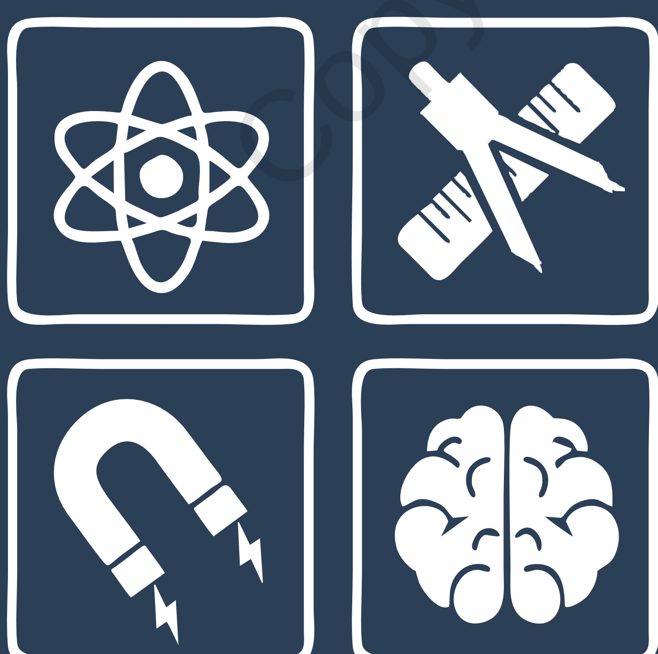
CBSE

10 LAST YEARS SOLVED PAPERS

CLASS X



2019 EXAMINATION

OSWAL

Mathematics 2018

Time allowed : 3 hours **Maximum marks : 80**

General Instructions :

(i) All questions are compulsory.

(ii) This question paper consists of **30** questions divided into four sections—A, B, C and D.

(iii) Section A contains **6** questions of **1** mark each. Section B contains **6** questions of **2** marks each, Section C contains **10** questions of **3** marks each. Section D contains **8** questions of **4** marks each.

(iv) There is no overall choice. However, an internal choice has been provided in **four** questions of **3** marks each and **3** questions of **4** marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculator is **not** permitted.

SECTION — A

1. If $x = 3$ is one root of the quadratic equation $x^2 - 2kx - 6 = 0$, then find the value of k. [1]

Solution : Given quadratic equation is,

$$x^2 - 2kx - 6 = 0$$

$x = 3$ is a root of above equation, then

$$(3)^2 - 2k(3) - 6 = 0$$
$$9 - 6k - 6 = 0$$
$$3 - 6k = 0$$
$$3 = 6k$$
$$\therefore \quad k = \frac{3}{6} = \frac{1}{2}$$
$$k = \frac{1}{2}$$ **Ans.**

2. What is the HCF of smallest prime number and the smallest composite number ? [1]

Solution : Smallest prime number = 2

Smallest composite number = 4

Prime factorisation of 2 is 1×2

Prime factorisation of 4 is 1×2^2

$\therefore$ HCF (2, 4) = 2 **Ans.**

3. Find the distance of a point $P(x,y)$ from the origin. [1]

Solution : The given point is $P(x, y)$.

The origin is $O(0, 0)$

Distance of point P from origin,

$$PO = \sqrt{x_2 - x_1 \;^2 \quad y_2 - y_1 \;^2}$$
$$= \sqrt{x - 0 \;^2 \quad y - 0 \;^2}$$
$$= \sqrt{x^2 \quad y^2} \text{ unit}$$ **Ans.**

4. In an AP, if the common difference (d) = –4 and the seventh term (a_7) is 4, then find the first term.

Solution : Given, **[1]**

$$d = -4$$
$$a_7 = 4$$
$$a + 6d = 4$$
$$a + 6(-4) = 4$$
$$a - 24 = 4$$
$$a = 4 + 24$$
$$a = 28$$ **Ans.**

5. What is the value of $(\cos^2 67° - \sin^2 23°)$? [1]

Solution : We have, $\cos^2 67° - \sin^2 23°$

$$= \cos^2 67° - \cos^2 (90° - 23°)$$
$$[\because \sin(90° - \theta) = \cos\theta]$$
$$= \cos^2 67° - \cos^2 67°$$
$$= 0$$ **Ans.**

6. Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then find $\frac{\text{ar} \quad ABC}{\text{ar} \quad PQR}$. [1]

Solution : Given, $\Delta ABC \sim \Delta PQR$

And $$\frac{AB}{PQ} = \frac{1}{3}$$

Now, $$\frac{\text{ar}(\quad ABC)}{\text{ar}(\quad PQR)} = \frac{AB^2}{PQ^2}$$
$$= \frac{1}{3}^2 = \frac{1}{9}$$ **Ans.**

SECTION — B

7. Given that $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number. [2]

Solution : Given, $\sqrt{2}$ is irrational number.

Let $\sqrt{2} = m$

Suppose, $5 + 3\sqrt{2}$ is a rational number.

So, $$5 + 3\sqrt{2} = \frac{a}{b} \qquad (a \neq b, b \neq o)$$
$$3\sqrt{2} = \frac{a}{b} - 5$$
$$3\sqrt{2} = \frac{a - 5b}{b}$$

or $$\sqrt{2} = \frac{a - 5b}{3b}$$

So, $$\frac{a - 5b}{3b} = m$$

But $\frac{a - 5b}{3b}$ is rational number, so m is rational number which contradicts the fact that $m = \sqrt{2}$ is irrational number.

So, our supposition is wrong.

Hence, $5 + 3\sqrt{2}$ is also irrational. **Hence Proved.**

8. In fig. 1, *ABCD* is a rectangle. Find the values of *x* and *y*. [2]

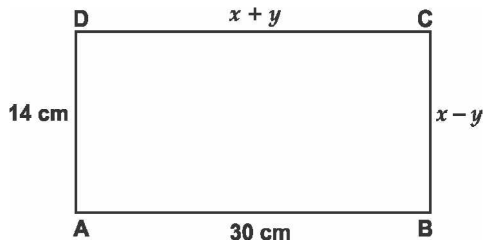


Figure 1

Solution : Given, *ABCD* is a rectangle.

$\therefore \quad AB = CD$

$\Rightarrow \quad 30 = x + y$

or $\quad x + y = 30 \quad$ ...(i)

Similarly, $\quad AD = BC$

$\Rightarrow \quad 14 = x - y$

or $\quad x - y = 14 \quad$ ...(ii)

On adding eq. (i) and (ii), we get

$2x = 44$

$\Rightarrow \quad x = 22$

Putting the value of x in eq. (i), we get

$22 + y = 30$

$\Rightarrow \quad y = 30 - 22$

$\Rightarrow \quad y = 8$

So, $x = 22$, $y = 8$. **Ans.**

9. Find the sum of first 8 multiples of 3. [2]

Solution : First 8 multiples of 3 are

3, 6, 9.............upto 8 terms

We can observe that the above series is an AP with $a = 3$, $d = 6 - 3 = 3$, $n = 8$

Sum of n terms of an A.P. is given by,

$$S_n = \frac{n}{2}[2a \quad (n-1)d]$$

$$\therefore \quad S_8 = \frac{8}{2}[2 \times 3 + (8 - 1)(3)]$$

$$= 4\,[6 + 7 \times 3]$$

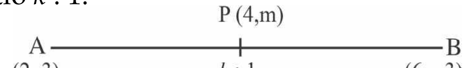
$$= 4\,[6 + 21]$$

$$= 4 \times 27$$

$$\Rightarrow \quad S_8 = 108$$ **Ans.**

10. Find the ratio in which *P* (4, *m*) divides the line segment joining the points *A* (2, 3) and *B* (6, –3). Hence find *m*. [2]

Solution : Let *P* divides line segment *AB* in the ratio *k* : 1.



Coordinates of P

$$P = \frac{m_1x_2 \quad m_2x_1}{m_1 \quad m_2}, \frac{m_1y_2 \quad m_2y_1}{m_1 \quad m_2}$$

$$(4, m) = \frac{k \times 6 \quad 1 \quad 2}{k \quad 1}, \frac{k \quad (-3) \quad 1 \quad 3}{k \quad 1}$$

$$(4, m) = \frac{6k \quad 2}{k \quad 1}, \frac{-3k \quad 3}{k \quad 1}$$

On comparing, we get

$$\frac{6k \quad 2}{k \quad 1} = 4$$

$\Rightarrow \quad 6k + 2 = 4 + 4k$

$\Rightarrow \quad 6k - 4k = 4 - 2$

$\Rightarrow \quad 2k = 2$

$\Rightarrow \quad k = 1$

Hence, *P* divides *AB* in the ratio 1 : 1. **Ans.**

From (i), $\dfrac{-3(1) \quad 3}{1 \quad 1} = m$

$\Rightarrow \quad \dfrac{-3 \quad 3}{2} = m$

$\Rightarrow \quad m = 0$ **Ans.**

11. Two different dice are tossed together. Find the probability :

(i) of getting a doublet.

(ii) of getting a sum 10, of the numbers on the two dice. [2]

Solution : Total outcomes on tossing two different dice = 36

(i) *A* : getting a doublet

$A = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$\therefore$ Number of favourable outcomes of $A = 6$

$$\therefore \quad P(A) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{6}{36} = \frac{1}{6}$$ **Ans.**

(ii) *B* : getting a sum 10.

$B = \{(4, 6), (5, 5), (6, 4)\}$

$\therefore$ Number of favourable outcomes of B = 3

$$\therefore \quad P(B) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$\therefore \quad = \frac{3}{36} = \frac{1}{12}$$ **Ans.**

12. An integer is chosen at random between 1 and 100. Find the probability that it is :

(i) divisible by 8.

(ii) not divisible by 8. [2]

Solution : Total numbers are 2, 3, 4, ..., 99

(i) Let *E* be the event of getting a number divisible by 8.

$E = \{8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96\}$
$= 12$

$$P(E) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{12}{98} = 0.1224$$

(ii) Let E' be the event of getting a number not divisible by 8.

Then, $P(E') = 1 - P(E)$

$= 1 - 0.1224$

$= 0.8756$ **Ans.**

SECTION — C

13. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers. [3]

Solution :

2	404
2	202
101	101
	1

2	96
2	48
2	24
2	112
2	6
3	3
	1

Prime factorization of 404 = 2 × 2 × 101

Prime factorization of 96 = 2 × 2 × 2 × 2 × 2 × 3

∴ HCF = 2 × 2 = 4

And LCM = 2 × 2 × 2 × 2 × 2 × 3 × 101

= 9696

∴ HCF = 4, LCM = 9696 **Ans.**

Verification

HCF × LCM = Product of two numbers

4 × 9696 = 404 × 96

38784 = 38784 **Hence Verified.**

14. Find all zeroes of the polynomial $(2x^4 - 9x^3 + 5x^2 + 3x - 1)$ if two of its zeroes are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$. [3]

Solution : Here, $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

And two of its zeroes are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$.

Quadratic polynomial with zeroes is given by,

$\{x - (2 + \sqrt{3})\}.\{x - (2 - \sqrt{3})\}$

$\Rightarrow \quad (x - 2 - \sqrt{3})(x - 2 + \sqrt{3})$

$\Rightarrow \quad (x - 2)^2 - (\sqrt{3})^2$

$\Rightarrow \quad x^2 - 4x + 4 - 3$

$\Rightarrow \quad x^2 - 4x + 1 = g(x)$ (say)

Now, $g(x)$ will be a factor of $p(x)$ so $g(x)$ will be divisible by $p(x)$

$$
\begin{array}{r|l}
 & 2x^2 - x - 1 \\
\hline
x^2 - 4x + 1 & 2x^4 - 9x^3 + 5x^2 + 3x - 1 \\
 & \underset{-}{2x^4} \underset{+}{\mp} 8x^3 \underset{-}{\pm} 2x^2 \\
\hline
 & -x^3 + 3x^2 + 3x \\
 & \underset{+}{\mp} x^3 \underset{-}{\pm} 4x^2 \underset{+}{\mp} x \\
\hline
 & -x^2 + 4x - 1 \\
 & \underset{+}{\mp} x^2 \underset{-}{\pm} 4x \underset{+}{\mp} 1 \\
\hline
 & \times \\
\hline
\end{array}
$$

For other zeroes,

$2x^2 - x - 1 = 0$

$2x^2 - 2x + x - 1 = 0$

or $2x(x - 1) + 1(x - 1) = 0$

$(x - 1)(2x + 1) = 0$

$x - 1 = 0 \quad 2x + 1 = 0$

$x = 1, x = \frac{-1}{2}$

Zeroes of $p(x)$ are

$1, \frac{-1}{2}, 2 + \sqrt{3}$ and $2 - \sqrt{3}$. **Ans.**

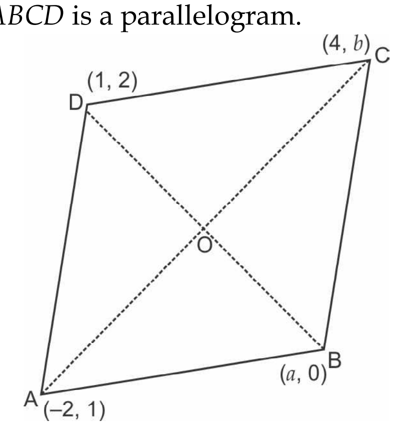
15. If A (–2, 1), B (a, 0), C (4, b) and D (1, 2) are the vertices of a parallelogram $ABCD$, find the values of a and b. Hence find the lengths of its sides. [3]

OR

If A (–5, 7), B (–4, –5), C (–1, –6) and D (4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral $ABCD$.

Solution :

Given, $ABCD$ is a parallelogram.



Midpoint of $AC = \frac{x_1 \quad x_2}{2}, \frac{y_1 \quad y_2}{2}$

$= \frac{-2 \quad 4}{2}, \frac{1 \quad b}{2}$

$= \frac{2}{2}, \frac{1 \quad b}{2}$

$= 1, \frac{1 \quad b}{2}$

Midpoint of $BD = \frac{x'_1 \quad x'_2}{2}, \frac{y'_1 \quad y'_2}{2}$

$= \frac{a \quad 1}{2}, \frac{0 \quad 2}{2}$

$= \frac{a \quad 1}{2}, \frac{2}{2}$

$= \frac{a \quad 1}{2}, 1$

Since, diagonals of a parallelogram bisect each other,

$$\therefore \quad 1, \frac{1 \quad b}{2} = \frac{a \quad 1}{2}, 1$$

On comparing, we get

$$\therefore \quad \frac{a \quad 1}{2} = 1 \quad \Bigg| \quad \frac{1 \quad b}{2} = 1$$

$$\Rightarrow \quad a + 1 = 2 \quad \Bigg| \quad \Rightarrow 1 + b = 2$$

$$\Rightarrow \quad a = 1 \quad \Bigg| \quad \Rightarrow b = 1$$

Ans.

Therefore, the coordinates of vertices of parallelogram $ABCD$ are $A(-2, 1)$, $B(1, 0)$, $C(4, 1)$ and $D(1, 2)$

Length of side $AB = DC = \sqrt{1 \quad 2^2 \quad 0 - 1^2}$

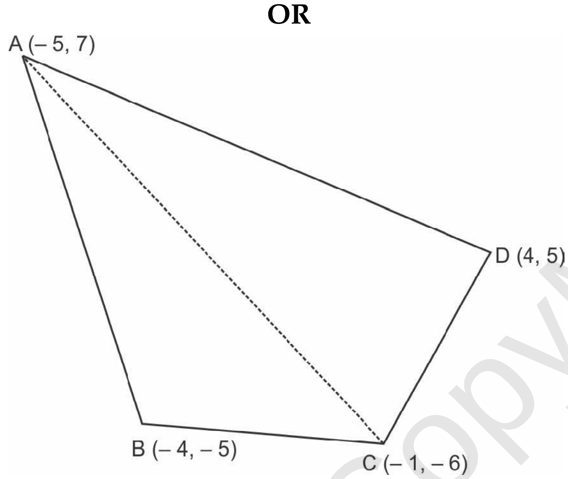
$$= \sqrt{9 \quad 1} = \sqrt{10} \text{ units}$$

And, $\quad AD = BC = \sqrt{1 \quad 2^2 \quad 2 - 1^2}$

$$= \sqrt{9 \quad 1} = \sqrt{10} \text{ units}$$

Ans.

OR



Given $ABCD$ is quadrilateral.

By joining points A and C, the quadrilateral is divided into two triangles.

Now, Area of quad. $ABCD$ = Area of $\Delta\, ABC$ + Area of $\Delta\, ACD$

Area of $\Delta\, ABC$

$$= \frac{1}{2}[x_1 \;\; y_2 - y_3 \quad x_2 \;\; y_3 - y_1 \quad x_3 \;\; y_1 - y_2 \;]$$

$$= \frac{1}{2}[-5 \;\; -5 \quad 6 \;\; -4 \;\; -6 - 7 \;\; -1 \;\; 7 \quad 5\;]$$

$$= \frac{1}{2}[-5(1) - 4(-13) - 1(12)]$$

$$= \frac{1}{2}(-5 \quad 52 - 12)$$

$$= \frac{1}{2}(35) = \frac{35}{2} \text{ sq. units.}$$

Area of $\Delta\, ADC$

$$= \frac{1}{2}[x_1 \;\; y_2 - y_3 \quad x_2 \;\; y_3 - y_1 \quad x_3 \;\; y_1 - y_2 \;]$$

$$= \frac{1}{2}[-5 \;\; 5 \quad 6 \quad 4 \;\; -6 - 7 \quad (-1) \;\; 7 - 5\;]$$

$$= \frac{1}{2}[-5(11) \quad 4(-13) - 1(2)]$$

$$= \frac{1}{2}(-55 \quad 52 - 12)$$

$$= \frac{1}{2}|-109| = \frac{109}{2} \text{ sq. units.}$$

Area of quadrilateral $ABCD$

$$= \frac{35}{2} \quad \frac{109}{2}$$

$$= \frac{144}{2} = 72 \text{ sq. units.}$$

Ans.

16. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed. [3]

Solution : Let the usual speed of plane be x km/h.

Increased speed = $(x + 100)$ km/h.

$\therefore$ Distance to cover = 1500 km.

Time taken by plane with usual speed = $\frac{1500}{x}$ hr.

Time taken by plane with increased speed $= \frac{1500}{(100 \quad x)}$ hrs.

According to the question,

$$\frac{1500}{x} - \frac{1500}{(100 \quad x)} = \frac{30}{60} = \frac{1}{2}$$

$$1500 \;\; \frac{1}{x} - \frac{1}{x \quad 100} = \frac{1}{2}$$

$$1500 \;\; \frac{x \quad 100 - x}{(x)(x \quad 100)} = \frac{1}{2}$$

$$\frac{1500 \quad 100}{x^2 \quad 100x} = \frac{1}{2}$$

$$x^2 + 100x = 300000$$

$$x^2 + 100x - 300000 = 0$$

$$x^2 + 600x - 500x - 300000 = 0$$

$$x(x + 600) - 500(x + 600) = 0$$

$$(x + 600)(x - 500) = 0$$

Either $\quad x + 600 = 0$

$x = -600 \quad$ (Rejected)

or $\quad x - 500 = 0$

$x = 500$

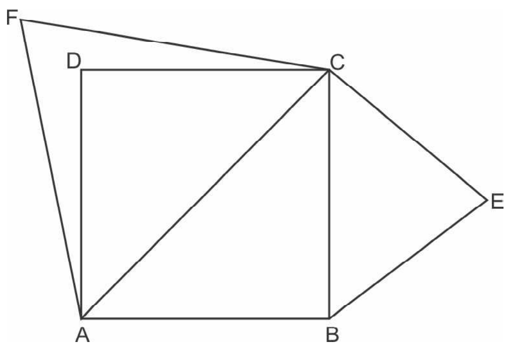
$\therefore$ Usual speed of plane = 500 km/hr. **Ans.**

17. Prove that the area of an equilateral triangle described on one side of the square is equal to half the area of the equilateral triangle described on one of its diagonal. [3]

OR

If the area of two similar triangles are equal, prove that they are congruent.

Solution : Let $ABCD$ be a square with side 'a'.



In ΔABC,

$$AC^2 = AB^2 + BC^2$$
$$= a^2 + a^2$$
$$= 2a^2$$
$$AC = \sqrt{2a^2} = \sqrt{2}\,a.$$

Area of equilateral ΔBEC (formed on side BC of square $ABCD$)

$$= \frac{\sqrt{3}}{4} \times (\text{side})^2$$
$$= \frac{\sqrt{3}}{4} a^2 \quad \text{....(i)}$$

Area of equilateral ΔACF (formed on diagonal AC of square $ABCD$)

$$= \frac{\sqrt{3}}{4} (\sqrt{2}\,a)^2$$
$$= \frac{\sqrt{3}}{4} (2a^2)$$
$$= 2\frac{\sqrt{3}}{4} a^2 \quad \text{....(ii)}$$

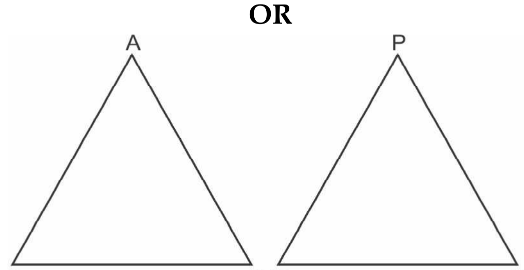
From eq. (i) and (ii),

$$\text{ar } \Delta ACF = 2 \times \text{ar } \Delta BCE$$

or $$\text{ar } (\Delta BCE) = \frac{1}{2} \text{ ar } (\Delta ACF)$$

i.e., area of triangle described on one side of square is half the area of triangle described on its diagonal. **Hence Proved.**

OR



Given, $\Delta ABC \sim \Delta PQR$

And $\text{ar } (\Delta ABC) = \text{ar } (\Delta PQR)$

To prove :

$$\Delta ABC \cong \Delta PQR$$

Proof :

Given, $\Delta ABC \sim \Delta PQR$

$$\therefore \quad \frac{\text{ar} (\ \ ABC)}{\text{ar} (\ \ PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

(Ratio of area of similar triangles is equal to the square of corresponding sides)

$$\text{But} \quad \frac{\text{ar} (\ \ ABC)}{\text{ar} (\ \ PQR)} = 1 \qquad \text{(Given)}$$

$$\therefore \quad \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} = 1$$

So, $AB^2 = PQ^2$ or $AB = PQ$

$BC^2 = QR^2$ or $BC = QR$

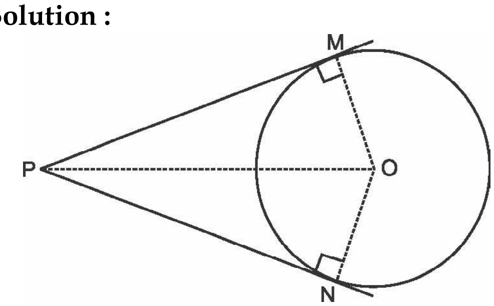
$AC^2 = QR^2$ or $AC = PR$

By SSS congruency axiom

$\Delta ABC \cong \Delta PQR$ **Hence Proved.**

18. Prove that the lengths of tangents drawn from an external point to a circle are equal. [3]

Solution :



Given : a circle with centre O on which two tangents PM and PN are drawn from an external point P.

To prove :

$$PM = PN$$

Construction : Join OM, ON and OP.

Proof : Since tangent and radius are perpendicular at point of contact,

$\therefore \quad \angle OMP = \angle ONP = 90°$

In ΔPOM and ΔPON,

$OM = ON$ (Radii)

$\angle OMP = \angle ONP$

$PO = OP$ (Common)

$\therefore \quad \Delta OMP \cong \Delta ONP$ (RHS cong.)

$\therefore \quad PM = PN$ (CPCT)

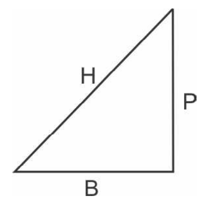
Hence Proved.

19. If 4 tan θ = 3, evaluate $\frac{4\sin \quad -\cos \quad 1}{4\sin \quad \cos \quad -1}$ **[3]**

OR

If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.

Solution : Given, 4 tan θ = 3,



$$\Rightarrow \qquad \tan\theta = \frac{3}{4} \qquad \frac{P}{B}$$

P = 3K, B = 4K,

Now,
$$H = \sqrt{P^2 \quad B^2}$$
$$= \sqrt{3K^{\ 2} \quad 4K^{\ 2}}$$
$$= \sqrt{9K^2 \quad 16K^2}$$
$$= \sqrt{25K^2}$$
$$\Rightarrow \qquad H = 5K$$
$$\therefore \qquad \sin\theta = \frac{P}{H} = \frac{3K}{5K} = \frac{3}{5}$$
and
$$\cos\theta = \frac{B}{H} = \frac{4K}{5K} = \frac{4}{5}$$

Now,
$$\frac{4\sin \quad -\cos \quad +1}{4\sin \quad +\cos \quad -1} = \frac{4 \quad \frac{3}{5} - \frac{4}{5} + 1}{4 \times \frac{3}{5} \quad \frac{4}{5} - 1}$$
$$= \frac{\frac{12}{5} - \frac{4}{5} \quad 1}{\frac{12}{5} \quad \frac{4}{5} - 1}$$
$$= \frac{\frac{12 - 4 \quad 5}{5}}{\frac{12 \quad 4 - 5}{5}}$$
$$= \frac{13/5}{11/5}$$
$$= \frac{13}{11} \qquad \textbf{Ans.}$$

OR

Given,
$$\tan 2A = \cot(A - 18°)$$
$$\Rightarrow \quad \cot(90° - 2A) = \cot(A - 18°)$$
$$[\because \tan\theta = \cot(90° - \theta)]$$
$$\Rightarrow \quad 90° - 2A = A - 18°$$
$$\Rightarrow \quad 90° + 18° = A + 2A$$
$$\Rightarrow \quad 108° = 3A$$
$$\Rightarrow \quad A = \frac{108}{3}$$
$$\Rightarrow \quad A = 36° \qquad \textbf{Ans.}$$

20. Find the area of the shaded region in Fig. 2, where arcs drawn with centres *A*, *B*, *C* and *D* intersect in pairs at mid-points *P*, *Q*, *R* and *S* of the sides *AB*, *BC*, *CD* and *DA* respectively of a square *ABCD* of side 12 cm. [Use π = 3.14] [3]

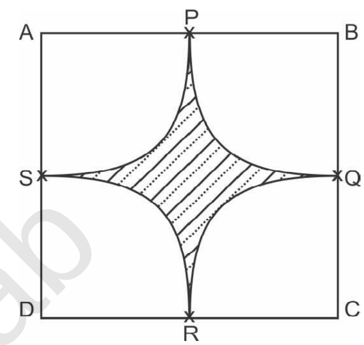
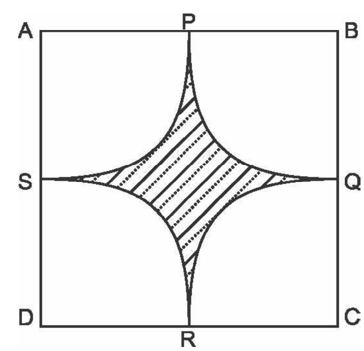


Figure 2

Solution :



Given, *ABCD* is a square of side 12 cm.

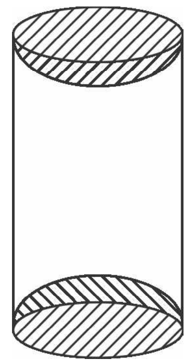
P, Q, R and *S* are the mid points of sides *AB*, *BC*, *CD* and *AD* respectively.

Area of shaded region

= Area of square – 4 × Area of quadrant

$$= a^2 - 4 \times \frac{1}{4}\pi r^2$$
$$= (12)^2 - 3.14 \times (6)^2$$
$$= 144 - 3.14 \times 36$$
$$= 144 - 113.04$$
$$= 30.96 \text{ cm}^2 \qquad \textbf{Ans.}$$

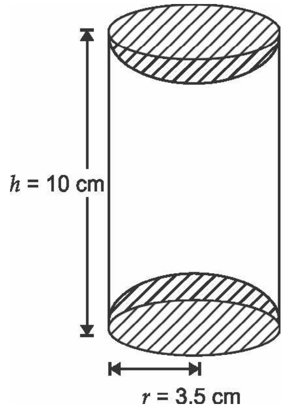
21. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. 3. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface area of the article. [3]



OR

A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap ?

Solution :



Given, Radius (r) of cylinder = Radius of hemisphere = 3.5 cm.

Total SA of article = CSA of cylinder + 2 × CSA of hemisphere

Height of cylinder, h = 10 cm

$$\text{TSA} = 2\pi rh + 2 \times 2\pi r^2$$
$$= 2\pi rh + 4\pi r^2$$
$$= 2\pi r(h + 2r)$$
$$= 2 \times \frac{22}{7} \times 3.5\,(10 + 2 \times 3.5)$$
$$= 2 \times 22 \times 0.5 \times (10 + 7)$$
$$= 2 \times 11 \times 17$$
$$= 374 \text{ cm}^2$$

OR

Base diameter of cone = 24 m.

$\therefore$ Radius r = 12 m

Height of cone, h = 3.5 m

Volume of rice in conical heap

$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5$$
$$= 528 \text{ m}^3$$

Now, slant height, $l = \sqrt{h^2 + r^2}$

$$= \sqrt{3.5^2 + 12^2}$$
$$= \sqrt{12.25 + 144}$$
$$= \sqrt{156.25}$$
$$= 12.5 \text{ m}$$

Canvas cloth required to just cover the heap = CSA of conical heap = πrl

$$= \frac{22}{7} \times 12 \times 12.5$$
$$= \frac{3300}{7} \text{ m}^2$$
$$= 471.43 \text{ m}^2.$$

Ans.

22. The table below shows the salaries of 280 persons : [3]

Salary (In thousand ₹)	No. of Persons
5 – 10	49
10 – 15	133
15 – 20	63
20 – 25	15
25 – 30	6
30 – 35	7
35 – 40	4
40 – 45	2
45 – 50	1

Calculate the median salary of the data.

Solution :

Salary	No. of Persons	Cumulative frequency (*c.f.*)
5 – 10	49	49
l 10 – 15	133 f	182
15 – 20	63	245
20 – 25	15	260
25 – 30	6	266
30 – 35	7	273
35 – 40	4	277
40 – 45	2	279
45 – 50	1	280
Total	280	

$$\frac{N}{2} = \frac{280}{2} = 140$$

The cumulative frequency just greater than 140 is 182.

$\therefore$ Median class is 10 – 15.

$\Rightarrow l = 10, h = 5, N = 280, c.f. = 49$ and $f = 133$

$$\text{Median} = l + \frac{\frac{N}{2} - c.f.}{f} \times h$$

$$= 10 + \frac{140 - 49}{133} \times 5$$

$$= 10 + \frac{91 \quad 5}{133}$$

$$= 10 + \frac{455}{133}$$

$$= 10 + 3.42$$

$$= 13.42$$ **Ans.**

SECTION — D

23. A motor boat whose speed is 18 km/hr in still water takes 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. [4]

OR

A train travels at a certain average speed for a distance of 63 km and then travels at a distance of 72 km at an average speed of 6 km/hr more than its original speed. If it takes 3 hours to complete total journey, what is the original average speed ?

Solution : Given, speed of motor boat in still water = 18 km/hr.

Let speed of stream = x km/hr.

$\therefore$ Speed of boat downstream = $(18 + x)$ km/hr.

And speed of boat upstream = $(18 - x)$ km/hr.

Time of the upstream journey $= \dfrac{24}{18 - x}$

Time of the downstream journey $= \dfrac{24}{18 \quad x}$

According to the question,

$$\frac{24}{18 - x} - \frac{24}{18 \quad x} = 1$$

$$\frac{24(18 \quad x) - 24(18 - x)}{18 - x \quad 18 \quad x} = 1$$

$$\frac{24 \quad 18 \quad 24x - 24 \quad 18 \quad 24x}{324 - x^2} = 1$$

$$\frac{48x}{324 - x^2} = 1$$

$$48x = 324 - x^2$$

$\Rightarrow$ $x^2 + 48x - 324 = 0$

$\Rightarrow$ $x^2 + 54x - 6x - 324 = 0$

$\Rightarrow$ $x(x + 54) - 6(x + 54) = 0$

$\Rightarrow$ $(x + 54)(x - 6) = 0$

Either $x + 54 = 0$

$x = -54$

Rejected, as speed cannot be negative

or $x - 6 = 0$

$x = 6$

Thus, the speed of the stream is 6 km/hr. **Ans.**

OR

Let original average speed of train be x km/hr.

$\therefore$ Increased speed of train = $(x + 6)$ km/hr.

Time taken to cover 63 km with average speed $= \dfrac{63}{x}$ hr.

Time taken to cover 72 km with increased speed $= \dfrac{72}{(x \quad 6)}$ hr.

According to the question,

$$\frac{63}{x} \quad \frac{72}{x \quad 6} = 3$$

$\Rightarrow$ $$\frac{63(x \quad 6) \quad 72(x)}{(x)(x \quad 6)} = 3$$

$\Rightarrow$ $$\frac{63x \quad 378 \quad 72x}{x^2 \quad 6x} = 3$$

$\Rightarrow$ $135x + 378 = 3(x^2 + 6x)$

$\Rightarrow$ $135x + 378 = 3x^2 + 18x$

$\Rightarrow$ $3x^2 + 18x - 135x - 378 = 0$

$\Rightarrow$ $3x^2 - 117x - 378 = 0$

$\Rightarrow$ $3(x^2 - 39x - 126) = 0$

$\Rightarrow$ $x^2 - 39x - 126 = 0$

$\Rightarrow$ $x^2 - 42x + 3x - 126 = 0$

$\Rightarrow$ $x(x - 42) + 3(x - 42) = 0$

$\Rightarrow$ $(x - 42)(x + 3) = 0$

Either $x - 42 = 0$

$x = 42$

or $x + 3 = 0$

$x = -3$

Rejected (as speed cannot be negative)

Thus, average speed of train is 42 km/hr. **Ans.**

24. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7 : 15. Find the numbers. [4]

Solution: Let the first term of AP be a and d be the common difference.

Let four consecutive terms of an AP be $a-3d$, $a - d$, $a + d$ and $a + 3d$

According to the question,

$a - 3d + a - d + a + d + a + 3d = 32$

$\Rightarrow$ $4a = 32$

$\Rightarrow$ $a = 8$ ...(i)

Also,

$$(a - 3d)(a + 3d) : (a - d)(a + d) = 7 : 15$$

$$\frac{a^2 - 9d^2}{a^2 - d^2} = \frac{7}{15}$$

$$\frac{64 - 9d^2}{64 - d^2} = \frac{7}{15}$$

[From (i) put $a = 8$]

$$15(64 - 9d^2) = 7(64 - d^2)$$

$$960 - 135d^2 = 448 - 7d^2$$

$$960 - 448 = 135d^2 - 7d^2$$

$$512 = 128d^2$$

$$d^2 = \frac{512}{128}$$

$$d^2 = 4$$

$$\Rightarrow \quad d = \pm 2$$

For $d = 2$, four terms of AP are,

$$a - 3d = 8 - 3(2) = 2$$
$$a - d = 8 - 2 = 6$$
$$a + d = 8 + 2 = 10$$
$$a + 3d = 8 + 3(2) = 14$$

For $d = -2$, four terms are

$$a - 3d = 8 - 3(-2) = 14$$
$$a - d = 8 - (-2) = 10$$
$$a + d = 8 + (-2) = 6$$
$$a + 3d = 8 + 3(-2) = 2$$

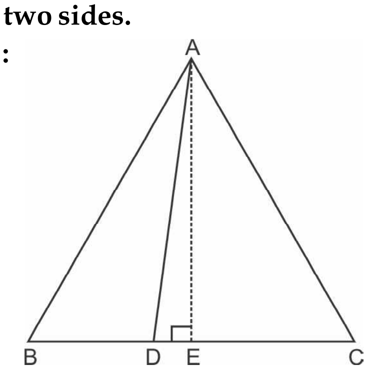
Thus, the four terms of AP series are 2, 6, 10, 14 or 14, 10, 6, 2. **Ans.**

25. In an equilateral $\Delta\, ABC$, D is a point on side BC such that $BD = \frac{1}{3} BC$. Prove that $9(AD)^2 = 7(AB)^2$. **[4]**

OR

Prove that, in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Solution :



Given, ABC is an equilateral triangle and D is a point on BC such that $BD = \frac{1}{3} BC$.

To prove :

$$9AD^2 = 7AB^2$$

Construction : Draw $AE \perp BC$

Proof : $BD = \frac{1}{3} BC$ ...(i)(Given)

$AE \perp BC$

We know that perpendicular from a vertex of equilateral triangle to the base divides base in two equal parts.

$$\therefore \quad BE = EC = \frac{1}{2} BC \quad \text{...(ii)}$$

In ΔAEB,

$$AD^2 = AE^2 + DE^2$$

(Pythagoras theorem)

$$\text{or} \quad AE^2 = AD^2 - DE^2 \quad \text{...(iii)}$$

Similarly, In ΔAEB,

$$AB^2 = AE^2 + BE^2$$

$$= AD^2 - DE^2 + \left(\frac{1}{2}BC\right)^2 \quad \text{[From (ii) and (iii)]}$$

$$= AD^2 - (BE - BD)^2 + \frac{1}{4}BC^2$$

$$= AD^2 - BE^2 - BD^2 + 2.BE.BD + \frac{1}{4}BC^2$$

$$= AD^2 - \left(\frac{1}{2}BC\right)^2 - \left(\frac{1}{3}BC\right)^2 + 2.\frac{1}{2}BC.\frac{1}{3}BC + \frac{1}{4}BC^2$$

$$AB^2 = AD^2 - \frac{1}{9}BC^2 + \frac{1}{3}BC^2$$

$$\Rightarrow \quad AB^2 = AD^2 + \frac{2}{9}BC^2$$

$$\Rightarrow \quad AB^2 = AD^2 + \frac{2}{9}AB^2 \quad (\because BC = AB)$$

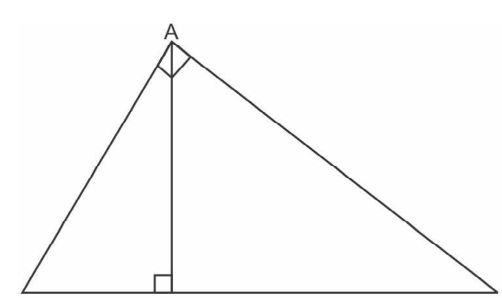
$$\Rightarrow \quad AB^2 - \frac{2}{9}AB^2 = AD^2$$

$$\Rightarrow \quad \frac{7}{9}AB^2 = AD^2$$

$$\Rightarrow \quad 7AB^2 = 9AD^2$$

Or $9(AD)^2 = 7(AB)^2$ **Hence Proved.**

OR



Given : ΔABC is a right angle triangle, right angled at A.

To prove : $BC^2 = AB^2 + AC^2$

Construction : Draw $AD \perp BC$.

Proof : In ΔADB and ΔBAC,

$$\angle B = \angle B \qquad \text{(Common)}$$
$$\angle ADB = \angle BAC \qquad \text{(Each 90°)}$$
$$\therefore \quad \Delta ADB \sim \Delta BAC$$
(By AA similarity axiom)
$$\therefore \quad \frac{AB}{BC} = \frac{BD}{AB} \qquad \text{(CPCT)}$$
$$AB^2 = BC \times BD \qquad \text{...(i)}$$

Similarly,

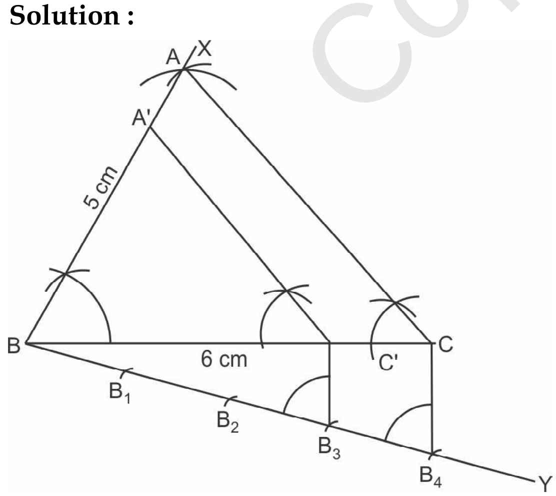
$$\Delta ADC \sim \Delta CAB$$
$$\frac{AC}{BC} = \frac{DC}{AC}$$
$$AC^2 = BC \times DC \qquad \text{...(ii)}$$

On adding eq. (i) and (ii)

$$AB^2 + AC^2 = BC \times BD + BC \times CD$$
$$= BC\,(BD + CD)$$
$$= BC \times BC$$
$$AB^2 + AC^2 = BC^2$$
$$\Rightarrow \quad BC^2 = AB^2 + AC^2$$
Hence Proved.

26. Draw a triangle ABC with $BC = 6$ cm, $AB = 5$ cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the ΔABC. **[4]**

Solution :



Steps of construction—

(i) Draw a line segment $BC = 6$ cm.

(ii) Construct $\angle XBC = 60°$.

(iii) With B as centre and radius equal to 5 cm, draw an arc intersecting XB at A.

(iv) Join AC. Thus, ΔABC is obtained.

(v) Draw an acute angle $\angle CBY$ below of B.

(vi) Mark 4-equal parts on BY as B_1, B_2, B_3 and B_4.

(vii) Join B_4 to C.

(viii) From B_3, draw a line parallel to B_4C intersecting BC at C'.

(ix) Draw another line parallel to CA from C′, intersecting AB at A.′

(x) $\Delta A'BC'$ is required triangle which is similar to ΔABC such that $BC' = \frac{3}{4} BC$.

27. Prove that : $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$. **[4]**

Solution : L.H.S. $= \frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A}$

$$= \frac{\sin A \;\; 1 - 2\sin^2 A}{\cos A\,(2\cos^2 A - 1)}$$

$$= \frac{\sin A}{\cos A} \frac{(1 - 2\sin^2 A)}{[2(1 - \sin^2 A) - 1]}$$

$[\because \cos^2 A = 1 - \sin^2 A]$

$$= \frac{\sin A}{\cos A} \frac{(1-2\sin^2 A)}{(2-2\sin^2 A-1)}$$

$$= \frac{\sin A}{\cos A} \frac{(1 - 2\sin^2 A)}{(1 - 2\sin^2 A)}$$

$= \tan A =$ R.H.S. **Hence Proved.**

28. The diameters of the lower and upper ends of a bucket in the form of a frustum of a cone are 10 cm and 30 cm respectively. If its height is 24 cm, find :

(i) The area of the metal sheet used to make the bucket.

(ii) Why we should avoid the bucket made by ordinary plastic ? [Use $\pi = 3.14$] **[4]**

Solution : Given, Height of frustum, $h = 24$ cm.

Diameter of lower end $= 10$ cm.

$\therefore$ Radius of lower end, $r = 5$ cm.

Diameter of upper end $= 30$ cm.

$\therefore$ Radius of upper end, $R = 15$ cm.

Slant height, $l = \sqrt{h^2 + (R - r)^2}$

$$= \sqrt{(24)^2 + (15 - 5)^2}$$
$$= \sqrt{576 + 100}$$
$$= \sqrt{676}$$
$$= 26 \text{ cm}$$

(i) Area of metal sheet used to make the bucket

$=$ CSA of frustum + Area of base

$= \pi l\,(R + r) + \pi r^2$

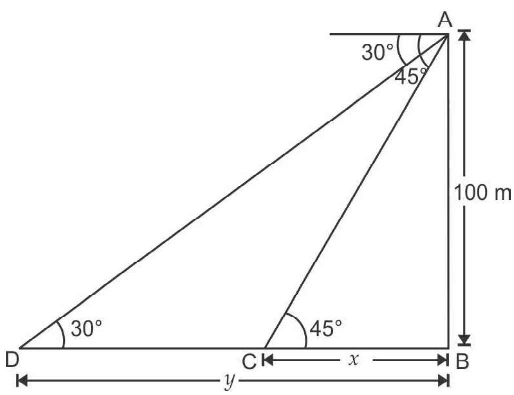
$= \pi[26\,(15 + 5) + (5)^2]$

$= 3.14\ (26 \times 20 + 25)$
$= 3.14\ (520 + 25)$
$= 3.14 \times 545$
$= 1711.3\ cm^2$ **Ans.**

(ii) We should avoid the bucket made by ordinary plastic because plastic is harmful to the environment and to protect the environment its use should be avoided.

29. As observed from the top of a 100 m high light house from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. [Use $\sqrt{3} = 1.732$] [4]

Solution :



Let AB be the light house and two ships be at C and D.

In $\Delta\ ABC$,

$$\frac{BC}{AB} = \cot 45°$$

$$\Rightarrow \quad \frac{x}{100} = 1$$

$$\Rightarrow \quad x = 100 \quad ...(i)$$

Similarly, in $\Delta\ ABD$,

$$\frac{BD}{AB} = \cot 30°$$

$$\Rightarrow \quad \frac{y}{100} = \sqrt{3}$$

$$\Rightarrow \quad y = 100\sqrt{3} \quad ...(ii)$$

Distance between two ships $= y - x$

$= 100\sqrt{3} - 100$ [from (i) and (ii)]

$= 100\ (\sqrt{3} - 1)$

$= 100\ (1.732 - 1)$

$= 100\ (0.732)$

$= 73.2$ m **Ans.**

30. The mean of the following distribution is 18. Find the frequency f of the class 19–21.

Class	**11 – 13**	**13 – 15**	**15 – 17**	**17 – 19**	**19 – 21**	**21 – 23**	**23 – 25**
Frequency	**3**	**6**	**9**	**13**	***f***	**5**	**4**

[4]

OR

The following distribution gives the daily income of 50 workers of a factory :

Daily Income (in ₹)	**100 – 120**	**120 – 140**	**140 – 160**	**160 – 180**	**180 – 200**
Number of workers	**12**	**14**	**8**	**6**	**10**

Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

Solution :

C.I.	Mid value x_i	f_i	$f_i x_i$
11–13	12	3	36
13–15	14	6	84
15–17	16	9	144
17–19	18	13	234
19–21	20	f	20f
21–23	22	5	110
23–25	24	4	96
Total		$\Sigma f_i = 40 + f$	$\Sigma f_i x_i = 704 + 20f$

Now, Mean = 18 (Given)

$$\Rightarrow \quad \frac{f_i x_i}{f_i} = 18$$

$$\therefore \quad \frac{704 \quad 20f}{40 \quad f} = 18$$

$$\Rightarrow \quad 704 + 20f = 18\ (40 + f)$$

$$\Rightarrow \quad 704 + 20f = 720 + 18f$$

$$\Rightarrow \quad 20f - 18f = 720 - 704$$

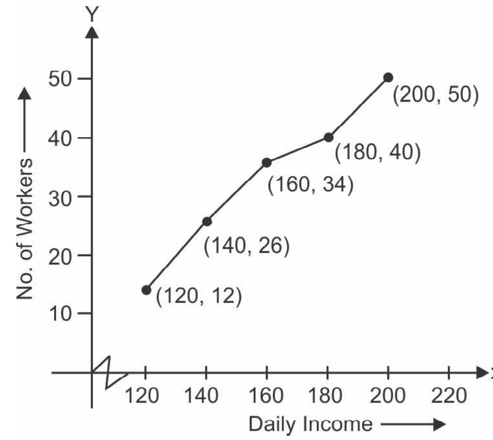
$$\Rightarrow \quad 2f = 16$$

$$\Rightarrow \quad f = 8$$ **Ans.**

OR

Less than type cumulative frequency distribution :

Daily Income	No. of workers
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50



Time allowed : 3 Hours **Maximum marks : 90**

SECTION — A

1. What is the common difference of an A.P. in which $a_{21} - a_7 = 84$? **[1]**

Solution : Given, $a_{21} - a_7 = 84$

$\Rightarrow \quad (a + 20d) - (a + 6d) = 84$

$\Rightarrow \quad a + 20d - a - 6d = 84$

$\Rightarrow \quad 20d - 6d = 84$

$\Rightarrow \quad 14d = 84$

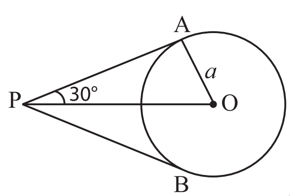
$$\Rightarrow \quad d = \frac{84}{14} = 6$$

Hence common difference = 6. **Ans.**

2. If the angle between two tangents drawn from an external point *P* to a circle of radius *a* and centre *O*, is 60°, then find the length of *OP*. **[1]**

Solution : Given, $\angle APB = 60°$

$\Rightarrow \quad \angle APO = 30°$



In right angle Δ *OAP*,

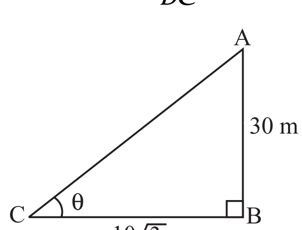
$$\frac{OP}{OA} = \text{cosec } 30°$$

$$\frac{OP}{a} = 2 \Rightarrow OP = 2a$$ **Ans.**

3. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun ? **[1]**

Solution : In Δ *ABC*,

$$\tan\theta = \frac{AB}{BC}$$



$$\tan\theta = \frac{30}{10\sqrt{3}} = \sqrt{3}$$

$\tan\theta = \tan 60° \quad \Rightarrow \quad \theta = 60°$

Hence angle of elevation is 60°. **Ans.**

4. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0·18. What is the number of rotten apples in the heap ? **[1]**

Solution : Total apples = 900

$P(E) = 0{\cdot}18$

$$\frac{\text{No. of rotten apples}}{\text{Total no. of apples}} = 0{\cdot}18$$

$$\frac{\text{No. of rotten apples}}{900} = 0{\cdot}18$$

No. of rotten apples $= 900 \times 0{\cdot}18$

$= 162$ **Ans.**

SECTION — B

5. Find the value of *p*, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other. **[2]**

Solution : Given, equation is $px^2 - 14x + 8 = 0$

Let one root $= \alpha$,

then other root $= 6\alpha$

$$\text{Sum of roots} = -\frac{b}{a};$$

$$\alpha + 6\alpha = \frac{-(-14)}{p}$$

$$7\alpha = \frac{14}{p};$$

$$\alpha = \frac{14}{p \times 7}$$

or $$\alpha = \frac{2}{p} \quad \text{...(i)}$$

$$\text{Product of roots} = \frac{c}{a}$$

$$(\alpha)(6\alpha) = \frac{8}{p}$$

$$6\alpha^2 = \frac{8}{p} \quad \text{...(ii)}$$

Putting value of α from eq. (i)

$$6\left(\frac{2}{p}\right)^2 = \frac{8}{p}$$

$$\Rightarrow \quad 6 \times \frac{4}{p^2} = \frac{8}{p}$$

$$\Rightarrow \quad 24p = 8p^2$$

$$\Rightarrow \quad 8p^2 - 24p = 0$$

$$\Rightarrow \quad 8p\,(p-3) = 0$$

$$\Rightarrow \quad \text{Either } 8p = 0 \Rightarrow p = 0$$

$$\text{or} \quad p - 3 = 0 \Rightarrow p = 3$$

For $p = 0$, given condition is not satisfied

$$\therefore \quad p = 3$$ **Ans.**

6. Which term of the progression 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$, ... is the first negative term ? **[2]**

Solution : Given, A.P. is 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$

$$= 20, \frac{77}{4}, \frac{37}{2}, \frac{71}{4}, \ldots$$

Here, $a = 20$, $d = \frac{77}{4} - 20 = \frac{77-80}{4} = \frac{-3}{4}$

Let a_n is first negative term

$$\Rightarrow \quad a_n + (n-1)d < 0$$

$$\Rightarrow \quad 20 + (n-1)\left(-\frac{3}{4}\right) < 0$$

$$\Rightarrow \quad 20 - \frac{3}{4}n + \frac{3}{4} < 0$$

$$\Rightarrow \quad 20 + \frac{3}{4} < \frac{3}{4}n$$

$$\Rightarrow \quad \frac{83}{4} < \frac{3}{4}n$$

$$\Rightarrow \quad n > \frac{83}{4} \times \frac{4}{3}$$

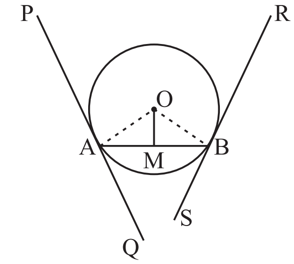
$$\Rightarrow \quad n > \frac{83}{3} = 27.66$$

28^{th} term will be first negative term of given A.P.

Ans.

7. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord. **[2]**

Solution : Given, a circle of radius OA and centred at O with chord AB and tangents PQ & RS are drawn from point A and B respectively.



Draw $OM \perp AB$, and join OA and OB.

In $\Delta\, OAM$ and $\Delta\, OMB$,

$$OA = OB \quad \text{(Radii)}$$

$$OM = OM \quad \text{(Common)}$$

$$\angle OMA = \angle OMB \quad \text{(Each } 90^\circ)$$

$$\therefore \quad \Delta\, OAM \cong \Delta\, OMB \quad \text{(R.H.S. cong.)}$$

$$\therefore \quad \angle OAM = \angle OBM \quad \text{(CPCT)}$$

Also, $\angle OAP = \angle OBR = 90^\circ$ (Line joining point of contact of tangent to centre is perpendicular on it)

On addition,

$$\angle OAM + \angle OAP = \angle OBM + \angle OBR$$

$$\Rightarrow \quad \angle PAB = \angle RBA$$

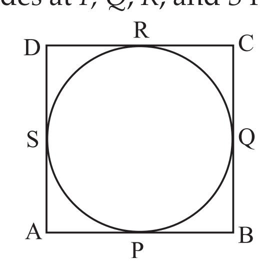
$$\Rightarrow \quad \angle PAQ - \angle PAB = \angle RBS - \angle RBA$$

$$\Rightarrow \quad \angle QAB = \angle SBA$$ **Hence Proved.**

8. A circle touches all the four sides of a quadrilateral *ABCD*. Prove that

$$\boldsymbol{AB + CD = BC + DA}$$ **[2]**

Solution : Given, a quad. $ABCD$ and a circle touches its all four sides at P, Q, R, and S respectively.



To prove : $AB + CD = BC + DA$

$$\text{L.H.S.} = AB + CD$$

$$= AP + PB + CR + RD$$

$$= AS + BQ + CQ + DS$$

(Tangents from same external point are always equal)

$$= (AS + SD) + (BQ + QC)$$

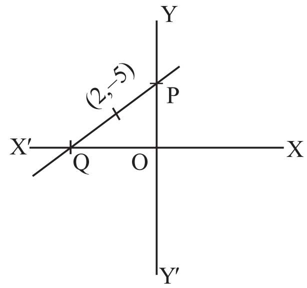
$$= AD + BC$$

$$= \text{R.H.S.}$$ **Hence Proved.**

9. A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, – 5) is the mid-point of PQ, then find the co-ordinates of P and Q. **[2]**

Solution : Let co-ordinate of P $(0, y)$

Co-ordinate of Q $(x, 0)$



Mid-point is (2, – 5)

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (2, -5)$$

$$\Rightarrow \quad \left(\frac{x+0}{2}, \frac{0+y}{2}\right) = (2, -5)$$

$$\Rightarrow \quad \frac{x}{2} = 2\,; \quad \frac{y}{2} = -5$$

$$\Rightarrow \quad x = 4\,; \quad y = -10$$

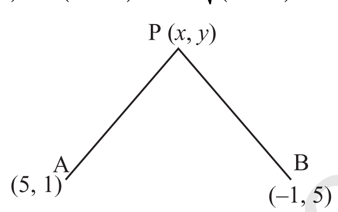
Co-ordinate of P (0, – 10)

Co-ordinate of Q (4, 0) **Ans.**

10. If the distances of $P(x, y)$, from $A(5, 1)$ and $B(-1, 5)$ are equal, then prove that $3x = 2y$. [2]

Solution : Given, $PA = PB$

$$\Rightarrow \quad \sqrt{(x-5)^2+(y-1)^2} = \sqrt{(x+1)^2+(y-5)^2}$$



Squaring both sides

$$(x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2$$

$$\Rightarrow x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 1 + 2x + y^2 + 25 - 10y$$

$$\Rightarrow \quad -10x - 2y = 2x - 10y$$

$$\Rightarrow \quad -10x - 2x = -10y + 2y$$

$$\Rightarrow \quad 12x = 8y$$

$$\Rightarrow \quad 3x = 2y$$ **Hence Proved.**

SECTION — C

11. If $ad \neq bc$, then prove that the equation $(a^2 + b^2)\,x^2 + 2\,(ac + bd)\,x + (c^2 + d^2) = 0$ has no real roots. [3]

Solution : Given, $ad \neq bc$

$$(a^2 + b^2)\,x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$$

$$\begin{aligned} D &= b^2 - 4ac \\ &= [2\,(ac + bd)]^2 - 4\,(a^2 + b^2)\,(c^2 + d^2) \\ &= 4\,[a^2c^2 + b^2d^2 + 2abcd] - 4\,(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) \\ &= 4\,[a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2] \\ &= 4\,[-a^2d^2 - b^2c^2 + 2abcd] \\ &= -4\,[a^2d^2 + b^2c^2 - 2abcd] \\ &= -4\,[ad - bc]^2 \end{aligned}$$

D is negative

Hence given equation has no real roots. **Hence Proved.**

12. The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P. [3]

Solution : Given, $a = 5$, $a_n = 45$, $S_n = 400$

We have, $$S_n = \frac{n}{2}\,[a + a_n]$$

$$\Rightarrow \quad 400 = \frac{n}{2}\,[5 + 45]$$

$$\Rightarrow \quad 400 = \frac{n}{2}\,[50]$$

$$\Rightarrow \quad 25n = 400 \Rightarrow n = \frac{400}{25}$$

$$\Rightarrow \quad n = 16$$

Now, $$a_n = a + (n-1)\,d$$

$$\Rightarrow \quad 45 = 5 + (16-1)\,d$$

$$\Rightarrow \quad 45 - 5 = 15d$$

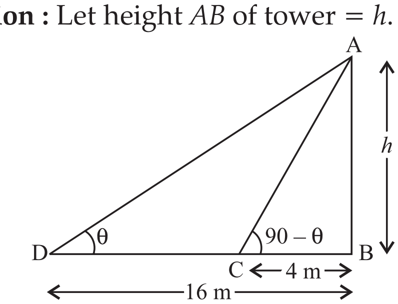
$$\Rightarrow \quad 15d = 40$$

$$\Rightarrow \quad d = \frac{8}{3}$$

So, $$n = 16 \text{ and } d = \frac{8}{3}$$ **Ans.**

13. On a straight line passing through the foot of a tower, two points C and D are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from C and D of the top of the tower are complementary, then find the height of the tower. [3]

Solution : Let height AB of tower = h.



