DATE Assignment 5.6 陈燕 12212231

Q1

(1) $W(d)$ (2) $\forall x, S(x) \to W(x)$ (3) $\exists x\ S(x) \to W(x)$

(4) $\neg \exists x, S(x) \to W(x)$ (5) $\exists x, W(x)$ (6) $\neg \exists x, W(x)$

(7) $\exists x, y. (x \neq y \land S(x) \land S(y))$ (8) $\exists x, y. (x \neq y \land (S(x) \land (S(y))$
$\land \forall z ((S(z)) \to (z = x \lor z = y)))$ (9) $\forall x (S(x) \to W(x)) \land S(d) \to W(d)$

Q2

(10) Not formula, P has only 2 candidates

(11) Not formula, function h can't accept predicate P

(12) Formula

(13) Not formula, P has only 1 candidate

(14) Not formula, model inplies predicate is not legal.

(15) Formula

Q3

(16) y and z in the first $P(y,z)$ are free variables, z in the latten $P(y,z)$ is free, x in $Q(y,x)$ is bounded by $\exists x$ and y is bounded by $\forall y$ in $Q(y,x)$ and latten $P(y,z)$

(17) y has free and bound occurrences

(18.1) $\exists x (P(y,z) \land \forall y (Q(y,w) \lor P(y,z)))$
$\exists x (P(w,z) \land \forall y (Q(y,x) \lor P(y,z)))$
$\exists x (P(f(x),z) \land \forall y (Q(y,x) \lor P(y,z)))$
$\exists x (P(y, g(y,z)) \land \forall y (Q(y,x) \lor P(y, g(y,z))))$

(18.2) w and $g(y,z)$

(18.3) $f(x)$ and w

(19) $P(y,z) \land \forall y (Q(y,x) \lor P(y,z)))$ is $\exists x$'s scope.

(20) $P(y,z)$ is the scope of $\exists x$.

DATE Q4

(1)
1. $(y=0)\land(y=x) \vdash (y=0)\land(y=x)$ Assumption
2. $(y=0)\land(y=x) \vdash y=0$ $\land$-E on 1
3. $(y=0)\land(y=x) \vdash y=x$ $\land$-E on 1
4. $(y=0)\land(y=x) \vdash 0=x$ $=$-E / EQSUB on 2, 3

(2)
1. $t_1=t_2 \vdash t_1=t_2$ Assumption
2. $t+t_2 \vdash t+t_2$ Assumption
3. $t=t \vdash t=t$ $=$-Intro
4. $t+t_2 \vdash t+t_1$ $=$-E on 1, 2
5. $t+t_2 \vdash (t+t_2)=(t+t_1)$ $=$-I on 2, 4, 3, 1
6. $t_1=t_2 \vdash (t+t_2)=(t+t_1)$ $=$-I on 5

Q5

(24)
1. $\{\forall x(P(x)\to Q(x)),\ \forall x\neg Q(x)\} \vdash \forall x(P(x)\to(Q(x)))$ Assumption
2. $\{\forall x(P(x)\to(Q(x)),\ \forall x\neg Q(x)\} \vdash \forall x(\neg Q(x))$ Assumption
3. 〃 $\vdash P(y)\to Q(y)$ $\forall$-E, assumption on 1
4. 〃 $\vdash \neg Q(y)$ $\forall$-E, on 2
5. 〃 $\vdash \neg P(y)$ Modus tollens on 3, 4
6. 〃 $\vdash \forall x\neg P(x)$ $\forall$-I, arbitrary y on 5
7. $\forall x(P(x)\to Q(x)) \vdash (\forall x\neg Q(x))\to(\forall x\neg P(x))$ $\to$ Intro on 6

(25)
1. $\{\forall x(P(x)\to\neg(Q(x)))\} \vdash \forall x(P(x)\to\neg(Q(x)))$ Assumption
2. 〃 $\vdash \forall x\ \neg P(x)\lor\neg Q(x)$ $\to$-Def on 1
3. 〃 $\vdash \forall x\ \neg(P(x)\land Q(x))$ $\neg$ distribution rule
4. 〃 $\vdash \cancel{\neg\exists x}\ \neg(P(x)\land Q(x))$ Assume arbitrary x
5. 〃 $\vdash \neg\exists x(P(x)\land Q(x))$ $\exists$-Intro
6. $\forall x(P(x)\to\neg(Q(x))) \vdash \neg(\exists x(P(x)\land Q(x))$ Conclusion

