CALCULUS – PAST PAPERS (QUESTIONS & SOLUTIONS)

November 2008

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -3x^2$. (5)

8.2 Determine, using the rules of differentiation:

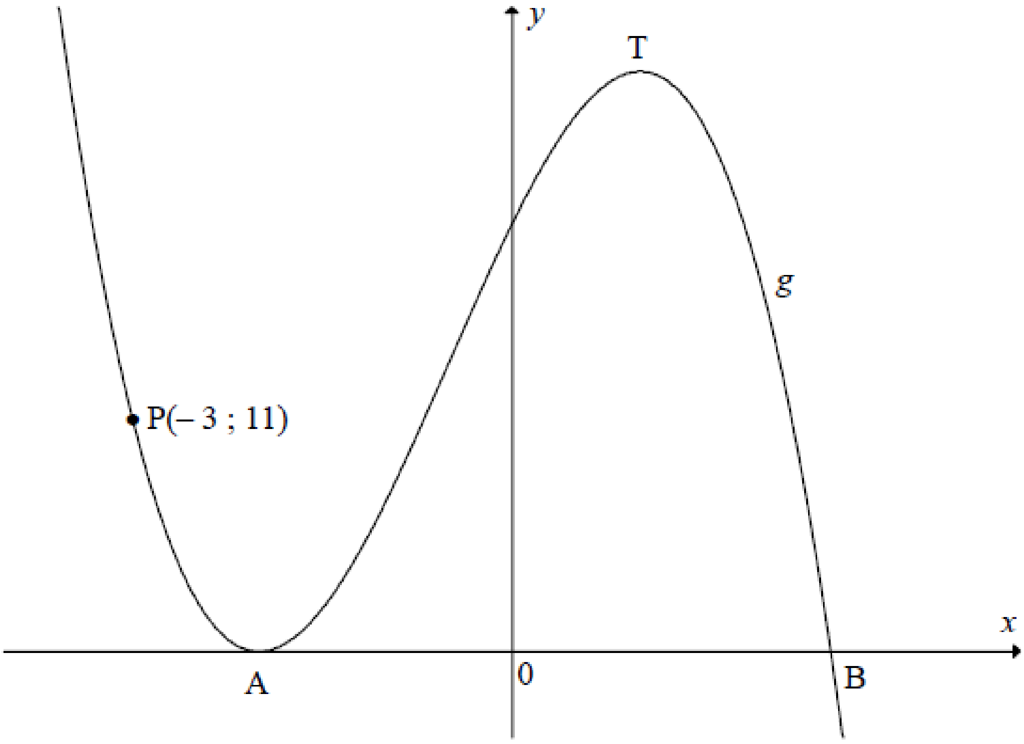
$$\frac{dy}{dx} \text{ if } y = \frac{\sqrt{x}}{2} - \frac{1}{6x^3}$$

Show ALL calculations. (3)

[8]

QUESTION 9

Sketched below is the graph of $g(x) = -2x^3 - 3x^2 + 12x + 20 = -(2x-5)(x+2)^2$
A and T are turning points of g. A and B are the x-intercepts of g.
P$(-3\ ;\ 11)$ is a point on the graph.



9.1 Determine the length of AB. (2)

9.2 Determine the x-coordinate of T. (4)

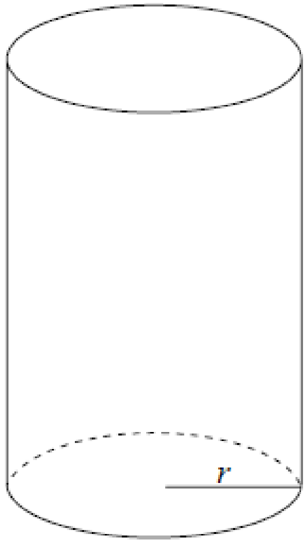
9.3 Determine the equation of the tangent to g at P$(-3\ ;\ 11)$, in the form $y = \ldots$ (5)

9.4 Determine the value(s) of k for which $-2x^3 - 3x^2 + 12x + 20 = k$ has three distinct roots. (3)

9.5 Determine the x-coordinate of the point of inflection. (4)
[18]

QUESTION 10

A drinking glass, in the shape of a cylinder, must hold 200 $m\ell$ of liquid when full.



10.1 Show that the height of the glass, h, can be expressed as $h = \dfrac{200}{\pi r^2}$. (2)

10.2 Show that the total surface area of the glass can be expressed as $S(r) = \pi r^2 + \dfrac{400}{r}$. (2)

10.3 Hence determine the value of r for which the total surface area of the glass is a minimum. (5)

[9]

QUESTION 8

8.1	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-3(x+h)^2-(-3x^2)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-3x^2-6xh-3h^2+3x^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{-6xh-3h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-6x-3h)}{h}$ $=\lim\limits_{h\to 0}(-6x-3h)$ $=-6x$	✓✓ definition $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ ✓ $-3(x+h)^2$ ✓substitution of $-3x^2$ ✓correct answer (5) Note: Penalty 1 for incorrect notation If a candidate has used the rules only: 0/5
8.2	$y=\dfrac{\sqrt{x}}{2}-\dfrac{1}{6x^3}$ $y=\dfrac{1}{2}.x^{\frac{1}{2}}-\dfrac{1}{6}.x^{-3}$ $\dfrac{dy}{dx}=\dfrac{1}{4}x^{-\frac{1}{2}}+\dfrac{3}{6}x^{-4}$ $\dfrac{dy}{dx}=\dfrac{1}{4}x^{-\frac{1}{2}}+\dfrac{1}{2}x^{-4}$ $\dfrac{dy}{dx}=\dfrac{1}{4\sqrt{x}}+\dfrac{1}{2x^4}$ Note: If removed coefficients, or moved the numbers from the denominator to the numerator: **Continued accuracy applies for each correct derivative** **Max 2/3** If leave out $\dfrac{dy}{dx}$ penalise 1 mark.	✓ Simplification ✓ $\dfrac{1}{4}x^{-\frac{1}{2}}$ ✓ $\dfrac{1}{2}x^{-4}$ or $\dfrac{3}{6}x^{-4}$ (3) **[8]**

QUESTION 9

9.1	$-(2x-5)(x+2)=0$ $x=\frac{5}{2} or -2$ AB = 4,5 units **OR** $-(2x-5)(x+2)=0$ $x=\frac{5}{2} or -2$ AB = $\sqrt{(2,5-(-2)^2+(0-0)^2}$ AB = 4,5 units	✓ $x=\frac{5}{2};x=-2$ ✓ answer (2) ✓ $x=\frac{5}{2};x=-2$ ✓ answer (2)
9.2	$g'(x)=0$ $-6x^2-6x+12=0$ $x^2+x-2=0$ $(x+2)(x-1)=0$ $x=-2$ or $x=1$ at T: $x=1$	✓ $g'(x)=0$ ✓ $g'(x)=-6x^2-6x+12$ ✓ factorisation ✓ answer (4)
9.3	$g'(x)=-6x^2-6x+12$ $g'(-3)=-6(-3)^2-6(-3)+12$ $g'(-3)=-54+18+12$ $g'(-3)=-24$ $y=ax+q$ $11=-24(-3)+q$ $q=-61$ $y=-24x-61$ **OR** $g'(x)=-6x^2-6x+12$ $g'(-3)=-6(-3)^2-6(-3)+12$ $g'(-3)=-54+18+12$ $g'(-3)=-24$ $y-11=-24(x+3)$ $y-11=-24x-72$ $y=-24x-61$	✓ $g'(-3)$ ✓ -24 ✓ method of setting up straight line equation ✓ substitution of point $(-3\ ;\ 11)$ ✓ answer in equation form (5) ✓ $g'(-3)$ ✓ -24 ✓ formula ✓ substitution of point $(-3\ ;\ 11)$ ✓ answer in equation form (5)

9.4	y-coordinate of T is $g(1) = -2(1)^3 - 3(1)^2 + 12(1) + 20$ $= 27$ T(1 ; 27) $\therefore 0 < k < 27$ **OR** $-2x^3 - 3x^2 + 12x + 20 = k$ $-2x^3 - 3x^2 + 12x + 20 - k = 0$ $-7 < 20 - k < 20$ $-27 < -k < 0$ $0 < k < 27$	✓ y-coordinate of T (27) ✓✓ answer (3) ✓ $-7 < 20 - k < 20$ ✓✓ answer (3) Answer Only: 3/3 $0 \le k \le 27 : 2/3$ $k > 0$: 1 / 3 $k < 27$: 1 / 3
9.5	$g'(x) = -6x^2 - 6x + 12$ $g''(x) = -12x - 6$ $12x + 6 = 0$ $x = -\frac{1}{2}$ $g''(x) < 0$ $\quad$ $g''(x) > 0$ ———•——— $x = -\frac{1}{2}$ $g''(x)$ changes sign at $x = -\frac{1}{2}$ $\therefore$ point of inflection at $x = -\frac{1}{2}$ **OR** Turning points A(-2;0); T(1;27) Now x co-ordinate of point of inflection is $x = -\frac{-2+1}{2} = -\frac{1}{2}$	✓ $-12x$ ✓ -6 ✓ $= 0$ ✓ $x = -\frac{1}{2}$ (4) ✓✓ points ✓✓ $x = -\frac{1}{2}$ (4) **[18]**

QUESTION 10

10.1	$V = \pi r^2 h$ $200 = \pi r^2 h$ $h = \dfrac{200}{\pi r^2}$	✓ formula ✓ substitution (2)
10.2	Surface Area $= 2\pi rh + \pi r^2$ $S(r) = \pi r^2 + \dfrac{200}{\pi r^2}.2\pi r$ $S(r) = \pi r^2 + \dfrac{400}{r}$	✓ formula ✓ substitution (2)
10.3	$S(r) = \pi r^2 + 400r^{-1}$ $\dfrac{dS}{dr} = 2\pi r - 400r^{-2}$ At minimum : $\dfrac{dS}{dr} = 0$ $2\pi r - \dfrac{400}{r^2} = 0$ $\pi r^3 - 200 = 0$ $r^3 = \dfrac{200}{\pi}$ $r = 3{,}99$ cm	✓ exponents correct ✓ $\dfrac{dS}{dr} = 2\pi r - 400r^{-2}$ ✓ $\dfrac{dS}{dr} = 0$ ✓ $r^3 = \dfrac{200}{\pi}$ ✓ $r = 3{,}99$ or $r = \sqrt[3]{\dfrac{200}{\pi}}$ (5) Note: If did not put = 0, penalise 1 mark If notation is $\dfrac{dy}{dx}$, ignore notation. **[9]**

November 2009

QUESTION 10

10.1 Differentiate $f(x)$ from first principles if $f(x) = -2x^2 + 3$. (5)

10.2 Evaluate: $\dfrac{dy}{dx}$ if $y = x^2 - \dfrac{1}{2x^3}$ (2)
[7]

QUESTION 11

Given: $f(x) = -x^3 + x^2 + 8x - 12$

11.1 Calculate the x-intercepts of the graph of f. (5)

11.2 Calculate the coordinates of the turning points of the graph of f. (5)

11.3 Sketch the graph of f, showing clearly all the intercepts with the axes and turning points. (3)

11.4 Write down the x-coordinate of the point of inflection of f. (2)

11.5 Write down the coordinates of the turning points of $h(x) = f(x) - 3$. (2)
[17]

QUESTION 12

A tourist travels in a car over a mountainous pass during his trip. The height above sea level of the car, after t minutes, is given as $s(t) = 5t^3 - 65t^2 + 200t + 100$ metres. The journey lasts 8 minutes.

12.1 How high is the car above sea level when it starts its journey on the mountainous pass? (2)

12.2 Calculate the car's rate of change of height above sea level with respect to time, 4 minutes after starting the journey on the mountainous pass. (3)

12.3 Interpret your answer to QUESTION 12.2. (2)

12.4 How many minutes after the journey has started will the rate of change of height with respect to time be a minimum? (3)
[10]

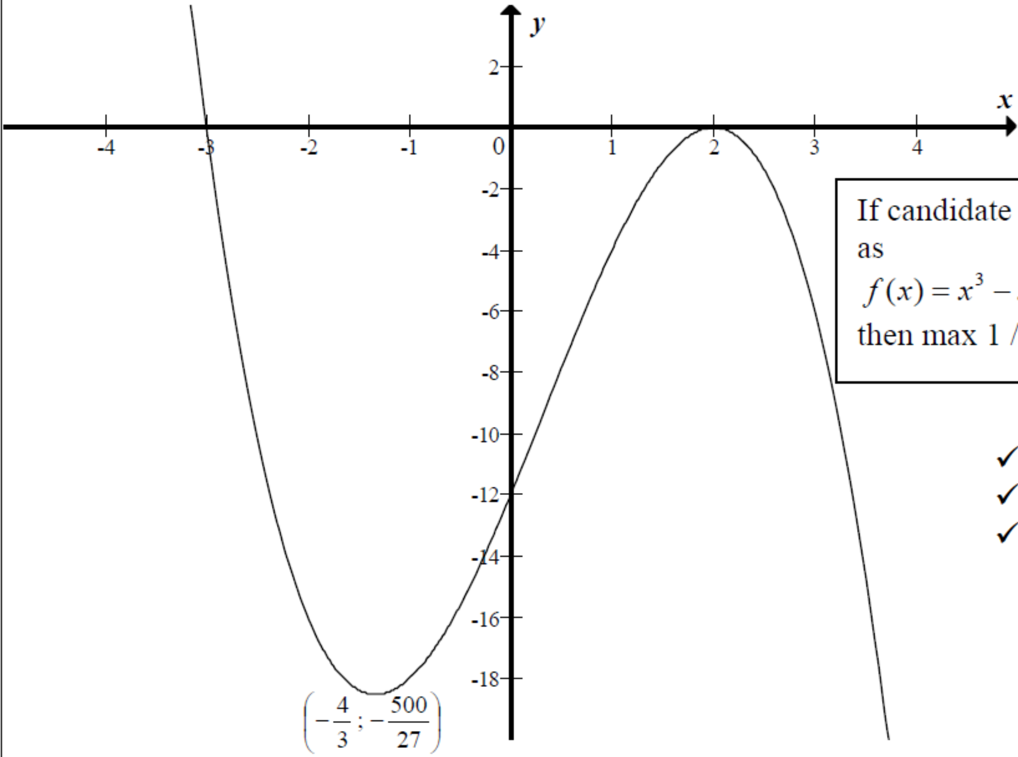
- Consistent Accuracy will apply as a general rule.
- If a candidate does a question twice and does not delete either, mark the FIRST attempt.
- If a candidate does a question, crosses it out and does not re-do it, mark the deleted attempt.

QUESTION 10

10.1	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{-2(x+h)^2+3-(-2x^2+3)}{h}$ $=\lim_{h\to 0}\frac{-2x^2-4xh-2h^2+3+2x^2-3}{h}$ $=\lim_{h\to 0}\frac{h(-4x-2h)}{h}$ $=\lim_{h\to 0}(-4x-2h)$ $=-4x$ **NOTE:** Penalty 1 mark only for incorrect notation (lim missing or = in incorrect place) Answer only : 0 / 5 Cannot give mark for answer if the answer is incorrect according to the working out, even if the answer is given as – 4*x*.	✓ $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ ✓ $-2(x+h)^2+3$ ✓ simplification ✓ simplification ✓ answer (5)
10.2	$y=x^2-\frac{1}{2x^3}$ $y=x^2-\frac{1}{2}x^{-3}$ $\frac{dy}{dx}=2x+\frac{3}{2}x^{-4}$ **OR** $\frac{dy}{dx}=2x+\frac{3}{2x^4}$ **OR** $\frac{dy}{dx}=2x-(-3)\frac{1}{2}x^{-4}$	✓ $2x$ ✓ $+\frac{3}{2}x^{-4}$ (2) [7]

QUESTION 11

11.1	$0 = -x^3 + x^2 + 8x - 12$ $x^3 - x^2 - 8x + 12 = 0$ $(x-2)(x^2 + x - 6) = 0$ $(x-2)(x-2)(x+3) = 0$ $x = 2$ or $x = -3$ x-intercepts are $(2\ ;\ 0)$ and $(-3\ ;\ 0)$ **OR** $0 = -x^3 + x^2 + 8x - 12$ $x^3 - x^2 - 8x + 12 = 0$ $(x+3)(x^2 - 4x + 4) = 0$ $(x+3)(x-2)(x-2) = 0$ $x = 2$ or $x = -3$ x-intercepts are $(2\ ;\ 0)$ and $(-3\ ;\ 0)$	✓ any one of factors ✓ quadratic factor ✓ linear factors ✓✓ x-answers (5)
11.2	$f'(x) = -3x^2 + 2x + 8$ $0 = 3x^2 - 2x - 8$ $0 = (x-2)(3x+4)$ $x = 2$ or $x = -\frac{4}{3}$ turning points are $(2\ ;\ 0)$ and $\left(-\frac{4}{3}\ ;\ -\frac{500}{27}\right)$ OR $(2\ ;\ 0)$ and $(-1{,}33\ ;\ -18{,}52)$ **NOTE:** If = 0 is omitted in 11.2: penalty 1 mark If not in coordinate form but coordinates implied: OK	✓ $f'(x) = 0$ ✓ $-3x^2 + 2x + 8 = 0$ or $3x^2 - 2x - 8 = 0$ ✓ factors ✓ x-values ✓ y-values (5)

11.3	 If candidate used function as $f(x)=x^3-x^2-8x+12$ then max 1 / 3	✓ shape ✓ y-intercept ✓ turning pts **(3)**
11.4	$f''(x)=0$ $6x-2=0$ $x=\frac{1}{3}$ or $f''(x)=0$ $-6x+2=0$ $x=\frac{1}{3}$ **OR** $x=\frac{2-\frac{4}{3}}{2}$ $x=\frac{1}{3}$ **Note:** If write down $f''(x)=6x-2$ or $f''(x)=-6x+2$ then 1 / 2	✓ method ✓ answer Answer only: Full marks (2)
11.5	$(2\ ;\ -3)$ and $\left(-\frac{4}{3};-\frac{581}{27}\right)$ **OR** $(2\ ;\ -3)$ and $(-1{,}33\ ;-21{,}52)$	✓✓ each answer (2) **[17]**

QUESTION 12

12.1	$s(0) = 5(0)^3 - 65(0)^2 + 200(0) + 100$ $= 100$ metres **NOTE:** If subs $t = 8$, then answer = 100: 0 / 2	✓ $t = 0$ ✓ answer (2) Answer only: full marks
12.2	$s(t) = 5t^3 - 65t^2 + 200t + 100$ $s'(t) = 15t^2 - 130t + 200$ $s'(4) = 15(4)^2 - 130(4) + 200$ $= -80$ metres per minute **NOTE:** If used average rate of change between $t = 0$ and $t = 4$: 0 / 3 If subs $t = 4$ into $s(t)$: 0 / 3	✓ $s'(t) = 15t^2 - 130t + 200$ ✓ substitution $t = 4$ ✓ answer (-80) (3)
12.3	The height of the car above sea level is decreasing at 80 metres per minute and the car is travelling downwards hence it is a negative rate of change. **OR** The **vertical** velocity of the car at $t = 4$ is 80 metres per minute. **NOTE:** Mark this CA even if answer to QUESTION 12.2 is completely inaccurate.	✓ speed 80 metres per minute ✓ downwards (2)
12.4	$s'(t) = 15t^2 - 130t + 200$ $s''(t) = 30t - 130$ $130 = 30t$ $t = 4{,}3\dot{3}$ minutes **OR** $t = \dfrac{-(-130)}{2(15)}$ $t = 4{,}3\dot{3}$ minutes	✓ $s''(t) = 30t - 130$ ✓ $s''(t) = 0$ ✓ answer (3) **[10]**

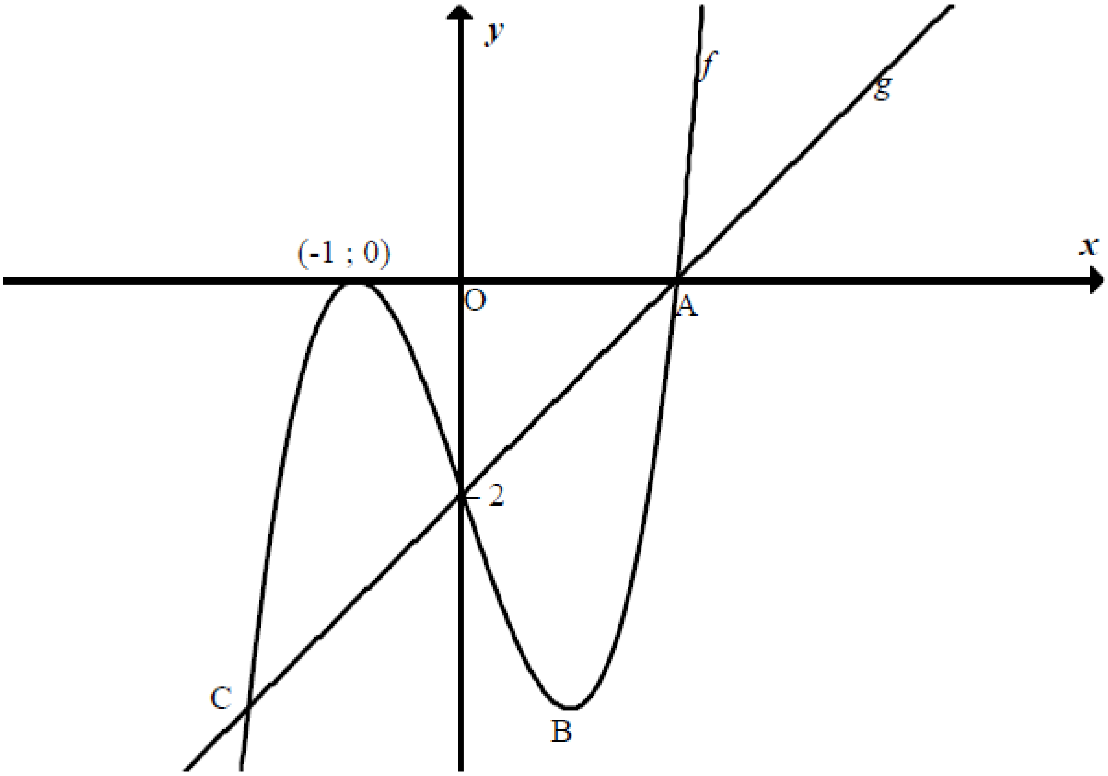
QUESTION 10

10.1 Differentiate f from first principles: $f(x) = \frac{1}{x}$ (4)

10.2 Use the rules of differentiation to determine $\frac{dy}{dx}$ if $y = (2-5x)^2$ (3)
[7]

QUESTION 11

The graph below represents the functions f and g with $f(x) = ax^3 - cx - 2$ and $g(x) = x - 2$.
A and $(-1\ ;\ 0)$ are the x-intercepts of f. The graphs of f and g intersect at A and C.



11.1	Determine the coordinates of A.	(1)
11.2	Show by calculation that $a = 1$ and $c = -3$.	(4)
11.3	Determine the coordinates of B, a turning point of f.	(3)
11.4	Show that the line BC is parallel to the x-axis.	(7)
11.5	Find the x-coordinate of the point of inflection of f.	(2)
11.6	Write down the values of k for which $f(x) = k$ will have only ONE root.	(3)
11.7	Write down the values of x for which $f'(x) < 0$.	(2) **[22]**

QUESTION 12

A wire, 4 metres long, is cut into two pieces. One is bent into the shape of a square and the other into the shape of a circle.

12.1 If the length of wire used to make the circle is x metres, write in terms of x the length of the sides of the square in metres. (1)

12.2 Show that the sum of the areas of the circle and the square is given by $f(x)=\left(\frac{1}{16}+\frac{1}{4\pi}\right)x^2-\frac{x}{2}+1$ square metres. (4)

12.3 How should the wire be cut so that the sum of the areas of the circle and the square is a minimum? (3)

[8]

QUESTION 10

10.1	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$ $=\lim_{h\to 0}\frac{\frac{x-(x+h)}{x(x+h)}}{h}$ $=\lim_{h\to 0}\frac{-h}{x(x+h)}\times\frac{1}{h}$ $=\lim_{h\to 0}-\frac{1}{x(x+h)}$ $=-\frac{1}{x^2}$	✓ substitution into correct formula ✓ expansion ✓ simplification ✓ answer (4)
10.2	$y=(2-5x)^2$ $y=4-20x+25x^2$ $\frac{dy}{dx}=-20+50x$ **OR** $y=(2-5x)^2$ By the chain rule $\frac{dy}{dx}=(2)(2-5x)(-5)$ $=-20+50x$	✓ simplification ✓✓ answers (3) ✓ simplification ✓✓ answers (3) [7]

QUESTION 11

11.1	$0 = x - 2$ $x = 2$ $A(2\ ;\ 0)$	✓ answer (1)
11.2	$f(-1) = 0: -a + c = 2$ $f(2) = 0: \quad 8a - 2c = 2$ $a = 1, \quad c = 3$ **OR** $a(x+1)(x+1)(x-2) = 0$ $a(0+1)(0+1)(0-2) = -2$ $-2a = -2$ $a = 1$ $f(x) = (x^2 + 2x + 1)(x - 2)$ $= x^3 - 3x - 2$ $c = -3$	✓ $-a + c = 2$ ✓ $8a - 2c = 2$ ✓ $a = 1$ ✓ $c = 3$ ✓ factors ✓ substitution ✓ a ✓ $c = -3$ (4)
11.3	$f'(x) = 0$ $3x^2 - 3 = 0$ $x^2 - 1 = 0$ $(x+1)(x-1) = 0$ $B(1\ ;\ -4)$	✓ $f'(x) = 0$ ✓ $x^2 - 1$ ✓ answer (3)
11.4	$x - 2 = x^3 - 3x - 2$ $0 = x^3 - 4x$ $0 = x(x^2 - 4)$ $0 = x(x-2)(x+2)$ $x_C = -2, \ y_C = (-2)^2 - 3(-2) - 2 = -4$ $C(-2\ ;\ -4)$ $m_{BC} = \dfrac{-4-(-4)}{1-(-2)}$ $= 0$ BC is parallel to the x-axis. **OR** Following from $C(-2\ ;\ -4)$, B and C have the same y – coordinate, viz. -4. So BC is parallel to the x-axis. **OR**	✓ equating f and g ✓ standard form ✓ factors ✓ $x_C = -2$ ✓ $y_C = -4$ ✓ $m = 0$ ✓ conclusion (7) (7)

	$(x-2)=(x-2)(x+1)^2$ $\therefore (x+1)^2=1$ for $x \neq 2$ $\therefore x+1=\pm 1$ $\therefore x=0$ *or* $x=-2$ $y=\ -4$	(7)
11.5	$f''(x)=0$ $6x=0$ $x=0$	✓ $f''(x)=0$ ✓ answer (2)
11.6	$k<-4$ or $k>0$	✓✓ answer ✓ or (3)
11.7	$f'(x)<0$ $-1<x<1$ **OR** $3(x^2-1)<0$ if $(x+1)(x-1)<0$ $-1<x<1$	✓✓answer (2) ✓✓answer (2) **[22]**

This alternative memo must be used for students who follow through using c = – 3. This marking memo must be used independently of the one provided in the existing memorandum.

