SCALER

Time Complexity

9PM
9:05 PM

TABLE OF CONTENTS

$b^c = a$

$c = \log_b a$

$\log_2 32 = 5$

$N = 2^k$

$\log_2 N = k$

$2^5 = \underbrace{2 \times 2 \times 2 \times 2 \times 2}_{5 \text{ times}} = 32$

Log Basics

1. $\log_2 64 =$ $2^? = 64 \Rightarrow 6$

2. $\log_3 27 =$ $3^? = 27 \Rightarrow 3$

3. $\log_2 32 =$ 5

4. $\log_2 10 =$ $2^? = 10$ $10 \to 5 \to 2 \to 1$, ans = 3

5. $\log_2 40 =$ $2^? = 40$ $40 \to 20 \to 10 \to 5 \to 2 \to 1$, ans = 5

6. $\log_2 2^6 =$ $2^? = 2^6 \Rightarrow 6$

7. $\log_3 3^5 =$ 5

$\log_a a^N = a^? = a^N$ ans = N

$\log_2 N$

< *Question* > : Given a positive integer N. How many times do we need to divide it by 2 until it reaches 1?

N = 100

100 ↓ 50 ↓ 25 ↓ 12 ↓ 6 ↓ 3 ↓ 1

ans = 6

N = 324

324 ↓ 162 ↓ 81 ↓ 40 ↓ 20 ↓ 10 ↓ 5 ↓ 2 ↓ 1

ans = 8

N = 9

9 ↓ 4 ↓ 2 ↓ 1

ans = 3

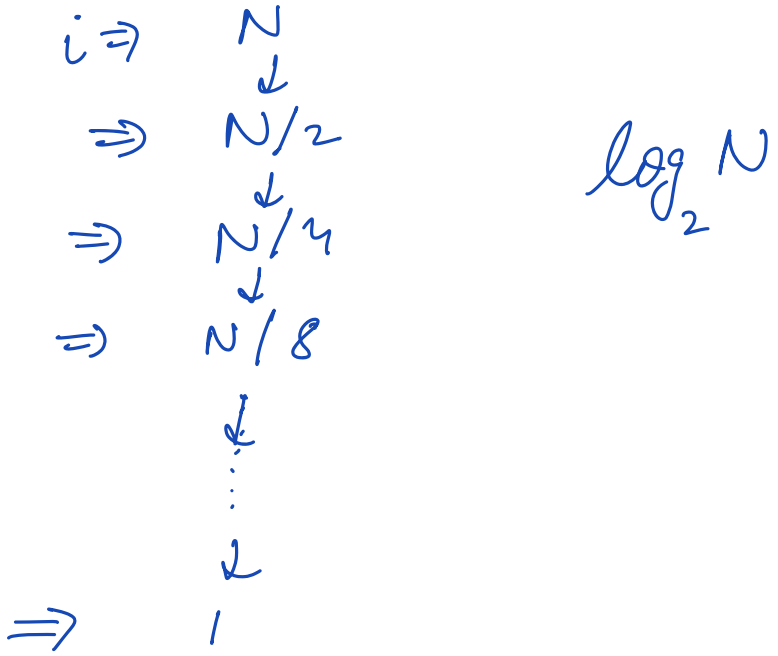
N = 27

27 ↓ 13 ↓ 6 ↓ 3 ↓ 1

ans = 4

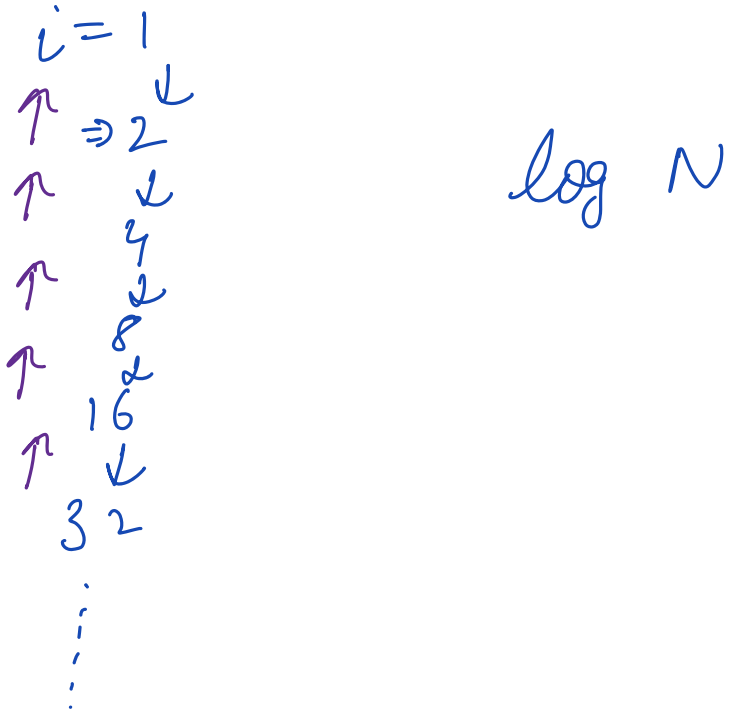
Quiz- 1

```
N>0
i=N;
while(i>1){
    i=i/2;
}
```



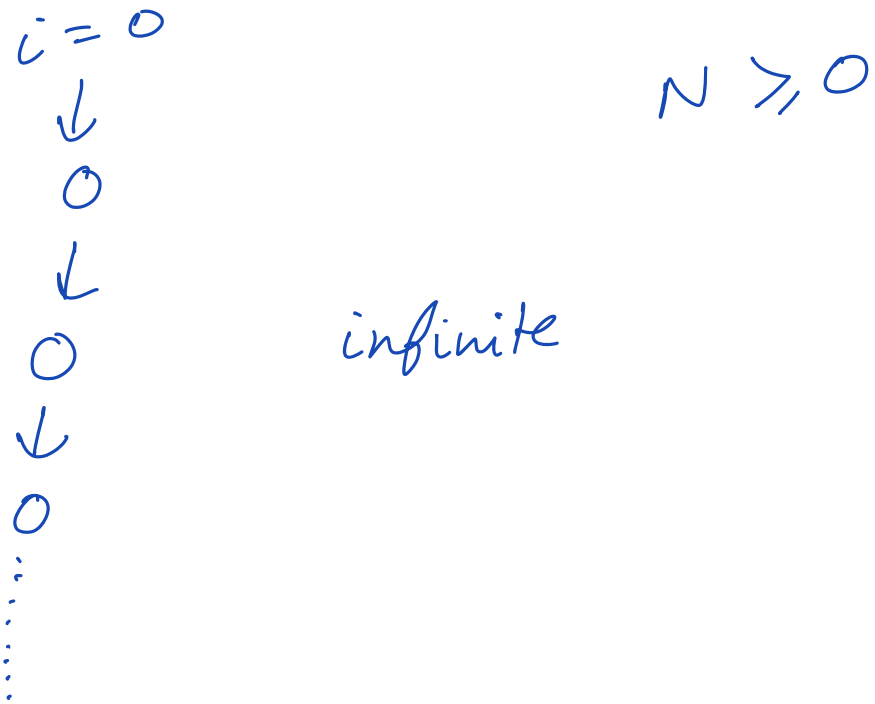
Quiz- 2

```
for(i=1; i<N; i=i*2){
    ----------
}
```