In Δ ABC,

$$\frac{AB}{BC} = \tan\,(90 - \theta)$$

$$\frac{h}{4} = \cot\theta \qquad \text{....(i)}$$

In Δ ABD,

$$\frac{AB}{BC} = \tan\theta$$

$$\frac{h}{16} = \tan\theta \qquad ...(ii)$$

Multiply eq. (i) and (ii)

$$\frac{h}{4}\times\frac{h}{16} = \cot\theta \times \tan\theta$$

$$\frac{h^2}{64} = 1$$

$$\left[\because \cot\theta \times \tan\theta = \frac{1}{\tan\theta} \times \tan\theta = 1\right]$$

$\Rightarrow h^2 = 64 \Rightarrow h = 8$ m

Height of tower = 8 m. **Ans.**

14. A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag. [3]

Solution : Given, no. of white balls = 15

Let no. of black balls $= x$

$\therefore$ Total balls $= (15 + x)$

According to the question,

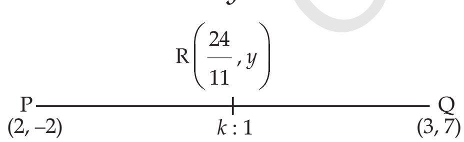
$$P \text{ (Black ball)} = 3 \times P \text{ (White ball)}$$

$$\Rightarrow \quad \frac{x}{(15+x)} = 3\times\frac{15}{(15+x)}$$

$$\Rightarrow \quad x = 45$$

$\therefore$ No. of black balls in bag = 45 **Ans.**

15. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points $P(2, -2)$ and $Q(3, 7)$? Also find the value of y. [3]



Solution : Let point R divides PQ in the ratio $k : 1$

$$R = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$\Rightarrow \quad \left(\frac{24}{11}, y\right) = \left(\frac{k(3)+1(2)}{k+1}, \frac{k(7)+1(-2)}{k+1}\right)$$

$$= \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$$

$$\Rightarrow \quad \frac{3k+2}{k+1} = \frac{24}{11}$$

$$\Rightarrow \quad 11(3k+2) = 24(k+1)$$

$$\Rightarrow \quad 33k + 22 = 24k + 24$$

$$\Rightarrow \quad 33k - 24k = 24 - 22$$

$$\Rightarrow \quad 9k = 2 \Rightarrow k = 2/9$$

$$\therefore \quad k : 1 = 2 : 9$$

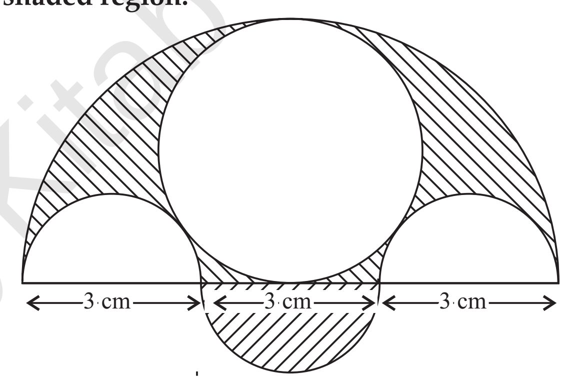
Now,
$$y = \frac{7k-2}{k+1} = \frac{7\left(\frac{2}{9}\right)-2}{\frac{2}{9}+1}$$

$$= \frac{\frac{14}{9}-2}{\frac{2}{9}+1} = \frac{\frac{14-18}{9}}{\frac{2+9}{9}} = \frac{-4}{11}$$

Line PQ divides in the ratio 2 : 9 and value of $y = \frac{-4}{11}$

Ans.

16. Three semicircles each of diameter 3 cm, a circle of diameter 4·5 cm and a semicircle of radius 4·5 cm are drawn in the given figure. Find the area of the shaded region. [3]



Solution : Given, radius of large semi-circle = 4·5 cm

$$\text{Area of large semi-circle} = \frac{1}{2}\pi R^2$$

$$= \frac{1}{2}\times\frac{22}{7} \times 4{\cdot}5 \times 4{\cdot}5$$

Diameter of inner circle = 4·5 cm

$$\Rightarrow \quad r = \frac{4{\cdot}5}{2} \text{ cm}$$

$$\text{Area of inner circle} = \pi r^2$$

$$= \frac{22}{7}\times\frac{4{\cdot}5}{2}\times\frac{4{\cdot}5}{2}$$

Diameter of small semi-circle = 3 cm

$$r = \frac{3}{2} \text{ cm}$$

$$\text{Area of small semi-circle} = \frac{1}{2}\pi r^2$$

$$= \frac{1}{2}\times\frac{22}{7}\times\frac{3}{2}\times\frac{3}{2}$$

Area of shaded region

= Area of large semi circle + Area of 1 small semi-circle – Area of inner circle – Area of 2 small semi circle

$$= \frac{1}{2} \times \frac{22}{7} \times 4{\cdot}5 \times 4{\cdot}5 + \frac{1}{2} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}$$

$$- \frac{22}{7} \times \frac{4{\cdot}5}{2} \times \frac{4{\cdot}5}{2} - 2 \times \frac{1}{2} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}$$

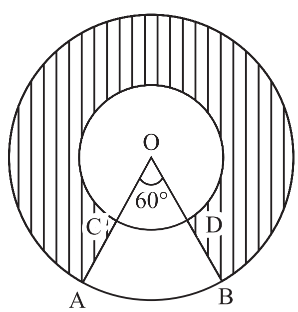
$$= \frac{1}{2} \times \left[20{\cdot}25 + \frac{9}{4}\right] - \frac{22}{7}\left[\frac{20{\cdot}25}{4} + \frac{9}{4}\right]$$

$$= \frac{11}{7} \times \frac{90}{4} - \frac{22}{7} \times \frac{29{\cdot}25}{4}$$

$$= \frac{990 - 643{\cdot}5}{28} = \frac{346{\cdot}5}{28}$$

$= 12{\cdot}37$ cm^2 (approx.) **Ans.**

17. In the given figure, two concentric circles with centre O have radii 21 cm and 42 cm. If $\angle AOB$ = 60°, find the area of the shaded region. [Use $\pi = \frac{22}{7}$] **[3]**



Solution : Angle for shaded region = 360° – 60°

= 300°

Area of shaded region

$$= \frac{\pi\theta}{360}(R^2 - r^2)$$

$$= \frac{22}{7} \times \frac{300}{360}[42^2 - 21^2]$$

$$= \frac{22}{7} \times \frac{5}{6} \times 63 \times 21$$

$= 3465$ cm^2 **Ans.**

18. Water in a canal, 5·4 m wide and 1·8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation ? **[3]**

Solution : Width of canal = 5·4 m

Depth of canal = 1·8 m

Length of water in canal for 1 hr = 25 km

= 25000 m

Volume of water flown out from canal in 1 hr

$= l \times b \times h$

$= 5{\cdot}4 \times 1{\cdot}8 \times 25000$

$= 243000$ m^3

Volume of water for 40 min $= 243000 \times \frac{40}{60} = 162000$ m^3

Area to be irrigated with 10 cm standing water in field

$$= \frac{\text{Volume}}{\text{Height}} = \frac{162000 \times 100}{10} \text{ m}^2$$

$= 1620000$ m^2

$= 162$ hectare **Ans.**

19. The slant height of a frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum. **[3]**

Solution : Slant height of frustum 'l' = 4 cm

Perimeter of upper top = 18 cm

$\Rightarrow \quad 2\pi R = 18 \text{ cm} \Rightarrow R = \frac{9}{\pi}$ cm

Perimeter of lower bottom = 6 cm

$\Rightarrow \quad 2\pi r = 6 \Rightarrow r = \frac{3}{\pi}$ cm

Curved S.A. of frustum $= \pi\, l\, [R + r]$

$$= \pi \times 4 \times \left[\frac{9}{\pi} + \frac{3}{\pi}\right]$$

$= \pi \times 4 \times \frac{12}{\pi} = 48$ cm^2 **Ans.**

20. The dimensions of a solid iron cuboid are 4·4 m × 2·6 m × 1·0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe. **[3]**

Solution : Inner radius of pipe 'r' = 30 cm

Thickness of pipe = 5 cm

$\therefore$ Outer radius = 30 + 5

$\Rightarrow$ R = 35 cm

Now, Vol. of hollow pipe = Vol. of cuboid

$\pi h (R^2 - r^2) = l \times b \times h$

$$\frac{22}{7} \times h\,[35^2 - 30^2] = 4{\cdot}4 \times 2{\cdot}6 \times 1 \times 100 \times 100 \times 100$$

$$\frac{22}{7} \times h \times 65 \times 5 = 44 \times 26 \times 1 \times 100 \times 100$$

$$h = \frac{44 \times 26 \times 100 \times 100 \times 7}{22 \times 65 \times 5}$$

$= 11200$ cm

$= 112$ m **Ans.**

SECTION — D

21. Solve for x :

$$\frac{1}{x+1}+\frac{3}{5x+1}=\frac{5}{x+4}, x \neq -1, -\frac{1}{5}, -4 \quad [4]$$

Solution : Given, $\frac{1}{x+1}+\frac{3}{5x+1}=\frac{5}{x+4}$

$$\Rightarrow \quad \frac{1}{x+1}-\frac{5}{x+4}=\frac{-3}{5x+1}$$

$$\Rightarrow \quad \frac{(x+4)-5(x+1)}{(x+1)(x+4)}=\frac{-3}{5x+1}$$

$$\Rightarrow \quad \frac{x+4-5x-5}{x^2+5x+4}=\frac{-3}{5x+1}$$

$$\Rightarrow \quad \frac{(-4x-1)}{x^2+5x+4}=\frac{-3}{5x+1}$$

$\Rightarrow \quad (4x + 1)(5x + 1) = 3(x^2 + 5x + 4)$

$\Rightarrow \quad 20x^2 + 4x + 5x + 1 = 3x^2 + 15x + 12$

$\Rightarrow \quad 17x^2 - 6x - 11 = 0$

$\Rightarrow \quad 17x^2 - 17x + 11x - 11 = 0$

$\Rightarrow \quad 17x(x - 1) + 11(x - 1) = 0$

$\Rightarrow \quad (x - 1)(17x + 11) = 0$

$\Rightarrow$ Either $x = 1$ or $x = \frac{-11}{17}$ **Ans.**

22. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank ? [4]

Solution : Let tank fill by one tap $= x$ hrs

Other tap $= (x + 3)$ hrs

Together they fill by $3\frac{1}{13} = \frac{40}{13}$ hrs

Now,

$$\Rightarrow \quad \frac{1}{x}+\frac{1}{x+3}=\frac{13}{40}$$

$$\Rightarrow \quad \frac{x+3+x}{(x)(x+3)}=\frac{13}{40}$$

$$\Rightarrow \quad \frac{2x+3}{x^2+3x}=\frac{13}{40}$$

$\Rightarrow \quad 13x^2 + 39x = 80x + 120$

$\Rightarrow \quad 13x^2 - 41x - 120 = 0$

$\Rightarrow \quad 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow \quad 13x(x - 5) + 24(x - 5) = 0$

$\Rightarrow \quad (x - 5)(13x + 24) = 0$

Either $x - 5 = 0$ or $13x + 24 = 0$

$x = 5, x = -24/13$ (Rejected)

One tap fill the tank in 5 hrs

So other tap fill the tank in 5 + 3 = 8 hrs **Ans.**

23. If the ratio of the sum of the first n terms of two A.P.s is $(7n + 1) : (4n + 27)$, then find the ratio of their 9^{th} terms. [4]

Solution : Ratio of sum of first n terms of two A.P.s are

$$\frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]}=\frac{7n+1}{4n+27}$$

Put $n = 17$

$$\Rightarrow \quad \frac{2a+(16)d}{2A+(16)D}=\frac{120}{95}$$

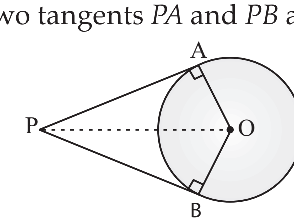
$$\frac{2a+(16)d}{2A+(16)D}=\frac{120}{95}=\frac{24}{19}$$

$$\frac{a+8d}{A+8D}=\frac{24}{19}$$

Hence ratio of 9^{th} terms of two A.P.s is 24 : 19 **Ans.**

24. Prove that the lengths of two tangents drawn from an external point to a circle are equal. [4]

Solution : Given, a circle with centre O and external point P. Two tangents PA and PB are drawn.



To prove : $PA = PB$

Const. : Join radius OA and OB also join O to P.

Proof : In ΔOAP and ΔOBP

$OA = OB$ (Radii)

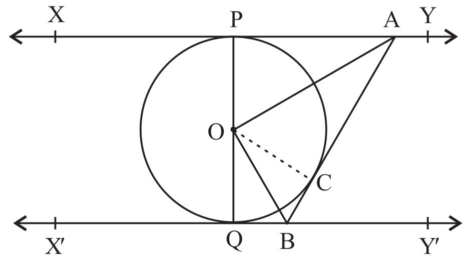
$\angle A = \angle B$ (Each 90°)

$OP = OP$ (Common)

$\therefore \quad \Delta AOP \cong \Delta BOP$ (RHS cong.)

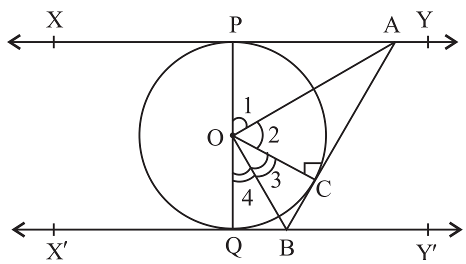
$\therefore \quad PA = PB$ (cpct) **Hence Proved.**

25. In the given figure, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, is intersecting XY at A and $X'Y'$ at B. Prove that $\angle AOB = 90°$. [4]



Solution : Given, XY & $X'Y'$ are parallel

Tangent AB is another tangent which touches the circle at C.



To prove : $\angle AOB = 90°$

Const. : Join OC.

Proof : In ΔOPA and ΔOCA

$$OP = OC \quad \text{(Radii)}$$

$$\angle OPA = \angle OCA \quad \text{(Radius} \perp \text{tangent)}$$

$$OA = OA \quad \text{(Common)}$$

$$\therefore \quad \Delta OPA \cong \Delta OCA \quad \text{(CPCT)}$$

$$\therefore \quad \angle 1 = \angle 2 \quad ...\text{(i)}$$

Similarly, $\Delta OQB \cong \Delta OCB$

$$\therefore \quad \angle 3 = \angle 4 \quad ...\text{(ii)}$$

Also, POQ is a diameter of circle

$$\therefore \quad \angle POQ = 180° \quad \text{(Straight angle)}$$

$$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$$

From eq. (i) and (ii)

$$\angle 2 + \angle 2 + \angle 3 + \angle 3 = 180°$$

$$2(\angle 2 + \angle 3) = 180°$$

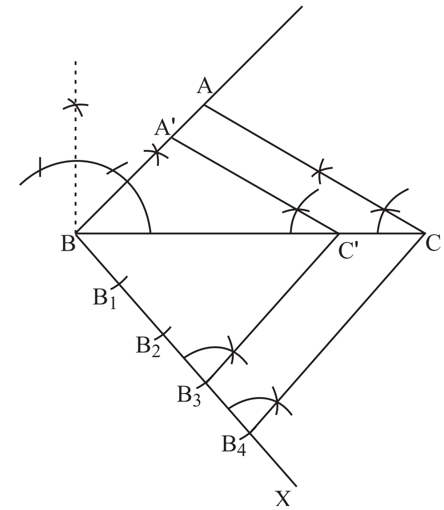
$$\angle 2 + \angle 3 = 90°$$

Hence, $\angle AOB = 90°$ **Hence Proved.**

26. Construct a triangle ABC with side $BC = 7$ cm, $\angle B = 45°$, $\angle A = 105°$. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the ΔABC. **[4]**

Solution : $BC = 7$ cm, $\angle B = 45°$, $\angle A = 105°$

$$\angle C = 180° - (\angle B + \angle A)$$
$$= 180° - (45° + 105°)$$
$$= 180° - 150°$$
$$= 30°$$



Steps of construction—

(i) Draw a line segment $BC = 7$ cm.

(ii) Draw an angle 45° at B and 30° at C. They intersect at A.

(iii) Draw an acute angle at B.

(iv) Divide angle ray in 4 equal parts as B_1, B_2, B_3 and B_4.

(v) Join B_4 to C.

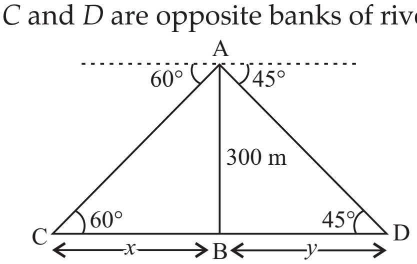
(vi) From B_3, draw a line parallel to B_4C intersecting BC at C'.

(vii) Draw another line parallel to CA from C' intersecting AB ray at A'.

(viii) $\Delta A'BC'$ is required triangle such that $\Delta A'BC' \sim \Delta ABC$ with $A'B = \frac{3}{4} AB$.

27. An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45° and 60° respectively. Find the width of the river. [Use $\sqrt{3} = 1{\cdot}732$] **[4]**

Solution : Let aeroplane is at A, 300 m high from a river. C and D are opposite banks of river.



In right ΔABC,

$$\frac{BC}{AB} = \cot 60°$$

$$\Rightarrow \quad \frac{x}{300} = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{300}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 100\sqrt{3} \text{ m}$$

$$= 100 \times 1{\cdot}732 = 173{\cdot}2 \text{ m}$$

In right $\Delta\, ABD$,

$$\Rightarrow \quad \frac{BD}{AB} = \cot 45°$$

$$\Rightarrow \quad \frac{y}{300} = 1 \Rightarrow y = 300$$

$$\text{Width of river} = x + y$$

$$= 173{\cdot}2 + 300$$

$$= 473{\cdot}2 \text{ m}$$ **Ans.**

28. If the points $A(k+1, 2k)$, $B(3k, 2k+3)$ and $C(5k-1, 5k)$ are collinear, then find the value of k. [4]

Solution : Since $A(k+1, 2k)$, $B(3k, 2k+3)$ and $C(5k-1, 5k)$ are collinear points, so area of triangle $= 0$.

$$\begin{vmatrix} k+1 & 2k \\ 3k & 2k+3 \\ 5k-1 & 5k \\ k+1 & 2k \end{vmatrix}$$

$$\Delta = \frac{1}{2}\,[(k+1)(2k+3) - 6k^2 + 15k^2 - (5k-1)(2k+3)$$
$$+ 2k(5k-1) - (k+1)(5k)]$$

$$0 = \frac{1}{2}\,[2k^2 + 5k + 3 - 6k^2 + 15k^2 - 10k^2 - 13k + 3$$
$$+ 10k^2 - 2k - 5k^2 - 5k]$$

$$0 = \frac{1}{2}\,[6k^2 - 15k + 6]$$

$$\Rightarrow \quad 6k^2 - 15k + 6 = 0$$

$$\Rightarrow \quad 6k^2 - 12k - 3k + 6 = 0$$

$$\Rightarrow \quad 6k(k-2) - 3(k-2) = 0$$

$$\Rightarrow \quad (k-2)(6k-3) = 0$$

$$k = 2 \text{ or } k = \frac{1}{2}$$ **Ans.**

29. Two different dice are thrown together. Find the probability that the numbers obtained have

(i) even sum, and

(ii) even product. [4]

Solution : When two different dice are thrown together

Total outcomes $= 6 \times 6 = 36$

(i) For even sum—Favourable outcomes are

(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)

No. of favourable outcomes = 18

$$P\text{ (even sum)} = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{18}{36} = \frac{1}{2}$$ **Ans.**

(ii) For even product—Favourable outcomes are

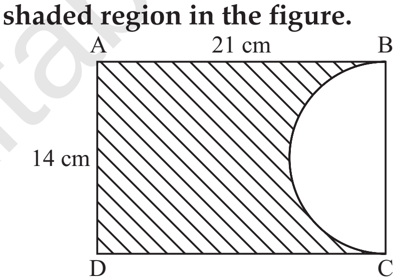
(1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6).

No. of favourable outcomes = 27

$$\text{P(even product)} = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$= \frac{27}{36} = \frac{3}{4}$$ **Ans.**

30. In the given figure, $ABCD$ is a rectangle of dimensions 21 cm × 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure. [4]



Solution : Area of shaded region

$$= \text{Area of rectangle} - \text{Area of semi circle}$$

$$= l \times b - \frac{1}{2}\pi r^2$$

$$= 21 \times 14 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

$$= 294 - 77$$

$$= 217 \text{ cm}^2$$

Perimeter of shaded region $= 2l + b + \pi r$

$$= 2 \times 21 + 14 + \frac{22}{7} \times 7$$

$$= 42 + 14 + 22$$

$$= 78 \text{ cm}$$ **Ans.**

31. In a rain-water harvesting system, the rain-water from a roof of 22 m × 20 m drains into a cylindrical tank having diameter of base 2 m and heigth 3·5 m. If the tank is full, find the rainfall in cm. Write your views on water conservation. [4]

Solution : Volume of water collected in system = Volume of cylindrical tank

$$L \times B \times H = \pi r^2 h$$

$$22 \times 20 \times H = \frac{22}{7} \times 1 \times 1 \times 3{\cdot}5$$

$$22 \times 20 \times H = 11$$

$$H = \frac{11}{22\times20} = \frac{1}{40} \text{ m}$$

$$= \frac{1}{40} \times 100 = \frac{5}{2} = 2{\cdot}5 \text{ cm}$$

Rainfall on system = 2·5 cm

Water conservation is very important as it solves the water problem in the absence of rain. **Ans.**

Mathematics 2017 (Outside Delhi) II — SET II

Note : Except for the following questions, all the remaining questions have been asked in previous set.

SECTION — B

10. Which term of the A.P. 8, 14, 20, 26, ... will be 72 more than its 41st term ? [2]

Solution : A.P. is 8, 14, 20, 26, ...

$$a = 8, d = 14 - 8 = 6$$

Let $a_n = a_{41} + 72$

$\Rightarrow \quad a + (n-1)d = a + 40d + 72$

$\Rightarrow \quad (n-1)\,6 = 40 \times 6 + 72$

$= 240 + 72$

$\Rightarrow \quad n - 1 = \frac{312}{6} = 52$

$\Rightarrow \quad n = 52 + 1 = 53^{rd}$ term **Ans.**

SECTION — C

18. From a solid right circular cylinder of height 2·4 cm and radius 0·7 cm, a right circular cone of same height and same radius is cut out. Find the total surface area of the remaining solid. [3]

Solution : Given, height of cylinder 'h' = 2·4 cm

Radius of base 'r' = 0·7 cm

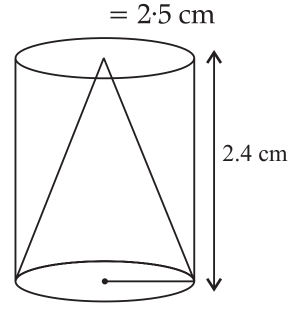
And slant height $l = \sqrt{h^2 + r^2}$

$= \sqrt{(2{\cdot}4)^2 + (0{\cdot}7)^2}$

$= \sqrt{5{\cdot}76 + 0{\cdot}49}$

$= \sqrt{6{\cdot}25}$

$= 2{\cdot}5$ cm



Total surface area of the remaining solid

= CSA of cylinder + CSA of cone + Area of top

$= 2\pi rh + \pi rl + \pi r^2$

$= \pi r\,[2h + l + r]$

$= \frac{22}{7} \times 0{\cdot}7\,[2 \times 2{\cdot}4 + 2{\cdot}5 + 0{\cdot}7]$

$= 2{\cdot}2\,[4{\cdot}8 + 2{\cdot}5 + 0{\cdot}7]$

$= 2{\cdot}2 \times 8 = 17{\cdot}6 \text{ cm}^2$ **Ans.**

19. If the 10th term of an A.P. is 52 and the 17th term is 20 more than the 13th term, find the A.P. [3]

Solution : Given, $a_{10} = 52$; $a_{17} = a_{13} + 20$

$\Rightarrow \quad a + 16d = a + 12d + 20$

$\Rightarrow \quad 16d = 12d + 20$

$\Rightarrow \quad 4d = 20$

$\Rightarrow \quad d = \frac{20}{4} = 5$

Also, $\quad a + 9d = 52$

$\Rightarrow \quad a + 9 \times 5 = 52$

$a + 45 = 52$

$a = 7$

Therefore A.P. = 7, 12, 17, 22, 27 . . . . **Ans.**

20. If the roots of the equation $(c^2 - ab)\,x^2 - 2\,(a^2 - bc)\,x + b^2 - ac = 0$ in x are equal, then show that either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$. [3]

Solution : $(c^2 - ab)\,x^2 - 2\,(a^2 - bc)\,x + b^2 - ac = 0$

For equal roots, $D = 0$

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad [-2(a^2 - bc)]^2 - 4\,(c^2 - ab)\,(b^2 - ac) = 0$

$\Rightarrow \quad 4\,[a^4 + b^2c^2 - 2a^2bc] - 4\,[b^2c^2 - ac^3 - ab^3 + a^2bc] = 0$

$\Rightarrow \quad 4\,[a^4 + b^2c^2 - 2a^2bc - b^2c^2 + ac^3 + ab^3 - a^2bc] = 0$

$\Rightarrow \quad 4a\,[a^3 - 3abc + c^3 + b^3] = 0$

Either $4a = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$

$a = 0$ or $a^3 + b^3 + c^3 = 3abc$. **Hence Proved**

SECTION — D

28. Solve for x :

$$\frac{1}{2x-3} + \frac{1}{x-5} = 1\frac{1}{9},\ x \neq \frac{3}{2}, 5 \qquad [4]$$

Solution : Given, $\frac{1}{2x-3} + \frac{1}{x-5} = 1\frac{1}{9}$

$$\frac{x-5+2x-3}{(2x-3)(x-5)} = \frac{10}{9}$$

$$\begin{aligned}
\frac{3x-8}{2x^2-13x+15} &= \frac{10}{9} \\
9\,(3x-8) &= 10\,(2x^2-13x+15) \\
27x-72 &= 20x^2-130x+150 \\
20x^2-157x+222 &= 0 \\
20x^2-120x-37x+222 &= 0 \\
20x\,(x-6)-37\,(x-6) &= 0 \\
(x-6)\,(20x-37) &= 0 \\
\text{Either } x-6 &= 0 \text{ or } 20x-37 = 0 \\
\Rightarrow \qquad x &= 6,\ x = \frac{37}{20}
\end{aligned}$$

Ans.

29. A train covers a distance of 300 km at a uniform speed. If the speed of the train is increased by 5 km/hour, it takes 2 hours less in the journey. Find the original speed of the train. [4]

Solution : Let original speed of train = x km/hr

$$\begin{aligned}
\text{Increased speed of train} &= (x+5) \text{ km/hr} \\
\text{Distance} &= 300 \text{ km}
\end{aligned}$$

According to the question,

$$\begin{aligned}
\frac{300}{x}-\frac{300}{x+5} &= 2 \\
\frac{300\,(x+5-x)}{(x)(x+5)} &= 2 \\
1500 &= 2\,(x^2+5x) \\
1500 &= 2x^2+10x \\
2x^2+10x-1500 &= 0 \\
x^2+5x-750 &= 0 \\
x^2+30x-25x-750 &= 0 \\
x\,(x+30)-25\,(x+30) &= 0 \\
(x+30)\,(x-25) &= 0
\end{aligned}$$

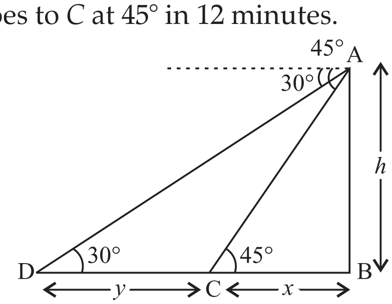
Either $x+30 = 0$ or $x-25 = 0$

$\Rightarrow \qquad x = -30$ (Rejected), so $x = 25$

Original speed of train is 25 km/hr. **Ans.**

30. A man observes a car from the top of a tower, which is moving towards the tower with a uniform speed. If the angle of depression of the car changes from 30° to 45° in 12 minutes, find the time taken by the car now to reach the tower. [4]

Solution : Let AB is a tower, car is at point D at 30° and goes to C at 45° in 12 minutes.



In $\Delta\, ABC$,

$$\begin{aligned}
\frac{AB}{BC} &= \tan 45^\circ \\
\Rightarrow \qquad \frac{h}{x} &= 1 \Rightarrow h = x \qquad \text{...(i)}
\end{aligned}$$

In $\Delta\, ABD$,

$$\begin{aligned}
\frac{AB}{BD} &= \tan 30^\circ \\
\Rightarrow \qquad \frac{h}{x+y} &= \frac{1}{\sqrt{3}} \Rightarrow h = \frac{x+y}{\sqrt{3}} \qquad \text{...(ii)}
\end{aligned}$$

Comparing eq. (i) & (ii), we get

$$x = \frac{x+y}{\sqrt{3}} \Rightarrow \sqrt{3}\,x = x+y$$

$$\Rightarrow \qquad \left(\sqrt{3}-1\right)x = y$$

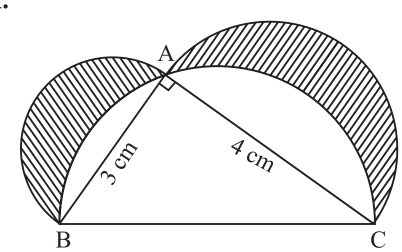
Car covers the distance y in time = 12 min

So $\left(\sqrt{3}-1\right)x$ distance covers in 12 min

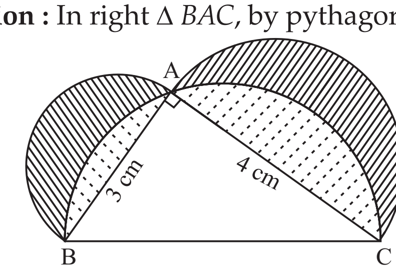
$$\begin{aligned}
\text{Distance } x \text{ covers in time} &= \frac{12}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1} \\
&= \frac{12\left(\sqrt{3}+1\right)}{3-1=2} \\
&= 6\left(\sqrt{3}+1\right) \text{ min} \\
&= 6 \times 2.732 = 16.39
\end{aligned}$$

Now car reaches to tower in 16.39 minutes. **Ans.**

31. In the given figure, $\Delta\, ABC$ is a right-angled triangle in which $\angle A$ is 90°. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region. [4]



Solution : In right $\Delta\, BAC$, by pythagoras theorem,



$$\begin{aligned}
BC^2 &= AB^2 + BC^2 \\
&= (3)^2 + (4)^2
\end{aligned}$$

$$= 9 + 16 = 25$$

$$BC = \sqrt{25} = 5 \text{ cm}$$

Area of semi-circle with diameter $BC = \frac{1}{2}\pi r^2$

$$= \frac{1}{2} \times \pi \left(\frac{5}{2}\right)^2 = \frac{25}{8}\pi \text{ cm}^2$$

Area of semi-circle with diameter $AB = \frac{1}{2}\pi r^2$

$$= \frac{1}{2}\pi \left(\frac{3}{2}\right)^2 = \frac{9}{8}\pi \text{ cm}^2$$

Area of semi-circle with diameter $AC = \frac{1}{2}\pi r^2$.

$$= \frac{1}{2}\pi \left(\frac{4}{2}\right)^2 = \frac{16}{8}\pi \text{ cm}^2$$

Area of rt $\Delta\, BAC = \frac{1}{2} \times AB \times AC$

$$= \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$$

Area of dotted region $= \left(\frac{25}{8}\pi - 6\right) \text{ cm}^2$

Area of shaded region $= \frac{16}{8}\pi + \frac{9}{8}\pi - \left(\frac{25}{8}\pi - 6\right)$

$$= \frac{16}{8}\pi + \frac{9}{8}\pi - \frac{25}{8}\pi + 6$$

$$= 6 \text{ cm}^2$$ **Ans.**

Mathematics 2017 (Outside Delhi) II SET III

Note : Except for the following questions, all the remaining questions have been asked in previous sets.

SECTION — B

10. For what value of n, are the n^{th} terms of two A.Ps 63, 65, 67, ... and 3, 10, 17, ... equal ? [2]

Solution : 1st A.P. is 63, 65, 67, ...

$a = 63, \quad d = 65 - 63 = 2$

$$a_n = a + (n-1)d$$
$$= 63 + (n-1)2$$
$$= 63 + 2n - 2 = 61 + 2n$$

2nd A.P. is 3, 10, 17 . . .

$a = 3, \quad d = 10 - 3 = 7$

$$a_n = a + (n-1)d$$
$$= 3 + (n-1)7$$
$$= 3 + 7n - 7$$
$$= 7n - 4$$

According to question,

$$61 + 2n = 7n - 4$$
$$61 + 4 = 7n - 2n$$
$$65 = 5n$$
$$n = \frac{65}{5} = 13$$
$$n = 13$$

Hence, 13th term of both A.P. is equal **Ans.**

SECTION — C

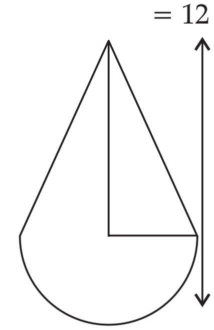
18. A toy is in the form of a cone of radius 3·5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15·5 cm. Find the total surface area of the toy. [3]

Solution : Given, radius of base 'r' = 3·5 cm

Total height of toy = 15·5 cm

Height of cone 'h' = 15·5 – 3·5

= 12 cm



Slant height 'l' $= \sqrt{h^2 + r^2}$

$$= \sqrt{12^2 + 3{\cdot}5^2}$$
$$= \sqrt{144 + 12{\cdot}25}$$
$$= \sqrt{156.25}$$
$$= 12{\cdot}5 \text{ cm}$$

Total S.A. of toy = CSA of cone + CSA of hemisphere

$$= \pi r l + 2\pi r^2$$
$$= \pi r\,[l + 2r]$$
$$= \frac{22}{7} \times 3{\cdot}5\,[12{\cdot}5 + 2 \times 3{\cdot}5]$$
$$= 22 \times 0{\cdot}5\,[12{\cdot}5 + 7]$$

$$= 11 \times 19{\cdot}5$$
$$= 214{\cdot}5 \text{ cm}^2 \quad \textbf{Ans.}$$

19. How many terms of an A.P. 9, 17, 25, ... must be taken to give a sum of 636 ? [3]

Solution : A.P. is 9, 17, 25, ..., $S_n = 636$

$a = 9, d = 17 - 9 = 8$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$636 = \frac{n}{2}[18 + 8n - 8]$$
$$636 = \frac{n}{2}[10 + 8n]$$
$$636 = n(5 + 4n)$$
$$636 = 5n + 4n^2$$
$$4n^2 + 5n - 636 = 0$$
$$4n^2 + 53n - 48n - 636 = 0$$
$$n(4n + 53) - 12(4n + 53) = 0$$
$$(n - 12)(4n + 53) = 0$$
$$n - 12 = 0 \left(\because n \neq \frac{-53}{4} \, as \; n > 0\right)$$
$$n = 12 \quad \textbf{Ans.}$$

20. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$. [3]

Solution : $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$

For equal roots, $D = 0$

$[-2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$

$4(ac + bd)^2 - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) = 0$

$4[a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 + b^2c^2 + b^2d^2] = 0$

$$-4[a^2d^2 + b^2c^2 - 2abcd] = 0$$
$$-4(ad - bc)^2 = 0$$
$$-4 \neq 0 \text{ so, } (ad - bc)^2 = 0$$
$$ad - bc = 0$$
$$ad = bc$$
$$\frac{a}{b} = \frac{c}{d} \quad \textbf{Hence Proved}$$

SECTION — D

28. Solve for x :

$$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 2, \text{ where } x \neq -\frac{1}{2}, 1 \quad [4]$$

Solution : $\dfrac{(x-1)^2 + (2x+1)^2}{(2x+1)(x-1)} = 2$

$$\frac{x^2 + 1 - 2x + 4x^2 + 1 + 4x}{2x^2 - x - 1} = \frac{2}{1}$$
$$5x^2 + 2x + 2 = 2(2x^2 - x - 1)$$
$$5x^2 + 2x + 2 = 4x^2 - 2x - 2$$
$$x^2 + 4x + 4 = 0$$
$$(x + 2)^2 = 0$$
$$\text{Either } x + 2 = 0 \text{ or } x + 2 = 0$$
$$x = -2, -2 \quad \textbf{Ans.}$$

29. *A* takes 6 days less than *B* to do a work. If both *A* and *B* working together can do it in 4 days, how many days will B take to finish it ? [4]

Solution : Let *B* can finish a work in x days

so, *A* can finish work in $(x - 6)$ days

Together they finish work in 4 days

Now,

$$\frac{1}{x} + \frac{1}{x-6} = \frac{1}{4}$$
$$\frac{x - 6 + x}{(x)(x-6)} = \frac{1}{4}$$
$$4(2x - 6) = x^2 - 6x$$
$$8x - 24 = x^2 - 6x$$
$$x^2 - 14x + 24 = 0$$
$$x^2 - 12x - 2x + 24 = 0$$
$$x(x - 12) - 2(x - 12) = 0$$
$$(x - 12)(x - 2) = 0$$
$$\text{Either } x - 12 = 0 \text{ or } x - 2 = 0$$
$$x = 12 \text{ or } x = 2, \text{(Rejected)}$$

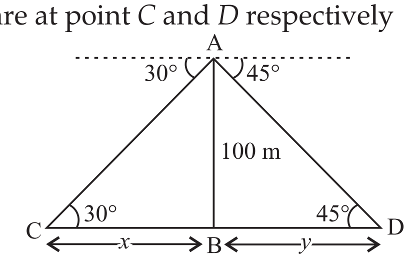
B can finish work in 12 days

A can finish work in 6 days **Ans.**

30. From the top of a tower, 100 m high, a man observes two cars on the opposite sides of the tower and in same straight line with its base, with angles of depression 30° and 45°. Find the distance between the cars. [Take $\sqrt{3} = 1{\cdot}732$] [4]

Solution : Let *AB* is a tower.

Cars are at point *C* and *D* respectively



In Δ *ABC*,

$$\frac{AB}{BC} = \tan 30°$$
$$\frac{100}{x} = \frac{1}{\sqrt{3}}$$
$$x = 100\sqrt{3}$$
$$= 100 \times 1{\cdot}732$$
$$= 173{\cdot}2 \text{ m}$$

In Δ *ABD*,

$$\frac{AB}{BD} = \tan 45°$$

$$\frac{100}{y} = 1$$

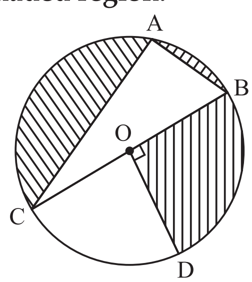
$$y = 100 \text{ m}$$

Distance between two cars $= x + y$

$$= 173{\cdot}2 + 100$$

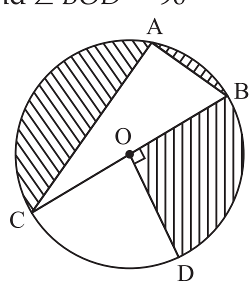
$$= 273{\cdot}2 \text{ m}$$ **Ans.**

31. In the given figure, O is the centre of the circle with AC = 24 cm, AB = 7 cm and $\angle BOD$ = 90°. Find the area of the shaded region. [4]



Solution : Given, $C(O, OB)$ with AC = 24 cm

AB = 7 cm and $\angle BOD$ = 90°



$\angle CAB$ = 90° (Angle in semi-circle)

Using pythagoras theorem in ΔCAB

$$BC^2 = AC^2 + AB^2$$
$$= (24)^2 + (7)^2$$
$$= 576 + 49$$
$$= 625$$
$$BC = 25 \text{ cm}$$

Radius of circle $= OB = OD = OC = \frac{25}{2}$ cm

Area of shaded region

= Area of semi-circle with diameter BC – Area of ΔCAB + Area of sector BOD

$$= \frac{1}{2}\pi\left(\frac{25}{2}\right)^2 - \frac{1}{2} \times 24 \times 7 + \frac{90}{360}\pi\left(\frac{25}{2}\right)^2$$

$$= \frac{3}{4} \times \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} - 84$$

$$= \frac{20625}{56} - 84$$

$$= \frac{20625 - 4704}{56}$$

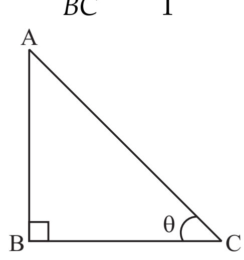
$$= \frac{15921}{56} = 284{\cdot}3 \text{ cm}^2 \text{ (approx.)}$$ **Ans.**

Mathematics 2017 (Delhi) Term II — SET I

SECTION — A

1. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}$: 1. What is the angle of elevation of the sun ? [1]

Solution : Given, $\frac{AB}{BC} = \frac{\sqrt{3}}{1}$



In ΔABC

$$\tan\theta = \frac{AB}{BC} = \frac{\sqrt{3}}{1}$$

$$\Rightarrow \qquad = 60°$$

Hence, the angle of elevation is 60°. **Ans.**

2. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere ? [1]

Solution : Let radius of hemisphere be r units

Volume of hemisphere = S.A. of hemisphere

$$\frac{2}{3}\pi r^3 = 3\pi r^2$$

$$\Rightarrow \qquad r = \frac{9}{2} \text{ or diameter} = 9 \text{ units}$$ **Ans.**

3. A number is chosen at random from the numbers – 3, – 2, – 1, 0, 1, 2, 3.

What will be the probability that square of this number is less then or equal to 1 ? [1]

Solution : Possible outcomes {– 3, – 2, – 1, 0, 1, 2, 3}, n = 7 and only three numbers – 1, 0, 1 fall under given condition so,

Required probability $= \frac{3}{7}$ **Ans.**

4. If the distance between the points (4, k) and (1, 0) is 5, then what can be the possible values of k ? [1]

Solution : Distance between (4, k) and (1, 0) = 5

$$\sqrt{(1-4)^2 + (0-k)^2} = 5$$

On squaring both sides

$$9 + k^2 = 25$$

or $\quad k^2 = 25 - 9 = 16$

So $\quad k = \pm 4$ **Ans.**

SECTION — B

5. Find the roots of the quadratic equation

$\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0.$ [2]

Solution : Given quadratic equation is,

$$\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$$

$$\sqrt{2}\,x^2 + 5x + 2x + 5\sqrt{2} = 0$$

[Splitting middle term]

$$\Rightarrow \quad x\left(\sqrt{2}x+5\right) + \sqrt{2}\left(\sqrt{2}x+5\right) = 0$$

$$\Rightarrow \quad \left(\sqrt{2}x+5\right)\left(x+\sqrt{2}\right) = 0$$

$$\Rightarrow \quad \text{either } \left(\sqrt{2}x+5\right) = 0 \text{ or } \left(x+\sqrt{2}\right) = 0$$

$$\Rightarrow \quad x = -\frac{5}{\sqrt{2}} \text{ or } x = -\sqrt{2}$$

Hence roots are $\frac{-5}{\sqrt{2}}$ and $-\sqrt{2}$ **Ans.**

6. Find how many integers between 200 and 500 are divisible by 8 ? [2]

Solution : Smallest divisible no. (by 8) in given range = 208

Last divisible no. (by 8) in range = 496

So, $a = 208, d = 8, n = ?, a_n = 496$

$$a_n = a + (n-1)d$$

$$= 208 + (n-1)\,8 = 496$$

$$\Rightarrow \quad 8n + 208 - 8 = 496$$

or $\quad 8n = 496 - 200 = 296$

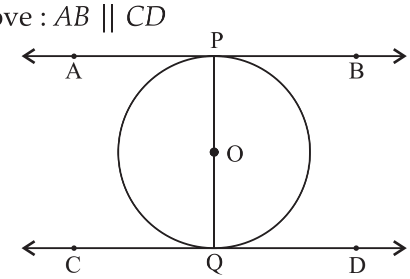
$$n = \frac{296}{8} = 37$$

So number of terms between 200 and 500 divisible by 8 are 37. **Ans.**

7. Prove that tangents drawn at the ends of a diameter of a circle are parallel to each other. [2]

Solution : Given, PQ is a diameter of a circle with centre O. The lines AB and CD are tangents at P and Q respectively.

To Prove : $AB \parallel CD$



Proof : AB is a tangent to the circle at P and OP is the radius through the point of contact

$\therefore \quad \angle OPA = 90°$

Similarly CD is a tangent to circle at Q and OQ is radius through the point of contact

$\therefore \quad \angle OQD = 90°$

$\Rightarrow \quad \angle OPA = \angle OQD$

But both form pair of alternate angles

$\therefore \quad AB \parallel CD$ **Hence Proved.**

8. Find the value of k for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ has real and equal roots. [2]

Solution : Given equation is,

$$x^2 + k(2x + k - 1) + 2 = 0$$

$$\Rightarrow x^2 + 2kx + k(k-1) + 2 = 0$$

Here $a = 1$, $b = 2k$ and $c = k(k-1) + 2$

For real and equal roots

(multiply) $\quad b^2 - 4ac = 0$

$$\Rightarrow (2k)^2 - 4\cdot 1\cdot(k\,(k-1) + 2) = 0$$

$$\Rightarrow \quad 4k^2 - 4\,(k^2 - k + 2) = 0$$

$$\Rightarrow \quad 4k^2 - 4k^2 + 4k - 8 = 0$$

$$\Rightarrow \quad 4k = 8$$

$$\Rightarrow \quad k = \frac{8}{4} = 2$$ **Ans.**

9. Draw a line segment of length 8 cm and divide it internally in the ratio 4 : 5. [2]

Solution : Steps of construction—

(i) Draw AB = 8 cm

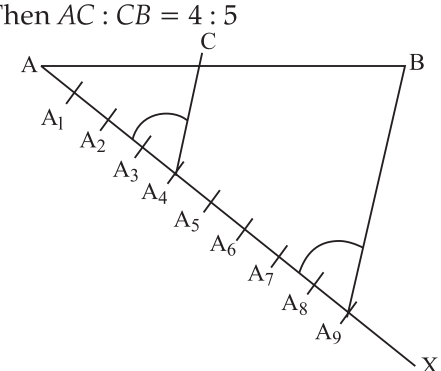
(ii) Draw any ray AX making an acute angle with AB

(iii) Draw 9(4 + 5) points on ray AX namely $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9$ at equal distance.

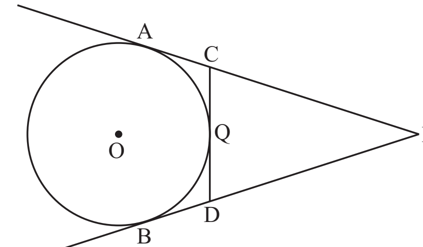
(iv) Join BA_9

(v) Through point A_4 draw a line parallel to A_9B inter- secting AB at the point C

Then $AC : CB = 4 : 5$



10. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, $QC = QD$ = 3 cm, then find $PC + PD$. [2]



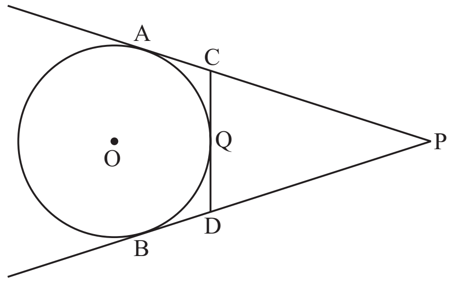
Solution : Given, $PA = PB = 12$ cm

[Tangent from an external point]

$AC = CQ = 3$ cm

$BD = QD = 3$ cm

[Tangent from external point]



So, $PC + PD$

$$= (PA - AC) + (PB - BD)$$
$$= (12 - 3) + (12 - 3)$$
$$= 9 + 9 = 18 \text{ cm}$$ **Ans.**

SECTION — C

11. If m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then find the sum of its first mn terms. [3]

Solution : Let a and d be the first term and common difference respectively of the given A.P.

Then, $\frac{1}{n} = m^{\text{th}}$ term $\Rightarrow \frac{1}{n} = a + (m-1)d$ ...(i)

$\frac{1}{m} = n^{\text{th}}$ term $\Rightarrow \frac{1}{m} = a + (n-1)d$ ...(ii)

By subtracting eq. (ii) from eq. (i),

$$\frac{1}{n} - \frac{1}{m} = (m-n)d$$

$$\Rightarrow \quad \frac{m-n}{mn} = (m-n)d$$

$$\Rightarrow \quad d = \frac{1}{mn}$$

Putting $d = \frac{1}{mn}$ in eq. (i)

We get, $$\frac{1}{n} = a + (m-1)\frac{1}{mn}$$

$$\Rightarrow \quad \frac{1}{n} = a + \frac{1}{n} - \frac{1}{mn}$$

$$\Rightarrow \quad a = \frac{1}{mn}$$

Sum of first mn terms

$$= \frac{mn}{2}[2a + (mn-1)d]$$

$$= \frac{mn}{2}\left[\frac{2}{mn} + (mn-1)\frac{1}{mn}\right] \qquad \left[\because a = \frac{1}{mn}, d = \frac{1}{mn}\right]$$

$$= \frac{mn}{2}\left[\frac{1}{mn} + 1\right]$$

$$= \frac{1+mn}{2}$$ **Ans.**

12. Find the sum of n terms of the series

$$\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \text{........}$$ **[3]**

Solution : In given series, $a = \left(4 - \frac{1}{n}\right)$

$$d = \left(4 - \frac{2}{n}\right) - \left(4 - \frac{1}{n}\right) = 4 - \frac{2}{n} - 4 + \frac{1}{n} = -\frac{1}{n}$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}\left[2\left(4 - \frac{1}{n}\right) + (n-1)\left(-\frac{1}{n}\right)\right]$$

$$= \frac{n}{2}\left[8 - \frac{2}{n} - \frac{(n-1)}{n}\right]$$

$$= \frac{n}{2}\left[7 - \frac{1}{n}\right]$$

$$= \frac{n}{2}\left[\frac{7n-1}{n}\right]$$

$$= \frac{7n-1}{2}$$ **Ans.**

13. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots then show that $c^2 = a^2(1 + m^2)$. [3]

Solution : The given equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots

Here, $A = 1 + m^2$, $B = 2mc$, $C = c^2 - a^2$

For equal roots, $D = 0 = B^2 - 4AC$

$$\Rightarrow \quad (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$$
$$\Rightarrow \quad 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$$
$$\Rightarrow \quad m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$$
$$\Rightarrow \quad -c^2 + a^2(1 + m^2) = 0$$
$$\Rightarrow \quad c^2 = a^2(1 + m^2)$$

Hence Proved.

14. The $\frac{3}{4}$ th part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel. [3]

Solution : According to the question,

$\frac{3}{4}$ volume of water in conical vessel = Volume of cylindrical vessel

$$\frac{3}{4} \times \frac{1}{3} \times \pi \times r^2_{cone} \times h_{cone} = \pi r^2_{cy} h_{cy}$$

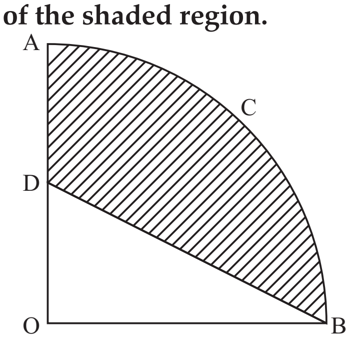
Or
$$h_{cy} = \frac{1}{4} \times \frac{r^2_{cone} \times h_{cone}}{r^2_{cy}}$$

$$\Rightarrow \quad = \frac{1}{4} \times \frac{5 \times 5 \times 24}{10 \times 10} = \frac{3}{2}$$

$$= 1.5 \text{ cm}$$

Hence, height of water in cylindrical vessel is 1·5 cm. **Ans.**

15. In the given figure, *OACB* is a quadrant of a circle with centre *O* and radius 3.5 cm. If *OD* = 2 cm, find the area of the shaded region. [3]



Solution : Area of shaded region = Area of quadrant *OACB* – Area of Δ *DOB*

$$= \frac{90}{360} \times \pi \times (3{\cdot}5)^2 - \frac{1}{2} \times 2 \times 3{\cdot}5$$

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} - 3{\cdot}5$$

$$= \frac{1925}{200} - 3{\cdot}5$$

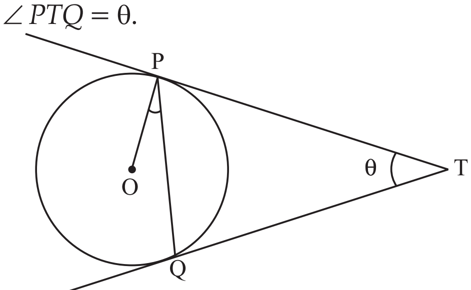
$$= 9{\cdot}625 - 3{\cdot}5 = 6{\cdot}125 \text{ cm}^2.$$

Hence, area of shaded region is 6·125 cm^2 **Ans.**

16. Two tangents *TP* and *TQ* are drawn to a circle with centre *O* from an external point *T*. Prove that ∠ *PTQ* = 2 ∠ *OPQ*. [3]

Solution : Given, a circle with centre *O*, an external point T and two tangents *TP* and *TQ*.

Let ∠ *PTQ* = θ.



To prove : ∠ *PTQ* = 2 ∠ *OPQ*.

Proof : *TP* = *TQ*

[Tangent from an external point]

So Δ*TPQ* is an isosceles triangle

∠ *TPQ* = ∠ *TQP*

[Angle opposite to equal sides of a Δ]

So, $\angle TPQ = \angle TQP = \frac{1}{2}(180 - \theta) = 90 - \frac{\theta}{2}$

But, $\angle TPO = 90°$

[Angle between tangent and radius]

$\therefore \quad \angle OPQ = \angle OPT - \angle TPQ = 90 - \left(90 - \frac{\theta}{2}\right)$

$$= \frac{\theta}{2} = \frac{1}{2} \angle PTQ$$

Or $\angle PTQ = 2 \angle OPQ$ **Hence Proved.**

17. Show that Δ *ABC*, where *A*(– 2, 0), *B*(2, 0), *C*(0, 2) and Δ *PQR* where *P*(– 4, 0), *Q*(4, 0), *R*(0, 4) are similar triangles. [3]

Solution : Coordinates of vertices are

A(– 2, 0), *B*(2, 0), *C*(0, 2)

P(– 4, 0), *Q*(4, 0), *R*(0, 4)

$$AB = \sqrt{(2+2)^2 + (0-0)^2} = 4 \text{ units}$$

$$BC = \sqrt{(0-2)^2 + (2-0)^2}$$

$$= \sqrt{4+4} = 2\sqrt{2} \text{ units}$$

$$CA = \sqrt{(-2-0)^2 + (0-2)^2}$$

$$= \sqrt{8} = 2\sqrt{2} \text{ units}$$

$$PR = \sqrt{(0+4)^2 + (4-0)^2}$$

$$= \sqrt{4^2 + (4)^2} = 4\sqrt{2} \text{ units}$$

$$QR = \sqrt{(0-4)^2 + (4-0)^2}$$

$$= \sqrt{4^2 + (4)^2} = 4\sqrt{2} \text{ units}$$

$$PQ = \sqrt{(4+4)^2 + (0-0)^2}$$

$$= \sqrt{(8)^2} = 8 \text{ units}$$

We see that sides of Δ *PQR* are twice the sides of Δ *ABC*.

