Refraction of light

The phenomenon of the change in the path of light when it goes from one medium to another is refraction.

<u>Cause of refraction</u>: It is due to the change in the velocity of light as it goes from one medium to another.

<u>Laws of refraction</u>

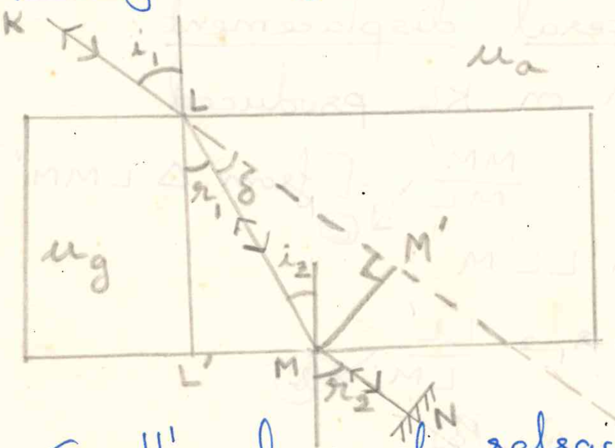
(a) The incident ray, the normal and the refracted ray all lie in the same plane.

(b). The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant. This constant is known as the relative refractive index of the second medium with respect to the first medium.

$$\frac{\sin i}{\sin r} = \mu_{21}$$

-X- <u>Note</u>: Whenever light travels from one medium to another, frequency of light does not change but velocity and wavelength of light changes.

<u>Refraction through a glass slab</u>:



Applying Snell's law of refraction at L

$$\frac{\sin i_1}{\sin r_1} = \mu_{ga} \quad \text{--- (1)}.$$

Similarly at M,

$$\frac{\sin i_2}{\sin r_2} = \mu_{ag} \text{ — } ②$$

On placing a plane mirror normal to MN the ray is found to retrace its path.

∴ For the reversed ray at M,

$$\frac{\sin r_2}{\sin i_2} = \mu_{ga} \text{ — } ③$$

× plying ② and ③

$$\mu_{ag} \times \mu_{ga} = 1$$

$$\boxed{\mu_{ga} = \frac{1}{\mu_{ag}}}$$

example $\mu_{ga} = 3/2$
then $\mu_{ag} = 2/3$

<u>Lateral displacement</u>

From ① and ③

$$\frac{\sin i_1}{\sin r_1} = \frac{\sin r_2}{\sin i_2}$$

Since $\angle i_2 = \angle r_1$ (alternate angles)

then $\angle i_1 = \angle r_2$.

⟹ KL is parallel to MN. (that is the ray is only displaced laterally).

<u>Expression for lateral displacement</u>:

Drop a ⊥ from M on KL produced

$$\sin \delta = \frac{MM'}{ML} \rightarrow ⓐ \quad [\text{from } \Delta LMM']$$

Also, from Δ LL'M

$$\cos r_1 = \frac{LL'}{LM} \rightarrow ⓑ$$

Substituting ⓑ in ⓐ

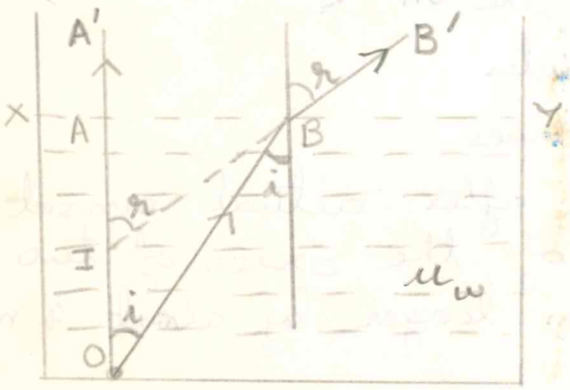
$$\sin \delta = \frac{MM'}{LL'} \cos r_1$$

$$MM' = \frac{LL' \sin \delta}{\cos r_1}$$

$$\therefore MM' = \frac{t \sin (i_1 - r_1)}{\cos r_1}$$

Thus the lateral displacement is proportional to the thickness of the glass slab. [Note: If $i_1 = 90°$, $MM' = t$]

REAL AND APPARENT DEPTH:



The point object O is at a depth OA below the free surface of water. From O, a ray of light incident normally on the surface XY would trace the path OAA'. From O, a ray of light along OB would be refracted along BB'.

$$\therefore \frac{\sin r}{\sin i} = \mu_{wa} \quad \text{— (1)}$$

From $\triangle$ OAB, $\sin i = \frac{AB}{OB}$ — (2)

and $\triangle$ IAB, $\sin r = \frac{AB}{IB}$ — (3)

From (1), (2) and (3) $\frac{\sin r}{\sin i} = \frac{OB}{IB} = \mu_{wa}$

Since points A and B are very close to each other

$$\frac{OB}{IB} = \frac{OA}{IA}$$

$$\therefore \boxed{\mu_{wa} = \frac{\text{Real depth}}{\text{Apparent depth}}}$$