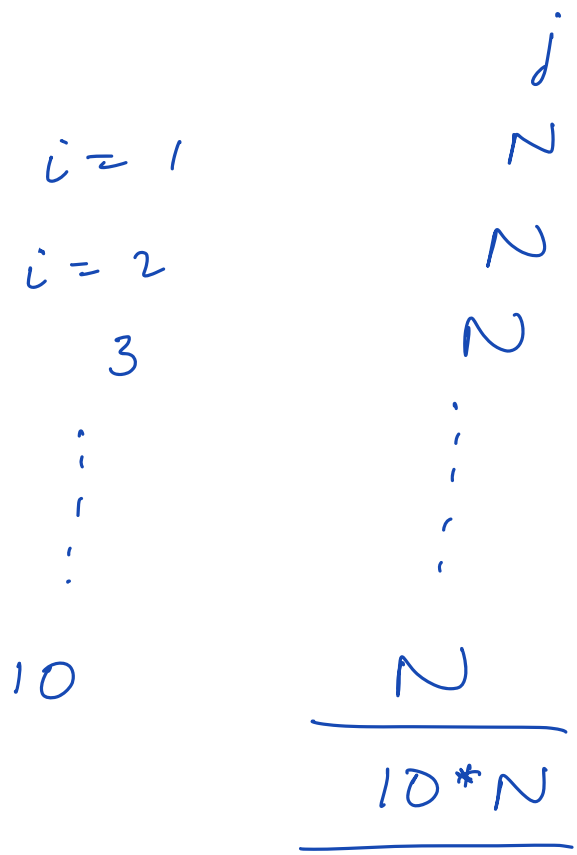
Quiz- 3

```
N≤0
for(i=0; i≤N; i=i*2){
        ------------
}
```



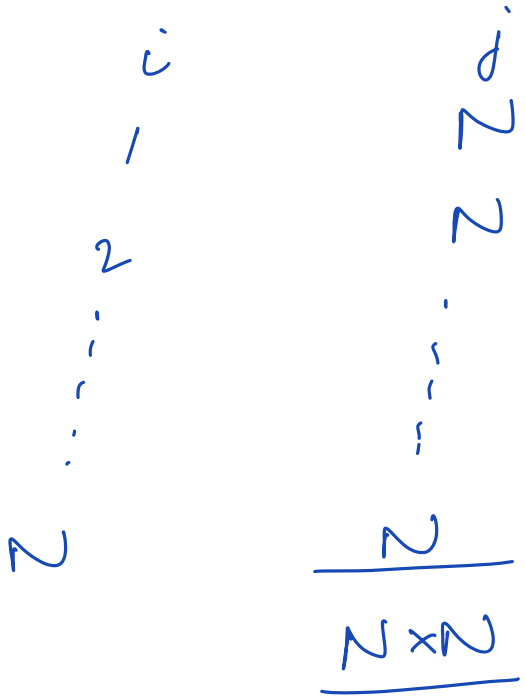
Quiz- 4

```
for(i=1; i≤10; i++){
    for(j=1; j≤N; j++){
       ---------------
    }
}
```



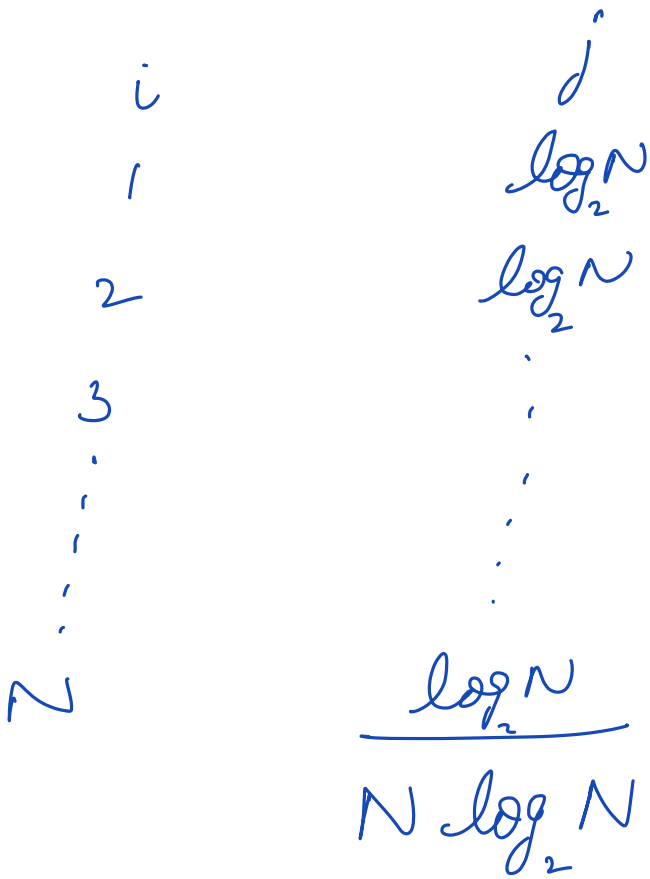
Quiz- 5

```
for(i=1; i≤N; i++){
    for(j=1; j≤N; j++){
       ----------------
    }
}
```



