

A complex network diagram with numerous nodes of varying sizes and colors (dark blue, light blue, grey) connected by thin grey lines. Some nodes are highlighted with larger concentric circles. The background is white with faint, larger-scale network patterns.

# CLUSTERING ANALYSIS

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DATA 3300 – Carly Fox, PhD

# OBJECTIVES

Define Cluster Analysis, data type requirements, and business applications

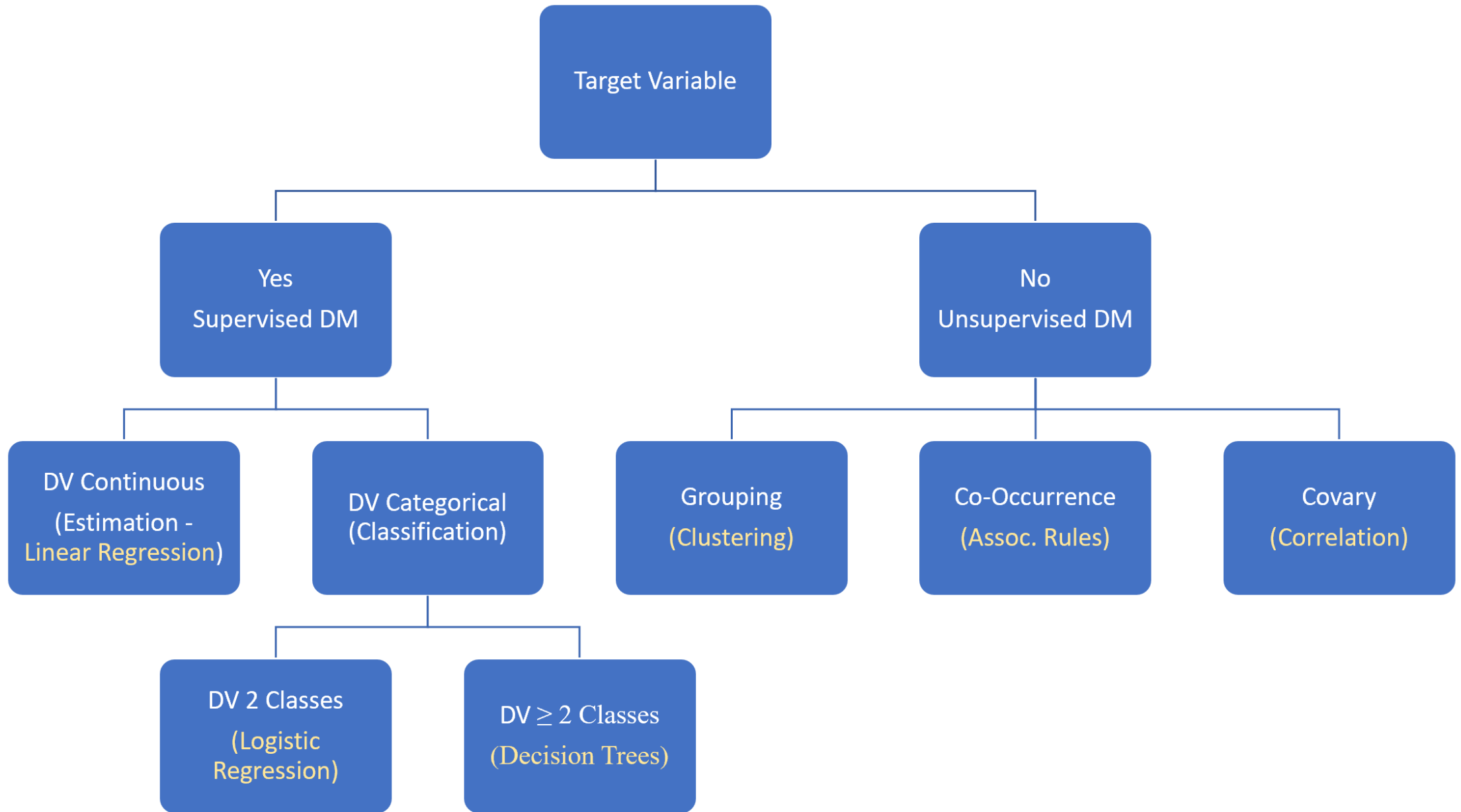
Understand what  $k$ -means clustering is and what  $k$  stands for

Calculate similarity/dissimilarity between observations (Euclidean Distance)

Describe the basic process of the  $k$ -means algorithm, including centroids

Understand the potential effect of outliers

Examine the limitations of Cluster Analysis



# CLUSTERING ANALYSIS OVERVIEW



**Type of Analysis:** Unsupervised; looking for natural relationships, not trying to predict a target variable

**Type of Data:** Quantitative (interval/ratio) and or qualitative (ordinal/nominal) – with additional preprocessing – may be used

**Type of Business Qs:** Do cases (e.g., customers, employees, etc.) tend to cluster into natural groups that we can use for an actionable purpose?

- Do certain groups of customers tend to display similar purchasing patterns?

- Are there certain clients who have a higher risk profile than others?

# APPLICATIONS OF CLUSTERING ANALYSIS

**Market/Customer Segmentation:** Grouping people according to their similarity across several dimensions (attributes) related to a product under consideration

**Sales Segmentation:** Clustering types of customers by which products purchased


**Credit Risk:** Clustering types of customers based on their credit history

**Operations:** Promoting based on a person's performance or segmenting high performers

**Insurance:** IDing groups of motor insurance policy holders with a high average claim cost

**City-Planning:** IDing groups of houses according to their house type, value, and geographical location

**Geographical:** IDing areas of similar land use



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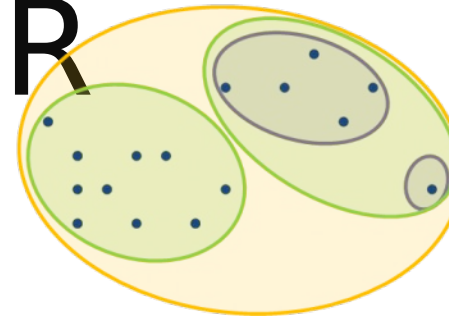
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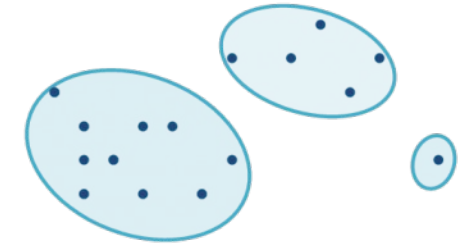
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# TYPES OF CLUSTER ANALYSIS

Hierarchical Clustering



Partitional Clustering



## Partitional (Non-Hierarchical)

A division of objects (data instances) into non-overlapping subsets (clusters) such that each object belongs to exactly one cluster

Divide the dataset of size  $N$  objects into  $M$  clusters

***K-Means Clustering*** ☾ most used non-hierarchical method in business analytics

## Hierarchical

A set of nested clusters organized as a hierarchical tree

Produces a set of nested clusters in which each pair of objects or clusters is progressively nested in a larger cluster until only one remains