Note: The shift $OI = OA - IA$

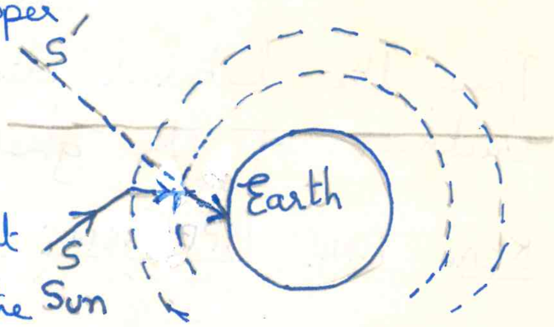
$$= OA\left[1 - \frac{IA}{OA}\right]$$

$$\boxed{\text{The distance through which an object appears to be raised} = t\left[1 - \frac{1}{{}^{a}\mu_w}\right]}$$

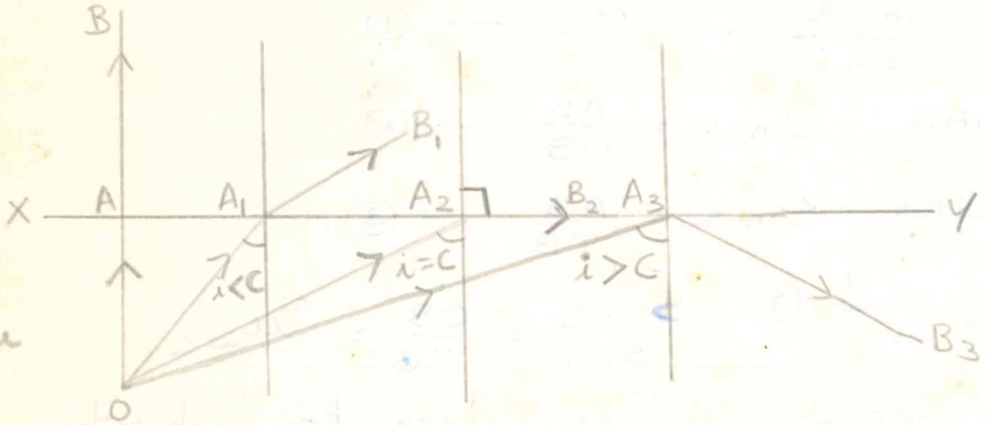
$\therefore$ The normal shift depends on the thickness of the refracting medium and refractive index of the refracting medium.

Refraction effects during sunrise and sunset

As the density of air is more near the surface of the earth than the upper layers of the atmosphere, a ray of light that originates from S bends towards the normal ∴ for an observer on earth the ray of light appears to come from S'. Hence the sun appears to rise a few minutes before actual sunrise and continues to be seen for a few minutes after actual sunset. Since the time difference is of the order of two minutes each, the day becomes longer by about 4 mins due to refraction effects.



Total internal reflection



1. A ray of light from a point object O incident normally on the surface XY passes straight along AB.
2. A ray of light along OA_1 is refracted along A_1B_1
3. As the angle of incidence is increased further and when the ray is incident along OA_2 the refracted ray is found to graze along the interface XY ie $r = 90°$
   The angle of incidence in the denser medium for which the angle of refraction is 90° is known as the <u>critical angle</u>.
4. As the angle of incidence is increased such that $i > C$, then the ray of light is reflected into the same medium. This is known as total internal reflection.

$\therefore$ When a ray of light travelling from a denser medium to rarer medium is incident at an angle greater than the critical angle, for the pair of media in contact, the ray is totally internally reflected into the denser medium. This phenomenon is known as <u>total internal reflection</u>

x. <u>Two essential conditions for total internal reflection</u>

(a) Light should travel from denser to rarer medium.

(b) The angle of incidence should be greater than the critical angle for the pair of media in contact.

<u>Relation between μ and $\sin C$</u>

$$\frac{\sin i}{\sin r} = \mu_{aw}$$

When $i = C$, $r = 90^\circ$

$$\therefore \frac{\sin C}{\sin 90} = \mu_{aw}$$

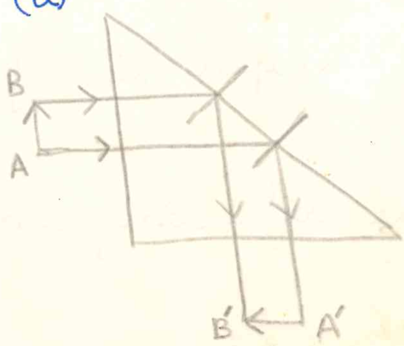
$$\boxed{\frac{1}{\sin C} = \mu_{wa}}$$

<u>Applications of total internal reflection</u>

(a) <u>Brilliance of diamond</u>: For diamond $\mu = 2.42$ and $C = 24.4^\circ$. The diamond is cut in such a manner that a ray of light from any face is incident at an angle greater than the critical angle. The light undergoes multiple total internal reflections at the various faces and remains within the diamond. $\therefore$ Diamond sparkles.