QUESTION 11

11.1	$0=x-2$ $x=2$ A(2 : 0)	✓ answer (1)
11.2	$f'(-1)=0$; $-a+c=2$ $f(2)=0$; $8a-2c=2$	✓✓ $f'(-1)=0$; $-a+c=2$ ✓✓ $f(2)=0$; $8a-2c=2$ (4)
11.3	$f(x)=x^2+3x-2$ $f'(x)=0$ $3x^2+3=0$	✓✓ $f'(x)=0$ ✓ $3x^2+3=0$ (3)

11.4	**If not attempted (3 marks)** ***OR*** To calculate C $x-2=x^3+3x-2$ $x^3+2x=0$ $x(x^2+2)=0$ $x=0$ $y=-2$	✓ ✓ ✓ $x-2=x^3+3x-2$ ✓ $x^3+2x=0$ ✓ $x(x^2+2)=0$ ✓ $x=0$ ✓ $y=-2$ (7)
11.5	$x=0$ (any method used)	✓✓ answer (2)
11.6	**Not Attempted : 0 marks** OR $k>0$	✓✓✓ answer (3)
11.7	If x > -1, **Maximum of 1 Mark** ***OR*** x is between -1 and (x – value) of B $f(x)=x^3+3\text{x}-2$ $f'(x)=3x^2+3<0$	✓✓ $f'(x)=3x^2+3<0$ (2) **[22]**

QUESTION 12

12.1	Length of sides of square $= \dfrac{4-x}{4} = 1 - \dfrac{x}{4}$	✓ answer (1)
12.2	$x = 2\pi r$ $r = \dfrac{x}{2\pi}$ Areas $= \left(\dfrac{4-x}{4}\right)^2 + \pi\left(\dfrac{x}{2\pi}\right)^2$ $= \dfrac{16-8x+x^2}{16} + \dfrac{x^2}{4\pi}$ $= 1 - \dfrac{1}{2}x + \left(\dfrac{1}{16} + \dfrac{1}{4\pi}\right)x^2$ **OR** $x = 2\pi r$ $r = \dfrac{x}{2\pi}$ $\left(1 - \dfrac{x}{4}\right)^2 + \pi\left(\dfrac{x}{2\pi}\right)^2$ $1 - \dfrac{1}{2}x + \dfrac{x^2}{16} + \dfrac{x^2}{4\pi}$ $1 - \dfrac{1}{2}x + \left(\dfrac{\pi+4}{16\pi}\right)x^2$	✓ $r = \dfrac{x}{2\pi}$ ✓ sum of areas ✓ simplification ✓ simplification (4) ✓ r ✓ sum of areas ✓ simplification ✓ simplification (4)
12.3	$x = \dfrac{-b}{2a}$ $= \dfrac{\frac{1}{2}}{2\left(\frac{\pi+4}{16\pi}\right)}$ $= 1{,}76$ meter **OR**	✓✓ substitution ✓ answer (3)

QUESTION 8

8.1 Differentiate $g(x) = x^2 - 5$ from first principles. (5)

8.2 Evaluate $\frac{dy}{dx}$ if $y = \frac{x^6}{2} + 4\sqrt{x}$. (3)

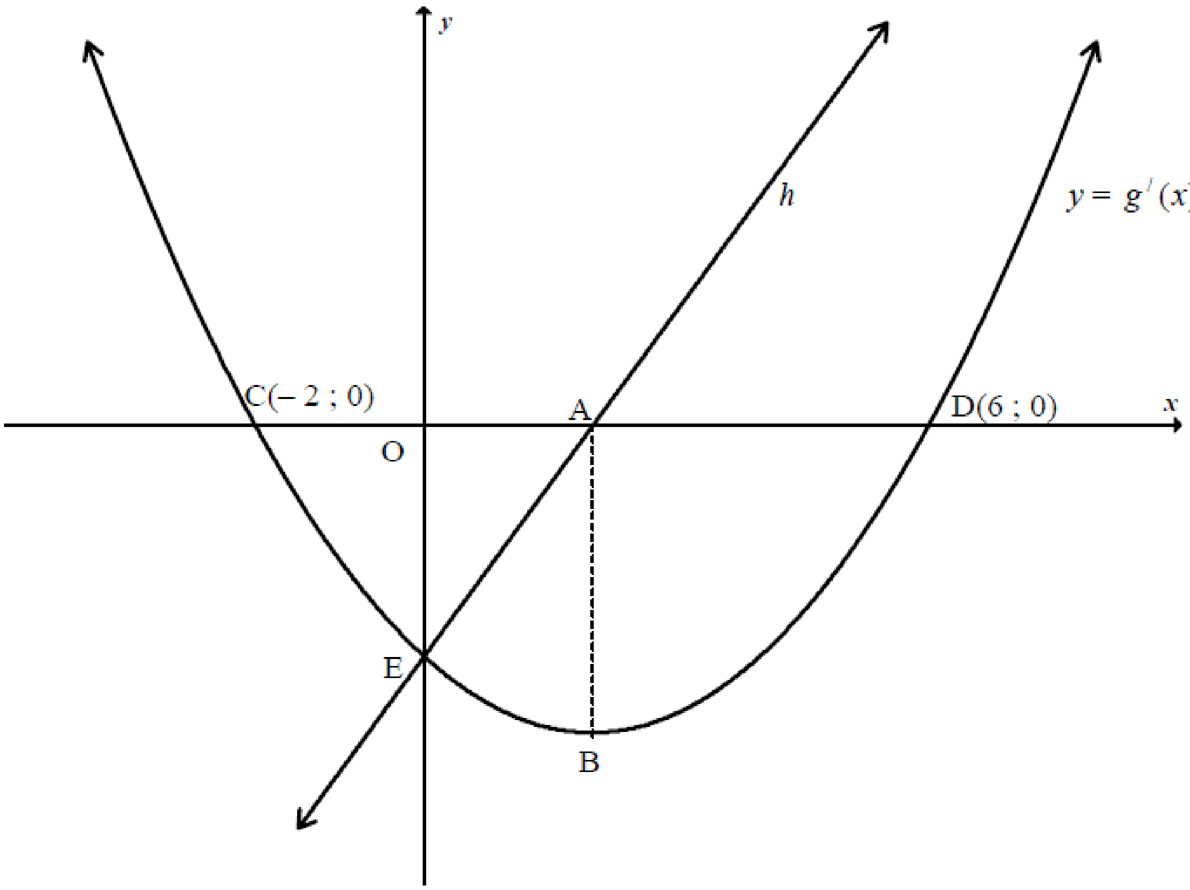
8.3 A function $g(x) = ax^2 + \frac{b}{x}$ has a minimum value at $x = 4$. The function value at $x = 4$ is 96. Calculate the values of a and b. (6)

[14]

QUESTION 9

The graphs of $y = g'(x) = ax^2 + bx + c$ and $h(x) = 2x - 4$ are sketched below. The graph of $y = g'(x) = ax^2 + bx + c$ is the derivative graph of a cubic function g.

The graphs of h and g' have a common y-intercept at E.
C$(-2\,;\,0)$ and D$(6\,;\,0)$ are the x-intercepts of the graph of g'.
A is the x-intercept of h and B is the turning point of g'.
AB || y-axis.

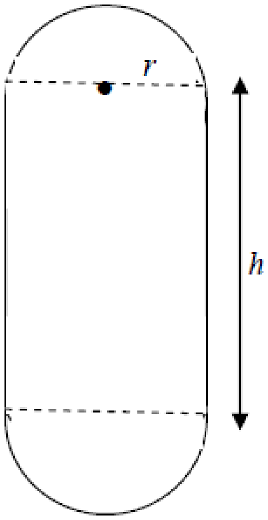


9.1	Write down the coordinates of E.	(1)
9.2	Determine the equation of the graph of g' in the form $y = ax^2 + bx + c$.	(4)
9.3	Write down the x-coordinates of the turning points of g.	(2)
9.4	Write down the x-coordinate of the point of inflection of the graph of g.	(2)
9.5	Explain why g has a local maximum at $x = -2$.	(3)
		[12]

QUESTION 10

A satellite is to be constructed in the shape of a cylinder with a hemisphere at each end. The radius of the cylinder is r metres and its height is h metres (see diagram below). The outer surface area of the satellite is to be coated with heat-resistant material which is very expensive.

The volume of the satellite has to be $\frac{\pi}{6}$ cubic metres.



Outer surface area of a sphere = $4\pi r^2$
Curved surface area of a cylinder = $2\pi rh$
Volume of a sphere = $\frac{4}{3}\pi r^3$
Volume of a cylinder = $\pi r^2 h$

10.1 Show that $h = \frac{1}{6r^2} - \frac{4r}{3}$. (3)

10.2 Hence, show that the outer surface area of the satellite can be given as $S = \frac{4\pi r^2}{3} + \frac{\pi}{3r}$. (3)

10.3 Calculate the minimum outer surface area of the satellite. (6)

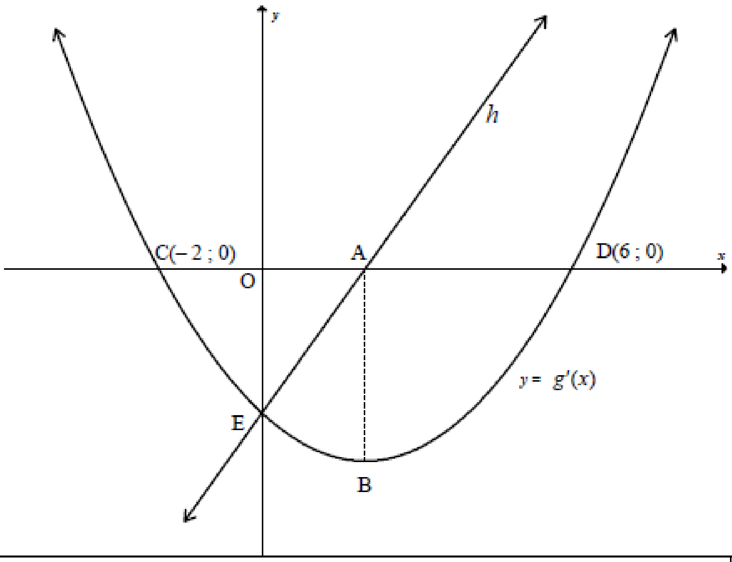
[12]

QUESTION 8

8.1	$g(x)=x^2-5$ $g'(x)=\lim\limits_{h\to 0}\dfrac{g(x+h)-g(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{(x+h)^2-5-(x^2-5)}{h}$ $=\lim\limits_{h\to 0}\dfrac{x^2+2xh+h^2-5-x^2+5}{h}$ $=\lim\limits_{h\to 0}\dfrac{2xh+h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(2x+h)}{h}$ $=\lim\limits_{h\to 0}(2x+h)$ $=2x$ **Note:** If the notation is incorrect, penalty 1 mark If candidate subtracts and gets $x^2+2xh+h^2-5-x^2-5$ in the numerator and then candidate corrects themselves, max 2 / 5 Answer only: 0 / 5	✓ formula ✓ substitution ✓ expansion ✓ $2x+h$ ✓ answer (5)
	OR $g(x)=x^2-5$ $g'(x)=\lim\limits_{h\to 0}\dfrac{g(x+h)-g(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{(x+h)^2-5-(x^2-5)}{h}$ $=\lim\limits_{h\to 0}\dfrac{(x+h)^2-x^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{(x+h+x)(x+h-x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(2x+h)}{h}$ $=\lim\limits_{h\to 0}(2x+h)$ $=2x$	✓ formula ✓ substitution ✓ expansion ✓ $2x+h$ ✓ answer (5)
8.2	$y=\dfrac{x^6}{2}+4\sqrt{x}$ $y=\dfrac{1}{2}x^6+4x^{\frac{1}{2}}$ $\dfrac{dy}{dx}=3x^5+2x^{-\frac{1}{2}}$ **Note:** If $\dfrac{dy}{dx}$ or y' is left out, penalty 1 mark If a candidate shows evidence of how to differentiate from an incorrect function which involves breakdown, then max 1 / 3	✓ $+4x^{\frac{1}{2}}$ ✓ $3x^5$ ✓ $2x^{-\frac{1}{2}}$ (3)

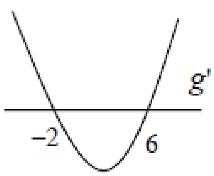
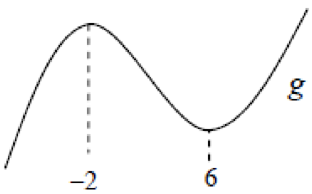
8.3	$g(x) = ax^2 + \frac{b}{x}$ $g(x) = ax^2 + bx^{-1}$ $g'(x) = 2ax - bx^{-2}$ $0 = 2a(4) - \frac{b}{(4)^2}$ $8a = \frac{b}{16}$ $b = 128a$ $96 = a(4)^2 + \frac{b}{4}$ $96 = 16a + \frac{1}{4}(128a)$ $96 = 48a$ $a = 2$ $b = 256$ **Note:** In the equation $g'(x) = 0$; $= 0$ must be shown in the equation. **OR** $g'(x) = 2ax - \frac{b}{x^2}$ $g'(4) = 8a - \frac{b}{16} = 0$ $g(4) = 16a + \frac{b}{4} = 96$ $32a - \frac{b}{4} = 0$ $48a = 96$ $a = 2$ $b = 256$	✓ $g'(x) = 2ax - bx^{-2}$ ✓ $0 = g'(x)$ ✓ $2a(4) - \frac{b}{(4)^2}$ ✓ subs $(4\ ;\ 96)$ ✓ $a = 2$ ✓ $b = 256$ (6) ✓ $g'(x) = 2ax - \frac{b}{x^2}$ ✓ $g'(4) = 8a - \frac{b}{16}$ ✓ $g'(x) = 0$ ✓ $g(4) = 16a + \frac{b}{4} = 96$ ✓ $a = 2$ ✓ $b = 256$ (6) **[14]**

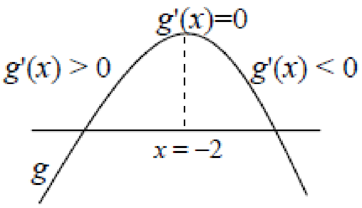
QUESTION 9



9.1	The y-intercept of g is E(0 ; – 4) **OR** $x = 0$ and $y = -4$	✓ answer (1)
9.2	$y = a(x+2)(x-6)$ $-4 = a(0+2)(0-6)$ $-4 = -12a$ $a = \frac{1}{3}$ $y = \frac{1}{3}(x+2)(x-6)$ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$ **OR** $g'(0) = -4 = c$ $g'(x) = ax^2 + bx - 4$ $g'(-2) = 0$ $4a - 2b - 4 = 0$ $b = 2a - 2$ $g''(2) = 0$ $2a(2) + b = 0$ $b = -4a$ $2a - 2 = -4a$ $a = \frac{1}{3}$ $b = -\frac{4}{3}$ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$	✓ setting up of equation ✓ subs (0 ; – 4) ✓ $a = \frac{1}{3}$ ✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$ (4) ✓ substitution $x = -2$ and $g'(x) = 0$ ✓ $g''(2) = 0$ ✓ $a = \frac{1}{3}$ ✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$ (4)

	OR $c=-4$ $4a-2b-4=0$ $36a+6b-4=0$ $48a-16=0$ $a=\frac{1}{3}$ $b=-\frac{4}{3}$ $y=\frac{1}{3}x^2-\frac{4}{3}x-4$	✓ setting up of equation ✓ simultaneous equation ✓ $a=\frac{1}{3}$ ✓ $y=\frac{1}{3}x^2-\frac{4}{3}x-4$ (4)
	OR $y=a(x+2)(x-6)$ $=a(x^2-4x-12)$ $=ax^2-4ax-12a$ $-12a=-4$ $a=\frac{1}{3}$ $y=\frac{1}{3}x^2-\frac{4}{3}x-4$	✓ setting up of equation ✓ $ax^2-4ax-12a$ ✓ $a=\frac{1}{3}$ ✓ $y=\frac{1}{3}x^2-\frac{4}{3}x-4$ (4)
	OR $\frac{dy}{dx}=2ax+b$ $0=2a(2)+b$ $b=-4a$ EITHER subs (6 ; 0) $0=36a+6b-4$ $4=36a+6b$ $2=18a+3b$ $2=18a+3(-4a)$ $2=6a$ $a=\frac{1}{3}$ $b=-\frac{4}{3}$ $y=\frac{1}{3}x^2-\frac{4}{3}x-4$ OR $0=4a-2b-4$ $0=4a-2(-4a)-4$ $12a=4$ $a=\frac{1}{3}$ $b=-\frac{4}{3}$ $y=\frac{1}{3}x^2-\frac{4}{3}x-4$	✓ $b=-4a$ ✓ simultaneous equation ✓ $a=\frac{1}{3}$ ✓ $y=\frac{1}{3}x^2-\frac{4}{3}x-4$ (4)

9.3	At turning point $g'(x) = 0$ $x = -2$ and $x = 6$ **Answer only:** Full marks If only 1 value given, max 1 / 2	✓ $g'(x) = 0$ ✓ $x = 6$ and $x = -2$ (2)
9.4	$x = \frac{-2+6}{2}$ $x = 2$ **Note:** Answer only Full marks **OR** x-value of point of inflection of g is at A. $g''(x) = 0$ $\frac{2x}{3} - \frac{4}{3} = 0$ $2x - 4 = 0$ $2x = 4$ $x = 2$ **OR** $x = -\frac{b}{2a}$ $x = \frac{\frac{4}{3}}{2\left(\frac{1}{3}\right)}$ $x = 2$ **OR** $g'(x) = \frac{1}{3}(x-2)^2 - \frac{16}{3}$ $x = 2$	✓ $x = \frac{-2+6}{2}$ ✓ answer (2) ✓ $2x - 4 = 0$ ✓ answer (2) ✓ $x = \frac{\frac{4}{3}}{2\left(\frac{1}{3}\right)}$ ✓ answer (2) ✓ $g'(x) = \frac{1}{3}(x-2)^2 - \frac{16}{3}$ ✓ answer (2)
9.5	$g'(x) > 0$ for $x < -2$, so g is increasing for $x < -2$. $g'(x) < 0$ for $x > -2$, so g is decreasing for $x > -2$. $\therefore$ g has a local maximum at $x = -2$ because the graph is increasing followed by decreasing **OR**   $\therefore$ g has a local maximum at $x = -2$ **OR**	✓ $g'(x) > 0$ ✓ g is incr for $x < -2$ ✓ g is decr for $x > -2$ (3) ✓ $g'(x) > 0$ for $x < -2$ ✓ $g'(x) < 0$ for $x > -2$ ✓ max at $x = -2$ (3)

		✓ $g^{/}(x)>0$ for $x<-2$ ✓ $g^{/}(x)<0$ for $x>-2$ ✓ max at $x=-2$ (3)
	OR $g'(-2)=0$ $g''(-2)<0$ so graph is concave down at $x=-2$, so g has a local maximum	✓ $g'(-2)=0$ ✓ $g''(-2)<0$ ✓ max at $x=-2$ (3) **[12]**

QUESTION 10

10.1	$V=\pi r^2h+2\times\frac{1}{2}\times\frac{4}{3}\pi r^3$ $V=\pi r^2h+\frac{4}{3}\pi r^3$ $\frac{\pi}{6}=\pi r^2h+\frac{4}{3}\pi r^3$ $\pi r^2h=\frac{\pi}{6}-\frac{4}{3}\pi r^3$ $h=\frac{\pi}{6\pi r^2}-\frac{4\pi r^3}{3\pi r^2}$ $h=\frac{1}{6r^2}-\frac{4r}{3}$	✓ volume equation ✓ substitution of $\frac{\pi}{6}$ ✓ $h=\frac{\pi}{6\pi r^2}-\frac{4\pi r^3}{\pi r^2}$ (3)
10.2	$S=2\times2\pi r^2+2\pi rh$ $S=4\pi r^2+2\pi rh$ $S=4\pi r^2+2\pi r\left(\frac{1}{6r^2}-\frac{4r}{3}\right)$ $S=4\pi r^2+\frac{\pi}{3r}-\frac{8\pi r^2}{3}$ $=\frac{4}{3}\pi r^2+\frac{\pi}{3r}$	✓ surface area equation ✓ substitution of h ✓ simplification (3)
10.3	$S=\frac{4}{3}\pi r^2+\frac{\pi}{3}r^{-1}$ $\frac{dS}{dr}=\frac{8\pi r}{3}-\frac{\pi}{3r^2}=0$ $8r=\frac{1}{r^2}$ $8r^3=1$ $r=\frac{1}{2}$ Then $S=\frac{4}{3}\pi\left(\frac{1}{2}\right)^2+\frac{\pi}{3}(2)$ $S=\pi$ square metres $=3{,}14$ square metres	✓ $\frac{\pi}{3}r^{-1}$ ✓ $\frac{dS}{dr}=\frac{\pi}{3}\left(8r-\frac{1}{r^2}\right)$ or $\frac{dS}{dr}=\frac{\pi}{3}\left(8r-r^{-2}\right)$ ✓ $\frac{dS}{dr}=0$ ✓ $8r=\frac{1}{r^2}$ ✓ $r=\frac{1}{2}$ ✓ S = π (6) **[12]**

QUESTION 9

9.1 Use the definition to differentiate $f(x)=1-3x^2$. (Use first principles.) (4)

9.2 Calculate $D_x\left[4-\frac{4}{x^3}-\frac{1}{x^4}\right]$. (3)

9.3 Determine $\frac{dy}{dx}$ if $y=\left(1+\sqrt{x}\right)^2$. (3)

[10]

QUESTION 10

Given: $g(x)=(x-6)(x-3)(x+2)$

10.1 Calculate the y-intercept of g. (1)

10.2 Write down the x-intercepts of g. (2)

10.3 Determine the turning points of g. (6)

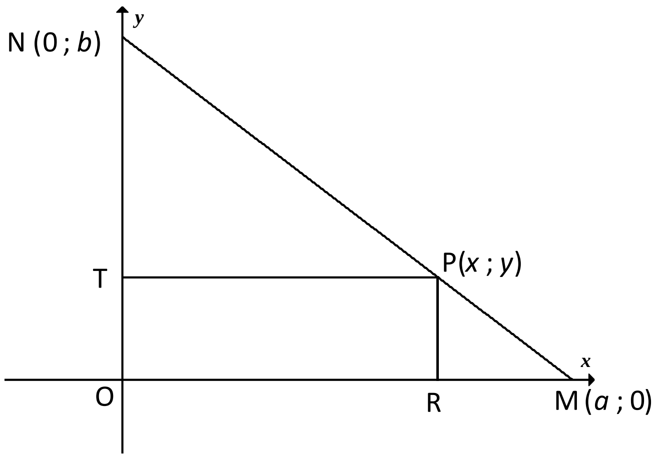
10.4 Sketch the graph of g on DIAGRAM SHEET 2. (4)

10.5 For which values of x is $g(x).g^{/}(x)<0$? (3)

[16]

QUESTION 11

A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land PTOR to his daughter, giving her the freedom to choose P anywhere along the boundary MN. Let OM = a, ON = b and P(x ; y) be any point on MN.



11.1 Determine an equation of MN in terms of a and b. (2)

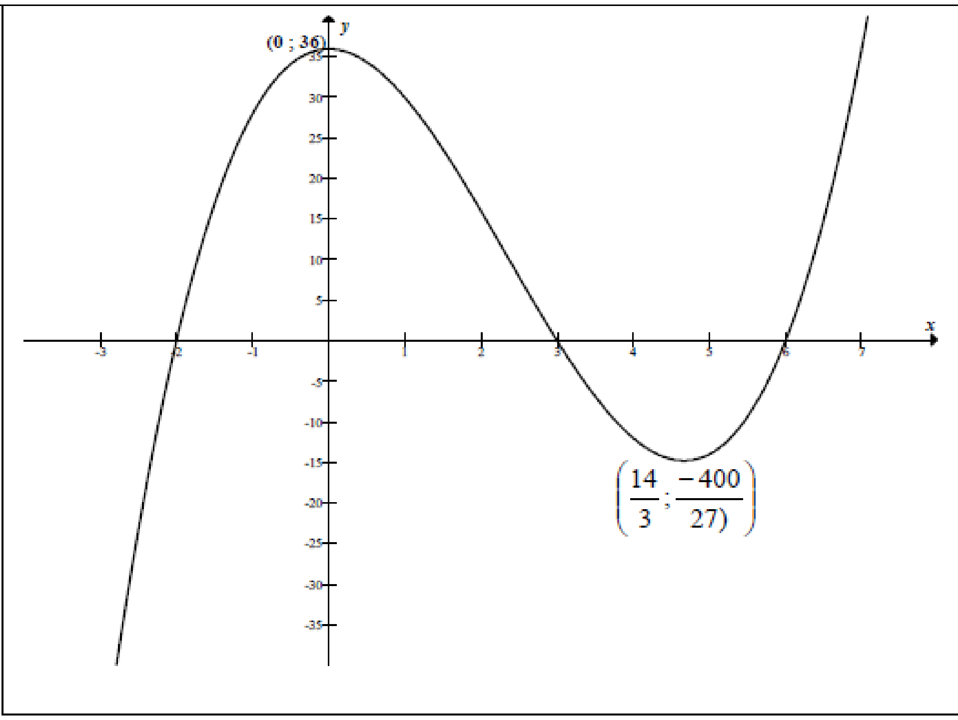
11.2 Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN. (6)

[8]

QUESTION 9

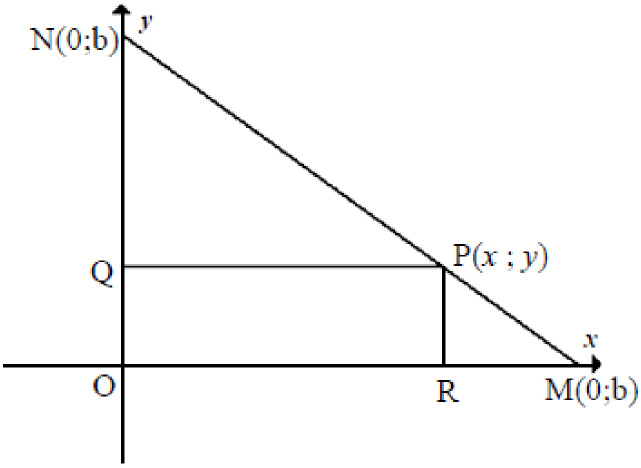
9.1	$f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0}\dfrac{1-3(x+h)^2-(1-3x^2)}{h}$ $= \lim_{h\to 0}\dfrac{1-3x^2-6xh-3h^2-1+3x^2}{h}$ $= \lim_{h\to 0}\dfrac{-6xh-3h^2}{h}$ $= \lim_{h\to 0}\dfrac{h(-6x-3h)}{h}$ $= \lim_{h\to 0}(-6x-3h)$ $= -6x$	✓ substitution into formula ✓ $1-3x^2-6xh-3h^2$ ✓ $h(-6x-3h)$ ✓ answer (4)
9.2	$D_x\left[4-\dfrac{4}{x^3}-\dfrac{1}{x^4}\right]$ $= D_x\left[4-4x^{-3}-x^{-4}\right]$ $= 12x^{-4}+4x^{-5}$	✓ simplification ✓✓ answer (3)
9.3	$y = \left(1+\sqrt{x}\right)^2$ $y = 1+2\sqrt{x}+x$ $y = 1+2x^{\frac{1}{2}}+x$ $\dfrac{dy}{dx} = x^{-\frac{1}{2}}+1$	✓ expansion ✓ $x^{-\frac{1}{2}}$ ✓ 1 (3) **[10]**

QUESTION 10

10.1	$(-6)(-3)(+2) = 36$ y-intercept is 36 **OR** $g(x) = (x-6)(x^2 - x - 6)$ $g(x) = x^3 - 7x^2 + 36$ y-intercept : $(0;36)$	✓ $(-6)(-3)(+2)$ ✓ y-intercept is 36 (1) ✓ trinomial ✓ 36 (1)
10.2	$g(x) = 0$ $x = 6$ or $x = 3$ or $x = -2$ intercepts are $(6 ; 0)$ and $(3 ; 0)$ and $(-2; 0)$	✓ $g(x) = 0$ ✓ all x-intercepts (2)
10.3	$g(x) = (x-6)(x^2 - x - 6)$ $= x^3 - 7x^2 + 36$ $g'(x) = 3x^2 - 14x$ $0 = x(3x - 14)$ $x = 0$ or $x = \frac{14}{3}$ Turning points are $(0 ; 36)$ and $\left(\frac{14}{3} ; -\frac{400}{27}\right)$	✓ $x^3 - 7x + 36$ ✓ $g'(x) = 3x^2 - 14x$ ✓ $g'(x) = 0$ ✓ answers ✓✓ points (6)
10.4		✓ x-intercepts ✓✓ turning points ✓ shape (4)

10.5	$g(x).g'(x<0$ $x<-2 \quad or \quad 0<x<3 \quad or \quad \frac{14}{3}<x<6$	1 mark for each inequality (3) **[16]**

QUESTION 11



11.1	$m=-\frac{b}{a}$ $y-b=\frac{-b}{a}(x-0)$ $y=\frac{-b}{a}x+b$ OR $y=mx+b$ $0=ma+b$ $m=\frac{-b}{a}$ $y=-\frac{b}{a}x+b$ OR $\frac{x}{a}+\frac{y}{b}=1$	✓ $m=-\frac{b}{a}$ ✓ answer (2)
11.2	A = xy $A=x\left(\frac{-bx}{a}+b\right)$ $=-\frac{b}{a}x^2+bx$ $\frac{dA}{dx}=-\frac{2b}{a}x+b$ $0=-\frac{2b}{a}x+b$ $-ba=-2bx$ $x=\frac{a}{2}$ $y=-\frac{b}{a}\left(\frac{a}{2}\right)+b$ $=\frac{b}{2}$ P$\left(\frac{a}{2};\frac{b}{2}\right)$ which is the midpoint of MN **OR**	✓ area formula ✓ substitution ✓ $\frac{dA}{dx}=-\frac{2b}{a}x+b$ ✓ $\frac{dA}{dx}=0$ ✓ x-value ✓ y-value (6)

	$\frac{x}{a}+\frac{y}{b}=1$ $\frac{y}{b}=1-\frac{x}{a}$ To maximise xy, we maximise $\frac{xy}{ab}=\frac{x}{a}\left(\frac{y}{b}\right)=\frac{x}{a}\left(1-\frac{x}{a}\right)$ This is a maximum when $\frac{x}{a}=\frac{1}{2}$ i.e. $x=\frac{a}{2}$ By the midpoint theorem, P is then the midpoint of MN.	**(6)** **[8]**

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -4x^2$. (5)

8.2 Evaluate:

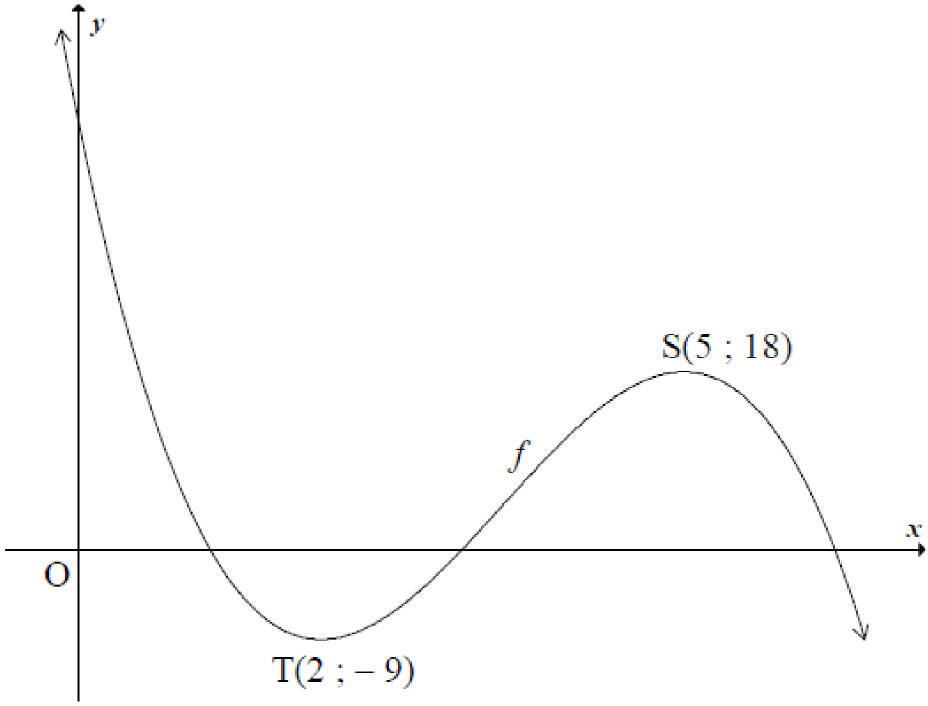
8.2.1 $\frac{dy}{dx}$ if $y = \frac{3}{2x} - \frac{x^2}{2}$ (3)

8.2.2 $f'(1)$ if $f(x) = (7x+1)^2$ (4)

[12]

QUESTION 9

The function $f(x) = -2x^3 + ax^2 + bx + c$ is sketched below.
The turning points of the graph of f are $T(2\,;-9)$ and $S(5\,;18)$.



9.1 Show that $a = 21$, $b = -60$ and $c = 43$. (7)

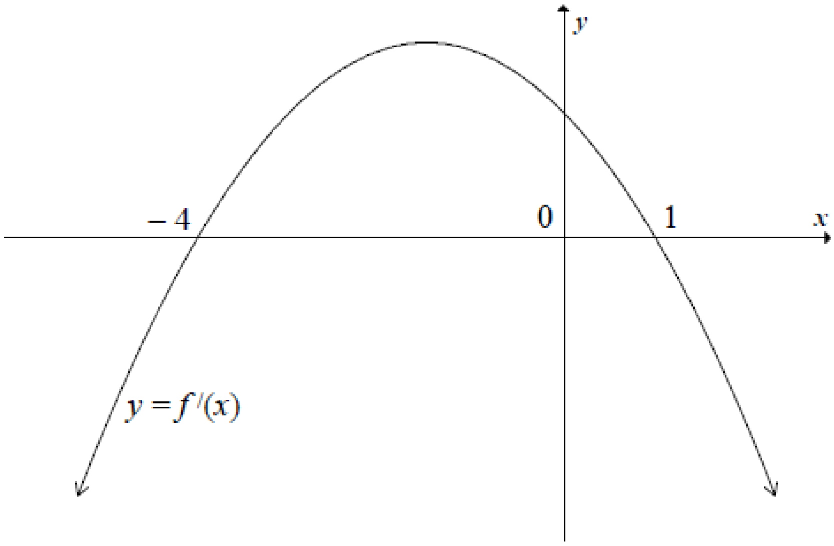
9.2 Determine an equation of the tangent to the graph of f at $x = 1$. (5)

9.3 Determine the x-value at which the graph of f has a point of inflection. (2)

[14]

QUESTION 10

The graph of $y = f'(x)$, where f is a cubic function, is sketched below.



Use the graph to answer the following questions:

10.1 For which values of x is the graph of $y = f'(x)$ decreasing? (1)

10.2 At which value of x does the graph of f have a local minimum? Give reasons for your answer. (3)

[4]

QUESTION 11

Water is flowing into a tank at a rate of 5 litres per minute. At the same time water flows out of the tank at a rate of k litres per minute. The volume (in litres) of water in the tank at time t (in minutes) is given by the formula $V(t) = 100 - 4t$.

11.1 What is the initial volume of the water in the tank? (1)

11.2 Write down TWO different expressions for the rate of change of the volume of water in the tank. (3)

11.3 Determine the value of k (that is, the rate at which water flows out of the tank). (2)

[6]

QUESTION 8

8.1	$f'(x) = \lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0}\dfrac{-4(x+h)^2-\left(-4x^2\right)}{h}$ $= \lim_{h\to 0}\dfrac{-4\left(x^2+2xh+h^2\right)+4x^2}{h}$ $= \lim_{h\to 0}\dfrac{-4x^2-8xh-4h^2+4x^2}{h}$ $= \lim_{h\to 0}\dfrac{-8xh-4h^2}{h}$ $= \lim_{h\to 0}\dfrac{h(-8x-4h)}{h}$ $= \lim_{h\to 0}(-8x-4h)$ $= -8x$ **OR**	**Note:** Incorrect notation: no lim written: penalty 2 marks lim written before equals sign: penalty 1 mark **Note:** A candidate who gives $-8x$ only: 0/5 marks **Note:** A candidate who omits brackets in the line $\lim_{h\to 0}(-8x-4h)$: NO penalty	✓ formula ✓ substitution ✓ expansion ✓ $-8x-4h$ ✓ answer (5)

	$f(x) = -4x^2$ $f(x+h) = -4(x+h)^2$ $= -4x^2 - 8xh - 4h^2$ $f(x+h) - f(x) = -8xh - 4h^2$ $f'(x) = \lim_{h\to 0} \frac{-8xh - 4h^2}{h}$ $= \lim_{h\to 0} \frac{h(-8x-4h)}{h}$ $= \lim_{h\to 0} (-8x - 4h)$ $= -8x$	✓ substitution ✓ expansion ✓ formula ✓ $-8x - 4h$ ✓ answer (5)
8.2.1	$y = \frac{3}{2x} - \frac{x^2}{2}$ $= \frac{3}{2}x^{-1} - \frac{1}{2}x^2$ $\frac{dy}{dx} = -\frac{3}{2}x^{-2} - x$ $= -\frac{3}{2x^2} - x$ **Note:** Incorrect notation in 8.2.1 and/or 8.2.2: Penalise 1 mark	✓ $\frac{3}{2}x^{-1}$ ✓ $-\frac{3}{2}x^{-2}$ ✓ $-x$ (3)
8.2.2	$f(x) = (7x+1)^2$ $= 49x^2 + 14x + 1$ $f'(x) = 98x + 14$ $f'(1) = 98(1) + 14$ $= 112$ **OR** $f(x) = (7x+1)^2$ $f'(x) = 2(7x+1)(7)$ By the chain rule $f'(x) = 98x + 14$ $f'(1) = 98(1) + 14$ $= 112$	✓multiplication ✓ $98x$ ✓14 ✓ answer (4) ✓✓ chain rule ✓✓ answer (4) **[12]**

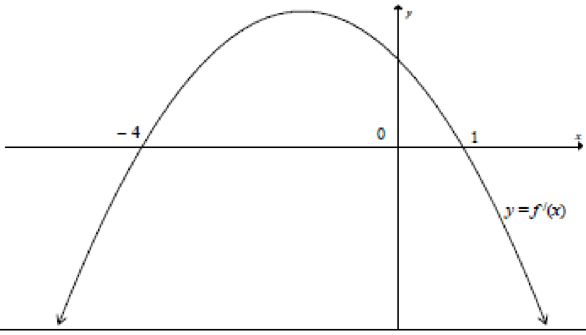
QUESTION 9

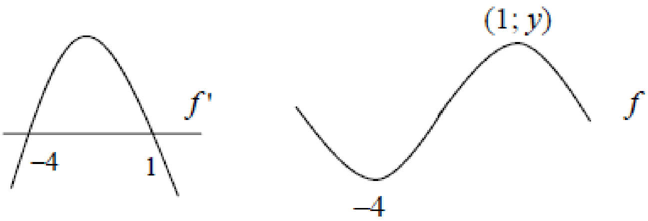
9.1	$f(x) = -2x^3 + ax^2 + bx + c$ $f'(x) = -6x^2 + 2ax + b$ $= -6(x-5)(x-2)$ $= -6(x^2 - 7x + 10)$ $= -6x^2 + 42x - 60$ **Note:** A candidate who substitutes the values of a, b and c and then checks (by substitution) that $T(2;-9)$ and $S(5;18)$ lie on the curve: award max 2/7 marks $2a = 42$ $a = 21$ $b = -60$ $f(5) = -2(5)^3 + 21(5)^2 - 60(5) + c$ $18 = -25 + c$ $c = 43$ OR $f(2) = -2(2)^3 + 21(2)^2 - 60(2) + c$ $-9 = -52 + c$ $c = 43$ $a = 21$; $b = -60$; $c = 43$	✓ $f'(x) = -6x^2 + 2ax + b$ ✓✓ $-6(x-5)(x-2)$ ✓ $b = -60$ ✓ $2a = 42$ ✓ subs (5 ; 18) or (2 ; -9) ✓ $c = 43$ (7)
	Note: A candidate who substitutes the values of a, b and c into the function i.e. gets $f(x) = -2x^3 - 21x^2 - 60x + 43$ and then shows by substitution that $T(2;-9)$ and $S(5;18)$ are on the curve **and** works out the derivative i.e. gets $f'(x) = -6x^2 - 42x - 60$ **and** shows (by substitution into the derivative) that the turning points are at $x = 2$ and $x = 5$ (assuming what s/he sets out to prove and proving what is given): **award max 4/7 marks** as follows: ✓ $x = 2$ from $f'(x) = 0$ OR subs $x = 2$ into the derivative and gets 0 ✓ $x = 5$ from $f'(x) = 0$ OR subs $x = 5$ into the derivative and gets 0 ✓substitution of $x = 2$ in f and gets -9 ✓substitution of $x = 5$ in f and gets 18	
	OR $f'(x) = -6x^2 + 2ax + b$ $f'(2) = -6(2)^2 + 2a(2) + b$ $0 = -24 + 4a + b$ $b = 24 - 4a$ $f'(5) = -6(5)^2 + 2a(5) + b$ $0 = -150 + 10a + b$ $0 = -150 + 10a + (24 - 4a)$ $0 = -126 + 6a$ $6a = 126$ $a = 21$ $b = -60$ **Note:** If derivative equal to zero is not written: penalize once only $f(5) = -2(5)^3 + 21(5)^2 - 60(5) + c$ $18 = -25 + c$ $c = 43$ OR $f(2) = -2(2)^3 + 21(2)^2 - 60(2) + c$ $-9 = -52 + c$ $c = 43$ $a = 21$; $b = -60$; $c = 43$	✓ $f'(x) = -6x^2 + 2ax + b$ ✓ $f'(2) = 0$ ✓ $f'(5) = 0$ ✓ $6a = 126$ ✓ $b = -60$ ✓ subs (5 ; 18) or (2 ; -9) ✓ $c = 43$ (7)

	OR $f(2) = -9$ i.e. $-16 + 4a + 2b + c = -9$ $4a + 2b + c = 7$ $f(5) = 18$ i.e. $-250 + 25a + 5b + c = 18$ $25a + 5b + c = 268$ $21a + 3b = 261$ $f'(x) = -6x^2 + 2ax + b$ and $f'(2) = 0$ OR $f'(5) = 0$ $4a + b = 24$ $10a + b = 150$ $12a + 3b = 72$ $9a = 189$ $a = \frac{189}{9}$ $a = 21$ OR $30a + 3b = 450$ $9a = 189$ $a = \frac{189}{9}$ $a = 21$ $12(21) + 3b = 72$ $3b = -180$ $b = -60$ $4a + 2b + c = 7$ $4(21) + 2(-60) + c = 7$ $c = 43$ OR $25a + 5b + c = 268$ $25(21) + 5(-60) + c = 268$ $c = 43$	✓ $-16 + 4a + 2b + c = -9$ and $-250 + 25a + 5b + c = 18$ ✓ $f'(x) = -6x^2 + 2ax + b$ ✓ $f'(2) = 0$ or $f'(5) = 0$ ✓ $9a = 189$ ✓ $b = -60$ ✓ subs (5 ; 18) or (2 ; -9) ✓ $c = 43$ (7)
9.2	$f'(x) = -6x^2 + 42x - 60$ $m_{\tan} = -6(1)^2 + 42(1) - 60$ $= -24$ $f(1) = -2(1)^3 + 21(1)^2 - 60(1) + 43$ $= 2$ Point of contact is (1 ; 2) $y - 2 = -24(x - 1)$ $y = -24x + 26$ **OR** $y = -24x + c$ $2 = -24(1) + c$ $c = 26$ $y = -24x + 26$	✓ $f'(x) = -6x^2 + 42x - 60$ ✓ subs $f'(1)$ ✓ $m_{\tan} = -24$ ✓ $f(1) = 2$ ✓ $y - 2 = -24(x - 1)$ OR $y = -24x + 26$ (5)
9.3	$f'(x) = -6x^2 + 42x - 60$ $f''(x) = -12x + 42$ $0 = -12x + 42$ $x = \frac{7}{2}$ **OR**	✓ $f''(x) = -12x + 42$ ✓ $x = \frac{7}{2}$ (2) ✓ $x = \frac{2+5}{2}$

	$x = \frac{2+5}{2}$ $x = \frac{7}{2}$ **OR** $x = \frac{-21}{3(-2)}$ $= \frac{7}{2}$	✓ $x = \frac{7}{2}$ (2) ✓ $x = \frac{-21}{3(-2)}$ ✓ $x = \frac{7}{2}$ (2) **[14]**

QUESTION 10



10.1	x-value of turning point: $x = \frac{-4+1}{2}$ $= -\frac{3}{2}$ $\therefore x > -\frac{3}{2}$ OR $\therefore x \in \left(-\frac{3}{2}\,;\infty\right)$	✓ $x > -\frac{3}{2}$ OR $\left(-\frac{3}{2}\,;\infty\right)$ (1)
10.2	f has a local minimum at $x = -4$ because:  **OR** $f^{/}(x) < 0$ for $x < -4$, so f is decreasing for $x < -4$. $f^{/}(x) > 0$ for $-4 < x < 1$, so f is increasing for $-4 < x < 1$. i.e.  $\therefore f$ has a local minimum at $x = -4$ **OR**	✓ $x = -4$ ✓✓ graph (3) ✓ $x = -4$ ✓ $f^{/}(x) < 0$ for $x < -4$ ✓ $f^{/}(x) > 0$ for $-4 < x < 1$ (3)

	OR Gradient of f changes from negative to positive at $x=-4$ **OR** $f'(-4)=0$ $f''(-4)>0$ so graph is concave up at $x=-4$, so f has a local minimum at $x=-4$.	✓ $x=-4$ ✓gradient negative for $x<-4$ ✓ gradient positive for $-4<x<1$ (3) ✓ $f'(-4)=0$ ✓ $f''(-4)>0$ ✓ $x=-4$ (3) **[4]**

QUESTION 11

11.1	$V(0)=100-4(0)$ $=100$ litres	✓ answer (1)
11.2	Rate in – rate out $=5-k \quad l/\text{min}$ $V'(t)=-4\ l/\text{min}$	✓ $5-k$ ✓ -4 ✓ units stated once (3)
11.3	$5-k=-4$ $k=9 \quad l/\text{min}$ **Note:** Answer only: award 2/2 marks **OR** Volume at any time t = initial volume + incoming total – outgoing total $100+5t-kt=100-4t$ $5t-kt=-4t$ $9t-kt=0$ $t(9-k)=0$ At 1 minute from start, $t=1$, $9-k=0$, so $k=9$ **OR** Since $\frac{dV}{dt}=-4$, the volume of water in the tank is decreasing by 4 litres every minute. So k is greater than 5 by 4, that is, $k=9$.	✓ $5-k=-4$ ✓ $k=9$ (2) ✓ $100+5t-kt=100-4t$ ✓ $k=9$ (2) ✓✓ $k=9$ (2) **[6]**

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 9 - x^2$. (5)

8.2 Evaluate:

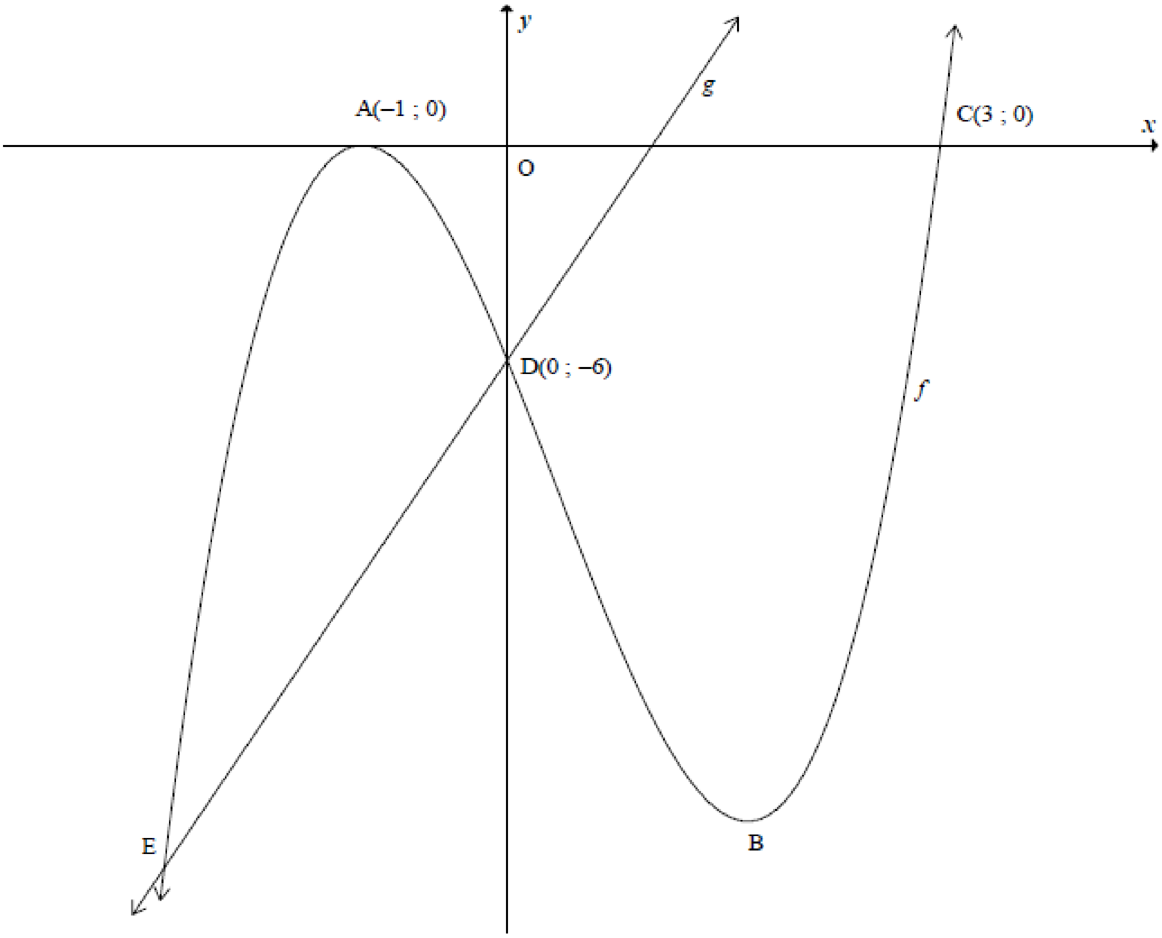
8.2.1 $D_x[1 + 6\sqrt{x}]$ (2)

8.2.2 $\dfrac{dy}{dx}$ if $y = \dfrac{8 - 3x^6}{8x^5}$ (4)

[11]

QUESTION 9

The graphs of $f(x) = ax^3 + bx^2 + cx + d$ and $g(x) = 6x - 6$ are sketched below.
A$(-1\ ;\ 0)$ and C$(3\ ;\ 0)$ are the x-intercepts of f.
The graph of f has turning points at A and B.
D$(0\ ;\ -6)$ is the y-intercept of f.
E and D are points of intersection of the graphs of f and g.