**EX:** doctors, nested within hospitals, nested within states, nested within the US

***CHAID tree*** most used in business analytics

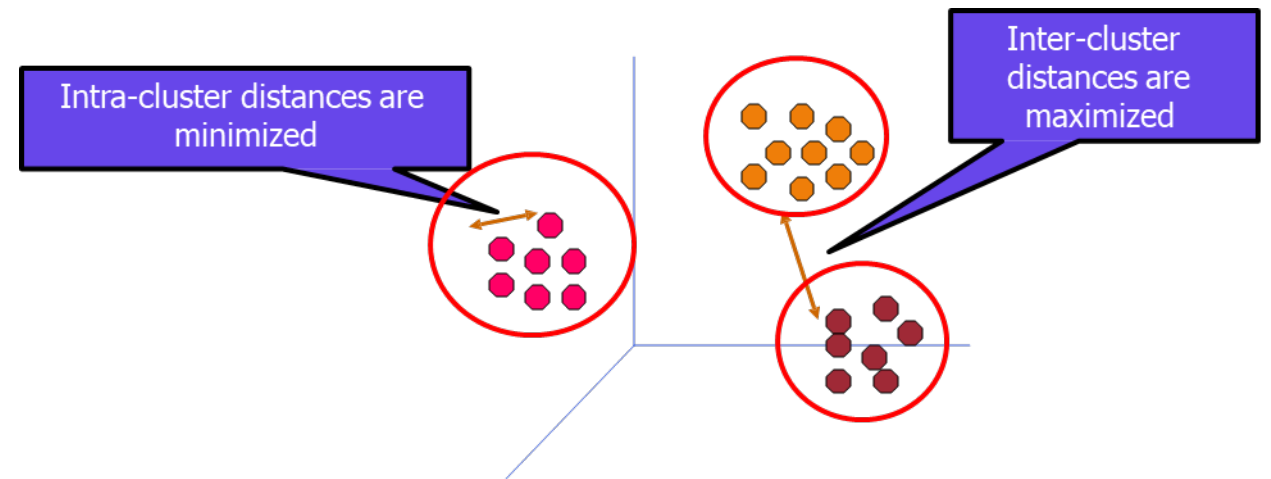
**What we're focusing on**

# IMPORTANT TERMS

**Cluster** ∈ group of objects (cases, points, observations, members, customers, etc.) – *not attributes* – that are similar to each other with respect to variable(s) of interest

**Clustering Analysis** ∈ Data mining method used to find naturally occurring groups of cases/observations/objects in the sample that are:

- **Homogenous** within groups (i.e., high intra-class similarity)
- **Heterogenous** between each group (i.e., low inter-class similarity)

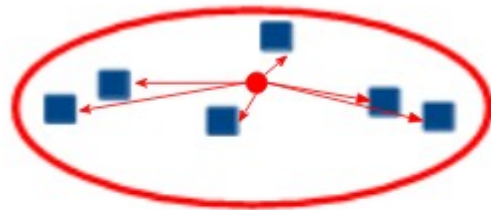


# CONSIDERATIONS IN CLUSTER ANALYSIS

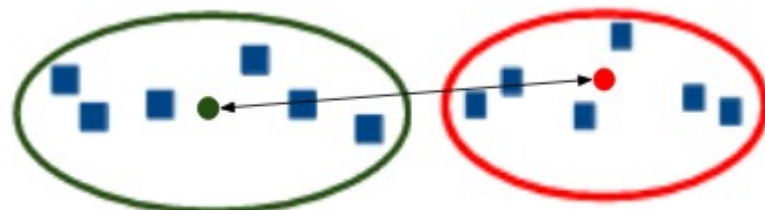
If the aim is to build clusters (e.g., divide a sample or population into groups of similar objects), how do we do that?

- What defines **similarity/dissimilarity**
- How do you define **distance** between clusters?

*Remember: trying to max similarity (min distance) within clusters and max dissimilarity (max distance) between clusters*



Intra cluster distance



Inter cluster distance



# SIMILARITY & DISSIMILARITY

	Weight
Cust1	68
Cust2	72
Cust3	100

Which two customers are similar?

	Weight	Age
Cust1	68	25
Cust2	72	70
Cust3	100	28

Which two customers are similar now?

	Weight	Age	Income
Cust1	68	25	60,000
Cust2	72	70	9,000
Cust3	100	28	62,000

Which two customers are similar in this case?

# QUANTIFYING SIMILARITY – MEASURES OF DISTANCE

The **similarity of two observations** can be quantified by calculating their ***distance*** from one another

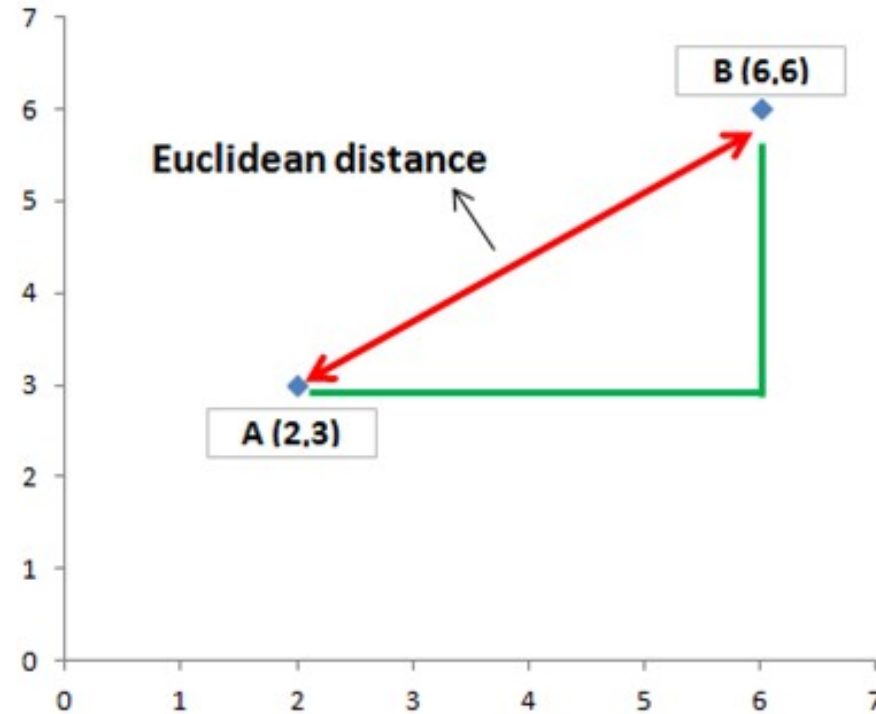
Straightforward when there's on a single variable

Multiple variables require an ***aggregate distance measure*** like ***Euclidean Distance*** (e.g., remember the cartesian coordinate system?)

$$distance = \sqrt{\sum_{i=0}^n (x_i - y_i)^2}$$

# FEELING NOSTALGI C ABOUT GEOMETR Y?

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$$\text{Euclidean distance } (a, b) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$$

Other types of distance metrics

# CALCULATING DISTANCE

	Weight
Cust1	68
Cust2	72
Cust3	100

- Cust1 vs Cust2 :  $(68-72)= 4$
- Cust2 vs Cust3 :  $(72-100) = \mathbf{28}$
- Cust3 vs Cust1 :  $(100-68) = \mathbf{32}$

	Weight	Age
Cust1	68	25
Cust2	72	70
Cust3	100	28

- Cust1 vs Cust2 :  $\text{sqrt}((68-72)^2 + (25-70)^2) = \mathbf{44.9}$
- Cust2 vs Cust3 :  $\mathbf{50.54}$
- Cust3 vs Cust1 :  $\mathbf{32.14}$

