Mathematics 2016 (Delhi) Term II

SET III

Note : Except for the following questions, all the remaining questions have been asked in previous sets.

SECTION — B

10. How many terms of the A.P. 65, 60, 55, ... be taken

so that their sum is zero? **[2]**

Solution : Given, A.P. is 65, 60, 55, ...

We have, $a = 65, d = 60 - 65 = 55 - 60 = -5$

Now, $S_n = 0$

Therefore, $S_n = \frac{n}{2}[2a + (n-1)d] = 0$

$\Rightarrow \quad [2(65) + (n-1)(-5)] = 0$

$\Rightarrow \quad 130 - 5n + 5 = 0$

$\Rightarrow \quad 135 - 5n = 0$

$\Rightarrow \quad 5n = 135$

$\therefore \quad n = 27$

Hence, the no. of terms are 27. **Ans.**

SECTION — C

18. A box consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Ramesh, a shopkeeper will buy only those shirts which are good but 'Kewal' another shopkeeper will not buy shirts with major defects. A shirt is taken out of the box at random. What is the probability that

(i) Ramesh will buy the selected shirt ?

(ii) 'Kewal' will buy the selected shirt ? **[3]**

Solution : Let E_1 be the event of selecting good shirts by Ramesh and E_2 be the event of selecting the shirts with no major defects by 'Kewal'.

Total no. of shirts in a box = 100

(i) $\because$ Number of good shirts = 88

$\therefore \quad P(E_1) = \frac{88}{100} = \frac{22}{25}$ **Ans.**

(ii) $\because$ Number of shirts with no major defect

$= 100 - 4 = 96$

$\therefore \quad P(E_2) = \frac{96}{100} = \frac{24}{25}$ **Ans.**

19. Solve the following quadratic equation for x :

$$x^2 + \left(\frac{a}{a+b} = \frac{a+b}{a}\right)x + 1 = 0$$ **[3]**

Solution : We have, $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$

$\Rightarrow \quad x^2 + \frac{a}{a+b}x + \frac{a+b}{a}x + 1 = 0$

$\Rightarrow x\left(x + \frac{a}{a+b}\right) + \frac{a+b}{a}\left(x + \frac{a}{a+b}\right) = 0$

$\Rightarrow \quad \left(x + \frac{a+b}{a}\right)\left(x + \frac{a}{a+b}\right) = 0$

$x = -\frac{a}{a+b}, -\frac{(a+b)}{a}$

$\therefore \quad x = -\frac{a}{a+b}$ or $x = -\frac{(a+b)}{a}$ **Ans.**

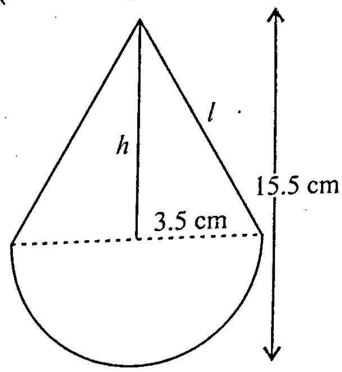
20. A toy is in the form of a cone of base radius 3.5 cm mounted on a hemisphere of base diameter 7 cm. If the total height of the toy is 15.5 cm, find the total surface area of the toy. $\left(\text{use } \pi = \frac{22}{7}\right)$ **[3]**

Solution : Given, the base radius of cone, $r = 3{\cdot}5$ cm

Total height of cone, $(h + r) = 15{\cdot}5$ cm

and base diameter of hemisphere = 7 cm

Now, $h = (15{\cdot}5 - 3{\cdot}5)$ cm = 12 cm



So, slant height, $l = \sqrt{h^2 + r^2} = \sqrt{(12)^2 + (3{\cdot}5)^2}$

$= \sqrt{144 + 12{\cdot}25}$

$= 12{\cdot}5$ cm

$\therefore$ Total Surface Area $= \pi rl + 2\pi r^2$

$= \frac{22}{7} \times 3{\cdot}5 \times 12{\cdot}5 + 2 \times \frac{22}{7} \times 3{\cdot}5 \times 3{\cdot}5$

$= \frac{22}{7} \times 3{\cdot}5\,(12{\cdot}5 + 2 \times 3{\cdot}5)$

$= 11\,(19{\cdot}5)$

$= 214{\cdot}5 \text{ cm}^2$ **Ans.**

SECTION — D

28. The sum of three numbers in A.P. is 12 and sum of their cubes is 288. Find the numbers. **[4]**

Solution : Let the three numbers in A.P. be $a - d, a, a + d$

Now, $a - d + a + a + d = 12$

$3a = 12$

$\therefore \quad a = 4$

Also, $(4 - d)^3 + 4^3 + (4 + d)^3 = 288$

$64 - 48d + 12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 = 288$

$192 + 24d^2 = 288$

$24d^2 = 288 - 192$

Equation (i) is divisible by 2, 11 and 256, which means it has more than 2 prime factors.