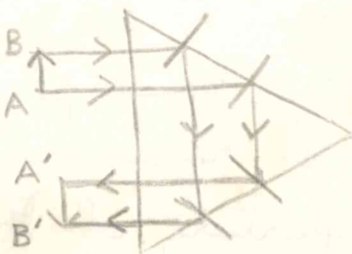
2. <u>Totally reflecting glass prisms</u>

(a)



(a) deviation is 90° on account of one TIR

(b)



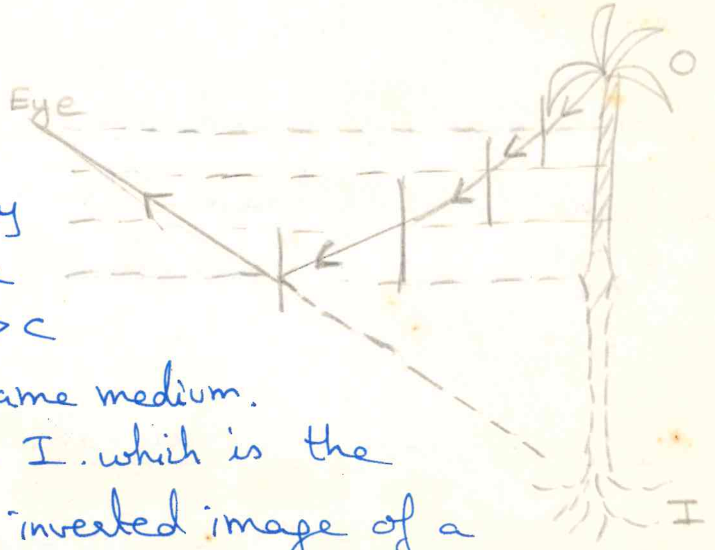
(b) deviation is 180° on account of two TIR

(c)

no deviation

(3) Optical fibre : Light incident at one end of the fibre greater than critical angle undergoes total internal reflection and emerges through the other end.

(4) Mirage : On a hot day the layers of air near the surface of earth is rarer than the top layers. A ray of light travels through the various layers and when $i > c$ light is reflected into the same medium. and appears to come from I. which is the mirror image of O. This inverted image of a tree creates the impression of reflection from a pond of water.



V.X. REFRACTION AT SPHERICAL SURFACES

There are two types of spherical refracting surfaces

(a) Convex spherical refracting surface
(b) Concave spherical refracting surface

Sign conventions

(a) All distances are measured from the pole of the refracting surface
(b) The distances measured in the direction of incident light are taken as positive and distances measured against the direction of incident light are taken as negative.

Assumptions

(a) The object is a point on the principal axis of the spherical refracting surface
(b) The aperture of the spherical refracting surface is small.

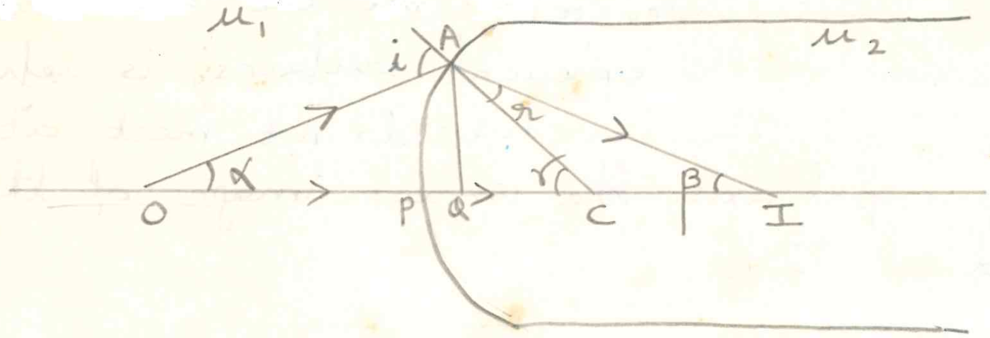
(c) The incident and refracted rays make small angles with the principal axis of the surface, so that $\sin i \simeq i$

I Refraction from rarer to denser medium at a convex refracting surface. (real image).

(a) Consider a spherical refracting surface XY separating a rarer medium of refractive index μ_1 from a denser medium of refractive index μ_2.

(b) O is the point object, P the pole, C the centre of curvature and PC = R the radius of curvature. A normal drawn to the spherical surface passes through the centre of curvature.

(c) A ray of light from the object passes through the optic centre without deviation. Another ray of light along OA, after refraction at the spherical surface, meets the principal axis at I. $\therefore$ AI is the refracted ray and a real image of the object is formed at I.



