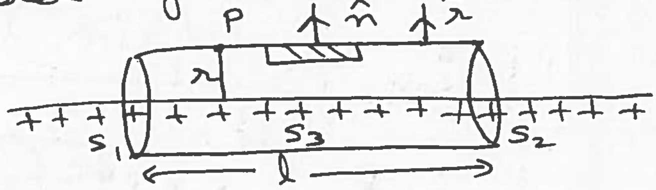
GAUSS's THEOREM STATEMENT: The total electric flux over a closed surface in vacuum is $1/\epsilon_0$ times the total charge q contained in the closed surface

$$\phi_E = \oint \vec{E} \cdot \vec{ds} = q/\epsilon_0$$

Applications

I Electric field intensity due to a line charge (infinitely long)

Consider a line charge with uniform charge density λ. To find the electric field intensity at any point P distance r from the line charge, imagine a gaussian surface to be a right circular cylinder of radius r and length l. The curved surface area contributes to the electric flux



$$\oint_S \vec{E} \cdot \vec{ds} = \int_{S_1} E\,ds \cos 90 + \int_{S_2} E\,ds \cos 90 + \oint_{S_3} E\,ds \cos 0$$

$$\therefore \oint \vec{E} \cdot \vec{ds} = \oint_{S_3} E\,ds$$

[$\hat{r} \cdot \hat{n} = 0$ for S_1 and S_2 or $\theta = 90°$

$\hat{r} \cdot \hat{n} = 1$ for S_3 or $\theta = 0$]

According to Gauss's theorem

$$\oint E\,ds = q/\epsilon_0$$

$$E \int_{S_3} ds = q/\epsilon_0$$

$$E(2\pi r l) = q/\epsilon_0$$

$$E = \frac{q}{2\pi r l \epsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi r \epsilon_0}}$$ directed radially outward from curved surface area of cylinder

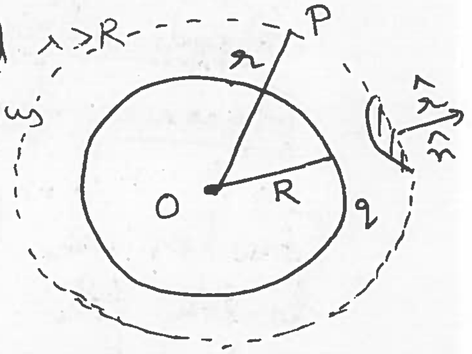
II Electric field intensity due to a uniformly charged spherical shell.

Consider a spherical shell of radius R and the charge q is distributed uniformly over its surface. In order to find the electric field intensity at any point P such that OP = r, imagine the gaussian surface to be a sphere of radius r.

a) At any point outside the spherical shell $r > R$

∴ The gaussian surface has a radius r

$\vec{E}$ is directed radially outward



According to Gauss's theorem

$$\oint \vec{E} \cdot \vec{ds} = q/\epsilon_0$$

$$E \oint ds = q/\epsilon_0 \quad [\because \hat{r} \cdot \hat{n} = 1 \text{ or } \theta = 0]$$

$$E(4\pi r^2) = q/\epsilon_0$$

$$\boxed{E = \frac{q}{4\pi \epsilon_0 r^2}}$$ directed radially outward.

Electric field intensity at any point outside the spherical shell is as if the entire charge is concentrated at the centre.

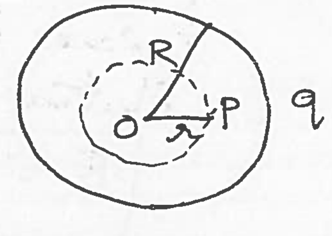
$$\boxed{E = \frac{\sigma R^2}{\epsilon_0 r^2}} \quad \text{since } \sigma = q/4\pi R^2$$

(b) At any point on the surface of the shell

$$r = R$$

$$\therefore E = \frac{q}{4\pi \epsilon_0 R^2} \Rightarrow E = \sigma/\epsilon_0$$

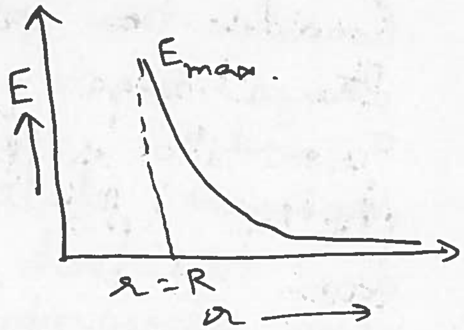
(c) At any point inside the spherical shell $r < R$



As charge inside Gaussian surface is $0 \Rightarrow E = 0$.

Electric field inside a spherical shell that is charged $= 0$.

Variation of electric field intensity with distance from centre of spherical shell.