Quiz- 6

```
for(i=1; i≤N; i++){
    for(j=1; j≤N; j*2){
       ----------------
    }
}
```



Quiz- 7

```
for(i=1; i≤4; i++){
    for(j=1; j≤i; j++){
        //print(i+j)
    }
}
```

1, 1

2, 1 2, 2

3, 1 3, 2 3, 3

4, 1 4, 2 4, 3 4, 4

i = 1

2

3

4

j

1

2

3

4

10

Quiz- 8

```
for(i=1; i≤N; i++){
    for(j=1; j≤i; j++){
        //print(i+j)
    }
}
```

i = 1

2

3

⋮

N

j

1

2

3

4

⋮

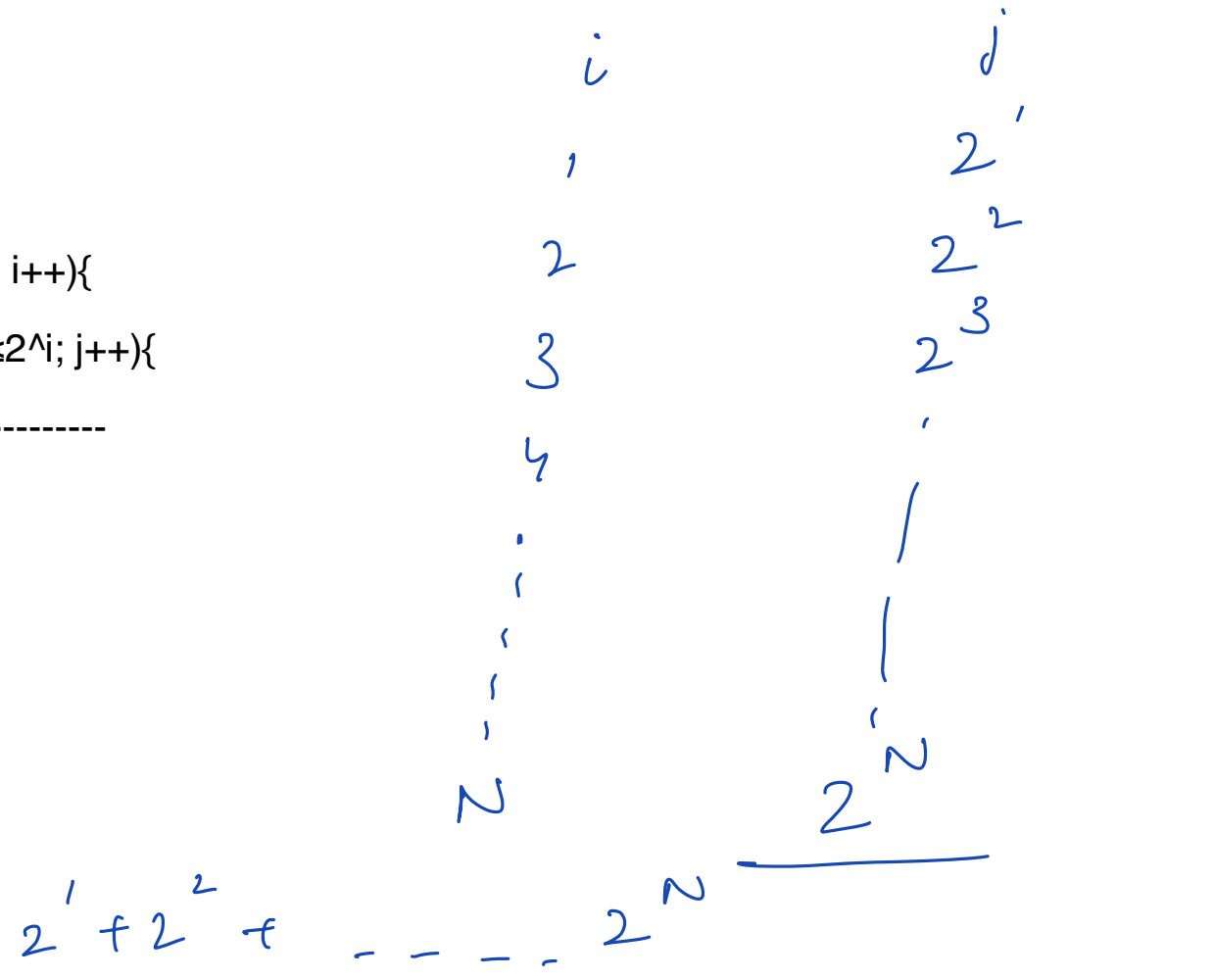
N

$1+2+3+4 \ldots\ldots+N$

$= \frac{N(N+1)}{2}$

Quiz- 9

```
for(i=1; i≤N; i++){
    for(j=1; j≤2^i; j++){
        ------------------
    }
}
```