From Δ ACI,

$$\gamma = r + \beta$$

$$\therefore r = \gamma - \beta.$$

From Δ AOC

$$i = \alpha + \gamma$$

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

Since the angles are small

$$\mu_1 i = \mu_2 r$$

$$\mu_1 [\alpha + \gamma] = \mu_2 [\gamma - \beta].$$

$$\mu_1\left[\frac{AQ}{OQ} + \frac{AQ}{QC}\right] = \mu_2\left[\frac{AQ}{QC} - \frac{AQ}{QI}\right]$$ [∵ angles are small $\alpha \simeq \tan\alpha$, $\beta \simeq \tan\beta$, $\gamma \simeq \tan\gamma$]

$$\mu_1\left[\frac{1}{OP} + \frac{1}{PC}\right] = \mu_2\left[\frac{1}{PC} - \frac{1}{PI}\right]$$ [$Q = P$ ∵ aperture is small]

$$\mu_1\left[\frac{1}{u} + \frac{1}{R}\right] = \mu_2\left[\frac{1}{R} - \frac{1}{v}\right]$$

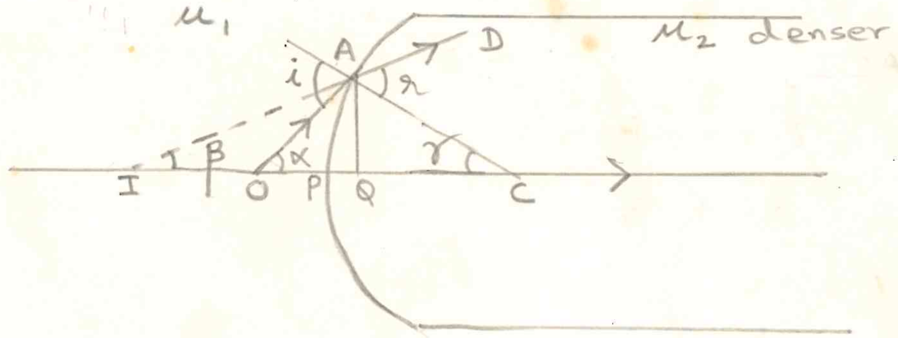
Applying sign convention.

$$\mu_1\left[\frac{1}{-u} + \frac{1}{R}\right] = \mu_2\left[\frac{1}{R} - \frac{1}{v}\right]$$

$$\boxed{-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$$

II <u>Refraction from rarer to denser medium at a convex spherical surface [virtual image]</u>.

(c) OA is the incident ray. A ray of light through the optic centre passes undeviated and the incident ray, after refraction at the spherical surface is refracted along AD. Since the two rays do not meet at a point on the principal axis, the virtual image of the object is at I.



From $\triangle$ AOC, $i = \alpha + \gamma$

From $\triangle$ AIC, $r = \gamma + \beta$

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

Since the angles are small

$$\mu_1 i = \mu_2 r.$$

$$\mu_1[\alpha+\gamma] = \mu_2[\gamma+\beta]$$

$$\mu_1\left[\frac{AQ}{OQ}+\frac{AQ}{QC}\right] = \mu_2\left[\frac{AQ}{QC}+\frac{AQ}{QI}\right]$$ [∵ angles are small $\alpha \simeq \tan\alpha$ . . .]

$$\mu_1\left[\frac{1}{OP}+\frac{1}{PC}\right] \simeq \mu_2\left[\frac{1}{PC}+\frac{1}{PI}\right]$$ [∵ P = Q ∵ aperture is small]

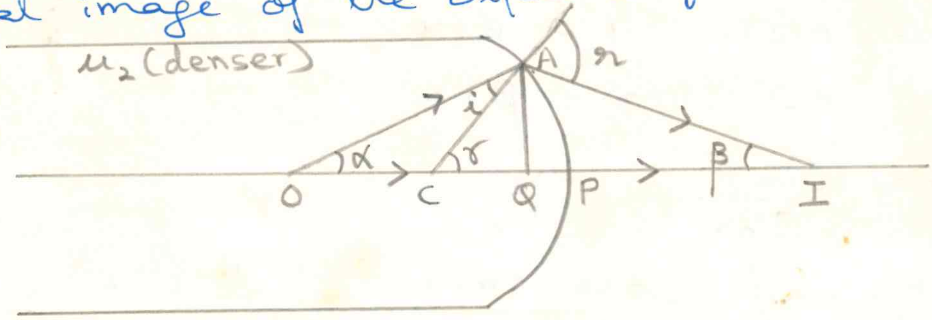
Applying sign convention.

$$\mu_1\left[\frac{1}{-u}+\frac{1}{R}\right] = \mu_2\left[\frac{1}{R}+\frac{1}{-v}\right]$$

$$\boxed{-\frac{\mu_1}{u}+\frac{\mu_2}{v} = \frac{\mu_2-\mu_1}{R}}$$

III Refraction from denser to rarer medium at a convex spherical surface (real image).

(c) A ray of light through the optic centre passes undeviated. Another ray of light along OA, after refraction at the spherical surface, is refracted along AI. The real image of the object is formed at I.