Hence both triangles are similar. **Hence Proved.**

18. The area of a triangle is 5 sq units. Two of its vertices are (2, 1) and (3, – 2). If the third vertex is $\left(\frac{7}{2}, y\right)$, find the value of *y*. [3]

Solution : Given,

$A(2, 1)$, $B(3, -2)$ and $C\left(\frac{7}{2}, y\right)$

Now, Area (ΔABC) $= \frac{1}{2}\,|x_1(y_2 - y_3) + x_2\,(y_3 - y_1) + x_3(y_1 - y_2)|$

$$5 = \frac{1}{2}\,|2(-2 - y) + 3\,(y - 1) + \frac{7}{2}\,(1 + 2)|$$

$$\Rightarrow \quad 10 = |-4 - 2y + 3y - 3 + \frac{7}{2} + 7|$$

$$\Rightarrow \quad 10 = \left|y + \frac{7}{2}\right|$$

$$\Rightarrow \quad 10 = y + \frac{7}{2} \quad \text{or} \quad -10 = \left(y + \frac{7}{2}\right)$$

$$\Rightarrow \quad y = \frac{13}{2} \quad \text{or} \quad y = \frac{-27}{2}$$ **Ans.**

19. Two different dice are thrown together. Find the probability that the numbers obtained
(i) have a sum less than 7
(ii) have a product less than 16
(iii) is a doublet of odd numbers. [3]

Solution : Total possible outcomes in each case $= 6 \times 6 = 36$

(i) Have a sum less than 7,
Possible outcomes are,
(1, 1) (1, 2) (1, 3) (1, 4) (1, 5)
(2, 1) (2, 2) (2, 3) (2, 4)
(3, 1) (3, 2) (3, 3)
(4, 1) (4, 2)
(5, 1)

$$n(E) = 15$$

$$\text{So, probability} = \frac{15}{36} = \frac{5}{12}$$ **Ans.**

(ii) Have a product less than 16,
Possible outcomes are,
(1,1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5)
(4, 1) (4, 2) (4, 3)
(5, 1) (5, 2) (5, 3)
(6, 1) (6, 2)

$$n(E) = 25$$

$$\text{So, probability} = \frac{25}{36}$$ **Ans.**

(iii) Is a doublet of odd no.,
Possible outcomes are
(1, 1), (3, 3), (5, 5)

$$n(E) = 3$$

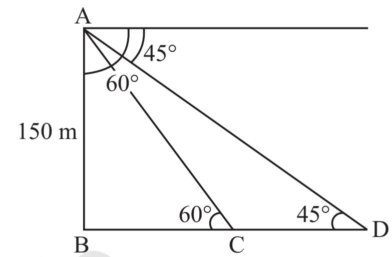
$$P(\text{doublet of odd no.}) = \frac{3}{36} = \frac{1}{12}$$ **Ans.**

20. A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/h. [3]

Solution : From $\Delta\, ABC$, $\frac{AB}{BC} = \tan 60°$

or $$BC = \frac{AB}{\tan 60°}$$

$$BC = \frac{150}{\sqrt{3}}\ \text{m}$$



From $\Delta\, ABD$, $\frac{AB}{BD} = \tan 45°$ or $AB = BD$ [$\because \tan 45° = 1$]

$$\Rightarrow \quad BD = 150\ \text{m}$$

Distance covered in 2 min $= BD - BC$

$$= 150 - \frac{150}{\sqrt{3}} = \frac{150\sqrt{3} - 150}{\sqrt{3}}$$

$$\text{Distance covered in 1 hour} = \frac{150\left(\sqrt{3} - 1\right)}{\sqrt{3} \times 2} \times 60\ \text{m}$$

$$\text{Speed} = \frac{4500(\sqrt{3} - 1)}{\sqrt{3}}$$

$$= 4500 - 1500\sqrt{3}$$

$$= 4500 - 2598 = 1902\ \text{m/hr}$$

Hence, the speed of boat is 1902 m/hr **Ans.**

SECTION — D

21. Construct an isosceles triangle with base 8 cm and altitude 4 cm. Construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of the isosceles triangle. [4]

Solution : Steps of construction—

(i) Draw $BC = 8$ cm.

(ii) Construct XY, the perpendicular bisector of line segment BC, meeting BC at M.

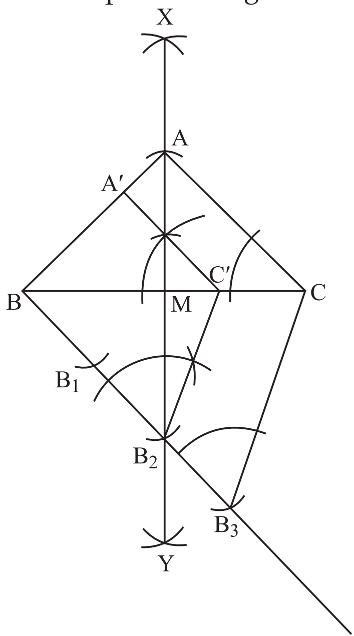
(iii) Cut $MA = 4$ cm on XM. Join BA and CA, $\Delta\, ABC$ is obtained.

(iv) At B, draw an acute angle in downward direction. Draw 3 arcs B_1, B_2 and B_3 on it.

(v) Join B_3C and at B_2 draw line parallel to B_3C, cutting BC at C'.

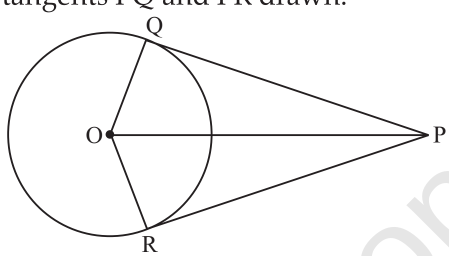
(vi) At C' draw $A'C'$ parallel to AC.

(vii) $\Delta A'C'B$ is required triangle.



22. Prove that the lengths of tangents drawn from an external point to a circle are equal. [4]

Solution : Given, a circle with centre O from point P with tangents PQ and PR drawn.



To prove : $PQ = PR$

Proof : In ΔPQO and ΔPRO

$$\angle PQO = \angle PRO = 90$$

(Angle between radius and tangent)

$$OQ = OR \text{ (Radius)}$$

OP is common

$\therefore$ By R.H.S. congruence rule

$$\Delta PQO \cong \Delta PRO$$

$PQ = PR$ (c.p.c.t.) **Hence Proved.**

23. The ratio of the sums of first m and first n terms of an A. P. is $m^2 : n^2$.

Show that the ratio of its m^{th} and n^{th} terms is $(2m-1) : (2n-1)$. [4]

Solution : Let a be first term and d is common difference.

Then,
$$\frac{S_m}{S_n} = \frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]} = \frac{m^2}{n^2}$$

$$\Rightarrow \quad n\,[2a + (m-1)\,d] = m\,[2a + (n-1\,d]$$

$$\Rightarrow \quad 2an + nd\,(m-1) = 2\,am + md\,(n-1)$$

$$\Rightarrow \quad 2a\,(n-m) = [m\,(n-1) - n\,(m-1)]\,d$$

$$= (mn - m - mn + n)\,d$$

$$2a\,(n-n) = (n-m)\,d$$

or $\quad 2a = d$

Now,
$$\frac{T_m}{T_n} = \frac{a+(m-1)d}{a+(n-1)d}$$

$$= \frac{a+(m-1)2a}{a+(n-1)2a} = \frac{2m-1}{2n-1}$$

Hence Proved.

24. Speed of a boat in still water is 15 km/h. It goes 30 km upstream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream. [4]

Solution : Let speed of stream be x

according to question,

$$\frac{30}{x+15} + \frac{30}{15-x} = 4 + \frac{30}{60}$$

$$30\,\frac{(15-x+x+15)}{(15)^2-(x)^2} = \frac{9}{2}$$

$$\frac{900}{225-x^2} = \frac{9}{2}$$

$$\Rightarrow \quad \frac{2}{9} \times 900 = 225 - x^2$$

$$200 = 225 - x^2$$

or $\quad -x^2 = 200 - 225 = -25$

or $\quad x = 5$ km/hr

Speed of stream is 5 km/hr **Ans.**

25. If $a \neq b \neq 0$, prove that the points (a, a^2), (b, b^2) $(0, 0)$ will not be collinear. [4]

Solution : Area $= \frac{1}{2}\left|a\,(b^2-c^2) + b(c^2-a^2) + c(a^2-b^2)\right|$

$$= \frac{1}{2}\left|a(b-c)\,(b+c) - a^2(b-c) - bc(b-c)\right|$$

$$= \frac{1}{2}\left|(b-c)\,(a\,(b+c) - a^2 - bc)\right|$$

$$= \frac{1}{2}\left|(b-c)\,(ab + ac - a^2 - bc)\right|$$

$$= \frac{1}{2}\left|(b-c)\,(a-b)\,(c-a)\right|$$

this can never be zero as $a \neq b \neq c$

Hence these point can never be collinear.

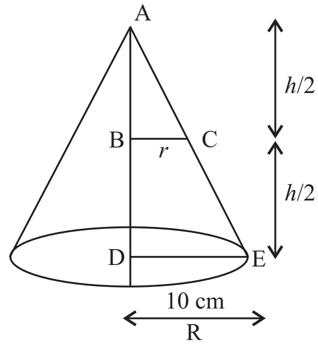
Hence Proved.

26. The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of the two parts. [4]

Solution : Let $BC = r$ cm & $DE = R$ cm

Since B is mid-point of AD & $BC \parallel DE$

$\therefore$ C is mid-point of AE or $AC = CE$



Also $\Delta ABC \sim \Delta ADE$

$$\therefore \quad \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE} = \frac{1}{2}$$

$$BC = \frac{1}{2} DE$$

$$r = \frac{1}{2} R \text{ or } R = 2r$$

$$\text{Now, } \frac{\text{Volume of cone}}{\text{Volume of frustum}} = \frac{\frac{1}{3}\pi r^2\left(\frac{h}{2}\right)}{\frac{1}{3}\pi\left(\frac{h}{2}\right)\left(R^2 + r^2 + Rr\right)}$$

$$= \frac{r^2}{\left(R^2 + r^2 + Rr\right)} = \frac{r^2}{4r^2 + r^2 + 2r \cdot r}$$

$$= \frac{r^2}{7r^2} = \frac{1}{7} \text{ or } 1:7 \qquad \textbf{Ans.}$$

27. Peter throws two different dice together and finds the product of the two numbers obtained. Rina throws a die and squares the number obtained.

Who has the better chance to get the number 25. [4]

Solution : Total possible events in case of peter is 36 favourable outcome is (5, 5)

So $\quad n(E) = 1$

So $\quad P(\text{getting 25 as product}) = \dfrac{1}{36}$

While total possible event in case of Rina is 6

Favourable outcome is 5

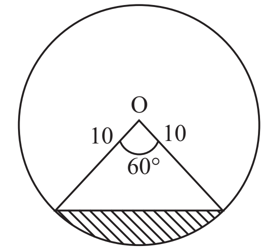
$$n(E) = 1$$

So $\quad P(\text{square is 25}) = \dfrac{1}{6}$

As $\dfrac{1}{6} > \dfrac{1}{36}$, so Rina has better chance. **Ans.**

28. A chord PQ of a circle of radius 10 cm subtends an angle of 60° at the centre of circle. Find the area of major and minor segments of the circle. [4]

Solution : $r = 10$ cm, $i = 60°$



Minor Segment

$$\text{Area of minor segment} = \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin i$$

$$= \frac{60}{360} \times 3{\cdot}14 \times 10 \times 10 - \frac{1}{2} \times 10 \times 10 \sin 60°$$

$$= \frac{1}{6} \times 3{\cdot}14 \times 100 - \frac{1}{2} \times 100 \times \frac{\sqrt{3}}{2}$$

$$= \frac{314}{6} - \frac{100}{4} \times 1{\cdot}73$$

$$= \frac{314}{6} - \frac{173}{4} = \frac{628 - 519}{12} = \frac{109}{12} \text{ cm}^2 \qquad \textbf{Ans.}$$

Area of major segment = Area of circle – Area of minor segment

$$= \pi r^2 - \frac{109}{12} \text{ cm}^2 = 3{\cdot}14 \times 10 \times 10 - \frac{109}{12}$$

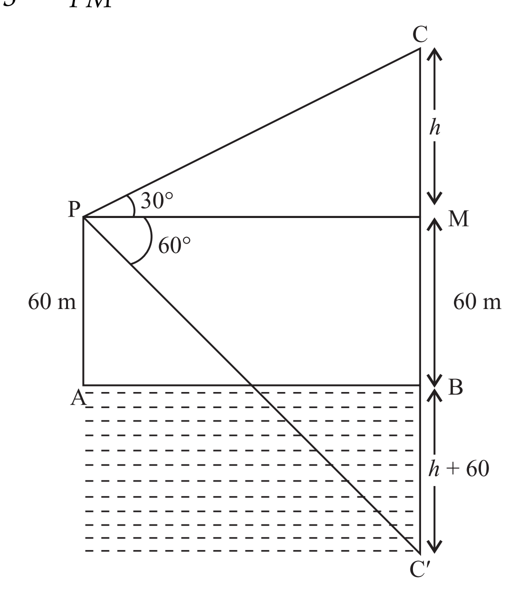
$$= 314 - \frac{109}{12} = \frac{3768 - 109}{12} = \frac{3659}{12} \text{ cm}^2 \qquad \textbf{Ans.}$$

29. The angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water of lake is 60°. Find the height of the cloud from the surface of water. [4]

Solution : In ΔCMP

$$\tan 30° = \frac{CM}{PM}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{PM} \text{ or } PM = \sqrt{3}\, h \qquad \text{...(i)}$$



In $\Delta PMC'$

$$\tan 60° = \frac{C'M}{PM}$$

$$= \frac{h+60+60}{PM} = \sqrt{3}$$

or $$PM = \frac{h+120}{\sqrt{3}} \quad ...(ii)$$

From (i) and (ii)

$$\sqrt{3}\,h = \frac{h+120}{\sqrt{3}}$$

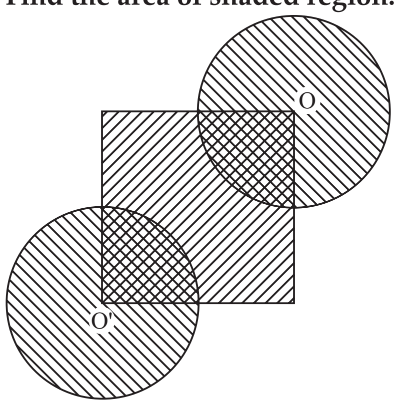
$$\Rightarrow \quad 3h = h + 120$$

$$2h = 120 \Rightarrow h = 60 \text{ m}$$

Height of cloud from surface of water $= h + 60$

$= 60 + 60 = 120$ m. **Ans.**

30. In the given figure, the side of square is 28 cm and radius of each circle is half of the length of the side of the square where *O* and *O'* are centres of the circles. Find the area of shaded region. [4]



Solution : $r = \frac{1}{2}$ (side) = 14 cm, $\because$ side = 28 cm

Area of shaded region

= 2 × (area of circle) + area of square

– 2 × (area of quadrant)

$$= 2 \times \pi r^2 + (\text{side})^2 - 2\left(\frac{1}{4}\times\pi r^2\right)$$

$$= 2\pi r^2 - \frac{1}{2}\pi r^2 + (\text{side})^2 = \frac{3}{2}\pi r^2 + (\text{side})^2$$

$$= \frac{3}{2}\times\frac{22}{7} \times 14 \times 14 + 28 \times 28 = 924 + 784$$

$= 1708$ cm^2 **Ans.**

31. In a hospital used water is collected in a cylindrical tank of diameter 2 m and height 5 m. After recycling, this water is used to irrigate a park of hospital whose length is 25 m and breadth is 20 m. If tank is filled completely then what will be height of standing water used for irrigating the park. Write your views on recycling of water. [4]

Solution : Given, diameter of cylinder $(d) = 2$ m

Radius of cylinder $(r) = 1$ m

Height of cylinder $(h_1) = 5$ m

Length of park $(l) = 25$ m

Breadth of park $(b) = 20$ m

Let height of standing water in the park $= h$

Volume of water used to irrigate the park = Volume stored in cylindrical tank

$$l \times b \times h = \pi r^2 h_1$$

$$25 \times 20 \times h = \frac{22}{7} \times 1 \times 1 \times 5$$

$$h = \frac{22\times5}{7\times25\times20} = \frac{3{\cdot}14}{100} = 0{\cdot}0314 \text{ m}$$

$= 3{\cdot}14$ cm

Recycling of water is very important as it saves wastage of fresh water for work like irrigation. Recycled water can be used for cleaning vehicles etc. also. **Ans.**

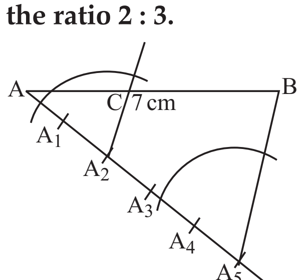
Mathematics 2017 (Delhi) Term II SET II

Note : Except for the following questions, all the remaining questions have been asked in previous set.

SECTION — B

10. Draw a line segment of length 7 cm and divide it internally in the ratio 2 : 3. [2]

Solution :



Steps of construction–

(i) Draw $AB = 7$ cm.

(ii) At A draw an acute angle with 5 equidistant marks A_1, A_2, A_3, A_4, A_5.

(iii) Join A_5B

(iv) Draw $A_2C \parallel A_5B$ to get point C on AB

(v) $AC : CB = 2 : 3$

SECTION — C

19. A metallic solid sphere of radius 10.5 cm is melted and recasted into smaller solid cones, each of radius 3.5 cm and height 3 cm. How many cones will be made ? [3]

Solution : Volume of metal in cones = Volume of solid sphere

Let n = number of cones

$n \times$ volume of each cone = volume of solid sphere

$$n = \frac{\text{Volume of sphere}}{\text{Volume of cone}} = \frac{\frac{4}{3}\pi r_{sp}^3}{\frac{1}{3}\pi r_{\text{cone}}^2 h}$$

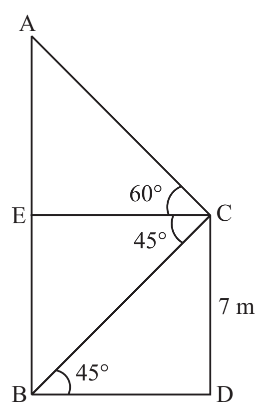
$$= \frac{4 \times 10{\cdot}5 \times 10{\cdot}5 \times 10{\cdot}5}{3{\cdot}5 \times 3{\cdot}5 \times 3}$$

$$= \frac{4 \times 105 \times 105 \times 105 \times 10 \times 10}{35 \times 35 \times 3 \times 10 \times 10 \times 10} = \frac{4 \times 3 \times 105}{10} = 126$$

So, 126 cones will be made. **Ans.**

20. From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower. [3]

Solution : Let C be top of a 7 m building CD and AB be tower. From C, draw $CE \perp AB$, so $EBDC$ is a rectangle.



From $\Delta\, CBD$, $\tan 45° = \dfrac{CD}{BD}$

or $\quad BD = CD = 7$ m

From $\Delta\, AEC$

$$\frac{AE}{EC} = \tan 60°$$

$$\Rightarrow \quad AE = EC \tan 60 = 7\sqrt{3}$$

[$\because$ EC = BD]

Height of tower is $AB = AE + EB = AE + DC$

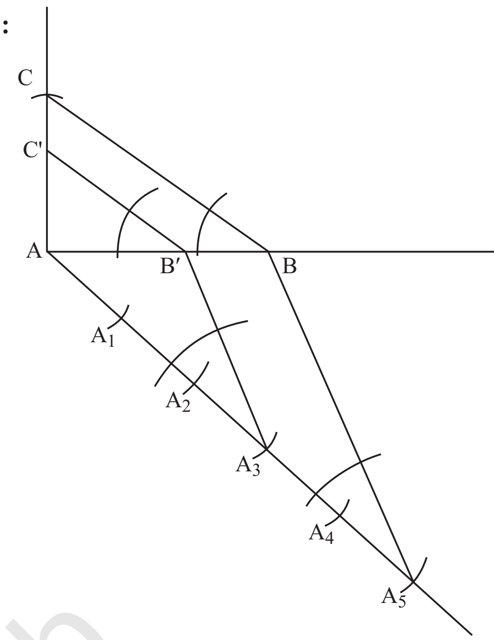
$$= 7\sqrt{3} + 7$$

$$= 7\left(\sqrt{3}+1\right) \text{ m.}$$ **Ans.**

SECTION — D

28. Draw a right triangle in which the sides (other than the hypotenuse) are of lengths 4 cm and 3 cm. Now construct another triangle whose sides are $\frac{3}{5}$ times the corresponding sides of the given triangle. [4]

Solution :



Steps of construction—

(i) Draw $AB = 4$ cm

(ii) Draw $AC \perp AB$ of 3 cm

(iii) Join BC

(iv) Draw an acute angle at A with 5 equidistant marks

(v) Join A_5B

(vi) Draw $A_3B' \parallel A_5B$.

(vii) Draw $B'C' \parallel BC$.

(viii) $AB'C'$ is required triangle.

29. If the sum of first m terms of an A.P. is the same as the sum of its first n terms, show that the sum of its first $(m + n)$ terms is zero. [4]

Solution : Let a be first term and d is common difference of given A.P. then,

$$S_m = S_n$$

$$\frac{m}{2}\{2a + (m-1)\,d\} = \frac{n}{2}\{2a + (n-1)\,d\}$$

$$\Rightarrow \quad \frac{2am}{2} + \frac{m}{2}(m-1)\,d - \frac{2an}{2} - \frac{n}{2}(n-1)\,d = 0$$

$$\Rightarrow \quad 2am - 2an + \{m(m-1) - n\,(n-1)\}\,d = 0$$

$$\Rightarrow \quad 2a\,(m-n) + (m^2 - m - n^2 + n)\,d = 0$$

$$\Rightarrow \quad 2a\,(m-n) + (m^2 - n^2 - (m-n)\,d = 0$$

$$\Rightarrow \quad 2a\,(m-n) + (m-n)\,(m+n-1)\,d = 0$$

$$\Rightarrow \quad (m-n)\,(2a + (m+n-1)\,d = 0$$

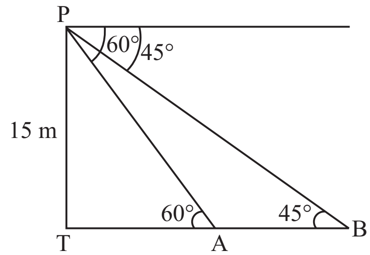
$$\Rightarrow \quad 2a + (m+n-1)\,d = 0$$

Now, $$S_{m+n} = \frac{m+n}{2}\{2a + (m+n-1)\,d\}$$

$$= \frac{m+n}{2} \times 0 = 0$$ **Hence Proved.**

30. Two points A and B are on the same side of a tower and in the same straight line with its base. The angles of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points. **[4]**

Solution : Let PT be tower



From $\Delta\, PTA$

$$\tan 60° = \frac{PT}{TA} \Rightarrow TA = \frac{15}{\sqrt{3}}$$

from $\Delta\, PTB$

$$\tan 45° = \frac{PT}{TB} \Rightarrow TB = PT = 15 \text{ m}$$

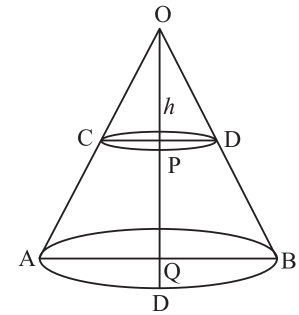
Distance between two points

$$AB = TB - TA$$

$$= 15 - \frac{15}{\sqrt{3}} = \frac{15(\sqrt{3}-1)}{\sqrt{3}} \text{ m}$$ **Ans.**

31. The height of a cone is 30 cm. From its topside a small cone is cut by a plane parallel to its base. If volume of smaller cone is $\frac{1}{27}$ of the given cone, then at what height it is cut from its base ? **[4]**

Solution : Volume of original cone OAB



$$= \frac{1}{3}\pi R^2 H \text{ cm}^3$$

$$= \frac{1}{3}\pi \times R^2 \times 30 \text{ cm}^3$$

$$= 10\pi R^2 \text{ cm}^3$$

Volume of small cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{27} \times \text{volume of cone OAB (given)}$$

$$\frac{1}{27} \times 10\pi R^2 = \frac{1}{3}\pi r^2 h$$

$$h = \frac{\frac{1}{27}\times 10\pi R^2}{\frac{1}{3}\pi r^2} = \frac{10}{9}\left(\frac{R}{r}\right)^2$$

From similar $\Delta\, OPD$ and $\Delta\, OQB$

$$\frac{QB}{PD} = \frac{OQ}{OP} = \frac{30}{h}$$

so $$\frac{R}{r} = \frac{30}{h}$$

$$h = \frac{10}{9}\left(\frac{30}{h}\right)^2 = \frac{9000}{9h^2}$$

$\Rightarrow$ $h^3 = 1000$ or $h = 10$ cm

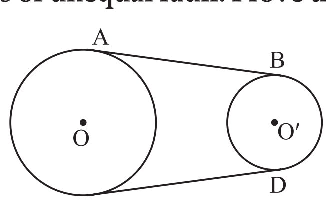
So height from base $= 30 - 10 = 20$ cm. **Ans.**

Mathematics 2017 (Delhi) Term II SET III

Note : Except for the following questions, all the remaining questions have been asked in previous sets.

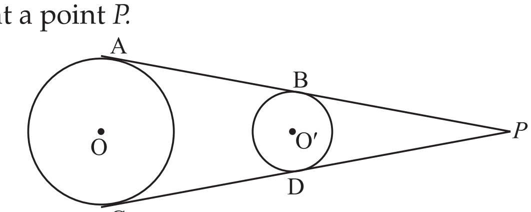
SECTION — B

10. In the figure, AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$.



[2]

Solution : Construction : Extend AB and CD to meet at a point P.



Now, PA and PC are tangents of circle with centre O

So $PA = PC$ ...(i)

PB and PD are tangent on circle with centre O'

So $PB = PD$ ...(ii)

On subtracting eq. (ii) from eq. (i)

$$PA - PB = PC - PD$$

$$AB = CD$$ **Hence Proved.**

SECTION — C

18. If the p^{th} term of an A.P. is q and q^{th} term is p, prove that its n^{th} term is $(p + q - n)$. [3]

Solution : Let a be first term and d be common difference.

Then, p^{th} term $= q \Rightarrow a + (p-1)d = q$ ...(i)

q^{th} term $= p \Rightarrow a + (q-1)d = p$ ...(ii)

On subtracting eq. (ii) from eq. (i)

$$(p-1)\,d - (q-1)d = q - p$$
$$pd - d - qd + d = q - p$$
$$(p-q)\,d = q - p \text{ or } d = \frac{q-p}{p-q} = -1$$

Putting value of d in eq. (i)

$$a + (p-1)(-1) = q$$
$$a = q + p - 1$$
$$n^{th} \text{ term} = a + (n-1)d$$
$$= q + p - 1 + (n-1)(-1)$$
$$= q + p - 1 + 1 - n = q + p - n$$
$$T_n = q + p - n$$ **Hence Proved.**

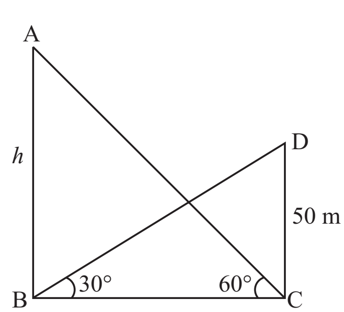
19. A solid metallic sphere of diameter 16 cm is melted and recasted into smaller solid cones, each of radius 4 cm and height 8 cm. Find the number of cones so formed. [3]

Solution :

$$\text{No. of cones formed} = \frac{\text{Volume of sphere melted}}{\text{Volume of cone}}$$
$$= \frac{\frac{4}{3}\pi r_{sp}^3}{\frac{1}{3}\pi r_{\text{cone}}^2 h} = \frac{4\times8\times8\times8}{4\times4\times8}$$
$$= 16$$ **Ans.**

20. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If height of the tower is 50 m, find the height of the hill. [3]

Solution : Let AB be hill and DC be tower.



From Δ ABC $\frac{AB}{BC} = \tan 60°$

$$h = BC \tan 60° = \sqrt{3}\ BC$$

From Δ DBC, $\frac{DC}{BC} = \tan 30° = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad BC = \sqrt{3}\ DC = 50\sqrt{3}$$
$$h = BC\sqrt{3}$$
$$= 50\sqrt{3} \times \sqrt{3} = 50 \times 3$$
$$= 150 \text{ m}$$ **Ans.**

SECTION — D

29. If the p^{th} term of an A.P. is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, prove that the sum of first pq terms of the A.P. is $\left(\frac{pq+1}{2}\right)$. [4]

Solution : Let a be first term and d is common difference

Then $a_p = \frac{1}{q} \Rightarrow a + (p-1)d = \frac{1}{q}$ ...(i)

$a_q = \frac{1}{p} \Rightarrow a + (q-1)d = \frac{1}{p}$ ...(ii)

Subtracting eq. (ii) from eq. (i)

$$pd - qd + = \frac{1}{q} - \frac{1}{p} = \frac{p-q}{pq}$$
$$(p-q)d = \frac{p-q}{pq} \text{ or } d = \frac{1}{pq}$$

Putting value of d in eq. (i)

$$a + (p-1)\frac{1}{pq} = \frac{1}{q} \Rightarrow a = \frac{1}{q} - \frac{p}{pq} + \frac{1}{pq}$$
$$a = \frac{1}{pq}$$

Now, $S_{pq} = \frac{pq}{2}\,(2a + (pq-1)\,d)$

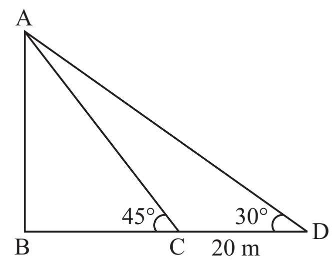
$$= \frac{pq}{2}\left(\frac{2}{pq} + (pq-1)\frac{1}{pq}\right)$$
$$= \frac{pq}{2}\left(\frac{2}{pq} + \frac{pq}{pq} - \frac{1}{pq}\right)$$
$$S_{pq} = \frac{pq}{2}\left(\frac{1+pq}{pq}\right)$$
$$= \frac{(pq+1)}{2}$$ **Hence Proved.**

30. An observer finds the angle of elevation of the top of the tower from a certain point on the ground as

30°. If the observer moves 20 m towards the base of the tower, the angle of elevation of the top increases by 15°, find the height of the tower. **[4]**

Solution : Let AB be tower of height h

From $\Delta\ ABC$, $\dfrac{AB}{BC} = \tan 45°$



$$h = BC$$

From $\Delta\ ABD$, $\dfrac{AB}{BD} = \tan 30°$

$$h = \frac{BD}{\sqrt{3}} \text{ or } BD = \sqrt{3}\, h$$

$$CD = BD - BC$$

$$\Rightarrow \quad = \sqrt{3}\, h - h = \left(\sqrt{3}-1\right) h$$

$$\Rightarrow \quad 20 \text{ m} = (\sqrt{3}-1)\, h$$

$$\Rightarrow \quad h = \frac{20}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{20(\sqrt{3}+1)}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{20(\sqrt{3}+1)}{2} = 10\left(\sqrt{3}+1\right) \text{m}$$ **Ans.**

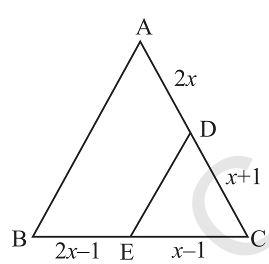
Mathematics 2016 Term I

Time allowed : 3 Hours **Maximum marks : 90**

SECTION — A

1. In ΔABC, D and E are points AC and BC respectively such that DE || AB. If AD = 2x, BE = 2x – 1, CD = x + 1 and CE = x – 1, then find the value of x. **[1]**

Solution :



DE || AB

So, $\dfrac{AD}{CD} = \dfrac{BE}{EC}$ [By B.P.T.]

$$\Rightarrow \quad \frac{2x}{x+1} = \frac{2x-1}{x-1}$$

$$\Rightarrow \quad 2x(x-1) = (x+1)(2x-1)$$

$$\Rightarrow \quad 2x^2 - 2x = 2x^2 + 2x - x - 1$$

$$\Rightarrow \quad -2x = x - 1$$

$$\Rightarrow \quad 1 = 3x$$

or $x = \dfrac{1}{3}$ **Ans.**

2. In A, B and C are interior angles of $\Delta\ ABC$, then prove that : sin $\dfrac{(A+C)}{2}$ = cos $\dfrac{B}{2}$. **[1]**

Solution : In $\Delta\ ABC$

$$\angle A + \angle B + \angle C = 180°$$

$$\angle A + \angle C = 180° - \angle B$$

Divide by 2 on both sides

$$\frac{\angle A + \angle C}{2} = \frac{180° - \angle B}{2}$$

$$\frac{\angle A + \angle C}{2} = 90° - \frac{\angle B}{2}$$

$$\sin\left(\frac{\angle A + \angle C}{2}\right) = \sin\left(90° - \frac{\angle B}{2}\right)$$

$$\sin\left(\frac{\angle A + \angle C}{2}\right) = \cos\frac{\angle B}{2}$$

$$\sin\frac{(A+C)}{2} = \cos\frac{B}{2}$$

Hence Proved.

3. If x = 3 sin θ and y = 4 cos θ, find the value of $\sqrt{16x^2+9y^2}$. **[1]**

Solution : $x = 3 \sin\theta$

$$\Rightarrow \quad x^2 = 9\sin^2\theta$$

$$\sin^2\theta = \frac{x^2}{9} \quad \text{...(i)}$$

And $y = 4\cos\theta$

$$y^2 = 16\cos^2\theta$$

$$\cos^2\theta = \frac{y^2}{16} \quad \text{...(ii)}$$

On adding eq. (i) and eq. (ii)

$$\sin^2\theta + \cos^2\theta = \frac{x^2}{9} + \frac{y^2}{16}$$

$$1 = \frac{x^2}{9} + \frac{y^2}{16}$$

$$1 = \frac{16x^2 + 9y^2}{144}$$

$$16x^2 + 9y^2 = 144$$
$$\sqrt{16x^2 + 9y^2} = \sqrt{144}$$
$$\sqrt{16x^2 + 9y^2} = 12$$ **Ans.**

4. If empirical relationship between mean, median and mode is expressed as mean = k(3 median – mode), then find the value of k. [1]

Solution : Given, mean = k(3 median – mode)

As we know, mode = 3 median – 2 mean

$\therefore$ mean = k[3 median – (3 median – 2 mean)]

mean = k[3 median – 3 medain + 2 mean]

mean = 2 k mean

$2k$ mean – mean = 0

mean $[2k - 1] = 0$

$2k - 1 = 0$

$2k = 0 + 1$

$k = 1/2$ **Ans.**

SECTION — B

5. Express 23150 as product of its prime factors. Is it unique ? [2]

Solution : Prime factors of 23150 = 2 × 5 × 5 × 463

As per the fundamental theorem of Arithmetic every number has a unique factorisation.

2	23150
5	11575
5	2315
463	463
	1

Ans.

6. State whether the real number 52.0521 is rational or not. If it is rational express it in the form $\frac{p}{q}$, where p, q are co-prime, integers and $q \neq 0$. What can you say about prime factorisation of q ? [2]

Solution : 52.0521

$$\Rightarrow \quad 52.0521 = \frac{520521}{10000}$$

Yes, it is rational number.

where $q = 10000 = 2^4 \times 5^4$

The given decimal expression is a terminating decimal as the factors of q consist only 2 and 5.

Ans.

7. Given the linear equation $x - 2y - 6 = 0$, write another linear equation in these two variables, such that the geometrical representation of the pair so formed is :

(i) coincident lines (ii) intersection lines [2]

Solution : (i) Given, $x - 2y - 6 = 0$

For line to be coincident

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{C_1}{C_2}$$

Thus one possible option will be

$$2x - 4y - 12 = 0$$

Here, $a_1 = 1, b_1 = -2, c_1 = -6$

$a_2 = 2, b_2 = -4, c_2 = -12$

$$\frac{a_1}{a_2} = \frac{1}{2};\ \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2};\ \frac{c_1}{c_2} = \frac{-6}{-12} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, it is showing coincident lines. **Ans.**

(ii) Given, $x - 2y - 6 = 0$

For intersecting lines

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Thus, one possible option will be,

$$2x - 7y - 13 = 0$$

Here, $a_1 = 1, b_1 = -2, c_1 = -6$

$a_2 = 2, b_2 = -7, c_2 = -13$

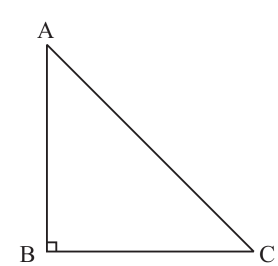
Here, $$\frac{a_1}{a_2} = \frac{1}{2};\ \frac{b_1}{b_2} = \frac{-2}{-7} = \frac{2}{7}$$

$$\Rightarrow \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, it is representing intersecting lines. **Ans.**

8. In an isosceles $\Delta\, ABC$ right angled at B, prove that $AC^2 = 2AB^2$. [2]

Solution : In ΔABC, AB = BC [$\because$ triangle is isosceles] ...(i)



In ΔABC by pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = AB^2 + AB^2$ [From (i)]

$AC^2 = 2AB^2$ **Hence Proved.**

9. Prove the following identity :

$$\left[\frac{1 - \tan A}{1 - \cot A}\right]^2 = \tan^2 A : \angle A \text{ is acute}$$ **[2]**

Solution : Given, $\left[\dfrac{1-\tan A}{1-\cot A}\right]^2 = \tan^2 A : \angle A$ is acute

$$\text{L.H.S.} = \left[\frac{1-\tan A}{1-\cot A}\right]^2$$

$$= \left[\frac{1-\dfrac{\sin A}{\cos A}}{1-\dfrac{\cos A}{\sin A}}\right]^2$$

$$= \left[\frac{\dfrac{\cos A-\sin A}{\cos A}}{\dfrac{\sin A-\cos A}{\sin A}}\right]^2$$

$$= \left[\frac{(\cos A-\sin A)\sin A}{-(\cos A-\sin A)\cos A}\right]^2$$

$$= \left[-\frac{\sin A}{\cos A}\right]^2$$

$$= [-\tan A]^2$$

$= \tan^2 A = \text{R.H.S.}$ **Hence Proved.**

10. Given below is a cumulative frequency distribution table. Corresponding to it, make an ordinary frequency distribution table. [2]

x	cf
More than or equal to 0	45
More than or equal to 10	38
More than or equal to 20	29
More than or equal to 30	17
More than or equal to 40	11
More than or equal to 50	6

Solution :

C.I.	Frequency
0 – 10	07 (45 – 38)
10 – 20	09 (38 – 29)
20 – 30	12 (29 – 17)
30 – 40	6(17 – 11)
40 – 50	5 (11 – 6)
50 – 60	6(6 – 0)

SECTION — C

11. Find LCM and HCF of 3930 and 1800 by prime factorisation method. [3]

Solution : By prime factorization method,

Factors of 3930 and 1800 are,

2	3930
3	1965
5	655
131	131
	1

2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

So, $3930 = 2 \times 3 \times 5 \times 131$

$1800 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

then, $\text{HCF} = 2 \times 3 \times 5 = 30$

and, $\text{LCM} = 2 \times 3 \times 5 \times 131 \times 2 \times 2 \times 3 \times 5$

$= 235800$ **Ans.**

12. Using division algorithm, find the quotient and remainder on dividing $f(x)$ by $g(x)$ where $f(x) = 6x^3 + 13x^2 + x - 2$ and $g(x) = 2x + 1$. [3]

Solution : Given, $f(x) = 6x^3 + 13x^2 + x - 2$

and $g(x) = 2x + 1$,

$f(x) \div g(x) \Rightarrow$

$$\begin{array}{r|l} 2x+1 & 6x^3 + 13x^2 + x - 2 \;(3x^2 + 5x - 2 \\ & \underline{6x^3 + 3x^2} \\ & \quad - \quad - \\ & \qquad 10x^2 + x - 2 \\ & \qquad \underline{10x^2 + 5x} \\ & \qquad - \quad - \\ & \qquad\qquad -4x - 2 \\ & \qquad\qquad \underline{-4x - 2} \\ & \qquad\qquad + \quad + \\ & \qquad\qquad\quad 0 \end{array}$$

quotient $= 3x^2 + 5x - 2$, remainder $= 0$ **Ans.**

13. If three zeroes of a polynomial $x^4 - x^3 - 3x^2 + 3x$ are 0, $\sqrt{3}$ and $-\sqrt{3}$, then find the fourth zero. [3]

Solution : Let $P(x) = x^4 - x^3 - 3x^2 + 3x$

Given, $0, \sqrt{3}, -\sqrt{3}$ are three zeroes, so

$x = 0,$

$x = \sqrt{3}$ and $x = -\sqrt{3}$

$\Rightarrow$ $(x - \sqrt{3}) = 0$ and $x + \sqrt{3} = 0$

Here, $x(x+\sqrt{3})(x-\sqrt{3})$ will also be the factor of $P(x)$.

Or, $x(x^2 - 3)$ will be the factor of $P(x)$.

then

$$\begin{array}{r|l} x^3 - 3x & x^4 - x^3 - 3x^2 + 3x \;(x - 1 \\ & \underline{x^4 - 3x^2} \\ & - \quad + \\ & \quad -x^3 + 3x \\ & \quad \underline{-x^3 + 3x} \\ & \quad + \quad - \\ & \qquad 0 \end{array}$$

quotient $= (x - 1)$

So fourth zero $\Rightarrow x - 1 = 0$

$$x = 1$$

Hence four zeroes will be $1, 0, \sqrt{3}, -\sqrt{3}$. **Ans.**

14. Solve the following pair of equations by reducing them to a pair of linear equations : [3]

$$\frac{1}{x} - \frac{4}{y} = 2$$

$$\frac{1}{x} + \frac{3}{y} = 9$$

Solution : Given, $\frac{1}{x} - \frac{4}{y} = 2$

$$\frac{1}{x} + \frac{3}{y} = 9$$

Let $\frac{1}{x} = u, \; \frac{1}{y} = v$

So, $u - 4v = 2$ ...(i)

$u + 3v = 9$ ...(ii)

On solving eq. (i) and eq. (ii)

$$\begin{array}{r} u - 4v = 2 \\ u + 3v = 9 \\ - \quad - \\ \hline -7v = -7 \\ v = 1 \end{array}$$

Putting the value of v in eq. (i)

$$u - 4v = 2$$
$$u - 4 \times 1 = 2$$
$$u - 4 = 2$$
$$u = 2 + 4$$
$$u = 6$$

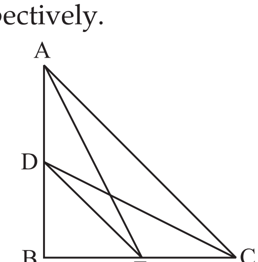
So $v = 1 \Rightarrow \frac{1}{y} = 1, y = 1$

$$u = 6 \Rightarrow \frac{1}{x} = 6, x = \frac{1}{6}$$

Hence, $x = \frac{1}{6}$ and $y = 1$ **Ans.**

15. ΔABC is a right angled triangle in which $\angle B = 90°$. D and E are any point on AB and BC respectively. Prove that $AE^2 + CD^2 = AC^2 + DE^2$. [3]

Solution : In ΔABC, $\angle B = 60°$and D, E are point of AB, BC respectively.



To prove :

$$AC^2 + DE^2 = AE^2 + CD^2$$

In ΔABC by using Pythagoras theorem,

$$AC^2 = AB^2 + BC^2 \quad ...(i)$$

In ΔABE by using Pythagoras theorem

$$AE^2 = AB^2 + BE^2 \quad ...(ii)$$

In ΔBCD by Pythagoras theorem

$$CD^2 = BD^2 + BC^2 \quad ...(iii)$$

In ΔDBE by Pythagoras theorem

$$DE^2 = DB^2 + BE^2 \quad ...(iv)$$

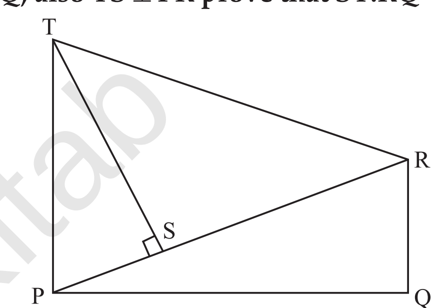
Adding eq. (i) and eq. (iv)

$$AC^2 + DE^2 = AB^2 + BC^2 + BD^2 + BE^2$$
$$= AB^2 + BE^2 + BC^2 + BD^2$$
$$AC^2 + DE^2 = AE^2 + CD^2$$

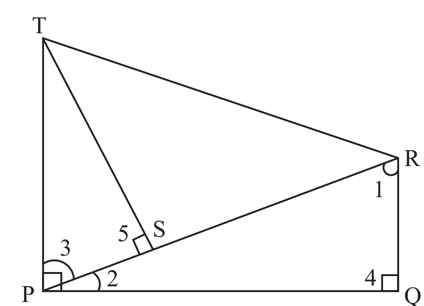
[From eq. (ii) and eq. (iii)]

Hence Proved.

16. In the given figure, RQ and TP are perpendicular to PQ, also TS ⊥ PR prove that ST.RQ = PS.PQ. [3]



Solution :



In Δ RPQ

$$\angle 1 + \angle 2 + \angle 4 = 180°$$
$$\angle 1 + \angle 2 + 90° = 180°$$
$$\angle 1 + \angle 2 = 180° - 90°$$
$$\angle 1 = 90° - \angle 2 \quad ...(i)$$

$\because$ TP ⊥ PQ

$\therefore$ $\angle TPQ = 90°$

$\Rightarrow$ $\angle 2 + \angle 3 = 90°$

$$\angle 3 = 90° - \angle 2 \quad ...(ii)$$

From eq. (i) and eq. (ii)

$$\angle 1 = \angle 3$$

Now in Δ RQP and Δ PST

$\angle 1 = \angle 3$ [Proved above]

$\angle 4 = \angle 5$ [Each 90°]

So by AA similarity

$$\Delta RQP \sim \Delta PST$$

$$\frac{ST}{QP} = \frac{PS}{RQ}.$$

[By c.p.c.t.]

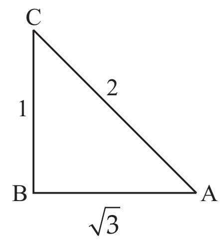
$\Rightarrow$ ST.RQ = PS.PQ **Hence Proved.**

17. If sec A = $\frac{2}{\sqrt{3}}$, find the value of

$$\frac{\tan A}{\cos A}+\frac{1+\sin A}{\tan A}$$

[3]

Solution : Given, sec A = $\frac{2}{\sqrt{3}}$



In Δ ABC

$$AC^2 = AB^2 + BC^2$$
$$2^2 = \left(\sqrt{3}\right)^2 = BC^2$$
$$4 = 3 + BC^2$$
$$BC^2 = 4 - 3$$
$$BC^2 = 1$$
$$BC = 1$$

So, $\tan A = \frac{1}{\sqrt{3}}$; $\cos A = \frac{\sqrt{3}}{2}$; $\sin A = \frac{1}{2}$

$$\frac{\tan A}{\cos A}+\frac{1+\sin A}{\tan A} = \frac{\frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{2}}+\frac{1+\frac{1}{2}}{\frac{1}{\sqrt{3}}}$$

$$= \frac{2}{3}+\frac{\frac{3}{2}}{\frac{1}{\sqrt{3}}}$$

$$= \frac{2}{3}+\frac{3\sqrt{3}}{2}$$

$$= \frac{4+9\sqrt{3}}{6}$$

Ans.

18. Prove that : **[3]**

$\sec^2\theta - \cot^2(90° - \theta) = \cos^2(90° - \theta) + \cos^2\theta$.

Solution : To prove : $\sec^2\theta - \cot^2(90° - \theta) = \cos^2(90° - \theta) + \cos^2\theta$.

$$\text{L.H.S.} = \sec^2\theta - \cot^2(90° - \theta)$$
$$= \sec^2\theta - [\cot(90 - \theta)]^2$$
$$= \sec^2\theta - (\tan\theta)^2$$
$$= \sec^2\theta - \tan^2\theta$$
$$= 1$$
$$\text{R.H.S.} = \cos^2(90 - \theta) + \cos^2\theta$$
$$= [\cos(90 - \theta)^2] + \cos^2\theta$$
$$= (\sin\theta)^2 + \cos^2\theta$$
$$= \sin^2\theta + \cos^2\theta$$
$$= 1$$

Hence, L.H.S. = R.H.S. **Hence Proved**

19. For the month of February, a class teacher of Class IX has the following absentee record for 45 students. Find the mean number of days, a student was absent.

Number of days of absent	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24
Number of students	18	3	6	2	0	1

[3]

Solution :

C.I.	f_i	x_i (mid-value)	$d = x_i - A$	$f_i \times d_i$
0 – 4	18	2	– 12	– 216
4 – 8	3	6	– 8	– 24
8 – 12	6	10	– 4	– 24
12 – 16	2	A = 14	0	00
16 – 20	0	18	4	00
20 – 24	1	22	8	08
	$\Sigma f_i = 30$			$\Sigma f_i d_i = -256$

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 14 + \left(\frac{-256}{30}\right)$$

$$= 14 - 8.53$$

$$= 5.47$$

Ans.

20. **Find the missing frequency (x) of the following distribution, if mode is 34.5 :**

Marks obtained	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of students	4	8	10	x	8

[3]

Solution :

C.I.	Frequency
0 – 10	4
10 – 20	$8 = f_0$
$l = 20 – 30$	$10 = f_1$
30 – 40	$x = f_2$
40 – 50	8

$$\text{Mode} = l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)h$$

$$34.5 = 20+\left(\frac{10-8}{20-8-x}\right)10$$

$$\Rightarrow \quad 34.5 = 20+\left(\frac{2}{12-x}\right)10$$

$$\Rightarrow \quad \frac{14.5}{1} = \frac{20}{12-x}$$

$$\Rightarrow \quad 20 = 14.5\,(12-x)$$

$$\Rightarrow \quad \frac{20}{14.5} = 12-x$$

$$\Rightarrow \quad \frac{40}{29} = 12-x$$

$$\Rightarrow \quad x = 12-\frac{40}{29}$$

$$\Rightarrow \quad x = \frac{348-40}{29}$$

$$\Rightarrow \quad x = \frac{308}{29}$$

$$\Rightarrow \quad x = 10.62$$

Ans.

SECTION — D

21. **Prove that $\sqrt{5}$ is an irrational number. Hence show that $3 + 2\sqrt{5}$ is also an irrational number.** **[4]**

Solution : Let $\sqrt{5}$ be a rational number.

So, $$\sqrt{5} = \frac{p}{q}$$

On squaring both sides

$$5 = \frac{p^2}{q^2}$$

$$q^2 = \frac{p^2}{5}$$

$\Rightarrow$ 5 is a factor of p^2

$\Rightarrow$ 5 is a factor of p.

Now, again let $p = 5c$.

So, $$\sqrt{5} = \frac{5c}{q}$$

On squaring both sides

$$5 = \frac{25c^2}{q^2}$$

$$q^2 = 5c^2$$

$$c^2 = \frac{q^2}{5}$$

$\Rightarrow$ 5 is factor of q^2

$\Rightarrow$ 5 is a factor of q.

Here 5 is a common factor of p, q which contradicts the fact that p, q are co-prime.

Hence our assumption is wrong, $\sqrt{5}$ is an irrational number.

Now we have to show that $3 + 2\sqrt{5}$ is an irrational number. So let us assume

$3 + 2\sqrt{5}$ is a rational number.

$$\Rightarrow \quad 3 + 2\sqrt{5} = \frac{p}{q}$$

$$\Rightarrow \quad 2\sqrt{5} = \frac{p}{q}-3$$

$$\Rightarrow \quad 2\sqrt{5} = \frac{p-3q}{q}$$

$$\Rightarrow \quad \sqrt{5} = \frac{p-3q}{2q}$$

$\frac{p-3q}{2q}$ is in the rational form of $\frac{p}{q}$ so $\sqrt{5}$ is a rational number but we have already proved that $\sqrt{5}$ is an irrational number so contradiction arises because we supposed wrong that $3+2\sqrt{5}$ is a rational number. So we can say that $3+2\sqrt{5}$ is an irrational number.

Hence Proved

22. **Obtain all other zeroes or the polynomial $x^4 + 6x^3 + x^2 – 24x – 20$, if two of its zeroes are $+ 2$ and $– 5$.** **[4]**

Solution : Given, 2, – 5 are the zeroes of polynomial

$$p(x) = x^4 + 6x^3 + x^2 - 24x - 20$$

So $(x-2)$ and $(x+5)$ are factors of $p(x)$
$\Rightarrow (x-2)(x+5)$ is also a factor of $p(x)$
So $(x-2)(x+5) = x^2+3x-10$

$$x^2+3x-10\overline{)x^4+6x^3+x^2-24x-20}(x^2+3x+2$$
$$\begin{array}{r} x^4+3x^3-10x^2 \\ -\quad -\quad + \\ \hline 3x^3+11x^2-24x-20 \\ 3x^3+9x^2-30x \\ -\quad -\quad + \\ \hline 2x^2+6x-20 \\ 2x^2+6x-20 \\ -\quad -\quad + \\ \hline 0 \end{array}$$

So, by remainder theorem,
Dividend = Divisor × Quotient + Remainder

$$x^4+6x^3+x^2-24x-20 = (x^2+3x-10)\times(x^2+3x+2)+0$$
$$= (x^2+3x-10)(x^2+2x+x+2)$$
$$= (x^2+3x-10)[x(x+2)+1(x+2)]$$
$$= (x^2+3x-10)(x+2)(x+1)$$

So other zeros are -2 and -1. **Ans.**

23. Draw graph of following pair of linear equations :
$y = 2(x-1)$
$4x+y = 4$
Also write the coordinate of the points where these lines meets x-axis and y-axis. [4]

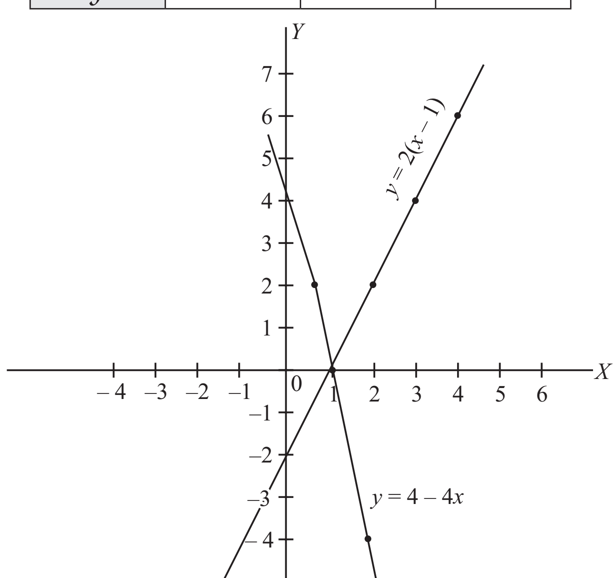
Solution : $y = 2(x-1)$

So,

x	1	2	3	4
y	0	2	4	6

And for, $4x+y=4$
or $y = 4-4x$

x	1	2	1/2
y	0	-4	2



Co-ordinates of point where lines meets
Line $y = 2(x-1)$: x-axis = (1, 0)
and y-axis = (0, – 2)
Line $4x+y=4$: x-axis = (1, 0)
y-axis = (0, 4) **Ans.**

24. A boat goes 30 km upstream and 44 km downstream in 10 hours. The same boat goes 40 km upstream and 55 km downstream in 13 hours. On this information some student guessed the speed of the boat in still water as 8.5 km/h and speed of the stream as 3.8 km/h. Do you agree with their guess ? Explain what do we learn from the incident ? [4]

Solution : Let the speed of boat = x km/hr.
Let the speed of stream = y km/hr.
Speed of boat in upstream = $(x-y)$ km/hr.
Speed of boat in downstream = $(x+y)$ km/hr.

Time taken to cover 30 km upstream = $\dfrac{30}{x-y}$ hrs.

Time taken to cover 40 km downstream = $\dfrac{44}{x+y}$ hrs.

According to question,
Total time taken = 10 hrs.

$$\frac{30}{x-y}+\frac{44}{x+y} = 10 \quad \text{...(i)}$$

Now, Time taken to cover 55 km downstream $= \dfrac{55}{x+y}$ hrs.

Time taken to cover 40 km upstream = $\dfrac{40}{x-y}$ hrs.

Total time taken = 13 hrs.

$$\frac{40}{x-y}+\frac{55}{x+y} = 13 \quad \text{...(ii)}$$

Solving eq. (i) and eq. (ii).

Let $\dfrac{1}{x-y} = u$, $\dfrac{1}{x+y} = v$.

$$30u+44v = 10$$
$$40u+55v = 13$$

or
$$15u+22v = 5 \quad \text{....(iii)}$$
$$8u+11v = \frac{13}{5} \quad \text{....(iv)}$$

Multiplying eq. (iii) by 8 and eq. (iv) by 15, we get

$$\begin{array}{r} 120u+176v = 40 \\ 120u+165v = 39 \\ -\qquad -\qquad - \\ \hline 11v = 1 \end{array}$$

$$v = \frac{1}{11}$$

Putting the value of v in eq. (iii)

$$15u+22v = 5$$

$$\Rightarrow \quad 15u + 22 \times \frac{1}{11} = 5$$

$$\Rightarrow \quad 15u + 2 = 5$$

$$\Rightarrow \quad 15u = 3$$

$$\Rightarrow \quad u = \frac{3}{15}$$

or $$u = \frac{1}{5}$$

Now,

$$v = \frac{1}{11}$$

$$\Rightarrow \quad \frac{1}{x+y} = \frac{1}{11}$$

$$\Rightarrow \quad x + y = 11 \qquad \text{...(v)}$$

And $$u = \frac{1}{5}$$

$$\frac{1}{x-y} = \frac{1}{5}$$

$$\Rightarrow$$

$$\Rightarrow \quad x - y = 5 \qquad \text{...(vi)}$$

On solving eq. (v) and (vi)

$$x + y = 11$$
$$x - y = 5$$
$$+ \quad - \quad +$$
$$2x \quad = 16$$

or $$x = 8$$

Put the value of x in eq. (v)

$$8 + y = 11$$
$$y = 11 - 8$$
$$y = 3$$

The speed of boat in still water = 8 km/hr.

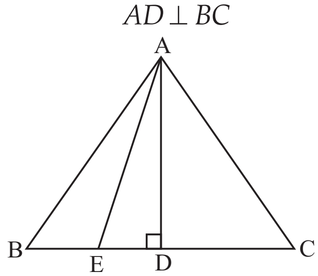
The speed of stream = 3 km/hr.

