Mathematics 2015 (Outside Delhi) II

Note : Except for the following questions, all the remaining questions have been asked in previous sets.

SECTION — B

10. Solve the following quadratic equation for x :

$$x^2 - 2ax - (4b^2 - a^2) = 0$$ **[2]**

Solution : We have, $x^2 - 2ax - (4b^2 - a^2) = 0$

$$x^2 - 2ax + a^2 - 4b^2 = 0$$

$$(x-a)^2 - (2b)^2 = 0$$

$\therefore (x - a + 2b)(x - a - 2b) = 0$

$\Rightarrow \quad x = a - 2b$ or $a + 2b$

Hence, $\quad x = a - 2b$ or $x = a + 2b$ **Ans.**

SECTION — C

18. The 13th term of an AP is four times its 3rd term. If its fifth term is 16, then find the sum of its first ten terms. **[3]**

Solution : In the given A.P., let first term $= a$ and common difference $= d$

Then, $\quad T_n = a + (n-1)d$

$\Rightarrow \quad T_{13} = a + (13-1)d = a + 12d$

and $\quad T_3 = a + (3-1)d = a + 2d$

Now, $\quad T_{13} = 4T_3$ (Given)

$$a + 12d = 4(a + 2d)$$

$$a + 12d = 4a + 8d$$

$$3a = 4d$$

$$a = \frac{4}{3}d \quad \text{...(i)}$$

Also, $\quad T_5 = a + (5-1)d$

$\Rightarrow \quad a + 4d = 16 \quad$ ...(ii)

Putting the value of a from eq. (i) in (ii), we get

$$\frac{4}{3}d + 4d = 16$$

$$4d + 12d = 48$$

$$16d = 48$$

$$d = 3$$

Substituting $d = 3$ in eq. (ii), we get

$$a + 4(3) = 16$$

$$a = 16 - 12$$

$$a = 4$$

$\therefore$ Sum of first ten terms is

$$S_{10} = \frac{n}{2}[2a + (n-1)d] \text{ where } n = 10$$

$$= \frac{10}{2}[2 \times 4 + (10-1)3]$$

$$= 5[8 + 27]$$

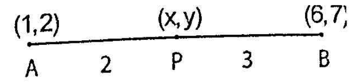
$$= 175$$ **Ans.**

19. Find the coordinates of a point P on the line segment joining $A(1, 2)$ and $B(6, 7)$ such that $AP = \frac{2}{5}AB$. **[3]**

Solution : Given, $A(1, 2)$ and $B(6, 7)$ are the given points of a line segment AB with a point P on it.

Let the co-ordinate of point P be (x, y)

$$AP = \frac{2}{5}AB \quad \text{(Given)}$$



$$AB = AP + PB$$

$\Rightarrow \quad \dfrac{AP}{PB} = \dfrac{2}{3}$

$\therefore \quad m = 2, n = 3$

Then, by section formula, we have

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

$$x = \frac{2 \times 6 + 3 \times 1}{2+3} \text{ and } y = \frac{2 \times 7 + 3 \times 2}{2+3}$$

$$x = \frac{15}{5} \text{ and } y = \frac{20}{5}$$

$\therefore x = 3$ and $y = 4$

Hence, the required point is $P(3, 4)$. **Ans.**

20. A bag contains white, black and red balls only. A ball is drawn at random from the bag. If the probability of getting a white ball is $\frac{3}{10}$ and that of a black ball is $\frac{2}{5}$, then find the probability of getting a red ball. If the bag contains 20 black balls, then find the total number of balls in the bag. **[3]**

Solution : Given, the probability of getting a white ball

$\Rightarrow \quad P(W) = \dfrac{3}{10}$

and the probability of getting a black ball

$\Rightarrow \quad P(B) = \dfrac{2}{5}$

then, the probability of getting a red ball

$\Rightarrow \quad P(R) = 1 - \dfrac{3}{10} - \dfrac{2}{5} = \dfrac{10-3-4}{10} = \dfrac{3}{10}$

Now, $\frac{2}{5}$ of total number of balls = 20

$$\text{Total number of balls} = \frac{20\times5}{2}$$

$$= 50$$

Hence, the total no. of balls in the bag is 50. **Ans.**

SECTION — D

8. A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at an average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, find the first speed of the truck. [4]

Solution : Let the average speed of the truck be x km/hr.