From ΔACO, $\gamma = \alpha + i$

From ΔAIC, $r = \gamma + \beta$.

$$\frac{\sin i}{\sin r} = \frac{\mu_1}{\mu_2}$$

Since the angles are small,

$$\mu_2 i = \mu_1 r$$

$$\mu_2[\gamma-\alpha] = \mu_1[\gamma+\beta]$$

$$\mu_2\left[\frac{AQ}{CQ}-\frac{AQ}{OQ}\right] = \mu_1\left[\frac{AQ}{CQ}+\frac{AQ}{QI}\right]$$

$$\mu_2\left[\frac{1}{CP}-\frac{1}{OP}\right] = \mu_1\left[\frac{1}{CP}+\frac{1}{PI}\right]$$

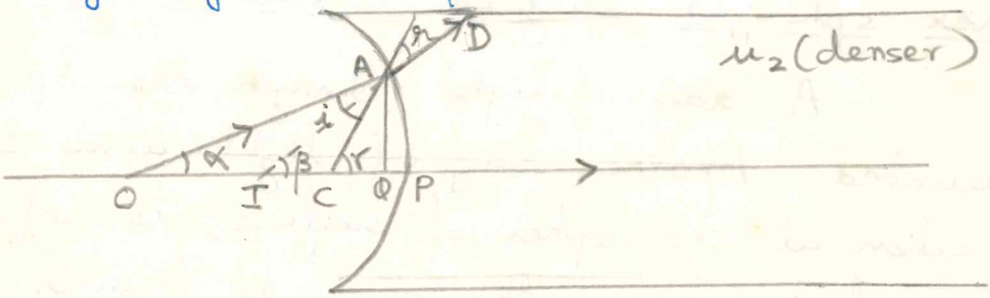
Applying sign convention

$$\mu_2\left[\frac{1}{-R} - \frac{1}{-u}\right] = \mu_1\left[\frac{1}{-R} + \frac{1}{v}\right]$$

$$\boxed{-\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}}$$

IV Refraction from rarer to denser medium at a concave spherical surface (virtual image)

(c) A ray of light passes through the optic centre without deviation. Another ray of light along OA is refracted along AD after refraction at the spherical surface. Since the two rays do not meet the virtual image of the object is at I.



From Δ AOC, $\gamma = \alpha + i$

From Δ AIC, $\gamma = \beta + r$.

Since $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

As the angles are small, $\mu_1 i = \mu_2 r$.

$$\mu_1[\gamma - \alpha] = \mu_2[\gamma - \beta]$$

$$\mu_1\left[\frac{AQ}{QC} - \frac{AQ}{OQ}\right] = \mu_2\left[\frac{AQ}{QC} - \frac{AQ}{QI}\right]$$

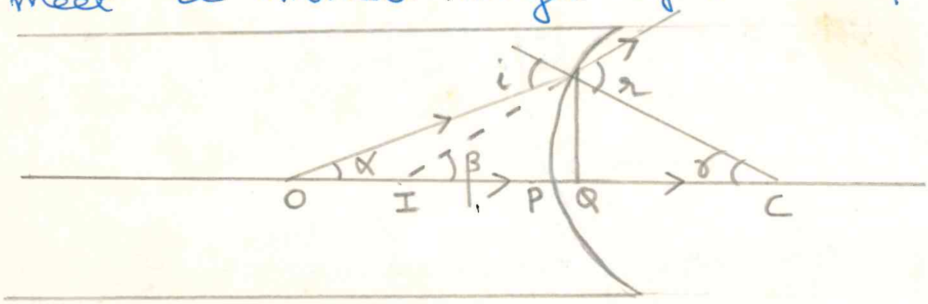
Applying sign convention

$$\mu_1\left[\frac{1}{-R} - \frac{1}{-u}\right] = \mu_2\left[\frac{1}{-R} - \frac{1}{-v}\right]$$

$$\boxed{\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$$

V. Refraction from denser to rarer medium at a concave spherical surface. (virtual image).

(c) A ray of light through the optic centre passes undeviated. Another ray of light along OA is refracted along AD. Since the two rays do not meet a virtual image of the object is at I.



From Δ AOC, $i = \alpha + \gamma$

From Δ AIC, $r = \beta + \gamma$

Also $\frac{\sin i}{\sin r} = \frac{\mu_1}{\mu_2}$

Since the angles are small,

$$\mu_2 i = \mu_1 r.$$

$$\mu_2 [\alpha + \gamma] = \mu_1 [\beta + \gamma]$$

$$\mu_2 \left[\frac{AQ}{OQ} + \frac{AQ}{QC}\right] = \mu_1 \left[\frac{AQ}{QI} + \frac{AQ}{QC}\right]$$

Applying sign convention.

$$\mu_2 \left[\frac{1}{-u} + \frac{1}{R}\right] = \mu_1 \left[\frac{1}{-v} + \frac{1}{R}\right]$$

$$\boxed{-\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}}$$

Note: When refraction is taking place at a spherical surface from a rarer to denser medium then,

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

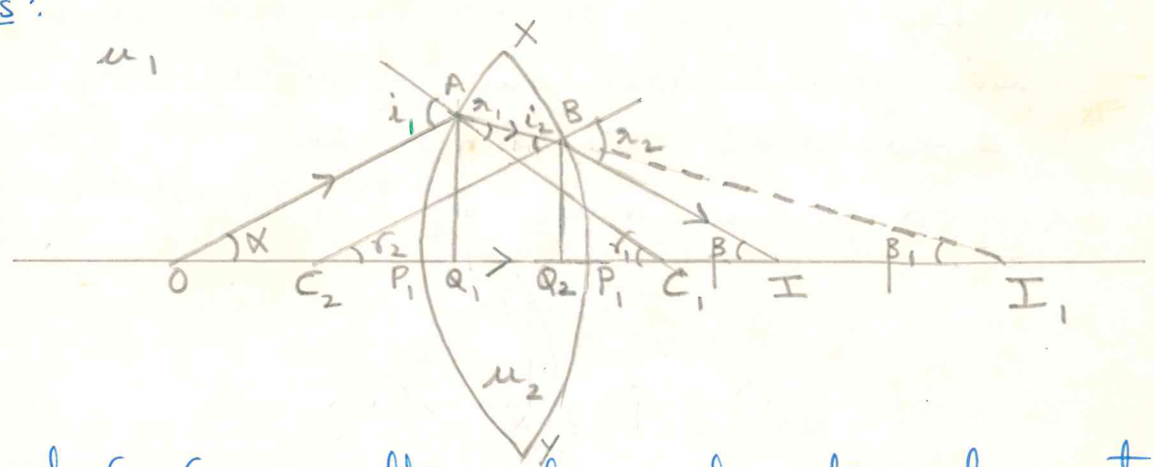
For denser to rarer medium, interchange μ_1 and μ_2

$$\therefore -\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$$

Lens maker's formula

Sign conventions and assumptions as given earlier

Convex lens :



1. P_1, P_2 and C_1, C_2 are the poles and centres of curvature of two surfaces of a thin convex lens. μ_2 is the refractive index of the material of the lens and μ_1 the refractive index of the rarer medium
2. A ray of light incident normally on XP_1Y passes straight and another ray along OA is refracted along AB and if the lens material were continuous the two rays would meet at I_1. $\therefore$ I_1 would be the real image of O after refraction at XP_1Y.

$$\frac{\sin i_1}{\sin r_1} = \frac{\mu_2}{\mu_1}$$

Since angles are small $\mu_1 i = \mu_2 r_1$.

For Δ AOC_1, $i_1 = \alpha + \gamma_1$

From Δ AC_1I_1, $\gamma_1 = r_1 + \beta_1$

$$\therefore \mu_1[\alpha + \gamma_1] = \mu_2[\gamma_1 - \beta_1]$$

Applying sign convention

$$\mu_1\left[\frac{1}{-u} + \frac{1}{R_1}\right] = \mu_2\left[\frac{1}{R_1} - \frac{1}{v_1}\right]$$

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v_1} = \frac{\mu_2 - \mu_1}{R_1} \quad \text{—} (1)$$

3. Since the lens material is not continuous, for refraction at the second surface XP_2Y, I_1 can be regarded as the virtual object whose real image is formed at I.

As refraction is now taking place from denser to rarer medium

$$-\frac{\mu_2}{v_1} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R_2} \quad — (2) \quad \left[\begin{array}{l}\text{Interchange } \mu_1 \text{ and} \\ \mu_2 \text{ in eqn (1)}\end{array}\right]$$

Adding eqns (1) and (2)

$$\mu_1\left[\frac{1}{v} - \frac{1}{u}\right] = (\mu_2 - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Rightarrow \left[\frac{1}{v} - \frac{1}{u}\right] = \left[\frac{\mu_2}{\mu_1} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

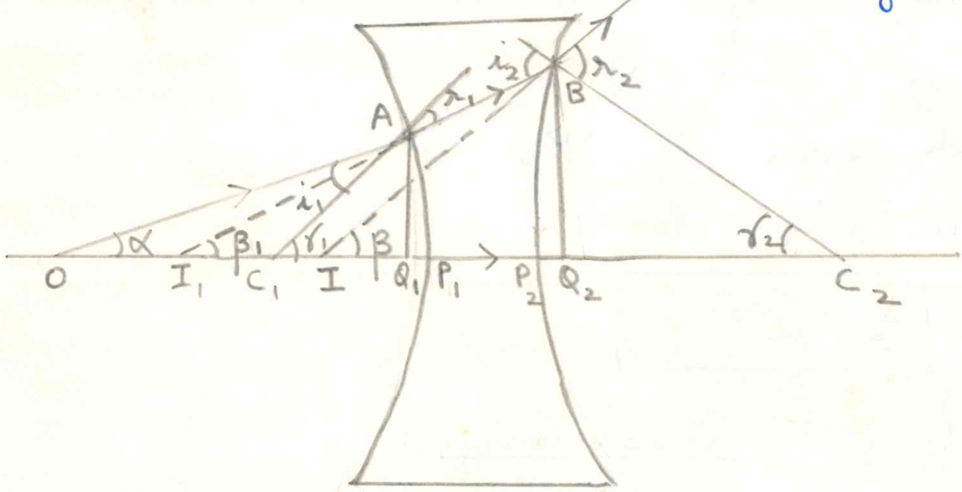
$\frac{\mu_2}{\mu_1} = \mu$, If $u = \infty$ then $v = f$.

$$\therefore \boxed{\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

μ is the refractive index of the material of the lens with respect to the surroundings

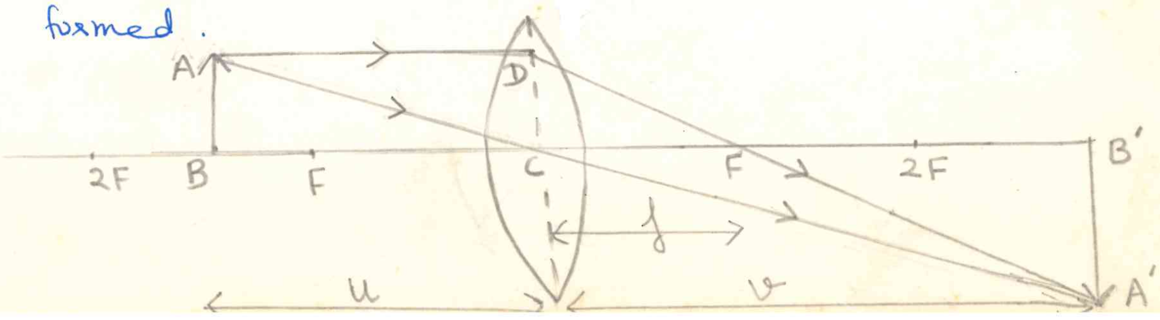
Note: Lens maker's formula can be derived for concave lens as well.

Lens maker formula for concave lens, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$



v.x. Lens formula:

(a) Convex lens (real image). When object is beyond 2F, at 2F and between F and 2F a real image of the object is formed.



(i) C is the optic centre, F the principal focus and CF the focal length f of the lens.

(ii) When an object AB is held perpendicular to the principal axis of the lens a real & inverted image of the object is formed (A'B').

Δs ABC and A'B'C are similar

$$\therefore \frac{A'B'}{AB} = \frac{CB'}{CB} \rightarrow ①a$$

Δ's A'B'F and CDF are similar

$$\therefore \frac{A'B'}{CD} = \frac{FB'}{CF} \rightarrow ① b$$

As CD = AB eqns ①a and ②a can be writter as

$$\frac{CB'}{CB} = \frac{FB'}{CF}$$

$$\Rightarrow \frac{CB'}{CB} = \frac{CB' - CF}{CF}$$

Applying sign convention

$$\frac{v}{-u} = \frac{v-f}{f}$$

$$vf = -uv + uf$$

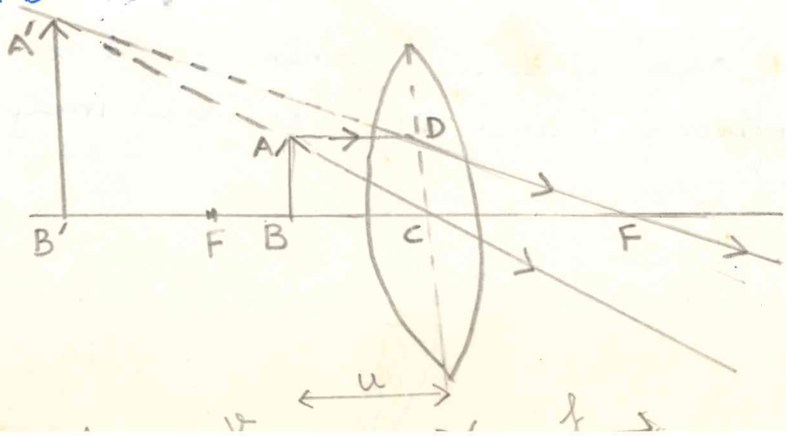
$\div$ both sides by uvf.

$$\frac{1}{u} = -\frac{1}{f} + \frac{1}{v}$$

$$\therefore \boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$$

(b) Convex lens (virtual image)

(ii) When an object AB is kept between C and F the image A'B' is virtual, erect and magnified.



Δ's ABC and A'B'C are similar

$$\therefore \frac{A'B'}{AB} = \frac{CB'}{CB} \rightarrow (2)a$$

Δ's A'B'F and CDF are similar.

$$\frac{A'B'}{CD} = \frac{B'F}{CF} \rightarrow (2)b$$

Since $AB = CD$, eqns (2)a and (2)b can be written as

$$\frac{CB'}{CB} = \frac{B'F}{CF}$$

$$\Rightarrow \frac{CB'}{CB} = \frac{CB' + CF}{CF}$$

Applying sign convention.