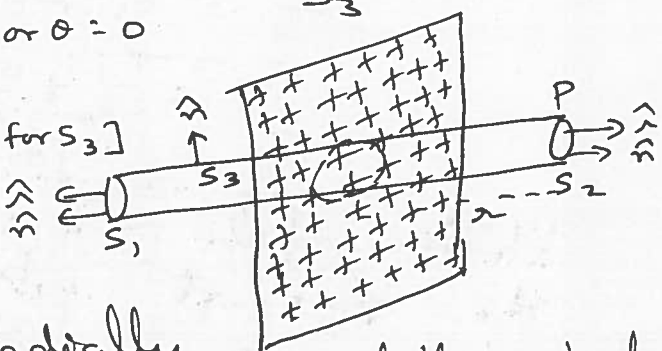


III Electric field intensity due to thin plane sheet of charge : Consider an infinite plane sheet of charge having charge density σ. To find the field intensity at any point P distance r from the plane sheet, consider a gaussian surface to be the right circular cylinder of cross sectional area A.

$$\oint \vec{E} \cdot d\vec{s} = \oint_{S_1} \vec{E} \cdot d\vec{s} + \oint_{S_2} \vec{E} \cdot d\vec{s} + \oint_{S_3} \vec{E} \cdot d\vec{s}$$

$= E\oint_{S_1} ds + E\oint_{S_2} ds$ [$\because \hat{n} \cdot \hat{n} = 1$ or $\theta = 0$ for S_1 and S_2, and $\hat{n} \cdot \hat{n} = 0$ or $\theta = 90°$ for S_3]

$= 2EA$



According to Gauss's theorem

$$2EA = q/\epsilon_0$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$ directed radially outward from the edges of the cylinder

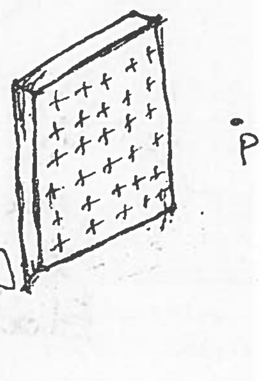
[E is independent of r]

(a) If the plane sheet of charge has uniform thickness.

The electric field intensity at any point P due to each surface is $E_1 = E_2 = \sigma/2\epsilon_0$

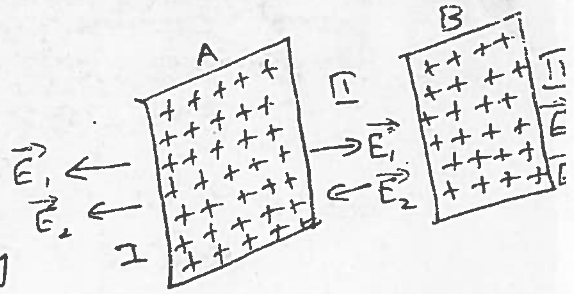
$\therefore$ Electric field intensity at P is

$$E = \sigma/2\epsilon_0 + \sigma/2\epsilon_0 = \sigma/\epsilon_0$$ [according to superposition principle]



$$\boxed{E = \sigma/\epsilon_0}$$

(b) Electric field intensity due to two thin infinite plane sheets of charge.



Consider two plane sheets of charge having charge densities σ_1 and σ_2. Let E_1 and E_2 represent electric field intensity due to sheets A & B, then using superposition principle the electric field intensity can be determined in regions I, II & III.

In region I

$$\vec{E}_I = -\vec{E}_1 - \vec{E}_2$$
$$= -\frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0}$$
$$E_I = \frac{-1}{2\epsilon_0}[\sigma_1 + \sigma_2]$$

In region II

$$\vec{E}_{II} = \vec{E}_1 - \vec{E}_2$$
$$= \frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0}$$
$$E_{II} = \frac{1}{2\epsilon_0}[\sigma_1 - \sigma_2]$$

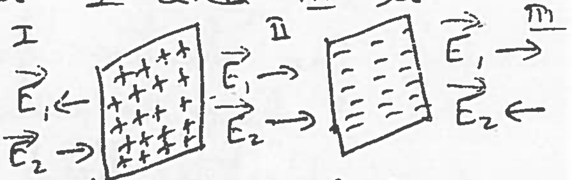
In region III

$$\vec{E}_{III} = \vec{E}_I + \vec{E}_2$$
$$= \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0}$$
$$\vec{E}_{III} = \frac{1}{2\epsilon_0}[\sigma_1 + \sigma_2]$$

(c) If $\sigma_1 = \sigma$ and $\sigma_2 = -\sigma$ [two equal and opposite plane sheets of charge]

The electric field in regions I and III is zero and in the region II

$$E = \sigma/\epsilon_0.$$



An uniform electric field is produced between two equal and opposite charged plates.