We learn that the speed of boat is slow in upstream and fast in downstream. **Ans.**

25. In an equilateral $\Delta\, ABC$, E is any point on BC such that $BE = \frac{1}{4}\, BC$. Prove that $16\, AE^2 = 13\, AB^2$. **[4]**

Solution : Given $\quad BE = \frac{1}{4}BC$

Draw $\quad AD \perp BC$



In $\Delta\, AED$ by pythagoras theorem,

$$AE^2 = AD^2 + DE^2 \qquad \text{...(i)}$$

In $\Delta\, ADB$

$$AB^2 = AD^2 + BD^2$$

$$AB^2 = AE^2 - DE^2 + BD^2 \qquad \text{[From (i)]}$$

$$= AE^2 - DE^2 + (BE + DE)^2$$

$$AB^2 = AE^2 - DE^2 + BE^2 + DE^2 + 2BE.DE$$

$$AB^2 = AE^2 + BE^2 + 2BE.DE$$

$$AB^2 = AE^2 + \left(\frac{BC}{4}\right)^2 + 2\frac{BC}{2}\cdot(BD - BE)$$

$$AB^2 = AE^2 \frac{BC^2}{16} + \frac{BC}{2}\left(\frac{BC}{2} - \frac{BC}{4}\right)$$

$$AB^2 = AE^2 + \frac{AB^2}{16} + \frac{AB}{2}\left[\frac{2\,AB - AB}{4}\right]$$

$$AB^2 = AE^2 + \frac{AB}{16} + \frac{AB}{2} \times \frac{AB}{4}$$

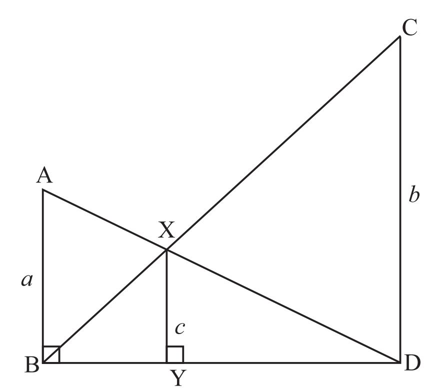
$$AB^2 + \frac{AB^2}{16} + \frac{AB^2}{8} = AE^2$$

$$\frac{16AB^2 - AB^2 - 2AB^2}{16} = AE^2$$

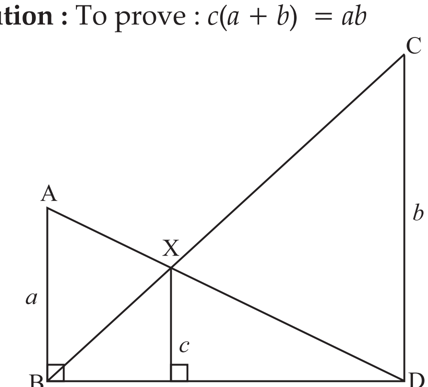
$$16AB^2 - 3AB^2 = 16AE^2$$

$$13AB^2 = 16AE^2$$ **Hence Proved.**

26. In the figure, if $\angle ABD = \angle XYD = \angle CDB = 90°$. $AB = a$, $XY = c$ and $CD = b$, then prove that $c\,(a + b) = ab$. **[4]**



Solution : To prove : $c(a + b) = ab$



In Δ ABD & Δ DXY

$$\angle B = \angle XYD \qquad \text{[Each 90°]}$$

$$\angle XDY = \angle ADB \qquad \text{[Common]}$$

So by AA similarity

$$\Delta\, DAB \sim \Delta\, DXY$$

$$\therefore \qquad \frac{DY}{DB} = \frac{XY}{AB}$$

$$DY = \frac{c}{a}(BD) \qquad ...(i)$$

In Δ BCD & Δ BYX

$$\angle XYB = \angle D \qquad \text{[Each 90°]}$$

$$\angle CBD = \angle XBY \qquad \text{[Common]}$$

So by AA similarity,

$$\Delta\, BYX \sim \Delta\, BDC$$

$$\frac{BY}{BD} = \frac{XY}{CD}$$

$$BY = \frac{c}{b}(\text{BD}) \qquad ...(ii)$$

Adding eq. (i) and eq. (ii)

$$DY + BY = -(\text{BD}) + \frac{c}{b}(BD)$$

$$BD = \text{BD}\left[\frac{c}{a} + \frac{c}{b}\right]$$

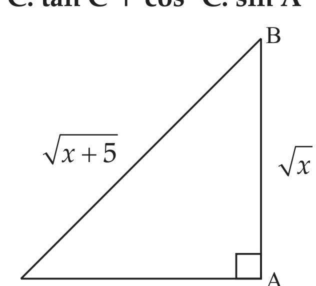
$$\frac{BD}{BD} = \left[\frac{cb \quad ca}{ab}\right]$$

$$1 = \frac{c(a+b)}{ab}$$

$$c(a + b) = ab \qquad \textbf{Hence Proved.}$$

27. In the ΔABC (see figure), $\angle$A = right angle, AB = $\sqrt{x}$ and BC = $\sqrt{x+5}$. Evaluate

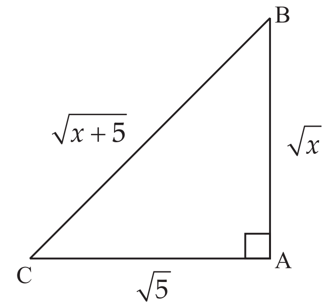
sin C. cos C. tan C + cos^2 C. sin A **[4]**



Solution : In Δ ABC, by pythagoras theorem,

$$\left(\sqrt{x+5}\right)^2 = \left(\sqrt{x}\right)^2 + AC^2$$

$$x + 5 = x + AC^2$$



$$5 = AC^2$$

or $\qquad$ $\text{AC} = \sqrt{5}$

$$\sin \text{C} = \frac{\sqrt{x}}{\sqrt{x+5}};\ \cos \text{C} = \frac{\sqrt{5}}{\sqrt{x+5}};$$

$$\tan \text{C} = \frac{\sqrt{x}}{\sqrt{5}}$$

and $\qquad$ $\sin \text{A} = \sin 90° = 1$

Then, sin C cos C tan C +cos^2 C sin A

$$= \frac{\sqrt{x}}{\sqrt{x+5}}\frac{\sqrt{5}}{\sqrt{x+5}}\frac{\sqrt{x}}{\sqrt{5}} + \left(\frac{\sqrt{5}}{\sqrt{x+5}}\right)^2 .1$$

$$= \frac{x}{x+5} + \frac{5}{x+5}$$

$$= \frac{x+5}{x+5}$$

$$= 1 \qquad \textbf{Ans.}$$

28. If $\frac{\cos B}{\sin A}$ = n and $\frac{\cos B}{\cos A}$ = m, then show that $(m^2 + n^2) \cos^2 A = n^2$. **[4]**

Solution : Given, $\qquad n = \frac{\cos \text{B}}{\sin \text{A}};\ m = \frac{\cos \text{B}}{\cos \text{A}}$

So, $\qquad n^2 = \frac{\cos^2 \text{B}}{\sin^2 \text{A}};m^2 = \frac{\cos^2 \text{B}}{\cos^2 \text{A}}$

$$\text{L.H.S.} = (m^2 + n^2)\cos^2 \text{A} = \left(\frac{\cos^2 \text{B}}{\cos^2 \text{A}} + \frac{\cos^2 \text{B}}{\sin^2 \text{A}}\right)\cos^2 \text{A}$$

$$= \frac{(\sin^2 \text{A} \cos^2\text{B} + \cos^2\text{A} \cos^2\text{B})}{\cos^2 \text{A} \sin^2\text{A}} \times \cos^2 \text{A}$$

$$= \frac{\cos^2 B(\sin^2 A + \cos^2 A)}{\sin^2 A}$$

$$= \frac{\cos^2 B}{\sin^2 A}$$

$= n^2 =$ R.H.S. **Hence Proved.**

29. Prove that :

$$\frac{\sec A - 1}{\sec A + 1} = \left(\frac{\sin A}{1 + \cos A}\right)^2 = (\cot A - \text{cosec } A)^2 \quad [4]$$

Solution : L.H.S. $= \frac{\sec A - 1}{\sec A + 1}$

$$= \frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1} = \frac{\frac{1 - \cos A}{\cos A}}{\frac{1 + \cos A}{\cos A}}$$

$$= \frac{1 - \cos A}{1 + \cos A}$$

$$= \frac{(1 - \cos A)(1 + \cos A)}{(1 + \cos A)(1 + \cos A)}$$

$$= \frac{1 - \cos^2 A}{(1 + \cos A)^2}$$

$$= \frac{\sin^2 A}{(1 + \cos A)^2}$$

$$= \left(\frac{\sin A}{1 + \cos A}\right)^2$$ **Hence Proved**

And, $\left(\frac{\sin A}{1 + \cos A}\right)^2 = \left[\left(\frac{\sin A}{1 + \cos A}\right) \times \frac{(1 - \cos A)}{(1 - \cos A)}\right]$

$$= \left[\frac{\sin A(1 - \cos A)}{1 - \cos^2 A}\right]^2$$

$$= \left[\frac{\sin A(1 - \cos A)}{\sin^2 A}\right]^2$$

$$= \left[\frac{1 - \cos A}{\sin A}\right]^2$$

$= (\text{cosec } A - \cot A)^2$

$= (-1)^2 [\cot A - \text{cosec } A]^2$

$= [\cot A - \text{cosec } A]^2 =$ R.H.S.

Hence Proved.

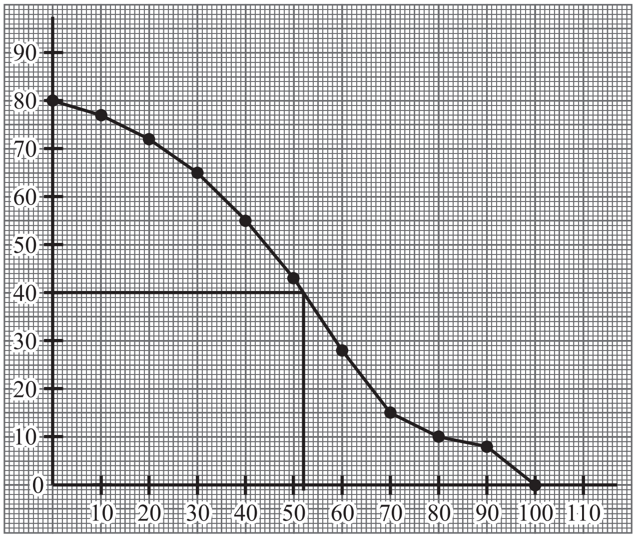
30. Following table shows marks (out of 100) of students in a class test :

Marks	No. of students
More than or equal to 0	80
More than or equal to 10	77
More than or equal to 20	72
More than or equal to 30	65
More than or equal to 40	55
More than or equal to 50	43
More than or equal to 60	28
More than or equal to 70	16
More than or equal to 80	10
More than or equal to 90	8
More than or equal to 100	0

Draw a 'more than type' ogive. From the curve, find the median. Also, check the value of the median by actual calculation. [4]

Solution :

More than type	C.I.	No. of Students	Frequency	*c.f.*
More than or equal to 0	0 – 10	80	3	3
More than or equal to 10	10 – 20	77	5	8
More than or equal to 20	20 – 30	72	7	15
More than or equal to 30	30 – 40	65	10	25
More than or equal to 40	40 – 50	55	12	37
More than or equal to 50	50 – 60	43	15	52
More than or equal to 60	60 – 70	28	12	64
More than or equal to 70	70 – 80	16	06	70
More than or equal to 80	80 – 90	10	02	72
More than or equal to 90	90 – 100	8	08	80
More than or equal to 100	100 – 110	0	00	



Median will be 52

Median by actual calculation :

$$N = 80 \text{ (even)}$$
$$= \frac{80}{2}$$
$$= 40$$

So modal class will be 50 – 60

$$l = 50, h = 10, f = 15,$$
$$c.f. = 37,$$
$$\text{Median} = l + \left[h \times \frac{\left(\frac{N}{2} - c.f.\right)}{f} \right]$$
$$= 50 + \left[10 \frac{(40-37)}{15} \right]$$
$$= 50 + 10 \times \frac{3}{15}$$
$$= 50 + 2$$
$$= 52$$

Hence Verified.

31. From the following data find the median age of 100 residents of a colony who took part in swachch bharat abhiyan :

Age (in yrs.) More than or equal to	No. of residents
0	50
10	46
20	40
30	20
40	10
50	3

[4]

Solution : First convert the given table into C.I. Table.

C.I.	Frequency	*c.f.*
0 – 10	4	4
10 – 20	6	10
20 – 30	20	30
30 – 40	10	40
40 – 50	7	47
50 – 60	3	50

$$\frac{N}{2} = \frac{50}{2} = 25$$
$$\text{Median} = l = \left[h \frac{\left(\frac{N}{2} - c.f.\right)}{f} \right]$$
$$= 20 + \left[10 \frac{(25-10)}{20} \right]$$
$$= 20 + \frac{15}{2}$$
$$= 27.5$$

Ans.

Mathematics 2016 (Outside Delhi) Term II SET I

SECTION — A

1. In fig. 1, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30°$, find $\angle PCA$. **[1]**

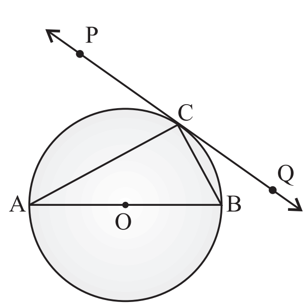


Figure 1

Solution : Given, $\angle CAB = 30°$ and PQ is a tangent at a point C to a circle with centre O.

Since, AB is a diameter.

$\therefore \quad \angle ACB = 90°$

Join OC

$$\angle CAO = \angle ACO = 30° \; (\quad OA = OC)$$

and, $\angle PCO = 90°$ (Tangent is perpendicular to the radius through the point of contact)

$$\therefore \quad \angle PCA = \angle PCO - \angle ACO$$
$$= 90° - 30° = 60°$$

Ans.

2. For what value of k will $k + 9$, $2k - 1$ and $2k + 7$ are the consecutive terms of an A.P. ? **[1]**

Solution : We have, $k + 9$, $2k - 1$ and $2k + 7$ as consecutive terms of an A.P

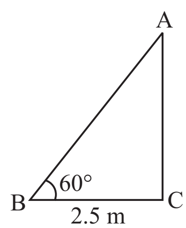
Then, $2(2k - 1) = k + 9 + 2k + 7$ [if a, b and c are in A.P. then $2b = a + c$]

$$4k - 2 = 3k + 16$$

$$k = 18$$ **Ans.**

3. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder. [1]

Solution : Let AB be the ladder leaning against a wall AC.



Then, $$\cos 60° = \frac{BC}{AB}$$

$$\frac{1}{2} = \frac{2{\cdot}5}{AB}$$

$$AB = 2{\cdot}5 \times 2 = 5 \text{ m}$$

∴ Length of ladder is 5 m. **Ans.**

4. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability of getting neither a red card nor a queen. [1]

Solution : Total number of possible outcomes = 52

Let E be the event of getting neither a red card nor a queen

∴ Number of favourable outcomes = 52 – 28 = 24

P(getting neither a red card nor a queen) = $P(E)$ =

$$\frac{24}{52} = \frac{6}{13}$$ **Ans.**

SECTION — B

5. If – 5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k. [2]

Solution : Given, – 5 is a root of $2x^2 + px - 15 = 0$

then, $f(-5) = 2(-5)^2 + p(-5) - 15 = 0$

$$50 - 5p - 15 = 0$$

$$35 - 5p = 0$$

$$5p = 35$$

$$p = 7$$

Now, putting the value of p, in, $p(x^2 + x) + k = 0$

we get $7x^2 + 7x + k = 0$

Now, $D = b^2 - 4ac = 0$ (∵ has the equal roots)

then, $49 - 28k = 0$

$$28k = 49$$

∴ $$k = \frac{49}{28} = \frac{7}{4}$$ **Ans.**

6. Let P and Q be the points of trisection of the line segment joining the points $A(2, -2)$ and $B(-7, 4)$ such that P is nearer to A. Find the coordinates of P and Q. [2]

Solution : Since, P and Q are the points of trisection of AB then, P divides AB in 1 : 2.

∴ Coordinates of P

$$= \left(\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right)$$

$$= \left(\frac{-3}{3}, \frac{0}{3}\right) = (-1, 0)$$

And, Q is the mid-point of PB

∴ Coordinates of Q $$= \left(\frac{-1+(-7)}{2}, \frac{0+4}{2}\right)$$

$$= (-4, 2)$$

So, $P \equiv (-1, 0), Q \equiv (-4, 2)$ **Ans.**

7. In Fig. 2, a quadrilateral $ABCD$ is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that $AB + CD = BC + DA$. [2]

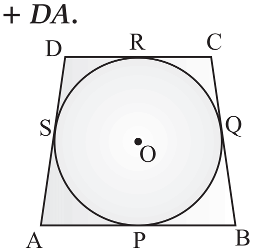


Figure 2

Solution : We have, AB, BC, CD and DA are the tangents touching the circle at P, Q, R and S respectively

Now, $AP = AS$, $BP = BQ$, $CR = CQ$ and $DR = DS$.

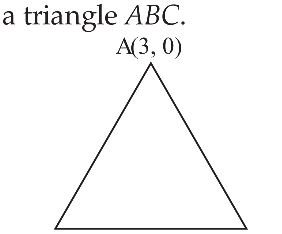
On adding we get

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$AB + CD = AD + BC$$ **Hence Proved.**

8. Prove that the points (3, 0), (6, 4) and (– 1, 3) are the vertices of a right angled isosceles triangle. [2]

Solution : Let $A(3, 0)$, $B(6, 4)$ and $C(-1, 3)$ be the vertices of a triangle ABC.



$$\text{Length of } AB = \sqrt{(6-3)^2+(4-0)^2}$$
$$= \sqrt{(3)^2+(4)^2}$$
$$= \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$
$$\text{Length of } BC = \sqrt{(-1-6)^2+(3-4)^2}$$
$$= \sqrt{(-7)^2+(-1)^2}$$
$$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2} \text{ units.}$$

And $\text{Length of } AC = \sqrt{(-1-3)^2+(3-0)^2}$

$$= \sqrt{(-4)^2+(3)^2}$$
$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ units}$$

$\therefore$ $AB = AC$

And $(AB)^2 + (AC)^2 = (BC)^2$

Hence, $\Delta\, ABC$ is a isosceles, right angled triangle.

Hence Proved

9. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term. [2]

Solution : We know that

$$T_n = a + (n-1)d$$

Given, $T_4 = a + (4-1)\,d = 0$

$$a + 3d = 0$$
$$a = -3d$$
$$T_{25} = a + (25-1)d$$
$$= a + 24d = (-3d) + 24d$$
$$= 21d$$

And, $T_{11} = a + (11-1)d$

$$= a + 10d$$

Then, $3\,T_{11} = 3(a + 10d)$

$$= 3a + 30d$$
$$= 3(-3d) + 30d \quad (\quad a = -3d)$$
$$= 30d - 9d = 21d = T_{25}$$

$\therefore$ $3\,T_{11} = T_{25}$ **Hence Proved.**

10. In Fig. 3, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If $OP = 2r$, show that $\angle OTS = \angle OST = 30°$. [2]

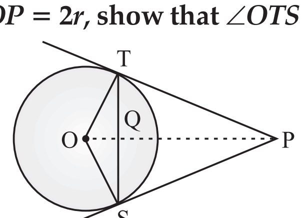


Figure 3

Solution : We have,

$$OP = 2r$$

Let $\angle TOP = \theta$

In $\Delta\, OTP$, $\cos\theta = \dfrac{OT}{OP} = \dfrac{r}{2r} = \dfrac{1}{2}$

$\because$ $\theta = 60°$

Hence, $\angle TOS = 2\theta = 2 \times 60° = 120°$

In $\Delta\, TOS$

$$\angle TOS + \angle OTS + \angle OST = 180°$$
$$120° + 2\,\angle OTS = 180° \; (\because \angle OTS = \angle OST)$$
$$2\,\angle OTS = 180° - 120°$$
$$\angle OTS = 30°$$

Hence, $\angle OTS = \angle OST = 30°$

Hence Proved.

SECTION — C

11. In fig. 4, O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region. (Take π = 3.14) [3]

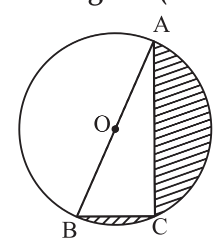


Figure 4

Solution : Given, AB is a diameter of length 13 cm and AC = 12 cm.

Then, by pythagoras theorem,

$$(BC)^2 = (AB)^2 - (AC)^2$$
$$(BC)^2 = (13)^2 - (12)^2$$
$$BC = \sqrt{169-144}$$
$$BC = \sqrt{25}$$

$\therefore$ $BC = 5$ cm

Now, Area of shaded region = Area of semi circle – Area of $\Delta\, ABC$

$$= \frac{\pi r^2}{2} - \frac{1}{2} \times BC \times AC$$
$$= \frac{1}{2} \times 3{\cdot}14 \times \left(\frac{13}{2}\right)^2 - \frac{1}{2} \times 5 \times 12$$
$$= \frac{1{\cdot}57 \times 169}{4} - 30$$
$$= 66{\cdot}33 - 30$$
$$= 36{\cdot}33 \text{ cm}^2$$

So, area of shaded region is $36{\cdot}33$ cm^2. **Ans.**

12. In fig. 5, a tent is in the shape of a cylinder surmounted by a conical top of same diameter. If the height and diameter of cylindrical part are 2.1 m and 3 m respectively and the slant height of conical part is 2.8 m, find the cost of canvas needed to make the tent if the canvas is available at the rate of ₹ 500/sq. metre. (Use $\pi = \frac{22}{7}$) [3]

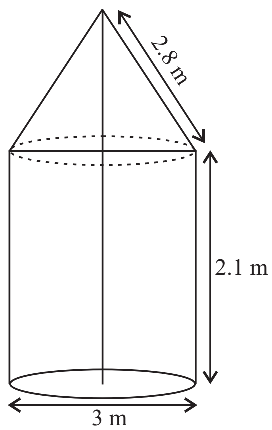


Figure 5

Solution : We have, height (h) and diameter (d) of cylinder as 2·1 m and 3 m respectively and slant height of conical part is 2·8 m.

Area of Canvas needed = C.S.A. of (cylinder + cone)

$$= 2\pi rh + \pi rl$$

$$= 2 \times \frac{22}{7} \times \frac{3}{4} \times 2{\cdot}1 + \frac{22}{7} \times \frac{3}{2} \times 2{\cdot}8$$

$$= \frac{22}{7}(6{\cdot}3 + 4{\cdot}2)$$

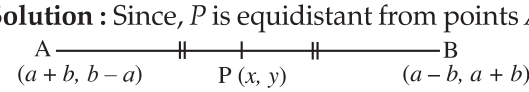
$$= \frac{22}{7} \times 10{\cdot}5 = 33 \text{ m}^2$$

$\therefore$ Cost of canvas needed at the rate of ₹ 500/m^2

= ₹ (33 × 500) = ₹ 16500 **Ans.**

13. If the point $P(x, y)$ is equidistant from the points $A(a + b, b - a)$ and $B(a - b, a + b)$. Prove that $bx = ay$. [3]

Solution : Since, P is equidistant from points A and B,



$$\therefore \quad PA = PB$$

or, $$(PA)^2 = (PB)^2$$

$$(a + b - x)^2 + (b - a - y)^2 = (a - b - x)^2 + (a + b - y)^2$$

$$(a + b)^2 + x^2 - 2ax - 2bx + (b - a)^2 + y^2 - 2by + 2ay$$
$$= (a - b)^2 + x^2 - 2ax + 2bx + (a + b)^2 + y^2 - 2ay - 2by$$

$$-2bx + 2ay = 2bx - 2ay$$

$$4ay = 4bx$$

$$ay = bx$$

or $$bx = ay$$ **Hence Proved.**

14. In fig. 6, find the area of the shaded region, enclosed between two concentric circles of radii 7 cm and 14 cm where $\angle AOC = 40°$. (Use $\pi = \frac{22}{7}$) [3]

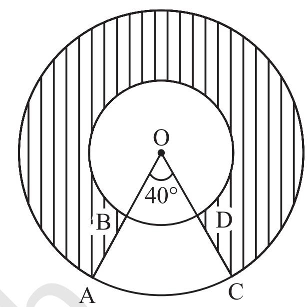


Figure 6

Solution : Given, $r = 7$ cm and $R = 14$ cm.

Area of shaded region $= \pi(R^2 - r^2)\dfrac{\theta}{360°}$

$$= \frac{22}{7}(14^2 - 7^2) \times \frac{(360° - 40°)}{360°}$$

$$= \frac{22}{7} \times 7 \times 21 \times \frac{320°}{360°}$$

$$= 410{\cdot}67 \text{ cm}^2$$ **Ans.**

15. If the ratio of the sum of first n terms of two A.P' s is $(7n + 1) : (4n + 27)$, find the ratio of their m^{th} terms. [3]

Solution : Let the sum of first n terms of two A.P's be S_n and S_n'.

then, $$\frac{S_n}{S_n'} = \frac{\frac{n}{2}\{2a + (n-1)d\}}{\frac{n}{2}\{2a' + (n-1)d'\}} = \frac{7n+1}{4n+27}$$

$$\frac{a + \left(\frac{n-1}{2}\right)d}{a' + \left(\frac{n-1}{2}\right)d'} = \frac{7n+1}{4n+27} \quad \text{...(i)}$$

Also, let m^{th} term of two A.P's be T_m and T_m'

$$\frac{T_m}{T_m'} = \frac{a + (m-1)d}{a' + (m-1)d'}$$

Replacing $\dfrac{n-1}{2}$ by $m - 1$ in (i), we get

$$\frac{a+(m-1)d}{a'+(m-1)d'} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$[\because n-1 = 2(m-1) \Rightarrow n = 2m-2+1 = 2m-1]$

$$\therefore \quad \frac{T_m}{T_m'} = \frac{14m-7+1}{8m-4+27} = \frac{14m-6}{8m+23}$$

$\therefore$ Ratio of m^{th} term of two A.P's is $14m-6 : 8m+23$

Ans.

16. Solve for x : $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}, x \neq 1, 2, 3$ **[3]**

Solution : We have,

$$\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}, x \neq 1, 2, 3.$$

$$\begin{aligned} 3(x-3)+3(x-1) &= 2(x-1)(x-2)(x-3) \\ 3x-9+3x-3 &= 2(x-1)(x-2)(x-3) \\ 6x-12 &= 2(x-1)(x-2)(x-3) \\ 6(x-2) &= 2(x-1)(x-2)(x-3) \\ 3 &= (x-1)(x-3) \\ 3 &= x^2-3x-x+3 \\ x^2-4x &= 0 \\ x(x-4) &= 0 \\ \therefore \quad x &= 0 \text{ or } 4 \end{aligned}$$

Ans.

17. A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. (Use $\pi = \frac{22}{7}$) **[3]**

Solution : Given, radius (r) and height (h) of conical vessel is 5 cm and 24 cm respectively.

Volume of water in conical vessel $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 24$$

$$= \frac{13200}{21} \text{ cm}^3$$

Since water is emptied into a cylindrical vessel.

$\because$ Volume of water in conical vessel = Volume of water in cylindrical vessel

$$\frac{13200}{21} = \pi R^2 H$$

$$\frac{13200}{21} = \frac{22}{7} \times 10 \times 10 \times H$$

$$H = \frac{13200 \times 7}{21 \times 22 \times 10 \times 10}$$

$$H = 2 \text{ cm}$$

$\therefore$ Height of water rise in cylindrical vessel is 2 cm.

Ans.

18. A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $\frac{32}{9}$ cm. Find the diameter of the cylindrical vessel. **[3]**

Solution : Given, diameter of sphere = 12 cm

then, radius of sphere (r) $= \frac{12}{2} = 6$ cm

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times (6)^3 \text{ cm}^3$$

Now, sphere is completely submerged in water and rise in water in cylindrical vessel is $3\frac{5}{9}$ cm.

Volume of sphere = Volume of cylindrical vessel

$$\frac{4}{3}\pi \times (6)^3 = \pi r^2 \times \frac{32}{9}$$

$$r^2 = \frac{4 \times 6 \times 6 \times 6 \times 9}{3 \times 32}$$

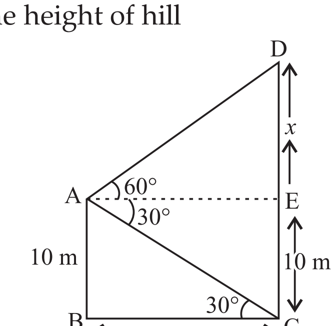
$$r = \sqrt{81}$$

$$r = 9 \text{ cm}$$

$\therefore$ Diameter of the cylindrical vessel is 18 cm. **Ans.**

19. A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of hill as 30°. Find the distance of the hill from the ship and the height of the hill. **[3]**

Solution : Let AB be the height of water level and CD be the height of hill



Then,

In ΔABC

$$\tan 30° = \frac{10}{y}$$

$$y = 10\sqrt{3} \quad ...(i)$$

In ΔADE

$$\tan 60° = \frac{x}{y}$$

$$y = \frac{x}{\sqrt{3}} \quad ...(ii)$$

From (i) and (ii), we get

$$\frac{x}{\sqrt{3}} = 10\sqrt{3}$$

$$x = 10 \times 3 = 30 \text{ m}$$

$\therefore$ Distance of the hill from this ship is $10\sqrt{3}$ m

and the height of the hill is 30 + 10 = 40 m. **Ans.**

20. Three different coins are tossed together. Find the probability of getting (i) exactly two heads, (ii) at least two heads (iii) at least two tails. [3]

Solution : Set of possible outcomes

= {$HHH, HHT, HTH, THH, HTT, THT, TTH, TTT$}.

(i) Let E_1 be the event of getting exactly two heads

$\therefore$ Favourable outcomes = {HHT, HTH, THH}

No. of favourable outcomes = 3

$$P(E_1) = \frac{3}{8}$$ **Ans.**

(ii) Let E_2 be the event of getting atleast two heads.

$\therefore$ Favourable outcomes = {HHT, HTH, THH, HHH}

No. of favourable outcomes = 4

$$P(E_2) = \frac{4}{8} = \frac{1}{2}$$ **Ans.**

(ii) Let E_3 be the event of getting atleast two tails.

$\therefore$ Favourable outcomes = {HTT, THT, TTH, TTT}

$$P(E_3) = \frac{4}{8} = \frac{1}{2}$$ **Ans.**

SECTION — D

21. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs ₹ 120 per sq. m, find the amount shared by each school to set up the tents. What value is generated by the above problem ? (Use $\pi = \frac{22}{7}$) [4]

Solution : Radius of the base of cylinder (r) = 2·8 m

Radius of the base of the cone (r) = 2·8 m

Height of the cylinder (h) = 3·5 m

Height of the cone (H) = 2·1 m.

Slant height of conical part (l) = $\sqrt{r^2 + H^2}$

$$= \sqrt{(2{\cdot}8)^2 + (2{\cdot}1)^2}$$

$$= \sqrt{7{\cdot}84 + 4{\cdot}41}$$

$$= \sqrt{12{\cdot}25}$$

$$= 3{\cdot}5 \text{ m}$$

Area of canvas used to make tent

= CSA of cylinder + CSA of cone

= $2 \times \pi \times 2{\cdot}8 \times 3{\cdot}5 + \pi \times 2{\cdot}8 \times 3{\cdot}5$

= 61·6 + 30·8

= 92·4 m^2

Cost of 1500 tents at ₹ 120 per sq. m

= 1500 × 120 × 92·4

= ₹ 16,632,000

Share of each school to set up the tents

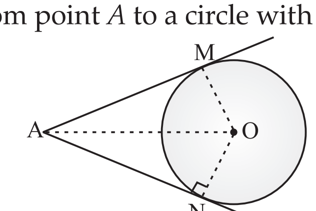
$$= \frac{16632000}{50}$$

= ₹ 332,640 **Ans.**

Value – Be kind and help others in need. **Ans.**

22. Prove that the lengths of the tangents drawn from an external point to a circle are equal. [4]

Solution : Given, Two tangents AM and AN are drawn from point A to a circle with centre O.



To Prove : $AM = AN$

Construction : Join OM, ON and OA.

Proof : Since, AM is a tangent and OM is a radius.

$\therefore \quad OM \perp AM$

Similarly, $\quad ON \perp AN$

Now, in ΔOMA and ΔONA

$OA = OA$ (Common)

$OM = ON$ (Radii of the circle)

$\angle OMA = \angle ONA = 90°$

$\therefore \quad \Delta OMA \cong \Delta ONA$ (By RHS congruence)

Hence, $\quad AM = AN$ **Hence Proved.**

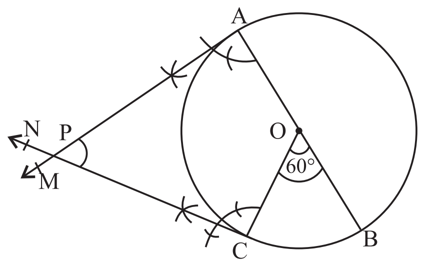
23. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other. **[4]**

Solution : Steps of construction :

(i) Draw a circle with O as centre and radius 4 cm.

(ii) Draw any diameter AOB of this circle.

(iii) Construct $\angle BOC = 60°$ such that radius OC meets the circle at C.



(iv) Draw $AM \perp AB$ and $CN \perp OC$.

Let AM and CN intersect each other at P

Then, PA and PC are the required tangents to the given circle and inclined at an angle of 60° to each other.

24. In Fig. 7, two equal circles, with centres O and O', touch each other at X. OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. $O'D$ is perpendicular AC. Find the value of $\dfrac{DO'}{CO}$. **[4]**

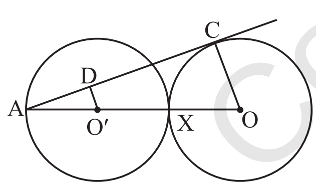


Figure 7

Solution : Given, AC is tangent to the circle with centre O and $O'D$ is perpendicular to AC.

then, $\angle ACO = 90°$

Also, $\angle ADO' = 90°$

$\angle CAO = \angle DAO'$ ($\because$ Common angle)

$\therefore$ $\Delta AO'D \sim \Delta AOC$

$\Rightarrow$ $\dfrac{AO'}{AO} = \dfrac{DO'}{CO}$

$\therefore$ $\dfrac{AO'}{3 \cdot AO'} = \dfrac{DO}{CO}$ $\left(\begin{matrix}\because AX = 2AO' \\ OX = AO'\end{matrix}\right)$

$\dfrac{DO'}{CO} = \dfrac{1}{3}$ **Ans.**

25. Solve for x : $\dfrac{1}{x+1} + \dfrac{2}{x+2} = \dfrac{4}{x+4}, x \neq -1, -2, -4$ **[4]**

Solution : We have, $\dfrac{1}{x+1} + \dfrac{2}{x+2} = \dfrac{4}{x+4}, x \neq -1, -2, -4$

$(x + 2)(x + 4) + 2(x + 1)(x + 4) = 4(x + 1)(x + 2)$

$x^2 + 2x + 4x + 8 + 2(x^2 + x + 4x + 4) = 4(x^2 + x + 2x + 2)$

$x^2 + 6x + 8 + 2x^2 + 10x + 8 = 4x^2 + 12x + 8$

$3x^2 + 16x + 16 = 4x^2 + 12x + 8$

$x^2 - 4x - 8 = 0$

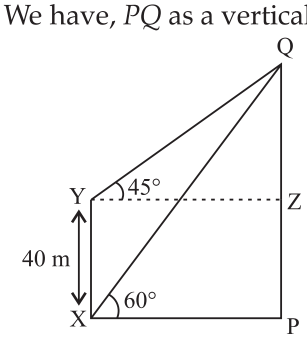
$\therefore$ $x = \dfrac{4 \pm \sqrt{16+32}}{2}$

$x = \dfrac{4 \pm \sqrt{48}}{2} = 2 \pm 4\sqrt{2}$

$\therefore$ $x = 2 \pm 2\sqrt{3}$ **Ans.**

26. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the tower PQ and the distance PX. (Use $\sqrt{3} = 1.73$) **[4]**

Solution : We have, PQ as a vertical tower



In ΔYZQ $\tan 45° = \dfrac{QZ}{YZ}$

$\dfrac{QZ}{YZ} = 1$

$QZ = YZ$ ...(i)

And, in ΔXPQ

$\tan 60° = \dfrac{QP}{XP}$

$\sqrt{3} = \dfrac{QZ+40}{XP}$

$\sqrt{3} = \dfrac{QZ+40}{YZ}$ ($\because XP = YZ$)

$\sqrt{3}\,QZ = QZ + 40$ [Using (i)]

$\sqrt{3}\,QZ - QZ = 40$

$$QZ\left(\sqrt{3}-1\right) = 40$$

$$QZ = \frac{40}{\sqrt{3}-1} = \frac{40}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 20\left(\sqrt{3}+1\right)$$

$$= 20\,(2{\cdot}73)$$

$$= 54{\cdot}60 \text{ m}$$

$$\therefore \quad PX = 54{\cdot}6 \text{ m}$$

And $PQ = (54{\cdot}6 + 40)$ m $= 94{\cdot}6$ m. **Ans.**

27. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses proceeding the house numbered X is equal to sum of the numbers of houses following X. **[4]**

Solution : Given, the houses in a row numbered consecutively from 1 to 49.

Now, sum of numbers preceeding the number X

$$= \frac{X(X-1)}{2}$$

And, sum of numbers following the number X

$$= \frac{49(50)}{2} - \frac{X(X-1)}{2} - X$$

$$= \frac{2450 - X^2 + X - 2X}{2}$$

$$= \frac{2450 - X^2 - X}{2}$$

According to the given condition,

Sum of no's preceeding X = Sum of no's following X

$$\frac{X(X-1)}{2} = \frac{2450 - X^2 - X}{2}$$

$$X^2 - X = 2450 - X^2 - X$$

$$2X^2 = 2450$$

$$X^2 = 1225$$

$$X = 35$$

Hence, at $X = 35$, sum of no. of houses preceeding the house no. X is equal to sum of the no. of houses following X. **Ans.**

28. In fig. 8, the vertices of ΔABC are $A(4, 6)$, $B(1, 5)$ and $C(7, 2)$. A line-segment DE is drawn to intersect the sides AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of $\Delta\, ADE$ and compare it with area of $\Delta\, ABC$. **[4]**

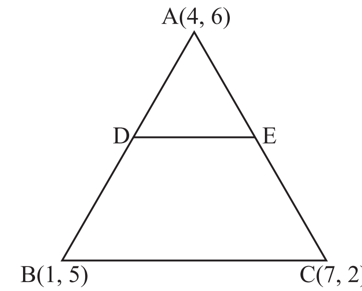
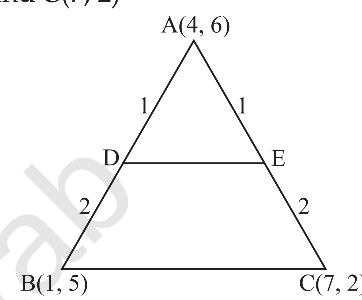


Figure 8

Solution : We have, the vertices of $\Delta\, ABC$ as $A(4, 6)$, $B(1, 5)$ and $C(7, 2)$



And $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$

Then, coordinates of D are

$$\left(\frac{1(1)+2(4)}{1+2}, \frac{1(5)+2(6)}{1+2}\right)$$

$$\left(\frac{1+8}{3}, \frac{5+12}{3}\right) \text{ i.e., } D\left(3, \frac{17}{3}\right)$$

and coordinates of E are

$$\left(\frac{1(7)+2(4)}{1+2}, \frac{1(2)+2(6)}{1+2}\right)$$

$$\left(\frac{7+8}{3}, \frac{2+12}{3}\right) \text{ i.e., } E\left(5, \frac{14}{3}\right)$$

Now, Area of $\Delta\, ADE$

$$= \frac{1}{2}\left[4\left(\frac{17}{3} - \frac{14}{3}\right) + 3\left(\frac{14}{3} - 6\right) + 5\left(6 - \frac{17}{3}\right)\right]$$

$$= \frac{1}{2}\left[4(1) + 3\left(-\frac{4}{3}\right) + 5\left(\frac{1}{3}\right)\right]$$

$$= \frac{5}{6} \text{ units}$$

and Area of $\Delta\, ABC$

$$= \frac{1}{2}\,[4(5-2) + 1\,(2-6) + 7\,(6-5)]$$

$$= \frac{1}{2}\,[4(3) + 1(-4) + 7(1)] = \frac{15}{2} \text{ units.}$$

$$\therefore \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{5/6}{15/2} = \frac{1}{9}$$

i.e., ar (ΔADE) : ar (ΔABC) = 1 : 9 **Ans.**

29. A number x is selected at random from the numbers 1, 2, 3 and 4. Another number y is selected at random from the numbers 1, 4, 9 and 16. Find the probability that product of x and y is less than 16. **[4]**

Solution : Let x be 1, 2, 3 or 4
and y be 1, 4, 9 or 16.

Now, xy = {1, 4, 9, 16, 2, 8, 18, 32, 3, 12, 27, 48, 4, 16, 36, 64}

Total number of possible outcomes = 16

Number of outcomes where product is less than 16 = 8

i.e., {1, 4, 9, 2, 8, 3, 12, 4}

$\therefore$ Required probability $= \frac{8}{16} = \frac{1}{2}$ **Ans.**

30. In Fig. 9, is shown a sector OAP of a circle with centre O, containing $\angle\theta$. AB is perpendicular to the radius OA and meets OP produced at B. Prove that the perimeter of shaded region is $r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right]$ **[4]**

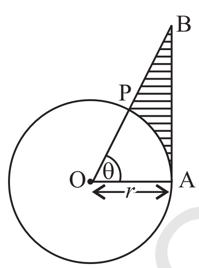


Figure 9

Solution : Given, the radius of circle with centre O is r.

$\angle POA = \theta$.

then, length of the arc $\widehat{PA} = \frac{2\pi r\theta}{360°} = \frac{\pi r\theta}{180°}$

And $\tan\theta = \frac{AB}{r}$

$AB = r\tan\theta$

And $\sec\theta = \frac{OB}{r}$

$OB = r\sec\theta$

Now, $PB = OB - OP$

$= r\sec\theta - r$

$\therefore$ Perimeter of shaded region

$$= AB + PB + \widehat{PA}$$

$$= r\tan\theta + r\sec\theta - r + \frac{\pi r\theta}{180°}$$

$$= r\left[\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right]$$

Hence Proved.

31. A motor boat whose speed is 24 km/h in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream. **[4]**

Solution : Let the speed of the stream be x km/hr.
Then speed upstream = $(24 - x)$ km/hr
and speed downstream = $(24 + x)$ km/hr.

Time taken to cover 32 km upstream $= \frac{32}{24-x}$ hrs.

Time taken to cover 32 km downstream $= \frac{32}{24+x}$ hrs.

$\therefore$ Time difference $= \frac{32}{24-x} - \frac{32}{24+x} = 1$

$$32[(24 + x) - (24 - x)] = (24 - x)(24 + x)$$
$$32(24 + x - 24 + x) = 576 - x^2$$
$$64x = 576 - x^2$$
$$x^2 + 64x - 576 = 0$$
$$x^2 + 72x - 8x - 576 = 0$$
$$x(x + 72) - 8(x + 72) = 0$$
$$(x + 72)(x - 8) = 0$$
$$x = 8 \text{ or } -72$$
$$\therefore \quad x = 8$$

(As speed can't be negative)

$\therefore$ Speed of the stream is 8 km/h. **Ans.**

Mathematics 2016 (Outside Delhi) Term II SET II

Note : Except for the following questions, all the remaining questions have been asked in previous set.

SECTION — B

10. Solve for x : $\sqrt{2x+9} + x = 13$ **[2]**

Solution : We have, $\sqrt{2x+9} + x = 13$.

$$\sqrt{2x+9} = 13 - x$$

On squaring both sides

$$\left(\sqrt{2x+9}\right)^2 = (13 - x)^2$$

$$2x + 9 = 169 + x^2 - 26x$$
$$x^2 - 28x + 160 = 0$$
$$x^2 - 20x - 8x + 160 = 0$$
$$x(x - 20) - 8(x - 20) = 0$$
$$(x - 8)(x - 20) = 0$$
$$x = 20 \text{ or } 8$$

$\therefore$ $x = 8$ (As $x = 20$ doesn't satisfy the given equation)

Ans.

SECTION — C

18. The digits of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number. **[3]**

Solution : Let the three digits of a positive number be

$a - d, a, a + d$

$\therefore$ $a - d + a + a + d = 3a = 15$

$\Rightarrow$ $a = 5$

Original number $= 100(a - d) + 10(a) + a + d$
$= 100a - 100d + 10a + a + d$
$= 111a - 99d$

And, number obtained by reversing the digits
$= 100(a + d) + 10(a) + a - d$
$= 100a + 100d + 10a + a - d$
$= 111a + 99d$

According to the given condition,

$(111a - 99d) - (111a + 99d) = 594$
$-198d = 594$
$d = -3$

$\therefore$ Original number is $111(5) - 99(-3)$

i.e., 852 **Ans.**

19. If the roots of the quadratic equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal, prove that $2a = b + c$. **[3]**

Solution : By comparing the given equation with $ax^2 + bx + c = 0$

$A = a - b, B = b - c, C = c - a$

Since the roots of the given quadratic equation are equal.

then, $(b - c)^2 - 4(c - a)(a - b) = 0$ $[\because B^2 - 4AC = 0]$

$$b^2 + c^2 - 2bc - 4(ac - a^2 - bc + ab) = 0$$
$$b^2 + c^2 - 2bc - 4ac + 4a^2 + 4bc - 4ab = 0$$
$$(b^2 + c^2 + 2bc) - 4a(b + c) + 4a^2 = 0$$
$$(b + c)^2 - 4a(b + c) + (2a)^2 = 0$$
$$((b + c) - 2a)^2 = 0$$
$$b + c - 2a = 0$$

i.e., $2a = b + c$ **Hence Proved.**

20. From a pack of 52 playing cards, Jacks, Queens and Kings of red colour are removed. From the remaining, a card is drawn at random. Find the probability that drawn card is : (i) a black King, (ii) a card of red colour, (iii) a card of black colour. **[3]**

Solution : Since, Jacks, Queens and Kings of red colour are removed. Then,

Total number of possible outcomes $= 52 - 6 = 46$

(i) Let E_1 be the event of getting a black king

$\therefore$ Favourable outcomes = king of spade and king of club.

No. of favourable outcomes = 2

$$P(E_1) = \frac{2}{46} = \frac{1}{23}$$ **Ans.**

(ii) Let E_2 be the event of getting a card of red colour

$\therefore$ Favourable outcomes = 10 cards of heart and 10 cards of diamond.

No. of favourable outcomes = 20

$$P(E_2) = \frac{20}{46} = \frac{10}{23}$$ **Ans.**

(iii) Let E_3 be the event of getting a card of black colour

$\therefore$ Favourable outcomes = 13 cards of spade and 13 cards of club.

No. of favourable outcomes = 26

$$P(E_3) = \frac{20}{46} = \frac{10}{23}$$ **Ans.**

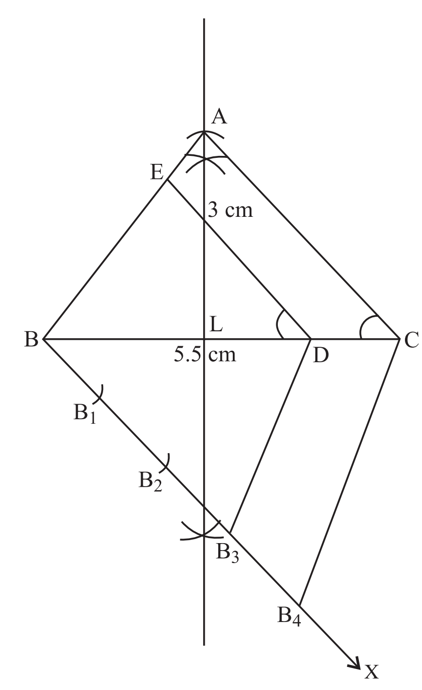
SECTION — D

28. Draw an isosceles ΔABC in which $BC = 5.5$ cm and altitude $AL = 3$ cm. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC. **[4]**

Solution : Steps of construction :

(i) Draw a line segment $BC = 5{\cdot}5$ cm.

(ii) Draw a perpendicular bisector of BC intersecting



BC at L such that $AL = 3$ cm.

(iii) Join AB and AC

Thus, $\Delta\, ABC$ is obtained.

(iv) Below BC, make an acute angle $\angle CBX$.

(v) Along BX, mark off four points B_1, B_2, B_3, B_4 such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

(vi) Join B_4C.

(vii) From B_3 draw $B_3D \parallel B_4C$, meeting BC at D.

(viii) From D, draw $DE \parallel CA$, meeting AB at E.

Then $\Delta\, EBD$ is the required triangle each of whose sides is $\frac{3}{4}$ of the corresponding side of $\Delta\, ABC$.

29. Prove that tangent drawn at any point of a circle is perpendicular to the radius through the point of contact. [4]

Solution : Given, a tangent AB at point P of the circle with centre O.

To prove : $OP \perp AB$.

Construction : Join OQ where Q is a point (other than P) on AB.

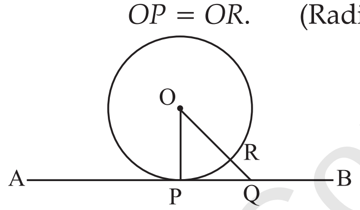
Proof : Since Q is a point on the tangent AB (other than P).

$\therefore$ Q lies outside the circle.

Let OQ intersect the circle at R.

$\Rightarrow$ $OR < OQ$.

But $OP = OR$. (Radii of the circle)



$\therefore$ $OP < OQ$.

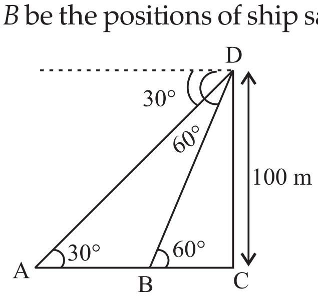
Thus, OP is the shortest distance than any other line segment joining O to any point of AB.

But, we know that the shortest distance between a point and a line is the perpendicular distance

$\therefore$ $OP \perp AB$ **Hence Proved.**

30. As observed from the top of a light house, 100 m high above sea level, the angles of depression of a ship, sailing directly towards it, changes from 30° to 60°. Find the distance travelled by the ship during the period of observation. (Use $\sqrt{3} = 1.73$) [4]

Solution : Let CD be a light house of length 100 m and A and B be the positions of ship sailing towards it.



Then, in $\Delta\, CBD$,

$$\tan 60^\circ = \frac{CD}{BC}$$

$$\sqrt{3} = \frac{100}{BC}$$

$$BC = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3}$$

And, in $\Delta\, CAD$

$$\tan 30^\circ = \frac{CD}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{AC}$$

$$AC = 100\sqrt{3}$$

$\therefore$ Distance travelled by the ship $(AB) = AC - BC$

$$= 100\sqrt{3} - \frac{100\sqrt{3}}{3}$$

$$= 100\sqrt{3}\left(\frac{3-1}{3}\right)$$

$$= \frac{200 \times 1{\cdot}73}{3}$$

$= 115{\cdot}33$ m **Ans.**

31. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park. [4]

Solution : Let the length of the rectangular park be x m then, breadth be $(x - 3)$ m

$\therefore$ Area of rectangular park $= x\,(x - 3)$ m^2

Area of isosceles triangular park $= \frac{1}{2}\,(x - 3) \times 12$ m^2

$= 6\,(x - 3)$ m^2

According to the given condition,

$$x\,(x - 3) - 6(x - 3) = 4$$

$$x^2 - 3x - 6x + 18 = 4$$

$$x^2 - 9x + 14 = 0$$

$$x^2 - 7x - 2x + 14 = 0$$

$$x\,(x - 7) - 2\,(x - 7) = 0$$

$$(x - 2)\,(x - 7) = 0$$

$$x = 2 \text{ or } 7$$

$\therefore$ $x = 7$ m

(As breadth can't be negative)

and $x - 3 = (7 - 3)$ m $= 4$ m

Hence, length and breadth of the rectangular park is 7 m and 4 m respectively. **Ans.**

Note : Except for the following questions, all the remaining questions have been asked in previous sets.

SECTION — B

10. Solve for x : $\sqrt{6x+7} - (2x-7) = 0$ [2]

Solution : We have, $\sqrt{2x+9} - (2x-7) = 0$

$$\sqrt{2x+9} = (2x-7)$$

On squaring both sides

$$\left(\sqrt{6x+7}\right)^2 = (2x-7)^2$$

$$\Rightarrow \quad 6x + 7 = 4x^2 + 49 - 28x$$

$$\Rightarrow \quad 4x^2 + 42 - 34x = 0$$

$$\Rightarrow \quad 2x^2 - 17x + 21 = 0$$

$$\Rightarrow \quad 2x^2 - 14x - 3x + 21 = 0$$

$$\Rightarrow \quad 2x(x-7) - 3(x-7) = 0$$

$$\Rightarrow \quad (2x-3)(x-7) = 0$$

$$\Rightarrow \quad x = \frac{3}{2} \text{ or } 7$$

$\therefore$ $x = 7$ (as $x = 3/2$ doesn't satisfy the given equation) **Ans.**

SECTION — C

18. There are 100 cards in a bag on which numbers from 1 to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card (i) is divisible by 9 and is a perfect square, (ii) is a prime number greater than 80. [3]

Solution : Number of possible outcomes = 100

(i) Let E_1 be the event of getting a number divisible by 9 and is a perfect square.

$\therefore$ Favourable outcomes = {9, 36, 81}

Number of favourable outcomes = 3

$$\therefore \quad P(E_1) = \frac{3}{100}$$ **Ans.**

(ii) Let E_2 be the event of getting a prime number greater than 80.

$\therefore$ Favourable outcomes = {83, 89, 97}

Number of favourable outcomes = 3

$$\therefore \quad P(E_2) = \frac{3}{100}$$ **Ans.**

19. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the squares of the other two by 60. Find the numbers. [3]

Solution : Let the three consecutive natural numbers be x, $x + 1$ and $x + 2$.

According to the given condition,

$$\therefore \quad (x+1)^2 - [(x+2)^2 - x^2] = 60$$

$$x^2 + 1 + 2x - [(x+2-x)(x+2+x)] = 60$$

$$x^2 + 2x + 1 - [2(2+2x)] = 60$$

$$x^2 + 2x + 1 - 4 - 4x = 60$$

$$x^2 - 2x - 63 = 0$$

$$x^2 - 9x + 7x - 63 = 0$$

$$x(x-9) + 7(x-9) = 0$$

$$(x+7)(x-9) = 0$$

$$\therefore \quad x = 9 \text{ or } -7$$

$$\therefore \quad x = 9$$

(neglect $x = -7$)

$\therefore$ Numbers are 9, 10, 11. **Ans.**

20. The sums of first n terms of three arithmetic progressions are S_1, S_2 and S_3 respectively. The first term of each A.P. is 1 and their common differences are 1, 2 and 3 respectively. Prove that $S_1 + S_3 = 2S_2$. [3]

Solution : Given, first term of each A.P. (a) = 1. and their common differences are 1, 2 and 3.

$$\therefore \quad S_1 = \frac{n}{2}[2a + (n-1)d_1]$$

$$= \frac{n}{2}(2 + (n-1)1) = \frac{n}{2}(n+1)$$

$$S_2 = \frac{n}{2}[2a + (n-1)d_2]$$

$$= \frac{n}{2}(2 + (n-1)2) = \frac{n}{2}(2n) = n^2$$

$$\text{and} \quad S_3 = \frac{n}{2}[2a + (n-1)d_3]$$

$$= \frac{n}{2}(2 + (n-1)3) = \frac{n}{2}(3n-1)$$

$$\text{Now,} \quad S_1 + S_3 = \frac{n}{2}(n+1) + \frac{n}{2}(3n-1)$$

$$= \frac{n}{2}(n + 1 + 3n - 1) = 4n \times \frac{n}{2} = 2n^2$$

$$= 2S_2$$

$$\therefore \quad S_1 + S_3 = 2S_2$$ **Hence Proved.**

SECTION — D

28. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately. [4]

Solution : Let the time taken by the one tap to fill the tank be x minutes.
then, other pipe takes $(x + 5)$ minutes to fill the tank.
According to the question,

$$\frac{1}{x}+\frac{1}{x+5}=\frac{1}{100/9}$$

$$\frac{x+5+x}{x(x+5)}=\frac{9}{100}$$

$$\begin{aligned}100\,(5 + 2x) &= 9x\,(x + 5)\\ 500 + 200x &= 9x^2 + 45x\\ 9x^2 + 45x - 200x - 500 &= 0\\ 9x^2 - 155x - 500 &= 0\\ 9x^2 - 180x + 25x - 500 &= 0\\ 9x\,(x - 20) + 25\,(x - 20) &= 0\\ (9x + 25)\,(x - 20) &= 0\\ x &= 20 \text{ or } -\frac{25}{9} \text{ (Neglect)}\end{aligned}$$

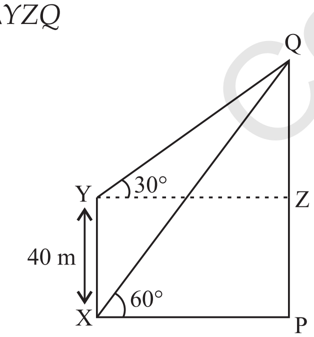
$$\therefore \quad x = 20$$

$\therefore$ Time in which each pipe would fill the tank separately are 20 mins and 25 mins respectively.

Ans.