Then, new average speed of truck = $(x + 20)$ km/hr.

Time taken by truck to cover 150 km = $\frac{150}{x}$ hrs.

and time taken by truck to cover 200 km = $\frac{200}{x+20}$ hrs.

$$\therefore \quad \frac{150}{x}+\frac{200}{x+20} = 5$$

$$\frac{150(x+20)+200x}{x(x+20)} = 5$$

$$150x + 3000 + 200x = 5x(x + 20)$$

$$350x + 3000 = 5x^2 + 100x$$

$$5x^2 - 250x - 3000 = 0$$

$$x^2 - 50x - 600 = 0$$

$$x^2 - 60x + 10x - 600 = 0$$

$$x(x - 60) + 10(x - 60) = 0$$

$$(x + 10)(x - 60) = 0$$

$$x = -10 \text{ or } 60$$

Since speed cannot be negative.

So, $x = 60$

$\therefore$ First speed of truck = 60 km/hr. **Ans.**

29. An arithmetic progression 5, 12, 19,..... has 50 terms. Find its last term. Hence find the sum of its last 15 terms. [4]

Solution : Given, AP is 5, 12, 19 ....

Here, $n = 50, a = 5, d = 12 - 5 = 19 - 12 = 7$

Now, $T_{50} = a + (50 - 1)d$

$\Rightarrow$ $T_{50} = 5 + (49)7 = 348$

15 terms from last = $(50 - 15 + 1)$ terms from starting

$$T_{36} = a + (36 - 1)d$$

$$= 5 + 35(7)$$

$$= 250$$

$\therefore$ Sum of last 15 terms $= \frac{n}{2}(a+l)$

$$= \frac{15}{2}(250+348)$$

$[\because a = 250 \text{ and } l = 348]$

$$= \frac{15}{2}\times598 = 4485$$ **Ans.**

30. Construct a triangle ABC in which AB = 5 cm, BC = 6 cm and $\angle ABC$ = 60°. Now construct another triangle whose sides are $\frac{5}{7}$ times the corresponding sides of ΔABC. [4]

Solution : Steps of construction :

(i) Draw a line segment AB = 5 cm.

(ii) Construct $\angle ABX$ = 60°.

(iii) From B, draw BC = 6 cm cutting BX at C.

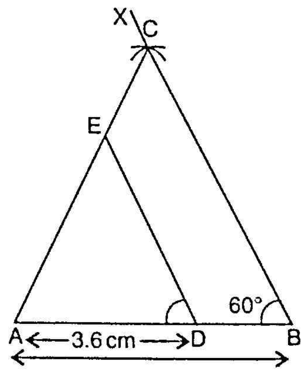
(iv) Join AC.

Thus, ΔABC is obtained

(v) **Step 5 :** Draw D on AB such that $AD = \frac{5}{7}AB$

$$= \left(\frac{5}{7}\times5\right)\text{cm} = 3.6 \text{ cm}$$

(vi) **Step 6 :** Draw $DE \parallel BC$ cutting AC at E. Then ΔADE is the required triangle similar to ΔABC such that each side of ΔADE is $\frac{5}{7}$ times the corresponding side of ΔABC.



31. Find the values of k for which the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k – 1, 5k) are collinear. [4]

Solution : Given, the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k – 1, 5k)

$\because$ The point to be collinear

$$\therefore x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$(k + 1)(2k + 3 - 5k) + 3k(5k - 2k) + (5k - 1)(2k - 2k - 3) = 0$$

$$(k + 1)(3 - 3k) + 3k(3k) + (5k - 1)(-3) = 0$$

$$3k + 3 - 3k^2 - 3k + 9k^2 - 15k + 3 = 0$$

$$6k^2 - 15k + 6 = 0$$

$$2k^2 - 5k + 2 = 0$$