$$D_{ij} = \sqrt{\sum_{k=1}^n (x_{ki} - x_{kj})^2}$$

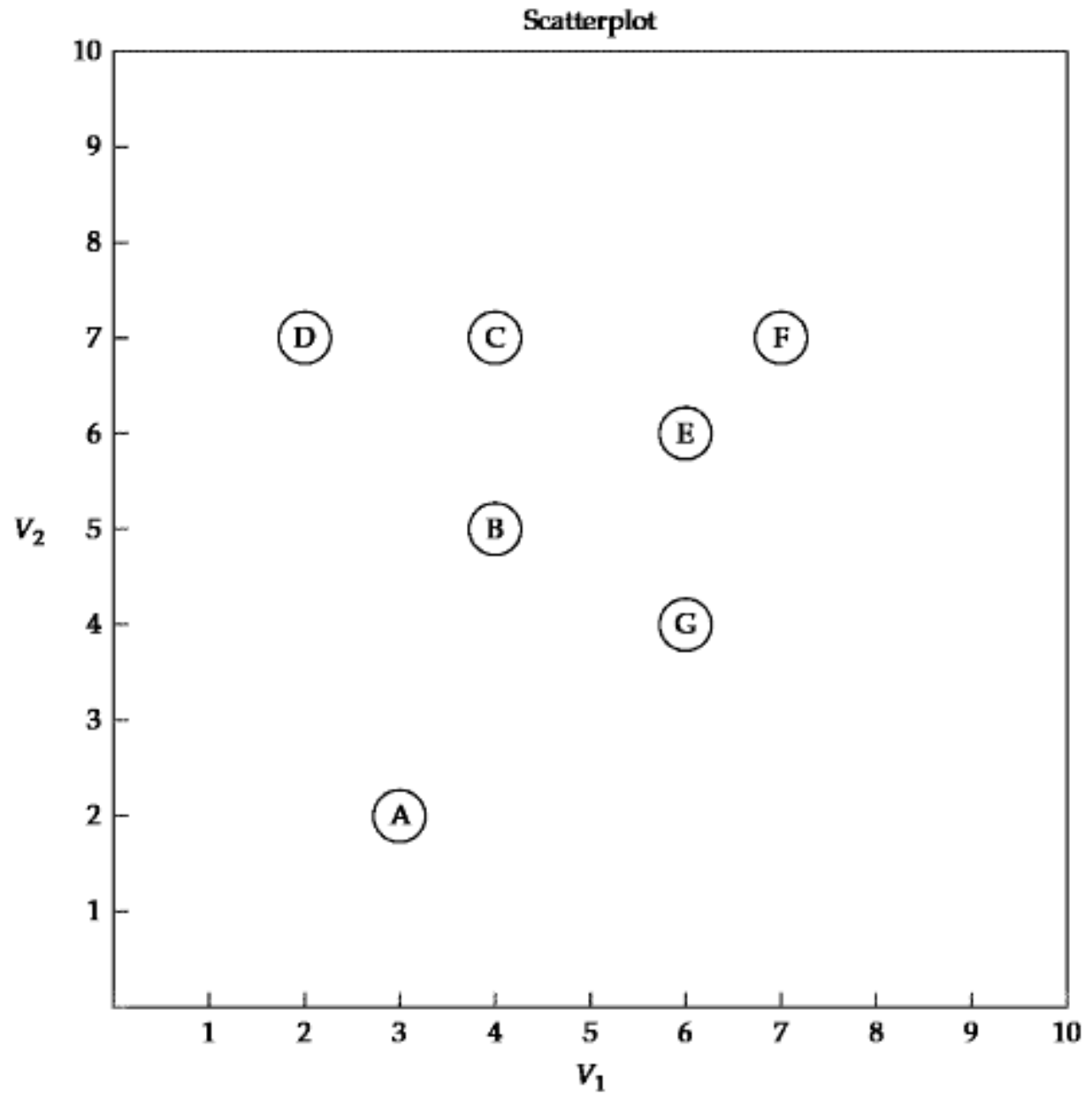
# EXAMPLE CASE

**Simple Example:** Suppose a marketing researcher wishes to determine market segments in a community based on patterns of loyalty to brands and stores. A small sample of seven respondents is selected as a pilot test of how cluster analysis is applied. Two measures of loyalty - V1(store loyalty) and V2(brand loyalty) - were measured for each respondent on a 0-10 scale.

Clustering Variable	Respondents						
	A	B	C	D	E	F	G
Store Loyalty	3	4	4	3	6	7	6
Brand Loyalty	2	4	7	7	6	7	4

# CALCULATING DISTANCE

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# CALCULATING DISTANCE

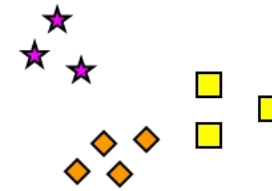
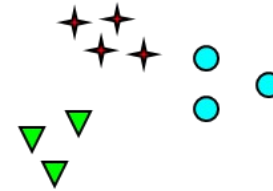
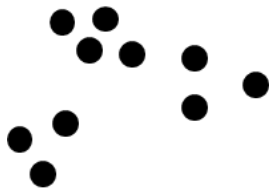
Proximity Matrix of Euclidean Distance Between Each Observation Set

Observations	Observations						
	A	B	C	D	E	F	G
A	-						
B	3.162	-					
C	5.099	2.000	-				
D	5.099	2.828	2.000	-			
E	5.000	2.236	2.236	4.123	-		
F	6.403	3.606	3.000	5.000	1.414	-	
G	3.606	2.236	3.606	5.000	2.000	3.162	-

$$d_{Euclidean}(A, B) = \sqrt{(V_{1(A)} - V_{1(B)})^2 + (V_{2(A)} - V_{2(B)})^2}$$

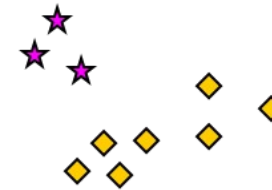
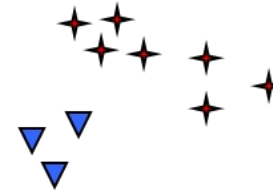
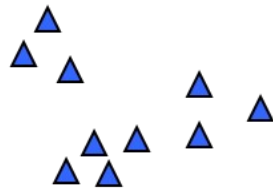
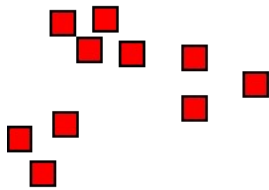
$$d_{Euclidean}(A, B) = \sqrt{(3 - 4)^2 + (2 - 5)^2} = 3.162$$

# CLUSTERING IS NOT AN EXACT SCIENCE



How many clusters?

Six Clusters



Two Clusters

Four Clusters



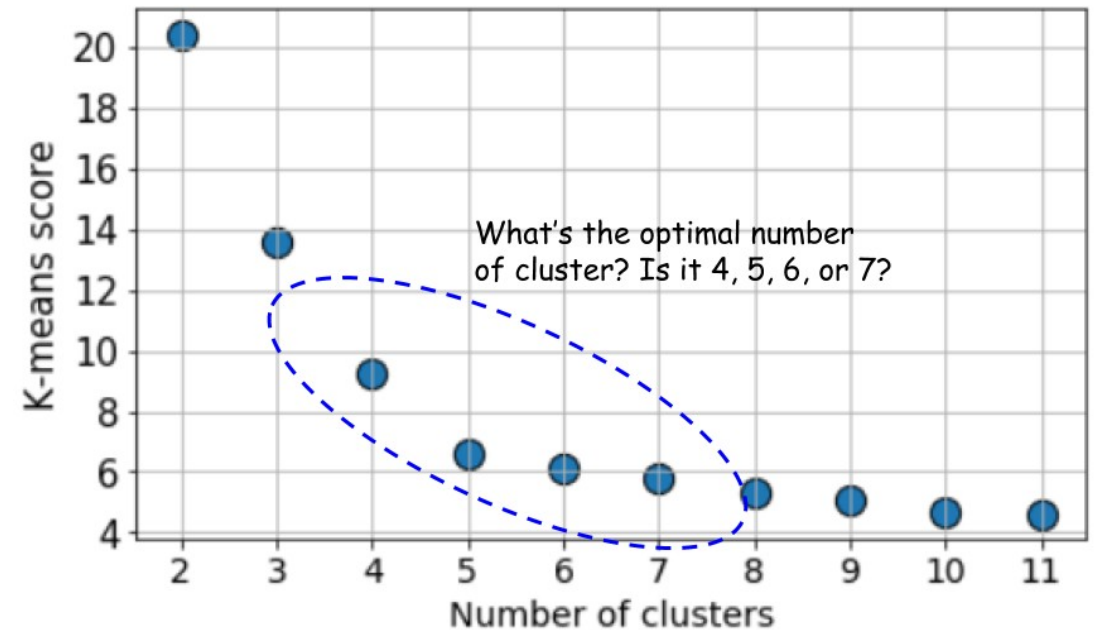
# HOW DO YOU DETERMINE CLUSTER SIZE ()?