29. From a point on the ground, the angle of elevation of the top of a tower is observed to be 60°. From a point 40 m vertically above the first point of observation, the angle of elevation of the top of the tower is 30°. Find the height of the tower and its horizontal distance from the point of observation. [4]

Solution : We have, PQ as a vertical tower.
Now, in ΔYZQ



$$\tan 30° = \frac{QZ}{YZ}$$

$$\frac{1}{\sqrt{3}}=\frac{QZ}{YZ}$$

$$YZ = QZ\sqrt{3} \qquad \text{...(i)}$$

And, in $\Delta\, XPQ$

$$\tan 60° = \frac{QP}{XP}$$

$$\sqrt{3} = \frac{QZ+40}{XP}$$

$$YZ\sqrt{3} = QZ + 40 \qquad (\because XP = YZ)$$

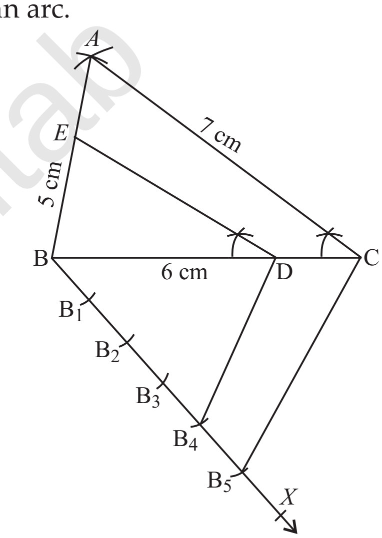
$$QZ\sqrt{3}\left(\sqrt{3}\right) = QZ + 40 \qquad \text{(using (i))}$$

$$\begin{aligned}3QZ &= QZ + 40\\ 2QZ &= 40\\ QZ &= 20\end{aligned}$$

$\therefore$ Height of tower = (40 + 20) m = 60 m
and Horizontal distance = $QZ\sqrt{3}$ = $20\sqrt{3}$ m **Ans.**

30. Draw a triangle with sides 5 cm, 6 cm, and 7 cm. Then draw another triangle whose sides are $\frac{4}{5}$ of the corresponding sides of first triangle. [4]

Solution : Steps of construction
(i) Draw a line segment BC = 6 cm.
(ii) With B as centre and radius equal to 5 cm, draw an arc.



(iii) With C as centre and radius equal to 7 cm, draw an arc.
(iv) Mark the point where the two arcs intersect as A. Join AB and AC.
Thus, $\Delta\, ABC$ is obtained.
(v) Below BC, make an acute $\angle CBX$.
(vi) Along BX, mark off five points B_1, B_2, B_3, B_4, B_5 such that $B\,B_1 = B_1\,B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
(vii) Join B_5C.
(viii) From B_4, draw $B_4D \parallel B_5C$, meeting BC at D.
(ix) From D, draw $DE \parallel CA$, meeting AB at E.
Then, $\Delta\, EBD$ is the required triangle each of whose sides is $\frac{4}{5}$ of the corresponding side of ΔABC.

31. A number x is selected at random from the numbers 1, 4, 9, 16 and another number y is selected at random from the numbers 1, 2, 3, 4. Find the probability that the value of xy is more than 16. [4]

Solution : Let x be 1, 4, 9, 16 and y be 1, 2, 3, 4.

Now, xy = {1, 2, 3, 4, 4, 8, 12, 16, 9, 18, 27, 36, 16, 32, 48, 64}

Total number of possible outcomes = 16

Number of outcomes where product is more than 16 = 6

i.e., {18, 27, 36, 32, 48, 64}

$\therefore$ Required probability $= \frac{6}{16} = \frac{3}{8}$ **Ans.**

Mathematics 2016 (Delhi) Term II SET I

SECTION — A

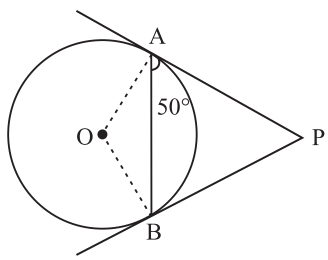
1. From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB = 50°$, then find $\angle AOB$. [1]

Solution : Since, tangents from an external point are equal.

i.e., $AP = BP$

Given, $\angle PAB = 50°$

$\therefore$ $\angle PBA = 50°$



In Δ APB

$$\angle APB = 180° - (50° + 50°) = 80°$$

$\therefore$ $\angle AOB = 180° - 80° = 100$ **Ans.**

2. In Fig. 1, AB is a 6 m high pole and CD is a ladder inclined at an angle of 60° to the horizontal and reaches up to a point D of pole. If $AD = 2.54$ m, find the length of the ladder. (Use $\sqrt{3} = 1.73$) [1]

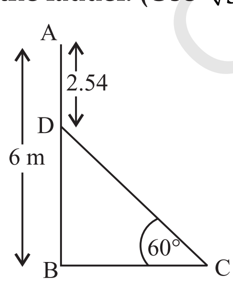


Fig. 1

Solution : Given, $AB = 6$ m and $AD = 2{\cdot}54$ m.

$\therefore$ $DB = (6 - 2{\cdot}54)$ m $= 3{\cdot}46$ m

In Δ DBC,

$$\sin 60° = \frac{DB}{DC}$$

$$\frac{\sqrt{3}}{2} = \frac{3{\cdot}46}{DC}$$

$\Rightarrow$ $$DC = \frac{3{\cdot}46 \times 2}{1{\cdot}732} = 3{\cdot}995 \text{ m} \simeq 4 \text{ m}$$

$\therefore$ The length of the ladder is 4 m. **Ans.**

3. Find the 9th term from the end (towards the first term) of the A.P. 5, 9, 13, ..., 185. [1]

Solution : Given, A.P. is 5, 9, 13, ..., 185

$l = 185$ and $d = 5 - 9 = 9 - 13 = -4$

then, $l_9 = l + (n-1)d$

$= 185 + (9-1)(-4)$

$= 185 + 8(-4)$

$\therefore$ $l_9 = 153$ **Ans.**

4. Cards marked with number 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the selected card bears a perfect square number. [1]

Solution : Total outcomes = 3, 4, 5, ..., 50

Total no. of outcomes = 48

Possible outcomes = 4, 9, 16, 25, 36, 49.

Let E be the event of getting a perfect square number

No. of possible outcomes = 6

$\therefore$ $$P(E) = \frac{6}{48} = \frac{1}{8}$$ **Ans.**

SECTION — B

5. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b. [2]

Solution : The given polynomial is, $p(x) = ax^2 + 7x + b$

$\therefore$ $$p\left(\frac{2}{3}\right) = a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$$

$$= \frac{4a}{9} + \frac{14}{3} + b = 0 \quad \text{...(i)}$$

and, $p(-3) = a(-3)^2 + 7(-3) + b = 0$

$\Rightarrow$ $9a - 21 + b = 0$ ...(ii)

Solving equation (i) and (ii), we get

$$4a + 42 + 9b = 0$$
$$81a - 189 + 9b = 0$$
$$- \quad + \quad -$$
$$-77a + 231 = 0$$

$$a = \frac{231}{77} = 3$$

Putting $a = 3$ in eq. (ii) we get,

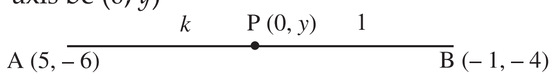
$$9(3) - 21 + b = 0$$

$\Rightarrow \qquad b = -6$

$\therefore \qquad a = 3$ and $b = -6$ **Ans.**

6. Find the ratio in which y-axis divides the line segment joining the points $A(5, -6)$ and $B(-1, -4)$. Also find the coordinates of the point of division. **[2]**

Solution : Let the required ratio be $k : 1$ and point on y-axis be $(0, y)$



$\therefore AP : PB = k : 1$

Then, by section formula

$$\frac{5-k}{k+1} = 0$$

$$5 - k = 0$$

$\Rightarrow \qquad k = 5$

Hence, required ratio is 5 : 1

$\therefore \qquad y = \dfrac{(-4)(5)+(-6)(1)}{5+1}$

$\therefore \qquad y = \dfrac{-26}{6} = -\dfrac{13}{3}$

Hence, point on y-axis is $\left(0, \dfrac{-13}{3}\right)$ **Ans.**

7. In Fig. 2, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF. **[2]**

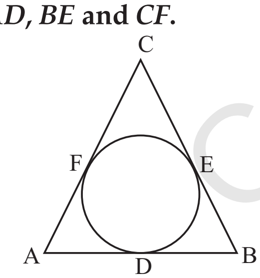


Fig. 2

Solution : Given, $AB = 12$ cm ; $BC = 8$ cm and $CA = 10$ cm

Let $\qquad AD = AF = x$

$\therefore \qquad DB = BE = 12 - x$

and, $\qquad CF = CE = 10 - x$

Now, $\qquad BC = BE + EC$

$\Rightarrow \qquad 8 = 12 - x + 10 - x$

$\Rightarrow \qquad 8 = 22 - 2x$

$\Rightarrow \qquad 2x = 14$

$\Rightarrow \qquad x = 7$ cm

$\therefore AD = 7$ cm, $BE = 5$ cm and $CF = 3$ cm **Ans.**

8. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from $Q(2, -5)$ and $R(-3, 6)$, find the coordinates of P. **[2]**

Solution : Let the coordinates of point P be $(2y, y)$

Since, P is equidistant from Q and R

$\therefore \qquad PQ = PR$

$\Rightarrow \qquad \sqrt{(2y-2)^2+(y+5)^2} = \sqrt{(2y+3)^2+(y-6)^2}$

$\Rightarrow \qquad (2y - 2)^2 + (y + 5)^2 = (2y + 3)^2 + (y - 6)^2$

$\Rightarrow 4y^2 + 4 - 8y + y^2 + 25 + 10y = 4y^2 + 9 + 12y + y^2 + 36 - 12y$

$\Rightarrow \qquad 2y + 29 = 45$

$\Rightarrow \qquad 2y = 45 - 29$

$\Rightarrow \qquad y = \dfrac{16}{2} = 8$

Hence, the co-ordinates of point P are (16, 8). **Ans.**

9. How many terms of the A.P. 18, 16, 14, ... be taken so that their sum is zero? **[2]**

Solution : Given, A.P. is 18, 16, 14 ....

We have, $a = 18$, $d = 16 - 18 = 14 - 16 = -2$

Now, $S_n = 0$

Therefore, $\qquad S_n = \dfrac{n}{2}[2a + (n-1)d] = 0$

$\Rightarrow \dfrac{n}{2}[2 \times 18 + (n-1)(-2)] = 0$

$\Rightarrow \qquad 36 - 2n + 2 = 0$

$\Rightarrow \qquad 2n = 38$

$\therefore \qquad n = 19$

Hence, the no. of terms are 19. **Ans.**

10. In Fig. 3, AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60°$. Find the length of chord AB. **[2]**

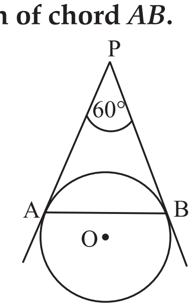


Fig. 3

Solution : Given, AP and BP are tangents to a circle with centre O.

$\therefore AP = BP$

Now, $\angle APB = 60°$ (Given)

$\therefore \angle PAB = \angle PBA = 60° \qquad (\because AP = BP)$

Thus, ΔAPB is an equilateral triangle.

Hence, the length of chord AB is equal to the length of AP *i.e.* 5 cm. **Ans.**

SECTION — C

11. In Fig. 4, *ABCD* is *a* square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region. $\left(\text{use } \pi = \frac{22}{7}\right)$ [3]

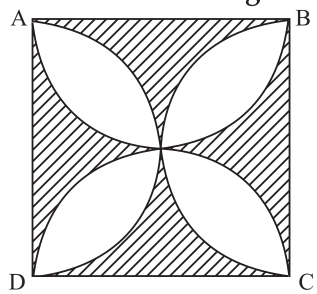


Fig. 4

Solution : Given, a square *ABCD* of side 14 cm

Then, Area of square $= (\text{side})^2$

$$= (14)^2 = 196 \text{ cm}^2$$

$$2 \text{ [Area of semicircle]} = \pi r^2$$

$$= \frac{22}{7} \times \frac{14}{2} \times \frac{14}{2} = 154 \text{ cm}^2$$

Now, Area of shaded region =

2[Area of square – 2 (Area of semicircle)]

$= 2\,[196 - 154] = 2 \times 42 = 84 \text{ cm}^2$ **Ans.**

12. In Fig. 5, is a decorative block, made up of two solids – a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block. $\left(\text{use } \pi = \frac{22}{7}\right)$ [3]

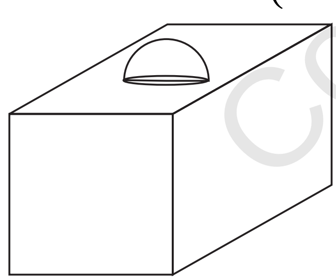


Fig. 5

Solution : Given, side of a cube = 6 cm

and the diameter of hemisphere = 3·5 cm

Now, total surface area of decorative block = total surface area of cube – surface area of base of hemisphere + CSA of hemisphere

$$= (6)^3 - \frac{22}{7} \times \frac{3\cdot5}{2} \times \frac{3\cdot5}{2} + 2 \times \frac{22}{7} \times \frac{3\cdot5}{2} \times \frac{3\cdot5}{2}$$

$$= 216 - \frac{22 \times 7}{16} + \frac{22 \times 7}{8}$$

$$= 216 + \frac{154}{16}$$

$= 225{\cdot}625 \text{ cm}^2$ **Ans.**

13. In Fig. 6, *ABC* is a triangle coordinates of whose vertex *A* are (0, – 1). *D* and *E* respectively are the mid-points of the sides *AB* and *AC* and their coordinates are (1, 0) and (0, 1) respectively. If *F* is the mid-point of *BC*, find the areas of Δ A*BC* and Δ *DEF*. [3]

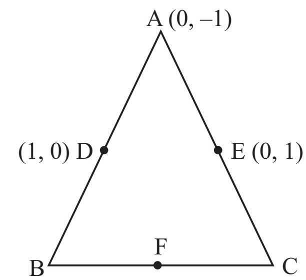


Fig. 6

Solution : Given, the coordinates of vertex *A* (0, – 1) and, mid points *D* (1,0) and *E*(0, 1) respectively.

Since, *D* is the mid-point of *AB*

Let, coordinates of *B* are (*x*, *y*)

then, $\left(\frac{x+0}{2}, \frac{y-1}{2}\right) = (1, 0)$

which gives *B* (2, 1)

Similarly, *E* is the mid-point of *AC*

Let, coordinates of *C* are (*x*', *y*')

then, $\left(\frac{x'+0}{2}, \frac{y'-1}{2}\right) = (0, 1)$

which gives *C* (0, 3)

Now, Area of Δ *ABC* $= \frac{1}{2} \left|[0\,(1-3) + 2\,(3+1) + 0\,(-1-1)]\right|$

= 4 sq units. **Ans.**

Now, *F* is the mid-point of *BC*.

$\Rightarrow$ Coordinates of *F* are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$

$\therefore$ Area of Δ *DEF* $= \frac{1}{2} \left|[1\,(1-2) + 0\,(2-0) + 1\,(0-1)]\right|$

$= \frac{|-2|}{2} = 1$ sq unit **Ans.**

14. In Fig. 7, are shown two arcs *PAQ* and *PBQ*. Arc *PAQ* is a part of circle with centre *O* and radius *OP* while arc *PBQ* is a semi-circle drawn on *PQ* as diameter with centre *M*. If *OP* = *PQ* = 10 cm show that area of shaded region is $25\left(\sqrt{3} - \frac{\pi}{6}\right)$ cm². [3]

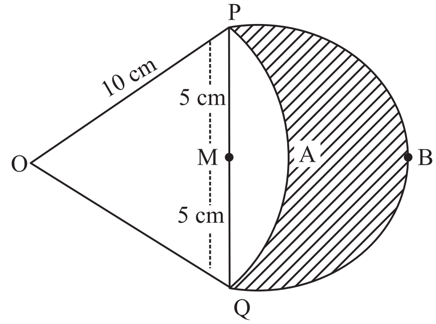


Fig. 7

Solution : Given, $OP = PQ = 10$ cm

Since, OP and OQ are radius of circle with centre O.

$\therefore$ $\Delta\, OPQ$ is equilateral.

$\Rightarrow \angle POQ = 60°$

Now, Area of segment $PAQM$

$$=(\text{Area of sector } OPAQO - \text{Area of } \Delta\, POQ)$$

$$= \frac{\pi r^2\theta}{360°} - \frac{1}{2} r^2 \sin i$$

$$= \frac{\pi \times (10)^2 \times 60°}{360°} - \frac{1}{2}(10)^2 \sin 60°$$

$$= \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right) \text{cm}^2$$

and, area of semicircle $PBQ = \frac{\pi r^2}{2} = \frac{\pi}{2}(5)^2 = \frac{25}{2}\pi \text{ cm}^2$

$\therefore$ Area of shaded region = Area of semicircle – Area of segment $PAQM$

$$= \frac{25}{2}\pi - \left(\frac{100\pi}{6} - \frac{100\sqrt{3}}{4}\right)$$

$$= \frac{25}{2}\pi - \frac{50\pi}{3} + 25\sqrt{3}$$

$$= \frac{75\pi - 100\pi}{6} + 25\sqrt{3}$$

$$= \frac{-25\pi}{6} + 25\sqrt{3}$$

$$= 25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{cm}^2$$

Hence Proved.

15. If the sum of first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first *n* terms of the A.P. [3]

Solution : Given, sum of first 7 terms of an A.P. (S_7) = 49

and sum of first 17 terms of an A.P. (S_{17}) = 289

i.e., $$S_7 = \frac{7}{2}[2a + (7-1)d] = 49$$

$$2a + 6d = 14 \quad \text{...(i)}$$

And, $$S_{17} = \frac{17}{2}[2a + (17-1)d] = 289$$

$$2a + 16d = 34 \quad \text{...(ii)}$$

Solving equations (i) and (ii), we get

$$\begin{array}{r} 2a + 16d = 34 \\ 2a + 6d = 14 \\ - \quad - \quad\quad - \\ \hline 10d = 20 \\ d = 2 \end{array}$$

Putting $d = 2$ in eq. (i), we get

$$a = 1$$

Hence, sum of first n term of A.P.,

$$S_n = \frac{n}{2}[2(1) + (n-1)2]$$

$$\Rightarrow \quad S_n = \frac{n}{2}[2 + (n-1)2]$$

$$\Rightarrow \quad S_n = n^2$$

Ans.

16. Solve for *x* :

$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = \mathbf{0},\ x \neq \mathbf{3}, -\mathbf{3/2}$$

[3]

Solution : We have, $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)}$

$$= 0,\ x \neq 3, -3/2$$

$$2x(2x+3) + (x-3) + (3x+9) = 0$$

$$4x^2 + 6x + x - 3 + 3x + 9 = 0$$

$$4x^2 + 10x + 6 = 0$$

$$2x^2 + 5x + 3 = 0$$

$$2x^2 + 2x + 3x + 3 = 0$$

$$2x(x+1) + 3(x+1) = 0$$

$$(2x+3)(x+1) = 0$$

$$x = -1, \frac{-3}{2}$$

$\therefore \quad x = -1 \quad [\because \text{Given } x \neq -3/2]$

Ans.

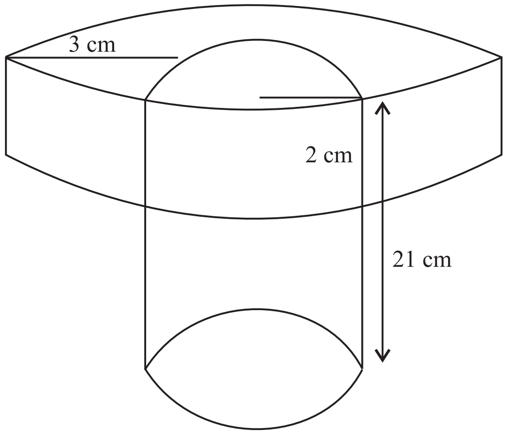
17. A well of diameter 4 m is dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment. [3]

Solution : Given, diameter and height of cylindrical well asre 4 m and 21 m respectively.

Now, the earth has been taken out to spread evenly all around.

Then, volume of earth dug out $= \frac{22}{7} \times \frac{3}{4} \times \frac{1}{3} \times 21$

$$= 264 \text{ m}^3$$



And the volume of embankment of width 3 m which forms a shape of circular ring $= \pi\,((5)^2 - (2)^2) \times h$

$$= \frac{22}{7}\,(25-4) \times h = 66\,h \text{ m}^3$$

[$\because$ Outer radius $= 2 + 3 = 5$ cm]

$\because$ Volume of earth dug out = Volume of embankment

$$\therefore \qquad 264 = 66h$$

$$\Rightarrow \qquad h = \frac{264}{66} = 4 \text{ m}$$

Hence, the height of the embankment is 4 m. **Ans.**

18. The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder. $\left(\text{use } \pi = \frac{22}{7}\right)$ **[3]**

Solution : Let the radius of base and height of a solid cylinder be r and h respectively.

Now, we have, $r + h = 37$ cm ...(i)

and, T.S.A. of solid cylinder $= 2\pi r\,(r + h) = 1628 \text{ cm}^2$

$$\Rightarrow \qquad 2\pi r\,(37) = 1628$$

$$\Rightarrow \qquad r = \frac{1628}{37 \times 2 \times \frac{22}{7}}$$

$$r = 7 \text{ cm}$$

$\therefore$ Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 30$$

(Using eq. (i), $h = 30$)

$$= 4620 \text{ cm}^3$$ **Ans.**

19. The angles of depression of the top and bottom of a 50 m high building from the top of a tower are 45° and 60° respectively. Find the height of the tower and the horizontal distance between the tower and the building. (use $\sqrt{3}$ = 1.73) **[3]**

Solution : Let AB and CD be the tower and high building respectively

Given, $CD = 50$ m

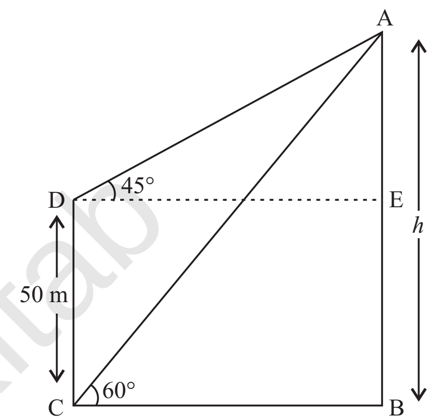
Let, $AB = h$ m

Then, in $\Delta\, ADE$

$$\tan 45^\circ = \frac{AE}{DE}$$

$$1 = \frac{h-50}{DE}$$

$$DE = h - 50 \qquad \text{...(i)}$$



and, in $\Delta\, ACB$

$$\tan 60^\circ = \frac{AB}{CB}$$

$$\sqrt{3} = \frac{h}{CB}$$

$$CB = \frac{h}{\sqrt{3}} \qquad \text{...(ii)}$$

Now, $CB = DE$

then from eq. (i) and (ii), we get

$$h - 50 = \frac{h}{\sqrt{3}}$$

$$h - \frac{h}{\sqrt{3}} = 50$$

$$\frac{\left(\sqrt{3}-1\right)}{\sqrt{3}}\,h = 50$$

$$h = \frac{50\sqrt{3}}{\sqrt{3}-1} = \frac{50\sqrt{3}}{\sqrt{3}-1} \times \frac{50\sqrt{3}}{\sqrt{3}-1}$$

$$= 25 \times 3 + 25\sqrt{3}$$

$$h = 75 + 25\,(1{\cdot}73)$$

$$= 118{\cdot}25 \text{ m}$$

Hence, the height of the tower is 118·25 m and the horizontal distance between the tower and the building is 68·25 m. **Ans.**

20. In a single throw of a pair of different dice, what is the probability of getting (i) a prime number on each dice ? (ii) a total of 9 or 11 ? [3]

Solution : Total outcomes = {(1, 1), (1, 2) (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

No. of outcomes = 36

(i) Let E_1 be the event of getting a prime number on each dice.

Favourable outcomes = {(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)}

$\Rightarrow$ No. of favourable outcomes = 9

$$\therefore \qquad P(E_1) = \frac{9}{36} = \frac{1}{4}$$

Ans.

(ii) Let E_2 be the event of getting a total of 9 or 11.

Favourable outcomes = {(3, 6), (4, 5), (5, 4), (6, 3), (5, 6), (6, 5)}

$\Rightarrow$ No. of favourable outcomes = 6

$$\therefore \qquad P(E_2) = \frac{6}{36} = \frac{1}{6}$$

Ans.

SECTION — D

21. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/hour than the usual speed. Find the usual speed of the plane.

What value is depicted in this question ? [4]

Solution : Let the usual speed of the plane be x km/h.

$\therefore$ Time taken by plane to reach 1500 km away $= \dfrac{1500}{x}$

and the time taken by plane to reach 1500 km with increased speed $= \dfrac{1500}{x+250}$

$$\text{Now, } \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2} \qquad \text{(Given)}$$

$$1500\,\frac{(x+250-x)}{x(x+250)} = \frac{1}{2}$$

$$3000 \times 250 = x^2 + 250x$$

$$x^2 + 250x - 750000 = 0$$

$$x^2 + 1000x - 750x - 750000 = 0$$

$$x(x + 1000) - 750(x + 1000) = 0$$

$$(x + 1000)(x - 750) = 0$$

$$x = -1000 \text{ or } x = 750$$

(As speed can't be negative)

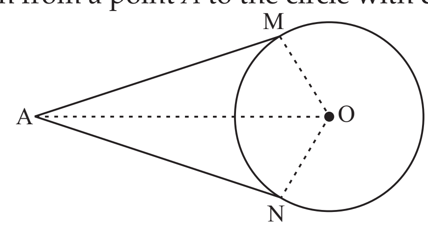
$$\therefore \qquad x = 750$$

$\therefore$ Speed of plane is 750 km/h. **Ans.**

Value : It shows his responsibility towards mankind and his work. **Ans.**

22. Prove that the lengths of tangents drawn from an external point to a circle are equal. [4]

Solution : Given, Two tangents AM and AN are drawn from a point A to the circle with centre O.



To prove : $AM = AN$

Construction : Join OM, ON and OA.

Proof : Since AM is a tangent at M and OM is radius

$\therefore$ $OM \perp AM$

Similarly, $ON \perp AN$

Now, in $\Delta\, OMA$ and $\Delta\, ONA$

$OM = ON$ (Radii of the circle)

$OA = OA$ (Common)

$\angle OMA = \angle ONA = 90°$

$\therefore$ $\Delta\, OMA \cong \Delta\, ONA$

(By RHS congruence)

Hence, $AM = AN$ (by c.p.c.t.)

Hence Proved.

23. Draw two concentric circles of radii 3 cm and 5 cm. Construct a tangent to smaller circle from a point on the larger circle. Also measure its length. [4]

Solution : Steps of construction—

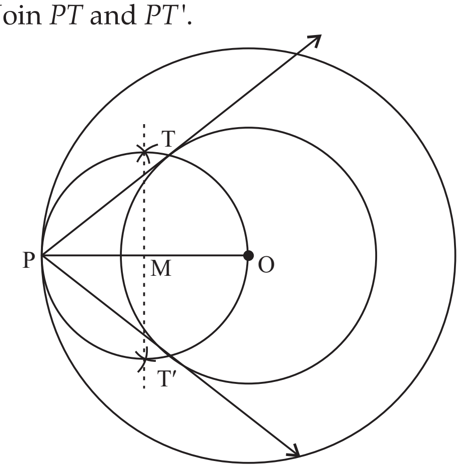
(i) Draw two concentric circles of radii 3 cm and 5 cm

(ii) Mark a point P on larger circle such that OP = 5 cm

(iii) Join OP and bisect it at M.

(iv) Draw a circle with M as centre and radius equal to MP to intersect the given circle at the points T and T'.

(v) Join PT and PT'.



Then, PT and PT' are the required tangents.

24. In Fig. 8, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle. **[4]**

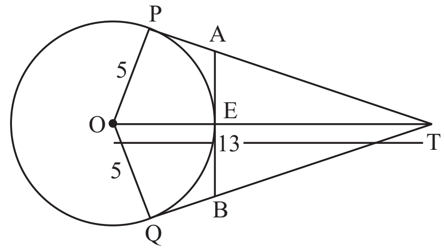


Fig. 8

Solution : Given, a circle with centre of radius 5 cm and OT = 13 cm

Since, PT is a tangent at P and OP is a radius through P

$\therefore \quad OP \perp PT$

In $\Delta\, OPT$

$$(PT)^2 = (OT)^2 - (OP)^2$$

$$\Rightarrow \quad PT = \sqrt{(13)^2 - (5)^2}$$

$$\Rightarrow \quad PT = \sqrt{169 - 25} = \sqrt{144}$$

$$\Rightarrow \quad PT = 12 \text{ cm}$$

And, $\quad TE = OT - OE = (13 - 5)$ cm = 8 cm

Now, $\quad PA = AE$

Let $\quad PA = AE = x$

Then, in $\Delta\, AET$

$$(AT)^2 = (AE)^2 + (ET)^2$$

$$(12 - x)^2 = (x)^2 + (8)^2$$

$$144 + x^2 - 24x = x^2 + 64$$

$$24x = 80$$

$$\Rightarrow \quad AE = x = 3{\cdot}33 \text{ cm}$$

$$\therefore \quad AB = 2AE = 2 \times 3{\cdot}33$$

$$= 6{\cdot}66 \text{ cm}$$ **Ans.**

25. Find x in terms of a, b and c :

$$\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a,b,c$$ **[4]**

Solution : We have, $\frac{a}{x-a} + \frac{b}{x-b} = \frac{2c}{x-c}, x \neq a,b,c$

$a\,(x - b)\,(x - c) + b\,(x - a)\,(x - c) = 2c\,(x - a)\,(x - b)$

$a\,(x^2 - bx - cx + bc) + b\,(x^2 - ax - cx + ac)$
$= 2c\,(x^2 - ax - bx + ab)$

$ax^2 - abx - acx + abc + bx^2 - abx - bcx + abc$
$= 2cx^2 - 2acx - 2bcx + 2abc$

$ax^2 + bx^2 - 2abx - acx - bcx + 2abc$
$= 2cx^2 - 2acx - 2bcx + 2abc$

$ax^2 + bx^2 - 2cx^2 - 2abx - acx - bcx + 2acx + 2bcx = 0$

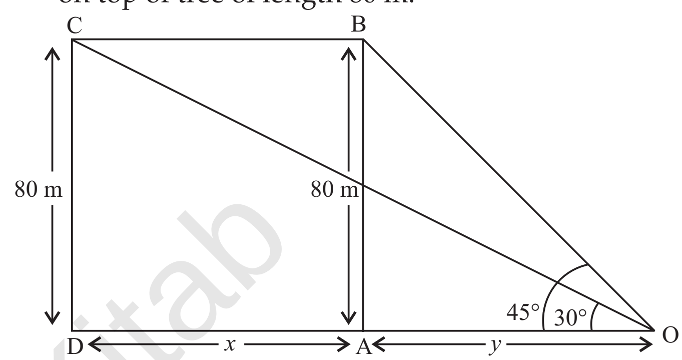
$(a + b - 2c)x^2 + (- 2ab + ac + bc)\,x = 0$

$x[(a + b - 2c)x + (ac + bc - 2ab)] = 0$

$$\therefore \quad x = 0, -\frac{(ac + bc - 2ab)}{a + b - 2c}$$ **Ans.**

26. A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird. (Take $\sqrt{3}$ = 1.732) **[4]**

Solution : Let B be the initial position of bird sitting on top of tree of length 80 m.



After 2 sec, the position of bird becomes C.

Let the distance travel by bird from B to C is x m.

Now, in $\Delta\, ABO$

$$\tan 45° = \frac{AB}{AO} = \frac{80}{y}$$

$$y = 80 \text{ m} \qquad \text{...(i)}$$

And, in $\Delta\, DCO$

$$\tan 30° = \frac{CD}{DO} = \frac{80}{x+y}$$

$$\frac{1}{\sqrt{3}} = \frac{80}{x+80} \qquad \text{[Using eq. (i)]}$$

$$x + 80 = 80\sqrt{3}$$

$$x = 80\left(\sqrt{3}-1\right) = 80 \times 0{\cdot}732$$

$$\therefore \quad x = 58{\cdot}56 \text{ m}$$

Hence, speed of flying of the bird $= \frac{58{\cdot}56}{2}$

$$\left(\text{Speed} = \frac{\text{Distance}}{\text{Time}}\right)$$

$$= 29{\cdot}28 \text{ m/s}$$ **Ans.**

27. A thief runs with a uniform speed of 100 m/minute. After one minute a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief. **[4]**

Solution : Let total time be n minutes

Since policeman runs after 1 minutes so he will catch the thief in $(n-1)$ minutes.

Total distance covered by thief = 100 m/minute × n minute

$$= (100\,n) \text{ m}$$

Now, total distance covered by the policeman = (100)m + (100 + 10)m + (100 + 10 + 10)m + ... + $(n-1)$ terms

i.e., 100 + 110 + 120 + ... + $(n-1)$ terms

$$\therefore S_{n-1} = \frac{n-1}{2}[2 \times 100 + (n-2)\,10]$$

$$\Rightarrow \frac{n-1}{2}[200 + (n-2)\,10] = 100\,n$$

$$\Rightarrow (n-1)(200 + 10n - 20) = 200\,n$$

$$\Rightarrow 200n - 200 + 10n^2 - 10n + 20 - 20n = 200n$$

$$\Rightarrow 10n^2 - 30n - 180 = 0$$

$$\Rightarrow n^2 - 3n - 18 = 0$$

$$\Rightarrow n^2 - (6-3)\,n - 18 = 0$$

$$\Rightarrow n^2 - 6n + 3n - 18 = 0$$

$$\Rightarrow n(n-6) + 3\,(n-6) = 0$$

$$\Rightarrow (n+3)(n-6) = 0$$

$$\therefore n = 6 \text{ or } n = -3 \text{ (Neglect)}$$

Hence, policeman will catch the thief in (6 – 1) *i.e.*, 5 minutes. **Ans.**

28. Prove that the area of a triangle with vertices $(t, t-2)$, $(t+2, t+2)$ and $(t+3, t)$ is independent of t. [4]

Solution : Given, the vertices of a triangle $(t, t-2)$, $(t+2, t+2)$ and $(t+3, t)$

$$\therefore \text{Area of the triangle} = \frac{1}{2}\,|[t\,(t+2-t) + (t+2)(t-t+2) + (t+3)(t-2-t-2)]|$$

$$= \frac{1}{2}\,|(2t + 2t + 4 - 4t - 12)|$$

$$= \frac{1}{2}\,|-8| = 4 \text{ sq units}$$

which is independent of t **Hence Proved.**

29. A game of chance consists of spinning an arrow on a circular board, divided into 8 equal parts, which comes to rest pointing at one of the numbers 1, 2, 3, ..., 8 (Fig. 9), which are equally likely outcomes. What is the probability that the arrow will point at (i) an odd number, (ii) a number greater than 3, (iii) a number less than 9. [4]

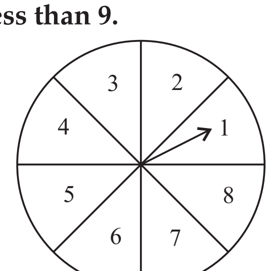


Fig. 9

Solution : Total no. of outcomes = 8

(i) Let E_1 be the event of getting an odd number

∴ Favourable outcomes = 1, 3, 5, 7

⇒ No. of favourable outcomes = 4

$$\therefore \quad P(E_1) = \frac{4}{8} = \frac{1}{2}$$

(ii) Let E_2 be the event of getting a number greater than 3.

∴ Favourable outcomes = 4, 5, 6, 7, 8

⇒ No. of favourable outcomes = 5

$$\therefore \quad P(E_2) = \frac{5}{8}$$

(iii) Let E_3 be the event of getting a number less than 9.

∴ Favourable outcomes = 1, 2, 3, 4, 5, 6, 7, 8

⇒ No. of favourable outcomes = 8

$$\therefore \quad P(E_3) = \frac{8}{8} = 1$$ **Ans.**

30. An elastic belt is placed around the rim of a pulley of radius 5 cm. (Fig. 10) From one point C on the belt, the elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area. (use $\pi = 3.14$ and $\sqrt{3} = 1.73$) [4]

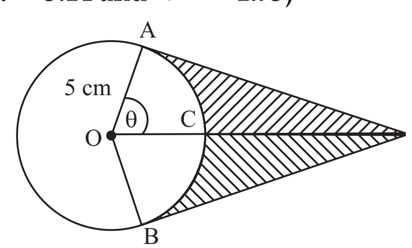


Fig. 10

Solution : Given, a circular pulley of radius 5 cm with centre O.

$$\therefore \quad AO = OB = OC = 5 \text{ cm}$$

and $OP = 10$ cm

Now, in right $\Delta\, AOP$

$$\cos i = \frac{AO}{OP} = \frac{5}{10} = \frac{1}{2}$$

$$\therefore \quad i = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

$\therefore \angle AOB = 2i = 120°$

$\Rightarrow$ Reflex $\angle AOB = 360° - 120° = 240°$

$$\text{Length of major arc } \widehat{AB} = \frac{2\pi r}{360°} \text{ reflex } \angle AOB$$

$$= \frac{2 \times 3{\cdot}14 \times 5 \times 240°}{360°}$$

$$= 20{\cdot}93 \text{ cm}$$

Hence, length of the belt that is still in contact with pulley = 20·93 cm

Now, by pythagorus theorem

$$(AP)^2 = (OP)^2 - (AO)^2$$
$$(AP)^2 = (10)^2 - (5)^2$$
$$AP = \sqrt{100 - 25}$$
$$= \sqrt{75} = 5\sqrt{3} \text{ cm}$$

$\therefore$ Area of $\Delta\, AOP = \dfrac{1}{2} \times 5 \times 5\sqrt{3}$

$$= \frac{25\sqrt{3}}{2} \text{ cm}^2$$

Also, Area of $\Delta\, BOP$ = Area of $\Delta\, AOP$

and, Area of quad. $AOBP$ = 2 (Area of $\Delta\, AOP$)

$$= 2 \times \frac{25\sqrt{3}}{2} = 25\sqrt{3} \text{ cm}^2$$
$$= 43{\cdot}25 \text{ cm}^2$$

Area of sector $ACBO = \dfrac{\pi r^2 \angle AOB}{360°}$

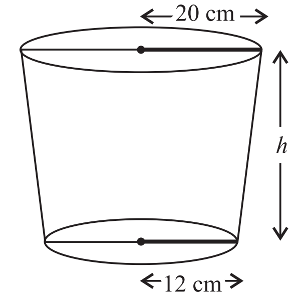
$$= \frac{3{\cdot}14 \times 5 \times 5 \times 120}{360°}$$
$$= 26{\cdot}16 \text{ cm}^2$$

$\therefore$ Area of shaded region = Area of quad. $AOBP$ – Area of sector $ACBO$

$$= (43{\cdot}25 - 26{\cdot}16) \text{ cm}^2$$
$$= 17{\cdot}09 \text{ cm}^2$$ **Ans.**

31. A bucket open at the top is in the form of frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of metal sheet used in making the bucket. (use π = 3.14) **[4]**

Solution : Given, the radii of top and bottom circular ends are 20 cm and 12 cm respectively.



And, volume of frustum (bucket) = 12308·8 cm^3

$$\Rightarrow \quad \frac{\pi h}{3} [R^2 + r^2 + Rr] = 12308{\cdot}8$$
$$\frac{3{\cdot}14 \times h}{3} [400 + 144 + 240] = 12308{\cdot}8$$
$$\therefore \quad \text{Height } (h) = \frac{12308{\cdot}8 \times 3}{3{\cdot}14 \times 784}$$
$$= \frac{36926{\cdot}4}{2461{\cdot}76} = 15 \text{ cm}$$

Slant height of the bucket $(l) = \sqrt{h^2 + (R - r)^2}$

$$= \sqrt{(15)^2 + (20 - 12)^2}$$
$$= \sqrt{225 + 64} = \sqrt{289}$$
$$= 17 \text{ cm}$$

$\therefore$ Area of metal sheet used in making the bucket

= Curved surface area of frustum + Base area

$= \pi\, l\, (R + r) + \pi\, r^2$

$= 3{\cdot}14 \times 17 \times (20 + 12) + 3{\cdot}14 \times 12 \times 12$

$= 3{\cdot}14 \times 17 \times 32 + 3{\cdot}14 \times 144$

$= 3{\cdot}14\, (544 + 144)$

$= 3{\cdot}14 \times 688$

$= 2160{\cdot}32$ cm^2 **Ans.**

Mathematics 2016 (Delhi) Term II — SET II

Note : Except for the following questions, all the remaining questions have been asked in previous set.

SECTION — B

10. How many terms of the A.P. 27, 24, 21, ... should be taken so that their sum is zero? **[2]**

Solution : Given, A.P. is 27, 24, 21, ...

We have, $a = 27$, $d = 24 - 27 = 21 - 24 = -3$

Now, $S_n = 0$

Therefore, $S_n = \dfrac{n}{2} [2a + (n - 1)d] = 0$

$$\Rightarrow \frac{n}{2} [2\,(27) + (n - 1)\,(-3)] = 0$$
$$\Rightarrow \quad 54 - 3n + 3 = 0$$
$$\Rightarrow \quad 57 - 3n = 0$$
$$\Rightarrow \quad 3n = 57$$
$$\therefore \quad n = 19$$

Hence, the no. of terms are 19 **Ans.**

SECTION — C

18. Solve for x :

$$\frac{x+1}{x+1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}; \; x \neq 1, -2, 2$$ **[3]**

Solution : We have, $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$; $x \neq 1, -2, 2$

$$\frac{(x+1)(x+2)+(x-2)(x-1)}{(x-1)(x+2)} = \frac{4(x-2)-(2x+3)}{x-2}$$

$(x-2)[x^2 + x + 2x + 2 + x^2 - 2x - x + 2] = [4x - 8 - 2x - 3](x^2 + x - 2)$

$(x-2)(2x^2 + 4) = (2x - 11)(x^2 + x - 2)$

$2x^3 + 4x - 4x^2 - 8 = 2x^3 + 2x^2 - 4x - 11x^2 - 11x + 22$

$4x - 4x^2 - 8 = -9x^2 - 15x + 22$

$5x^2 + 19x - 30 = 0$

$5x^2 + 25x - 6x - 30 = 0$

$5x(x + 5) - 6(x + 5) = 0$

$(5x - 6)(x + 5) = 0$

$$x = -5, \frac{6}{5}$$

$\therefore \quad x = -5$ or $x = \frac{6}{5}$ **Ans.**

19. Two different dice are thrown together. Find the probability of :

(i) getting a number greater than 3 on each die

(ii) getting a total of 6 or 7 of the numbers on two dice **[3]**

Solution : Total outcomes = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3) (6, 4), (6, 5), (6, 6)}

$\Rightarrow$ Total no. of outcomes = 36

(i) Let E_1 be the event of getting a number greater than 3 on each die.

Favourable outcomes = {(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)}

No. of favourable outcomes = 9

$\therefore \quad P(E_1) = \frac{9}{96} = \frac{1}{4}$ **Ans.**

(ii) Let E_2 be the event of getting a total of 6 or 7 of the numbers on two dice.

Favourable outcomes = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

$\Rightarrow$ No. of favourable outcomes = 11

$\therefore \quad P(E_2) = \frac{11}{36}$ **Ans.**

20. A right circular cone of radius 3 cm, has a curved surface area of 47.1 cm^2. Find the volume of the cone. (use π = 3.14) **[3]**

Solution : Given, radius of right circular cone = 3 cm and, curved surface area = 47·1 cm^2

$$\therefore \quad \pi rl = 47{\cdot}1$$

$$l = \frac{47{\cdot}1}{3{\cdot}14 \times 3} = 5 \text{ cm}$$

$$\therefore \quad h = \sqrt{l^2 - r^2}$$

$$= \sqrt{(5)^2 - (3)^2}$$

$$= \sqrt{25 - 9} = 4 \text{ cm}$$

Now, Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times 3{\cdot}14 \times 3 \times 3 \times 4$$

$= 37{\cdot}68$ cm^3 **Ans.**

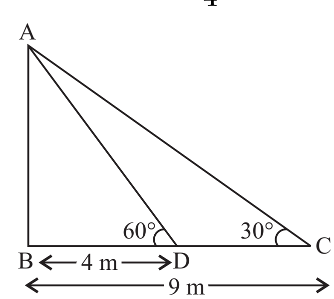
SECTION — D

28. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are 60° and 30° respectively. Find the height of the tower. **[4]**

Solution : Let length of tower is h

In ΔABD

$$\tan 60° = \frac{h}{4} \quad \text{...(i)}$$



In ΔABC

$$\tan 30° = \frac{h}{9}$$

$$\cot(90° - 30°) = \frac{h}{9}$$

$$\cot 60° = \frac{h}{9} \quad \text{...(ii)}$$

Multiplying eq. (i) and (ii), we get

$$\tan 60°.\cot 60° = \frac{h}{4} \times \frac{h}{9}$$

$$1 = \frac{h^2}{36}$$

$h = 6$ m **Ans.**

Note : In this question, it has not been specified whether two points from tower are taken in same or opposite side we have taken these points on the same side of tower.

29. Construct a triangle ABC in which $BC = 6$ cm, $AB = 5$ cm and $\angle ABC = 60°$.

Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of $\Delta\, ABC$. **[4]**

Solution : Steps of Construction—

(i) Draw a line segment $BC = 6$ cm.

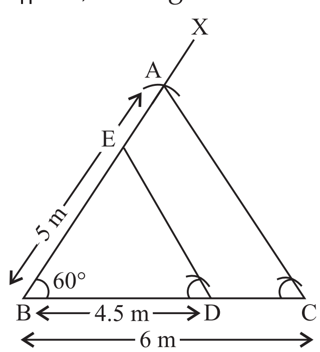
(ii) Construct $\angle XBC = 60°$

(iii) With B as centre and radius equal to 5 cm draw an arc which intersect XB at A.

(iv) Join AC. Thus, $\Delta\, ABC$ is obtained.

(v) Draw D on BC such that $BD = \frac{3}{4}\, BC = \left(\frac{3}{4}\times 6\right)$ cm $= \frac{9}{2}$ cm $= 4{\cdot}5$ cm

(vi) Draw $DE \parallel CA$, cutting BA at E.

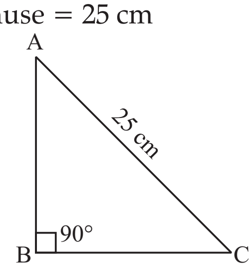


Then, $\Delta\, BDE$ is the required triangle similar to $\Delta\, ABC$ such that each side of $\Delta\, BDE$ is $\frac{3}{4}$ times the corresponding side of $\Delta\, ABC$.

30. The perimeter of a right triangle is 60 cm. Its hypotenuse is 25 cm. Find the area of the triangle. **[4]**

Solution : Given, the perimeter of right triangle = 60 cm

and hypotenuse = 25 cm



$$\therefore \quad AB + BC + CA = 60 \text{ cm}$$

$$AB + BC + 25 = 60$$

$$\therefore \quad AB + BC = 35 \quad \text{...(i)}$$

Now, by pythagoras theorem,

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(25)^2 = (AB)^2 + (BC)^2$$

$$\therefore \quad AB^2 + BC^2 = 625 \quad \text{...(ii)}$$

we, know that, $(a + b)^2 = a^2 + b^2 + 2ab$

then, $\quad (AB + BC)^2 = (AB)^2 + (BC)^2 + 2AB{\cdot}BC$

$$(35)^2 = 625 + 2\, AB{\cdot}BC$$

$$\therefore \quad 2AB{\cdot}BC = 1225 - 625$$

$$2AB{\cdot}BC = 600$$

$$\therefore \quad AB{\cdot}BC = 300$$

$$\therefore \quad \text{Area of } \Delta\, ABC = \frac{1}{2} \times AB \times BC$$

$$= \frac{1}{2} \times 300 = 150 \text{ cm}^2 \quad \textbf{Ans.}$$

31. A thief, after committing a theft, runs at a uniform speed of 50 m/ minute. After 2 minutes, a policeman runs to catch him. He goes 60 m in first minute and increases his speed by 5 m/minute every succeeding minute. After how many minutes, the policeman will catch the thief? **[4]**

Solution : Let total time be n minutes

Since policeman runs after two minutes he will catch the thief in $(n - 2)$ minutes.

Total distance covered by thief = 50 m /min × n min = $(50\, n)$ m

Now, total distance covered by the policeman = (60) + (60 + 5) + (60 + 5 + 5) + ... + $(n - 2)$ terms

i.e., 60 + 65 + 70 + ... + $(n - 2)$ terms

$$\therefore\ S_{n-2} = \frac{n-2}{2}\,[2 \times 60 + (n - 3)\, 5]$$

$$\Rightarrow \quad \frac{n-2}{2}\,[120 + (n - 3)\, 5] = 50n$$

$$\Rightarrow \quad n - 2\,(120 + 5n - 15) = 100n$$

$$\Rightarrow\ 120n - 240 + 5n^2 - 10n - 15n + 30 = 100n$$

$$\Rightarrow \quad 5n^2 - 5n - 210 = 0$$

$$\Rightarrow \quad n^2 - n - 42 = 0$$

$$\Rightarrow \quad n^2 - (7 - 6)\, n - 42 = 0$$

$$\Rightarrow \quad n^2 - 7n + 6n - 42 = 0$$

$$\Rightarrow \quad n(n - 7) + 6\,(n - 7) = 0$$

$$\Rightarrow \quad (n + 6)\,(n - 7) = 0$$

$$n = 7 \text{ or } n = -6 \text{ (neglect)}$$

Hence, policeman will catch the thief in $(7 - 2)$ *i.e.*, 5 minutes.

Ans.

Mathematics 2016 (Delhi) Term II — SET III

Note : Except for the following questions, all the remaining questions have been asked in previous sets.

SECTION — B

10. How many terms of the A.P. 65, 60, 55, ... be taken

so that their sum is zero? **[2]**

Solution : Given, A.P. is 65, 60, 55, ...

We have, $a = 65$, $d = 60 - 65 = 55 - 60 = -5$

Now, $S_n = 0$

Therefore, $$S_n = \frac{n}{2}[2a + (n-1)d] = 0$$

$$\Rightarrow \quad [2(65) + (n-1)(-5)] = 0$$

$$\Rightarrow \quad 130 - 5n + 5 = 0$$

$$\Rightarrow \quad 135 - 5n = 0$$

$$\Rightarrow \quad 5n = 135$$

$$\therefore \quad n = 27$$

Hence, the no. of terms are 27. **Ans.**

SECTION — C

18. A box consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Ramesh, a shopkeeper will buy only those shirts which are good but 'Kewal' another shopkeeper will not buy shirts with major defects. *A* shirt is taken out of the box at random. What is the probability that

(i) Ramesh will buy the selected shirt ?

(ii) 'Kewal' will buy the selected shirt ? **[3]**

Solution : Let E_1 be the event of selecting good shirts by Ramesh and E_2 be the event of selecting the shirts with no major defects by 'Kewal'.

Total no. of shirts in a box = 100

(i) $\because$ Number of good shirts = 88

$$\therefore \quad P(E_1) = \frac{88}{100} = \frac{22}{25}$$ **Ans.**

(ii) $\because$ Number of shirts with no major defect

$$= 100 - 4 = 96$$

$$\therefore \quad P(E_2) = \frac{96}{100} = \frac{24}{25}$$ **Ans.**

19. Solve the following quadratic equation for x :

$$x^2 + \left(\frac{a}{a+b} = \frac{a+b}{a}\right)x + 1 = 0$$ **[3]**

Solution : We have, $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$

$$\Rightarrow \quad x^2 + \frac{a}{a+b}x + \frac{a+b}{a}x + 1 = 0$$

$$\Rightarrow x\left(x + \frac{a}{a+b}\right) + \frac{a+b}{a}\left(x + \frac{a}{a+b}\right) = 0$$

$$\Rightarrow \quad \left(x + \frac{a+b}{a}\right)\left(x + \frac{a}{a+b}\right) = 0$$

$$x = -\frac{a}{a+b}, -\frac{(a+b)}{a}$$

$$\therefore \quad x = -\frac{a}{a+b} \text{ or } x = -\frac{(a+b)}{a}$$ **Ans.**

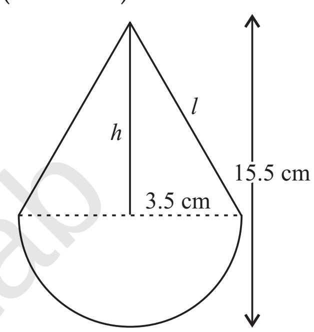
20. A toy is in the form of a cone of base radius 3.5 cm mounted on a hemisphere of base diameter 7 cm. If the total height of the toy is 15.5 cm, find the total surface area of the toy. $\left(\text{use } \pi = \frac{22}{7}\right)$ **[3]**

Solution : Given, the base radius of cone, $r = 3{\cdot}5$ cm

Total height of cone, $(h + r) = 15{\cdot}5$ cm

and base diameter of hemisphere = 7 cm

Now, $h = (15{\cdot}5 - 3{\cdot}5)$ cm = 12 cm



So, slant height, $$l = \sqrt{h^2 + r^2} = \sqrt{(12)^2 + (3{\cdot}5)^2}$$

$$= \sqrt{144 + 12{\cdot}25}$$

$$= 12{\cdot}5 \text{ cm}$$

$\therefore$ Total Surface Area $= \pi rl + 2\pi r^2$

$$= \frac{22}{7} \times 3{\cdot}5 \times 12{\cdot}5 + 2 \times \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5$$

$$= \frac{22}{7} \times 3{\cdot}5\,(12{\cdot}5 + 2 \times 3{\cdot}5)$$

$$= 11\,(19{\cdot}5)$$

$$= 214{\cdot}5 \text{ cm}^2$$ **Ans.**

SECTION — D

28. The sum of three numbers in A.P. is 12 and sum of their cubes is 288. Find the numbers. **[4]**

Solution : Let the three numbers in A.P. be $a - d$, a, $a + d$

Now, $a - d + a + a + d = 12$

$$3a = 12$$

$$\therefore \quad a = 4$$

Also, $(4 - d)^3 + 4^3 + (4 + d)^3 = 288$

$$64 - 48d + 12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 = 288$$

$$192 + 24d^2 = 288$$

$$24d^2 = 288 - 192$$

$$d^2 = \frac{96}{24} = 4$$

$$d = \pm 2$$

$\therefore$ The numbers are 2, 4, 6 or 6, 4, 2. **Ans.**

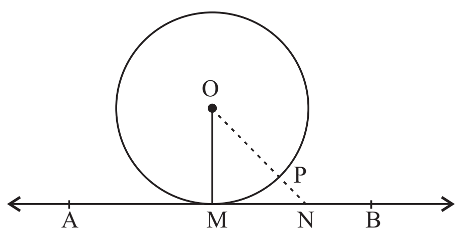
29. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. [4]

Solution : Given, a circle with centre O and a tangent AB at a point M on circle.

To prove : $OM \perp AB$.

Construction : Take point N (other than M) on AB. Join ON.

Proof : Since N is a point on the tangent AB other than P



$\therefore$ N lies outside the circle.

Let ON passes through point P.

Then, $OP < ON$ ...(i)

But, $OM = OP$ (Radii) ...(ii)

$\therefore$ $OM < ON$ (From eq. (i) and (ii))

Thus, OM is the shortest distance between the point O and the line AB.

But, it is known that the shortest distance between a point and a line is the perpendicular distance

$\therefore$ $OM \perp AB$. **Hence Proved.**

30. The time taken by a person to cover 150 km was $2\frac{1}{2}$ hours more than the time taken is return journey. If he returned at a speed of 10 km/hour more than the speed while going, find the speed per hour in each direction. [4]

Solution : Let the speed while going be x km/h

Time taken by a person to cover 150 km = $\frac{150}{x}$ hours

Time taken by a person in return journey = $\frac{150}{x+10}$ hours

Now, according to the given condition,

$$\frac{150}{x} - \frac{150}{x+10} = \frac{5}{2}$$

$$\frac{150(x+10-x)}{x(x+10)} = \frac{5}{2}$$

$$300 \times 10 = 5x(x+10)$$

$$3000 = 5x^2 + 50x$$

$$5x^2 + 50x - 3000 = 0$$

$$x^2 + 10x - 600 = 0$$

$$x^2 + 30x - 20x - 600 = 0$$

$$x(x+30) - 20(x+30) = 0$$

$$(x-20)(x+30) = 0$$

$$x = 20 \text{ or } x = -30 \text{ (neglect)}$$

Hence, the speed while going is 20 km/h and the speed while returning is 30 km/h **Ans.**

31. Draw a triangle ABC with $BC = 7$ cm, $\angle B = 45°$ and $\angle A = 105°$. Then construct a triangle whose sides are $\frac{4}{5}$ times the corresponding sides of ΔABC. [4]

Solution :

$$\angle B = 45° \text{ and } \angle A = 105°$$

$\because$ Sum of angles of triangle is 180°

$\therefore$ $\angle A + \angle B + \angle C = 180°$

$105° + 45° + \angle C = 180°$

$\Rightarrow$ $\angle C = 180° - (105° + 45°)$

$\Rightarrow$ $\angle C = 30°$

Steps of construction :

(i) Draw a line segment $BC = 7$ cm

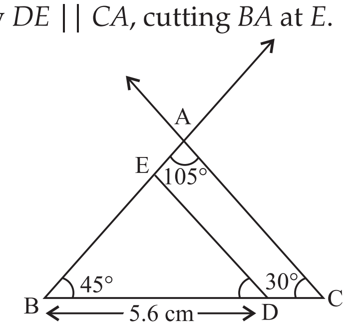
(ii) Construct $\angle B = 45°$ and $\angle C = 30°$

(iii) A is the intersecting point of ray through B and C.

Thus, ΔABC is obtained.