9.1 Show that $a = 2$; $b = -2$; $c = -10$ and $d = -6$. (5)

9.2 Calculate the coordinates of the turning point B. (5)

9.3 $h(x)$ is the vertical distance between $f(x)$ and $g(x)$, that is $h(x) = f(x) - g(x)$. Calculate x such that $h(x)$ is a maximum, where $x < 0$. (5)

[15]

QUESTION 10

The tangent to the curve of $g(x) = 2x^3 + px^2 + qx - 7$ at $x = 1$ has the equation $y = 5x - 8$.

10.1 Show that $(1 ; -3)$ is the point of contact of the tangent to the graph. (1)

10.2 Hence or otherwise, calculate the values of p and q. (6)

[7]

QUESTION 11

A cubic function f has the following properties:

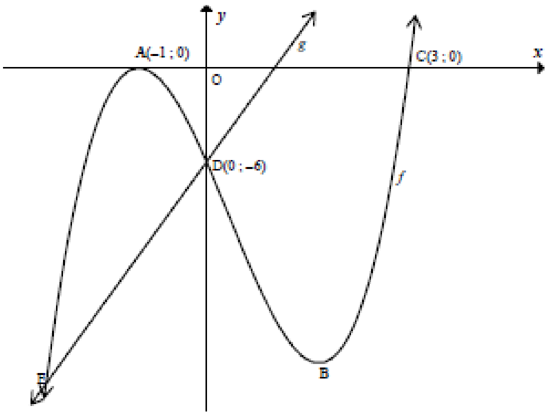
- $f\left(\frac{1}{2}\right) = f(3) = f(-1) = 0$
- $f'(2) = f'\left(-\frac{1}{3}\right) = 0$
- f decreases for $x \in \left[-\frac{1}{3} ; 2\right]$ only

Draw a possible sketch graph of f, clearly indicating the x-coordinates of the turning points and ALL the x-intercepts. **[4]**

QUESTION 8

8.1	$f(x)=9-x^2$ $f(x+h)=9-(x+h)^2$ $=9-x^2-2xh-h^2$ $f(x+h)-f(x)=-2xh-h^2$ $f'(x)=\lim_{h\to 0}\frac{-2xh-h^2}{h}$ $=\lim_{h\to 0}\frac{h(-2x-h)}{h}$ $=\lim_{h\to 0}(-2x-h)$ $=-2x$	✓ substitution ✓ simplification ✓ formula ✓ common factor ✓ answer (5)
	OR $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{9-(x+h)^2-(9-x^2)}{h}$ $=\lim_{h\to 0}\frac{9-(x^2+2xh+h^2)-9+x^2}{h}$ $=\lim_{h\to 0}\frac{-2xh-h^2}{h}$ $=\lim_{h\to 0}\frac{h(-2x-h)}{h}$ $=\lim_{h\to 0}(-2x-h)$ $=-2x$	✓ formula ✓ substitution ✓ simplification ✓ common factor ✓ answer (5)
8.2.1	$D_x[1+6\sqrt{x}]$ $=D_x\left[1+6x^{\frac{1}{2}}\right]$ $=3x^{-\frac{1}{2}}$	✓ $6x^{\frac{1}{2}}$ ✓ answer (2)
8.2.2	$y=\frac{8-3x^6}{8x^5}$ $=\frac{1}{x^5}-\frac{3}{8}x$ $=x^{-5}-\frac{3}{8}x$ $\frac{dy}{dx}=-5x^{-6}-\frac{3}{8}$	✓ x^{-5} ✓ $\frac{3}{8}x$ ✓ $-5x^{-6}$ ✓ $-\frac{3}{8}$ (4) **[11]**

QUESTION 9

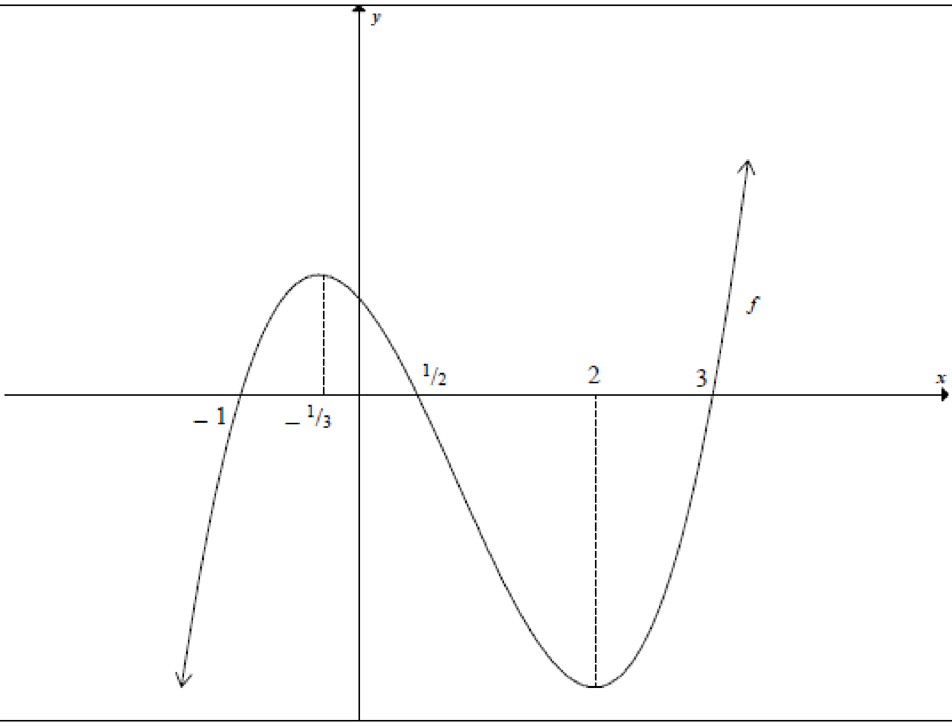


9.1	$f(x)=a(x+1)^2(x-3)$ $-6=a(0+1)^2(0-3)$ $-6=-3a$ $a=2$ $f(x)=2(x^2+2x+1)(x-3)$ $=2x^3-2x^2-10x-6$	✓✓ substitution of x-values ✓ subs $(0\ ;-6)$ ✓ $a=2$ ✓ simplification (5)
9.2	$f'(x)=6x^2-4x-10$ $6x^2-4x-10=0$ $3x^2-2x-5=0$ $(3x-5)(x+1)=0$ $x=\frac{5}{3}\quad or\quad x=-1$ $B\left(\frac{5}{3}\ ;-\frac{512}{27}\right)$ **OR** $B(1{,}67\ ;-18{,}96)$	✓ $f'(x)=6x^2-4x-6$ ✓ $f'(x)=0$ ✓ factors ✓ x-value ✓ y-value (5)
9.3	$h(x)=2x^3-2x^2-10x-6-(6x-6)$ $=2x^3-2x^2-16x$ $h'(x)=6x^2-4x-16$ $0=3x^2-2x-8$ $0=(3x+4)(x-2)$ $x=-\frac{4}{3}\quad or\quad x=2$ $\therefore x=-\frac{4}{3}$	✓ $h(x)=2x^3-2x^2-16x$ ✓ $h'(x)=6x^2-4x-16$ ✓ $h'(x)=0$ ✓ factors ✓ correct x-value (5) **[15]**

QUESTION 10

10.1	$y = 5(1) - 8$ $= -3$ Point of contact is $(1\,;-3)$	✓ subs 1 (1)
10.2	$-3 = 2(1)^3 + p(1)^2 + q(1) - 7$ $2 = p + q$ $g'(x) = 6x^2 + 2px + q$ $g'(1) = 5$ $5 = 6(1)^2 + 2p(1) + q$ $-1 = 2p + q$ $p = -3$ $q = 5$	✓ subs $(1\,;-3)$ ✓ $g'(x) = 6x^2 + 2px + q$ ✓ subs $x = 1$ and $y = 5$ ✓ simplification ✓ p-value ✓ q-value (6) [7]

QUESTION 11

		✓ x-intercepts ✓ turning point ✓✓ shape **[4]**

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 2x^2 - 5$. (5)

8.2 Evaluate $\frac{dy}{dx}$ if $y = x^{-4} + 2x^3 - \frac{x}{5}$. (3)

8.3 Given: $g(x) = \frac{x^2 + x - 2}{x - 1}$

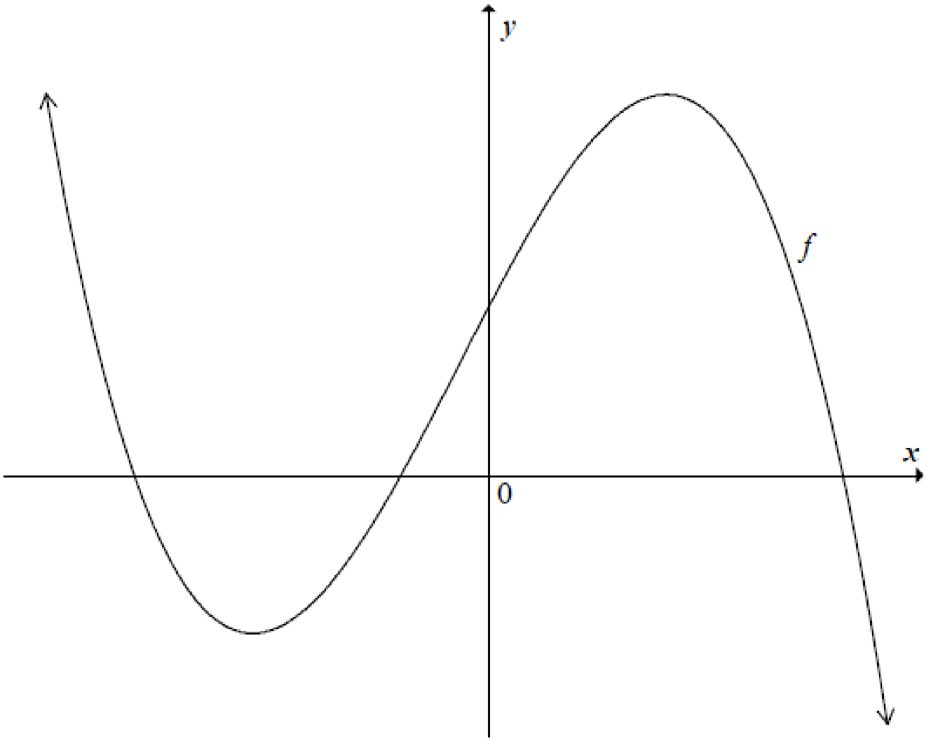
8.3.1 Calculate $g'(x)$ for $x \neq 1$. (2)

8.3.2 Explain why it is not possible to determine $g'(1)$. (1)

[11]

QUESTION 9

9.1 The graph of the function $f(x) = -x^3 - x^2 + 16x + 16$ is sketched below.



9.1.1 Calculate the x-coordinates of the turning points of f. (4)

9.1.2 Calculate the x-coordinate of the point at which $f'(x)$ is a maximum. (3)

9.2 Consider the graph of $g(x) = -2x^2 - 9x + 5$.

9.2.1 Determine the equation of the tangent to the graph of g at $x = -1$. (4)

9.2.2 For which values of q will the line $y = -5x + q$ not intersect the parabola? (3)

9.3 Given: $h(x) = 4x^3 + 5x$

Explain if it is possible to draw a tangent to the graph of h that has a negative gradient. Show ALL your calculations. (3)

[17]

QUESTION 10

A particle moves along a straight line. The distance, s, (in metres) of the particle from a fixed point on the line at time t seconds ($t \geq 0$) is given by $s(t) = 2t^2 - 18t + 45$.

10.1 Calculate the particle's initial velocity. (Velocity is the rate of change of distance.) (3)

10.2 Determine the rate at which the velocity of the particle is changing at t seconds. (1)

10.3 After how many seconds will the particle be closest to the fixed point? (2)

[6]

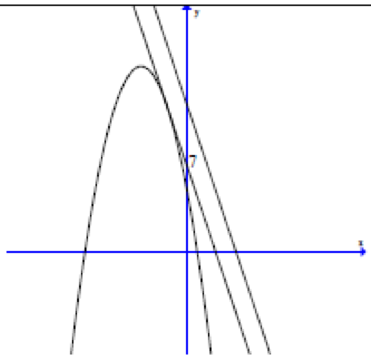
QUESTION 8

8.1	$f(x)=2x^2-5$ $f(x+h)=2(x+h)^2-5$ $=2x^2+4xh+2h^2-5$ $f(x+h)-f(x)=4xh+2h^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(4x+2h)}{h}$ $=\lim\limits_{h\to 0}(4x+2h)$ $=4x$ **OR** **Note:** If candidate makes a notation error Penalise 1 mark **Note:** If candidate uses differentiation rules Award 0/5 marks	✓ substitution of of $x+h$ ✓ simplification to $4xh+2h^2$ ✓ formula ✓ $\lim\limits_{h\to 0}(4x+2h)$ ✓ answer (5)

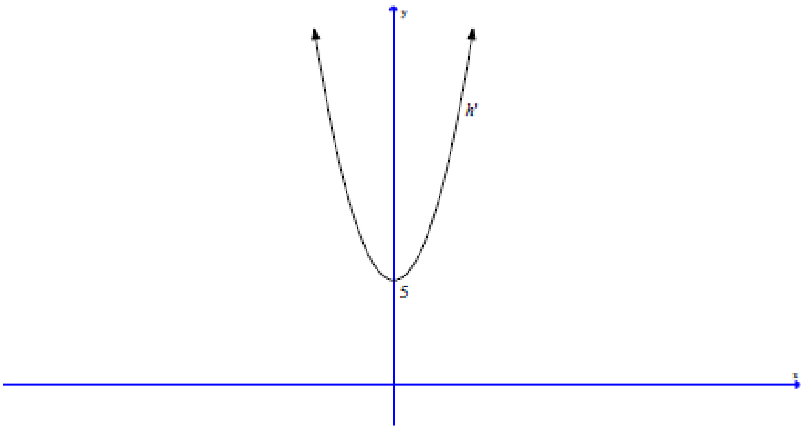
	$f'(x)=\lim\limits_{h\to0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to0}\dfrac{\left[2(x+h)^2-5\right]-\left(2x^2-5\right)}{h}$ $=\lim\limits_{h\to0}\dfrac{\left[2\left(x^2+2xh+h^2\right)-5\right]-2x^2+5}{h}$ $=\lim\limits_{h\to0}\dfrac{[2x^2+4xh+2h^2-5]-2x^2+5}{h}$ $=\lim\limits_{h\to0}\dfrac{4xh+2h^2}{h}$ $=\lim\limits_{h\to0}\dfrac{h(4x+2h)}{h}$ $=\lim\limits_{h\to0}(4x+2h)$ $=4x$	✓ formula ✓ substitution of $x+h$ ✓ simplification to $\dfrac{4xh+2h^2}{h}$ ✓ $\lim\limits_{h\to0}(4x+2h)$ ✓ answer (5)
8.2	$\dfrac{dy}{dx}=-4x^{-5}+6x^2-\dfrac{1}{5}$ $=\dfrac{-4}{x^5}+6x^2-\dfrac{1}{5}$ **Note:** notation error penalise 1 mark **Note:** candidates do NOT need to give their answer with positive exponents	✓ $-4x^{-5}$ ✓ $6x^2$ ✓ $-\dfrac{1}{5}$ (3)
8.3.1	$g(x)=\dfrac{x^2+x-2}{x-1}$ $=\dfrac{(x+2)(x-1)}{x-1}$ $=x+2 \quad (x\neq1)$ $g'(x)=1 \quad (x\neq1)$	✓ simplification ✓ answer (2)
8.3.2	The function is undefined at $x=1$. **OR** Division by zero is undefined. **OR** The denominator cannot be zero. **OR** In the definition of the derivative , $g'(1)=\lim\limits_{h\to0}\dfrac{g(1+h)-g(1)}{h}$, but $g(1)$ does not exist.	✓ answer (1) **[11]**

QUESTION 9

9.1.1	$f(x) = -x^3 - x^2 + 16x + 16$ $f'(x) = -3x^2 - 2x + 16$ $0 = -3x^2 - 2x + 16$ $3x^2 + 2x - 16 = 0$ $(3x+8)(x-2) = 0$ $x = -\frac{8}{3}$ or $x = 2$ **Note:** if neither $f'(x) = 0$ nor $0 = -3x^2 - 2x + 16$ explicitly stated, award maximum 3/4 marks **OR** $f(x) = -x^3 - x^2 + 16x + 16$ $f'(x) = -3x^2 - 2x + 16$ $0 = -3x^2 - 2x + 16$ $0 = 3x^2 + 2x - 16$ $x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-16)}}{2(3)}$ $x = -\frac{8}{3}$ or $x = 2$	✓ $f'(x) = -3x^2 - 2x + 16$ ✓ $f'(x) = 0$ or $0 = -3x^2 - 2x + 16$ ✓ factors ✓ x values (4) ✓ $f'(x) = -3x^2 - 2x + 16$ ✓ $f'(x) = 0$ or $0 = -3x^2 - 2x + 16$ ✓ subs into formula ✓ x values (4)
9.1.2	$f''(x) = 0$ $-6x - 2 = 0$ $x = -\frac{1}{3}$ **OR** $x = \frac{-\frac{8}{3} + 2}{2}$ $x = -\frac{1}{3}$ **OR** $f'(x) = -3x^2 - 2x + 16$ $x = \frac{-(-2)}{2(-3)}$ $= -\frac{1}{3}$ **OR**	✓ $f''(x) = -6x - 2$ ✓ $-6x - 2 = 0$ ✓ answer (3) ✓ $x = \frac{-\frac{8}{3} + 2}{2}$ ✓✓ answer (3) ✓✓ $x = \frac{-(-2)}{2(-3)}$ ✓ answer (3)

	$f(x)=-x^3-x^2+16x+16$ $x=\dfrac{-(-1)}{3(-1)}$ $=-\dfrac{1}{3}$	✓✓ $x=\dfrac{-(-1)}{3(-1)}$ ✓ answer (3)
9.2.1	$g(x)=-2x^2-9x+5$ $g(-1)=-2(-1)^2-9(-1)+5$ $=12$ $g'(x)=-4x-9$ $m_{\tan}=-4(-1)-9$ $=-5$ $y=-5x+c$ $12=-5(-1)+c$ $c=7$ $y=-5x+7$ **OR** $g(x)=-2x^2-9x+5$ $g(-1)=-2(-1)^2-9(-1)+5$ $=12$ $g'(x)=-4x-9$ $m_{\tan}=-4(-1)-9$ $=-5$ $y-12=-5(x+1)$ $y=-5x+7$	✓ $g(-1)=12$ ✓ $g'(x)=-4x-9$ ✓ $m_{\tan}=-5$ ✓ answer (4) ✓ $g(-1)=12$ ✓ $g'(x)=-4x-9$ ✓ $m_{\tan}=-5$ ✓ answer (4)
9.2.2	 $q>7$ **OR** $y=-5x+q$ and $y=-2x^2-9x+5$ $-5x+q=-2x^2-9x+5$ $q=-2(x+1)^2+7$ $\therefore q>7$	✓ sketch ✓ 7 ✓ correct inequality (3) ✓ method ✓ 7 ✓ correct inequality (3)

	OR $y=-5x+q$ and $y=-2x^2-9x+5$ $-5x+q=-2x^2-9x+5$ $2x^2+4x+q-5=0$ $x=\dfrac{-4\pm\sqrt{16-4(2)(q-5)}}{2(2)}$ $x=\dfrac{-4\pm\sqrt{56-8q}}{4}$ $56-8q<0$ $q>7$ **OR** Since $g(-1)=12$ and at $x=-1$, tangent equation is $y=-5x+7$, $y=-5x+q$ not intersecting g $\Rightarrow$ $12<-5(-1)+q$ $12-5<q$ $7<q$	✓ method ✓ 7 ✓correct inequality (3) ✓ method ✓ 7 ✓correct inequality (3)
9.3	$h'(x)=12x^2+5$ For all values of x: $x^2\geq 0$ $12x^2\geq 0$ $12x^2+5\geq 5$ $12x^2+5>0$ For all values of x: $h'(x)>0$ All tangents drawn to h will have a positive gradient. It will never be possible to draw a tangent with a negative gradient to the graph of h. **OR** $h'(x)=12x^2+5$ Suppose $h'(x)<0$ and try to solve for x : $12x^2+5<0$ $x^2<-\dfrac{5}{12}$ but x^2 is always positive $\therefore$ no solution for x $\therefore h'(x)\geq 0$ for all $x\in R$ i.e. there are no tangents with negative slopes	✓ $h'(x)=12x^2+5$ ✓ clearly argues that $h'(x)>0$ ✓ conclusion (3) ✓ $h'(x)=12x^2+5$ ✓ clearly argues that $h'(x)<0$ is impossible ✓ conclusion (3)

	OR $h'(x) = 12x^2 + 5$  Since clearly $h'(x) > 0$ for all $x \in R$, it will never be possible to draw a tangent with a negative gradient to the graph of h.	✓ $h'(x) = 12x^2 + 5$ ✓ argues $h'(x) > 0$ by drawing a sketch ✓ conclusion (3) **[17]**

QUESTION 10

10.1	$s(t)=2t^2-18t+45$ $s'(t)=4t-18$ $s'(0)=4(0)-18$ $=-18\ m/s$ **Note:** answer only award 0/3 marks	✓ $s'(t)$ ✓ subs $t=0$ into $s'(t)$ formula ✓ answer (3)
10.2	$s''(t)=4\ \text{m/s}^2$	✓ answer (1)
10.3	$4t-18=0$ $4t=18$ $t=\frac{9}{2}$ seconds or 4,5 seconds **OR** $s(t)=2\left(t-\frac{9}{2}\right)^2+\frac{9}{2}$ $t=\frac{9}{2}$ seconds or 4,5 seconds **OR** $s(t)=2t^2-18t+45$ $t=-\frac{-18}{2(2)}$ $t=\frac{9}{2}$ seconds or 4,5 seconds	✓ $s'(t)=0$ ✓ answer (2) ✓ $s(t)=2\left(t-\frac{9}{2}\right)^2+\frac{9}{2}$ ✓ answer (2) ✓ $t=-\frac{-18}{2(2)}$ ✓ answer (2) **[6]**

QUESTION 9

9.1 Use the definition of the derivative (first principles) to determine $f'(x)$ if $f(x) = 2x^3$ (5)

9.2 Determine $\frac{dy}{dx}$ if $y = \frac{2\sqrt{x}+1}{x^2}$ (4)

9.3 Calculate the values of a and b if $f(x) = ax^2 + bx + 5$ has a tangent at $x = -1$ which is defined by the equation $y = -7x + 3$ (6)

[15]

QUESTION 10

Given: $f(x) = -x^3 - x^2 + x + 10$

10.1 Write down the coordinates of the y-intercept of f. (1)

10.2 Show that (2 ; 0) is the only x-intercept of f. (4)

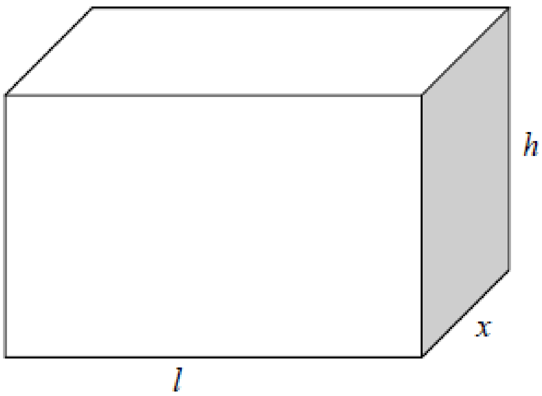
10.3 Calculate the coordinates of the turning points of f. (6)

10.4 Sketch the graph of f in your ANSWER BOOK. Show all intercepts with the axes and all turning points. (3)

[14]

QUESTION 11

A rectangular box is constructed in such a way that the length (l) of the base is three times as long as its width. The material used to construct the top and the bottom of the box costs R100 per square metre. The material used to construct the sides of the box costs R50 per square metre. The box must have a volume of 9 m^3. Let the width of the box be x metres.



11.1 Determine an expression for the height (h) of the box in terms of x. (3)

11.2 Show that the cost to construct the box can be expressed as $C = \frac{1200}{x} + 600x^2$. (3)

11.3 Calculate the width of the box (that is the value of x) if the cost is to be a minimum. (4)

[10]

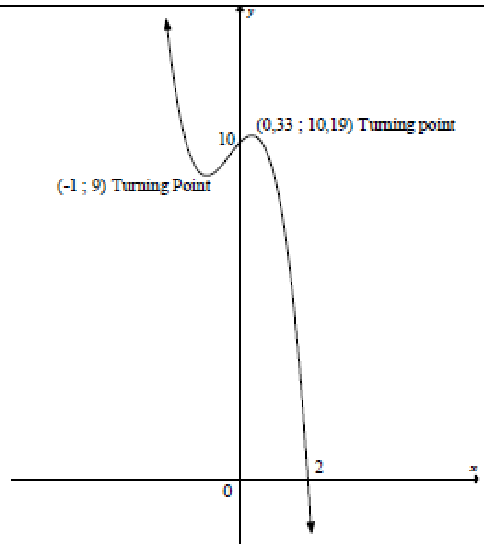
QUESTION 9

9.1	$f(x)=2x^3$ $f(x+h)=2(x+h)^3$ $=2(x^3+3x^2h+3xh^2+h^3)$ $=2x^3+6x^2h+6xh^2+2h^3$ $f(x+h)-f(x)=2x^3+6x^2h+6xh^2+2h^3-2x^3$ $=6x^2h+6xh^2+2h^3$ $f'(x)=\lim\limits_{h\to 0}\dfrac{6x^2h+6xh^2+2h^3}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(6x^2+6xh+2h^2)}{h}$ $=\lim\limits_{h\to 0}(6x^2+6xh+2h^2)$ $f'(x)=6x^2$	✓ substitution ✓ expansion ✓ formula ✓ $6x^2+6xh+2h^2$ ✓ answer (5)
	OR $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{2(x+h)^3-2x^3}{h}$ $=\lim\limits_{h\to 0}\dfrac{2(x^3+3x^2h+3xh^2+h^3)-2x^3}{h}$ $=\lim\limits_{h\to 0}\dfrac{6x^2h+6xh^2+2h^3}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(6x^2+6xh+2h^2)}{h}$ $=\lim\limits_{h\to 0}(6x^2+6xh+2h^2)$ $f'(x)=6x^2$	✓ formula ✓ substitution ✓ expansion ✓ $6x^2+6xh+2h^2$ ✓ answer (5)
9.2	$y=\dfrac{2\sqrt{x}+1}{x^2}$ $=2x^{-\frac{3}{2}}+x^{-2}$ $\dfrac{dy}{dx}=-3x^{-\frac{5}{2}}-2x^{-3}$	✓ $2x^{-\frac{3}{2}}$ ✓ x^{-2} ✓ $-3x^{-\frac{5}{2}}$ ✓ $-2x^{-3}$ (4)

9.3	$f'(-1) = -7$ $f'(x) = 2ax + b$ $-7 = -2a + b$ $f(-1) = -7(-1) + 3$ $= 10$ $\therefore\ a - b + 5 = 10$ $a - b = 5$............[1] $-2a + b = -7$.........[2] $-a = -2$.........[1]+[2] $a = 2$ $b = -3$	✓ $f'(x) = 2ax + b$ ✓ substitution of $x = -1$ ✓ $-7 = -2a + b$ ✓ $f(-1) = 10$ ✓ $a = 2$ ✓ $b = -3$ (6) **[15]**

QUESTION 10

$$f(x) = -x^3 - x^2 + x + 10$$

10.1	$(0;10)$	✓ $(0;10)$ (1)
10.2	$0 = -x^3 - x^2 + x + 10$ $0 = -(x-2)(x^2+3x+5)$ $x - 2 = 0$ or $x^2 + 3x + 5 = 0$ $x = 2$ $x = \dfrac{-3 \pm \sqrt{3^2 - 4(1)(5)}}{2(1)}$ $= \dfrac{-3 \pm \sqrt{-11}}{2}$ which has no solution Therefore the only x-intercept of f is $(2;0)$	✓ $(x-2)$ ✓ (x^2+3x+5) ✓ $x = \dfrac{-3 \pm \sqrt{-11}}{2}$ ✓ no solution (4)
10.3	$f'(x) = -3x^2 - 2x + 1$ $0 = -3x^2 - 2x + 1$ $0 = (3x-1)(x+1)$ $x = \dfrac{1}{3}$ or $x = -1$ $y = -\left(\dfrac{1}{3}\right)^3 - \left(\dfrac{1}{3}\right)^2 + \left(\dfrac{1}{3}\right) + 10$ or $y = -(-1)^3 - (-1)^2 + (-1) + 10$ $= \dfrac{275}{27}$ $\quad = 9$ $\left(\dfrac{1}{3}; 10\dfrac{5}{27}\right)$ $\quad (-1;9)$	✓ $f'(x) = -3x^2 - 2x + 1$ ✓ $f'(x) = 0$ ✓ factors ✓ x-values ✓ $\left(\dfrac{1}{3}; 10\dfrac{5}{27}\right)$ ✓ $(-1;9)$ (6)
10.4		✓ shape ✓ intercepts ✓ turning points (3) **[14]**

QUESTION 11

11.1	Length of box $= 3x$ Volume $= l \times b \times h$ $9 = 3x \cdot x \cdot h$ $9 = 3x^2 h$ $h = \dfrac{3}{x^2}$	✓ length of box $= 3x$ ✓ $9 = 3x \cdot x \cdot h$ ✓ $h = \dfrac{3}{x^2}$ (3)
11.2	$C = (2(3xh) + 2xh) \times 50 + (2 \times 3x^2) \times 100$ $= 8x\left(\dfrac{3}{x^2}\right) \times 50 + 600x^2$ $= \dfrac{1200}{x} + 600x^2$ **OR** $C = (h \times 8x) \times 50 + (2 \times 3x^2) \times 100$ $= 8x\left(\dfrac{3}{x^2}\right) \times 50 + 600x^2$ $= \dfrac{1200}{x} + 600x^2$	✓ $(2(3xh) + 2xh) \times 50$ ✓ $(2 \times 3x^2) \times 100$ ✓ substitution of $h = \dfrac{3}{x^2}$ (3) ✓ $(h \times 8x) \times 50$ ✓ $(2 \times 3x^2) \times 100$ ✓ substitution of $h = \dfrac{3}{x^2}$ (3)
11.3	$C = 1200x^{-1} + 600x^2$ $\dfrac{dC}{dx} = -1200x^{-2} + 1200x$ $0 = -1200x^{-2} + 1200x$ $1200x^3 = 1200$ $x^3 = 1$ $x = 1$ Therefore the width of the box is 1 metre.	✓ $\dfrac{dC}{dx} = -1200x^{-2} + 1200x$ ✓ $\dfrac{dC}{dx} = 0$ ✓ $x^3 = 1$ ✓ $x = 1$ (4) **[10]**

QUESTION 8

8.1 Given: $f(x) = 3x^2 - 4$

8.1.1 Determine $f'(x)$ from first principles. (5)

8.1.2 A(x ; 23), where $x > 0$, and B(– 2 ; y) are points on the graph of f. Calculate the numerical value of the average gradient of f between A and B. (5)

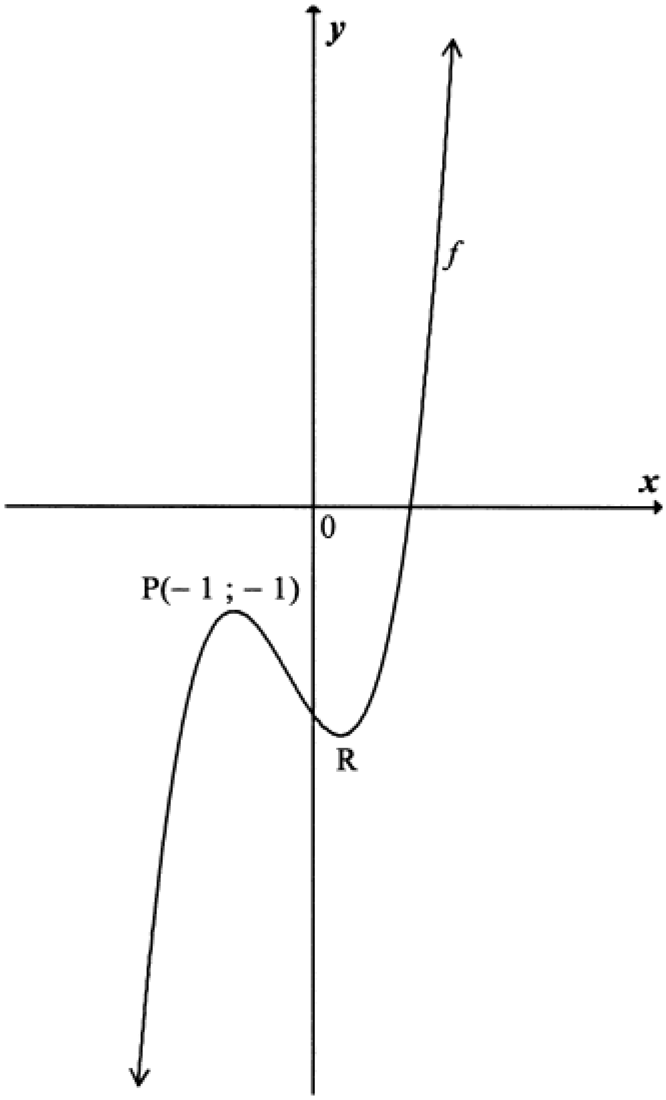
8.2 Differentiate $y = \dfrac{x+5}{\sqrt{x}}$ with respect to x. (3)

8.3 Determine the gradient of the tangent of the graph of $f(x) = -3x^3 - 4x + 5$ at $x = -1$. (4)

[17]

QUESTION 9

The function defined by $f(x) = x^3 + ax^2 + bx - 2$ is sketched below.
P(– 1 ; – 1) and R are the turning points of f.



9.1 Show that $a = 1$ and $b = -1$. (6)

9.2 Hence, or otherwise, determine the x-coordinate of R. (3)

9.3 Write down the coordinates of a turning point of h if h is defined by $h(x) = 2f(x) - 4$. (2)

[11]

QUESTION 10

An industrial process requires water to flow through its system as part of the cooling cycle. Water flows continuously through the system for a certain period of time.

The relationship between the time (t) from when the water starts flowing and the rate (r) at which the water is flowing through the system is given by the equation:

$$r = -0{,}2t^2 + 10t$$

where t is measured in seconds.

10.1 After how long will the water be flowing at the maximum rate? (3)

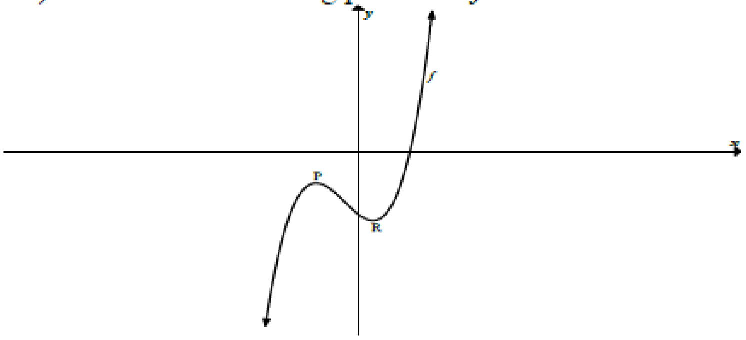
10.2 After how many seconds does the water stop flowing? (3)
[6]

QUESTION/*VRAAG* 8

8.1.1	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{3(x+h)^2-4-(3x^2-4)}{h}$ $=\lim\limits_{h\to 0}\dfrac{3x^2+6xh+3h^2-4-3x^2+4}{h}$ $=\lim\limits_{h\to 0}\dfrac{6xh+3h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(6x+3h)}{h} \quad ; h\neq 0$ $=\lim\limits_{h\to 0}(6x+3h)$ $=6x$ **NOTE:** • Incorrect notation: max 4/5 marks (Leaving out limit / incorrect use of limit, leaving out $f'(x)$, = in the wrong place constitute incorrect notation) • If $\lim\limits_{h\to 0}\dfrac{3x^2+6xh+3h^2-3x^2-8}{h}$, take out common factor, then correct to the final answer: max 3/5 marks. **OR** $f(x)=3x^2-4$ $f(x+h)=3(x+h)^2-4$ $=3x^2+6xh+3h^2-4$ $f(x+h)-f(x)=6xh+3h^2$ **NOTE:** If candidate uses differentiation rules: 0/5 marks $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{6xh+3h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(6x+3h)}{h}$ $=\lim\limits_{h\to 0}(6x+3h)$ $=6x$	✓ formula ✓ substitution of of $x+h$ ✓ simplification to $\dfrac{6xh+3h^2}{h}$ ✓ $\lim\limits_{h\to 0}(6x+3h)$ ✓ answer (5) ✓ substitution of of $x+h$ ✓ simplification to $6xh+3h^2$ ✓ formula ✓ $\lim\limits_{h\to 0}(6x+3h)$ ✓ answer (5)
8.1.2	$f(x)=3x^2-4$ average gradient of f between $A(-2\ ;\ y)$ and $B(x\ ;\ 23)$ $y=3(-2)^2-4=8$ $23=3x^2-4$ $27=3x^2$ $9=x^2$ $x=3$	✓ $y=8$ ✓ $23=3x^2-4$ ✓ $x=3$

	Average gradient $= \frac{23-y}{x-(-2)}$ $= \frac{23-8}{3+2}$ $= 3$	✓ $\frac{23-y}{x-(-2)}$ ✓answer (5)
8.2	$y = \frac{x+5}{x^{\frac{1}{2}}}$ $= \frac{x}{x^{\frac{1}{2}}} + \frac{5}{x^{\frac{1}{2}}}$ $= x^{\frac{1}{2}} + 5x^{-\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - \frac{5}{2}x^{-\frac{3}{2}}$ **OR** $y = \frac{x+5}{x^{\frac{1}{2}}}$ By the quotient rule $\frac{dy}{dx} = \frac{1.x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}(x+5)}{\left(x^{\frac{1}{2}}\right)^2}$ $= \frac{x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{-\frac{1}{2}}}{x}$ $= \frac{1}{2x^{\frac{1}{2}}} - \frac{5}{2x^{\frac{3}{2}}}$	**NOTE:** There is a maximum penalty of 1 mark for incorrect notation in question 8. ✓ $x^{\frac{1}{2}} + 5x^{-\frac{1}{2}}$ ✓ $\frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{1}{2\sqrt{x}}$ ✓ $-\frac{5}{2}x^{-\frac{3}{2}}$ or $\frac{-5}{2\sqrt{x^3}}$ (3) ✓✓ $\frac{1.x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}(x+5)}{\left(x^{\frac{1}{2}}\right)^2}$ ✓ $\frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{1}{2\sqrt{x}}$ or $-\frac{5}{2}x^{-\frac{3}{2}}$ or $\frac{-5}{2\sqrt{x^3}}$ (3)
8.3	$f(x) = -3x^3 - 4x + 5$ $f'(x) = -9x^2 - 4$ $m_{\tan} = -9(-1)^2 - 4$ $= -13$	✓ $-9x^2$ ✓ -4 ✓ substitution of $x = -1$ ✓ answer (4) **[17]**

QUESTION/*VRAAG* 9

	Given: $f(x)=x^3+ax^2+bx-2$ P$(-1\,;-1)$ and R are the turning points of f. 	
9.1	$f(x)=x^3+ax^2+bx-2$ $-1=(-1)^3+a(-1)^2+b(-1)-2$ $2=a-b \quad \ldots(1)$ $f'(x)=3x^2+2ax+b$ $0=3(-1)^2+2a(-1)+b$ $-3=-2a+b \quad \ldots(2)$ $-1=-a \quad \ldots(1)+(2)$ $a=1$ $b=-1$ **NOTE:** • The $=0$ must be explicitly stated • If the candidate starts with what is given and then prove $(1\,;1)$ is a TP: 0/6 marks	✓ $-1=(-1)^3+a(-1)^2+b(-1)-2$ ✓ $2=a-b$ ✓ $f'(x)=3x^2+2ax+b$ ✓ $f'(-1)=0$ ✓ $-3=-2a+b$ ✓ method (6)
9.2	R is a turning point of f, hence at R, $f'(x)=0$ i.e. $3x^2+2x-1=0$ $(3x-1)(x+1)=0$ $x=\frac{1}{3}$ or -1 $\therefore x=\frac{1}{3}$ **NOTE:** Answer only: 1/3 marks	✓ $f'(x)=0$ ✓ $f'(x)=3x^2+2x-1$ ✓ selection of $x=\frac{1}{3}$ (3)
9.3	$(-1\,;2f(-1)-4)$ $=(-1\,;-6)$ **OR** $\left(\frac{1}{3}\,;2f\left(\frac{1}{3}\right)-4\right)$ $=\left(\frac{1}{3}\,;-\frac{226}{27}\right)$ or $(0{,}33\,;-8{,}37)$	✓ x-coordinate ✓ y-coordinate (2) ✓ x-coordinate ✓ y-coordinate (2) **[11]**

QUESTION/*VRAAG* 10

10.1	$\frac{dr}{dt}=-0{,}4t+10$ $0=-0{,}4t+10$ $0{,}4t=10$ $t=\frac{10}{0{,}4}$ $=25$ seconds	✓ $\frac{dr}{dt}=-0{,}4t+10$ ✓ $0=-0{,}4t+10$ ✓ t value (3)
	OR $t=-\frac{b}{2a}$ $=-\frac{10}{2(-0{,}2)}$ $=25$ seconds	✓ $t=-\frac{b}{2a}$ ✓ substitution ✓ t value (3)
	OR $r=\frac{1}{5}t(50-t)$ $0=\frac{t}{5}(50-t)$ $t=0 \quad or \quad t=50$ Fastest at $t=\frac{0+50}{2}$ $t=25$ seconds	✓ $t=0$ or $t=50$ ✓ $t=\frac{0+50}{2}$ ✓ t value (3)
10.2	$-0{,}2t^2+10t=0$ $t(-0{,}2t+10)=0$ $-0{,}2t+10=0$ or $t=0$ $t=\frac{-10}{-0{,}2}$ $=50$ sec Hence the water stops flowing 50 seconds after it started. **NOTE:** Answer only: 3/3 marks	✓ $-0{,}2t^2+10t=0$ ✓ factors ✓ answer (3)
	OR $-0{,}2t^2+10t=0$ $t^2-50t=0$ $t(t-50)=0$ $t=0$ or $t=50$ Hence the water stops flowing 50 seconds after it started.	✓ $-0{,}2t^2+10t=0$ ✓ factors ✓ answer (3) **[6]**

QUESTION 10

10.1 Given: $f(x) = -\frac{2}{x}$

10.1.1 Determine $f'(x)$ from first principles. (5)

10.1.2 For which value(s) of x will $f'(x) > 0$? Justify your answer. (2)

10.2 Evaluate $\frac{dy}{dx}$ if $y = \frac{1}{4}x^2 - 2x$. (2)

10.3 Given: $y = 4\left(\sqrt[3]{x^2}\right)$ and $x = w^{-3}$

Determine $\frac{dy}{dw}$. (4)

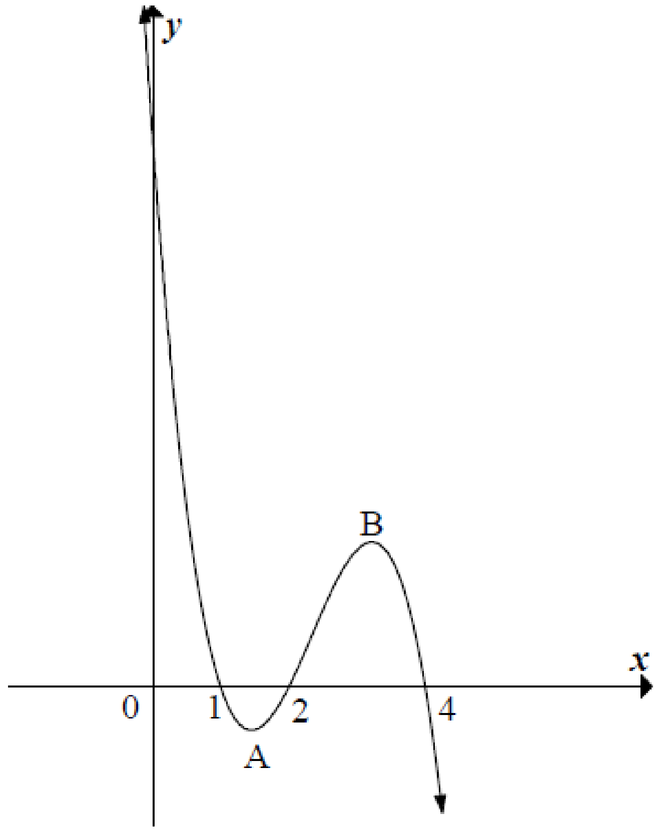
10.4 Given: $f(x) = ax^3 + bx^2 + cx + d$

Draw a possible sketch of $y = f'(x)$ if a, b and c are all NEGATIVE real numbers. (4)

[17]

QUESTION 11

The graph of $f(x) = -x^3 + ax^2 + bx + c$ is sketched below. The x-intercepts are indicated.



11.1 Calculate the values of a, b and c. (4)

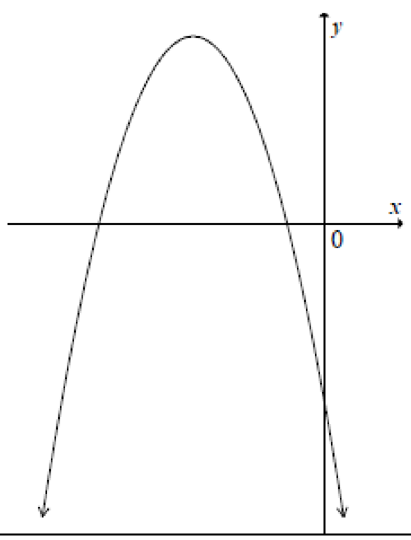
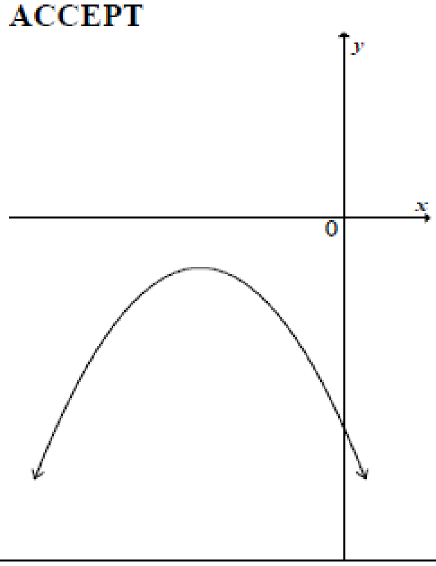
11.2 Calculate the x-coordinates of A and B, the turning points of f. (5)

11.3 For which values of x will $f'(x) < 0$? (3)

[12]

QUESTION/*VRAAG* 10

10.1.1	$f(x) = -\frac{2}{x}$ $f(x+h) = -\frac{2}{(x+h)}$ $f(x+h) - f(x) = -\frac{2}{(x+h)} - \left(-\frac{2}{x}\right)$ $= \frac{-2x + 2(x+h)}{x(x+h)}$ $= \frac{-2x + 2x + 2h}{x(x+h)}$ $= \frac{2h}{x(x+h)}$ $f'(x) = \lim_{h \to 0} \frac{\frac{2h}{x(x+h)}}{h}$ $= \lim_{h \to 0} \left(\frac{2}{x^2 + xh}\right)$ $= \frac{2}{x^2}$	✓ substitution ✓ simplification ✓ formula ✓ common factor ✓ answer (5)
	OR $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{\left[-\frac{2}{(x+h)}\right] - \left(-\frac{2}{x}\right)}{h}$ $= \lim_{h \to 0} \frac{\frac{-2x + 2(x+h)}{x(x+h)}}{h}$ $= \lim_{h \to 0} \frac{\frac{-2x + 2x + 2h}{x(x+h)}}{h}$ $= \lim_{h \to 0} \frac{\frac{2h}{x(x+h)}}{h}$ $= \lim_{h \to 0} \left(\frac{2}{x^2 + xh}\right)$ $= \frac{2}{x^2}$	✓ formula ✓ substitution ✓ simplification ✓ common factor ✓ answer (5)

10.1.2	$f'(x)=\dfrac{2}{x^2}$ $x^2 \geq 0$ for $x \in R$ $f'(x)>0$ for $x \in R;\ x \neq 0$	✓ $x^2 \geq 0$ or $\dfrac{2}{x^2} \geq 0$ for $x \in R$ ✓ $f'(x)>0$ for $x \in R;\ x \neq 0$ (2)
10.2	$y=\dfrac{1}{4}x^2-2x$ $\dfrac{dy}{dx}=\dfrac{1}{2}x-2$	✓ $\dfrac{1}{2}x$ ✓ -2 (2)
10.3	$y=4\left(\sqrt[3]{x^2}\right)$ $y=4x^{\frac{2}{3}}$ and $x=w^{-3}$ $y=4(w^{-3})^{\frac{2}{3}}$ $=4w^{-2}$ $\dfrac{dy}{dw}=-8w^{-3}$ $=-\dfrac{8}{w^3}$	✓ $y=4x^{\frac{2}{3}}$ ✓ subs: $4(w^{-3})^{\frac{2}{3}}$ ✓simplification ✓ answer (4)
10.4	$f'(x)=3ax^2+2bx+c$ $a<0$ shape (max TP) $c<0$ y - intercept is negative b <0 axis of symmetry on LHS of y - axis **ACCEPT** 	✓ $f'(x)=3ax^2+2$ ✓ shape (max TP) ✓ axis of symmetry on LHS if y-axis ✓y – intercept is below x-axis (4) **[17]**

QUESTION/*VRAAG* 11

11.1	$f(x) = -(x-1)(x-2)(x-4)$ $f(x) = -(x^2 - 3x + 2)(x-4)$ $f(x) = -x^3 + 7x^2 - 14x + 8$	✓ $-(x-1)(x-2)(x-4)$ ✓ $a = 7$ ✓ $b = -14$ ✓ $c = 8$ (4)
11.2	$f(x) = -x^3 + 7x^2 - 14x + 8$ $f'(x) = 0$ $-3x^2 + 14x - 14 = 0$ $3x^2 - 14x + 14 = 0$ $x = \frac{14 \pm \sqrt{14^2 - 4(3)(14)}}{2(3)}$ $= \frac{14 \pm \sqrt{28}}{6}$ $= \frac{7 \pm \sqrt{7}}{3}$ $x = 1{,}45$ or $x = 3{,}22$	✓ $f'(x) = 0$ ✓ $-3x^2 + 14x - 14 = 0$ ✓subs into formula ✓x –value ✓x – value (5)
11.3	$x < 1{,}45$ or $x > 3{,}22$	✓critical values ✓✓notation (3) **[12]**

November 2014

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = x^3$. (5)

8.2 Determine the derivative of: $f(x) = 2x^2 + \frac{1}{2}x^4 - 3$ (2)

8.3 If $y = (x^6 - 1)^2$, prove that $\frac{dy}{dx} = 12x^5\sqrt{y}$, if $x > 1$. (3)

8.4 Given: $f(x) = 2x^3 - 2x^2 + 4x - 1$. Determine the interval on which f is concave up. (4)

[14]

QUESTION 9

Given: $f(x) = (x+2)(x^2 - 6x + 9)$
$\quad\quad = x^3 - 4x^2 - 3x + 18$

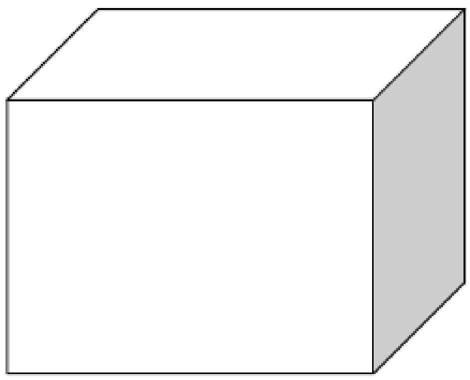
9.1 Calculate the coordinates of the turning points of the graph of f. (6)

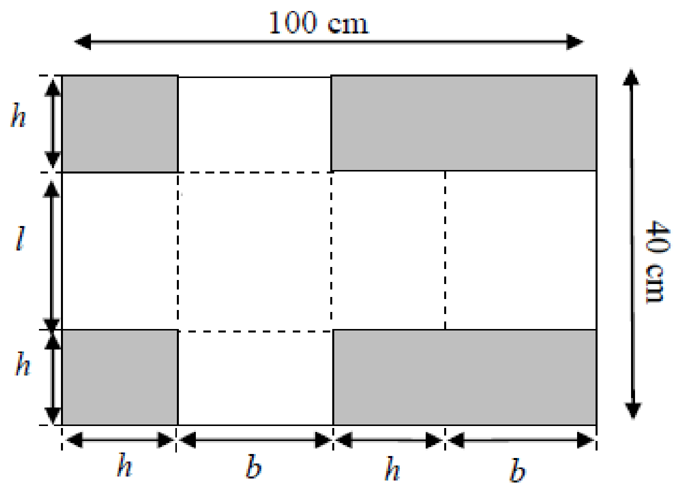
9.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

9.3 For which value(s) of x will $x.f'(x) < 0$? (3)

[13]

QUESTION 10





A box is made from a rectangular piece of cardboard, 100 cm by 40 cm, by cutting out the shaded areas and folding along the dotted lines as shown in the diagram above.

10.1 Express the length l in terms of the height h. (1)

10.2 Hence prove that the volume of the box is given by $V = h(50-h)(40-2h)$ (3)

10.3 For which value of h will the volume of the box be a maximum? (5)

[9]

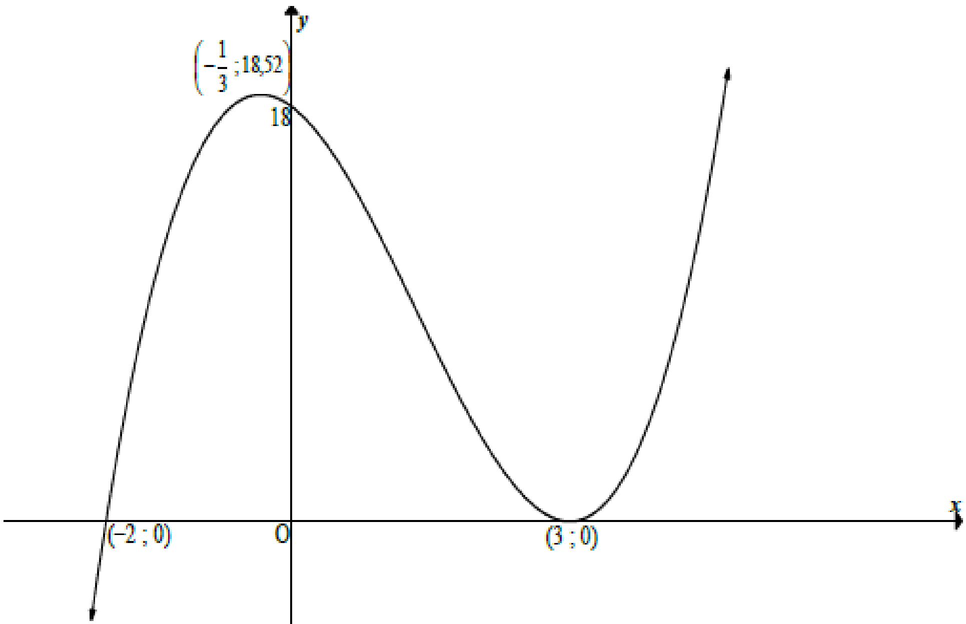
QUESTION/*VRAAG* 8

8.1	$f(x+h)=(x+h)^3=(x^2+2xh+h^2)(x+h)$ $=x^3+x^2h+2x^2h+2xh^2+h^2x+h^3$ $=x^3+3x^2h+3xh^2+h^3$ $f(x+h)-f(x)=x^3+3x^2h+3xh^2+h^3-x^3$ $=3x^2h+3xh^2+h^3$	✓simplifying/*vereenvouding*
	$f'(x)=\lim\limits_{h\to0}\frac{f(x+h)-f(x)}{h}$	✓formula/*formule*
	$=\lim\limits_{h\to0}\frac{3x^2h+3xh^2+h^3}{h}$	✓subst. into formula/*subst. in formule*
	$=\lim\limits_{h\to0}\frac{h(3x^2+3xh+h^2)}{h}$	✓factorization/*faktorisering*
	$=\lim\limits_{h\to0}(3x^2+3xh+h^2)$ $=3x^2$	✓answer/*antwoord* (5)
	OR/OF $f'(x)=\lim\limits_{h\to0}\frac{f(x+h)-f(x)}{h}$	✓formula/*formule*
	$=\lim\limits_{h\to0}\frac{(x+h)^3-x^3}{h}$	✓subst. into formula/*subst. in formule*
	$=\lim\limits_{h\to0}\frac{(x+h)(x+h)^2-x^3}{h}$ $=\lim\limits_{h\to0}\frac{(x+h)(x^2+2xh+h^2)-x^3}{h}$	
	$=\lim\limits_{h\to0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$	✓simplifying/*vereenvoudiging*
	$=\lim\limits_{h\to0}\frac{h(3x^2+3xh+h^2)}{h}$	✓factorization/*faktorisering*
	$=\lim\limits_{h\to0}(3x^2+3xh+h^2)$ $=3x^2$	✓answer/*antwoord* (5)
	OR	

	$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$ $=\lim_{h\to 0}\frac{(x+h-x)(x^2+2xh+h^2+x^2+xh+x^2)}{h}$ $=\lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h}$ $=\lim_{h\to 0}\left(3x^2+3xh+h^2\right)$ $=3x^2$	✓formula/*formule* ✓subst. into formula/*subst. in formule* ✓factorization/*faktorisering* ✓simplifying/*vereenvoudiging* ✓answer/*antwoord* (5)

8.2	$f'(x)=4x+2x^3$	✓$4x$ ✓ $2x^3$ (2)
8.3	$y=x^{12}-2x^6+1$ $\frac{dy}{dx}=12x^{11}-12x^5$ $=12x^5\left(x^6-1\right)$ $=12x^5\sqrt{y}$	✓simplification/*vereenvoudiging* ✓derivative/*afgeleide* ✓factors/*faktore* (3)
8.4	$f(x)=2x^3-2x^2+4x-1$ $f'(x)=6x^2-4x+4$ $f''(x)=12x-4$ f is concave up when/*is konkaaf op as* $f''(x)>0$ $\therefore 12x-4>0$ $12x>4$ $x>\frac{1}{3}$	✓first derivative/*eerste afgeleide* ✓ second derivative/*tweede afgeleide* ✓ $f''(x)>0$ ✓ $x>\frac{1}{3}$ (4) **[14]**

QUESTION/*VRAAG* 9

9.1	$f'(x)=3x^2-8x-3=0$ $(3x+1)(x-3)=0$ $x=-\frac{1}{3}$ or $x=3$ $y=\frac{500}{27}$ $\left(\text{or } y=18\frac{14}{27} \text{ or } 18{,}52\right)$ $y=0$ Turning points are/*Draaipunte is* $\left(-\frac{1}{3};\frac{500}{27}\right)$ and $(3;0)$	✓derivative/*afgeleide* ✓derivative/ *afgeleide* = 0 ✓factors/*faktore* ✓*x*-values/*waardes* ✓✓each *y*-values/*elke y-waarde* (6)
9.2		✓*x*-intercepts/*afsnitte* ✓*y*-intercept/*afsnit* ✓turning points/ *draaipunte* ✓shape/*vorm* (4)
9.3	$x<\frac{-1}{3}$ or $0<x<3$ **OR** $(-\infty;-\frac{1}{3})\cup(0;3)$	✓ $x<\frac{-1}{3}$ ✓both critical points/ *beide kritieke-punte* ✓notation/*notasie* (3)

QUESTION/*VRAAG* 10

10.1	$l + 2h = 40$ $l = 40 - 2h$	✓answer (1)
10.2	$2b + 2h = 100$ $b = 50 - h$ $V = lbh$ $V = h(40-2h)(50-h)$	✓ $2b + 2h = 100$ ✓ $b = 50 - h$ ✓volume formula (3)
10.3	$V = (50h - h^2)(40-2h)$ $V = 2h^3 - 140h^2 + 2000h$ $V' = 6h^2 - 280h + 2000 = 0$ $h = \dfrac{280 \pm \sqrt{(-280)^2 - 4(6)(2000)}}{2(6)}$ $h \neq 37{,}86$ or $h = 8{,}80$ $\therefore$ for a box as large as possible, $h = 8{,}80$ cm *vir die grootste moontlike boks = 8,80 cm*	✓simplifying/*vereenvoudig* ✓derivative / *afgeleide* ✓✓h-values in any form / *h-waardes in enige vorm* ✓answer/*antwoord* (5) **[9]**

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 2$. (5)

8.2 Determine $\dfrac{dy}{dx}$ if $y = 2x^{-4} - \dfrac{x}{5}$. (2)

[7]

QUESTION 9

Given: $f(x) = x^3 - 4x^2 - 11x + 30$.

9.1 Use the fact that $f(2) = 0$ to write down a factor of $f(x)$. (1)

9.2 Calculate the coordinates of the x-intercepts of f. (4)

9.3 Calculate the coordinates of the stationary points of f. (5)

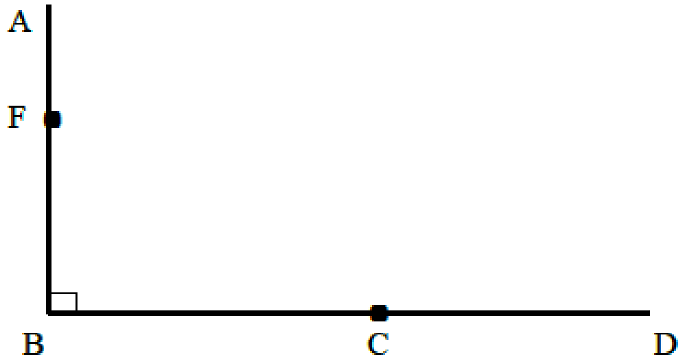
9.4 Sketch the curve of f in your ANSWER BOOK. Show all intercepts with the axes and turning points clearly. (3)

9.5 For which value(s) of x will $f'(x) < 0$? (2)

[15]

QUESTION 10

Two cyclists start to cycle at the same time. One starts at point B and is heading due north to point A, whilst the other starts at point D and is heading due west to point B. The cyclist starting from B cycles at 30 km/h while the cyclist starting from D cycles at 40 km/h. The distance between B and D is 100 km. After time t (measured in hours), they reach points F and C respectively.



10.1 Determine the distance between F and C in terms of t. (4)

10.2 After how long will the two cyclists be closest to each other? (4)

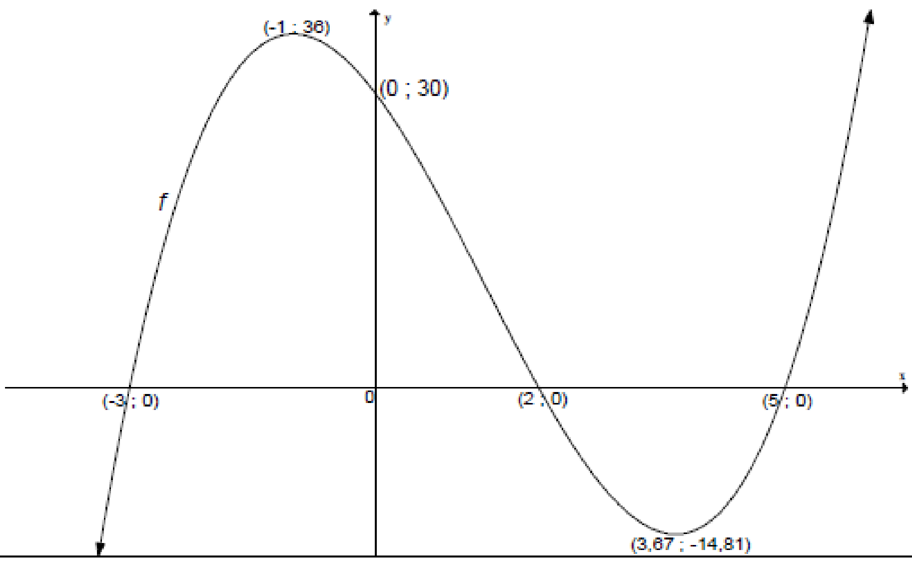
10.3 What will the distance between the cyclists be at the time determined in QUESTION 10.2? (2)

[10]

QUESTION/VRAAG 8

8.1	$f(x)=3x^2-2$ $f(x+h)=3(x+h)^2-2$ $=3x^2+6xh+3h^2-2$ $f(x+h)-f(x)=6xh+3h^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{6xh+3h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(6x+3h)}{h}$ $=\lim\limits_{h\to 0}(6x+3h)$ $=6x$	✓ substitution of of $x+h$ ✓ simplification to $6xh+3h^2$ ✓ formula ✓taking out common factor ✓ answer (5)
	OR $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{[3(x+h)^2-2]-(3x^2-2)}{h}$ $=\lim\limits_{h\to 0}\dfrac{[3(x^2+2xh+h^2)-2]-3x^2+2}{h}$ $=\lim\limits_{h\to 0}\dfrac{[3x^2+6xh+3h^2-2]-3x^2+2}{h}$ $=\lim\limits_{h\to 0}\dfrac{6xh+3h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(6x+3h)}{h}$ $=\lim\limits_{h\to 0}(6x+3h)$ $=6x$	✓ formula ✓ substitution of $x+h$ ✓ simplification to $\dfrac{6xh+3h^2}{h}$ ✓ taking out common factor ✓ answer (5)
8.2	$y=2x^{-4}-\dfrac{x}{5}$ $\dfrac{dy}{dx}=-8x^{-5}-\dfrac{1}{5}$	✓ $-8x^{-5}$ ✓ $-\dfrac{1}{5}$ (2) [7]

QUESTION/VRAAG 9

9.1	$(x-2)$ is a factor of *f* / *is 'n faktor van f.*	✓ answer (1)
9.2	$f(x)=x^3-4x^2-11x+30$ $=(x-2)(x^2-2x-15)$ $=(x-2)(x+3)(x-5)$ $f(x)=0$ $(x+3)(x-2)(x-5)=0$ $x=-3$ or $x=2$ or $x=5$ x-intercepts: $(-3;\ 0);\ (2;0);\ (5;0)$	✓ $(x^2-2x-15)$ ✓$(-3;\ 0)$ ✓$(2;\ 0)$ ✓$(5;\ 0)$ (4)
9.3	$f(x)=x^3-4x^2-11x+30$ $f'(x)=3x^2-8x-11$ At turning points $f'(x)=0$ $(3x-11)(x+1)=0$ $x=-1$ or $x=\frac{11}{3}$ $y=36$ $\quad y=-\frac{400}{27}$ $\quad (-14{,}81)$ TP's are $(-1;36)$ and $\left(\frac{11}{3};-14{,}81\right)$	✓ $f'(x)=3x^2-8x-11$ ✓ $f'(x)=0$ ✓x - value ✓x - value ✓y - values (5)
9.4		✓y and x - intercepts ✓shape ✓turning points (3)

9.5	$f'(x) < 0$ if $-1 < x < 3{,}67$ **OR** $(-1\ ;\ 3{,}67)$	✓extreme values ✓notation (2) ✓extreme values ✓notation (2) **[15]**

QUESTION/VRAAG 10

10.1	After t hours : $BF = 30t$ km and $CD = 40t$ km $\therefore BC = 100 - 40t$ $FC = \sqrt{(30t)^2 + (100-40t)^2}$ $= \sqrt{900t^2 + 10000 - 8000t + 1600t^2}$ $= \sqrt{2500t^2 - 8000t + 10000}$	✓ $BF = 30t$ ✓ $BC = 100 - 40t$ ✓Pythagoras ✓answer (4)
10.2	FC is a minimum when FC^2 is a minimum. $FC^2 = 2500t^2 - 8000t + 10000$ $\frac{dFC^2}{dt} = 5000t - 8000 = 0$ $t = \frac{8000}{5000} = 1{,}6\text{hrs}$ (96 minutes)	✓$FC^2 =$ $2500t^2 - 8000t + 10000$ ✓ $\frac{dFC^2}{dt} = 5000t - 8000$ ✓ $\frac{dFC^2}{dt} = 0$ ✓answer (4)
10.3	$FC = \sqrt{2500t^2 - 8000t + 10000}$ $= \sqrt{2500(1.6)^2 - 8000(1.6) + 10000}$ $= 60$ They will be 60km apart.	✓subs into equation ✓answer (2) **[10]**

QUESTION 8

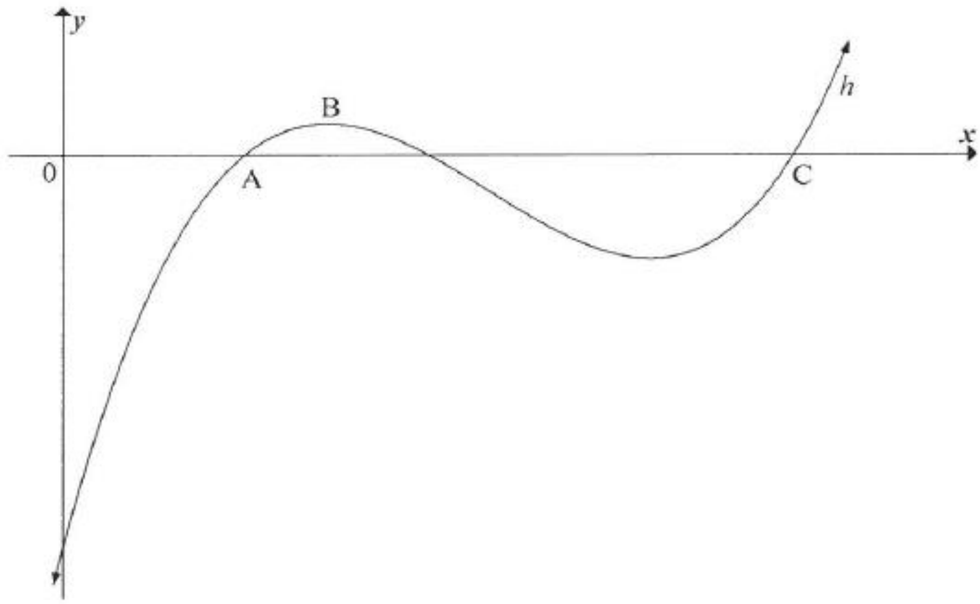
8.1 Determine the derivative of $f(x) = 2x^2 + 4$ from first principles. (4)

8.2 Differentiate:

8.2.1 $f(x) = -3x^2 + 5\sqrt{x}$ (3)

8.2.2 $p(x) = \left(\frac{1}{x^3} + 4x\right)^2$ (4)

8.3 The sketch below shows the graph of $h(x) = x^3 - 7x^2 + 14x - 8$. The x-coordinate of point A is 1. C is another x-intercept of h.



8.3.1 Determine $h'(x)$. (1)

8.3.2 Determine the x-coordinate of the turning point B. (3)

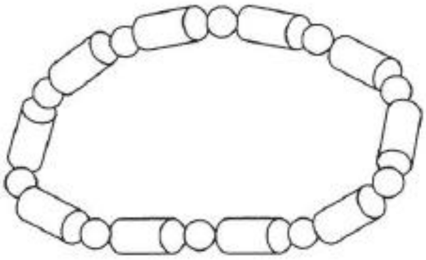
8.3.3 Calculate the coordinates of C. (4)

8.3.4 The graph of h is concave down for $x < k$. Calculate the value of k. (3)

[22]

QUESTION 9

A necklace is made by using 10 wooden spheres and 10 wooden cylinders. The radii, r, of the spheres and the cylinders are exactly the same. The height of each cylinder is h. The wooden spheres and cylinders are to be painted. (Ignore the holes in the spheres and cylinders.)



$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi r h$
$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$

9.1 If the volume of a cylinder is 6 cm^3, write h in terms of r. (1)

9.2 Show that the total surface area (S) of all the painted surfaces of the necklace is equal to $S = 60\pi r^2 + \frac{120}{r}$ (4)

9.3 Determine the value of r so that the least amount of paint will be used. (4)

[9]

QUESTION/*VRAAG* 8

8.1	$f(x+h)=2(x+h)^2+4$ $=2x^2+4xh+2h^2+4$ $f(x+h)-f(x)=2x^2+4xh+2h^2+4-2x^2-4$ $=4xh+2h^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{4xh+2h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(4x+2h)}{h}$ $=\lim\limits_{h\to 0}(4x+2h)$ $=4x$	✓ $2x^2+4xh+2h^2+4$ ✓ $4xh+2h^2$ ✓ $\lim\limits_{h\to 0}\dfrac{h(4x+2h)}{h}$ ✓ $4x$ (4)
8.2.1	$f(x)=-3x^2+5\sqrt{x}$ $f(x)=-3x^2+5x^{\frac{1}{2}}$ $f'(x)=-6x+\dfrac{5}{2}x^{-\frac{1}{2}}$	✓ $5x^{\frac{1}{2}}$ ✓ $-6x$ ✓ $\dfrac{5}{2}x^{-\frac{1}{2}}$ (3)
8.2.2	$p(x)=\left(\dfrac{1}{x^3}+4x\right)^2$ $=\dfrac{1}{x^6}+\dfrac{8}{x^2}+16x^2$ $=x^{-6}+8x^{-2}+16x^2$ $p'(x)=-6x^{-7}-16x^{-3}+32x$ **OR/*OF*** $p(x)=(x^{-3}+4x)^2$ by making use of the chain rule: $p'(x)=2(x^{-3}+4x)(-3x^{-4}+4)$ $p'(x)=-6x^{-7}-16x^{-3}+32x$	✓ $\dfrac{1}{x^6}+\dfrac{8}{x^2}+16x^2$ ✓ $x^{-6}+8x^{-2}+16x^2$ ✓✓ answer/*antwoord* (4) ✓✓ $2(x^{-3}+4x)$ ✓✓ $(-3x^{-4}+4)$ (4)
8.3.1	$h'(x)=3x^2-14x+14$	✓ finding/*kry* $h'(x)$ (1)
8.3.2	At/*By* B: $h'(x)=0$ $3x^2-14x+14=0$ $x=\dfrac{14\pm\sqrt{(-14)^2-4(3)(14)}}{2(3)}$ $=1{,}45$ or $3{,}22$ n/a	✓ derivative equal to/ *afgeleide gelyk aan* 0 ✓ substitution into correct formula/*substitusie in korrekte formule* ✓ x-value of/x-*waarde van* 1,45 (3)

8.3.3	$x^3 - 7x^2 + 14x - 8 = (x-1)(x^2 - 6x + 8)$ $= (x-1)(x-2)(x-4)$ $C(4;0)$ **OR/*OF*** $x_c > 3{,}22$ $h(4) = (4)^3 - 7(4)^2 + 14(4) - 8 = 0$ $\therefore x_c = 4$	✓ $(x-1)$ ✓ $x^2 - 6x + 8$ ✓ $(x-2)(x-4)$ ✓coordinates of/*koördinate van* C (4) ✓ $x_c > 3{,}22$ ✓substitution of/ *substitusie van* 4 ✓ $h(4) = 0$ ✓ x_c (4)
8.3.4	$h^{/}(x) = 3x^2 - 14x + 14$ $h^{//}(x) = 6x - 14$ $6x - 14 < 0$ $6x < 14$ $\therefore x < \frac{7}{3}$ $\therefore k = \frac{7}{3}$	✓ $h^{//}(x) = 6x - 14$ ✓ $6x - 14 < 0$ ✓ $k = \frac{7}{3}$ (3) **[22]**

QUESTION/*VRAAG* 9

9.1	$\pi r^2 h = 6$ $h = \dfrac{6}{\pi r^2}$	✓ $h = \dfrac{6}{\pi r^2}$ (1)
9.2	$S = 10\left(2\pi r^2 + 2\pi rh + 4\pi r^2\right)$ $= 10\left[2\pi rh + 6\pi r^2\right]$ $= 20\pi rh + 60\pi r^2$ $= 20\pi r\left(\dfrac{6}{\pi r^2}\right) + 60\pi r^2$ $= 60\pi r^2 + \dfrac{120}{r}$ **OR/*OF*** Area of/*van* 10 spheres/*sfere* $= 10 \times 4 \times \pi \times r^2 = 40\pi r^2$ Area of/*van* 10 cylinders/*silinders* $= 10(2\pi r^2 + 2\pi r h)$ $= 10(2\pi r^2 + 2\pi r \dfrac{6}{\pi r^2})$ $= 20\pi r^2 + \dfrac{120}{r}$ Total area/*Totale area* $= 40\pi r^2 + 20\pi r^2 + \dfrac{120}{r}$ $= 60\pi r^2 + \dfrac{120}{r}$	✓✓ $10\left(2\pi r^2 + 2\pi rh + 4\pi r^2\right)$ ✓ $20\pi rh + 60\pi r^2$ ✓ substitution/*substitusie* (4) ✓area of 10 spheres/ *area van 10 sfere* ✓ area of 10 cylinders/ *area van 10 silinders* ✓ substitution/*substitusie* ✓simplification/*vereen-voudiging* (4)
9.3	$S' = 120\pi r - 120 r^{-2} = 0$ $120\pi r - \dfrac{120}{r^2} = 0$ $120\pi r^3 - 120 = 0$ $r^3 = \dfrac{120}{120\pi}$ $\therefore r = \dfrac{1}{\pi^{\frac{1}{3}}} = 0{,}68$ cm	✓ $120\pi r - 120 r^{-2}$ ✓ $= 0$ ✓ $r^3 = \dfrac{120}{120\pi}$ ✓answer/*antwoord* (4) **[9]**

QUESTION 8

8.1 If $f(x) = x^2 - 3x$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\dfrac{dy}{dx}$ if $y = \left(x^2 - \dfrac{1}{x^2}\right)^2$ (3)

8.2.2 $D_x\left(\dfrac{x^3 - 1}{x - 1}\right)$ (3)

[11]

QUESTION 9

Given: $h(x) = -x^3 + ax^2 + bx$ and $g(x) = -12x$. P and Q(2 ; 10) are the turning points of h. The graph of h passes through the origin.

9.1 Show that $a = \dfrac{3}{2}$ and $b = 6$. (5)

9.2 Calculate the average gradient of h between P and Q, if it is given that $x = -1$ at P. (4)

9.3 Show that the concavity of h changes at $x = \dfrac{1}{2}$. (3)

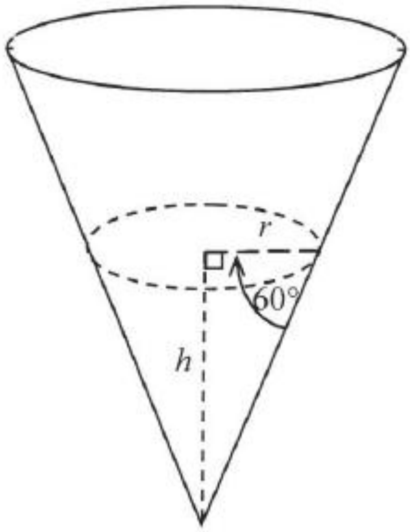
9.4 Explain the significance of the change in QUESTION 9.3 with respect to h. (1)

9.5 Determine the value of x, given $x < 0$, at which the tangent to h is parallel to g. (4)

[17]

QUESTION 10

A rain gauge is in the shape of a cone. Water flows into the gauge. The height of the water is h cm when the radius is r cm. The angle between the cone edge and the radius is 60°, as shown in the diagram below.



Formulae for volume:

$V = \pi r^2 h$ $\qquad$ $V = \frac{1}{3}\pi r^2 h$

$V = lbh$ $\qquad$ $V = \frac{4}{3}\pi r^3$

10.1 Determine r in terms of h. Leave your answer in surd form. (2)

10.2 Determine the derivative of the volume of water with respect to h when h is equal to 9 cm. (5)

[7]

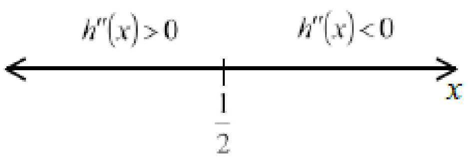
QUESTION/*VRAAG* 8

8.1	$f(x+h)=(x+h)^2-3(x+h)$ $=x^2+2xh+h^2-3x-3h$ $f(x+h)-f(x)=x^2+2xh+h^2-3x-3h-(x^2-3x)$ $=2xh+h^2-3h$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2-3h}{h}$ $=\lim_{h\to 0}\frac{h(2x+h-3)}{h}$ $=\lim_{h\to 0}(2x+h-3)$ $=2x-3$ **OR/*OF*** $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{(x+h)^2-3(x+h)-(x^2-3x)}{h}$ $=\lim_{h\to 0}\frac{x^2+2xh+h^2-3x-3h-x^2+3x}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2-3h}{h}$ $=\lim_{h\to 0}\frac{h(2x+h-3)}{h}$ $=\lim_{h\to 0}(2x+h-3)$ $=2x-3$	✓finding $f(x+h)$ ✓ $2xh+h^2-3h$ ✓formula ✓factorisation ✓answer (5) ✓formula ✓finding $f(x+h)$ ✓ $2xh+h^2-3h$ ✓factorisation ✓answer (5)
8.2.1	$y=\left(x^2-\frac{1}{x^2}\right)^2$ $y=x^4-2+\frac{1}{x^4}$ $=x^4-2+x^{-4}$ $\frac{dy}{dx}=4x^3-4x^{-5}$ **OR/*OF***	✓ $x^4-2+\frac{1}{x^4}$ ✓ $4x^3$ ✓ $-4x^{-5}$ (3)

	By using the chain rule (which is not part of CAPS): $y=\left(x^2-x^{-2}\right)^2$ $\frac{dy}{dx}=2\left(x^2-x^{-2}\right)\left(2x+2x^{-3}\right)$ $=2\left(2x^3+2x^{-1}-2x^{-1}-2x^{-5}\right)$ $=2\left(2x^3-2x^{-5}\right)$ $=4x^3-4x^{-5}$	✓✓✓ $2\left(x^2-x^{-2}\right)\left(2x+2x^{-3}\right)$ (3)
8.2.2	$D_x\left[\frac{(x-1)(x^2+x+1)}{x-1}\right]$ $=D_x\left[x^2+x+1\right]$ $=2x+1$ **OR/*OF*** By using the quotient rule (with is not part of CAPS): $D_x\left[\frac{x^3-1}{x-1}\right]$ $=\frac{3x^2(x-1)-\left(x^3-1\right)}{(x-1)^2}$	✓factorisation ✓ x^2+x+1 ✓ $2x+1$ (3) ✓✓✓ $\frac{3x^2(x-1)-\left(x^3-1\right)}{(x-1)^2}$ (3) **[11]**

QUESTION/*VRAAG* 9

9.1	Substitute Q(2; 10) into $h(x) = -x^3 + ax^2 + bx$ $-2^3 + a(2^2) + b(2) = 10$ $-8 + 4a + 2b = 10$ $2a + b = 9$ ....line 1 $h'(x) = -3x^2 + 2ax + b$ At Q : $h'(2) = 0$ $-3(2)^2 + 2a(2) + b = 0$ $-12 + 4a + b = 0$ $4a + b = 12$ ....line 2 line 2 − line 1 : $2a = 3$ $a = \frac{3}{2}$ Substitute in line 1 : $b = 6$	✓substitute Q into h ✓finding derivative ✓ $h'(2)$ ✓equating derivative to 0 ✓solving simultaneously for a and b (5)
9.2	$f(-1) = -(-1)^3 + \frac{3}{2}(-1)^2 + 6(-1)$ $= -3{,}5$ Average gradient/*Gemiddelde gradiënt* $= \dfrac{f(x_Q) - f(x_P)}{x_Q - x_P}$ Average gradient/ *Gemiddelde gradiënt* $= \dfrac{10-(-3{,}5)}{2-(-1)}$ $= 4{,}5$	✓ $f(-1) = -3{,}5$ ✓formula ✓substitution ✓answer (4)

9.3	$h'(x)=-3x^2+3x+6$ $h''(x)=-6x+3$ $=-3(2x-1)$  For $x<\frac{1}{2}$, h is concave up and for $x>\frac{1}{2}$, h is concave down *Vir $x<\frac{1}{2}$, is h konkaaf na bo en vir $x>\frac{1}{2}$, is h konkaaf na onder* $\therefore$ concavity changes at $x=\frac{1}{2}$ / $\therefore$ *konkwiteit verander by* $x=\frac{1}{2}$	✓$h'(x)=-3x^2+3x+6$ ✓$h''(x)=-6x+3$ ✓explanation using $h''(x)$ (3)
9.4	The graph of h has a point of inflection at $x=\frac{1}{2}$ / *Die grafiek van h het 'n buigpunt by* $x=\frac{1}{2}$. **OR/*OF*** The graph of h changes from concave up to concave down at $x=\frac{1}{2}$ / *Die grafiek van h verander by $x=\frac{1}{2}$ van konkaaf op na konkaaf af*	✓answer (1) ✓answer (1)
9.5	Gradient of g is -12/*Gradiënt van g is -12* Gradient of tangent is/*Gradiënt van die raaklyn is:* $h'(x)=-3x^2+3x+6$ $h'(x)=-12$ $-3x^2+3x+6=-12$ $3x^2-3x+18=0$ $x^2-x+6=0$ $(x-3)(x+2)=0$ $x=-2$ only	✓$h'(x)=-3x^2+3x+6$ ✓$h'(x)=-12$ ✓factors ✓selection of x-value (4) **[17]**

QUESTION/*VRAAG* 10

10.1	$\frac{h}{r} = \tan 60^0$ $r = \frac{h}{\tan 60^0}$ $\therefore r = \frac{h}{\sqrt{3}}$	✓ $\frac{h}{r} = \tan 60^0$ ✓answer (2)
10.2	$V_{cone} = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h$ $= \frac{1}{9}\pi h^3$ $\frac{dV}{dh} = \frac{1}{3}\pi h^2$ $\left.\frac{dV}{dh}\right\|_{h=9} = \frac{1}{3}\pi (9)^2$ $= 27\pi$ or $84{,}82\ \text{cm}^3/\text{cm}$	✓formula ✓substitution of the value of r in terms of h ✓simplified volume answer ✓derivative ✓answer (5) [7]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = -x^2 + 4$. (5)

8.2 Determine the derivative of:

8.2.1 $y = 3x^2 + 10x$ (2)

8.2.2 $f(x) = \left(x - \frac{3}{x}\right)^2$ (3)

8.3 Given: $f(x) = 2x^3 - 23x^2 + 80x - 84$

8.3.1 Prove that $(x-2)$ is a factor of f. (2)

8.3.2 Hence, or otherwise, factorise $f(x)$ fully. (2)

8.3.3 Determine the x-coordinates of the turning points of f. (4)

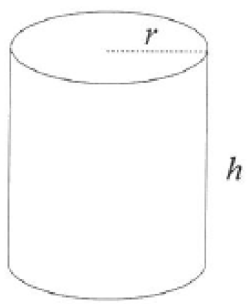
8.3.4 Sketch the graph of f, clearly labelling ALL turning points and intercepts with the axes. (3)

8.3.5 Determine the coordinates of the y-intercept of the tangent to f that has a slope of 40 and touches f at a point where the x-coordinate is an integer. (6)

[27]

QUESTION 9

A soft drink can has a volume of 340 cm^3, a height of h cm and a radius of r cm.



9.1 Express h in terms of r. (2)

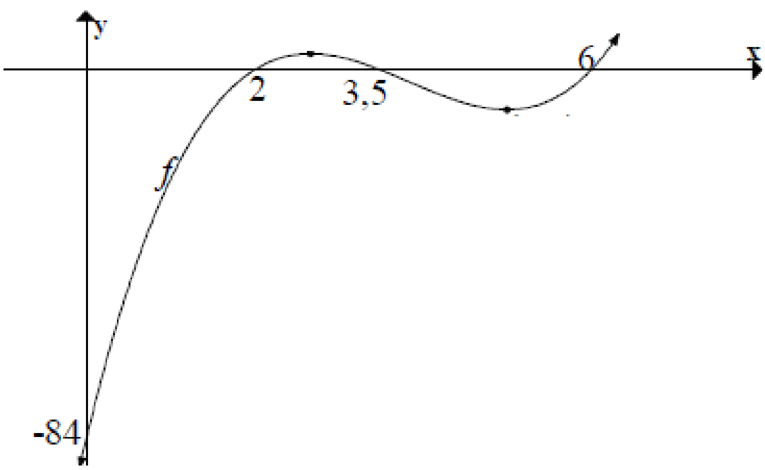
9.2 Show that the surface area of the can is given by $A(r) = 2\pi r^2 + 680r^{-1}$. (2)

9.3 Determine the radius of the can that will ensure that the surface area is a minimum. (4)

[8]

QUESTION/*VRAAG* 8

8.1	$f(x+h)=-(x+h)^2+4=-(x^2+2xh+h^2)+4$ $=-x^2-2xh-h^2+4$ $f(x+h)-f(x)=-2xh-h^2$ $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\dfrac{-2xh-h^2}{h}$ $=\lim_{h\to 0}\dfrac{h(-2x-h)}{h}$ $=\lim_{h\to 0}(-2x-h)$ $=-2x$ **OR/*OF*** $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\dfrac{-(x+h)^2+4-\left(-x^2+4\right)}{h}$ $=\lim_{h\to 0}\dfrac{-x^2-2xh-h^2+4+x^2-4}{h}$ $=\lim_{h\to 0}\dfrac{-2xh-h^2}{h}$ $=\lim_{h\to 0}\dfrac{h(-2x-h))}{h}$ $=\lim_{h\to 0}(-2x-h)$ $=-2x$	✓ finding $f(x+h)$ ✓ $-2xh-h^2$ ✓formula ✓factorisation ✓answer (5) ✓formula ✓ finding $f(x+h)$ ✓ $-2xh-h^2$ ✓factorisation ✓answer (5)
8.2.1	$y=3x^2+10x$ $\dfrac{dy}{dx}=6x+10$	✓ $6x$ ✓ 10 (2)
8.2.2	$f(x)=\left(x-\dfrac{3}{x}\right)^2$ $=x^2-6+\dfrac{9}{x^2}$ $=x^2-6+9x^{-2}$ $f'(x)=2x-18x^{-3}$	✓ $x^2-6+\dfrac{9}{x^2}$ ✓ $9x^{-2}$ ✓ $2x-18x^{-3}$ (3)

8.3.1	$f(2)=2(2)^3-23(2)^2+80(2)-84$ $=0$ $\therefore (x-2)$ is a factor	✓substitution of 2 into *f* ✓value of 0 (2)
8.3.2	$f(x)=2x^3-23x^2+80x-84$ $=(x-2)(2x^2-19x+42)$ $=(x-2)(2x-7)(x-6)$	✓ $2x^2-19x+42$ ✓ $(x-2)(2x-7)(x-6)$ (2)
8.3.3	$f'(x)=6x^2-46x+80$ $6x^2-46x+80=0$ $3x^2-23x+40=0$ $(3x-8)(x-5)=0$ $x=\frac{8}{3}$ or $x=5$	✓ $f'(x)=6x^2-46x+80$ ✓ $f'(x)=0$ ✓factors ✓*x*-values (4)
8.3.4		✓*x*-intercepts ✓*y*-intercept ✓shape (3)
8.3.5	$6x^2-46x+80=40$ $6x^2-46x+40=0$ $3x^2-23x+20=0$ $(3x-20)(x-1)=0$ $x=\frac{20}{3}$ or $x=1$ But *x* must be an integer, so $x=1$ at the point where tangent touches *f* /*x moet heelgetal wees so x = 1 by punt waar die raaklyn f raak:* $y=f(1)=2(1)^3-23(1)^2+80(1)-84=-25$ $y=mx+c$ $-25=40(1)+c$ $-65=c$ $(0;-65)$	✓ $6x^2-46x+80=40$ ✓factors ✓$x=1$ ✓ *y*-value ✓ $-25=40(1)+c$ ✓answer (6) **[27]**

QUESTION/*VRAAG* 9

9.1	$340 = \pi r^2 h$ $\therefore h = \dfrac{340}{\pi r^2}$	✓substitution into volume formula ✓answer (2)
9.2	$A = 2\pi r^2 + 2\pi rh$ $= 2\pi r^2 + 2\pi r\left(\dfrac{340}{\pi r^2}\right)$ $= 2\pi r^2 + 680r^{-1}$	✓formula ✓substitution of h (2)
9.3	$A(r) = 2\pi r^2 + 680r^{-1}$ $A'(r) = 4\pi r - 680r^{-2}$ $4\pi r - 680r^{-2} = 0$ $4\pi r = \dfrac{680}{r^2}$ $r^3 = \dfrac{680}{4\pi}$ $r = \sqrt[3]{\dfrac{680}{4\pi}}$ cm or $3{,}78$ cm	✓ $4\pi r$ ✓ $-680r^{-2}$ ✓ $r^3 = \dfrac{680}{4\pi}$ ✓answer (4) **[8]**

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2$ (5)

8.2 John determines $g'(a)$, the derivative of a certain function g at $x = a$, and arrives at the answer: $\lim_{h\to 0}\frac{\sqrt{4+h}-2}{h}$
Write down the equation of g and the value of a. (2)

8.3 Determine $\frac{dy}{dx}$ if $y = \sqrt{x^3} - \frac{5}{x^3}$ (4)

8.4 $g(x) = -8x + 20$ is a tangent to $f(x) = x^3 + ax^2 + bx + 18$ at $x = 1$. Calculate the values of a and b. (5)
[16]

QUESTION 9

For a certain function f, the first derivative is given as $f'(x) = 3x^2 + 8x - 3$

9.1 Calculate the x-coordinates of the stationary points of f. (3)

9.2 For which values of x is f concave down? (3)

9.3 Determine the values of x for which f is strictly increasing. (2)

9.4 If it is further given that $f(x) = ax^3 + bx^2 + cx + d$ and $f(0) = -18$, determine the equation of f. (5)
[13]

QUESTION 10

The number of molecules of a certain drug in the bloodstream t hours after it has been taken is represented by the equation $M(t) = -t^3 + 3t^2 + 72t$, $0 < t < 10$.

10.1 Determine the number of molecules of the drug in the bloodstream 3 hours after the drug was taken. (2)

10.2 Determine the rate at which the number of molecules of the drug in the bloodstream is changing at exactly 2 hours after the drug was taken. (3)

10.3 How many hours after taking the drug will the rate at which the number of molecules of the drug in the bloodstream is changing, be a maximum? (3)
[8]

***** **SOLUTIONS TO FOLLOW** *****

QUESTION 7

7.1 Determine $f'(x)$ from first principles if $f(x) = x^2 - 5$. (5)

7.2 Determine the derivative of: $g(x) = 5x^2 - \frac{2x}{x^3}$ (3)

7.3 Given: $h(x) = ax^2$, $x > 0$.
Determine the value of a if it is given that $h^{-1}(8) = h'(4)$. (6)

[14]

QUESTION 8

Given: $f(x) = 2x^3 - 5x^2 + 4x$

8.1 Calculate the coordinates of the turning points of the graph of f. (5)

8.2 Prove that the equation $2x^3 - 5x^2 + 4x = 0$ has only one real root. (3)

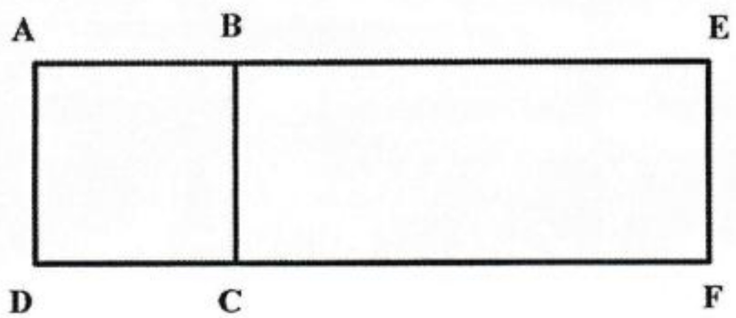
8.3 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (3)

8.4 For which values of x will the graph of f be concave up? (3)

[14]

QUESTION 9

A piece of wire 6 metres long is cut into two pieces. One piece, x metres long, is bent to form a square ABCD. The other piece is bent into a U-shape so that it forms a rectangle BEFC when placed next to the square, as shown in the diagram below.



Calculate the value of x for which the sum of the areas enclosed by the wire will be a maximum. **[7]**

******* SOLUTIONS TO FOLLOW *******

IEB - 2012

QUESTION 4

(a) Given: $f(x) = -2x^2$. Determine $f'(x)$ from first principles. (5)

(b) Find $\frac{dy}{dx}$ for $y = 3\sqrt{x^3} + \frac{4}{\sqrt{x}} - \sqrt{2}$.
The exponents in your answer must be positive values. (5)

(c) Given $f(x) = x^3 + 3x^2 + x + 1$.

(1) Show that the tangent to the curve $y = f(x)$ at the point where $x = -2$ is $y = x + 5$. (6)

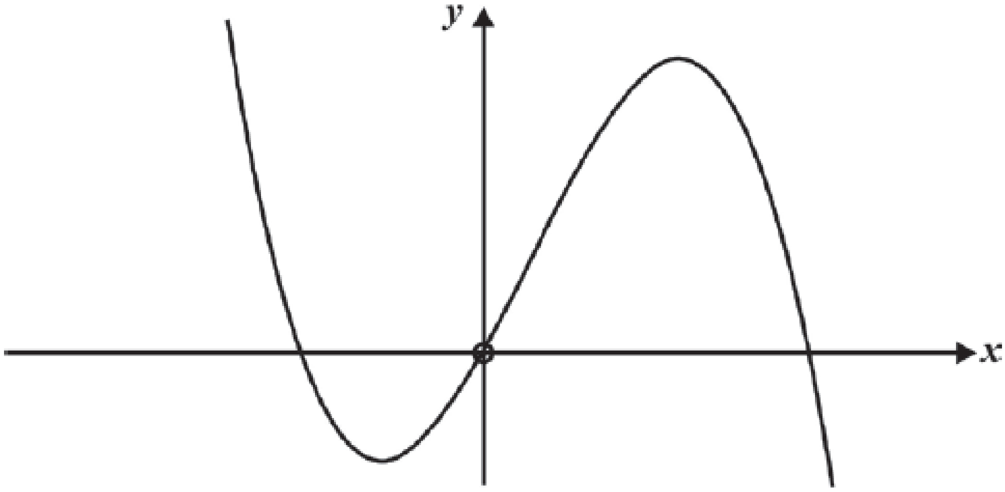
(2) Determine the x-coordinate of the point where this tangent intersects the curve again. (5)

[21]

QUESTION 8

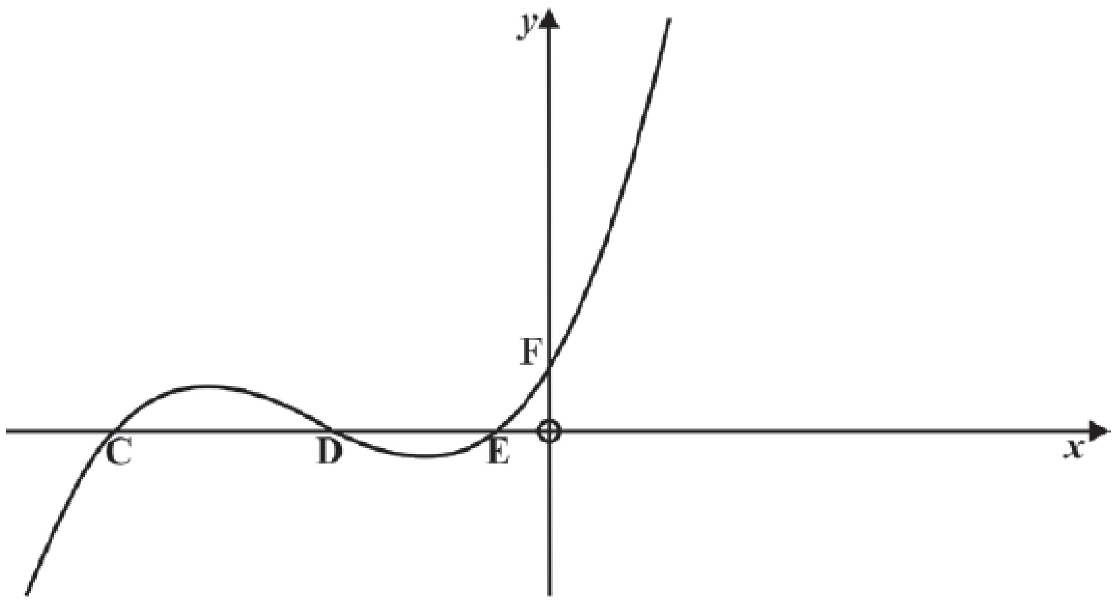
(a) Find the values of x and y if $x + y = 60$ and $K = xy^3$ is a maximum. (5)

(b) Refer to the figure showing the graph of $y = f(x)$.



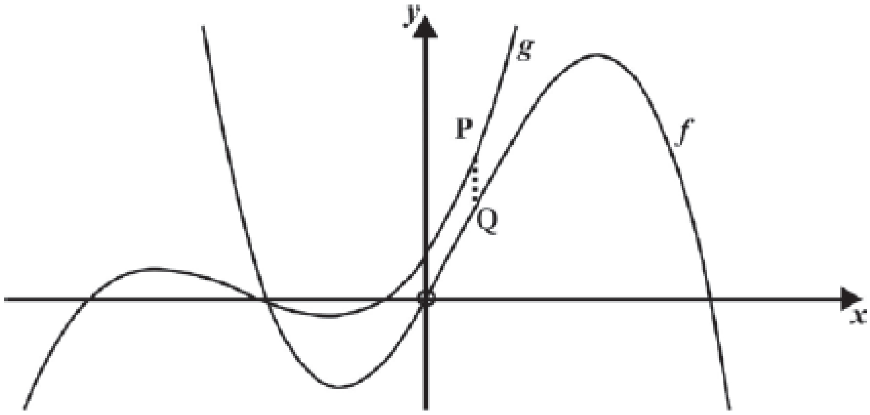
In your answer book, draw a copy of the above graph, then on the same axes, add a possible sketch of $y = f'(x)$. (3)

(c) Refer to the figure showing the graph of cubic function $y = g(x)$ that has intercepts with axes C(– 4 ; 0), D(– 2 ; 0), E(– 0,5 ; 0) and F(0 ; 8).



Determine the equation of the graph. You do not need to simplify your answer. (3)

(d) Refer to the figure showing the graphs of cubic functions $f(x) = -4x^3 + 6x^2 + 26x$ and $g(x) = 2x^3 + 13x^2 + 22x + 8$ with PQ the vertical distance between the graphs.

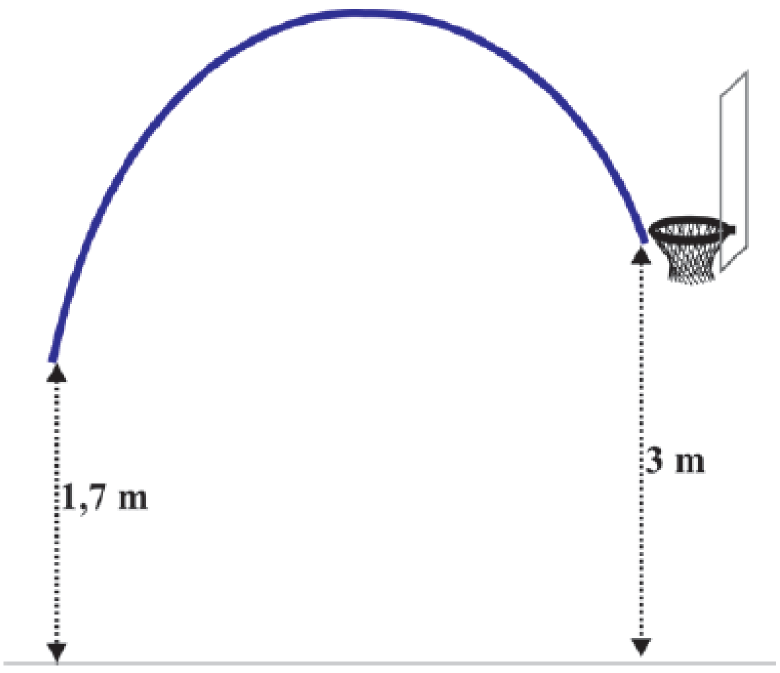


Calculate the minimum length of PQ for $x > 0$. (7)

[18]

QUESTION 9

Tashmira, an enthusiastic basketball player, is practising her shooting.



She throws from a point 1,7 m from the floor. Each throw follows the path of a parabola. On one of her throws, the ball reaches its maximum height of 3,1625 m when it has covered a horizontal distance of 3 m. Unfortunately, the ball does not go into the basket but hits the front of the rim which is 3 m above the floor.

Determine how far Tashmira is from the rim, that is: the horizontal distance between Tashmira's hand and the front of the rim.

[6]

QUESTION 4

(a)

$$f(x) = -2x^2$$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{-2(x+h)^2-(-2x^2)}{h}$$

$$= \lim_{h\to 0}\frac{-2(x^2+2hx+h^2)+2x^2}{h}$$

$$= \lim_{h\to 0}\frac{-2h(2x+h)}{h}$$

$$= \lim_{h\to 0} -2(2x+h)$$

$$= -4x$$ (5)

(b)

$$y = 3\sqrt{x^3}+\frac{4}{\sqrt{x}}-\sqrt{2}$$

$$= 3x^{\frac{3}{2}}+4x^{-\frac{1}{2}}-\sqrt{2}$$

$$\frac{dy}{dx} = 3\times\frac{3}{2}.x^{\frac{1}{2}}+4\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$$

$$= \frac{9\sqrt{x}}{2}-\frac{2}{\sqrt{x^3}} \quad \text{or} \quad \frac{9x^{\frac{1}{2}}}{2}-\frac{2}{x^{\frac{3}{2}}}$$ (5)

(c) $f(x) = x^3+3x^2+x+1$

(1)

$$f(-2) = (-2)^3+3(-2)^2+(-2)+1$$

$$= 3$$

$$f'(x) = 3x^2+6x+1$$

$$f'(-2) = 3(-2)^2+6(-2)+1$$

$$= 1$$

Eqn. of Tangent: $y-3 = 1(x+2)$

$$y = x+2+3$$

$$y = x+5$$ (6)

(2)

$$x^3+3x^2+x+1 = x+5$$

$$x^3+3x^2-4 = 0$$

$$(x+2)^2(x-1) = 0$$

Other intersection point has $x=1$ (5)

[21]

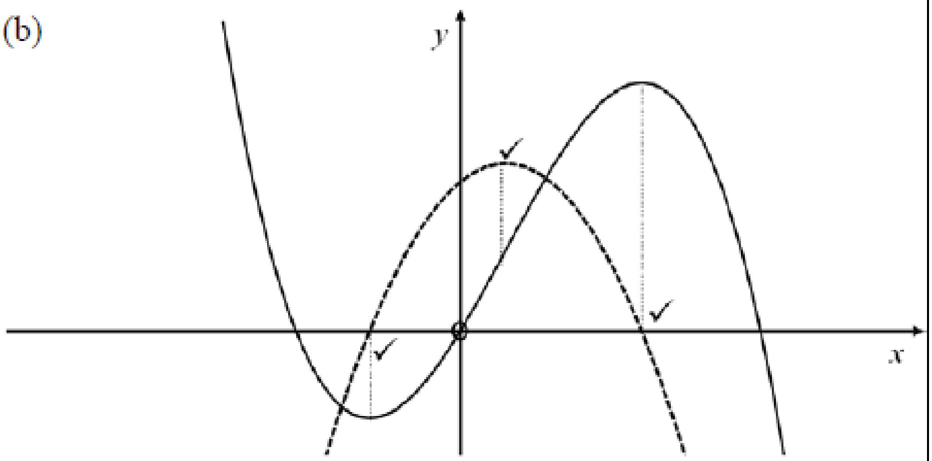
QUESTION 8

(a)
$$x + y = 60$$
$$x = 60 - y$$

$$K = xy^3$$
$$= (60 - y)y^3$$
$$= 60y^3 - y^4$$
$$\frac{dK}{dy} = 180y^2 - 4y^3 = 0$$
$$4y^2(45 - y) = 0$$
$y = 0$ or $y = 45$

✗ Gives Min $x = 15$ (5)

(b)



(3)

(c)
$$y = a(x+4)(x+2)(x+0,5)$$
$$8 = a(0+4)(0+2)(0+0,05)$$
$$= a(4)$$
$$a = 2$$
$$y = 2(x+4)(x+2)(x+0,5)$$ (3)

(d)
$$PQ = (2x^3 + 13x^2 + 22x + 8) - (-4x^3 + 6x^2 + 26x)$$
$$= 6x^3 + 7x^2 - 4x + 8$$
$$\frac{dPQ}{dx} = 18x^2 + 14x - 4$$
$$9x^2 + 7x - 2 = 0$$
$$(9x - 2)(x + 1) = 0$$
$\therefore x = \frac{2}{9}$ or $x = -1$

✗ (N.V. $x > 0$)

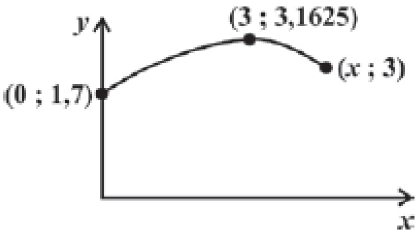
Min. $PQ = 6\left(\frac{2}{9}\right)^3 + 7\left(\frac{2}{9}\right)^2 - 4\left(\frac{2}{9}\right) + 8$

$= 7,5226 \ldots$

$\approx 7,5$ (7)

QUESTION 9

Place Tashmira on y-axis as shown in diagram.



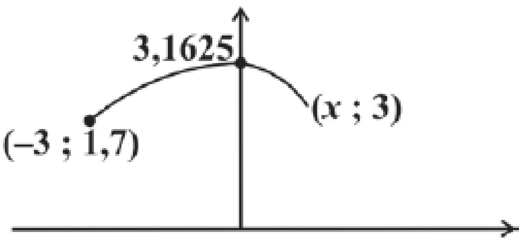
$$
\begin{aligned}
y &= a(x-3)^2 + 3{,}1625 \\
1{,}7 &= a(9) + 3{,}1625 \\
9a &= -1{,}4625 \\
a &= -0{,}1625
\end{aligned}
$$

$$-0{,}1625(x-3)^2 = -0{,}1625$$

$$
\begin{aligned}
-0{,}1625(x-3)^2 &= -0{,}1625 \\
(x-3)^2 &= 1 \\
x-3 &= \pm 1 \\
x &= 3+1 \text{ or } x = 3-1 \\
&= 4 \quad \text{or} \quad = 2
\end{aligned}
$$

N.V. (Before T.P)

ALTERNATIVELY
Place y-axis to go through T.P.



$$
\begin{aligned}
y &= ax^2 + 3{,}1625 \\
1{,}7 &= a(-3)^2 + 3{,}1625 \\
&= 9a + 3{,}1625 \\
9a &= -1{,}4625 \\
a &= -0{,}1625 \\
y &= -0{,}1625x^2 + 3{,}1625 \\
3 &= -0{,}1625x^2 + 3{,}1625 \\
0{,}1625x^2 &= 0{,}1625 \\
x^2 &= 1 \\
x &= 1 \quad (\text{Net is right of } y\text{-axis})
\end{aligned}
$$

$\therefore$ Distance from Tashmira to net is 4 m

[6]

QUESTION 3

(a) Given $f(x) = 6x^2$, determine $f'(x)$ from first principles. (4)

(b) Determine $f'(x)$ given $f(x) = \dfrac{3x^4 + 7x^2 - 5x}{2x^2}$.
Leave your answer with positive exponents. (4)

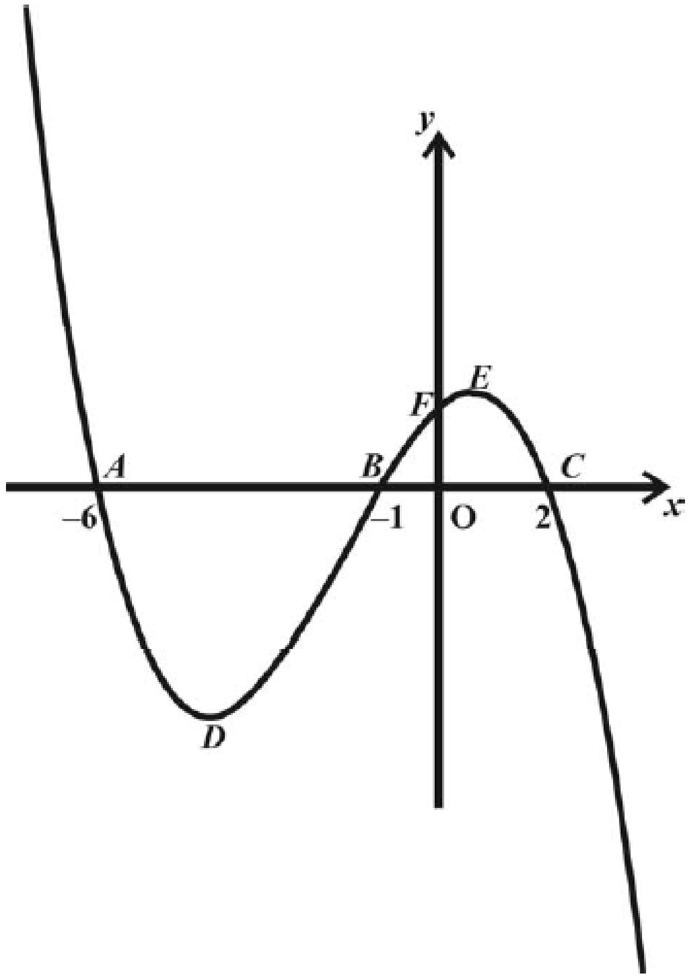
(c) Given: $f(x) = x^3 - 7x^2 + 7x + 15$
Determine the average gradient of the curve between the points where $x = -1$ and $x = 1$. (3)

[11]

QUESTION 8

Refer to the figure showing the graph of a cubic function $f(x) = ax^3 + bx^2 + cx + d$.

A(–6 ; 0), B(–1 ; 0), C(2 ; 0) and F(0 ; 24) are intercepts with the axes, with D and E as turning points.



(a) Show that $a = -2$, $b = -10$, $c = 16$ and $d = 24$. (5)

(b) Determine the coordinates of D. (6)

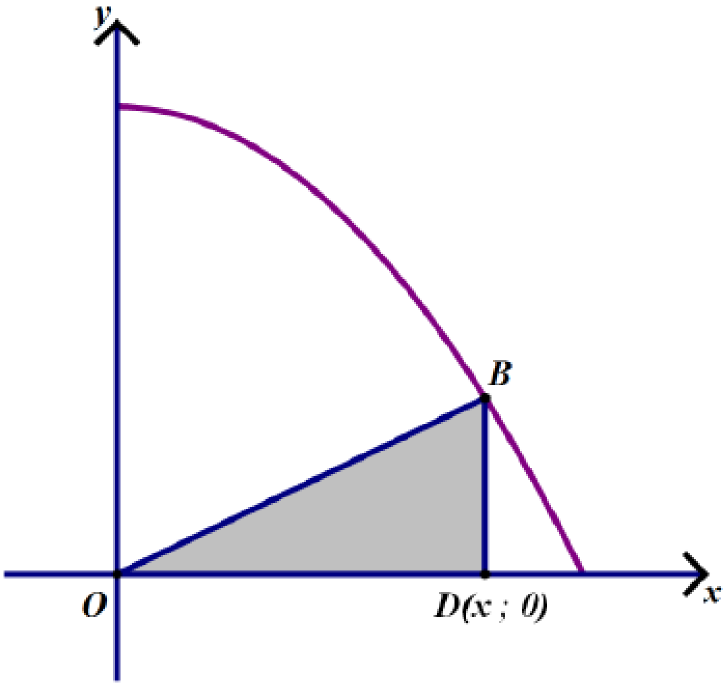
(c) Suppose that the graph is translated in such a way that the point D is moved to the origin. That is, the new graph has equation $y = f(x - p) + q$, where p and q are constants.
Write down the values of p and q. (2)

[13]

QUESTION 9

Refer to the figure showing the parabola given by $f(x) = 4 - \frac{x^2}{4}$ with $0 \le x \le 4$.

D is the point (x ; 0) and DB is parallel to the y-axis, with B on the graph of f.



(a) Write down the coordinates of B in terms of x. (2)

(b) Show that the area, A, of ΔOBD is given by: $A = 2x - \frac{x^3}{8}$. (3)

(c) Determine how far D should be from O in order that the area of ΔOBD is as large as possible. (5)

(d) Hence, calculate the area of ΔOBD when D is at the point determined in (c). (2)

[12]

QUESTION 3

(a) $f(x) = 6x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{6(x+h)^2 - 6x^2}{h} \text{ A}$$

$$= \lim_{h \to 0} \frac{6}{h}\left[x^2 + 2hx + h^2 - x^2\right] \text{ M (expanding)}$$

$$= \lim_{h \to 0} \frac{6h}{h}\left[2x + h\right]$$

$$= \lim_{h \to 0} (12x + 6h) \text{ A}$$

$$= 12x \text{ CA}$$

P1N (penalty for notation)

Answer only : 0 (4)

(b) $$f(x) = \frac{3x^4 + 7x^2 - 5x}{2x^2}$$

$$= \frac{3x^2}{2} + \frac{7}{2} - \frac{5x^{-1}}{2} \text{ MA}$$

$$f'(x) = \frac{3}{2}.2x + \frac{5}{2}.x^{-2} \text{ M (Finding derivative)}$$

$$= 3x + \frac{5}{2x^2} \text{ A}$$

No marks for $\frac{12x^3 + 14x - 5}{4x}$ (4)

(c) $$\text{Average Gradient } = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{f(1) - f(-1)}{1 - (-1)} \text{ M (Gradient formula)}$$

$$= \frac{16 - 0}{2} \text{ A (for 16 and 0)}$$

$$= 8 \text{ CA}$$

Beware of average of 2 derivatives (3)

[11]

QUESTION 8

(a)

$$f(x) = a(x+6)(x+1)(x-2) \quad ^{\text{A}}$$
$$24 = a(6)(1)(-2) \quad ^{\text{M (Sub (0;24) into expression)}}$$
$$= -12a$$
$$a = -2$$
$$f(x) = -2(x+6)(x+1)(x-2) \quad ^{\text{CA}}$$
$$= -2(x+6)(x^2 - x - 2)$$
$$= -2(x^3 - x^2 - 2x + 6x^2 - 6x - 12) \quad ^{\text{M (Expanding)}}$$
$$= -2(x^3 + 5x^2 - 8x - 12) \quad ^{\text{A}}$$
$$= -2x^3 - 10x^2 + 16x + 24$$
$$\therefore a = -2,\ b = -10,\ c = 16 \text{ and } d = 24$$

(5)

(b)

At D and E, $f'(x) = 0$ $^{\text{M (Derivative = 0)}}$

$$-6x^2 - 20x + 16 = 0 \quad ^{\text{A (Derivative)}}$$
$$3x^2 + 10x - 8 = 0$$
$$(3x-2)(x+4) = 0 \quad ^{\text{A}}$$
$$x = \frac{2}{3} \quad \text{or} \quad x = -4 \quad ^{\text{CA}}$$
$$f(-4) = -2(-4)^3 - 10(-4)^2 + 16(-4) + 24$$

M (Sub <u>negative</u> value)

ALTERNATIVELY

$$f(-4) = -2(2)(-3)(-6)$$
$$= -72 \quad ^{\text{A}}$$
$$\therefore \mathrm{D}(-4\,;-72)$$

(6)

(c)

$$-4 + p = 0,\ -72 + q = 0$$
$$\therefore p = 4 \text{ and } q = 72 \quad ^{\text{A A}}$$

(2)

[13]

QUESTION 9

(a) $B\left(x\,;\,4-\dfrac{x^2}{4}\right)$

A A (2)

(b) Area $\Delta OBD = \dfrac{1}{2}\times(OD)(DB)$ M

$= \dfrac{1}{2}x\left(4-\dfrac{x^2}{4}\right)$ CA

$= 2x-\dfrac{x^3}{8},$ A $\quad x\in[0\,;4]$ (3)

(c) Max Area when $\dfrac{d\text{Area}}{dx}=0$ M

$2-\dfrac{3x^2}{8}$ A $=0$

$\dfrac{3x^2}{8}=2$ M (Solving)

$x^2=\dfrac{16}{3}$ A

$x=\dfrac{4\sqrt{3}}{3}$ CA (2,3 units)

$x\in[0\,;4]$ Only positive (5)

(d) Max Area $= 2\times\dfrac{4\sqrt{3}}{3}-\left(\dfrac{4\sqrt{3}}{3}\right)^3\div 8$

M (Sub ans from c into given formula from b)

$=\dfrac{16\sqrt{3}}{9}$ A (3,1) sq. units

ALTERNATIVELY

Max. area $=\dfrac{1}{2}\bullet\dfrac{4\sqrt{3}}{3}\left(4-\left(\dfrac{4\sqrt{3}}{3}\right)^2\div 4\right)$ M

$=\dfrac{16\sqrt{3}}{9}$ A

(2)

[12]

QUESTION 5

(a) Given: $f(x)=\dfrac{x^2}{4}$, determine $f'(x)$ from first principles. (5)

(b) Find $\dfrac{dy}{dx}$ for $y=\dfrac{15x^2+x-2}{3x-1}$ $x \neq \dfrac{1}{3}$ (3)

(c) Given: $f(x)=\sqrt{x}+\dfrac{1}{x^2}-3$ $x>0$

Evaluate $f'(4)$. (5)

(d) Given: $f(x)=x^3-3x^2+kx+8$ where k is a constant. The graph has a turning point at $x=1$. Find the value of k. (4)

[17]

QUESTION 9

(a) The radioactive decay of a substance is given by the formula:

$m(t)=500(0,92)^t$ where $m(t)$ is the mass(in grams) of the radioactive substance and t is its age in years.

(1) Write down the initial mass of the radioactive material. (1)

(2) Write down the percentage by which the radioactivity decreases each year. (1)

(3) Determine the mass of the substance after 50 years. (2)

(4) Determine the least number of years it takes for the substance's mass to be less than one gram. Give your answer to the nearest year. (3)

(b) Given: $f(x)=\dfrac{x}{2}$ when x is rational

but $f(x)=x^2$ when x is irrational

Evaluate $f\left(\sqrt{4}\right)+f\left(\sqrt{8}\right)$. (3)

QUESTION 5

(a) $f(x)=\dfrac{x^2}{4}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{(x+h)^2}{4}-\frac{x^2}{4}}{h} \checkmark$$

$$=\lim_{h\to 0}\frac{1}{4h}\left[x^2+2hx+h^2-x^2\right] \checkmark$$

$$=\lim_{h\to 0}\frac{h}{4h}\left[2x+h\right] \checkmark$$

$$=\lim_{h\to 0}\left(\frac{x}{2}+\frac{h}{4}\right) \checkmark$$

$$=\frac{x}{2} \checkmark$$

(5)

(b) $y=\dfrac{15x^2+x-2}{3x-1}$

$$=\frac{(5x+2)(3x-1)}{3x-1} \checkmark$$

$$=5x+2 \checkmark$$

$$\frac{dy}{dx}=5 \checkmark$$

(3)

(c) $f(x)=\sqrt{x}+\dfrac{1}{x^2}-3$

$$=x^{\frac{1}{2}}+x^{-2}-3 \checkmark$$

$$f'(x)=\tfrac{1}{2}x^{-\frac{1}{2}}-2x^{-3} \checkmark\checkmark$$

$$f'(4)=\tfrac{1}{2}4^{-\frac{1}{2}}-2.4^{-3} \checkmark$$

$$=\frac{1}{2.2}-\frac{2}{64}$$

$$=\frac{7}{32} \qquad (0{,}2) \checkmark$$

(5)

(d) $f(x)=x^3-3x^2+kx+8$

$$f'(x)=3x^2-6x+k \checkmark$$

$$f'(1)=3-6+k=0 \checkmark \checkmark$$

$$k=3 \checkmark$$

(4)

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QUESTION 9

(a) $m(t) = 500(0,92)^t$

(1) 500 g ✓ (1)

(2) 8% ✓ (1)

(3) $m(50) = 500(0,92)^{50}$ ✓

$= 7,7332...$

$\approx 7,7$ g ✓ (2)

(4) $500(0,92)^t < 1$

$0,92^t < \frac{1}{500}$ ✓

$t > 74,5321...$ ✓

i.e. 75 years ✓ (3)

(b) $f(\sqrt{4}) + f(\sqrt{8})$

$= \frac{\sqrt{4}}{2} + \left(\sqrt{8}\right)^2$ ✓✓

$= 1 + 8$

$= 9$ ✓ (3)