$\therefore$ $(17 \times 5 \times 11 \times 3 \times 2 + 2 \times 11)$ is a composite number. **Ans.**

7. Find whether the following pair of linear equations is consistent or inconsistent :

$$3x + 2y = 8$$
$$6x - 4y = 9$$ **[2]**

Solution : Here, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{2}{-4} = \frac{-1}{2}$

$$\frac{1}{2} \neq \frac{-1}{2}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, which will give a unique solution.

Hence, given pair of linear equations is consistent. **Ans.**

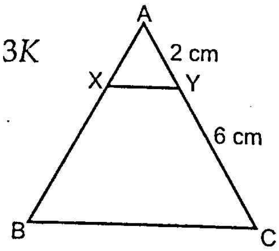
8. X and Y are points on the sides AB and AC respectively of a triangle ABC such that $\frac{AX}{AB}$, AY = 2 cm and YC = 6 cm. Find whether $XY \parallel BC$ or not. **[2]**

Solution : $\frac{AX}{AB} = \frac{1}{4}$

i.e., $AX = 1K, AB = 4K$ (K– constant)

$\therefore$ $BX = AB - AX$

$= 4K - 1K = 3K$



Now, $\frac{AX}{XB} = \frac{1K}{3K} = \frac{1}{3}$

And, $\frac{AY}{YC} = \frac{2}{6} = \frac{1}{3}$

$\frac{AX}{XB} = \frac{AY}{YC}$

$\therefore$ $XY \parallel BC$ (By converse of Thales' theorem) **Ans.**

9. Prove the following identity :

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = 1 - \sin\theta.\cos\theta.$$ **[2]**

Solution : L.H.S. $= \frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta}$

$$= \frac{(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta.\cos\theta)}{(\sin\theta + \cos\theta)}$$

$[a^3 + b^3 = (a + b)(a^2 + b^2 - ab)]$

$= 1 - \sin\theta.\cos\theta$ = R.H.S.

$[\because \sin^2\theta + \cos^2\theta = 1]$ **Hence Proved.**

10. Show that the mode of the series obtained by combining the two series S_1 and S_2 given below is different from that of S_1 and S_2 taken separately:

S_1 : 3, 5, 8, 8, 9, 12, 13, 9, 9

S_2 : 7, 4, 7, 8, 7, 8, 13 **[2]**

Solution : Mode of S_1 series = 9

Mode of S_2 series = 7

After combining S_1 and S_2, the new series will be = 3, 5, 8, 8, 9, 12, 13, 9, 9, 7, 4, 7, 8, 7, 8, 13.

Mode of combined series = 8 (maximum times)

Mode of (S_1, S_2) is different from mode of S_1 and mode of S_2 separately. **Hence Proved.**

SECTION — C

11. The length, breadth and height of a room are 8 m 50 cm, 6 m 25 cm and 4 m 75 cm respectively. Find the length of the longest rod that can measure the dimensions of the room exactly. **[3]**

Solution : To find the length of the longest rod that can measure the dimensions of the room exactly, we have to find HCF.

Length, L = 8 m 50 cm = 850 cm = $2^1 \times 5^2 \times 17$

Breadth, B = 6 m 25 cm = 625 cm = 5^4

Height, H = 4 m 75 cm = 475 cm = $5^2 \times 19$

$\therefore$ HCF of L, B and H is 5^2 = 25 cm

$\therefore$ Length of the longest rod = 25 cm **Ans.**

12. Solve by elimination :

$3x - y = 7$

$2x + 5y + 1 = 0$ **[3]**

Solution : $3x - y = 7$ ...(i)

$2x + 5y = -1$ ...(ii)

Multiplying equation (i) by 5 and solving it with equation (ii), we get

$$\begin{array}{r} 2x + 5y = -1 \\ 15x - 5y = 35 \\ \hline 17x \quad = 34 \end{array}$$ (Adding)

$\Rightarrow$ $x = \frac{34}{17} = 2$

Putting the value of x in (i), we have

$3(2) - y = 7$

$\Rightarrow$ $6 - y = 7 \Rightarrow -y = 7 - 6$

$\Rightarrow$ $y = -1$

$\therefore$ $x = 2, y = -1$ **Ans.**