DATE Q6

(a) 1. $\{S, \exists x(S \to Q(x))\} \vdash S$ Assumption

2. $\{S, \exists x(S \to Q(x))\} \vdash \exists x(S \to Q(x))$ Assumption

3. 〃 $\vdash S \to Q(x)$ assume x

4. 〃 $\vdash Q(x)$ $\to$-Elim on 1, 3

5. 〃 $\vdash \exists x, Q(x)$ $\exists$-Intro

6. $\exists x(S \to Q(x)) \vdash S \to \exists x, Q(x)$

(b) 1. $\neg \forall x \neg P(x) \vdash \neg \forall x \neg P(x)$ Assumption

2. $\neg \forall x \neg P(x) \vdash \neg(\neg P(x))$ $\forall$-Elim

3. $\neg \forall x \neg P(x) \vdash \neg\neg P(x)$ remove parenthesis

4. $\neg \forall x \neg P(x) \vdash P(x)$ DouNeg-Elim

5. $\neg \forall x \neg P(x) \vdash \forall x\, P(x)$ $\forall$-intro

(c) 1. $\{\forall x \neg P(x)\} \vdash \forall x \neg P(x)$ Assumption

2. $\{\exists x P(x)\} \vdash \exists x\, P(x)$ Assumption

3. $\{\forall x \neg P(x)\} \vdash \neg P(x)$ $\forall$-Elim on 1

4. $\{\exists x P(x)\} \vdash P(x)$ for some x on 2

5. $\{\forall x \neg P(x)\} \vdash \neg \exists x\, P(x)$ Contradicton on on 3, 4

6. $\forall x \neg P(x) \vdash \neg \exists x\, P(x)$ conclusion

Q7

(a) 1. $\{P(b), x=b\} \vdash x=b$ Assumption

2. $\{P(b), x=b\} \vdash P(b)$ Assumption

3. $\{P(b), x=b\} \vdash P(x) = P(b)$ $=$-Substition on 1

4. $\{P(b), x=b\} \vdash P(x)$ $=$-Elim on 2, 3

5. $\{P(b)\} \vdash x=b \to P(x)$ $\to$Intro no 4

6. $\{P(b)\} \vdash \forall x(x=b \to P(x))$ $\forall$ Intro

7. $P(b) \vdash \forall x(x=b \to P(x))$ Conclusion

DATE (b)/

1. $\{p(b), p(x), \forall x \forall y (p(x) \wedge p(y) \to x=y)\} \vdash p(b)$ Assumption

2. $\quad " \vdash p(x)$ Assumption

3. $\quad " \vdash \forall x, y (p(x) \wedge p(y) \to x=y)$ Assumption

4. $\quad " \vdash p(x) \wedge p(b) \to x=b$ assume $x=x$, $y=b$

5. $\quad " \vdash p(x) \wedge p(b)$ $\wedge$-Intro on 1.2.

6. $\quad " \vdash x=b$ $\to$-Elim on 4.5

7 $\{p(b), \forall x \forall y (p(x) \wedge p(y) \to x=y)\} \vdash p(x) \to x=b$

8. $\{p(b), x=b, \forall x \forall y (p(x) \wedge p(y) \to x=y)\} \vdash p(b)$ Assumption

9. $\quad " \vdash x=b$ Assumption

10. $\quad " \vdash \forall x, y (p(x) \wedge p(y) \to x=y)$ Assumption

11. $\quad " \vdash p(x) = p(b)$ =-Substitution on 9

12. $\quad " \vdash p(x)$ =-Elim on 8.11

13. $\{p(b), \forall x \forall y (p(x) \wedge p(y) \to x=y)\} \vdash x=b \to p(x)$ $\to$-Intro 12

14. $\{p(b), \forall x \forall y (p(x) \wedge p(y) \to x=y)\} \vdash p(x) \leftrightarrow x=b$ Double Impli

14 $\{p(b), \forall x \forall y (p(x) \wedge p(y) \to x=y)\} \vdash \forall x\, p(x) \leftrightarrow x=b$ $\forall$-Intro

(d) 1. $\forall x (p(x) \leftrightarrow x=b) \vdash \forall x (p(x) \leftrightarrow x=b$ Assumption.