(iv) Draw D on BC such that $BD = \frac{4}{5}$ BC = $\left(\frac{4}{5}\times 7\right)$ cm = $\frac{28}{5}$ cm = 5·6 cm

(v) Draw $DE \,||\, CA$, cutting BA at E.



Then, Δ BDE is the required triangle similar to Δ ABC such that each side of Δ BDE is $\frac{4}{5}$ times the corresponding side of Δ ABC.

Mathematics 2015 Term I

Time allowed : 3 Hours **Maximum marks : 90**

SECTION — A

1. In Δ *DEW*, AB $||$ *EW*. If *AD* = 4 cm, *DE* = 12 cm and *DW* = 24 cm, then find the value of *DB*. [1]

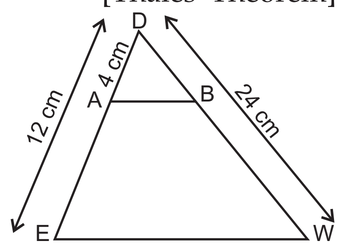
Solution : Let $BD = x$ cm,

$DW = 24$ cm,

Then, $BW = (24 - x)$ cm, $AE = 12 - 4 = 8$ cm

In ΔDEW, $AB \parallel EW$

$\therefore \quad \frac{AD}{AE} = \frac{BD}{BW}$ [Thales' Theorem]



$\Rightarrow \quad \frac{4}{8} = \frac{x}{24-x}$

$\Rightarrow \quad 8x = 96 - 4x$

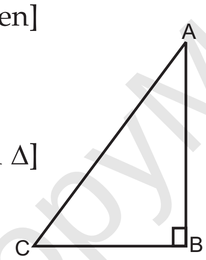
$\Rightarrow \quad 12x = 96$

$\Rightarrow \quad x = \frac{96}{12} = 8$ cm

$\therefore \quad DB = 8$ cm **Ans.**

2. If Δ *ABC* is right angled at *B*, what is the value of sin (*A* + *C*). [1]

Solution : $\angle B = 90°$ [Given]



We know that in Δ ABC,

$\angle A + \angle B + \angle C = 180°$

[Angle sum property of a Δ]

$\Rightarrow \angle A + \angle C + 90° = 180°$

$\Rightarrow \quad \angle A + \angle C = 180° - 90°$

$= 90°$

$\therefore \quad \sin (A + C) = \sin 90° = 1$ **Ans.**

3. If $\sqrt{3} \sin \theta = \cos \theta$, find the value of $\frac{3\cos^2\theta + 2\cos\theta}{3\cos\theta + 2}$. [1]

Solution : $\sqrt{3} \sin \theta = \cos \theta$ [Given]

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$ or $\tan \theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan \theta = \tan 30° \Rightarrow \theta = 30°$

Now,

$$\frac{3\cos^2\theta + 2\cos\theta}{3\cos\theta + 2} = \frac{\cos\theta(3\cos\theta + 2)}{(3\cos\theta + 2)}$$

$= \cos \theta$

Put $\quad \theta = 30°$

$\Rightarrow \quad \cos 30° = \frac{\sqrt{3}}{2}$ **Ans.**

4. From the following frequency distribution, find the median class :

Cost of living index	1400-1550	1550-1700	1700-1850	1850-2000
Number of weeks	8	15	21	8

[1]

Solution :

Cost of living index	No. of weeks (f)	*c.f.*
1400-1550	8	8
1550-1700	15	23
1700-1850	21	44
1850-2000	8	52
	$\Sigma f = 52$	

Here, $\quad n = 52$

$\Rightarrow \quad \frac{n}{2} = \frac{52}{2} = 26,$

26 will lie in the class interval 1700-1850.

$\therefore$ Median class is 1700-1850. **Ans.**

SECTION — B

5. Show that $3\sqrt{7}$ is an irrational number. [2]

Solution : Let us assume, to the contrary, that $3\sqrt{7}$ is rational.

That is, we can find co-prime a and b ($b \neq 0$) such that $3\sqrt{7} = \frac{a}{b}$

Rearranging, we get $\sqrt{7} = \frac{a}{3b}$

Since 3, a and b are integers, $\frac{a}{3b}$ can be written in the form of $\frac{p}{q}$, so $\frac{a}{3b}$ is rational, and so $\sqrt{7}$ is rational.

But this contradicts the fact that $\sqrt{7}$ is irrational. So, we conclude that $3\sqrt{7}$ is irrational.

Hence Proved.

6. Explain why (17 × 5 × 11 × 3 × 2 + 2 × 11) is a composite number ? [2]

Solution : 17 × 5 × 11 × 3 × 2 + 2 × 11 = 17 × 5 × 3 × 22 + 22 ...(i)

= 22 (17 × 5 × 3 + 1)

= 22 (255 + 1) = 2 × 11 × 256

Equation (i) is divisible by 2, 11 and 256, which means it has more than 2 prime factors.

$\therefore$ $(17 \times 5 \times 11 \times 3 \times 2 + 2 \times 11)$ is a composite number. **Ans.**

7. Find whether the following pair of linear equations is consistent or inconsistent :

$$3x + 2y = 8$$
$$6x - 4y = 9 \qquad \textbf{[2]}$$

Solution : Here, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{2}{-4} = \frac{-1}{2}$

$$\frac{1}{2} \neq \frac{-1}{2}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, which will give a unique solution.

Hence, given pair of linear equations is consistent. **Ans.**

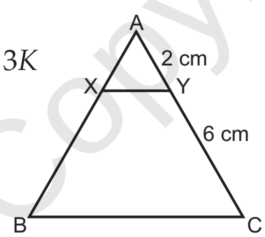
8. X and Y are points on the sides AB and AC respectively of a triangle ABC such that $\frac{AX}{AB}$, AY = 2 cm and YC = 6 cm. Find whether $XY \parallel BC$ or not. **[2]**

Solution : $\frac{AX}{AB} = \frac{1}{4}$

i.e., $AX = 1K$, $AB = 4K$ (K– constant)

$\therefore$ $BX = AB - AX$

$= 4K - 1K = 3K$



Now, $\frac{AX}{XB} = \frac{1K}{3K} = \frac{1}{3}$

And, $\frac{AY}{YC} = \frac{2}{6} = \frac{1}{3}$

$\frac{AX}{XB} = \frac{AY}{YC}$

$\therefore$ $XY \parallel BC$ (By converse of Thales' theorem) **Ans.**

9. Prove the following identity :

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = 1 - \sin\theta.\cos\theta. \qquad \textbf{[2]}$$

Solution : L.H.S. $= \frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta}$

$$= \frac{(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta.\cos\theta)}{(\sin\theta + \cos\theta)}$$

$[a^2 + b^3 = (a + b)(a^2 + b^2 - ab)]$

$= 1 - \sin\theta.\cos\theta =$ R.H.S.

$[\because \sin^2\theta + \cos^2\theta = 1]$ **Hence Proved.**

10. Show that the mode of the series obtained by combining the two series S_1 and S_2 given below is different from that of S_1 and S_2 taken separately:

S_1 : 3, 5, 8, 8, 9, 12, 13, 9, 9

S_2 : 7, 4, 7, 8, 7, 8, 13 **[2]**

Solution : Mode of S_1 series = 9

Mode of S_2 series = 7

After combining S_1 and S_2, the new series will be = 3, 5, 8, 8, 9, 12, 13, 9, 9, 7, 4, 7, 8, 7, 8, 13.

Mode of combined series = 8 (maximum times)

Mode of (S_1, S_2) is different from mode of S_1 and mode of S_2 separately. **Hence Proved.**

SECTION — C

11. The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly. **[3]**

Solution : To find the length of the longest rod that can measure the dimensions of the room exactly, we have to find HCF.

Length, L = 8 m 50 cm = 850 cm = $2^1 \times 5^2 \times 17$

Breadth, B = 6 m 25 cm = 625 cm = 5^4

Height, H = 4 m 75 cm = 475 cm = $5^2 \times 19$

$\therefore$ HCF of L, B and H is 5^2 = 25 cm

$\therefore$ Length of the longest rod = 25 cm **Ans.**

12. Solve by elimination :

$3x - y = 7$

$2x + 5y + 1 = 0$ **[3]**

Solution : $3x - y = 7$ ...(i)

$2x + 5y = -1$ ...(ii)

Multiplying equation (i) by 5 and solving it with equation (ii), we get

$$\begin{array}{rcl} 2x + 5y &=& -1 \\ 15x - 5y &=& 35 \\ \hline 17x &=& 34 \end{array}$$

(Adding)

$\Rightarrow$ $x = \frac{34}{17} = 2$

Putting the value of x in (i), we have

$3(2) - y = 7$

$\Rightarrow$ $6 - y = 7 \Rightarrow -y = 7 - 6$

$\Rightarrow$ $y = -1$

$\therefore$ $x = 2, y = -1$ **Ans.**

13. Find a quadratic polynomial, the sum and product of whose zeroes are 0 and $-\frac{3}{5}$ respectively. Hence find the zeroes. **[3]**

Solution : Quadratic polynomial

$$= x^2 - (\text{Sum of zeroes})\, x + \text{Product of zeroes}$$

$$= x^2 - (0)\, x + \left(\frac{-3}{5}\right) = x^2 - \frac{3}{5}$$

$$= (x)^2 - \left(\sqrt{\frac{3}{5}}\right)^2$$

$$= \left(x - \sqrt{\frac{3}{5}}\right)\left(x + \sqrt{\frac{3}{5}}\right) \left[\begin{array}{l}\text{By applying} \\ (a^2 - b^2) = (a+b)(a-b)\end{array}\right]$$

Zeroes are, $x - \sqrt{\frac{3}{5}} = 0$ or $x + \sqrt{\frac{3}{5}} = 0$

$$\Rightarrow \quad x = \sqrt{\frac{3}{5}} \quad \text{or} \quad x = -\sqrt{\frac{3}{5}}$$

$$x = \sqrt{\frac{3}{5} \times \frac{5}{5}} \quad \text{or} \quad x = -\sqrt{\frac{3}{5} \times \frac{5}{5}}$$

$$x = \frac{\sqrt{15}}{5} \quad \text{or} \quad x = \frac{-\sqrt{15}}{5}$$ **Ans.**

14. The sum of the digits of a two digit number is 8 and the difference between the number and that formed by reversing the digits is 18. Find the number. **[3]**

Solution : Let unit digit $= x$

Tens digit $= y$

So, original number = unit digit + 10 × tens digit

$$1 = x + 10y$$

According to question,

$$\text{Sum of digits} = 8$$

so, $$x + y = 8 \quad \text{...(i)}$$

On reversing the digits, unit digit $= y$

Tens digit $= x$

so, New number $= 10x + y$

According to question,

$$\text{Difference} = 18$$

$$\Rightarrow x + 10y - (10x + y) = 18$$

$$\Rightarrow x + 10y - 10x - y = 18$$

$$\Rightarrow 9y - 9x = 18$$

$$\Rightarrow y - x = 2 \quad \text{...(ii)}$$

By adding eq. (i) and (ii)

$$2y = 10$$

$$y = \frac{10}{2} \Rightarrow y = 5$$

Put the value of y in eq. (i)

$$x + y = 8$$

$$\Rightarrow x + 5 = 8$$

$$\Rightarrow x = 8 - 5$$

$$\Rightarrow x = 3$$

$\therefore$ Original number $= 10y + x$

$$= 10 \times 5 + 3$$

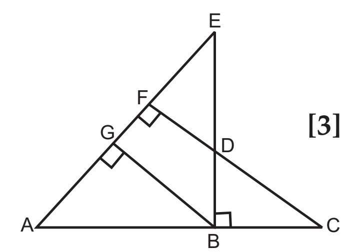
$$= 50 + 3$$

$$= 53$$ **Ans.**

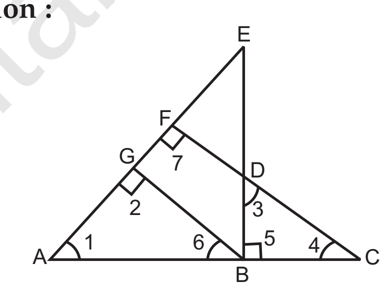
15. In given figure, EB ⊥ AC, BG ⊥ AE and CF ⊥ AE Prove that :

(i) ΔABG ~ ΔDCB

(ii) $\frac{BC}{BD} = \frac{BE}{BA}$ **[3]**



Solution :



Given : EB ⊥ AC, BG ⊥ AE and CF ⊥ AE

To prove : (i) ΔABG ~ ΔDCB

(ii) $$\frac{BC}{BD} = \frac{BE}{BA}$$

Proof : (i) In Δ ABG and Δ DCB, BG || CF as corresponding angles are equal.

$$\angle 2 = \angle 5 \quad \text{[Each 90°]}$$

$$\angle 6 = \angle 4 \quad \text{[Corresponding angles]}$$

$\therefore$ Δ ABG ~ Δ DCB **Hence Proved.**

[By AA similarity]

$$\angle 1 = \angle 3 \quad \text{[CPCT]}$$

(ii) In Δ ABE and Δ DBC

$$\angle 1 = \angle 3 \quad \text{[Proved above]}$$

$$\angle ABE = \angle 5$$

[Each is 90°, EB ⊥ AC (Given)]

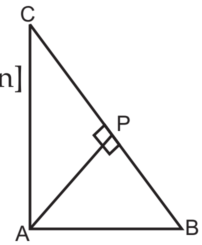
Δ ABE ~ Δ DBC [By AA similarity]

In similar triangles, corresponding sides are proportional

$$\therefore \quad \frac{BC}{BD} = \frac{BE}{BA}$$ **Hence Proved.**

16. In triangle ABC, if AP ⊥ BC and $AC^2 = BC^2 - AB^2$, then prove that $PA^2 = PB \times CP$. **[3]**

Solution : $AC^2 = BC^2 - AB^2$ [Given]



$$AC^2 + AB^2 = BC^2$$

$$\therefore \quad \angle BAC = 90°$$

[By converse of Pythagoras' theorem]

$$\Delta APB \sim \Delta CPA$$

[If a perpendicular is drawn from the vertex of the right angle of a triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other]

$$\Rightarrow \quad \frac{AP}{CP} = \frac{PB}{PA}$$

[In similar triangle, corresponding sides are proportional]

$$\Rightarrow \quad PA^2 = PB \,.\, CP$$

Hence Proved.

17. If $\sin\theta = \frac{12}{13}$, $0° < \theta < 90°$, find the value of :

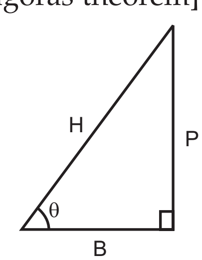
$$\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta.\cos\theta} \times \frac{1}{\tan^2\theta}$$ **[3]**

Solution : Given, $\sin\theta = \frac{12}{13}$

$$\Rightarrow \quad \frac{P}{H} = \frac{12}{13}$$

Let, P = 12K, H = 13K

$$P^2 + B^2 = H^2 \quad \text{[Pythagoras theorem]}$$



$$(12K)^2 + B^2 = (13K)^2$$

$$144K^2 + B^2 = 169K^2$$

$$B^2 = 169K^2 - 144K^2$$

$$= 25K^2$$

$$B = 5K$$

$$\therefore \quad \cos\theta = \frac{B}{H} = \frac{5K}{13K} = \frac{5}{13}$$

$$\tan\theta = \frac{P}{B} = \frac{12K}{5K} = \frac{12}{5}$$

Now, $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta.\cos\theta} \times \frac{1}{\tan^2\theta}$

On solving,

$$= \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2}$$

$$= \frac{\frac{144-25}{169}}{\frac{120}{169}} \times \frac{25}{144}$$

$$= \frac{119}{120} \times \frac{25}{144} = \frac{595}{3456}$$

Ans.

18. If $\sec\theta + \tan\theta = p$, prove that $\sin\theta = \frac{p^2-1}{p^2+1}$ **[3]**

Solution : R.H.S. $= \frac{p^2-1}{p^2+1}$

$$= \frac{(\sec\theta + \tan\theta)^2 - 1}{(\sec\theta + \tan\theta)^2 + 1}$$

$$= \frac{\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta - 1}{\sec^2\theta + \tan^2 + 2\sec\theta\,\tan\theta + 1}$$

[By $(a + b)^2 = a^2 + b^2 + 2ab$]

$$= \frac{(\sec^2\theta - 1) + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + (1 + \tan^2\theta) + 2\sec\theta\tan\theta}$$

$$= \frac{\tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta}{\sec^2\theta + \sec^2\theta + 2\sec\theta\tan\theta}$$

$$\left[\begin{matrix}\sec^2\theta - 1 = \tan^2\theta \\ \sec^2\theta = 1 + \tan^2\theta\end{matrix}\right]$$

$$= \frac{2\tan^2\theta + 2\sec\theta\tan\theta}{2\sec^2\theta + 2\sec\theta\tan\theta}$$

$$= \frac{2\tan\theta(\tan\theta + \sec\theta)}{2\sec\theta\,(\sec\theta + \tan\theta)} = \frac{\tan\theta}{\sec\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$= \sin\theta =$ L.H.S. **Hence Proved.**

19. Find the mean of the following distribution by Assumed Mean Method :

Class interval	Frequency
10-20	8
20-30	7
30-40	12
40-50	23
50-60	11

60-70	13
70-80	8
80-90	6
90-100	12

[3]

Solution :

Class interval	Frequency (f_i)	X_i	$d_i = X_i - 55$	$f_i d_i$
10-20	8	15	-40	-320
20-30	7	25	-30	-210
30-40	12	35	-20	-240
40-50	23	45	-10	-230
50-60	11	55	0	0
60-70	13	65	10	130
70-80	8	75	20	160
80-90	6	85	30	180
90-100	12	95	40	480
	$\Sigma f_i = 100$			$\Sigma f_i d_i = -50$

Let A = 55

$$\text{Mean} = A + \frac{\sum f_i d_i}{\sum f_i} = 55 + \left(\frac{-50}{100}\right)$$

$$= 55 - \frac{50}{100} = 55 - 0.5 = 54.5 \quad \textbf{Ans.}$$

20. The average score of boys in the examination of a school is 71 and that of the girls is 73. The average score of the school in the examination is 71.8. Find the ratio of number of boys in the number of girls who appeared in the examination. **[3]**

Solution : Let the number of boys = n_1

and number of girls = n_2

Average boys' score = $71 = \overline{X_1}$ (Let)

Average girls' score = $73 = \overline{X_2}$ (Let)

$$\text{Combined mean} = \frac{n_1\overline{X}_1 + n_2\overline{X}_2}{n_1 + n_2}$$

$$71.8 = \frac{n_1(71) + n_2(73)}{n_1 + n_2}$$

$$71n_1 + 73n_2 = 71.8n_1 + 71.8n_2$$

$$71n_1 - 71.8n_1 = 71.8n_2 - 73n_2$$

$$-0.8n_1 = -1.2n_2$$

$$\frac{n_1}{n_2} = \frac{1.2}{0.8} \Rightarrow \frac{n_1}{n_2} = \frac{3}{2}$$

$\Rightarrow \quad n_1 : n_2 = 3 : 2$

$\therefore$ No. of boys : No. of girls = 3 : 2. **Ans.**

SECTION — D

21. Find HCF of numbers 134791, 6341 and 6339 by Euclid's division algorithm. **[4]**

Solution : First we find HCF of 6339 and 6341 by Euclid's division method

```
6339) 6341 (1
      6339
      ----
        2) 6339 (3169
           6
           --
            3
            2
           --
            13
            12
           --
             19
             18
           ----
           1) 2 (2
              2
             --
              0
```

$$6341 > 6339$$

$\Rightarrow \quad 6341 = 6339 \times 1 + 2$

Also, $\quad 6339 = 2 \times 3169 + 1$

$$2 = 1 \times 2 + 0$$

$\therefore$ HCF of 6341 and 6339 is 1.

Now, we find the HCF of 134791 and 1

$$134791 = 1 \times 134791 + 0$$

$\therefore$ HCF of 134791 and 1 is 1.

Hence, HCF of given three numbers is 1. **Ans.**

22. Draw the graph of the following pair of linear equations :
$x + 3y = 6$ and $2x - 3y = 2$
Find the ratio of the areas of the two triangles formed by first line, $x = 0$, $y = 0$ and second line, $x = 0$, $y = 0$. **[4]**

Solution : **First Line** | **Second Line**

$x + 3y = 6$ | $2x - 3y = 12$

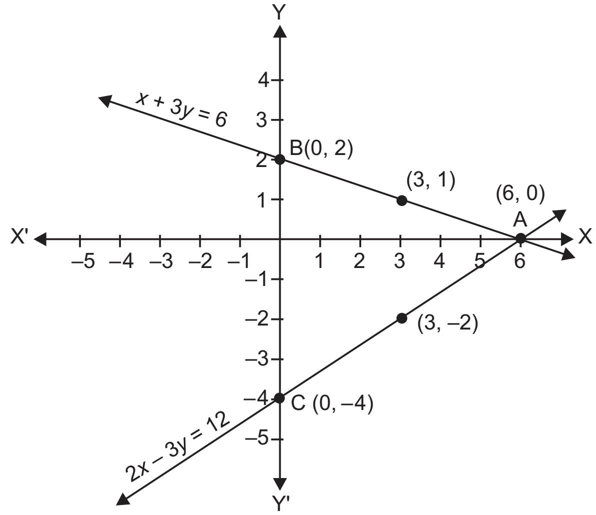
$\Rightarrow \quad x = 6 - 3y$ | $\Rightarrow 2x = 12 + 3y$

$$\Rightarrow x = \frac{12 + 3y}{2}$$

x	6	3	0
y	0	1	2

x	6	3	0
y	0	–2	–4

(6, 0), (3, 1), (0, 2)   (6, 0), (3, –2), (0, –4)



Area of triangle

$= \frac{1}{2} \times$ base $\times$ corresponding altitude

$$\therefore \quad \frac{\text{Area of } \Delta\, AOB}{\text{Area of } \Delta\, AOC} = \frac{1/2 \times \text{OA} \times \text{OB}}{1/2 \times \text{OA} \times \text{OC}}$$

$$\Rightarrow \quad \frac{\text{OB}}{\text{OC}} = \frac{2}{4} = \frac{1}{2}$$

$\therefore$ Required ratio = 1 : 2 **Ans.**

23. If the polynomial ($x^4 + 2x^3 + 8x^2 + 12x + 18$) is divided by another polynomial ($x^2 + 5$), the remainder comes out to be ($px + q$), find the values of p and q. [4]

Solution :

$$
\begin{array}{rl}
 & x^2+2x+3 \\ \hline
x^2+5\,) & x^4+2x^3+8x^2+12x+18 \\
 & +x^4 \qquad\quad +5x^2 \\
 & - \qquad\qquad - \\ \hline
 & \quad 2x^3+3x^2+12x+18 \\
 & \quad +2x^3 \qquad +10x \\
 & \quad - \qquad\qquad - \\ \hline
 & \qquad 3x^2+2x+18 \\
 & \qquad +3x^2 \qquad +15 \\
 & \qquad - \qquad\qquad - \\ \hline
 & \qquad\qquad 2x+3 \\ \hline
\end{array}
$$

Remainder $= 2x + 3$

i.e., $px + q = 2x + 3$

$\therefore$ $p = 2, q = 3$ **Ans.**

24. What must be subtracted from $p(x) = 8x^4 + 14x^3 - 2x^2 + 8x - 12$ so that $4x^2 + 3x - 2$ is factor of $p(x)$? This quesiton was given to group of students for working together. Do you think teacher should promote group work ? [4]

Solution : For this,

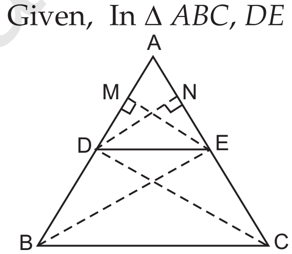
$$
\begin{array}{rl}
 & 2x^2+2x-1 \\ \hline
4x^2+3x-2\,) & 8x^4+14x^3-2x^2+8x-12 \\
 & +8x^4+\ 6x^3\ -4x^2 \\
 & -\qquad -\qquad + \\ \hline
 & \qquad 8x^3\ +2x^2+8x-12 \\
 & \qquad +8x^3\ +6x^2-4x \\
 & \qquad -\qquad -\qquad + \\ \hline
 & \qquad\qquad -4x^2+12x-12 \\
 & \qquad\qquad -4x^2-3x+2 \\
 & \qquad\qquad +\qquad +\qquad - \\ \hline
 & \qquad\qquad\qquad 15x-14 \\ \hline
\end{array}
$$

Polynomial to be subtracted is ($15x - 14$).

Value : Yes, as it increases confidence and team spirit among students. **Ans.**

25. Prove "If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio". [4]

Solution : Given, In $\Delta\, ABC$, $DE \parallel BC$



To prove : $\frac{AD}{DB} = \frac{\text{AE}}{\text{EC}}$

Construction : Draw $EM \perp AB$ and $DN \perp$ AC. Join B to E and C to D.

Proof : In $\Delta\, ADE$ and $\Delta\, BDE$

$$\frac{\text{ar}\,(\Delta\, ADE)}{\text{ar}\,(\Delta\, BDE)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times DB \times EM} = \frac{AD}{DB} \qquad ...(i)$$

[Area of $\Delta = \frac{1}{2}$ × base × corresponding altitude]

In $\Delta\, ADE$ and $\Delta\, CDE$

$$\frac{\text{ar}\,(\Delta\, ADE)}{\text{ar}\,(\Delta\, CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN} = \frac{AE}{EC} \qquad ...(ii)$$

Since, $DE \parallel BC$ [Given]

$\therefore$ ar (ΔBDE) = ar (ΔCDE) ...(iii)

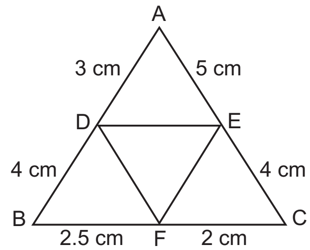
[Δs on the same base and between the same parallel sides are equal in area]

From eq. (i), (ii) and (iii)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence Proved.

26. In the given figure, $AD = 3$ cm, $AE = 5$ cm, $BD = 4$ cm, $CE = 4$ cm, $CF = 2$ cm, $BF = 2.5$ cm, then find the pair of parallel lines and hence their lengths. [4]



Solution : $\frac{EC}{EA} = \frac{CF}{FB}$ and $\frac{CF}{FB} = \frac{2}{2.5} = \frac{4}{5}$

$\Rightarrow \quad \frac{EC}{EA} = \frac{CF}{FB}$

In $\Delta\, ABC$, $EF \parallel AB$

[Converse of Thales' theorem]

Also, $\quad \frac{CE}{CA} = \frac{4}{4+5} = \frac{4}{9} \quad$ ...(i)

$$\frac{CF}{CB} = \frac{2}{2+2.5} = \frac{2}{4.5} = \frac{4}{9}$$

$$\frac{EC}{EA} = \frac{CF}{CB}$$

$\angle ECF = \angle ACB \quad$ [Common]

$\Delta\, CFE \sim \Delta\, CBA \quad$ [SAS similarity]

$\Rightarrow \quad \frac{EF}{AB} = \frac{CE}{CA}$

[In similar Δ's, corresponding sides are proportional]

$\Rightarrow \quad \frac{EF}{7} = \frac{4}{9}$

[$\because AB = 3 + 4 = 7$ cm]

$\therefore \quad EF = \frac{28}{9}$ cm and $AB = 7$ cm **Ans.**

27. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$, where $0 < A + B < 90°$, $A > B$, find A and B. Also calculate $\tan A . \sin(A + B) + \cos A . \tan(A - B)$. [4]

Solution : Given, $\tan(A + B) = \sqrt{3}$, $\tan(A - B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \tan(A + B) = \tan 60°$

$(A + B) = 60° \quad$ ...(i)

And, $\tan(A - B) = \tan 30°$

$(A - B) = 30° \quad$ ...(ii)

On adding eq. (i) & (ii)

$$A + B = 60°$$
$$\underline{A - B = 30°}$$
$$\underline{2A \quad = 90°} \quad \text{[By adding]}$$

$\Rightarrow \quad A = \frac{90°}{2} = 45°$

From eq. (i), $A + B = 60°$

$45° + B = 60°$

$B = 15°$

$\therefore \quad A = 45°, B = 15°$

Now, $\tan A . \sin(A + B) + \cos A . \tan(A - B)$

$$= \tan 45° . \sin(60°) + \cos 45° . \tan(30°)$$

$$= 1 \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{3}}{2} + \frac{\sqrt{6}}{6}$$

$$= \frac{3\sqrt{3} + \sqrt{6}}{6}$$ **Ans.**

28. Prove that :

$$\mathbf{(1 + \cot A + \tan A).(\sin A - \cos A) = \frac{\sec^3 A - \mathrm{cosec}^3 A}{\sec^2 A . \mathrm{cosec}^2 A}} \quad [4]$$

Solution : L.H.S. $= (1 + \cot A + \tan A)(\sin A - \cos A)$

$$= \left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A)$$

$$= \left(\frac{\sin A \cos A + \cos^2 A + \sin^2 A}{\sin A . \cos A}\right)(\sin A - \cos A)$$

$$= \frac{\sin^3 A - \cos^3 A}{\sin A . \cos A}$$

[Using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$$= \frac{\frac{\sin^3 A}{\sin^3 A . \cos^3 A} - \frac{\cos^3 A}{\sin^3 A . \cos^3 A}}{\frac{\sin A \cos A}{\sin^3 A . \cos^3 A}}$$

[Dividing Num. & Denom. by $\sin^3 A . \cos^3 A$]

$$= \frac{\sec^3 A - \mathrm{cosec}^3 A}{\sec^2 A . \mathrm{cosec}^2 A} = \text{R.H.S.}$$ **Hence Proved.**

29. Prove the identity :

$$\mathbf{\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{1 - 2\cos^2 A}} \quad [4]$$

Solution : L.H.S. $= \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$

$$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}$$

$$\frac{\sin^2 A + \cos^2 A + 2 \sin A \cos A + \sin^2 A + \cos^2 A - 2 \sin A \cos A}{\sin^2 A - \cos^2 A}$$

$$= \frac{1+1}{1-\cos^2 A - \cos^2 A} \quad [\because \sin^2 A + \cos^2 A = 1, \sin^2 A = 1 - \cos^2 A]$$

$$= \frac{2}{1-2\cos^2 A} = \text{R.H.S.}$$

Hence Proved.

30. The following table gives the daily income of 50 workers of a factory. Draw both types ("less than type" and " greater than type") ogives.

Daily income (in ₹)	No. of workers
100 – 120	12
120 – 140	14
140 – 160	8
160 – 180	6
180 – 200	10

[4]

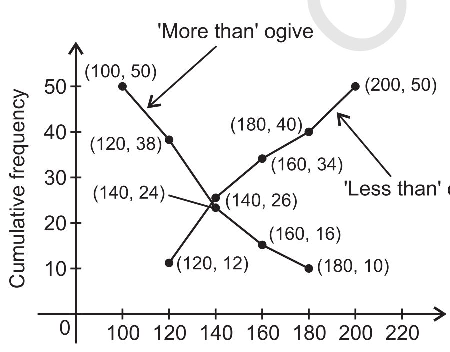
Solution :

Less than ogive

Daily Income (in ₹)	No. of workers (*c.f.*)
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

More than ogive

Daily Income (in ₹)	No. of workers (*c.f.*)
More than 100	50
More than 120	38
More than 140	24
More than 160	16
More than 180	10



31. In a class test, marks obtained by 120 students are given in the following frequency distribution. If it is given that mean is 59, find the missing frequencies x and y.

Marks	No. of students
0 – 10	1
10 – 20	3
20 – 30	7
30 – 40	10
40 – 50	15
50 – 60	x
60 – 70	9
70 – 80	27
80 – 90	18
90 – 100	y

[4]

Solution :

Marks	No. of Students f_i	X_i	$d_i = \frac{X_i - 55}{10}$	$f_i d_i$
0-10	1	5	–5	–5
10-20	3	15	–4	–12
20-30	7	25	–3	–21
30-40	10	35	–2	–20
40-50	15	45	–1	–15
50-60	x	A= 55	0	0
60-70	9	65	1	9
70-80	27	75	2	54
80-90	18	85	3	54
90-100	y	95	4	$4y$
	$\Sigma f_i = 90 + x + y$			$\Sigma f_i d_i = 44 + 4y$

$$\Sigma f_i = 90 + x + y$$

But $\Sigma f_i = 120$ [Given]

$$\therefore \quad 90 + x + y = 120$$

$$x = 120 - 90 - y = 30 - y \quad \text{...(i)}$$

$$\text{Mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i} \times h$$

$$\Rightarrow \quad 59 = 55 + \left(\frac{44+4y}{120} \times 10\right)$$

$[A = 55, h = 10, \Sigma f_i = 120]$

$$\Rightarrow \quad 59 - 55 = \frac{4(11+y)}{12}$$

$$\Rightarrow \quad 4 \times 3 = 11 + y$$

$$\Rightarrow \quad y = 12 - 11 = 1$$

From eq. (i), $x = 30 - 1 = 29$

$\therefore \quad x = 29, y = 1$ **Ans.**

SECTION — A

1. If the quadratic equation $px^2 - 2\sqrt{5}px + 15 = 0$, has two equal roots then find the value of p. [1]

Solution : The given quadratic equation is,

$$px^2 - 2\sqrt{5}px + 15 = 0$$

This is of the form

$$ax^2 + bx + c = 0$$

Where, $a = p$, $b = -2\sqrt{5}p$, $c = 15$

We have,
$$\begin{aligned} D &= b^2 - 4ac \\ &= (-2\sqrt{5}p)^2 - 4 \times p \times 15 \\ &= 20p^2 - 60p \\ &= 20p(p-3) \end{aligned}$$

For real and equal roots, we must have :

$$D = 0, \quad \Rightarrow 20p(p-3) = 0$$
$$\Rightarrow p = 0, p = 3$$

$p = 0$, is not possible as whole equation will be zero.

Hence, 3 is the required value of p. **Ans.**

2. In figure 1, a tower AB is 20 m high and BC, its shadow on the ground, is $20\sqrt{3}$ m long. Find the sun's altitude. [1]

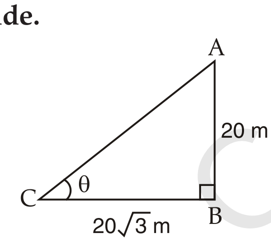


Figure 1

Solution : Given, AB is the tower and BC is its shadow.

$\therefore$ $$\tan\theta = \frac{AB}{BC}$$

$$\left[\because \tan\theta = \frac{\text{Perpendicular}}{\text{Base}}\right]$$

$\Rightarrow$ $$\tan\theta = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $$\tan\theta = \tan 30° \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$\Rightarrow$ $$\theta = 30°$$ **Ans.**

3. Two different dice are tossed together. Find the probability that the product of two numbers on the top of the dice is 6. [1]

Solution : When two dice are thrown simultaneously, all possible outcomes are :

$$S = \begin{bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{bmatrix}$$

Total number of all outcomes $= 6 \times 6 = 36$

Favourable outcomes of getting the product as 6 are :

(2, 3), (3, 2), (1, 6), (6, 1)

Hence, number of favourable outcomes getting product as 6 is 4.

Probability that the product of the two numbers on the top of the die is 6

$$= \frac{4}{36}$$
$$= \frac{1}{9}$$ **Ans.**

4. In figure 2, PQ is a chord of a circle with centre O and PT is a tangent. If $\angle QPT = 60°$, find $\angle PRQ$. [1]

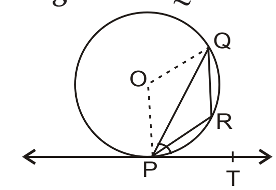


Figure 2

Solution : Given, O is the centre of the given circle

$\therefore$ OQ and OP are the radius of circle.

$\because$ PT is a tangent

$\therefore$ $$OP \perp PT$$

So, $$\angle OPT = 90°$$

$\therefore$ $$\angle OPQ = 90° - \angle QPT$$
$$\angle OPQ = 90° - 60°$$
[Given, $\angle QPT = 60°$]
$$\angle OPQ = 30°$$

$\therefore$ $$\angle OQP = 30°$$ [$\because$ ΔOPQ is isosceles triangle]

Now, in ΔOPQ

$$\angle POQ + \angle OPQ + \angle OQP = 180°$$
$$\angle POQ + 30° + 30° = 180°$$
$$\angle POQ = 120°$$

reflex $\angle POQ = 360° - 120° = 240°$

$\therefore \qquad \angle PRQ = \frac{1}{2}$ reflex $\angle POQ$

[$\because$ The angle substended by an arc of a circle at the centre is double the angle substended by it at any point on the remaining part of the circle]

$$\angle PRQ = \frac{1}{2} \times 240°$$

Hence, $\qquad \angle PRQ = 120°$ **Ans.**

SECTION — B

5. In figure 3, two tangents *RQ* and *RP* are drawn from an external point *R* to the circle with centre *O*. If $\angle PRQ = 120°$, then prove that, $OR = PR + RQ$. [2]

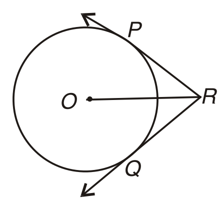
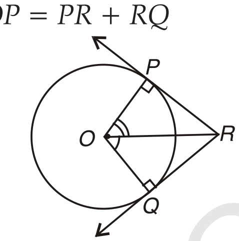


Figure 3

Solution : *O* is the centre of the circle and $\angle PRQ = 120°$

Construction : Join *OP*, *OQ*

To prove : $OP = PR + RQ$



Proof : We know that,

Tangent to a circle is perpendicular to the radius at the point of tangent *i.e.*, $OP \perp RP$ and $OQ \perp RQ$.

$\therefore \qquad \angle OPR = \angle OQR = 90°$

Now, in ΔOPR and ΔOQR,

$OP = OQ$ [Radius of circle]

$OR = OR$ [Common]

$\angle OPR = \angle OQR = 90°$ [Each 90°]

$\therefore \qquad \Delta OPR \cong \Delta OQR$

[By SSA congruence]

So, $\qquad PR = QR$ [By c.p.c.t.]

and $\qquad \angle ORP = \angle ORQ$

$$= \frac{120°}{2} = 60°$$

Now, in Δ *OPR*

$$\cos 60° = \frac{PR}{OR} \quad \left[\because \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}\right]$$

$$\frac{1}{2} = \frac{PR}{OR}$$

$OR = 2PR$

$OR = PR + PR$

$OR = PR + RQ$ [$\because PR = RQ$]

Hence, $\qquad OR = PR + RQ.$ **Hence Proved**

6. In figure 4, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm^2, then find the lengths of sides AB and AC. [2]

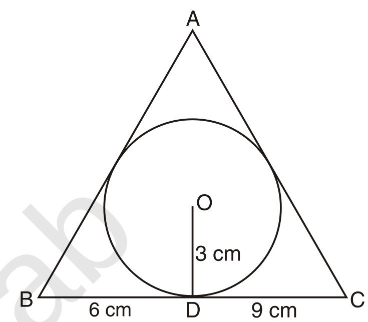
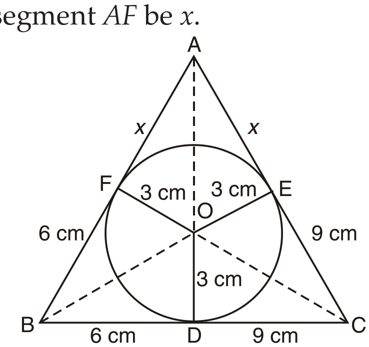


Figure 4

Solution : Given, in Δ *ABC*, circle touch the triangle at point *D*, *F* and *E* respectively and let the lengths of the segment *AF* be *x*.



So, $\qquad BF = BD = 6$ cm [Tangent from point *B*]

$CE = CD = 9$ cm [Tangent from point *C*]

and $\qquad AE = AF = x$ cm [Tangent from point *A*]

Now, $\qquad$ Area of Δ *OBC* $= \frac{1}{2} \times BC \times OD$

$$= \frac{1}{2} \times (6 + 9) \times 3$$

$$= \frac{45}{2} \text{ cm}^2$$

Area of Δ *OCA* $= \frac{1}{2} \times AC \times OE$

$$= \frac{1}{2} \times (9 + x) \times 3$$

$$= \frac{3}{2}(9 + x) \text{ cm}^2$$

$$\text{Area of } \Delta BOA = \frac{1}{2} \times AB \times OF$$
$$= \frac{1}{2} \times (6 + x) \times 3$$
$$= \frac{3}{2} (6 + x) \text{ cm}^2$$

Area of $\Delta ABC = 54 \text{ cm}^2$ [Given]

$\because$ Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔBOA

$$54 = \frac{45}{2} + \frac{3}{2}(9+x) + \frac{3}{2}(6+x)$$
$$\Rightarrow \quad 54 \times 2 = 45 + 27 + 3x + 18 + 3x$$
$$\Rightarrow \quad 108 - 45 - 27 - 18 = 6x$$
$$\Rightarrow \quad 6x = 18$$
$$\Rightarrow \quad x = 3$$

So, $AB = AF + FB = x + 6 = 3 + 6 = 9$ cm

and $AC = AE + EC = x + 9 = 3 + 9 = 12$ cm

Hence, lengths of AB and AC are 9 cm and 12 cm respectively. **Ans.**

7. Solve the following quadratic equation for x :

$4x^2 + 4bx - (a^2 - b^2) = 0$ **[2]**

Solution : The given equation is

$$4x^2 + 4bx - (a^2 - b^2) = 0 \quad \ldots(i)$$

Comparing equation (i) with quadratic equation

$Ax^2 + Bx + C = 0$, we get

$A = 4, B = 4b, C = -(a^2 - b^2)$

By quadratic formula

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$
$$x = \frac{-4b \pm \sqrt{16b^2 + 4 \times 4 \times (a^2 - b^2)}}{2 \times 4}$$
$$x = \frac{-4b \pm \sqrt{16b^2 + 16a^2 - 16b^2}}{8}$$
$$x = \frac{-4b \pm 4a}{8}$$
$$x = \frac{-b \pm a}{2}$$

Therefore, $x = \dfrac{-b-a}{2} \Rightarrow -\left(\dfrac{a+b}{2}\right)$

or $x = \dfrac{-b+a}{2} \Rightarrow \dfrac{a-b}{2}$

Hence, $x = -\left(\dfrac{a+b}{2}\right)$ and $x = \dfrac{a-b}{2}$. **Ans.**

8. In an A.P., if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the A.P., where S_n denotes the sum of its first n terms. **[2]**

Solution : Given, $S_5 + S_7 = 167$

$$\Rightarrow \quad \frac{5}{2}(2a+4d) + \frac{7}{2}(2a+6d) = 167$$
$$\Rightarrow \frac{5}{2} \times 2(a+2d) + \frac{7}{2} \times 2(a+3d) = 167$$
$$\Rightarrow \quad 5a + 10d + 7a + 21d = 167$$
$$\Rightarrow \quad 12a + 31d = 167 \quad \ldots(i)$$
$$\Rightarrow \quad \frac{10}{2}(2a+9d) = 235$$
$$\Rightarrow \quad 10a + 45d = 235$$
$$\Rightarrow \quad 2a + 9d = 47 \quad \ldots(ii)$$

On multiplying equation (ii) by 6, we get :

$$12a + 54d = 282 \quad \ldots(iii)$$

On subtracting equation (i) from (iii), we get :

$$\begin{array}{r} 12a + 54d = 282 \\ 12a + 31d = 167 \\ - \quad - \quad\quad - \\ \hline 23d = 115 \end{array}$$
$$\Rightarrow \quad d = 5$$

Substituting value of d in equation (i), we get

$$12a + 31 \times 5 = 167$$
$$12a + 155 = 167$$
$$\Rightarrow \quad 12a = 12$$
$$\Rightarrow \quad a = 1$$

Hence A.P. is 1, 6, 11.... **Ans.**

9. The points A(4, 7), B(p, 3) and C(7, 3) are the vertices of a right triangle, right-angled at B. Find the value of p. **[2]**

Solution : The given points are $A(4, 7)$, $B(p, 3)$ and $C(7, 3)$.

Since A, B and C are the vertices of a right angled triangle

then, $(AB)^2 + (BC)^2 = (AC)^2$

[By Pythagoras theorem]

$$[(p-4)^2 + (3-7)^2] + [(7-p)^2 + (3-3)^2] = [(7-4)^2 + (3-7)^2]$$
$$(p-4)^2 + (-4)^2 + (7-p)^2 = (3)^2 + (-4)^2$$
$$p^2 + 16 - 8p + 16 + 49 + p^2 - 14p = 9 + 16$$
$$2p^2 - 22p + 56 = 0$$
$$p^2 - 11p + 28 = 0$$
$$p^2 - 7p - 4p + 28 = 0$$
$$p(p-7) - 4(p-7) = 0$$

$p = 4$ or 7

$p \neq 7$ (As B and C will coincide)

So, $p = 4$. **Ans.**

10. Find the relation between x and y if the points $A(x, y)$, B(– 5, 7) and C(– 4, 5) are collinear. [2]

Solution : Given that the points $A(x, y)$, $B(-5, 7)$ and C(– 4, 5) are collinear.

So, the area formed by the vertices are 0.

Therefore,

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[x(7-5) - 5(5-y) - 4(y-7)] = 0$$

$$\Rightarrow \quad \frac{1}{2}[x(2) - 5(5-y) - 4(y-7)] = 0$$

$$\Rightarrow \quad 2x - 25 + 5y - 4y + 28 = 0$$

$$\Rightarrow \quad 2x + y + 3 = 0$$

$$-2x - 3 = y$$

which is the required, relation between x and y *i.e.*, $y = -2x - 3$. **Ans.**

SECTION — C

11. The 14th term of an AP is twice its 8th term. If its 6th term is – 8, then find the sum of its first 20 terms. [3]

Solution : In the given AP, let first term = a and common difference = d

Then, $\quad T_n = a + (n-1)d$

$\Rightarrow \quad T_{14} = a + (14-1)d = a + 13d$

and $\quad T_8 = a + (8-1)d = a + 7d$

Now, $\quad T_{14} = 2T_8$ (Given)

$$a + 13d = 2(a + 7d)$$

$$a + 13d = 2a + 14d$$

$$a = -d \quad \text{...(i)}$$

Also, $\quad T_6 = a + (6-1)d$

$\Rightarrow \quad a + 5d = -8 \quad$ ...(ii)

Putting the value of a from eq. (i), we get

$$-d + 5d = -8$$

$$4d = -8$$

$$d = -2$$

Substituting $d = -2$ in eq. (ii), we get

$$a + 5(-2) = -8$$

$$a = 10 - 8$$

$$a = 2$$

$\therefore$ Sum of first 20 terms is

$$S_{20} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{20}{2}[2 \times 2 + (20-1)(-2)]$$

$$= 10[4 - 38]$$

$$= -340$$ **Ans.**

12. Solve for x :

$$\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0 \quad [3]$$

Solution : We have, $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$$\Rightarrow \quad \sqrt{3}x^2 - 3\sqrt{2}x + \sqrt{2}x - 2\sqrt{3} = 0$$

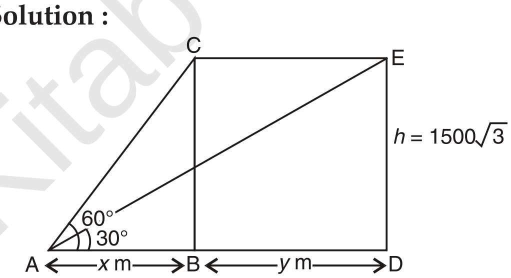
$$\Rightarrow \quad \sqrt{3}x(x - \sqrt{6}) + \sqrt{2}(x - \sqrt{6}) = 0$$

$$\Rightarrow \quad (\sqrt{3}x + \sqrt{2})(x - \sqrt{6}) = 0$$

$$\Rightarrow \quad x = -\sqrt{\frac{2}{3}} \text{ or } \sqrt{6}$$ **Ans.**

13. The angle of elevation of an aeroplane from point A on the ground is 60°. After flight of 15 seconds, the angle of elevation change to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the plane in km/hr. [3]

Solution :



Let BC be the height at which the aeroplane flying.

Then, $\quad BC = 1500\sqrt{3}$ m

In 15 seconds, the aeroplane moves from C to E and makes angle of elevation 30°.

Let $AB = x$ m, $BD = y$ m

So, $\quad AD = (x + y)$ m

In $\Delta\, ABC$,

$$\tan 60° = \frac{BC}{AB}$$

$$\sqrt{3} = \frac{1500\sqrt{3}}{x} \quad [\because \tan 60° = \sqrt{3}]$$

$$x = 1500 \text{ m} \quad \text{...(i)}$$

In $\Delta\, EAD$

$$\tan 30° = \frac{ED}{AD} \quad \left[\because \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$\frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{x + y}$$

$$x + y = 1500 \times 3$$

$$y = 4500 - 1500 = 3000 \text{ m}$$

[Using equation (i)]

$$\text{Speed of aeroplane} = \frac{\text{Distance}}{\text{Time}}$$

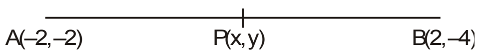
$$= \frac{3000}{15}$$

$$= 200 \text{ m/s} \text{ or } 720 \text{ km/hr} \quad \textbf{Ans.}$$

14. If the coordinates of points A and B are (– 2, – 2) and (2, – 4) respectively find the coordinates of P such that AP = $\frac{3}{7}$ AB, where P lies on the line segment AB. [3]

Solution : Here P (x, y) divides line segment AB

such that AP $= \frac{3}{7}$ AB



$$\Rightarrow \quad \frac{AP}{AB} = \frac{3}{7}$$

$$\Rightarrow \quad \frac{AB}{AP} = \frac{7}{3}$$

$$\Rightarrow \quad \frac{AB}{AP} - 1 = \frac{7}{3} - 1$$

$$\Rightarrow \quad \frac{AB - AP}{AP} = \frac{4}{3}$$

$$\Rightarrow \quad \frac{BP}{AP} = \frac{4}{3}$$

$$\Rightarrow \quad \frac{AP}{BP} = \frac{3}{4}$$

$\therefore$ P divides AB in the ratio 3 : 4 $(m : n)$

The coordinates of P are (x, y)

Therefore,

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

$$x = \frac{3 \times 2 + 4(-2)}{3+4}, y = \frac{3(-4) + 4(-2)}{3+4}$$

$$x = \frac{6-8}{7}, y = \frac{-12-8}{7}$$

$$x = \frac{-2}{7}, y = \frac{-20}{7}$$

Therefore, co-ordinates of $P(x, y)$ are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$

Ans.

15. A probability of selecting a red ball at random from a jar that contains only red, blue and orange is $\frac{1}{4}$. The probability of selecting a blue ball at random from the same jar is $\frac{1}{3}$. If the jar contains 10 orange balls, find the total number of balls in the jar. [3]

Solution : Given, a jar contains red, blue and orange balls.

Let the number of red balls $= x$

and the number of blue balls $= y$

Number of orange balls $= 10$

Then, total number of balls $= x + y + 10$

Let P be the probability of selecting a red ball from the jar

$$\text{P} = \frac{x}{x+y+10}$$

But P(a red balls) $= \frac{1}{4}$ (Given)

$$\therefore \quad \frac{1}{4} = \frac{x}{x+y+10}$$

$$x + y + 10 = 4x$$

$$3x - y = 10 \quad \ldots\text{(i)}$$

Similarly, P(a blue ball) $= \frac{y}{x+y+10}$

But P(a blue ball) $= \frac{1}{3}$

$$\therefore \quad \frac{1}{3} = \frac{y}{x+y+10}$$

$$x + y + 10 = 3y$$

$$x - 2y = -10 \quad \ldots\text{(ii)}$$

On multiplying equation (ii) by 3, we get

$$3x - 6y = -30 \quad \ldots\text{(iii)}$$

On subtracting equation (iii) from (i)

$$\begin{array}{rl} 3x - y &= 10 \\ 3x - 6y &= -30 \\ - \quad + & \quad + \\ \hline 5y &= 40 \\ y &= 8 \end{array}$$

On putting the value of y in (iii), we get

$$3x - 6 \times 8 = -30$$

$$3x = -30 + 48$$

$$x = \frac{18}{3}$$

$$x = 6$$

Total number of balls $= x + y + 10$

$= 6 + 8 + 10$

$= 24$

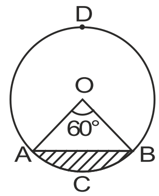
Hence, total number of balls in the jar is 24. **Ans.**

16. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is 60°. Also find the area of the corresponding major segment.

[Use $\pi = \frac{22}{7}$] [3]

Solution : Let ACB be the given arc subtending an angle of 60° at the centre.

Here, $r = 14$ cm and $\theta = 60°$.



Area of the minor segment ACBA

= (Area of the sector $OACBO$) – (Area of ΔOAB)

$$= \frac{\pi r^2\theta}{360°} - \frac{1}{2}r^2 \sin\theta$$

$$= \frac{22}{7}\times 14\times 14\times \frac{60°}{360°} - \frac{1}{2}\times 14\times 14\times \sin 60°$$

$$= \frac{308}{3} - 7\times 14\times \frac{\sqrt{3}}{2}$$

$$= \frac{308}{3} - 49\sqrt{3}$$

$$= 17.89 \text{ cm}^2$$

Area of the major segment $BDAB$

= Area of circle – Area of minor segment $ACBA$

$$= \pi r^2 - 17.89$$

$$= \frac{22}{7}\times 14\times 14 - 17.89$$

$$= 616 - 17.89$$

$$= 598.11 \approx 598 \text{ cm}^2$$ **Ans.**

17. Due to sudden floods, some welfare associations jointly requested the government to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4 m with the conical upper part of same diameter but of height 2.8 m and the canvas to be used costs ₹ 100 per sq. m. Find amount the associations will have to pay. What values are shown by these associations ? [3]

Solution : Diameter of the tent (d) = 4.2 m

$\therefore$ Radius of the tent (r) = 2.1 m $\left[\because r = \frac{d}{2}\right]$

Height of the cylindrical part of tent (h) = 4 m

Height of conical part (H) = 2.8 m

Slant height of conical part (l) = $\sqrt{H^2 + r^2}$

$$l = \sqrt{(2.8)^2 + (2.1)^2}$$

$$l = \sqrt{7.84 + 4.41}$$

$$l = \sqrt{12.25}$$

$$l = 3.5 \text{ m}$$

Curved surface area of the cylinder = $2\pi rh$

$$= 2\times\frac{22}{7}\times 2.1\times 4 \quad \left[\because \pi = \frac{22}{7}\right]$$

$$= 2 \times 22 \times 0{\cdot}3 \times 4$$

$$= 52.8 \text{ m}^2$$

Curved surface area of conical tent = πrl

$$= \frac{22}{7} \times 2.1 \times 3.5$$

$$= 22 \times 0.3 \times 3.5$$

$$= 23.1 \text{ m}^2$$

Total area of cloth required for building one tent

= C.S.A. of cylinder + C.S.A. of conical tent

$= (52.8 + 23.1)\text{m}^2$

$= 75.9 \text{ m}^2$

Cost of building one tent = 75.9×100

= ₹ 7590

Total cost of 100 tents = ₹ (7590 × 100)

= ₹ 7,59,000

Cost to be borne by the associations (50% of the cost)

$$= \frac{759000\times 50}{100}$$

= ₹ 379500

Hence, the association will have to pay ₹ 379500.

Values shown by associations are helping the flood victims and showing concern for humanity. **Ans.**

18. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. Find the height of each bottle, if 10% liquid is wasted in this transfer. [3]

Solution : Internal diameter of hemispherical bowl = 36 m

$\therefore$ Radius of hemispherical bowl (r) = 18 cm

Volume of liquid $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3}\times\pi\times 18^3$$

$\because$ Diameter of bottle = 6 cm

$\therefore$ Radius of bottle = 3 cm

Now, volume of a cylindrical bottle $= \pi R^2 h$

$$= \pi\, 3^2\, h$$
$$= 9\pi h$$

Volume of liquid to be transfer = volume of liquid –10% volume of liquid

$$= \frac{2}{3}\pi 18^3 - \frac{10}{100}\left(\frac{2}{3}\pi 18^3\right)$$
$$= \frac{2}{3}\pi 18^3\left(1-\frac{10}{100}\right)$$
$$= \frac{2}{3}\pi 18^3 \times \frac{9}{10}$$
$$= \pi \times 18^3 \times \frac{3}{5}$$

Number of cylindrical bottles

$$= \frac{\text{Volume of liquid to be transfered}}{\text{Volume of a bottle}}$$
$$72 = \frac{\pi \times 18 \times 18 \times 18 \times \frac{3}{5}}{9\pi h}$$
$$h = \frac{27}{5} = 5.4 \text{ cm}$$

Hence, height of each bottle will be 5.4 cm. **Ans.**

19. A cubical block of side 10 cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have ? Find the cost of painting the total surface area of the solid so formed, at the rate of ₹ 5 per 100 sq. cm [Use π = 3.14] [3]

Solution : Side of the cubical block (a) = 10 cm

Longest diagonal of the cubical block $= a\sqrt{3}$

$$= 10\sqrt{3}\text{ cm}$$

Since the cube is surmounted by a hemisphere, therefore the side of the cube should be equal to the diameter of the hemisphere.

