DATE Assignment 7 & 8 陈责 12212231

Q2

(a) $\forall z \forall u \exists w\ \neg Q(x,y,z) \vee P(w,x,y,u)$

(b) $\forall z \exists w (\neg R(x,z) \vee \neg R(x,y) \vee (R(x,w) \wedge R(y,w) \wedge R(x,y)))$

(c) $\exists z \exists u \exists v\, (S(y,z) \wedge S(z,u) \wedge S(x,v) \wedge S(v,u))$

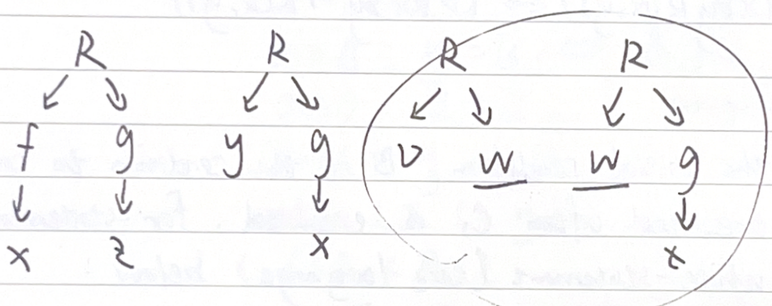
Q3

(a) $\forall z \forall u\ \ \neg Q(x,y,z) \vee P(f(u),x,y,u)$

(b) $\forall z (\neg R(x,z) \vee \neg R(x,y) \vee (R(x,f(z)) \wedge R(y,f(z)) \wedge R(x,y)))$

(c) $S(y,a) \wedge S(a,b) \wedge S(x,c) \wedge S(c,b)$

Q4



is conflict with $R(f(x), g(z))$. w can't be $f(c)$ and $g(c)$ at the same time.

~~$\sigma = \{w \mapsto g(z), z \mapsto x,$~~ Not unifiable because ↗

Q5

(a) $\neg((\exists x \forall y Q(x,y) \wedge \forall x (Q(x,x) \to \exists y R(y,x))) \to \exists y \exists x R(x,y))$

CNF:

$(Q(f(c), y)) \wedge (\neg Q(x,x) \vee R(w,x)) \wedge (\neg R(u,v))$

Resolution:

$$\frac{Q(f(c),y) \qquad \neg Q(x,x)}{\perp\sigma} \quad \sigma = \{y \mapsto f(c), x \mapsto f(c)\}$$

$$\frac{R(w,x) \qquad \neg R(u,v)}{\perp\sigma} \quad \sigma = \{w \mapsto u, v \mapsto x\}$$

Cond: $\vdash (\exists x \forall y Q(x,y) \wedge \forall x (Q(x,x) \to \exists y R(y,x))) \to \exists y \exists x R(x,y)$

DATE (b) $\neg((\exists x \forall y R(x,y)) \leftrightarrow (\neg \forall x \exists y \neg R(x,y)))$

CNF:

$$(R(c,y) \wedge \neg R(x,f(w))) \vee (R(a,b) \wedge \neg R(x,f(w)))$$

Resolution:

$$\frac{R(c,y) \wedge \neg R(x,f(w))}{\perp_{\sigma}} \quad \sigma = \{x \mapsto c,\ y \mapsto f(w)\}$$

$$\frac{R(a,b) \quad \neg R(x,f(w))}{\perp_{\sigma}} \quad \sigma = \{x \mapsto a,\ b \mapsto f(w)\}$$

Conclusion:

$$\phi \vdash (\exists x \forall y R(x,y)) \leftrightarrow (\neg \forall x \exists y \neg R(x,y))$$

Q6

Since C_1 is the initial condition, B is the condition to continue the loop, C_2 is executed after C_3 is executed, for-statement is equilivant to the while-statement (core language) below:

```
for(C1 ; B ; C2){            C1;
    C3;           =>        while(B){
}                              C3; C2;
                            }
```

Q7

(a) Proof: $(|x > 0|)$

$x = A \Leftrightarrow A > 0$ assignment

$y = x+1 = A+1 \Leftrightarrow A+1 > 1$

$(| y > 1 |)$

(b) Proof: $(|T|)$ $x = A$ assignment

$y = x \Leftrightarrow y = A$

$y = x+x+y \Leftrightarrow y = A+A+A = 3 \cdot A$

DATE /

$(| y = 3 \cdot x |)$

(C) Proof

$(| x > 1 |)$

$x = A ; \iff A > 1$ assignmet

$y = x ; \iff y = A$

$y = y - a \land a = 1 \iff y = y - 1 = A - 1 > 0$

$(x = A) \land (y = A - 1) \iff x > y$

$(| x > y \land y > 0 |)$

Q8

$(| y \geq 0 |)$

$x = A ; y = B ; \iff B \geq 0$ assignmet

$a = 0 ; z = 0 ;$ assignmet

$(| a < y = B |)$

while $(a != y)$ {

$(| a < B |)$

$z = z + x \iff z = A + (a-1) A = a \cdot A$

$a = a + 1 \iff a \leq B$

}

$(| a = B |)$

$z = a \cdot A \iff z = B \cdot A$

$(| z = x \cdot y |)$

Q9

Consider variant $t(a+y) = (y - a)$

Pre-cond : $y - a \geq 0 \iff a = 0 \land y \geq 0$

$(y-a)$ decrease by 1 because a increases by 1 during every iteration.

Post-cond : $y - a = 0$ when $y - a = 0$ the loop

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is terminated. Since the equlivant loop is

```
while (y-a != 0) { // y != a <=> y-a != 0
    z = z + x;
    a = a+1; // increases.
}
```

From Q8,

$\vdash_{par} (| y \geq 0 |)$ multi $(| z = x \cdot y |)$

and ~~Q9~~ above.

$\vdash_{term} (| y \geq 0 |)$ multi $(| z = x \cdot y |)$

Thus,

$\vdash_{tot} (| y \geq 0 |)$ multi $(| z = x \cdot y |)$