2. $\forall x (p(x) \leftrightarrow x=b) \vdash b=b$ =-reflex

3. $\forall x (p(x) \leftrightarrow x=b) \vdash p(b) \leftrightarrow b=b$ $\forall$-Elim, assum $x=b$

4. $\forall x (p(x) \leftrightarrow x=b) \vdash p(b)$ Double Impli on 2, 3

5. $\forall x (p(x) \leftrightarrow x=b) \vdash p(x) \leftrightarrow x=b$ $\forall$-Elim on 1

6. $\forall x (p(x) \leftrightarrow x=b) \vdash p(y) \leftrightarrow y=b$ $\forall$-Elim on 1

7. $\forall x (p(x) \leftrightarrow x=b) \vdash p(x) \wedge p(y) \leftrightarrow x=y$ Substition

8. $\forall x (p(x) \leftrightarrow x=b) \vdash \forall y\, p(x) \wedge p(y) \to x=y$ Dou Imp -Elim $\forall$-Intro

9. $\forall x (p(x) \leftrightarrow x=b) \vdash p(b) \wedge \forall y\, p(x) \wedge p(y) \to x=y$ $\wedge$-Intro on 4.8

Q8

(a) 1. $\{\forall x\, P(a,x,x), \forall xyz (P(x,y,z) \to P(f(x), y, f(z)))\} \vdash \forall x\, P(a,x,x)$

2. $\quad " \vdash \forall xyz (p(x,y,z) \to p(f(x), y, f(z)))$ Assumption

3. $\quad " \vdash p(a,a,a)$ $\forall$-Elim on 1, assume $x=a$

4. $\quad " \vdash p(a,a,a) \to p(f(a), a, f(a)))$ $\forall$-Elim on 2 assume $x=y=z=a$

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5. " $\vdash p(f(a), a, f(a))$ $\to$-Elim on 3, 4

6. $\forall x\, p(a,x,x)$, $\forall xyz\, P(x,y,z) \to p(f(x),y,f(z)) \vdash p(f(a),a,f(a))$ conclusion

(b) 1. $\{\forall x(P(a,x,x)), \forall x,y,z(p(x,y,z) \to p(f(x),y,f(z)))\} \vdash \forall x\, P(a,x,x)$

2. " $\vdash \forall xyz\,(P(x,y,z) \to p(f(x),y,f(z)))$ Assumption

3. " $\vdash p(a,z,f(a)) \to p(f(a),z,f(f(a)))$ $\forall$-Elim on 2 assume $z=f(a)$, $y=z$, $x=a$

4. " $\vdash p(a,z,z)$ $\forall$-Elim on 1

5. " $\vdash p(a,z,f(a))$ =EqSub on 4

6. " $\vdash p(f(a),z,f(f(a)))$ $\to$-Elim on 3, 5

7. " $\vdash \exists z,\, p(f(a),z,f(f(a)))$ $\exists$-Intro, when $z=f(a)$

(c) 1. $\forall y\, Q(b,y)$, $\forall x \forall y(Q(x,y) \to Q(s(x),s(y))) \vdash \forall y\, Q(b,y)$

2. " $\vdash \forall xy(Q(x,y) \to Q(s(x),s(y)))$ Assumption

3. " $\vdash Q(b, s(b))$ $\forall$ Elim, assume $y=s(b)$ on 1

4. " $\vdash Q(b,s(b)) \to Q(s(b), s(s(b)))$ $\forall$-Elim on 2

5. " $\vdash Q(s(b), s(s(b)))$ $\to$-Elim on 3, 4

6. " $\vdash Q(b,s(b)) \wedge Q(s(b), s(s(b)))$ $\wedge$-Intro on 3, 5

7. " $\vdash \exists z(Q(b,z) \wedge Q(z, s(s(b))))$ $\exists$-Intro, assume $z=s(b)$

(d) 1. $\{S(x,y), \forall xyz(S(x,y) \wedge S(y,z) \to S(x,z)), \forall x, \neg S(x,x)\}$ Assumption
$\vdash \forall xyz(S(x,y) \wedge S(y,z) \to S(x,z))$

2. " $\vdash S(x,y)$ Assumption

3. " $\vdash \forall x, \neg S(x,x)$ Assumption

4. " $\vdash S(x,y) \wedge S(y,x) \to S(x,x)$ $\forall$-Elim on 1

5. " $\vdash \neg(S(x,x))$ $\forall x$-Elim on 3

6. " $\vdash \neg(S(x,y) \wedge S(y,x))$ Monus Pollus on 4 5

7. " $\vdash \cancel{S(x,y)} \to \neg S(y,x)$ resolution on 2, 6 $\to$-Intro

8. $\forall xyz(S(x,y) \wedge S(y,z) \to S(x,z))$, $\forall x\, \neg S(x,x) \vdash S(x,y) \to \neg S(y,x)$

9. $\forall xyz(S(x,y) \wedge S(y,z) \to S(x,z))$, $\forall x \neg S(x,x) \vdash \forall x,y(S(x,y) \to \neg S(y,x))$
$\forall$-Intro on 8