$\because$ Diameter of the sphere = 10 cm

$\therefore$ Radius of the sphere (r) = 5 cm

$$\left[\because \text{Radius} = \frac{\text{Diameter}}{2}\right]$$

Total surface area of solid = T.S.A. of the cube + C.S.A. of hemisphere – Inner cross-section area of hemisphere

$$= 6a^2 + 2\pi r^2 - \pi r^2$$
$$= 6a^2 + \pi r^2$$
$$= 6(10)^2 + 3.14(5)^2$$
$$[\because \pi = 3.14]$$
$$= 600 + 25 \times 3.14$$
$$= 600 + 78.5$$
$$= 678.5 \text{ cm}^2$$

Cost of painting per square metre is ₹ 5

$$\text{Total cost for painting} = \frac{₹\ 678.5}{100} \times 5$$
$$= ₹\ 33.92$$

Hence, total cost for painting will be ₹ 33.92 **Ans.**

20. 504 cones each of diameter 3.5 cm and height 3 cm are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area. $\left[\pi = \frac{22}{7}\right]$ [3]

Solution : Diameter of each cone (d) = 3.5 cm

$$\text{Radius of each cone } (r) = \frac{3.5}{2} = \frac{7}{4}\text{ cm} \qquad \left[\because r = \frac{d}{2}\right]$$

Height of each cone (h) = 3 cm

Volume of 504 cones = 504 × Volume of one cone

$$= 504 \times \frac{1}{3}\pi r^2 h$$
$$= 504 \times \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 3$$

Let radius of sphere be R cm

$\therefore$ Volume of sphere = Volume of 504 cones

$$\frac{4}{3} \times \frac{22}{7} \times R^3 = 504 \times \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 3$$
$$R = \sqrt[3]{\frac{3 \times 3 \times 7 \times 7 \times 7 \times 3}{2 \times 2 \times 2}}$$
$$R = \frac{21}{2}\text{ cm}$$

Hence, diameter of sphere = 2R = 21 cm. **Ans.**

Now, surface area of sphere $= 4\pi R^2$

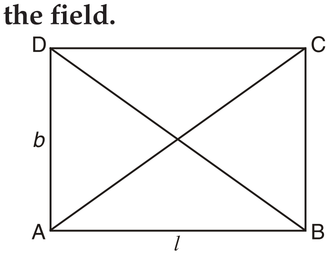
$$= 4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$$
$$= 63 \times 22$$
$$= 1386 \text{ cm}^2$$

Hence, surface area of sphere is 1386 cm^2. **Ans.**

SECTION — D

21. The diagonal of a rectangular field is 16 m more than the shorter side. If the longer side is 14 m more than the shorter side, then find the lengths of the sides of the field. [4]

Solution :



Let l be the length of the longer side and b be the length of the shorter side.

Given that the length of the diagonal of the rectangular field is 16 m more than shorter side.

Thus, Diagonal $= 16 + b$

Since longer side is 14 m more than shorter side,

$\therefore\ l = 14 + b.$

We know,

$$(\text{Diagonal})^2 = (\text{Length})^2 + (\text{Breadth})^2$$

[By Pythagoras theorem]

$$\therefore \quad (16 + b)^2 = (14 + b)^2 + b^2$$
$$256 + b^2 + 32b = 196 + b^2 + 28b + b^2$$
$$b^2 - 4b - 60 = 0$$
$$b^2 - 10b + 6b - 60 = 0$$
$$b(b - 10) + 6(b - 10) = 0$$
$$(b + 6)(b - 10) = 0$$
$$\Rightarrow \quad b = -6 \text{ or } +10$$

As breadth cannot be negative

$\therefore$ Breadth $(b) = 10$ m.

Now, length of rectangular field $= (14 + b)$ m
$= (14 + 10)$ m
$= 24$ m

Thus, length of rectangular field is 24 cm and breadth is 10 m. **Ans.**

22. Find the 60th term of the A.P. 8, 10, 12.... if it has a total of 60 terms and hence find the sum of its last 10 terms. [4]

Solution : Consider the given A.P. 8, 10, 12, ...

Hence the first term is 8

And the common difference

$$d = 10 - 8 = 2$$

or $12 - 10 = 2$

Therefore, 60th term is

$$a_{60} = 8 + (60 - 1)\,2$$
$$\Rightarrow \quad a_{60} = 8 + 59 \times 2$$
$$\Rightarrow \quad a_{60} = 126$$

We need to find the sum of last 10 terms

Since, sum of last 10 terms = sum of first 60 terms – sum of first 50 terms.

$$S_{10} = \frac{60}{2}[2\times8+(60-1)2]-\frac{50}{2}[2\times8+(50-1)2]$$
$$= \frac{60}{2}\times2[8+59]-\frac{50}{2}\times2[8+49]$$
$$= 60 \times 67 - 50 \times 57$$
$$= 4020 - 2850$$
$$= 1170$$

Hence, the sum of last 10 terms is 1170. **Ans.**

23. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed ? [4]

Solution : Let the average speed of the train be x km/hr.

Then, new average speed of the train $= (x + 6)$ km/hr

Time taken by train to cover 54 km $= \frac{54}{x}$ hrs

And time taken by train to cover 63 km

$$= \frac{63}{(x+6)} \text{ hrs}$$

According to the question,

$$\frac{54}{x}+\frac{63}{x+6} = 3$$
$$\frac{54(x+6)+63x}{x(x+6)} = 3$$
$$54x + 324 + 63x = 3x(x + 6)$$
$$324 + 117x = 3x^2 + 18x$$
$$3x^2 - 99x - 324 = 0$$
$$x^2 - 33x - 108 = 0$$
$$x^2 - 36x + 3x - 108 = 0$$
$$x(x - 36) + 3(x - 36) = 0$$
$$(x + 3)(x - 36) = 0$$
$$x = -3 \text{ or } 36$$

Since, speed cannot be negative

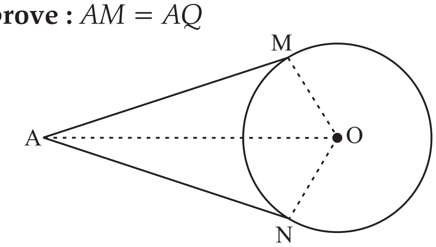
$$\therefore \quad x = 36$$

so, First speed of train = 36 km/hr **Ans.**

24. Prove that the lengths of the tangents drawn from an external point to a circle are equal. [4]

Solution : Given, Tangents AM and AN are drawn from point A to a circle with centre O.

To prove : $AM = AQ$



Construction : Join OM, ON and OA

Proof : Since AM is a tangent at M and OM is radius

$$\therefore \quad OM \perp AM$$

Similarly, $ON \perp AN$

Now, in ΔOMA and ΔONA.

$OM = ON$ (radii of same circle)

$OA = OA$ (common)

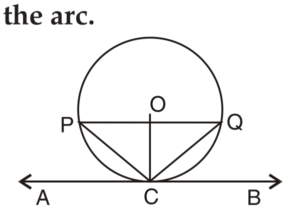
$\angle OMA = \angle ONA = 90°$

$\therefore$ $\Delta OMA \cong \Delta ONA$ (By RHS congruence)

Hence, $AM = AN$ (By cpct) **Hence Proved.**

25 . Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc. [4]

Solution :



Given, C is the mid-point of the minor arc PQ and O is the centre of the circle and AB is tangent to the circle through point C.

Construction : Draw PC and QC.

To prove : $PQ \parallel AB$

Proof : It is given that C is the mid-point of the arc PQ.

So, Minor arc PC = Minor arc QC

$\Rightarrow$ $PC = QC$

Hence ΔPQC is an isosceles triangle.

Thus the perpendicular bisector of the side PQ of ΔPQC passes through vertex C.

But we know that the perpendicular bisector of a chord passes through centre of the circle.

So, the perpendicular bisector of PQ passes through the center O of the circle.

Thus, the perpendicular bisector of PQ passes through the points O and C.

$\Rightarrow$ $PQ \perp OC$ ...(i)

AB is perpendicular to the circle through the point C on the circle

$\Rightarrow$ $AB \perp OC$ ...(ii)

From equations (i) and (ii), the chord PQ and tangent AB of the circle are perpendicular to the same line OC.

Hence, $AB \parallel PQ$

or $PQ \parallel AB$ **Hence Proved.**

26. Construct a $\Delta\, ABC$ in which AB = 6 cm, $\angle A = 30°$ and $\angle B = 60°$. Construct another $\Delta\, AB'C'$ similar to $\Delta\, ABC$ with base AB' = 8 cm. [4]

Solution : Steps of construction :

(i) Draw a line segment AB= 6 cm.

(ii) Construct $\angle ABP = 60°$ and $\angle QAB = 30°$

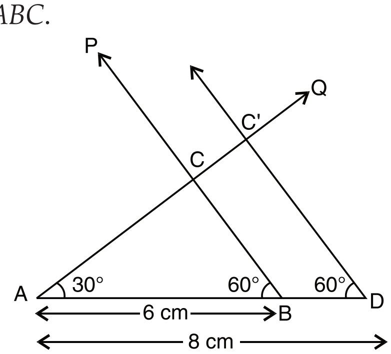
(iii) Join AC and BC such that C is the intersection point of BP and AQ.

Thus, ΔABC is the required triangle.

(iv) Extend AB to B', such that AB' = 8 cm.

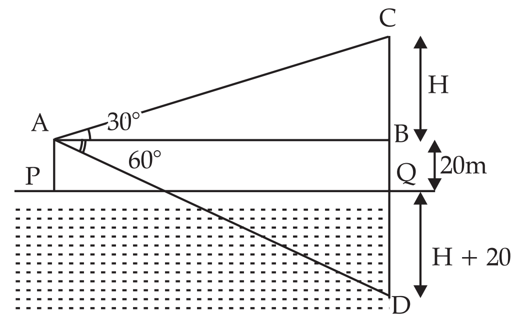
(v) Draw $B'C' \parallel BC$ cutting AC produced at C'.

Then, $\Delta AB'C'$ is the required triangle similar to ΔABC.



27. At a point A, 20 m above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A. [4]

Solution :



Let PQ be the surface of the lake. A is the point vertically above P such that AP = 20 m.

Let C be the position of the cloud and D be its reflection in the lake.

Let $BC = H$ metres

Now, In $\Delta\, ABD$

$$\tan 60° = \frac{BD}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{H+20+20}{AB}$$

$$\Rightarrow \quad \sqrt{3}.\, AB = H + 40$$

$$\Rightarrow \quad AB = \frac{H+40}{\sqrt{3}} \qquad ...(i)$$

And, in $\Delta\, ABC$

$$\tan 30° = \frac{BC}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{H}{AB}$$

$$AB = \sqrt{3}H \qquad ...(ii)$$

From eq. (i) and (ii)

$$\frac{H+40}{\sqrt{3}} = \sqrt{3}\, H$$

$$\Rightarrow \quad 3H = H + 40$$

$$\Rightarrow \quad 2H = 40 \Rightarrow H = 20$$

Putting the value of H in eq. (ii), we get

$$AB = 20\sqrt{3}$$

Again, in $\Delta\, ABC$

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$= \left(20\sqrt{3}\right)^2 + (20)^2$$

$$= 1200 + 400$$

$$= 1600$$

$$AC = \sqrt{1600} = 40$$

Hence, the distance of cloud from A is 40 m.

Ans.

28. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability that the card drawn is

(i) A card of spade or an ace

(ii) A black king

(iii) Neither a jack nor a king

(iv) Either a king or a queen **[4]**

Solution : (i) Let S be the sample space of drawing a card from a well-shuffled deck

Then, $S = 52$

There are 13 spade cards and 4 aces in a deck.

As a ace of spade is included in 13 spade cards,

so, there are 13 spade cards and 3 aces.

A card of spade or an ace can be drawn in

$$13 + 4 - 1 = 16 \text{ (ways)}$$

Probability of drawing a card of spade or an ace.

$$P = \frac{16}{52} = \frac{4}{13}$$

(ii) There are 2 black king cards in a deck.

Probability of drawing a black king

$$P = \frac{2}{52}$$

$$P = \frac{1}{26}$$

(iii) There are 4 jack and 4 king cards in a deck.

So, there are 52 – 8= 44 cards which are neither jack nor king

Probability of drawing a card which is neither a jack nor a king

$$P = \frac{44}{52}$$

$$P = \frac{11}{13}$$

(iv) There are 4 queen and 4 king cards in a deck.

So, there are 8 cards which are either king or queen.

Probability of drawing a card which is either king or a queen

$$P = \frac{8}{52}$$

$$P = \frac{2}{13}$$ **Ans.**

29. Find the values of k so that the area of the triangle with vertices (1, – 1), (– 4, 2k) and (– k, – 5) is 24 sq. units. **[4]**

Solution : The vertices of the given $\Delta\, ABC$ are $A(1, -1)$, $B(-4, 2k)$ and $C(-k, -5)$

$$\therefore \text{ Area of } \Delta ABC = \frac{1}{2}\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$= \frac{1}{2}[1(2k + 5) + (-4)(-5 + 1) + (-k)(-1 - 2k)]$$

$$= \frac{1}{2}[2k + 5 + 16 + k + 2k^2]$$

$$= \frac{1}{2}[2k^2 + 3k + 21]$$

Area of ΔABC = 24 sq. units (Given)

$$\therefore \quad \frac{1}{2}[2k^2 + 3k + 21] = 24$$

$$[2k^2 + 3k + 21] = 48$$

$$\Rightarrow \quad 2k^2 + 3k + 21 = 48$$

$$\Rightarrow \quad 2k^2 + 3k - 27 = 0$$

$$\Rightarrow \quad 2k^2 + 9k - 6k - 27 = 0$$

$$\Rightarrow \quad k(2k + 9) - 3(2k + 9) = 0$$

$$\Rightarrow \quad (k - 3)(2k + 9) = 0$$

$$k = 3 \text{ or } -\frac{9}{2}$$

Hence, $k = 3$ or $k = -\dfrac{9}{2}$. **Ans.**

30. In figure 5, $PQRS$ is a square lawn with side PQ = 42 m Two circular flower beds are there on the sides PS and QR with center at O, the intersection of its diagonals. Find the total area of the two flower beds (shaded parts). **[4]**

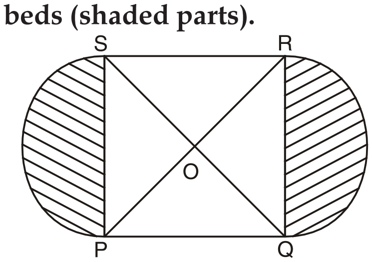
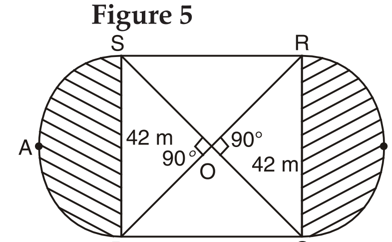


Figure 5

Solution :



Given, $PQRS$ is a square with side 42 m.

Let its diagonals intersect at O.

Then, $OP = OQ = OR = OS$

and $\angle POS = \angle QOR = 90°$

$$PR^2 = PQ^2 + QR^2$$

$$PR = (\sqrt{2} \times 42)\,\text{m}$$

Now, $OP = \frac{1}{2}\times(\text{diagonal}) = 21\sqrt{2}$ m

$\because$ Area of flower bed PAS = Area of flower bed QBR

$\therefore$ Total area of the two flower beds = Area of flower bed PAS + Area of flower bed QBR

$= 2\times[\text{Area of sector } OPAS - \text{Area of } \Delta\, POS]$

$= 2\times\left[\pi r^2 \frac{\theta}{360°} - \frac{1}{2}r^2 \sin\theta\right]$

[Where, $\theta = 90°$]

$= 2\times\left[\frac{22}{7}\times(21\sqrt{2})^2 \frac{90°}{360°} - \frac{1}{2}\times 21\sqrt{2}\times 21\sqrt{2}\right]$

[$\because \sin 90° = 1$]

$= 2\times\left[\frac{22}{7}\times 21\times 21\times 2\times\frac{1}{4} - \frac{1}{2}\times 21\times 21\times 2\right]$

$= 2[33 \times 21 - 441]$

$= 2[693 - 441]$

$= 504 \text{ m}^2$

Hence area of flower beds is 504 m^2. **Ans.**

31. From each end of a solid metal cylinder, metal was scooped out in hemispherical form of same diameter. The height of the cylinder is 10 cm and its base is of radius 4.2 cm. The rest of the cylinder is melted and converted into a cylindrical wire of 1.4 cm thickness. Find the length of the wire. [4]

Solution : Height of the cylinder (h) = 10 cm

Radius of base of cylinder (r) = 4.2 cm

Now,

Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 4.2 \times 4.2 \times 10$

$= 554.4 \text{ cm}^3$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3}\times\frac{22}{7} \times 4{\cdot}2 \times 4{\cdot}2 \times 4{\cdot}2$

$= 155.232 \text{ cm}^3$

Volume of the rest of the cylinder after scooping out the hemisphere from each end = Volume of cylinder – 2 × Volume of hemisphere

$= 554.4 - 2 \times 155.232$

$= 554.4 - 310.464$

$= 243.936 \text{ cm}^3$.

The remaining cylinder is melted and converted into a new cylindrical wire of 1.4 cm thickness.

So, radius of cylindrical wire = 0.7 cm

Volume of remaining cylinder = Volume of new cylindrical wire

$243.936 = \pi R^2 H$

$243{\cdot}936 = \frac{22}{7} \times 0{\cdot}7 \times 0{\cdot}7 \times H$

$\Rightarrow \quad H = 158.4$ cm **Ans.**

Mathematics 2015 (Outside Delhi) Term II SET II

Note : Except for the following questions, all the remaining questions have been asked in previous set.

SECTION — B

10. If $A(4, 3)$, $B(-1, y)$ and $C(3, 4)$ are the vertices of a right triangle ABC, right-angled at A, then find the value of y. [2]

Solution : Given the triangle ABC, right angled at A.

Now,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$AB = \sqrt{(-1-4)^2 + (y-3)^2}$$
$$AB = \sqrt{(-5)^2 + (y-3)^2}$$
$$AB = \sqrt{25 + (y-3)^2}$$
$$AB = \sqrt{25 + y^2 + 9 - 64}$$
$$AB = \sqrt{34 + y^2 - 6y}$$

$$BC = \sqrt{(3-(-1))^2 + (4-y)^2}$$
$$BC = \sqrt{(4)^2 + (4-y)^2}$$
$$BC = \sqrt{16 + 16 + y^2 - 8y}$$
$$BC = \sqrt{32 + y^2 - 8y}$$

And
$$AC = \sqrt{(3-4)^2 + (4-3)^2}$$
$$AC = \sqrt{(-1)^2 + (1)^2}$$
$$AC = \sqrt{1+1}$$
$$AC = \sqrt{2} \text{ units}$$

Given, $\Delta\, ABC$ is a right angled triangle

So, by Pythagoras theorem

$$BC^2 = AC^2 + AB^2$$
$$(\sqrt{32 + y^2 - 8y})^2 = (\sqrt{2})^2 + (\sqrt{34 + y^2 - 6y})^2$$
$$32 + y^2 - 8y = 2 + 34 + y^2 - 6y$$

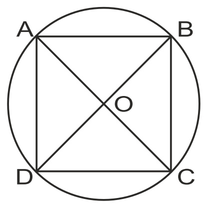
$$-2y = 4$$
$$y = -2$$

Hence, the value of y is -2. **Ans.**

SECTION — C

18. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if the area of the circle is 1256 cm^2. [Use π = 3.14] [3]

Solution :



Given that the area of the circle is 1256 cm^2.

$\because$ Area of the circle $= \pi r^2$

$$1256 = \frac{3.14}{100} \times r^2$$

$$r^2 = \frac{1256 \times 100}{314}$$

$$r = \sqrt{400}$$

$$r = 20 \text{ cm}$$

Now, $ABCD$ are the vertices of a rhombus.

$\therefore$ $\angle A = \angle C$ ...(i)

[opposite angles of rhombus]

But $ABCD$ lie on the circle.

So, $ABCD$ is called cyclic quadrilateral

$\therefore$ $\angle A + \angle C = 180°$ ...(ii)

On using equation (i), we get

$$\angle A + \angle A = 180°$$
$$2\angle A = 180°$$
$$\angle A = 90°$$

so, $\angle C = 90°$ [From eq. (i)]

$\therefore$ $ABCD$ is square.

So, BD is a diameter of circle.

[$\because$ The angle in a semicircle is a right angle triangle]

Now, Area of rhombus $= \frac{1}{2} \times$ product of diagonals

$$= \frac{1}{2} \times 40 \times 40$$

$$= 800 \text{ cm}^2$$

Hence, Area of rhombus is 800 cm^2. **Ans.**

19. Solve for x :

$$\mathbf{2x^2 + 6\sqrt{3}x - 60 = 0} \quad \textbf{[3]}$$

Solution : Consider the given equation

$$2x^2 + 6\sqrt{3}x - 60 = 0$$

$\Rightarrow$ $x^2 + 3\sqrt{3}x - 30 = 0$ ...(i)

Comparing equation (i) by

$$ax^2 + bx + c = 0$$

We get $a = 1, b = 3\sqrt{3}, c = -30.$

By quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3\sqrt{3} \pm \sqrt{27 + 120}}{2}$$

$$x = \frac{-3\sqrt{3} \pm \sqrt{147}}{2}$$

Hence value for $x = \dfrac{-3\sqrt{3} \pm \sqrt{147}}{2}$ **Ans.**

20. The 16th term of an AP is five times its third term. If its 10th term is 41, then find the sum of its first fifteen terms. [3]

Solution : Given that 16th term of an A.P. is five time its 3rd term.

i.e., $a + (16-1)d = 5[a + (3-1)d]$

$\Rightarrow$ $a + 15d = 5[a + 2d]$

$\Rightarrow$ $a + 15d = 5a + 10d$

$\Rightarrow$ $4a - 5d = 0$ ...(i)

Also given that,

$$a_{10} = 41$$

$\Rightarrow$ $a + (10-1)d = 41$

$\Rightarrow$ $a + 9d = 41$ ...(ii)

On multiplying equation (ii) by 4, we get

$4a + 36d = 164$ ...(iii)

Subtracting equation (iii) from (i), we get

$$\begin{array}{rcrcl} 4a & - & 5d & = & 0 \\ 4a & + & 36d & = & 164 \\ - & & - & & - \\ \hline & & -41d & = & -164 \end{array}$$

$$d = 4$$

On putting the value of d in *eq.* (i), we get

$$4a - 5 \times 4 = 0$$
$$4a = 20$$
$$a = 5$$

Now, $S_{15} = \frac{15}{2}[2a + (15-1)d]$

$$S_{15} = \frac{15}{2}(2 \times 5 + 14 \times 4)$$

$$= \frac{15}{2} 2(5 + 14 \times 2)$$

$$= 15(5 + 28)$$
$$= 15 \times 33$$
$$S_{15} = 495$$

Hence, sum of first fifteen terms is 495. **Ans.**

SECTION — D

28. A bus travels at a certain average speed for a distance of 75 km and then travels a distance of 90

km at an average speed of 10 km/h more than the first speed. If it takes 3 hours to complete the total journey, find its first speed. **[4]**

Solution : Let x be the initial speed of the bus we know that

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

or
$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Thus, we have

$$3 = \frac{75}{x} + \frac{90}{x+10}$$

$$\Rightarrow \quad 3 = \frac{75(x+10)+90x}{x(x+10)}$$

$$\Rightarrow \quad 3(x)(x+10) = 75x + 750 + 90x$$

$$\Rightarrow \quad 3x^2 + 30x = 75x + 750 + 90x$$

$$\Rightarrow \quad 3x^2 - 135x - 750 = 0$$

$$\Rightarrow \quad x^2 - 45x - 250 = 0$$

$$\Rightarrow \quad x^2 - 50x + 5x - 250 = 0$$

$$\Rightarrow \quad x(x-50) + 5(x-50) = 0$$

$$\Rightarrow \quad (x+5)(x-50) = 0$$

$$\Rightarrow \quad x = -5 \text{ or } x = 50$$

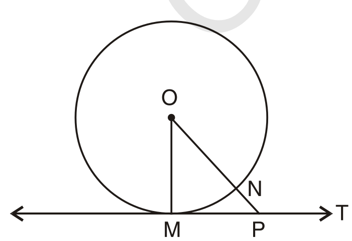
Since, speed cannot be negative

So, $\quad x = 50$

Hence, the initial speed of bus is 50 km/hr. **Ans.**

29. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. **[4]**

Solution :



Given,

A circle with centre O and a tangent T at a point M of the circle.

To prove : $OM \perp T$

Construction : Take a point P, other than M on T. Join OP.

Proof : P is a point on the tangent T, other than the point of contact M.

$\therefore$ P lies outside the circle.

Let OP intersect the circle at N.

Then, $\quad ON < OP \quad$ ...(i)

[$\because$ a part is less than whole]

But $\quad OM = ON \quad$ ...(ii)

[Radii of the same circle]

$\therefore \quad OM < OP \quad$ [Using (ii)]

Thus, OM is shorter than any other line segment joining O to any point T, other than M.

But a shortest distance between a point and a line is the perpendicular distance.

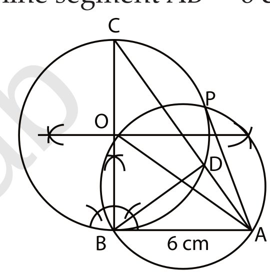
$\therefore \quad OM \perp T$

Hence, OM is perpendicular on T. **Hence Proved.**

30. Construct a right triangle ABC with AB = 6 cm, BC = 8 cm and $\angle B$ = 90°. Draw BD, the perpendicular from B on AC. Draw the circle through B, C and D and construct the tangents from A to this circle. **[4]**

Solution : Steps of construction :

(i) Draw a line segment AB = 6 cm.



(ii) Make a right angle at point B and draw BC = 8 cm.

(iii) Draw a perpendicular BD to AC.

(iv) Taking BC as diameter, draw a circle which passes through points B, C and D.

(v) Join A to O and taking AO as diameter, draw second circle.

(vi) From point A, draw tangents AB and AP.

31. Find the values of k so that the area of the triangle with vertices (k + 1, 1), (4, – 3) and (7, – k) is 6 sq. units. **[4]**

Solution : Given, the vertices are (k + 1, 1), (4, – 3) and (7, – k) and the area of the triangle is 6 square units.

Therefore,

$$\text{Area} = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$6 = \frac{1}{2}[(k+1)(-3+k) + 4(-k-1) + 7(1+3)]$$

$$12 = (k+1)(k-3) + 4(-k-1) + 28$$

$$12 = k^2 - 3k + k - 3 - 4k - 4 + 28$$

$$k^2 - 6k + 9 = 0$$

$$k^2 - 3k - 3k + 9 = 0$$

$$k(k-3) - 3(k-3) = 0$$

$$(k-3)(k-3) = 0$$

$$\therefore \quad k = 3, 3$$

Hence, value of k is 3. **Ans.**

Mathematics 2015 (Outside Delhi) II SET III

Note : Except for the following questions, all the remaining questions have been asked in previous sets.

SECTION — B

10. Solve the following quadratic equation for x :

$$x^2 - 2ax - (4b^2 - a^2) = 0 \quad [2]$$

Solution : We have, $x^2 - 2ax - (4b^2 - a^2) = 0$

$$x^2 - 2ax + a^2 - 4b^2 = 0$$

$$(x - a)^2 - (2b)^2 = 0$$

$\therefore (x - a + 2b)(x - a - 2b) = 0$

$\Rightarrow \quad x = a - 2b$ or $a + 2b$

Hence, $\quad x = a - 2b$ or $x = a + 2b$ **Ans.**

SECTION — C

18. The 13th term of an AP is four times its 3rd term. If its fifth term is 16, then find the sum of its first ten terms. [3]

Solution : In the given A.P., let first term = a and common difference = d

Then, $\quad T_n = a + (n - 1)d$

$\Rightarrow \quad T_{13} = a + (13 - 1)d = a + 12d$

and $\quad T_3 = a + (3 - 1)d = a + 2d$

Now, $\quad T_{13} = 4T_3$ (Given)

$$a + 12d = 4(a + 2d)$$

$$a + 12d = 4a + 8d$$

$$3a = 4d$$

$$a = \frac{4}{3}d \quad \text{...(i)}$$

Also, $\quad T_5 = a + (5 - 1)d$

$\Rightarrow \quad a + 4d = 16$ ...(ii)

Putting the value of a from eq. (i) in (ii), we get

$$\frac{4}{3}d + 4d = 16$$

$$4d + 12d = 48$$

$$16d = 48$$

$$d = 3$$

Substituting $d = 3$ in eq. (ii), we get

$$a + 4(3) = 16$$

$$a = 16 - 12$$

$$a = 4$$

$\therefore$ Sum of first ten terms is

$$S_{10} = \frac{n}{2}[2a + (n-1)d] \text{ where } n = 10$$

$$= \frac{10}{2}[2 \times 4 + (10 - 1)3]$$

$$= 5[8 + 27]$$

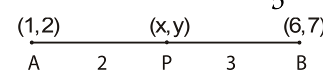
$$= 175$$ **Ans.**

19. Find the coordinates of a point P on the line segment joining $A(1, 2)$ and $B(6, 7)$ such that $AP = \frac{2}{5}AB$. [3]

Solution : Given, $A(1, 2)$ and $B(6, 7)$ are the given points of a line segment AB with a point P on it.

Let the co-ordinate of point P be (x, y)

$$AP = \frac{2}{5}AB \quad \text{(Given)}$$



$$AB = AP + PB$$

$\Rightarrow$ $$\frac{AP}{PB} = \frac{2}{3}$$

$\therefore \quad m = 2, n = 3$

Then, by section formula, we have

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

$$x = \frac{2\times6 + 3\times1}{2+3} \text{ and } y = \frac{2\times7 + 3\times2}{2+3}$$

$$x = \frac{15}{5} \text{ and } y = \frac{20}{5}$$

$\therefore x = 3$ and $y = 4$

Hence, the required point is $P(3, 4)$. **Ans.**

20. A bag contains white, black and red balls only. A ball is drawn at random from the bag. If the probability of getting a white ball is $\frac{3}{10}$ and that of a black ball is $\frac{2}{5}$, then find the probability of getting a red ball. If the bag contains 20 black balls, then find the total number of balls in the bag. [3]

Solution : Given, the probability of getting a white ball

$\Rightarrow$ $$P(W) = \frac{3}{10}$$

and the probability of getting a black ball

$\Rightarrow$ $$P(B) = \frac{2}{5}$$

then, the probability of getting a red ball

$\Rightarrow$ $$P(R) = 1 - \frac{3}{10} - \frac{2}{5} = \frac{10 - 3 - 4}{10} = \frac{3}{10}$$

Now, $\frac{2}{5}$ of total number of balls = 20

$$\text{Total number of balls} = \frac{20\times5}{2}$$

$$= 50$$

Hence, the total no. of balls in the bag is 50. **Ans.**

SECTION — D

28. A truck covers a distance of 150 km at a certain average speed and then covers another 200 km at an average speed which is 20 km per hour more than the first speed. If the truck covers the total distance in 5 hours, find the first speed of the truck. [4]

Solution : Let the average speed of the truck be x km/hr.

Then, new average speed of truck = $(x + 20)$ km/hr.

Time taken by truck to cover 150 km = $\frac{150}{x}$ hrs.

and time taken by truck to cover 200 km = $\frac{200}{x+20}$ hrs.

$$\therefore \quad \frac{150}{x}+\frac{200}{x+20} = 5$$

$$\frac{150(x+20)+200x}{x(x+20)} = 5$$

$$150x + 3000 + 200x = 5x(x + 20)$$

$$350x + 3000 = 5x^2 + 100x$$

$$5x^2 - 250x - 3000 = 0$$

$$x^2 - 50x - 600 = 0$$

$$x^2 - 60x + 10x - 600 = 0$$

$$x(x - 60) + 10 (x - 60) = 0$$

$$(x + 10) (x - 60) = 0$$

$$x = -10 \text{ or } 60$$

Since speed cannot be negative.

So, $x = 60$

$\therefore$ First speed of truck = 60 km/hr. **Ans.**

29. An arithmetic progression 5, 12, 19,..... has 50 terms. Find its last term. Hence find the sum of its last 15 terms. [4]

Solution : Given, AP is 5, 12, 19 ....

Here, $n = 50, a = 5, d = 12 - 5 = 19 - 12 = 7$

Now, $T_{50} = a + (50 - 1)d$

$\Rightarrow$ $T_{50} = 5 + (49)7 = 348$

15 terms from last = (50 – 15 + 1) terms from starting

$$T_{36} = a + (36 - 1)d$$

$$= 5 + 35(7)$$

$$= 250$$

$\therefore$ Sum of last 15 terms $= \frac{n}{2}(a+l)$

$$= \frac{15}{2}(250+348)$$

$[\because a = 250 \text{ and } l = 348]$

$$= \frac{15}{2}\times598 = 4485$$ **Ans.**

30. Construct a triangle ABC in which AB = 5 cm, BC = 6 cm and $\angle ABC$ = 60°. Now construct another triangle whose sides are $\frac{5}{7}$ times the corresponding sides of ΔABC. [4]

Solution : Steps of construction :

(i) Draw a line segment AB = 5 cm.

(ii) Construct $\angle ABX$ = 60°.

(iii) From B, draw BC = 6 cm cutting BX at C.

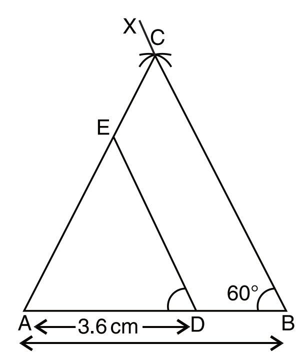
(iv) Join AC.

Thus, ΔABC is obtained

(v) Step 5 : Draw D on AB such that $AD = \frac{5}{7}AB$

$$= \left(\frac{5}{7}\times5\right)\text{cm} = 3.6 \text{ cm}$$

(vi) Step 6 : Draw $DE \parallel BC$ cutting AC at E. Then ΔADE is the required triangle similar to ΔABC, such that each side of ΔADE is $\frac{5}{7}$ times the corresponding side of ΔABC.



31. Find the values of k for which the points A(k + 1, 2k), B(3k, 2k + 3) and C(5k – 1, 5k) are collinear. [4]

Solution : Given, the points A(k +1, 2k), B(3k, 2k + 3) and C(5k – 1, 5k)

$\because$ The point to be collinear

$\therefore x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$$(k + 1) (2k + 3 - 5k) + 3k (5k - 2k) + (5k - 1) (2k - 2k - 3) = 0$$

$$(k + 1) (3 - 3k) + 3k(3k) + (5k - 1) (-3) = 0$$

$$3k + 3 - 3k^2 - 3k + 9k^2 - 15k + 3 = 0$$

$$6k^2 - 15k + 6 = 0$$

$$2k^2 - 5k + 2 = 0$$

$$2k^2 - 4k - k + 2 = 0$$
$$2k(k-2) - 1(k-2) = 0$$
$$(2k-1)(k-2) = 0$$
$$k = 2 \text{ or } \frac{1}{2}$$

Hence, $k = 2$ or $k = \frac{1}{2}$. **Ans.**

Mathematics 2015 (Delhi) Term II — SET I

SECTION — A

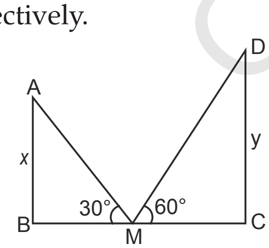
1. If $x = -\frac{1}{2}$, is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k. [1]

Solution : Since $x = \frac{-1}{2}$ is a solution of $3x^2 + 2kx - 3 = 0$, it must satisfy the equation.

$$\therefore \quad 3 \times \left(-\frac{1}{2}\right)^2 + 2k\left(-\frac{1}{2}\right) - 3 = 0$$
$$\frac{3}{4} - k - 3 = 0$$
$$k = \frac{3}{4} - 3$$
$$k = \frac{-9}{4}$$ **Ans.**

2. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$. [1]

Solution : Let AB and CD be two towers of height x and y respectively.



M is the mid-point of BC *i.e.*, $BM = MC$

In $\Delta\, ABM$, we have

$$\frac{AB}{BM} = \tan 30°$$
$$BM = \frac{x}{\tan 30°} \qquad \text{...(i)}$$

In $\Delta\, CDM$, we have

$$\frac{DC}{MC} = \tan 60°$$
$$\frac{y}{MC} = \tan 60°$$
$$MC = \frac{y}{\tan 60°} \qquad \text{...(ii)}$$

From *eq.* (i) and (ii), we get

$$\frac{x}{\tan 30°} = \frac{y}{\tan 60°}$$
$$\frac{x}{y} = \frac{\tan 30°}{\tan 60°}$$
$$\frac{x}{y} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$
$$\therefore \quad x : y = 1 : 3.$$ **Ans.**

3. A letter of English alphabet is chosen at random. Determine the probability that the chosen letter is a consonant. [1]

Solution : Total number of all possible outcomes = 26

Number of consonants = 21

Let E be the event of getting a consonant

$\therefore$ P (getting a consonant) $= P(\text{E}) = \frac{21}{26}$ **Ans.**

4. In Fig. 1, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$, Write the measure of $\angle OAB$. [1]

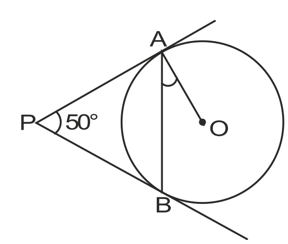


Figure 1

Solution : Since PA and PB are tangents to the circle with centre O then,

$$PA = PB$$

and $\angle APO = \angle BPO = 25°$

Join OP and $OA \perp PA$.

In $\Delta\, APO$,

$$\angle APO + \angle POA + \angle OAP = 180°$$
$$25° + \angle POA + 90° = 180°$$
$$\angle POA = 65°$$

Join OB, then

In Δ AOB

$$\angle OAB + \angle OBA + \angle AOB = 180°$$

$$2\angle OAB + 2\angle POA = 180°$$

[$\because \angle OAB = \angle OBA$
OA & OB are radii]

$$2\angle OAB + 2 \times 65° = 180°$$

$$\angle OAB = 90° - 65°$$

$\therefore$ $\angle OAB = 25°$ **Ans.**

SECTION — B

5. In Fig. 2, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58°$, find $\angle ATQ$. [2]

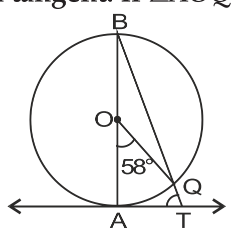


Figure 2

Solution : Given, AB is a diameter of a circle with centre O and AT is a tangent, then

$$BA \perp AT$$

Also $\angle ABQ = \frac{1}{2}\angle APQ$

($\because$ Angle subtended on the arc is half of the angle subtended at centre)

$\Rightarrow$ $\angle ABQ = \frac{1}{2} \times 58° = 29°$

Now, $\angle ATQ = 180° - (\angle ABQ + \angle BAT)$
$= 180° - (29° + 90°)$

$\therefore$ $\angle ATQ = 61°$ **Ans.**

6. Solve the following quadratic equation for x :

$4x^2 - 4a^2x + (a^4 - b^4) = 0.$ [2]

Solution : We have $4x^2 - 4a^2x + (a^4 - b^4) = 0$

$$(4x^2 - 4a^2x + a^4) - b^4 = 0$$

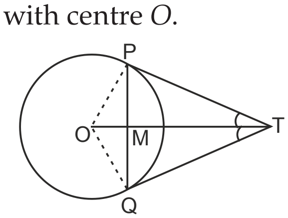
$$(2x - a^2)^2 - (b^2)^2 = 0$$

$\therefore$ $(2x - a^2 + b^2)(2x - a^2 - b^2) = 0$

$x = \frac{a^2 - b^2}{2}$ or $\frac{a^2 + b^2}{2}$ **Ans.**

7. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ. [2]

Solution : Given, TP and TQ are the tangents drawn on a circle with centre O.



To prove : OT is the right bisector of PQ.

Proof : In ΔTPM and ΔTQM

$TP = TQ$ (Tangents drawn from external point are equal)

$TM = TM$ (Common)

$\angle PTM = \angle QTM$ (TP and TQ are equally inclined to OT)

$\therefore$ $\Delta TPM \cong \Delta TQM$

(By SAS congruence)

$\therefore$ $PM = MQ$

and $\angle PMT = \angle QMT$ (By CPCT)

Since, PMQ is a straight line, then

$$\angle PMT + \angle QMT = 180°$$

$\therefore$ $\angle PMT = \angle QMT = 90°$

$\therefore$ OT is the right bisector of PQ. **Hence Proved.**

8. Find the middle term of the A.P. 6, 13, 20, ..., 216. [2]

Solution : Given A.P. is 6, 13, 20, ..., 216

Here, $a = 6$, $d = 13 - 6 = 20 - 13 = 7$

Let n be the number of terms.

then $T_n = a + (n - 1)d$

$$216 = 6 + (n - 1)7$$

$$216 = 6 + 7n - 7$$

$$217 = 7n$$

$\therefore$ $n = 31$

and middle term is $\frac{(n+1)\text{th}}{2}$ term *i.e.*, 16th term

$\therefore$ $T_{16} = 6 + (16 - 1)\ 7$
$= 6 + 15 \times 7$

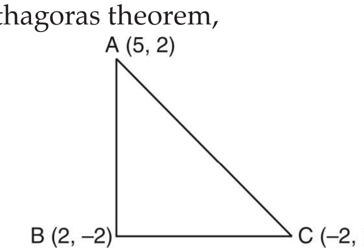
$\therefore$ $T_{16} = 111$

$\therefore$ Middle term of the A.P. is 111. **Ans.**

9. If A (5, 2), B(2, – 2) and C(– 2, t) are the vertices of a right angled triangle with $\angle B = 90°$. then find the value of t. [2]

Solution : Given, ABC are the vertices of a right angled triangle, then,

By Pythagoras theorem,



$(AC)^2 = (BC)^2 + (AB)^2$ ...(i)

Now, $(AC)^2 = (5 + 2)^2 + (2 - t)^2$
$= 49 + (2 - t)^2$

$(BC)^2 = (2 + 2)^2 + (-2 - t)^2$
$= 16 + (t + 2)^2$

And $\quad (AB)^2 = (5-2)^2 + (2+2)^2$

$\qquad\qquad = 9 + 16 = 25$

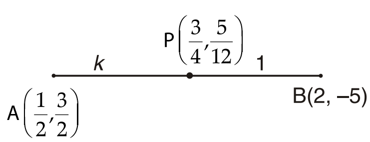
Putting these values in (i)

$$49 + (2-t)^2 = 16 + (t+2)^2 + 25$$
$$49 + (2-t)^2 = 41 + (t+2)^2$$
$$8 = (t+2)^2 - (2-t)^2$$
$$8 = t^2 + 4 + 4t - 4 - t^2 + 4t$$
$$8 = 8t$$
$$\therefore \quad t = 1$$

Ans.

10. Find the ratio in which the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and $B(2, -5)$. **[2]**

Solution : Let point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line AB in ratio $k : 1$.



Then, by section formula, coordinates of P are

$$\frac{2k + \frac{1}{2}}{k+1} = \frac{3}{4}$$

and
$$\frac{-5k + \frac{3}{2}}{k+1} = \frac{5}{12}$$

$\Rightarrow \quad 8k + 2 = 3k + 3$

and $\quad -60k + 18 = 5k + 5$

$\Rightarrow \quad 8k - 3k = 3 - 2$

and $\quad 65k = 18 - 5$

$\Rightarrow \quad 5k = 1$

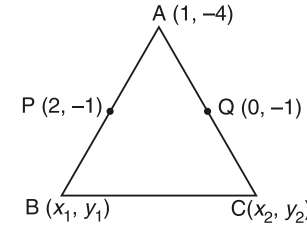
and $\quad 65k = 13 \Rightarrow k = \frac{1}{5}$ in each case

Hence the required ratio is 1 : 5. **Ans.**

SECTION — C

11. Find the area of the triangle ABC with A(1, – 4) and mid-points of sides through A being (2, – 1), and (0, – 1). **[3]**

Solution : Let $A(1, -4)$, $B(x_1, y_1)$ and $C(x_2, y_2)$ be the vertices of a triangle ABC and let $P(2, -1)$ and $Q(0, -1)$ be the mid-points of AB and AC respectively.



$\because$ P is the mid-point of AB.

$$\therefore \quad \frac{1+x_1}{2} = 2, \frac{-4+y_1}{2} = -1$$
$$x_1 = 3, y_1 = 2$$

So, $\quad B(x_1, y_1) \equiv B(3, 2)$

Similary, Q is the mid-point of AC.

$$\therefore \quad \frac{1+x_2}{2} = 0, \frac{-4+y_2}{2} = -1$$
$$x_2 = -1, y_2 = 2$$

So, $\quad C(x_2, y_2) \equiv C(-1, 2)$

Thus, Area of ΔABC

$$= \frac{1}{2}[1(2-2) + 3(2+4) - 1(-4-2)]$$
$$= \frac{1}{2} \times 24 = 12 \text{ sq. units.}$$

Ans.

12. Find that non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k-1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation. **[3]**

Solution : The given equation can be written as

$$(k+1)x^2 - 2(k-1)x + 1 = 0$$

Since the equation has equal roots

$$4(k-1)^2 - 4(k+1) = 0$$
$$4(k^2 + 1 - 2k) - 4(k+1) = 0$$
$$4k^2 + 4 - 8k - 4k - 4 = 0$$
$$4k^2 - 12k = 0$$
$$4k(k-3) = 0$$
$$k = 0, 3$$

$\therefore$ Non zero value of k is 3.

And the equation becomes,

$$4x^2 - 4x + 1 = 0$$
$$(2x-1)^2 = 0$$

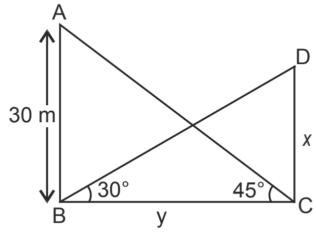
$x = \frac{1}{2}, \frac{1}{2}$ which are the required roots of the given equation. **Ans.**

13. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 45°. If the tower is 30 m high, find the height of the building. **[3]**

Solution : Let AB be the tower and CD be a building of height 30 m and x m respectively.

Let the distance between the two be y m.

Then , in $\Delta\, ABC$



$$\frac{30}{y} = \tan 45°$$

$$\frac{30}{y} = 1 \Rightarrow y = 30$$

And, in $\Delta\, BDC$

$$\frac{x}{y} = \tan 30°$$

$$x = y \tan 30°$$

$$x = 30 \times \frac{1}{\sqrt{3}} = 10\sqrt{3}$$

Hence, the height of the building is $10\sqrt{3}$ m. **Ans.**

14. Two different dice are rolled together. Find the probability of getting :

(i) the sum of numbers on two dice to be 5.

(ii) even numbers on both dice. **[3]**

Solution : Total possible outcomes when two dices are rolled together = 36

(i) Let E_1 be the event of getting the sum of 5 on two dice.

Then, the favourable outcomes are (2, 3), (3, 2), (1, 4), (4, 1).

Number of favourable outcomes = 4

$\therefore$ P(getting the sum of 5) = $P(E_1) = \frac{4}{36} = \frac{1}{9}$ **Ans.**

(ii) Let E_2 be the event of getting even numbers on both dice.

Then, the favourable outcomes are (2, 2), (2, 4), (2, 6), (4, 2), (4, 4) (4, 6), (6, 2), (6, 4), (6, 6)

Number of favourable outcomes = 9

$\therefore$ P(getting even numbers on both dice) = $P(E_2)$

$$= \frac{9}{36} = \frac{1}{4}$$ **Ans.**

15. If S_n, denotes the sum of first n terms of an A.P., prove that $S_{12} = 3(S_8 - S_4)$ **[3]**

Solution : Let a be the first term and d be the common difference.

We know, $S_n = \frac{n}{2}[2a + (n-1)d]$

Then, $S_{12} = \frac{12}{2}[2\,a + (12-1)]$

$= 6\,(2a + 11d) = 12a + 66d$

$S_8 = \frac{8}{2}[2\,a + (8-1)d]$

$= 4(2a + 7d) = 8a + 28d$

and, $S_4 = \frac{4}{2}[2a + (4-1)d]$

$= 2(2a + 3d) = 4a + 6d$

Now,

$$\begin{aligned} 3(S_8 - S_4) &= 3\,(8a + 28d - 4a - 6d) \\ &= 3\,(4a + 22d) \\ &= 12a + 66d \\ &= S_{12} \end{aligned}$$

Hence Proved.

16. In Fig. 3, APB and AQO are semicircles, and $AO = OB$. If the perimeter of the figure is 40 cm, find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$ **[3]**

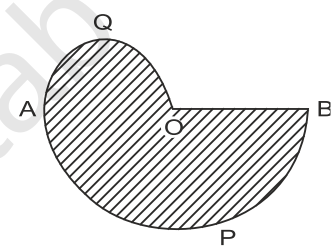


Figure 3

Solution : Given, $OA = OB = r$(say)

We have, perimeter of the figure $= \pi r + \frac{\pi r}{2} + r$

$$\therefore \quad 40 = \frac{22}{7} \times r + \frac{22}{7} \times \frac{r}{2} + r$$

$$280 = 22r + 11r + 7r$$

$$40r = 280$$

$$\therefore \quad r = 7$$

Now, area of the shaded region $= \frac{\pi r^2}{2} + \frac{\pi}{2}\left(\frac{r}{2}\right)^2$

$$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 + \frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 77 + \frac{77}{4}$$

$$= \frac{77 \times 5}{4} = \frac{385}{4} = 96\frac{1}{4} \text{ cm}^2$$ **Ans.**

17. In Fig. 4, from the top of a solid cone of height 12 cm base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid. $\left[\text{Use } \pi = \frac{22}{7} \text{ and } \sqrt{5} = 2.236\right]$ **[3]**

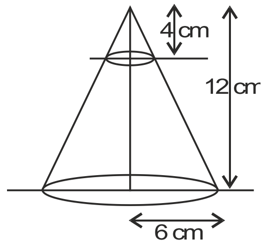
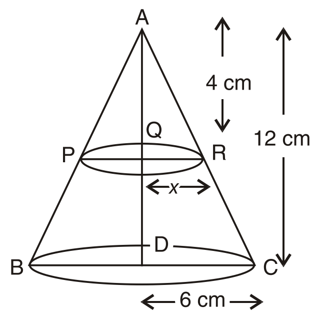


Figure 4

Solution : Height of the given cone = 12 cm
and the radius of the base = 6 cm
Let the radius of the base of the smaller cone be x cm and height is 4 cm.



Now, $\Delta ARQ \sim \Delta ACD$

$$\therefore \quad \frac{DC}{QR} = \frac{AD}{AQ}$$

$$\frac{6}{x} = \frac{12}{4} \Rightarrow x = 2 \text{ cm}$$

$$\therefore \quad l = \text{RC} = \sqrt{h^2 + (R-r)^2}$$

$$= \sqrt{8^2 + (6-2)^2}$$

$$= \sqrt{64+16}$$

$$= 4\sqrt{5}$$

Total surface area of frustum $PRCB$

$$= [\pi l(\text{R} + r) + \pi r^2 + \pi \text{R}^2]$$

$$= \frac{22}{7} \times 4\sqrt{5}(6+2) + \frac{22}{7} \times (2)^2 + \frac{22}{7} + (6)^2$$

$$= \frac{22}{7}[32 \times 2.236 + 4 + 36]$$

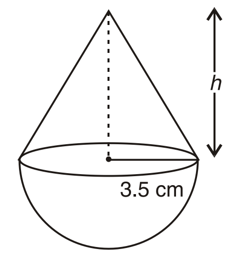
$$= \frac{22}{7}(111.552)$$

$$= 350.592 \text{ cm}^2$$ **Ans.**

18. A solid wooden toy is in the form of hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}$ cm^3. Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of ₹ 10 per cm^2. **[3]**

$$\left[\textbf{Use } \pi = \frac{22}{7}\right]$$

Solution : Given, the radius of hemisphere is 3.5 cm and let the height of the cone be h cm.



Now, Volume of wood $= 166\frac{5}{6}$ cm^3

$$\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = 166\frac{5}{6}$$

$$\frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5 + \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times h$$

$$= \frac{1001}{6}$$

$$\frac{22}{7} \times 3.5 \times 3.5 \left(\frac{2}{3} \times \frac{7}{2} + \frac{1}{3} \times h\right) = \frac{1001}{6}$$

$$\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\left(\frac{7+h}{3}\right) = \frac{1001}{6}$$

$$7 + h = \frac{1001 \times 7 \times 2 \times 2 \times 3}{6 \times 7 \times 7 \times 22}$$

$$h = \frac{1001}{77} - 7$$

$$h = 6 \text{ cm}$$

Area of hemispherical part of the toy $= 2\pi r^2$

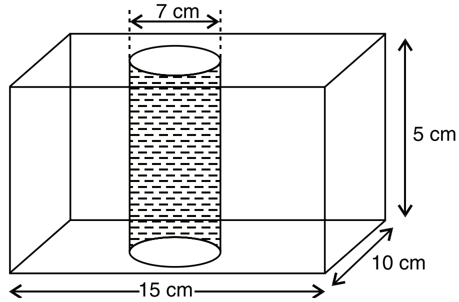
$$= 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$$

$$= 77 \text{ cm}^2$$

$\therefore$ The cost of painting the hemispherical part of the toy $= ₹(77 \times 10)$

$= ₹\,770$ **Ans.**

19. In Fig. 5, from a cuboidal solid metallic block, of dimensions 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. $\left[\textbf{Use } \pi = \frac{22}{7}\right]$



[3]

Figure 5

Solution : We have cuboidal solid metallic block having dimensions 15 cm × 10 cm × 5 cm.
and diameter of cylinder is 7 cm.
Now, Total surface area of cuboidal block

$$= 2(lb + bh + hl)$$
$$= 2(15 \times 10 + 10 \times 5 + 5 \times 15)$$
$$= 2\,(150 + 50 + 75)$$
$$= 2 \times 275 = 550\text{ cm}^2.$$

2 (Area of circular base) $= 2 \times \pi r^2$

$$= 2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$
$$= 77\text{ cm}^2$$

And, curved surface area of cylinder $= 2\,\pi rh$

$$= 2\times\frac{22}{7}\times\frac{7}{2}\times 5$$
$$= 110\text{ cm}^2$$

Hence, required surface area = T.S.A. of block – Area of base + C.S.A. of cylinder

$$= 550 - 77 + 110$$
$$= 583\text{ cm}^2$$ **Ans.**

20. In Fig. 6, find the area of the shaded region [Use π = 3.14] **[3]**

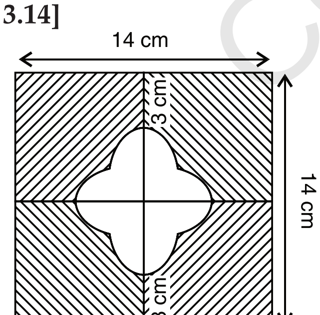
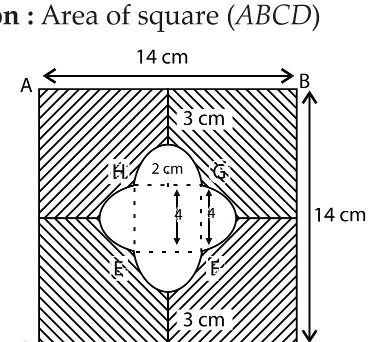


Figure 6

Solution : Area of square (*ABCD*)



$$= 14 \times 14$$
$$= 196\text{ cm}^2$$

Area of small square (*EFGH*)

$$= (7 - 3)^2$$
$$= 4 \times 4 = 16\text{ cm}^2$$

4 (Area of semicircle) $= 4\times\frac{\pi r^2}{2}$

$$= 2 \times 3.14 \times 2 \times 2$$
$$= 25.12\text{ cm}^2$$

$\therefore$ Required area (shaded) = Area of square – Area of small square – 4 × Area of semicircle

$$= (196 - 16 - 25.12)\text{ cm}^2$$
$$= 154.88\text{ cm}^2$$ **Ans.**

SECTION — D

21. The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is $\frac{29}{20}$. Find the original fraction. **[4]**

Solution : Let the denominator of the fraction be x then numerator is $x - 3$ and fraction is $\frac{x-3}{x}$.

If 2 is added to both numerator and denominator then new fraction is

$$\frac{x-3+2}{x+2} = \frac{x-1}{x+2}$$

According to the question,

$$\frac{x-3}{x}+\frac{x-1}{x+2} = \frac{29}{20}$$
$$\Rightarrow \quad \frac{(x-3)(x+2)+x(x-1)}{x(x+2)} = \frac{29}{20}$$
$$\Rightarrow 20(x^2 - 3x + 2x - 6 + x^2 - x) = 29(x^2 + 2x)$$
$$\Rightarrow \quad 40x^2 - 40x - 120 = 29x^2 + 58x$$
$$\Rightarrow \quad 11x^2 - 98x - 120 = 0$$
$$\Rightarrow \quad 11x^2 - 110x + 12x - 120 = 0$$
$$\Rightarrow \quad 11x(x - 10) + 12\,(x - 10) = 0$$
$$\Rightarrow \quad (11x + 12)\,(x - 10) = 0$$
$$x = 10 \text{ or } -\frac{12}{11} \text{ (neglect)}$$

Hence, the fraction is $\frac{10-3}{10}$ *i.e.,* $\frac{7}{10}$. **Ans.**

22. Ramkali required ₹ 2500 after 12 weeks to send her daughter to school. She saved ₹ 100 in the first week and increased her weekly saving by ₹ 20 every week. Find whether she will be able to send her daughter to school after 12 weeks.

What value is generated in the above situation ? **[4]**

Solution : Money required by Ramkali = ₹ 2500

We have, $a = 100$, $d = 20$ and $n = 12$

$\therefore$ A.P. formed is 100, 120, 140 .... upto 12 terms.

Sum of money after 12 weeks

$$S_{12} = \frac{12}{2}[2\times 100+(12-1)20]$$

$$= 6[200 + 11 \times 20] = 6(200 + 220)$$

$$= 6 \times 420 = ₹\,2520$$

Hence, Ramkali will be able to send her daughter to school after 12 weeks.