**Tractability:** The client identifies the number of clusters, typically based on the ease with which something can be managed

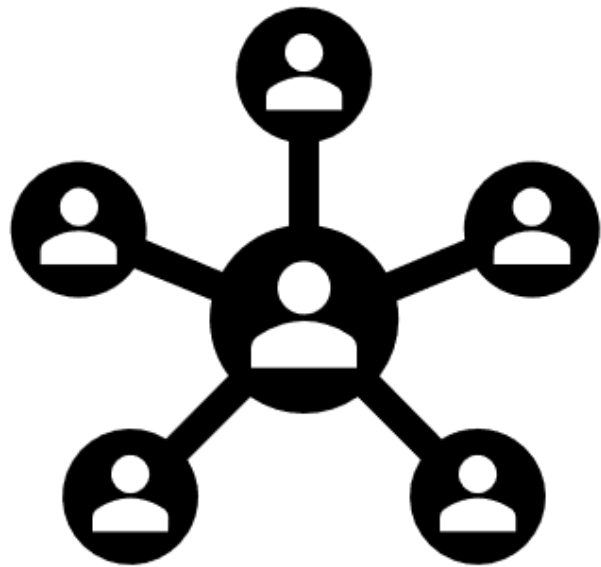
**Post-hoc evaluation:** Practice of performing several analyses using different values for  $k$ , then reviewing the results to determine which  $k$  value is most suitable

**Elbow rule:** Identify the value of  $k$  after which increasing the value of  $k$  adds very little to the clusters in terms of further decreasing the within-cluster distance or increasing the between-cluster distance

The elbow method for determining number of clusters



# CLUSTERING ANALYSIS P2



# - MEANS CLUSTERING - ALGORITHM STEPS

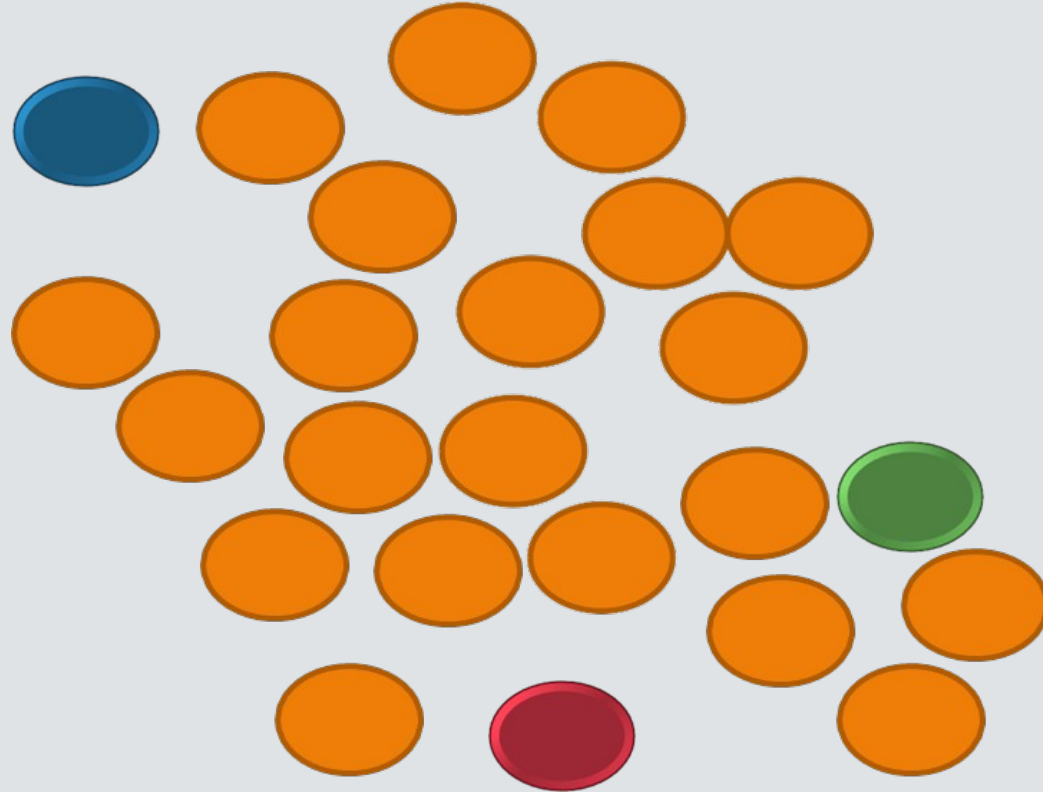
1. Number of clusters is set by the analyst
2. An initial set of “seeds” (aggregation centroids) is provided
  - Starts with the first  $k$  elements
  - Other seeds (randomly selected or explicitly defined)
3. Given a certain fixed threshold value, all units are assigned to the nearest cluster’s seed (centroid)
4. New seeds are computed
5. Repeat steps 3-4 until no reclassification of observations is necessary