$2^1 + 2^2 + \dots\dots 2^N$

$\frac{a(r^N-1)}{r-1}$

$a = 2 \qquad r = 2$

$\frac{2(2^N-1)}{2-1}$

$2(2^N-1)$

Algo.1

100*logN

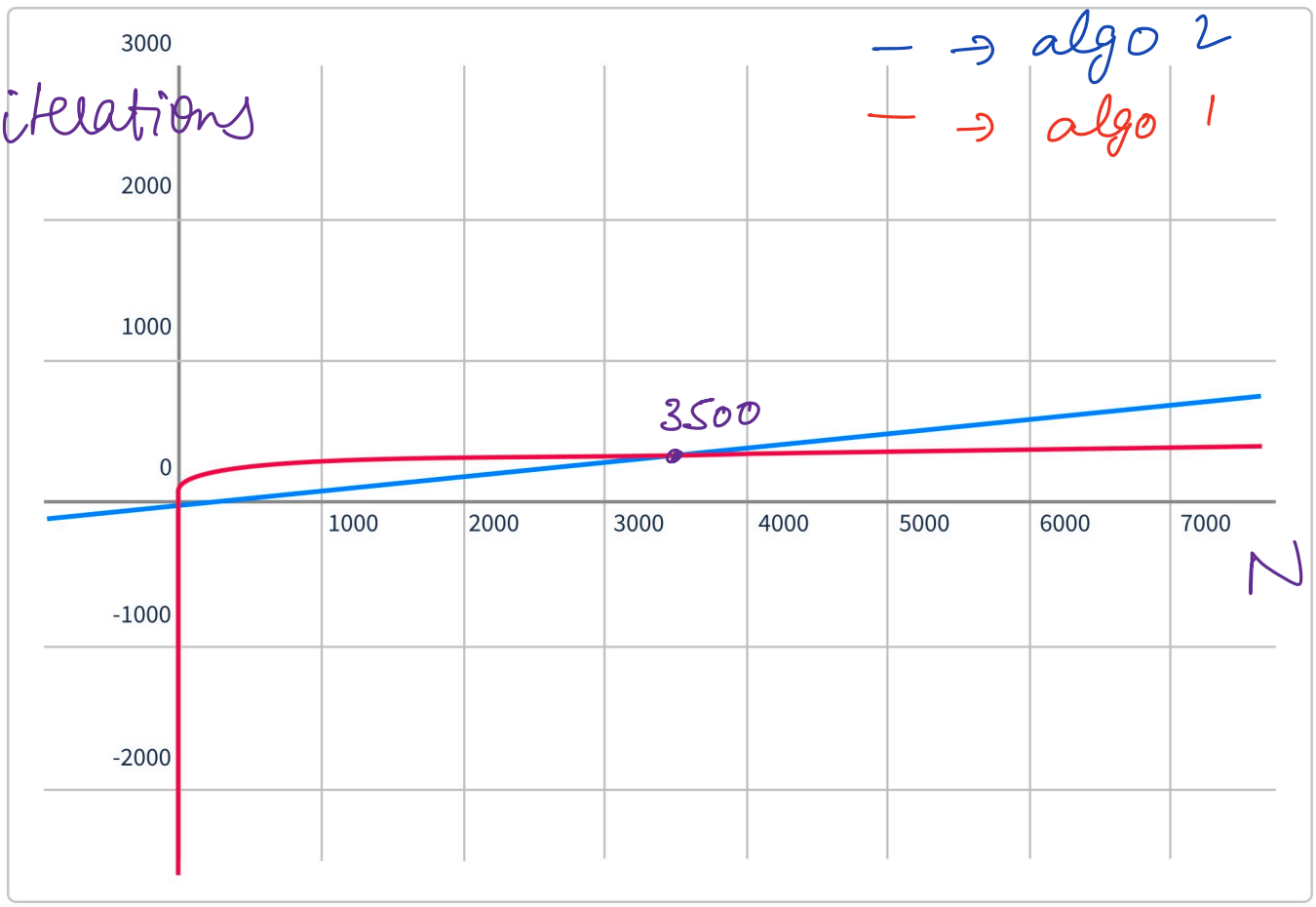
$\Rightarrow O(\log N)$

Algo.2

N/10

$\frac{1}{10} \times N$

$O(N)$



Asymptotic analysis of Algorithms

Analysis for very large scale / numbers

Big-0 notation

1) Calculate iterations of your algo
2) Ignore lower order terms
3) Ignore constant multiplier.

$$3N^2 + 4N + 55$$

$$\Rightarrow 3N^2$$

$$\Rightarrow N^2 \qquad \Rightarrow O(N^2)$$

$$\log(N) < \text{sqrt}(N) < N < N\log N <$$

$$N\,\text{sqrt} < N^2 < N^3 < 2^N < N^N$$

$$N^2$$

Why do we ignore lower order terms?

Iterations → $N^2 + 10.N$

100 + 100

N	$N^2 + 10.N$ (Total iterations)	Percentage of 10.N in total iterations
10	200	50%
100		9 %
1000		0.1 %

Why to neglect co-efficient / constants?

$10 \log N$ N

$100 \log N$ N

$9N$ N^2

$19N$ N^2

2.

```
for(int i=1; i≤N; i++){
        if(i%2!=0){
            c=c+1;
        }
}
```

$N \Rightarrow O(N)$

```
for(int i=1; i≤N; i=i+2){
    c=c+1;
}
```

$N/2 \Rightarrow O(N)$

$i = 1 \quad 3 \quad 5 \quad 7 \quad 9 \quad \ldots \quad N$

Online Editors and T.L.E

$10^8 \Rightarrow 1$ sec

$N = 10^5$ N^2 10^{10}

$N = 10^4$ N^2 10^8

Constraints

$N \leq 10^5$

$10^5 \times 10^5 \Rightarrow 10^{10}$

N^2 ×

$N \log N$ ