SECTION — A

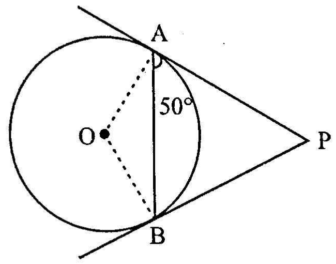
1. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50°$, then find $\angle AOB$. **[1]**

Solution : Since, tangents from an external point are equal.

i.e., $AP = BP$

Given, $\angle PAB = 50°$

$\therefore$ $\angle PBA = 50°$



In ΔAPB

$$\angle APB = 180° - (50° + 50°) = 80°$$

$$\therefore \quad \angle AOB = 180° - 80° = 100$$ **Ans.**

2. In Fig. 1, AB is a 6 m high pole and CD is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point D of pole. If $AD = 2.54$ m, find the length of the ladder. (Use $\sqrt{3} = 1.73$) **[1]**

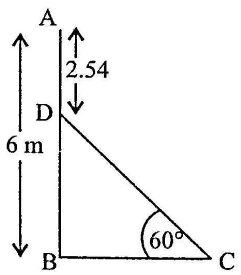


Fig. 1

Solution : Given, $AB = 6$ m and $AD = 2{\cdot}54$ m.

$\therefore DB = (6 - 2{\cdot}54)$ m $= 3{\cdot}46$ m

In ΔDBC,

$$\sin 60° = \frac{DB}{DC}$$

$$\frac{\sqrt{3}}{2} = \frac{3{\cdot}46}{DC}$$

$$\Rightarrow \quad DC = \frac{3{\cdot}46 \times 2}{1{\cdot}732} = 3{\cdot}995 \text{ m} \simeq 4 \text{ m}$$

$\therefore$ The length of the ladder is 4 m. **Ans.**

3. Find the 9th term from the end (towards the first term) of the A.P. 5, 9, 13, ..., 185. **[1]**

Solution : Given, A.P. is 5, 9, 13, ..., 185

$l = 185$ and $d = 5 - 9 = 9 - 13 = -4$

then,
$$l_9 = l + (n-1)d$$
$$= 185 + (9-1)(-4)$$
$$= 185 + 8(-4)$$
$$\therefore \quad l_9 = 153$$ **Ans.**

4. Cards marked with number 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the selected card bears a perfect square number. **[1]**

Solution : Total outcomes = 3, 4, 5, ..., 50

Total no. of outcomes = 48

Possible outcomes = 4, 9, 16, 25, 36, 49.

Let E be the event of getting a perfect square number

No. of possible outcomes = 6

$$\therefore \quad P(E) = \frac{6}{48} = \frac{1}{8}$$ **Ans.**

SECTION — B

5. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b. **[2]**

Solution : The given polynomial is, $p(x) = ax^2 + 7x + b$

$$\therefore \quad p\left(\frac{2}{3}\right) = a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$$

$$= \frac{4a}{9} + \frac{14}{3} + b = 0 \quad \text{...(i)}$$

and, $p(-3) = a(-3)^2 + 7(-3) + b = 0$

$$\Rightarrow \quad 9a - 21 + b = 0 \quad \text{...(ii)}$$

Solving equation (i) and (ii), we get

$$4a + 42 + 9b = 0$$
$$81a - 189 + 9b = 0$$
$$- \quad + \quad -$$
$$-77a + 231 = 0$$

$$a = \frac{231}{77} = 3$$

Putting $a = 3$ in eq. (ii) we get,

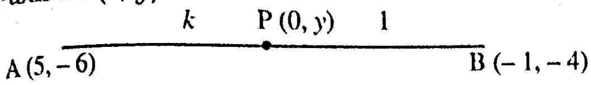
$$9(3) - 21 + b = 0$$

$\Rightarrow$ $b = -6$

$\therefore$ $a = 3$ and $b = -6$ **Ans.**

6. Find the ratio in which y-axis divides the line segment joining the points $A(5, -6)$ and $B(-1, -4)$. Also find the coordinates of the point of division. **[2]**

Solution : Let the required ratio be $k : 1$ and point on y-axis be $(0, y)$



$\therefore AP : PB = k : 1$

Then, by section formula

$$\frac{5-k}{k+1} = 0$$

$$5 - k = 0$$

$\Rightarrow$ $k = 5$

Hence, required ratio is 5 : 1

$\therefore$ $$y = \frac{(-4)(5)+(-6)(1)}{5+1}$$

$\therefore$ $$y = \frac{-26}{6} = -\frac{13}{3}$$

Hence, point on y-axis is $\left(0, \frac{-13}{3}\right)$ **Ans.**

7. In Fig. 2, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF. **[2]**

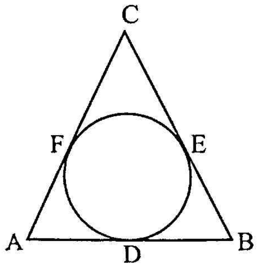


Fig. 2

Solution : Given, $AB = 12$ cm ; $BC = 8$ cm and $CA = 10$ cm

Let $AD = AF = x$

$\therefore$ $DB = BE = 12 - x$

and, $CF = CE = 10 - x$

Now, $BC = BE + EC$

$\Rightarrow$ $8 = 12 - x + 10 - x$

$\Rightarrow$ $8 = 22 - 2x$

$\Rightarrow$ $2x = 14$

$\Rightarrow$ $x = 7$ cm

$\therefore AD = 7$ cm, $BE = 5$ cm and $CF = 3$ cm **Ans.**

8. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from $Q(2, -5)$ and $R(-3, 6)$, find the coordinates of P. **[2]**

Solution : Let the coordinates of point P be $(2y, y)$

Since, P is equidistant from Q and R

$\therefore$ $PQ = PR$

$\Rightarrow$ $$\sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$$

$\Rightarrow$ $(2y-2)^2 + (y+5)^2 = (2y+3)^2 + (y-6)^2$

$\Rightarrow 4y^2 + 4 - 8y + y^2 + 25 + 10y = 4y^2 + 9 + 12y + y^2 + 36 - 12y$

$\Rightarrow$ $2y + 29 = 45$

$\Rightarrow$ $2y = 45 - 29$

$\Rightarrow$ $$y = \frac{16}{2} = 8$$

Hence, the co-ordinates of point P are (16, 8). **Ans.**

9. How many terms of the A.P. 18, 16, 14, ... be taken so that their sum is zero? **[2]**

Solution : Given, A.P. is 18, 16, 14 ....

We have, $a = 18$, $d = 16 - 18 = 14 - 16 = -2$

Now, $S_n = 0$

Therefore, $$S_n = \frac{n}{2}[2a + (n-1)d] = 0$$

$\Rightarrow$ $$\frac{n}{2}[2 \times 18 + (n-1)(-2)] = 0$$

$\Rightarrow$ $36 - 2n + 2 = 0$

$\Rightarrow$ $2n = 38$

$\therefore$ $n = 19$

Hence, the no. of terms are 19. **Ans.**

10. In Fig. 3, AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60°$. Find the length of chord AB. **[2]**

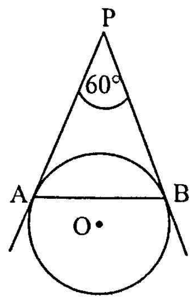


Fig. 3

Solution : Given, AP and BP are tangents to a circle with centre O.

$\therefore AP = BP$

Now, $\angle APB = 60°$ (Given)

$\therefore \angle PAB = \angle PBA = 60°$ $(\because AP = BP)$

Thus, ΔAPB is an equilateral triangle.

Hence, the length of chord AB is equal to the length of AP *i.e.* 5 cm. **Ans.**

SECTION — C

11. In Fig. 4, $ABCD$ is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region. $\left(\text{use } \pi = \frac{22}{7}\right)$ [3]

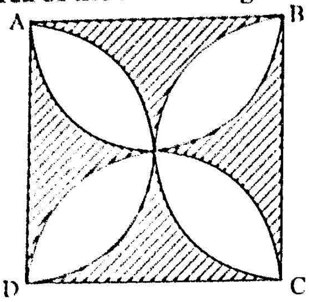


Fig. 4

Solution : Given, a square $ABCD$ of side 14 cm

Then, Area of square $= (\text{side})^2$

$= (14)^2 = 196 \text{ cm}^2$

2 [Area of semicircle] $= \pi r^2$

$= \frac{22}{7} \times \frac{14}{2} \times \frac{14}{2} = 154 \text{ cm}^2$

Now, Area of shaded region =

2[Area of square – 2 (Area of semicircle)]

$= 2\,[196 - 154] = 2 \times 42 = 84 \text{ cm}^2$ **Ans.**

12. In Fig. 5, is a decorative block, made up of two solids – a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block. $\left(\text{use } \pi = \frac{22}{7}\right)$ [3]

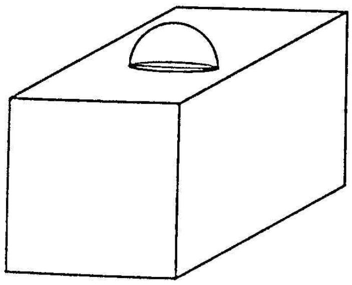


Fig. 5

Solution : Given, side of a cube = 6 cm

and the diameter of hemisphere = 3·5 cm

Now, total surface area of decorative block =

total surface area of cube – surface area of base of hemisphere + CSA of hemisphere

$= (6)^3 - \frac{22}{7} \times \frac{3{\cdot}5}{2} \times \frac{3{\cdot}5}{2} + 2 \times \frac{22}{7} \times \frac{3{\cdot}5}{2} \times \frac{3{\cdot}5}{2}$

$= 216 - \frac{22 \times 7}{16} + \frac{22 \times 7}{8}$

$= 216 + \frac{154}{16}$

$= 225{\cdot}625 \text{ cm}^2$ **Ans.**

13. In Fig. 6, ABC is a triangle coordinates of whose vertex A are $(0, -1)$. D and E respectively are the mid-points of the sides AB and AC and their coordinates are $(1, 0)$ and $(0, 1)$ respectively. If F is the mid-point of BC, find the areas of $\Delta\, ABC$ and $\Delta\, DEF$. [3]

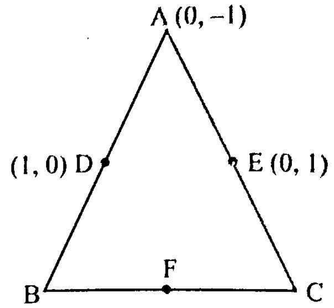


Fig. 6

Solution : Given, the coordinates of vertex A $(0, -1)$ and, mid points D (1,0) and E(0, 1) respectively.

Since, D is the mid-point of AB

Let, coordinates of B are (x, y)

then, $\left(\frac{x+0}{2}, \frac{y-1}{2}\right) = (1, 0)$

which gives B (2, 1)

Similarly, E is the mid-point of AC

Let, coordinates of C are (x', y')

then, $\left(\frac{x'+0}{2}, \frac{y'-1}{2}\right) = (0, 1)$

which gives C (0, 3)

Now, Area of $\Delta\, ABC = \frac{1}{2}\,|[0\,(1-3) + 2\,(3+1) + 0\,(-1-1)]|$

$= 4$ sq units. **Ans.**

Now, F is the mid-point of BC.

$\Rightarrow$ Coordinates of F are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$

$\therefore$ Area of $\Delta\, DEF = \frac{1}{2}\,|[1\,(1-2) + 0\,(2-0) + 1\,(0-1)]|$

$= \frac{|-2|}{2} = 1$ sq unit **Ans.**

14. In Fig. 7, are shown two arcs PAQ and PBQ. Arc PAQ is a part of circle with centre O and radius OP while arc PBQ is a semi-circle drawn on PQ as diameter with centre M. If $OP = PQ = 10$ cm show that area of shaded region is $25\left(\sqrt{3} - \frac{\pi}{6}\right)$ cm². [3]

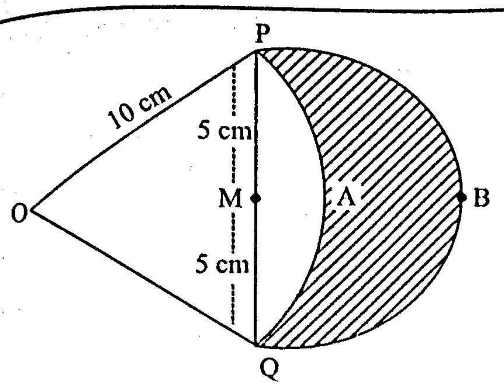


Fig. 7

Solution : Given, $OP = PQ = 10$ cm

Since, OP and OQ are radius of circle with centre O.

$\therefore \Delta\, OPQ$ is equilateral.

$\Rightarrow \angle POQ = 60°$

Now, Area of segment $PAQM$

$$=(\text{Area of sector } OPAQO - \text{Area of } \Delta\, POQ)$$

$$= \frac{\pi r^2 \theta}{360°} - \frac{1}{2} r^2 \sin i$$

$$= \frac{\pi \times (10)^2 \times 60°}{360°} - \frac{1}{2}(10)^2 \sin 60°$$

$$= \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right) \text{cm}^2$$

and, area of semicircle $PBQ = \frac{\pi r^2}{2} = \frac{\pi}{2}(5)^2 = \frac{25}{2}\pi \text{ cm}^2$

$\therefore$ Area of shaded region $=$ Area of semicircle $-$ Area of segment $PAQM$

$$= \frac{25}{2}\pi - \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right)$$

$$= \frac{25}{2}\pi - \frac{50\pi}{3} + 25\sqrt{3}$$

$$= \frac{75\pi - 100\pi}{6} + 25\sqrt{3}$$

$$= \frac{-25\pi}{6} + 25\sqrt{3}$$

$$= 25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{cm}^2$$

Hence Proved.

15. If the sum of first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first n terms of the A.P. **[3]**

Solution : Given, sum of first 7 terms of an A.P. $(S_7) = 49$

and sum of first 17 terms of an A.P. $(S_{17}) = 289$

i.e.,
$$S_7 = \frac{7}{2}[2a + (7-1)d] = 49$$

$$2a + 6d = 14 \quad \text{...(i)}$$

And,
$$S_{17} = \frac{17}{2}[2a + (17-1)d] = 289$$

$$2a + 16d = 34 \quad \text{...(ii)}$$

Solving equations (i) and (ii), we get

$$2a + 16d = 34$$
$$2a + 6d = 14$$
$$- \quad - \qquad -$$
$$10d = 20$$
$$d = 2$$

Putting $d = 2$ in eq. (i), we get

$$a = 1$$

Hence, sum of first n term of A.P.,

$$S_n = \frac{n}{2}[2(1) + (n-1)2]$$

$$\Rightarrow \quad S_n = \frac{n}{2}[2 + (n-1)2]$$

$$\Rightarrow \quad S_n = n^2$$

Ans.

16. Solve for x :

$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, -3/2$$

[3]

Solution : We have, $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)}$

$$= 0, x \neq 3, -3/2$$

$$2x(2x+3) + (x-3) + (3x+9) = 0$$
$$4x^2 + 6x + x - 3 + 3x + 9 = 0$$
$$4x^2 + 10x + 6 = 0$$
$$2x^2 + 5x + 3 = 0$$
$$2x^2 + 2x + 3x + 3 = 0$$
$$2x(x+1) + 3(x+1) = 0$$
$$(2x+3)(x+1) = 0$$
$$x = -1, \frac{-3}{2}$$

$$\therefore \quad x = -1 \qquad [\because \text{Given } x \neq -3/2]$$

Ans.

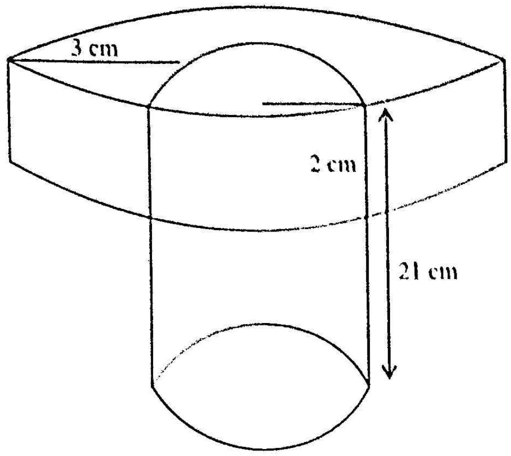
17. A well of diameter 4 m is dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment. **[3]**

Solution : Given, diameter and height of cylindrical well asre 4 m and 21 m respectively.

Now, the earth has been taken out to spread evenly all around.

Then, volume of earth dug out $= \frac{22}{7} \times \frac{3}{4} \times \frac{1}{3} \times 21$

$$= 264 \text{ m}^3$$



And the volume of embankment of width 3 m which forms a shape of circular ring $= \pi\,((5)^2 - (2)^2) \times h$

$$= \frac{22}{7}(25-4) \times h = 66\,h \text{ m}^3$$

[$\because$ Outer radius $= 2 + 3 = 5$ cm]

$\because$ Volume of earth dug out = Volume of embankment

$\therefore \quad 264 = 66h$

$\Rightarrow \quad h = \frac{264}{66} = 4$ m

Hence, the height of the embankment is 4 m. **Ans.**

18. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\left(\text{use } \pi = \frac{22}{7}\right)$ [3]

Solution : Let the radius of base and height of a solid cylinder be r and h respectively.

Now, we have, $r + h = 37$ cm ...(i)

and, T.S.A. of solid cylinder $= 2\pi r(r + h) = 1628$ cm^2

$\Rightarrow \quad 2\pi r(37) = 1628$

$\Rightarrow \quad r = \frac{1628}{37 \times 2 \times \frac{22}{7}}$

$r = 7$ cm

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 30$$

(Using eq. (i), $h = 30$)

$= 4620$ cm^3 **Ans.**

19. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. (use $\sqrt{3} = 1.73$) [3]

Solution : Let AB and CD be the tower and high building respectively

Given, $CD = 50$ m

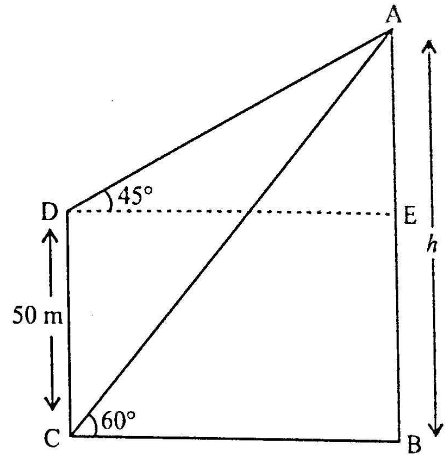
Let, $AB = h$ m

Then, in ΔADE

$$\tan 45° = \frac{AE}{DE}$$

$$1 = \frac{h-50}{DE}$$

$$DE = h - 50 \quad ...(i)$$



and, in ΔACB

$$\tan 60° = \frac{AB}{CB}$$

$$\sqrt{3} = \frac{h}{CB}$$

$$CB = \frac{h}{\sqrt{3}} \quad ...(ii)$$

Now, $CB = DE$

then from eq. (i) and (ii), we get

$$h - 50 = \frac{h}{\sqrt{3}}$$

$$h - \frac{h}{\sqrt{3}} = 50$$

$$\frac{(\sqrt{3}-1)}{\sqrt{3}} h = 50$$

$$h = \frac{50\sqrt{3}}{\sqrt{3}-1} = \frac{50\sqrt{3}}{\sqrt{3}-1} \times \frac{50\sqrt{3}}{\sqrt{3}-1}$$

$$= 25 \times 3 + 25\sqrt{3}$$

$$h = 75 + 25(1{\cdot}73)$$

$$= 118{\cdot}25 \text{ m}$$

Hence, the height of the tower is 118·25 m and the horizontal distance between the tower and the building is 68·25 m. **Ans.**

20. In a single throw of a pair of different dice, what is the probability of getting (i) a prime number on each dice ? (ii) a total of 9 or 11 ? [3]

Solution : Total outcomes = {(1, 1), (1, 2) (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

No. of outcomes = 36

(i) Let E_1 be the event of getting a prime number on each dice.

Favourable outcomes = {(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)}

$\Rightarrow$ No. of favourable outcomes = 9

$$\therefore \quad P(E_1) = \frac{9}{36} = \frac{1}{4}$$ **Ans.**

(ii) Let E_2 be the event of getting a total of 9 or 11.

Favourable outcomes = {(3, 6), (4, 5), (5, 4), (6, 3), (5, 6), (6, 5)}

$\Rightarrow$ No. of favourable outcomes = 6

$$\therefore \quad P(E_2) = \frac{6}{36} = \frac{1}{6}$$ **Ans.**

SECTION — D

21. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/hour than the usual speed. Find the usual speed of the plane.

What value is depicted in this question ? [4]

Solution : Let the usual speed of the plane be x km/h.

$\therefore$ Time taken by plane to reach 1500 km away $= \frac{1500}{x}$

and the time taken by plane to reach 1500 km with increased speed $= \frac{1500}{x+250}$

$$\text{Now, } \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2} \quad \text{(Given)}$$

$$1500\,\frac{(x+250-x)}{x(x+250)} = \frac{1}{2}$$

$$3000 \times 250 = x^2 + 250x$$

$$x^2 + 250x - 750000 = 0$$

$$x^2 + 1000x - 750x - 750000 = 0$$

$$x(x+1000) - 750(x+1000) = 0$$

$$(x+1000)(x-750) = 0$$

$$x = -1000 \text{ or } x = 750$$

(As speed can't be negative)

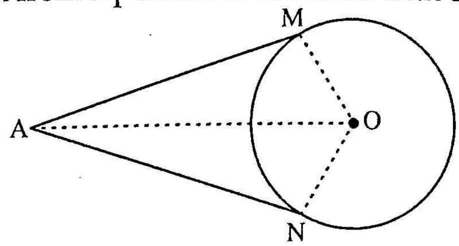
$$\therefore \quad x = 750$$

$\therefore$ Speed of plane is 750 km/h. **Ans.**

Value : It shows his responsibility towards mankind and his work. **Ans.**

22. Prove that the lengths of tangents drawn from an external point to a circle are equal. [4]

Solution : Given, Two tangents AM and AN are drawn from a point A to the circle with centre O.



To prove : $AM = AN$

Construction : Join OM, ON and OA.

Proof : Since AM is a tangent at M and OM is radius

$\therefore OM \perp AM$

Similarly, $ON \perp AN$

Now, in $\Delta\, OMA$ and $\Delta\, ONA$

$OM = ON$ (Radii of the circle)

$OA = OA$ (Common)

$\angle OMA = \angle ONA = 90°$

$\therefore \quad \Delta\, OMA \cong \Delta\, ONA$

(By RHS congruence)

Hence, $AM = AN$ (by c.p.c.t.)

Hence Proved.

23. Draw two concentric circles of radii 3 cm and 5 cm. Construct a tangent to smaller circle from a point on the larger circle. Also measure its length. [4]

Solution : Steps of construction—

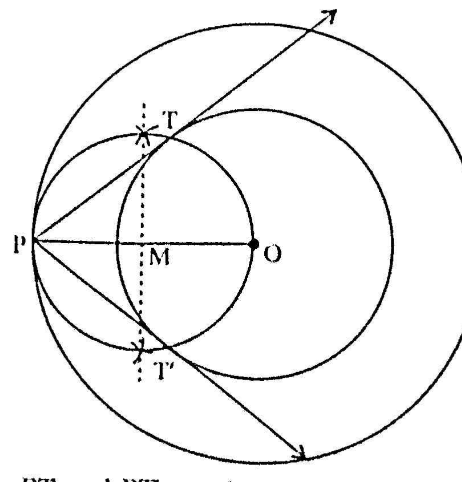
(i) Draw two concentric circles of radii 3 cm and 5 cm

(ii) Mark a point P on larger circle such that OP = 5 cm

(iii) Join OP and bisect it at M.

(iv) Draw a circle with M as centre and radius equal to MP to intersect the given circle at the points T and T'.

(v) Join PT and PT'.



Then, PT and PT' are the required tangents.

24. In Fig. 8, O is the centre of a circle of radius 5 cm. T is a point such that $OT = 13$ cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle. [4]

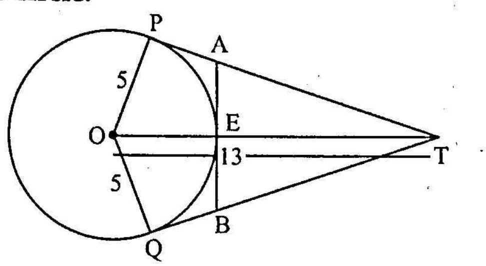


Fig. 8

Solution : Given, a circle with centre of radius 5 cm and $OT = 13$ cm

Since, PT is a tangent at P and OP is a radius through P

$\therefore \quad OP \perp PT$

In $\Delta\, OPT$

$$(PT)^2 = (OT)^2 - (OP)^2$$

$\Rightarrow \quad PT = \sqrt{(13)^2 - (5)^2}$

$\Rightarrow \quad PT = \sqrt{169 - 25} = \sqrt{144}$

$\Rightarrow \quad PT = 12$ cm

And, $\quad TE = OT - OE = (13 - 5)$ cm $= 8$ cm

Now, $\quad PA = AE$

Let $\quad PA = AE = x$

Then, in $\Delta\, AET$

$$(AT)^2 = (AE)^2 + (ET)^2$$
$$(12 - x)^2 = (x)^2 + (8)^2$$
$$144 + x^2 - 24x = x^2 + 64$$
$$24x = 80$$

$\Rightarrow \quad AE = x = 3{\cdot}33$ cm

$\therefore \quad AB = 2AE = 2 \times 3{\cdot}33$

$\quad = 6{\cdot}66$ cm **Ans.**

25. Find x in terms of a, b and c :

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a,b,c \qquad [4]$$

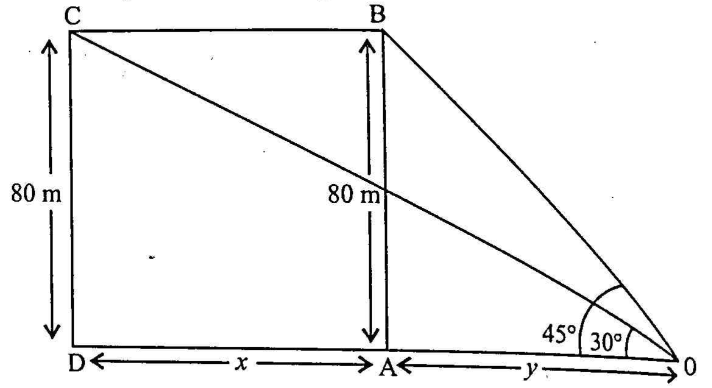
Solution : We have, $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a,b,c$

$$a(x-b)(x-c) + b(x-a)(x-c) = 2c(x-a)(x-b)$$
$$a(x^2 - bx - cx + bc) + b(x^2 - ax - cx + ac) = 2c(x^2 - ax - bx + ab)$$
$$ax^2 - abx - acx + abc + bx^2 - abx - bcx + abc = 2cx^2 - 2acx - 2bcx + 2abc$$
$$ax^2 + bx^2 - 2abx - acx - bcx + 2abc = 2cx^2 - 2acx - 2bcx + 2abc$$
$$ax^2 + bx^2 - 2cx^2 - 2abx - acx - bcx + 2acx + 2bcx = 0$$
$$(a + b - 2c)x^2 + (-2ab + ac + bc)x = 0$$
$$x[(a + b - 2c)x + (ac + bc - 2ab)] = 0$$

$$\therefore \quad x = 0, -\frac{(ac + bc - 2ab)}{a + b - 2c} \quad \textbf{Ans.}$$

26. A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird. (Take $\sqrt{3} = 1.732$) [4]

Solution : Let B be the initial position of bird sitting on top of tree of length 80 m.



After 2 sec, the position of bird becomes C.

Let the distance travel by bird from B to C is x m.

Now, in $\Delta\, ABO$

$$\tan 45° = \frac{AB}{AO} = \frac{80}{y}$$

$$y = 80 \text{ m} \qquad ...(i)$$

And, in $\Delta\, DCO$

$$\tan 30° = \frac{CD}{DO} = \frac{80}{x+y}$$

$$\frac{1}{\sqrt{3}} = \frac{80}{x+80} \qquad \text{[Using eq. (i)]}$$

$$x + 80 = 80\sqrt{3}$$
$$x = 80\left(\sqrt{3} - 1\right) = 80 \times 0{\cdot}732$$

$\therefore \quad x = 58{\cdot}56$ m

Hence, speed of flying of the bird $= \frac{58{\cdot}56}{2}$

$$\left(\text{Speed} = \frac{\text{Distance}}{\text{Time}}\right)$$

$= 29{\cdot}28$ m/s **Ans.**

27. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief. [4]

Solution : Let total time be n minutes

Since policeman runs after 1 minutes so he will catch the thief in $(n-1)$ minutes.

Total distance covered by thief = 100 m/minute × n minute

$$= (100\,n) \text{ m}$$

Now, total distance covered by the policeman = (100)m + (100 + 10)m + (100 + 10 + 10)m + ... + $(n-1)$ terms

i.e., $100 + 110 + 120 + ... + (n-1)$ terms

$$\therefore S_{n-1} = \frac{n-1}{2}[2 \times 100 + (n-2)\,10]$$

$$\Rightarrow \frac{n-1}{2}[200 + (n-2)\,10] = 100\,n$$

$$\Rightarrow (n-1)(200 + 10n - 20) = 200\,n$$

$$\Rightarrow 200n - 200 + 10n^2 - 10n + 20 - 20n = 200n$$

$$\Rightarrow \quad 10n^2 - 30n - 180 = 0$$

$$\Rightarrow \quad n^2 - 3n - 18 = 0$$

$$\Rightarrow \quad n^2 - (6-3)\,n - 18 = 0$$

$$\Rightarrow \quad n^2 - 6n + 3n - 18 = 0$$

$$\Rightarrow \quad n(n-6) + 3\,(n-6) = 0$$

$$\Rightarrow \quad (n+3)(n-6) = 0$$

$$\therefore \quad n = 6 \text{ or } n = -3 \text{ (Neglect)}$$

Hence, policeman will catch the thief in $(6-1)$ *i.e.,* 5 minutes. **Ans.**

28. Prove that the area of a triangle with vertices $(t, t-2)$, $(t+2, t+2)$ and $(t+3, t)$ is independent of t. [4]

Solution : Given, the vertices of a triangle $(t, t-2)$, $(t+2, t+2)$ and $(t+3, t)$

$$\therefore \text{Area of the triangle} = \frac{1}{2}\,|[t\,(t+2-t) + (t+2)(t-t+2) + (t+3)(t-2-t-2)]|$$

$$= \frac{1}{2}\,|(2t + 2t + 4 - 4t - 12)|$$

$$= \frac{1}{2}\,|-8| = 4 \text{ sq units}$$

which is independent of t **Hence Proved.**

29. A game of chance consists of spinning an arrow on a circular board, divided into 8 equal parts, which comes to rest pointing at one of the numbers 1, 2, 3, ..., 8 (Fig. 9), which are equally likely outcomes. What is the probability that the arrow will point at (i) an odd number, (ii) a number greater than 3, (iii) a number less than 9. [4]

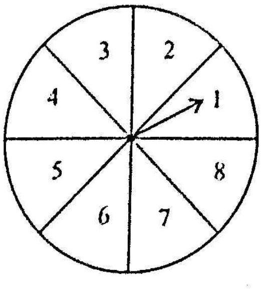


Fig. 9

Solution : Total no. of outcomes = 8

(i) Let E_1 be the event of getting an odd number

$\therefore$ Favourable outcomes = 1, 3, 5, 7

$\Rightarrow$ No. of favourable outcomes = 4

$$\therefore \quad P(E_1) = \frac{4}{8} = \frac{1}{2}$$

(ii) Let E_2 be the event of getting a number greater than 3.

$\therefore$ Favourable outcomes = 4, 5, 6, 7, 8

$\Rightarrow$ No. of favourable outcomes = 5

$$\therefore \quad P(E_2) = \frac{5}{8}$$

(iii) Let E_3 be the event of getting a number less than 9.

$\therefore$ Favourable outcomes = 1, 2, 3, 4, 5, 6, 7, 8

$\Rightarrow$ No. of favourable outcomes = 8

$$\therefore \quad P(E_3) = \frac{8}{8} = 1$$

Ans.

30. An elastic belt is placed around the rim of a pulley of radius 5 cm. (Fig. 10) From one point C on the belt, the elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area. (use π = 3.14 and $\sqrt{3}$ = 1.73) [4]

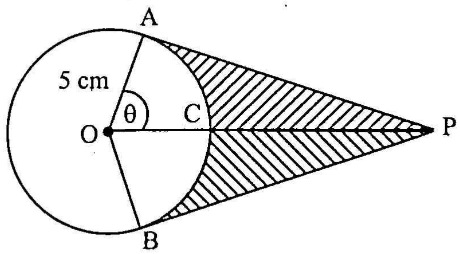


Fig. 10

Solution : Given, a circular pulley of radius 5 cm with centre O.

$$\therefore \quad AO = OB = OC = 5 \text{ cm}$$

$$\text{and} \quad OP = 10 \text{ cm}$$

Now, in right $\Delta\,AOP$

$$\cos i = \frac{AO}{OP} = \frac{5}{10} = \frac{1}{2}$$

$$\therefore \quad i = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

$\therefore \angle AOB = 2i = 120^\circ$

$\Rightarrow$ Reflex $\angle AOB = 360^\circ - 120^\circ = 240^\circ$

$$\text{Length of major arc } \widehat{AB} = \frac{2\pi r}{360^\circ} \text{ reflex } \angle AOB$$

$$= \frac{2 \times 3{\cdot}14 \times 5 \times 240^\circ}{360^\circ}$$

$$= 20{\cdot}93 \text{ cm}$$

Hence, length of the belt that is still in contact with pulley = 20·93 cm

Now, by pythagorus theorem

$$(AP)^2 = (OP)^2 - (AO)^2$$
$$(AP)^2 = (10)^2 - (5)^2$$
$$AP = \sqrt{100 - 25}$$
$$= \sqrt{75} = 5\sqrt{3} \text{ cm}$$

$$\therefore \quad \text{Area of } \Delta AOP = \frac{1}{2} \times 5 \times 5\sqrt{3}$$
$$= \frac{25\sqrt{3}}{2} \text{ cm}^2$$

Also, Area of ΔBOP = Area of ΔAOP

and, Area of quad. $AOBP$ = 2 (Area of ΔAOP)

$$= 2 \times \frac{25\sqrt{3}}{2} = 25\sqrt{3} \text{ cm}^2$$
$$= 43{\cdot}25 \text{ cm}^2$$

$$\text{Area of sector } ACBO = \frac{\pi r^2 \angle AOB}{360°}$$
$$= \frac{3{\cdot}14 \times 5 \times 5 \times 120}{360°}$$
$$= 26{\cdot}16 \text{ cm}^2$$

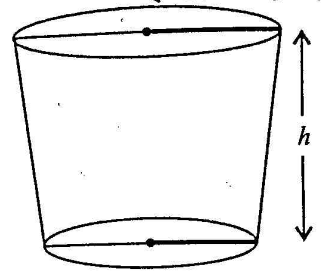
$\therefore$ Area of shaded region = Area of quad. $AOBP$ – Area of sector $ACBO$

$$= (43{\cdot}25 - 26{\cdot}16) \text{ cm}^2$$
$$= 17{\cdot}09 \text{ cm}^2 \qquad \text{Ans.}$$

31. A bucket open at the top is in the form of frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of metal sheet used in making the bucket.

(use π = 3.14) [4]

Solution : Given, the radii of top and bottom circular ends are 20 cm and 12 cm respectively.



And, volume of frustum (bucket) = 12308·8 cm^3

$$\Rightarrow \quad \frac{\pi h}{3}[R^2 + r^2 + Rr] = 12308{\cdot}8$$
$$\frac{3{\cdot}14 \times h}{3}[400 + 144 + 240] = 12308{\cdot}8$$
$$\therefore \quad \text{Height } (h) = \frac{12308{\cdot}8 \times 3}{3{\cdot}14 \times 784}$$
$$= \frac{36926{\cdot}4}{2461{\cdot}76} = 15 \text{ cm}$$

$$\text{Slant height of the bucket } (l) = \sqrt{h^2 + (R-r)^2}$$
$$= \sqrt{(15)^2 + (20-12)^2}$$
$$= \sqrt{225 + 64} = \sqrt{289}$$
$$= 17 \text{ cm}$$

$\therefore$ Area of metal sheet used in making the bucket

= Curved surface area of frustum + Base area

$= \pi l (R + r) + \pi r^2$

$= 3{\cdot}14 \times 17 \times (20 + 12) + 3{\cdot}14 \times 12 \times 12$

$= 3{\cdot}14 \times 17 \times 32 + 3{\cdot}14 \times 144$

$= 3{\cdot}14 (544 + 144)$

$= 3{\cdot}14 \times 688$

$= 2160{\cdot}32$ cm^2 Ans.