# $k$ -Means Clustering



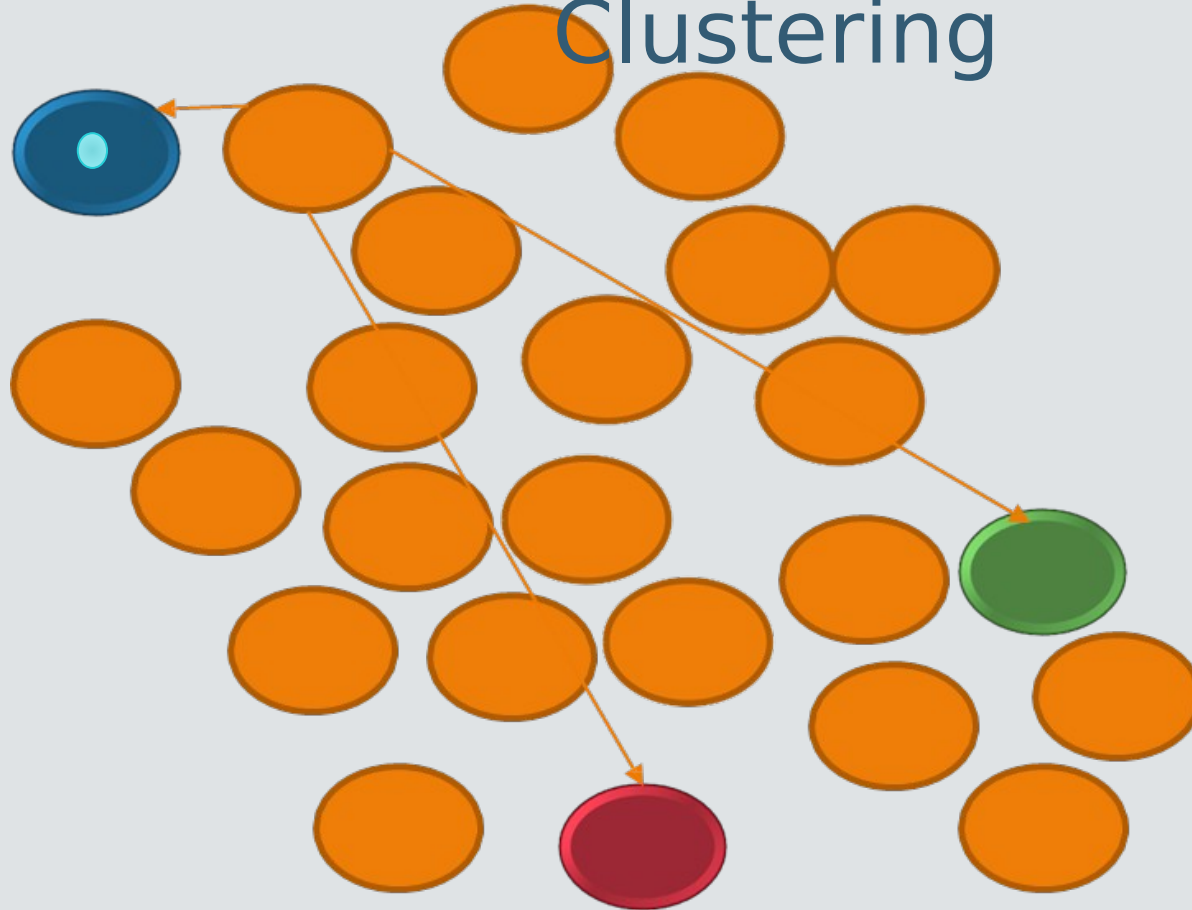
Overall sample

# $k$ -Means Clustering



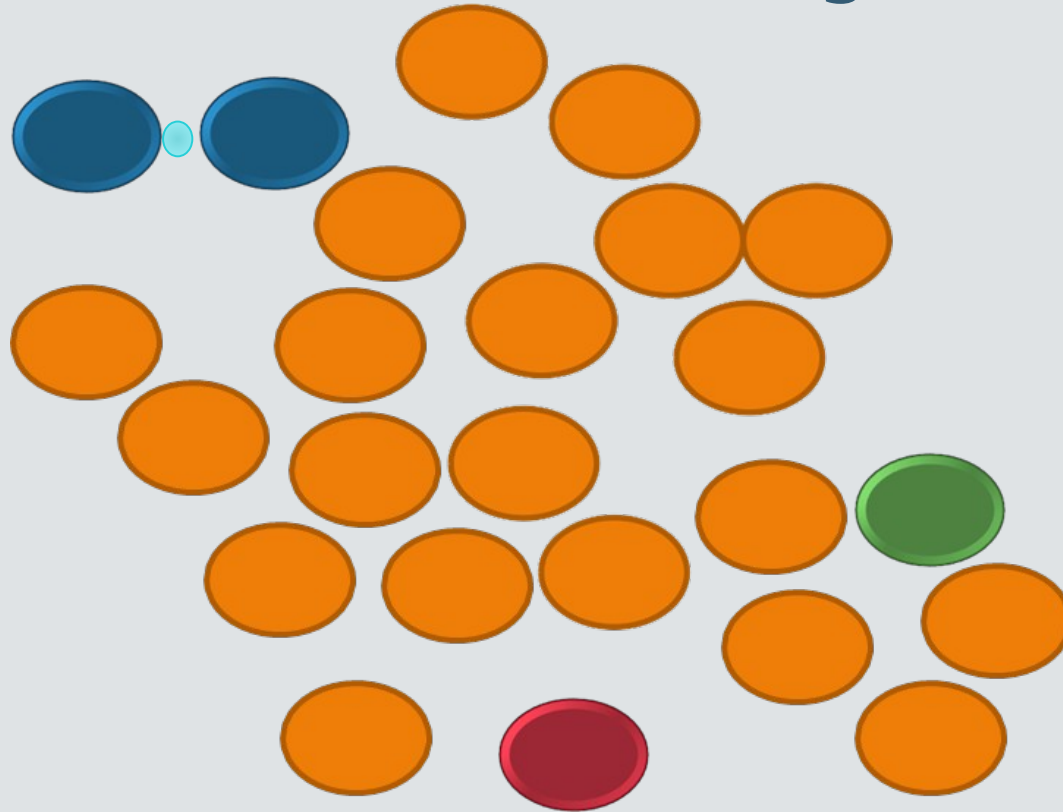
Fix the Number  
of Cluster **seeds**  
or **centroids**

# $k$ -Means Clustering



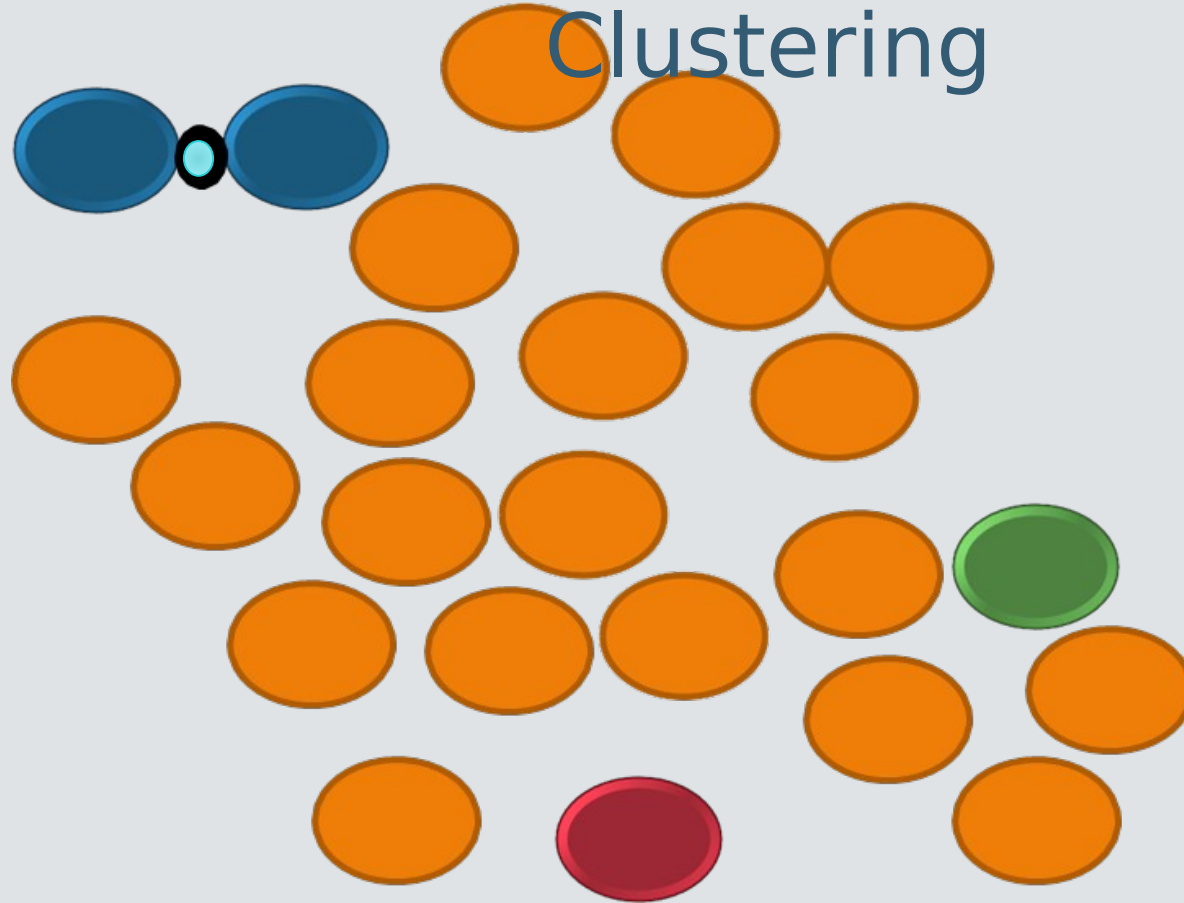
Calculate the  
distance of  
each case from  
all centroids

# $k$ -Means Clustering



Assign each  
case to nearest  
cluster centroid

# $k$ -Means Clustering



Recalculate the  
cluster  
centroids

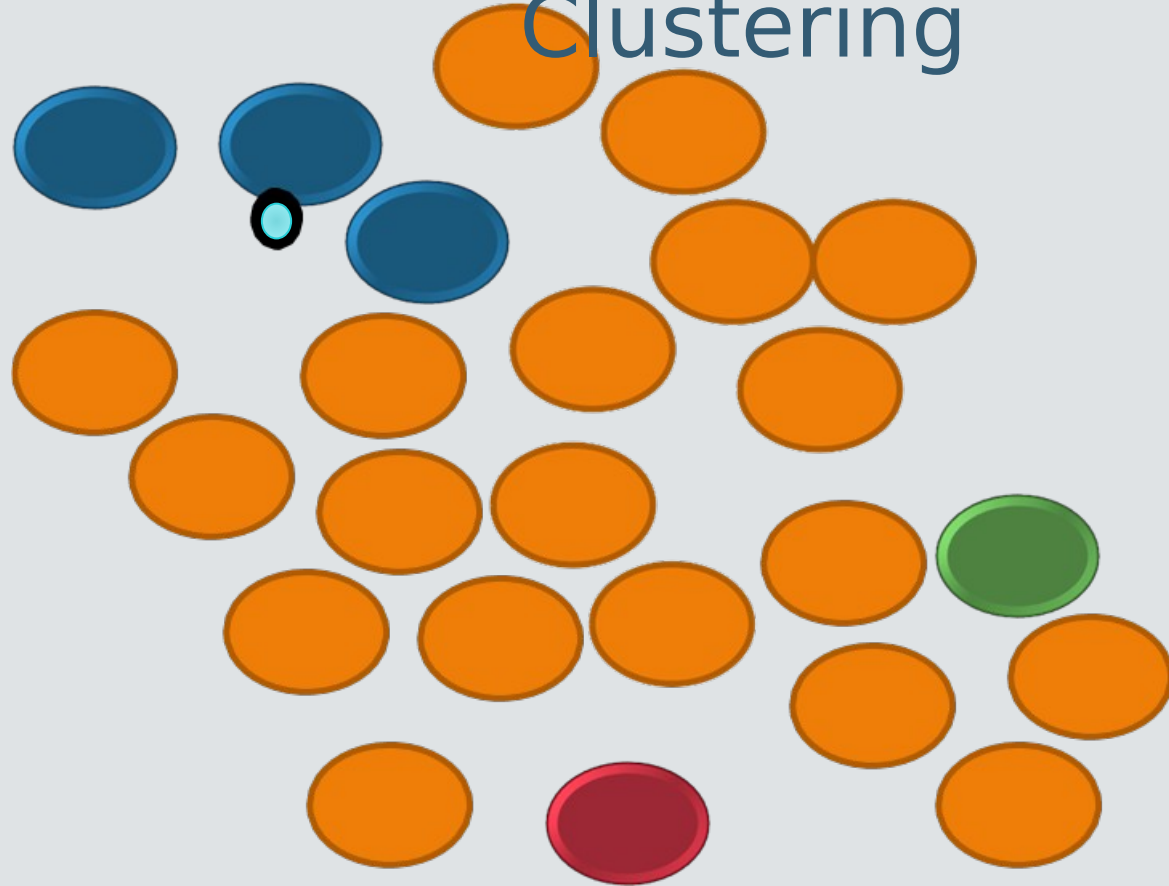


# $k$ -Means Clustering

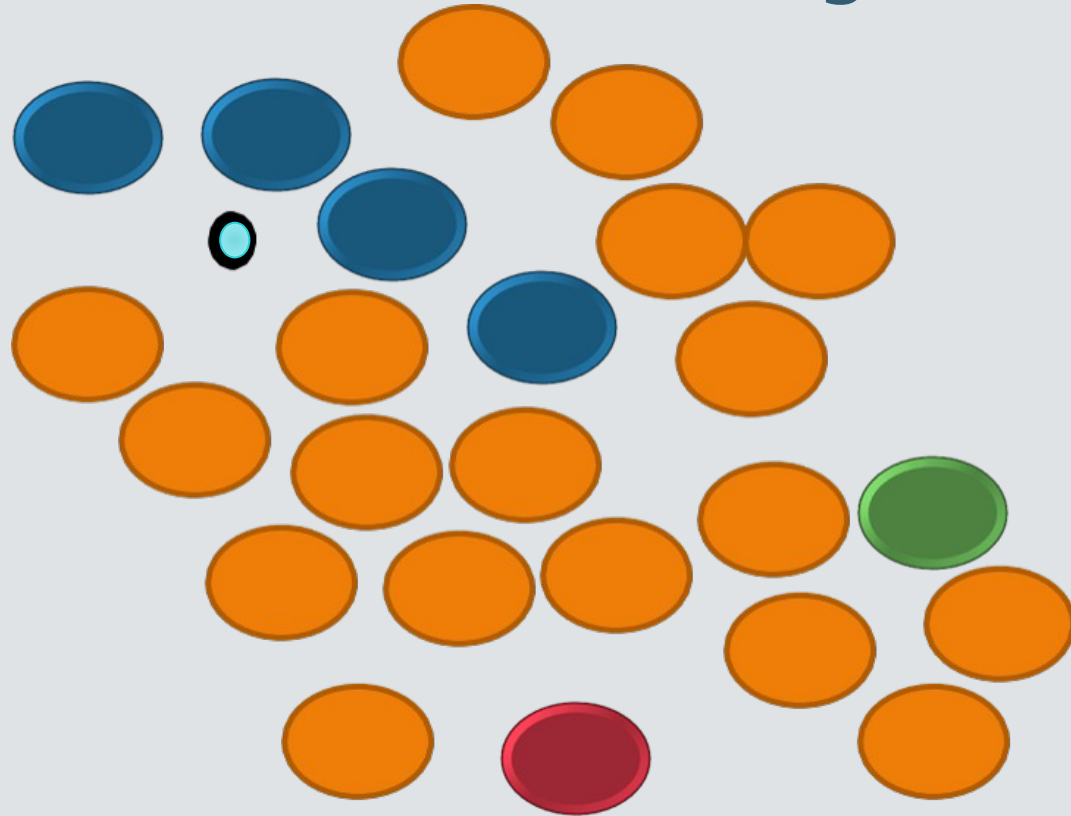


Repeat...

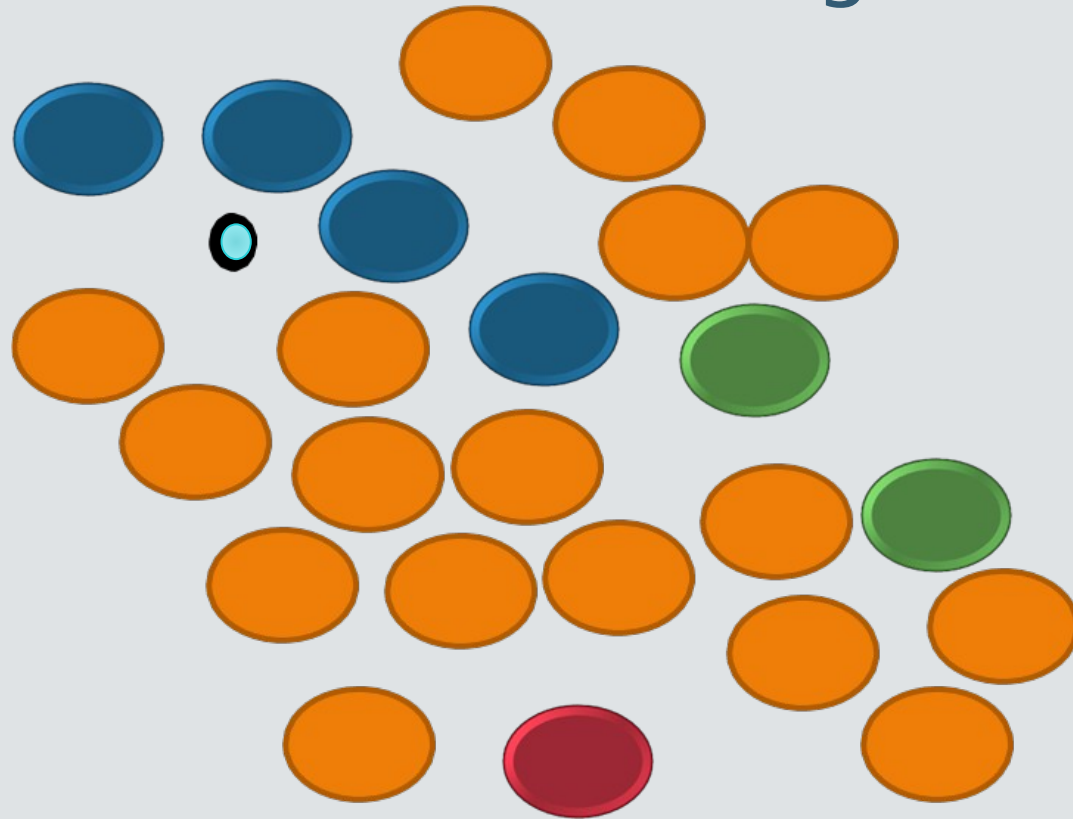
# $k$ -Means Clustering



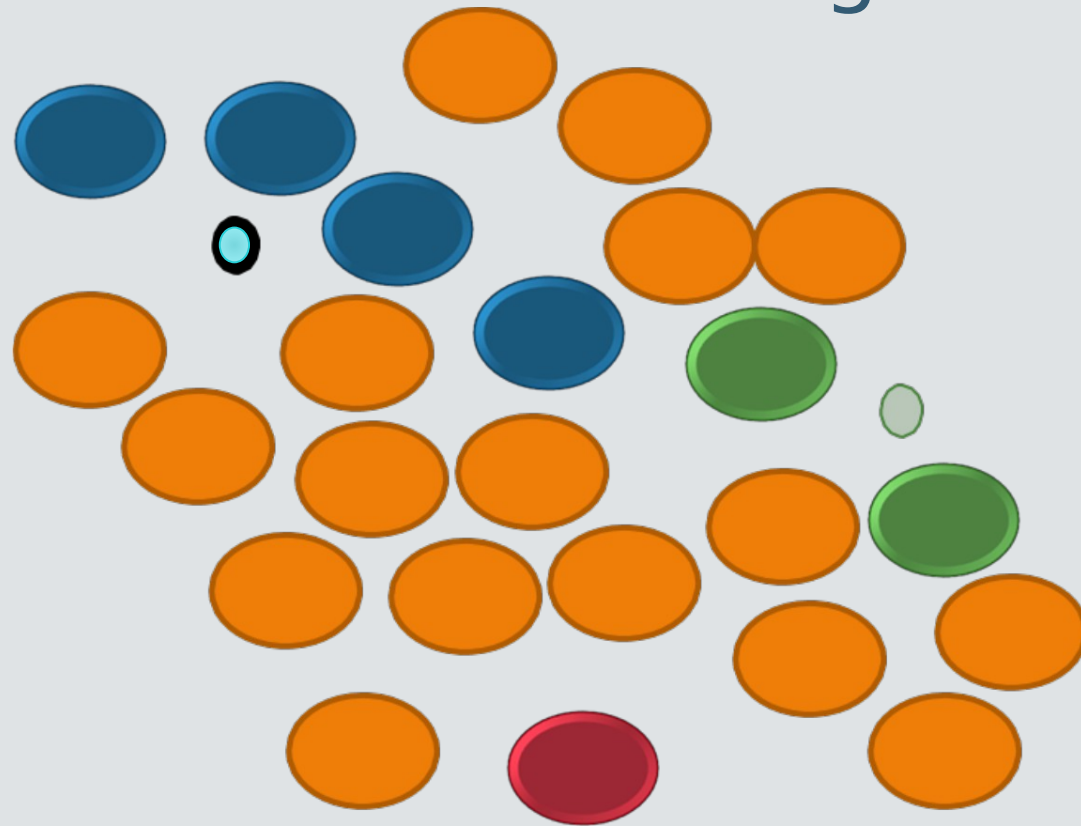
# $k$ -Means Clustering



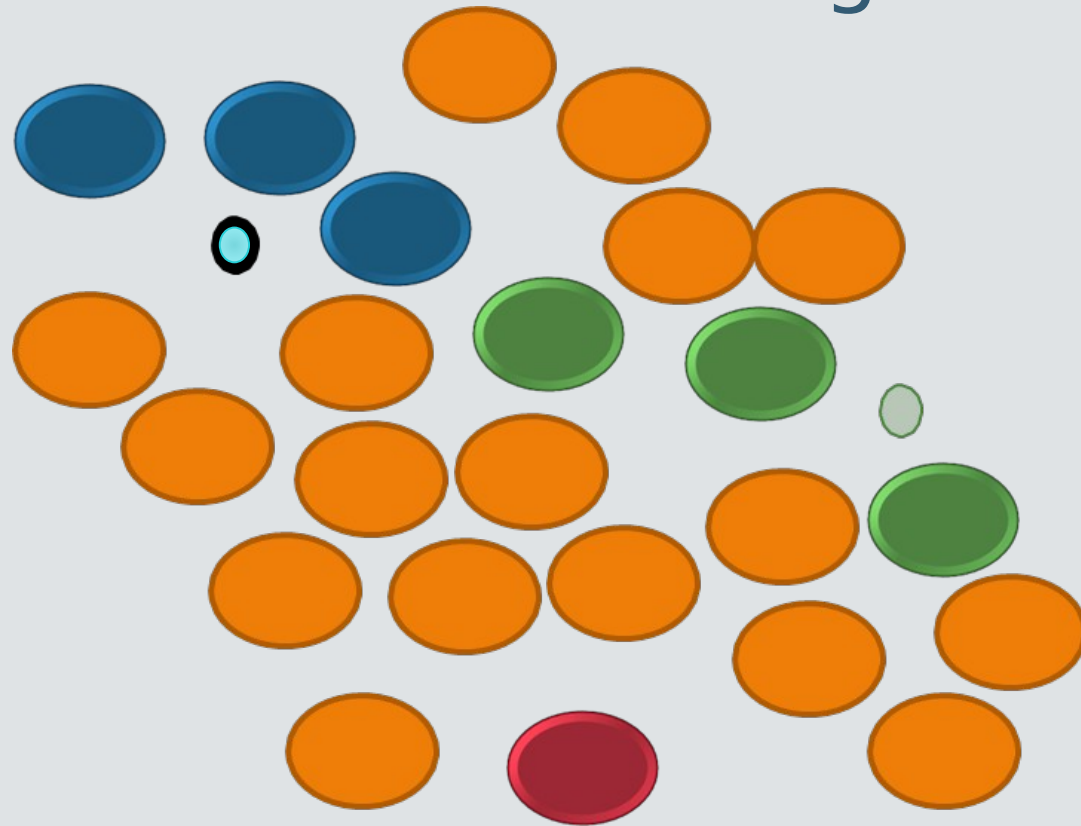
# $k$ -Means Clustering



# $k$ -Means Clustering

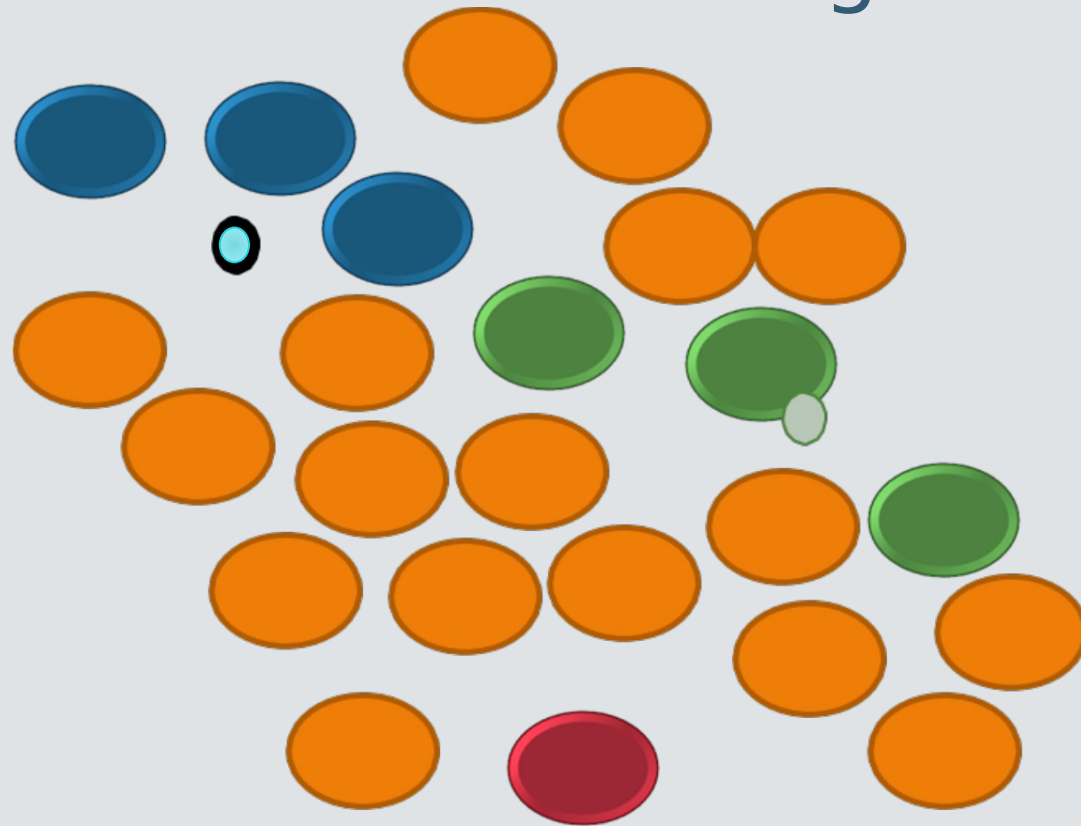


# $k$ -Means Clustering



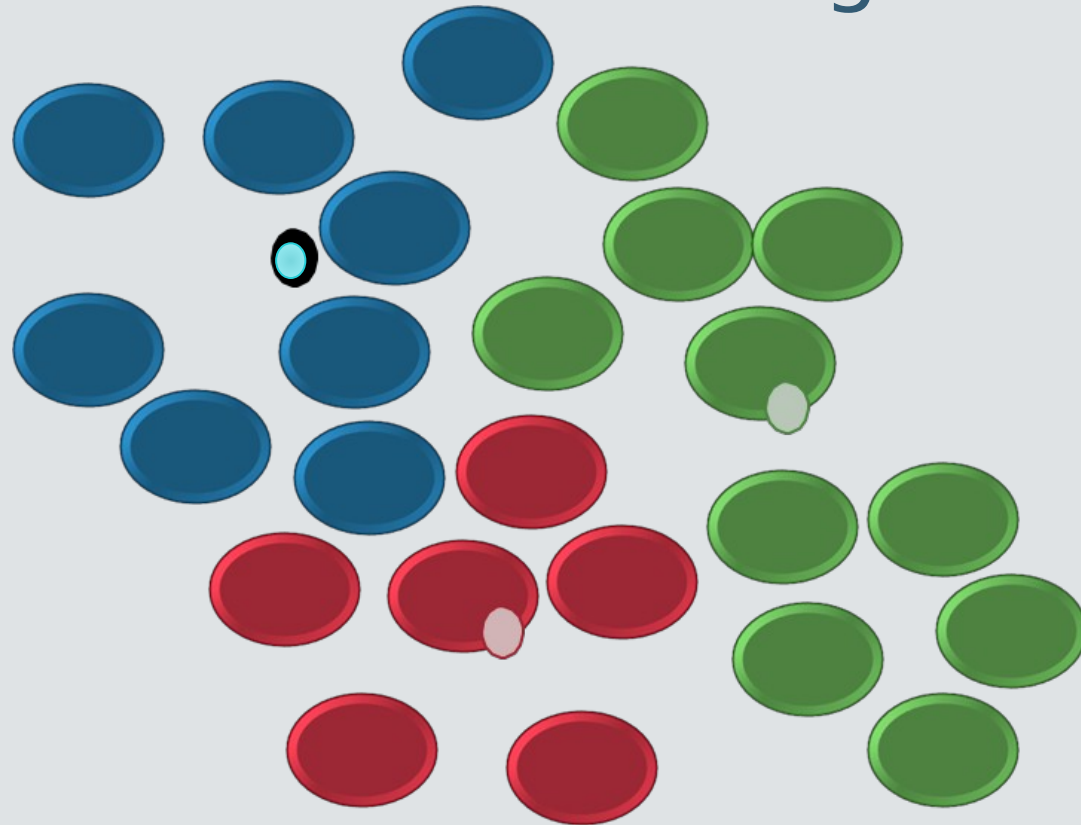
Reassign cases  
as needed after  
changing  
cluster  
centroids

# $k$ -Means Clustering



Re-compute  
cluster  
centroids as  
new cases are  
assigned

# $k$ -Means Clustering



Continue until  
there is no  
significant  
change  
between each  
iteration



# THE BANE OF OUR (ANALYST) EXISTENCE: OUTLIERS



**Outliers can severely distort the representativeness of the results of cluster analysis**

They **should be removed** **IF** the outliers represent:

- Aberrant observations not representative of the population

- Observations of small or insignificant segments within the population and of no interest to the analysis objectives

They **should be NOT be removed** **IF:**

- There is undersampling/poor representation of relevant groups in the population

- The sample should be augmented to ensure representation of these groups

## **How to detect outliers?**

Their appearance in cluster solutions as single-member or small clusters

Using the 68-95-99 rule

Seeing if a value is two or more standard deviations from the mean

# ISSUES OF SCALE

**Some analyses (e.g., k-means) work better when vars have similar ranges/are on similar scales to prevent overweighting certain variables**

This can be accomplished via:

**Normalization** ☾ Every var is fit into the same range (e.g., between 0-1) or on the same scale

**Standardization** ☾ Every value is calculated in terms of SD from the mean; typically, though a *Z-score conversion*, sets the mean to 0 with an SD of 1

$$z = \frac{x - \mu}{\sigma}$$

$\mu$  = Mean  
 $\sigma$  = Standard Deviation

[Read more here about differences between Normalization & Standardization](#)

# INTERPRETATION OF CLUSTERS

## Centroid Table

Variable	Mean	SD	Cluster_1	Cluster_2	Cluster_3	Cluster_4
Gender = F	0.48	0.50	1.00	0.54	0.37	0.00
Age_C	47.26	12.194	43.04	63.36	43.66	44.02
Dog_wt	67.49	26.107	57.67	72.82	90.73	57.31
Visits	8.45	5.934	8.09	13.80	4.99	7.96

**What can be interpreted about cluster 1? What about clusters 2,3,4?**

*Are these differences substantial, or do they fail to show much variation?*

*Do the cluster centroids fit in with prior expectations?*

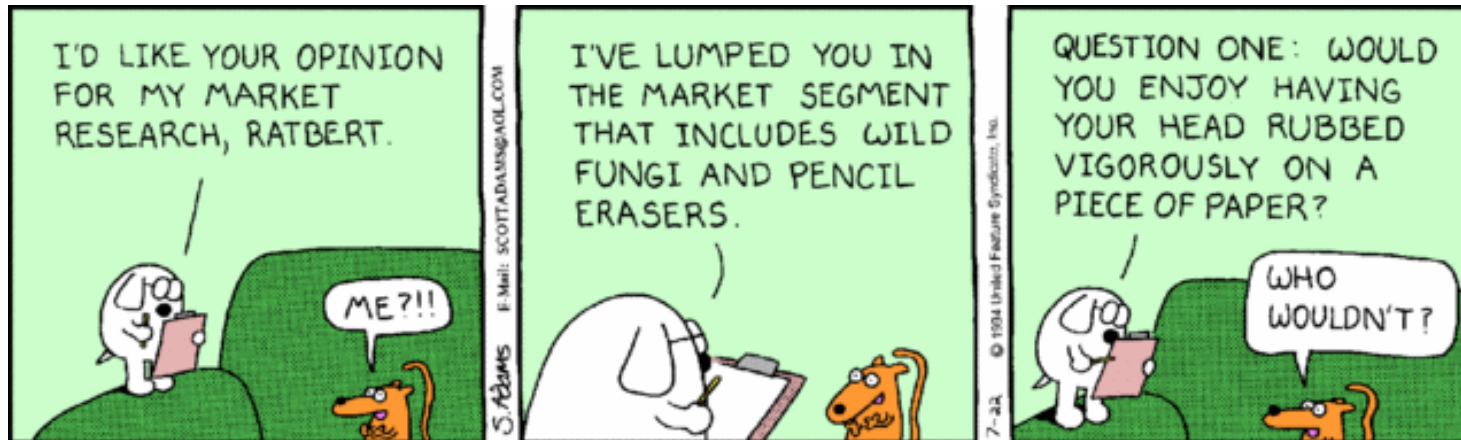
# VALIDATION & LIMITATIONS OF CLUSTERS

Important to **validate** your cluster analysis because its descriptive in nature and requires additional support

Limitations:

- No statistical basis upon which to draw inferences from your sample to the population
- Will always create  $k$  clusters, whether they exist or not
- Cluster solution is not generalizable to the data outside of the sample used to develop the cluster solution
- Humans like to ID patterns where they don't necessarily exist, and beware of cognitive bias or bias in the data set

# WEB COMICS (MAYBE) FOR THOUGHT







Moment of Chill