Value : Many a little makes a mickle. **Ans.**

23. Solve for x :

$$\frac{2}{x+1}+\frac{3}{2(x-2)} = \frac{23}{5x}, x \neq 0, -1, 2 \quad [4]$$

Solution : We have, $\frac{2}{x+1}+\frac{3}{2(x-2)} = \frac{23}{5x}$ $x \neq 0, -1, 2$

$\Rightarrow$ $2(10x)(x-2) + 3(5x)(x+1) = 23(2)(x+1)(x-2)$

$\Rightarrow$ $20x(x-2) + 15x(x+1) = 46(x+1)(x-2)$

$\Rightarrow$ $20x^2 - 40x + 15x^2 + 15x = 46(x^2 + x - 2x - 2)$

$\Rightarrow$ $20x^2 - 40x + 15x^2 + 15x = 46x^2 - 46x - 92$

$\Rightarrow$ $11x^2 - 21x - 92 = 0$

$\Rightarrow$ $x = \frac{21 \pm \sqrt{441+4048}}{22}$

$\Rightarrow$ $x = \frac{21 \pm \sqrt{4489}}{22}$

$\Rightarrow$ $x = \frac{21 \pm 67}{22}$

$\Rightarrow$ $x = \frac{21+67}{22}$ or $\frac{21-67}{22}$

$\Rightarrow$ $x = \frac{88}{22}$ or $-\frac{46}{22}$

$\therefore$ $x = 4$ or $-\frac{23}{11}$ **Ans.**

24. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact. **[4]**

Solution : Given, a circle with centre O and a tangent AB at point P on circle.

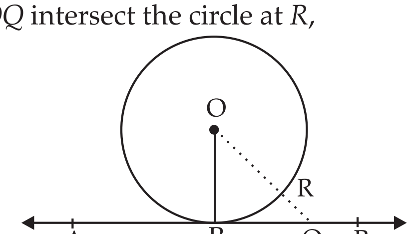
To prove : $OP \perp AB$.

Construction : Take another point Q on AB and join OQ.

Proof : Since Q is a point on AB (other than P)

$\therefore$ Q lies outside the circle.

Let OQ intersect the circle at R,



Then, $OR < OQ$ ...(i)

But $OP = OR$ (radii of circle) ...(ii)

$\therefore$ $OP < OQ$ (from (i) and (ii))

Thus, OP is shorter than any other line segment joining O to any point on AB.

But the shortest distance between a point and a line is the perpendicular distance.

$\therefore$ $OP \perp AB$ **Hence Proved.**

25. In Fig. 7, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle RPQ = 30°$. A chord RS is drawn parallel to the tangent PQ. Find $\angle RQS$. **[4]**

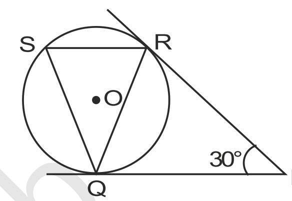


Figure 7

Solution : We have, $PR = PQ$

and $\angle PRQ = \angle PQR$

In Δ PQR,

$\angle PRQ + \angle PQR + \angle RPQ = 180°$

$2\angle PRQ + 30° = 180°$

$\angle PRQ = \frac{180° - 30°}{2} = 75°$

$\because$ $SR \parallel QP$ and QR is a transversal

$\angle SRQ = \angle PQR = 75°$

Join OR, OQ.

$\therefore$ $\angle ORQ = \angle RQO = 90° - 75° = 15°$

$\therefore$ $\angle QOR = (180° - 2 \times 15°)$

$= 180° - 30° = 150°$

$\angle QSR = \frac{1}{2}\angle QOR$

$= 75°$

(Angle subtended on arc is half the angle subtended on centre)

$\therefore$ In Δ SQR

$\angle RQS = 180° - (\angle SRQ + \angle RSQ)$

$= 180° - (75° + 75°)$

$\therefore$ $\angle RQS = 30°$ **Ans.**

26. Construct a triangle ABC with $BC = 7$ cm, $\angle B = 60°$ and $AB = 6$ cm. Construct another triangle whose sides are $\frac{3}{4}$ times the corresponding side of ΔABC.

[4]

Solution : Steps of Construction—

(i) Draw a line segment $AB = 6$ cm.

(ii) Construct $\angle ABX = 60°$.

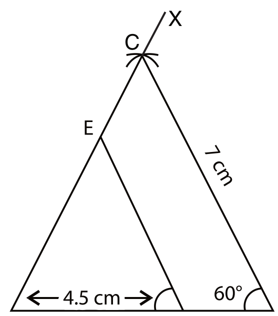
(iii) From B as centre draw an arc of 7 cm cutting BX at C.

(iv) Join AC.

Thus $\Delta\, ABC$ is obtained.

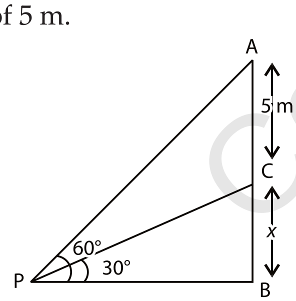
(v) Take point D on AB such that $AD = \frac{3}{4}\, AB = \left(\frac{3}{4}\times 6\right)\text{cm} = \frac{9}{2}\text{cm} = 4.5 \text{ cm}$

(vi) Draw $DE \;||\; BC$, meeting AC at E. Then, $\Delta\, ADE$ is the required triangle similar to $\Delta\, ABC$ such that each side of $\Delta\, ADE$ is $\frac{3}{4}$ times the corresponding side of $\Delta\, ABC$.



27. From a point P on the ground the angle of elevation of the top of a tower is 30° and that of the top of a flag staff fixed on the top of the tower, is 60°. If the length of the flag staff is 5 m, find the height of the tower. [4]

Solution : Let CB be the tower of x m and AC be the flag staff of 5 m.



Then, in $\Delta\, CPB$

$$\tan 30° = \frac{x}{PB}$$

$$PB = \frac{x}{\tan 30°} = \sqrt{3}x \qquad \text{...(i)}$$

In Δ APB

$$\tan 60° = \frac{5+x}{PB}$$

$$PB = \frac{5+x}{\sqrt{3}} \qquad \text{...(ii)}$$

From eq. (i) and (ii)

$$\sqrt{3}x = \frac{x+5}{\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{x+5}{3}$$

$$\Rightarrow \quad 3x - x = 5$$

$$\Rightarrow \quad 2x = 5$$

$$\Rightarrow \quad x = 5/2 = 2.5$$

$\therefore$ Height of the tower is 2.5 m. **Ans.**

28. A box contains 20 cards numbered from 1 to 20. A card is drawn at random from the box. Find the probability that the number on the drawn card is :

(i) divisible by 2 or 3

(ii) a prime number [4]

Solution : Total number of outcomes = 20

(i) Let E_1 be the event of getting a number divisible by 2 or 3.

Then, favourable outcomes = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 3, 9, 15.

Number of favourable outcomes = 13

$\therefore$ P(getting a no. divisible by 2 or 3)

$$= P(E_1) = \frac{13}{20}$$ **Ans.**

(ii) Let E_2 be the event of getting a prime number.

Then, favourable outcomes = 2, 3, 5, 7, 11, 13, 17, 19.

Number of favourable outcomes = 8

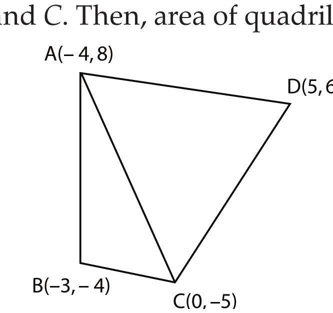
$\therefore$ P(getting a prime number) = $P(E_2) = \frac{8}{20} = \frac{2}{5}$

Ans.

29. If A(– 4, 8), B(– 3, – 4), C(0, – 5) and D(5, 6) are the vertices of a quadrilateral $ABCD$, find its area. [4]

Solution : We have, $A(-4, 8)$, $B(-3, -4)$, $C(0, -5)$ and $D(5, 6)$ are the vertices of a quadrilateral.

Join A and C. Then, area of quadrilateral $ABCD$



= (area of ΔABC) + (area of ΔACD)

Area of ΔABC

$$= \frac{1}{2}[-4(-4+5) - 3(-5-8) + 0(8+4)]$$

$$= \frac{1}{2}[-4+39] = \frac{35}{2} \text{ sq. units}$$

And, area of ΔACD

$$= \frac{1}{2}[-4(-5-6)+0(6-8)+5(8+5)]$$

$$= \frac{1}{2}[44+65]$$

$$= \frac{109}{2} \text{ sq. units.}$$

$\therefore$ Area of quadilateral $ABCD$ = Area of $\Delta\, ABC$ + Area of $\Delta\, ACD$

$$= \frac{35}{2}+\frac{109}{2}$$

$$= \frac{144}{2} \text{ sq. units} = 72 \text{ sq. units.}$$ **Ans.**

30. A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 cm high embankment. Find the width of the embankment. [4]

Solution : We have, diameter of well = 4 m and height = 14 m.

Volume of earth taken out after digging the well

$$= \frac{22}{7}\times\frac{4}{2}\times\frac{4}{2}\times 14$$

$$= 176 \text{ m}^3$$

Let x be the width of the embankment formed by the earth taken out.

Volume of embankment

$$\frac{22}{7}[(2+x)^2-(2)^2]\times\frac{40}{100} = 176$$

$$\Rightarrow \quad \frac{22}{7}[4+x^2+4x-4]\times\frac{2}{5} = 176$$

$$\Rightarrow \quad x^2+4x = \frac{176\times5\times7}{22\times2}$$

$$\Rightarrow \quad x^2+4x-140 = 0$$

$$\Rightarrow \quad x^2+14x-10x-140 = 0$$

$$\Rightarrow \quad x(x+14)-10(x+14) = 0$$

$$\Rightarrow \quad (x+14)(x-10) = 0$$

$$\Rightarrow \quad x = -14 \text{ or } 10$$

$$x = -14 \text{ (neglect)}$$

$$\therefore \quad x = 10$$

Hence, width of embankment = 10 m. **Ans.**

31. Water is flowing at the rate of 2.52 km/h through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm, If the increase in the level of water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe. [4]

Solution : Let the internal radius of the pipe be x m.

Radius of base of tank = 40 cm = $\frac{2}{5}$ m

Speed of water flowing through the pipe

$$= 2.52 \text{ km/hr}$$

$$= \frac{2.52}{2}\times 1000$$

$$= 1260 \text{ m in half an hour}$$

Volume of water flown in half an hour

$$= \pi r^2 h$$

$$= \frac{22}{7}\times x\times x\times 1260$$

$$= 3960\, x^2$$

Level of water raised in the tank

$$= 3.15 \text{ m}$$

$$= \frac{315}{100} \text{ m}$$

Now, $$\pi\times\frac{2}{5}\times\frac{2}{5}\times\frac{315}{100} = 3960\, x^2$$

$$x^2 = \frac{22\times2\times2\times315}{7\times5\times5\times100\times3960}$$

$$x^2 = \frac{4}{10000}$$

$$x = \frac{2}{100} = 0.02 \text{ m}$$

Internal diameter of the pipe = 0.04 m = 4 cm

Ans.

Mathematics 2015 (Delhi) Term II — SET II

Note : Except for the following questions, all the remaining questions have been asked in previous set.

SECTION — B

10. Find the middle term of the A.P. 213, 205, 197, ..., 37. [2]

Solution : Given A.P. is 213, 205, 197 ...., 37

Here $a = 213$, $d = 205 - 213 = 197 - 205 = -8$

Let n be the number of terms

Then,
$$T_n = a+(n-1)d$$
$$37 = 213+(n-1)(-8)$$
$$37 = 213-8n+8$$
$$8n = 184$$
$$n = 23$$

And middle term is $\frac{(n+1)^{\text{th}}}{2}$ term *i.e.* 12^{th} term

$\therefore \quad T_{12} = 213 + (12 - 1)(-8)$
$= 213 + 11(-8)$
$= 213 - 88$
$\therefore \quad T_{12} = 125$

$\therefore$ Middle term of the A.P. is 125. **Ans.**

SECTION — C

18. If the sum of the first n terms of an A.P. is $\frac{1}{2}(3n^2+7n)$, then find its n^{th} term. Hence write its 20^{th} term. [3]

Solution : Given, $S_n = \frac{1}{2}(3n^2+7n)$

Now, $S_1 = \frac{1}{2}[3(1)^2+7(1)] = 5 = a$

(First term)

And, $S_2 = \frac{1}{2}[3(2)^2+7(2)] = 13$

Second term $(a_2) = 13 - 5 = 8$

$\therefore \quad a = 5, d = 3$

We know, $T_n = a + (n-1)d$
$= 5 + (n-1)(3)$
$= 5 + 3n - 3$

$\therefore \quad T_n = 3n + 2$ **Ans.**

And $T_{20} = 5 + (20-1)3$
$= 5 + 19 \times 3$

$\therefore \quad T_{20} = 62$ **Ans.**

19. Three distinct coins are tossed together. Find the probability of getting

(i) at least 2 heads

(ii) at most 2 heads [3]

Solution : Total number of possible outcomes = 8

(i) Let E_1 be the event of getting atleast two heads

Favourable outcomes = $(H, H, T), (T, H, H), (H, T, H), (H, H, H)$

Number of favourable outcomes = 4

P(getting atleast two heads) = $P(E_1) = \frac{4}{8} = \frac{1}{2}$ **Ans.**

(ii) Let E_2 be the event of getting atmost two heads.

Favourable outcomes = $(H, T, T), (T, H, T), (T, T, H), (H, H, T), (H, T, H), (T, H, H), (T, T, T)$

Number of favourable outcomes = 7

P(getting atmost two heads) = $P(E_2) = \frac{7}{8}$ **Ans.**

20. Find that value of p for which the quadratic equation $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0$, $p \neq -1$ has equal roots. Hence find the roots of the equation. [3]

Solution : Given, $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0, p \neq -1$.

For equation to have equal roots

$$[6(p+1)]^2 - 4(p+1).3(p+9) = 0$$
$$36(p+1)^2 - 12(p+1)(p+9) = 0$$
$$12(p+1)[3p+3-p-9] = 0$$
$$12(p+1)(2p-6) = 0$$
$$24(p+1)(p-3) = 0$$
$$p = -1 \text{ or } 3$$

So, $p = 3$

As $p \neq -1$

Now the given equation become

$$4x^2 - 24x + 36 = 0$$
$$x^2 - 6x + 9 = 0$$
$$x^2 - 3x - 3x + 9 = 0$$
$$x(x-3) - 3(x-3) = 0$$
$$(x-3)(x-3) = 0$$
$$x = 3, 3$$

$\therefore$ Roots are 3, 3. **Ans.**

SECTION — D

28. To fill a swimming pool two pipes are to be used. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool. [4]

Solution : Let the pipe of larger diameter fills the pool in x hours.

Then, the pipe with smaller diameter fills the pool in $(x + 10)$ hours.

According to the condition,

$$\frac{4}{x} + \frac{9}{x+10} = \frac{1}{2}$$

$$\Rightarrow \frac{4(x+10)+9x}{x(x+10)} = \frac{1}{2}$$

$$\Rightarrow 2(4x + 40 + 9x) = x^2 + 10x$$
$$\Rightarrow 8x + 80 + 18x = x^2 + 10x$$
$$\Rightarrow 26x + 80 = x^2 + 10x$$
$$\Rightarrow x^2 - 16x - 80 = 0$$
$$\Rightarrow x^2 - 20x + 4x - 80 = 0$$
$$\Rightarrow x(x-20) + 4(x-20) = 0$$
$$\Rightarrow (x-20)(x+4) = 0$$
$$\Rightarrow x = 20 \quad [\text{As } x \neq -4]$$

Hence, the pipe with larger diameter fills the tank in 20 hours.

And, the pipe with smaller diameter fills the tank in 30 hours. **Ans.**

30. Construct an isosceles triangle whose base is 6 cm and altitude 4 cm. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of isosceles triangle. [4]

Solution : Steps of construction—

(i) Draw a line segment $AB = 6$ cm.

(ii) Draw a perpendicular bisector PQ of AB.

(iii) Draw an arc at a distance 4 cm (from AB) intersecting PQ at C.

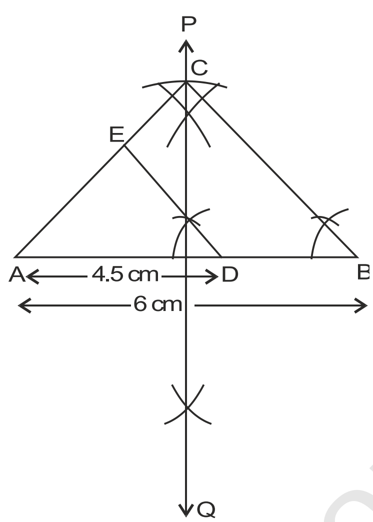
(iv) Join CA and CB.

Thus, ΔABC is obtained.

(v) Mark D on AB, such that $AD = \frac{3}{4} AB = \left(\frac{3}{4}\times 6\right)$ cm $= \frac{9}{2}$ cm $= 4.5$ cm

(vi) Draw DE || BC, cutting AC at E.

Then, ΔADE is the required triangle similar to ΔABC such that each side of ΔADE is $\frac{3}{4}$ times the corresponding side of ΔABC.



31. If P(– 5, – 3), Q(– 4, – 6), R(2, – 3) and S(1, 2) are the vertices of a quadrilateral PQRS, find its area. [4]

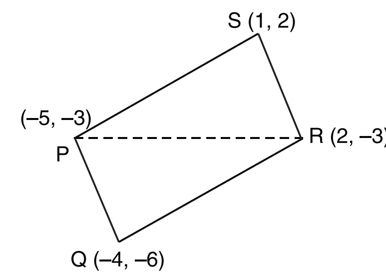
Solution : We have P(– 5, – 3), Q(– 4, – 6), R(2, – 3) and S(1, 2) are the vertices of a quadrilateral PQRS.

Join P and R. Then,

Area of quad $PQRS$ = (Area of ΔPQR) + (Area of ΔPRS)

Area of ΔPQR

$$= \frac{1}{2} |-5(-6+3) - 4(-3+3) + 2(-3+6)|$$



$$= \frac{1}{2} |-5(-3) - 4(0) + 2(3)|$$

$$= \frac{1}{2} |15 + 6| = \frac{21}{2} \text{ sq. units}$$

And, area of ΔPRS

$$= \frac{1}{2} |-5(-3-2) + 2(2+3) + 1(-3+3)|$$

$$= \frac{1}{2} = |-5(-5) + 2(5) + 1(0)|$$

$$= \frac{1}{2} |25 + 10| = \frac{35}{2} \text{ sq. units}$$

Hence, area of quad. $PQRS$ = Area of ΔPQR + Area of ΔPRS

$$= \left(\frac{21}{2}+\frac{35}{2}\right) \text{ sq. units}$$

$$= \frac{56}{2} \text{ sq. units}$$

$= 28$ sq. units **Ans.**

Mathematics 2015 (Delhi) Term II SET III

Note : Except for the following questions, all the remaining questions have been asked in previous sets.

SECTION — B

10. Solve the following quadratic equation for x :

$$9x^2 - 6b^2x - (a^4 - b^4) = 0 \quad [2]$$

Solution : We have, $9x^2 - 6b^2x - (a^4 - b^4) = 0$

$$(9x^2 - 6b^2x + b^4) - a^4 = 0$$

$$(3x - b^2)^2 - (a^2)^2 = 0$$

$$(3x - b^2 + a^2)(3x - b^2 - a^2) = 0$$

$$x = \frac{b^2 - a^2}{3} \text{ or } \frac{b^2 + a^2}{3}$$

Ans.

SECTION — C

18. All red face cards are removed from a pack of playing cards. The remaining cards were well shuffled and then a card is drawn at random from them. Find the probability that the drawn card is :

(i) a red card

(ii) a face card

(iii) a card of clubs [3]

Solution : Total number of possible outcomes $= 52 - 6 = 46$

[$\because$ No. of red face cards = 6]

(i) Let E_1 be the event of getting a red card.

Favourable outcomes = 10 of heart + 10 of diamond

$\therefore$ No. of favourable outcomes = 20

$$\therefore P(\text{getting a red card}) = P(E_1) = \frac{20}{46} = \frac{10}{23} \quad \textbf{Ans.}$$

(ii) Let E_2 be the event of getting a face card

Favourable outcomes = 3 of club + 3 of spade

$\therefore$ No. of favourable outcomes = 6

$$\therefore P\ (\text{getting a face card}) = P(E_2) = \frac{6}{46} = \frac{3}{23} \quad \textbf{Ans.}$$

(iii) Let E_3 be the event of getting a card of clubs

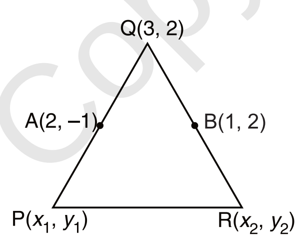
Favourable outcomes= 13 of clubs

$\therefore$ No. of favourable outcomes = 13

$$\therefore P(\text{getting a card of clubs}) = P(E_3) = \frac{13}{46} \quad \textbf{Ans.}$$

19. Find the area of the triangle PQR with $Q(3, 2)$ and the mid-points of the sides through Q being (2, – 1) and (1, 2). **[3]**

Solution : Let $P(x_1, y_1)$, $Q(3, 2)$ an $R(x_2, y_2)$ be the vertices of a triangle PQR and let $A(2, -1)$ and $B(1, 2)$ be the mid-points of PQ and QR respectively.



$\because A$ is the mid-point of PQ

$$\therefore \quad \frac{3+x_1}{2} = 2, \ \frac{2+y_1}{2} = -1$$

$$\Rightarrow \quad x_1 = 1, y_1 = -4$$

So, $P(1, -4)$

$\because B$ is the mid-point of QR

$$\therefore \quad \frac{3+x_2}{2} = 1, \ \frac{2+y_2}{2} = 2$$

$$\Rightarrow \quad x_2 = -1, y_2 = 2$$

So, R(– 1, 2)

$$\text{Thus, Area of } \Delta\, PQR = \frac{1}{2}\,|[1(2-2) - 1(2+4) + 3(-4-2)]|$$

$$= \frac{1}{2}|[1(0) - 1(6) + 3(-6)]|$$

$$= \frac{1}{2}|[-6-18]|$$

$$= \frac{24}{2} = 12 \text{ sq. units.} \quad \textbf{Ans.}$$

20. If S_n denotes the sum of first n terms of an A.P., prove that $S_{30} = 3[S_{20} - S_{10}]$ **[3]**

Solution : Let a be the first term and d be the common difference of the A.P.

$$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_{30} = \frac{30}{2}[2a + (30-1)d]$$

$$= 15(2a + 29d)$$

$$= 30a + 435d$$

$$S_{20} = \frac{20}{2}[2a + (20-1)d]$$

$$= 10[2a + 19d]$$

$$= 20a + 190d$$

$$\text{And,} \quad S_{10} = \frac{10}{2}[2a + (10-1)d]$$

$$= 5[2a + 9d]$$

$$= 10a + 45d$$

Now, $3[S_{20} - S_{10}]$

$$= 3[20a + 190d - 10a - 45d]$$

$$= 3[10a + 145d]$$

$$= 30a + 435d$$

$$= S_{30}$$

$$\therefore \quad S_{30} = 3[S_{20} - S_{10}] \quad \textbf{Hence Proved.}$$

SECTION — D

28. A 21 m deep well with diameter 6 m is dug and the earth from digging is evenly spread to form a platform 27 m × 11 m. Find the height of the platform. $\left[\text{Use } \pi = \frac{22}{7}\right]$ **[4]**

Solution : Volume of earth taken out after digging the well of height 21 m and diameter 6 m.

$$= \frac{22}{7} \times \frac{6}{2} \times \frac{6}{2} \times 21$$

$$= 594 \text{ m}^3$$

Let h be the height of the platform formed by the earth dug out.

$\therefore$ Volume of platform = Volume of earth dug out

$$27 \times 11 \times h = 594$$

$$h = \frac{594}{27 \times 11}$$

$$= 2{\cdot}02 \text{ m} \approx 2 \text{ m}$$

$\therefore$ Height of the platform = 2 m **Ans.**

29. A bag contains 25 cards numbered from 1 to 25. A card is drawn at random from the bag. Find the probability that the number on the drawn card is :

(i) divisible by 3 or 5

(ii) a perfect square number. [4]

Solution : Total number of possible outcomes = 25

(i) Let E_1 be the event of getting a number divisible by 3 or 5.

Favourable outcomes = {3, 6, 9, 12, 15, 18, 21, 24, 5, 10, 20, 25}

$\therefore$ Number of favourable outcomes = 12

$$P(\text{getting a no. divisible by 3 or 5}) = P(E_1) = \frac{12}{25}$$

Ans.

(ii) Let E_2 be the event of getting a perfect square number.

Favourable outcomes= {1, 4, 9, 16, 25}

$\therefore$ Number of favourable outcomes = 5

P (getting a perfect square number)

$$= P(E_2) = \frac{5}{25} = \frac{1}{5}$$ **Ans.**

30. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other cirlce. [4]

Solution : Steps of construction :

(i) Draw a line segment AB = 7 cm.

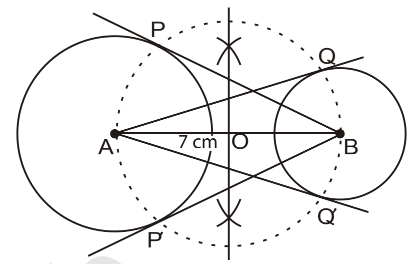
(ii) Take A as centre, draw a circle of radius 3 cm.

(iii) Take B as centre, draw a circle of radius 2 cm.

(iv) Bisect AB at O.

(v) Draw a circle with O as centre and radius equal to AO (= OB) to intersect the circle of radius 3 cm at P and P′, and the circle of radius 2 cm at Q and Q'.

(vi) Join BP and BP′. Also, join AQ and AQ′.
Then, BP, BP′, AQ and AQ′ are the required tangents.



31. Solve for x :

$$\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1};\ x \neq 1, -1, \frac{1}{4}$$

[4]

Solution : We have,

$$\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1};\ x \neq 1, -1, \frac{1}{4}$$

$\Rightarrow 3(x-1)(4x-1) + 4(x+1)(4x-1) = 29(x+1)(x-1)$

$\Rightarrow 3(4x^2 - 4x - x + 1) + 4(4x^2 + 4x - x - 1) = 29(x^2 - 1)$

$\Rightarrow 12x^2 - 15x + 3 + 16x^2 + 12x - 4 = 29x^2 - 29$

$\Rightarrow \quad 28x^2 - 3x - 1 = 29x^2 - 29$

$\Rightarrow \quad x^2 + 3x - 28 = 0$

$\Rightarrow \quad x^2 + 7x - 4x - 28 = 0$

$\Rightarrow \quad x(x+7) - 4(x+7) = 0$

$\Rightarrow \quad (x-4)(x+7) = 0$

$\Rightarrow \quad x = -7 \text{ or } 4.$ **Ans.**

Mathematics 2014 Term I

Time allowed : 3 hours **Maximum Marks : 90**

SECTION – A

1. In the given figure if $DE \,||\, BC$, AE = 8 cm, EC = 2 cm and BC = 6 cm, then find DE. **[1]**

Solution : In ΔADE and ΔABC,

$$\angle DAE = \angle BAC \quad \text{[Common]}$$

$$\angle ADE = \angle ABC \text{[Corresponding angles]}$$

By AA axiom

$$\Delta ADE \sim \Delta ABC$$

$$\therefore \quad \frac{AE}{AC} = \frac{DE}{BC} \quad \text{[CPCT]}$$

$$\Rightarrow \quad \frac{8}{8+2} = \frac{DE}{6}$$

$$\Rightarrow \quad 10\,DE = 48$$

$$\Rightarrow \quad DE = 4.8 \text{ cm}$$ **Ans.**

2. Evaluate : $10.\dfrac{1-\cot^2 45°}{1+\sin^2 90°}$. **[1]**

Solution :
$$10.\frac{1-\cot^2 45}{1 \quad \sin^2 90} = 10.\frac{1-(1)^2}{1 \quad (1)^2}$$

$$= 10.\ \frac{0}{2} \ = 0$$ **Ans.**

3. If cosec $\theta = \dfrac{5}{4}$, find the value of cot θ. **[1]**

Solution : We know that,

$$\cot^2\theta = \text{cosec}^2\theta - 1$$

$$= \frac{5}{4}^{\ 2} - 1$$

$$= \frac{25}{16} - 1 = \frac{25-16}{16} = \frac{9}{16}$$

$$\Rightarrow \quad \cot^2\theta = \frac{9}{16}$$

$$\Rightarrow \quad \cot\theta = \frac{3}{4}$$ **Ans.**

4. Following table shows sale of shoes in a store during one month :

Size of shoe	3	4	5	6	7	8
Number of pairs sold	4	18	25	12	5	1

Find the model size of the shoes sold. **[1]**

Solution : Maximum number of pairs sold = 25 (size 5)

$\therefore$ Modal size of shoes = 5 **Ans.**

SECTION – B

5. Find the prime factorisiation of the denominator of rational number expressed as $6.\overline{12}$ in simplest form. **[2]**

Solution : Let $x = 6.\overline{12}$ ...(i)

$\Rightarrow$ $100x = 6.\overline{12}$ ...(ii)

Substracting eq. (i) from (ii), we get

$$99x = 606$$

$$x = \frac{606}{99} = \frac{202}{33}$$

$\therefore$ Denominator = 33

Prime factorisation = 3 × 11 **Ans.**

6. Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{3}$ and $\dfrac{1}{\sqrt{3}}$ respectively. **[2]**

Solution : Given, sum of zeroes, (S) = $\sqrt{3}$

Product of zeroes, (P) = $\dfrac{1}{\sqrt{3}}$

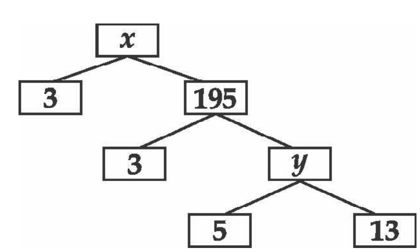
Quadratic polynomial is given as, $x^2 - Sx + P$

$$= x^2 - \sqrt{3}\,x + \frac{1}{\sqrt{3}}$$

$$= \frac{\sqrt{3}x^2 - 3x + 1}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}(\sqrt{3}\,x^2 - 3x + 1)$$ **Ans.**

7. Complete the following factor tree and find the composite number x. **[2]**

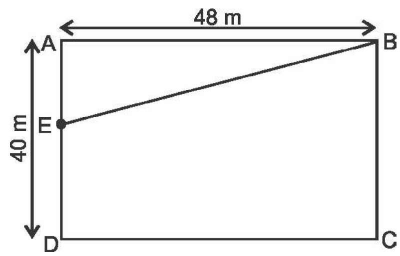


Solution : $y = 5 \times 13 = 65$

$x = 3 \times 195 = 585$ **Ans.**

8. In a rectangle $ABCD$, E is middle point of AD. If AD = 40 m and AB = 48 m, then find EB. **[2]**

Solution :



Given, E is the mid-point of AD

$\therefore \quad AE = \frac{40}{2} = 20$ m

$\angle A = 90°$ [Angle of a rectangle]

$\therefore$ In rt. ΔBAE,

$EB^2 = AB^2 + AE^2$

[Pythagoras' theorem]

$= (48)^2 + (20)^2$

$= 2304 + 400 = 2704$

$EB = \sqrt{2704} = 52$ m **Ans.**

9. If $x = p \sec\theta + q \tan\theta$ and $y = p \tan\theta + q \sec\theta$, then prove that $x^2 - y^2 = p^2 - q^2$. [2]

Solution : L.H.S. $= x^2 - y^2$

$= (p \sec\theta + q \tan\theta)^2 - (p \tan\theta + q \sec\theta)^2$

$= (p^2\sec^2\theta + q^2\tan^2\theta + 2pq \sec\theta \tan\theta - (p^2\tan^2\theta + q^2\sec^2\theta + 2pq \sec\theta \tan\theta)$

$= p^2\sec^2\theta + q^2\tan^2\theta + 2pq \sec\theta \tan\theta - p^2\tan^2\theta - q^2\sec^2\theta - 2pq \sec\theta \tan\theta$

$= p^2(\sec^2\theta - \tan^2\theta) - q^2(\sec^2\theta - \tan^2\theta)$

$= p^2 - q^2$ [$\because \sec^2\theta - \tan^2\theta = 1$]

= R.H.S. **Hence Proved.**

10. Given below is the distribution of weekly pocket money recieved by students of a class. Calculate the pocket money that is received by most of the students. [2]

Pocket Money (in ₹)	**0–20**	**20–40**	**40–60**	**60–80**	**80–100**	**100–120**	**120–140**
No. of Students	**2**	**2**	**3**	**12**	**18**	**5**	**2**

Solution :

Pocket Money (in ₹)	**Number of Students**	
0–20	2	
20–40	2	
40–60	3	
60–80	12 f_0	
80–100	18 f_1	*(Maximum)*
100–120	5 f_2	
120–140	2	

Maximum frequency is 18

$\therefore$ Modal class = 80 – 100

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 80 + \frac{18-12}{36-12-5} \times 20$$

$$= 80 + \frac{6}{19} \times 20$$

$$= 80 + \frac{120}{19} = 80 + 6.32$$

$$= 86.32 \text{ (approx.)}$$

$\therefore$ Required pocket money = ₹ 86.32 (approx.)

Ans.

SECTION – C

11. Prove that $3 + 2\sqrt{3}$ is an irrational number. [3]

Solution : Let us assume to the contrary, that $3 + 2\sqrt{3}$ is rational.

So that we can find co-prime positive integers a and b $(b \neq 0)$, such that $3 + 2\sqrt{3} = \frac{a}{b}$

Rearranging the equation, we get

$$2\sqrt{3} = \frac{a}{b} - 3 = \frac{a-3b}{b}$$

$$\sqrt{3} = \frac{a-3b}{2b} = \frac{a}{2b} - \frac{3b}{2b}$$

$$\sqrt{3} = \frac{a}{2b} - \frac{3}{2}$$

Since a and b are integer, we get $\frac{a}{2b} - \frac{3}{2}$ is rational and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational. So we conclude that $3 + 2\sqrt{3}$ is irrational.

Hence Proved.

12. Solve by elimination :

$3x = y + 5$

$5x - y = 11$ [3]

Solution : Given equations are,

$3x = y + 5$ ...(i)

$5x - y = 11$ ...(ii)

On subtracting eq. (i) and (ii), we get

$$\begin{aligned} 3x - y &= 5 \\ 5x - y &= 11 \\ - \quad + \; &\; - \\ \hline -2x &= -6 \\ x &= 3 \end{aligned}$$

Putting the value of x in eq. (i)

$$3(3) - y = 5$$
$$9 - 5 = y$$
$$\Rightarrow \quad y = 4$$

$\therefore$ $x = 3, y = 4$ **Ans.**

13. A man earns ₹ 600 per month more than his wife. One-tenth of the man's salary and one-sixth of the wife's salary amount to ₹ 1,500, which is saved every month. Find their incomes. [3]

Solution : Let wife's monthly income = ₹ x

Then man's monthly income = ₹ $(x + 600)$

According to the question,

$$\frac{1}{10}(x + 600) + \frac{1}{6}(x) = 1{,}500$$
$$\frac{3(x+600)+5x}{30} = 1{,}500$$
$$3x + 1{,}800 + 5x = 45{,}000$$
$$8x = 45{,}000 - 1{,}800$$
$$x = \frac{43{,}200}{8} = 5{,}400$$

Wife's income = ₹ x = ₹ 5,400

Man's income = ₹ $(x + 600)$ = ₹ 6,000

Ans.

14. Check whether polynomial $x - 1$ is a factor of the polynomial $x^3 - 8x^2 + 19x - 12$. Verify by division algorithm. [3]

Solution : Let $P(x) = x^3 - 8x^2 + 19x - 12$

Put $x = 1$,

$$P(1) = (1)^3 - 8(1)^2 + 19(1) - 12$$
$$= 1 - 8 + 19 - 12$$
$$= 20 - 20$$
$$= 0$$

$\therefore$ $(x - 1)$ is a factor of $P(x)$.

Verification :

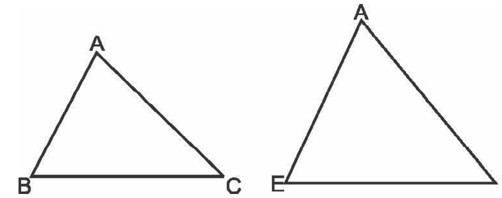
$$\begin{array}{r|l} & x^2 - 7x + 12 \\ \hline x - 1 & x^3 - 8x^2 + 19x - 12 \\ & x^3 - x^2 \\ & - \quad + \\ \hline & -7x^3 + 19x - 12 \\ & -7x^3 + 7x \\ & + \quad - \\ \hline & 12x - 12 \\ & 12x - 12 \\ & - \quad + \\ \hline & \times \end{array}$$

Since remainder = 0.

$(x - 1)$ is a factor of $P(x)$. **Hence Verified.**

15. If the perimeters of two similar triangles *ABC* and *DEF* are 50 cm and 70 cm respectively and one side of ΔABC = 20 cm, then find the corresponding side of ΔDEF. [3]

Solution :



Given, $\Delta ABC \sim \Delta DEF$,

Perimeter of ΔABC = 50 cm

Perimeter ΔDEF = 70 cm

One side of ΔABC = 20 cm

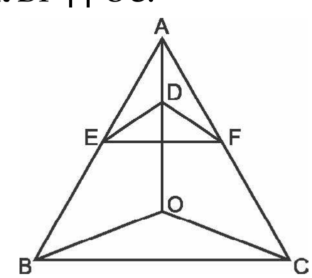
Let AB = 20 cm

$$\Delta ABC \sim \Delta DEF \quad \text{[Given]}$$

$$\therefore \quad \frac{\text{Peri}(\Delta ABC)}{\text{Peri}(\ DEF)} = \frac{AB}{DE}$$
$$\frac{50}{70} = \frac{20}{DE}$$
$$\Rightarrow \quad 5DE = 140$$
$$DE = 28 \text{ m}$$

The corresponding side of ΔDEF = 28 cm. **Ans.**

16. In the figure if $DE \parallel OB$ and $EF \parallel BC$, then prove that $DF \parallel OC$. [3]



Solution : Given, In ΔABC, $DE \parallel OB$ and $EF \parallel BC$

To Prove : $DF \parallel OC$

Proof : In ΔAOB,

$$DE \parallel OB$$

$$\therefore \quad \frac{AE}{EB} = \frac{AD}{DO} \quad \text{...(i) [Thales' Theorem]}$$

Similarly, in ΔABC,

$$EF \parallel BC$$

$$\therefore \quad \frac{AE}{EB} = \frac{AF}{FC} \quad \text{...(ii)[Thales' Theorem]}$$

From (i) and (ii),

$$\frac{AD}{DO} = \frac{AF}{FC}$$

$$\therefore \quad DF \parallel OC$$

[By Converse of Thales' Theorem]

Hence Proved.

17. Prove the identify :

(sec A – cos A).(cot A + tan A) = tan A.sec A. [3]

Solution :

L.H.S. = (sec A – cos A) (cot A + tan A)

$$= \frac{1}{\cos A} - \cos A \quad \frac{\cos A}{\sin A} \quad \frac{\sin A}{\cos A}$$

$$= \frac{1-\cos^2 A}{\cos A} \quad \frac{\cos^2 A \quad \sin^2 A}{\sin A \ \cos A}$$

$$= \frac{\sin^2 A}{\cos A} \quad \frac{1}{\sin A \cos A}$$

$[\because \cos^2 A + \sin^2 A = 1]$

$$= \frac{\sin A}{\cos A} \quad \frac{1}{\cos A}$$

$= \tan A.\sec A =$ R.H.S.

Hence Proved.

18. Given $2 \cos 3\theta = \sqrt{3}$, find the value of θ. [3]

Solution : Given, $2 \cos 3\theta = \sqrt{3}$

$$\Rightarrow \quad \cos 3\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \cos 3\theta = \cos 30°$$

$$3\theta = 30°$$

$$\Rightarrow \quad \theta = 10°$$

Ans.

19. For helping poor girls of their class, students saved pocket money as shown in the following table :

Money saved (in ₹)	5–7	7–9	9–11	11–13	13–15
Number of students	6	3	9	5	7

Find mean and median for this data. [3]

Solution :

Money saved (in ₹)	No. of Students (f_i)	X_i	$d_i = \frac{X_i - 10}{2}$	$f_i d_i$	c.f.
5–7	6	6	–2	–12	6
7–9	3	8	–1	–3	9
9–11	9	$a = 10$	0	0	18
11–13	5	12	1	5	23
13–15	7	14	2	14	30
	$\Sigma f_i = 30$			$\Sigma f_i d_i = 4$	

(i) $$\text{Mean} = a + \frac{f_i d_i}{f_i} \times h$$

$$= 10 + \frac{4}{30} \times 2 = 10 + 0.27$$

$= ₹\, 10.27$ **Ans.**

(ii) $N = \Sigma f_i = 30$

$$\frac{N}{2} = \frac{30}{2} = 15,$$

$\therefore$ Median class is 9 – 11.

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \quad h$$

$$= 9 + \frac{15-9}{9} \times 2 = 9 + \frac{6}{9} \times 2$$

$= 9 + 1.33 = ₹\, 10.33$ **Ans.**

20. Monthly pocket money of students of a class is given in the following frequency distribution : [3]

Pocket money (in ₹)	100–125	125–150	150–175	175–200	200–225
Number of students	14	8	12	5	11

Find mean pocket money using step deviation method.

Solution :

Pocket Money (in ₹)	No. of Students (f_i)	X_i	$d_i = \frac{X_i - 162.5}{25}$	$f_i d_i$
100–125	14	112.5	–2	–28
125–150	8	137.5	–1	–8
150–175	12	$a = 162.5$	0	0
175–200	5	187.5	1	5
200–225	11	212.5	2	22
	$\Sigma f_i = 50$			$\Sigma f_i d_i = –9$

$$\text{Mean} = a + \frac{f_i d_i}{f_i} \quad h$$

$$= 162.5 + \frac{-9}{50} \quad 25$$

$= 162.5 - 4.5 = ₹\, 158$ **Ans.**

SECTION — D

21. If two positive integers x and y are expressible in terms of primes as $x = p^2q^3$ and $y = p^3q$, what can you say about their LCM and HCF. Is LCM a multiple of HCF ? Explain. [4]

Solution : Given,

$$x = p^2 q^3$$

$$= p \times p \times q \times q \times q$$

And $$y = p^3 q$$

$$= p \times p \times p \times q$$

$\therefore$ $$\text{HCF} = p \times p \times q = p^2 q$$

And $$\text{LCM} = p \times p \times p \times q \times q \times q = p^3 q^3$$

$\Rightarrow$ $$\text{LCM} = pq^2 \text{ (HCF)}$$

Yes, LCM is a multiple of HCF.

Explanation :

Let $a = 12 = 2^2 \times 3$

$b = 18 = 2 \times 3^2$

$\therefore$ HCF $= 2 \times 3 = 6$ ...(i)

LCM $= 2^2 \times 3^2 = 36$

LCM $= 6 \times 6$

LCM $= 6$ (HCF) [From (i)]

Here LCM is 6 times HCF. **Ans.**

22. Sita Devi wants to make a rectangular pond on the road side for the purpose of providing drinking water for street animals. The area of the pond will be decreased by 3 square feet if its length is decreased by 2 ft. and breadth is increased by 1 ft. Its area will be increased by 4 square feet if the length is increased by 1 ft. and breadth remains same. Find the dimensions of the pond.

What motivated Sita Devi to provide water point for street animals? [4]

Solution : Let length of rectangular pond = x ft.

and breadth of rectangular pond = y ft.

Area of rectangular pond = xy

According to the question,

$$(x - 2)(y + 1) = (xy - 3)$$
$$xy + x - 2y - 2 = xy - 3$$
$$x - 2y = -1 \quad ...(i)$$
$$(x + 1).y = (xy + 4)$$
$$xy + y = xy + 4$$
$$y = 4 \quad ...(ii)$$

Putting the value of y in eq. (i), we get

$$x - 2(4) = -1$$
$$x - 8 = -1$$
$$\Rightarrow \quad x = -1 + 8 = 7$$

Length of rectangular pond = 7 ft.

Breadth of rectangular pond = 4 ft.

Values :

1. Water is essential for the survival of all living things including street animals.
2. Water is the base of life and no one can live with out it.

23. If a polynomial $x^4 + 5x^3 + 4x^2 - 10x - 12$ has two zeroes as –2 and –3, then find the other zeroes. [4]

Solution : Given, polynomial is $x^4 + 5x^3 + 4x^2 - 10x - 12$.

Since two zeroes are –2 and –3

$$\therefore \quad (x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$$

Dividing the polynomial with $x^2 + 5x + 6$,

$$\begin{array}{r|l} & x^2 - 2 \\ x^2 + 5x + 6 & x^4 + 5x^3 + 4x^2 - 10x - 12 \\ & \underline{\overset{-}{x^4} \overset{-}{+ 5x^3} \overset{-}{+ 6x^2}} \\ & -2x^2 - 10x - 12 \\ & \underline{\overset{+}{-2x^2} \overset{+}{- 10x} \overset{+}{- 12}} \\ & \times \end{array}$$

$\therefore \ x^4 + 5x^3 + 4x^2 - 10x - 12$

$$= (x^2 + 5x + 6)(x^2 - 2)$$
$$= (x + 2)(x + 3)(x - \sqrt{2})(x + \sqrt{2})$$

Other zeroes : $x - \sqrt{2} = 0$ or $x + \sqrt{2} = 0$

$x = \sqrt{2}$ or $x = -\sqrt{2}$

The zeros of the polynomial are –2, –3, $\sqrt{2}$ and $-\sqrt{2}$. **Ans.**

24. Find all the zeroes of the polynomial $8x^4 + 8x^3 - 18x^2 - 20x - 5$, if it is given that two of its zeroes are $\sqrt{\frac{5}{2}}$ and $-\sqrt{\frac{5}{2}}$. [4]

Solution : Given polynomial is $8x^4 + 8x^3 - 18x^2 - 20x - 5$.

Since two zeroes are $\sqrt{\frac{5}{2}}$ and $-\sqrt{\frac{5}{2}}$

$$\therefore \quad x - \sqrt{\frac{5}{2}} \quad x \quad \sqrt{\frac{5}{2}} = (x)^2 - \sqrt{\frac{5}{2}}^{\,2}$$
$$= x^2 - \frac{5}{2}$$

Dividing the polynomial with $x^2 - \frac{5}{2}$

$$\begin{array}{r|l} & 8x^2 + 8x + 2 \\ x^2 - 5/2 & 8x^4 + 8x^3 - 18x^2 - 20x - 5 \\ & \underline{\overset{-}{8x^4} \quad \overset{+}{- 20x^2}} \\ & 8x^3 + 2x^2 - 20x - 5 \\ & \underline{\overset{-}{- 8x^3} \quad \overset{+}{- 20x}} \\ & 2x^2 \quad - 5 \\ & \underline{\overset{-}{2x^2} \quad \overset{+}{- 5}} \\ & \times \end{array}$$

$\therefore \ 8x^4 + 8x^3 - 18x^2 - 20x - 5$

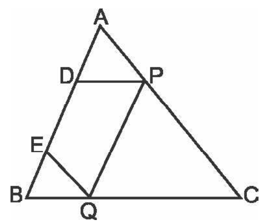
$$= \quad x \ - \frac{5}{2} \ (8x^2 \quad 8x \quad 2)$$
$$= \quad x \ - \frac{5}{2} \ .2(4x^2 \quad 4x \quad 1)$$
$$= 2 \ x \ - \frac{5}{2} \ (4x^2 \quad 2x + 2x \quad 1)$$

$$= 2\ \ x\ \ -\frac{5}{2}\ \ [2x(2x + 1) + 1\ (2x + 1)]$$

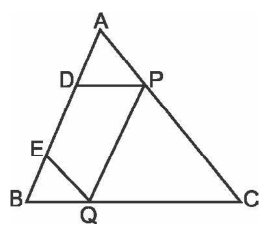
$$= 2\ \ x\ \ -\frac{5}{2}\ \ (2x + 1)\ (2x + 1)$$

All the zeroes are $\sqrt{\frac{5}{2}}, -\sqrt{\frac{5}{2}}, \frac{-1}{2}$ and $\frac{-1}{2}$. **Ans.**

25. In the figure, there are two points D and E on side AB of ΔABC such that AD = BE. If DP | | BC and EQ | | AC, then prove that PQ | | AB. [4]



Solution :



In ΔABC,

$DP \parallel BC$ (Given)

$\Rightarrow \quad \frac{AD}{DB} = \frac{AP}{PC}$ ... (i) [Thales' Theorem]

Also, $EQ \parallel AC$ (Given)

$\Rightarrow \quad \frac{BE}{EA} = \frac{BQ}{QC}$ [Thales' Theorem]

$\Rightarrow \quad \frac{AD}{DB} = \frac{BQ}{QC}$

... (ii) [$\because AD = BE;\ \therefore EA = DB$]

From eq. (i) and (ii)

$$\frac{AP}{PC} = \frac{BQ}{PC}$$

$\therefore \quad PQ \parallel AB$

(Inverse of Thales theorem)

Hence Proved.

26. In Δ*ABC*, altitudes *AD* and *CE* intersect each other at the point P. Prove that

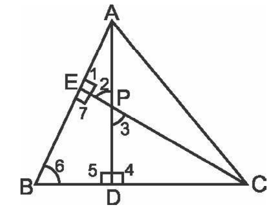
(i) $\Delta APE \sim \Delta CPD$

(ii) $AP \times PD = CP \times PE$

(iii) $\Delta ADB \sim \Delta CEB$

(iv) $AB \times CE = BC \times AD$ [4]

Solution :



Given, In ΔABC, $AD \perp BC$ and $CE \perp AB$

(i) In ΔAPE and ΔCPD

$\angle 1 = \angle 4$ [Each 90°]

$\angle 2 = \angle 3$ [Vertically opposite angles]

By AA axiom

$\Delta APE \sim \Delta CPD$ **Hence Proved.**

(ii) $\Delta APE \sim \Delta CPD$ [Proved above]

$\therefore \quad \frac{AP}{CP} = \frac{PE}{PD}$ [CPCT]

$\Rightarrow AB \times PD = CP \times PE$ **Hence Proved**

(iii) In $\Delta ADB \sim \Delta CEB$

$\therefore \quad \frac{AB}{CB} = \frac{AD}{CE}$ [cpct]

$AB \times CE = BC \times AD$ **Hence Proved.**

$= (\cot A + \sec B)^2 - (\tan B - \text{cosec } A)^2$

$\angle 5 = \angle 7$ (Each 90°)

$\angle 6 = \angle 6$ (Common)

By AA axiom,

$\Delta ADB \sim \Delta CEB$ **Hence Proved.**

(iv) $\Delta ADB \sim \Delta CEB$ [Proved Above]

$\frac{AB}{CB} = \frac{AD}{CE}$ [cpct]

$\Rightarrow$ AB × CE = BC × AD **Hence Proved**

27. Prove that : (cot A + sec B)^2 – (tan B – cosec A)^2 = 2 (cot A . sec B + tan B. cosec A). [4]

Solution : L.H.S.

$$= (\cot A + \sec B)^2 - (\tan B - \text{cosec } A)^2$$

$$= (\cot^2 A + \sec^2 B + 2 \cot A \sec B) - - (\tan^2 B + \text{cosec}^2 A - 2 \tan B \text{ cosec } A)$$

$$= \cot^2 A + \sec^2 B + 2 \cot A \sec B - \tan^2 B - \text{cosec}^2 A + 2 \tan B \text{ cosec } A$$

$$= (\sec^2 B - \tan^2 B) - (\text{cosec}^2 A - \cot^2 A) + 2(\cot A \sec B + \tan B \text{ cosec } A)$$

$$= 1 - 1 + 2(\cot A \sec B + \tan B \text{ cosec } A)$$

$$\because \sec^2 B - \tan^2 B = 1,$$
$$\text{cosec}^2 A - \cot^2 A = 1$$

$$= 2(\cot A \sec B + \tan B \text{ cosec } A) = \text{R.H.S.}$$

Hence Proved.

28. Prove that : (sin θ + cos θ + 1) . (sin θ – 1 + cos θ) . sec θ . cosec θ = 2. [4]

Solution : L.H.S. $= (\sin\theta + \cos\theta + 1).(\sin\theta - 1 + \cos\theta) . \sec\theta \text{ cosec } \theta$

$= [(\sin\theta + \cos\theta) + 1].[(\sin\theta + \cos\theta) - 1)].\sec\theta.\text{cosec}\,\theta$

$= [(\sin\theta + \cos\theta)^2 - (1)^2] \sec\theta \text{ cosec}\,\theta$

$[\because (a + b)(a - b) = a^2 - b^2]$

$= [\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1].\sec\theta \text{ cosec}\,\theta$

$= (1 + 2\sin\theta.\cos\theta - 1).\sec\theta \text{ cosec}\,\theta$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= (2\sin\theta\cos\theta).\dfrac{1}{\cos}.\dfrac{1}{\sin}$

$= 2 = \text{R.H.S.}$ **Hence Proved.**

29. If tan (20° – 3α) = cot (5α – 20°), then find the value of α and hence evaluate :
sin α . sec α . tan α – cosec α . cos α . cot α. [4]

Solution : $\tan(20° - 3\alpha) = \cot(5\alpha - 20°)$

$\Rightarrow \tan(20° - 3\alpha) = \tan[90 - (5\alpha - 20°)]$

$[\because \cot\theta = \tan(90° - \theta)]$

$\Rightarrow 20° - 3\alpha = 90° - 5\alpha + 20°$

$\Rightarrow -3\alpha + 5\alpha = 90° + 20° - 20°$

$\Rightarrow 2\alpha = 90°$

$\Rightarrow \alpha = 45°$ **Ans.**

Now,

$\sin\alpha.\sec\alpha.\tan\alpha - \text{cosec}\,\alpha.\text{cosec}\,\alpha.\cot\alpha$

$= \sin 45°.\sec 45°.\tan 45° - \text{cosec}\,45°.\cos 45°.\cot 45°$

$= \dfrac{1}{\sqrt{2}} \times \sqrt{2} \times 1 - \sqrt{2} \times \dfrac{1}{\sqrt{2}} \times 1$

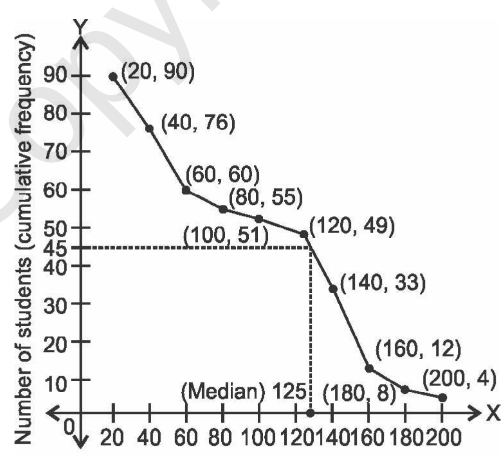
$= 1 - 1 = 0$ **Ans.**

30. The frequency distribution of weekly pocket money received by a group of students is given below :

Pocket money in (₹)	More than or equal to 20	More than or equal to 40	More than or equal to 60	More than or equal to 80	More than or equal to 100	More than or equal to 120	More than or equal to 140	More than or equal to 160	More than or equal to 180	More than or equal to 200
Number of Students	90	76	60	55	51	49	33	12	8	4

Draw a 'more than type' ogive and from it, find median. Verify median by actual calculations. [4]

Solution :



Pocket money (in ₹)	No. of Students	*c.i.*	f_i	cf_i
More than or equal to 20	90	20–40	14	14
More than or equal to 40	76	40–60	16	30
More than or equal to 60	60	60–80	5	35
More than or equal to 80	55	80–100	4	39
More than or equal to 100	51	100–120	2	41
More than or equal to 120	49	120–140	16	57
More than or equal to 140	33	140–160	21	78

More than or equal to 160	12	160–180	4	82
More than or equal to 180	8	180–200	4	86
More than or equal to 200	4	200–220	4	90
$n = 90$			90	

$$\frac{n}{2} = \frac{90}{2} = 45$$

∴ Median class is 120 – 140

$$\text{Median} = l \quad \frac{\frac{n}{2} - cf}{f} \quad h$$

$$= 120 + \frac{45 - 41}{16} \times 20$$

$$= 120 + \frac{4 \quad 20}{16} = 120 + 5 = ₹\ 125$$

Hence Verified.

31. Cost of living Index for some period is given in the following frequency distribution :

Index	**1500–1600**	**1600–1700**	**1700–1800**	**1800–1900**	**1900–2000**	**2000–2100**	**2100–2200**
No. of weeks	**3**	**11**	**12**	**7**	**9**	**8**	**2**

Find the mode and median for above data. [4]

Solution :

Index	Number of weeks (f_i)	cf_i
1500–1600	3	3
1600–1700	11 f_0	14
1700–1800	12 f_1	26
1800–1900	7 f_2	33
1900–2000	9	42
2000–2100	8	50
2100–2200	2	52
	$\Sigma f_i = 52$	

$$n = 52$$

$$\frac{n}{2} = \frac{52}{2} = 26$$

∴ Median class is 1700–1800

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 1700 + \frac{26 - 14}{12} \times 100$$

$$\frac{n}{2} = 1700 + \frac{12}{12} \quad 100 \quad = 1800$$

Maximum frequency is 12

∴ Modal class is 1700–1800

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 1700 + \frac{12 - 11}{24 - 11 - 7} \times 100$$

$$= 1700 + \frac{1}{6} \times 100$$

$$= 1700 + 16.67$$

$$= 1716.67$$

Ans.