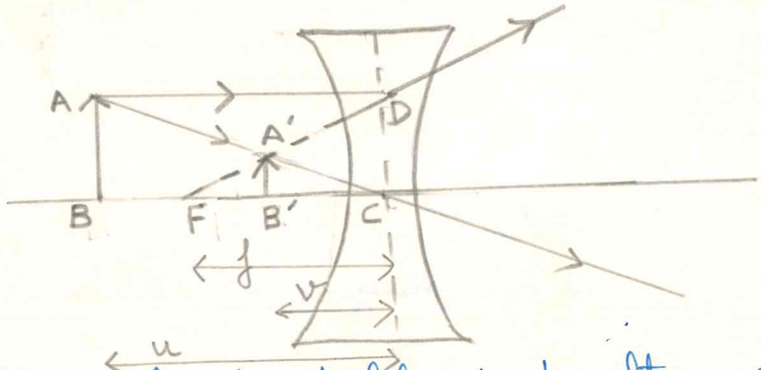
$$\frac{-v}{-u} = \frac{-v+f}{f}$$

$$-vf = uv - uf$$

$\div$ both sides by uvf

$$\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$$

(c) Concave lens (always forms virtual image)



AB is the object held $\perp$ to the principal axis of the lens. A virtual, erect and diminished image A'B' is due to refraction through concave lens.

Δ's ABC and A'B'C are similar,

$$\frac{A'B'}{AB} = \frac{CB'}{CB} \rightarrow (3)a$$

Δ's CDF and A'B'F are similar

$$\frac{A'B'}{CD} = \frac{B'F}{CF} \rightarrow (3)b$$

Since $CD = AB$, 3(a) and 3(b) can be written as

$$\Rightarrow \frac{CB'}{CB} = \frac{B'F}{CF}.$$

$$\frac{CB'}{CB} = \frac{CF - CB'}{CF}$$

Applying sign convention,

$$\frac{-v}{-u} = \frac{-f + v}{-f}$$

$$vf = uf - uv$$

÷ both sides by uvf

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

Linear magnification (m) : It is defined as the ratio of size of image (h_2) to the size of the object (h_1).

$$m = \frac{\text{Size of image } (A'B')}{\text{Size of object } (AB)} = \frac{h_2}{h_1} = \frac{v}{u}$$

[m is −ve for real image and +ve for virtual image]

Using the lens formula show that a convex lens produces virtual and enlarged image when object is placed between optic centre and focus

Given: $f > 0$, $u < 0$, $u < f$

Lens formula is $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\therefore \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

Applying sign convention $\frac{1}{v} = \frac{1}{f} - \frac{1}{|u|}$

$$\therefore \frac{1}{v} = \frac{|u| - f}{f|u|}$$

Since $u < f$, v is negative $\Rightarrow$ virtual image

$$\frac{1}{f} = \frac{1}{-|v|} + \frac{1}{|u|}$$

Since $f > 0$, $\frac{1}{u} > \frac{1}{v} \Rightarrow v > u$

$\therefore$ Enlarged image

Hence a virtual and magnified image is formed when the object is placed between focus and optic centre.

Power of a lens: It is the ability of the lens to converge or diverge a beam of light falling on the lens.

$$\boxed{P = \frac{1}{f\,(\text{in metre})}}$$

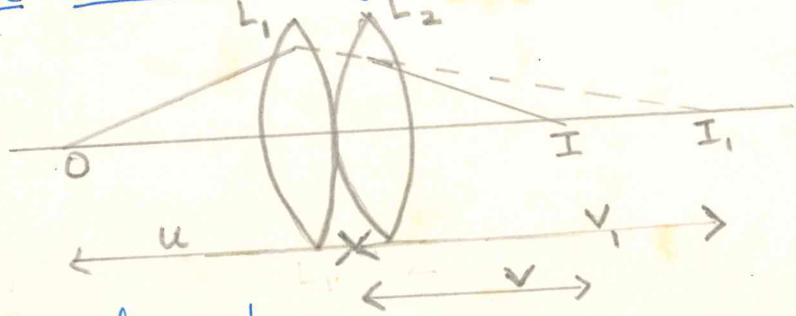
SI unit of power is dioptre (D).

According to lens maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \boxed{P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Power of a combination of two lens.



For the first lens L_1,

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u} \quad \text{--- (1)}$$

If v_1 is the virtual object for the second lens L_2 then

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v_1} \quad \text{--- (2)}$$

Adding (1) and (2)

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u}.$$

$$\therefore \boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}}.$$

where F is the focal length of the combination of the two lens.

$$\therefore \boxed{P = P_1 + P_2}$$

Practice question: Using the lens formula show that (a) convex lens produces virtual and enlarged image when object is between the optic centre & focus (b) concave lens produces virtual and diminished image for any position of the object.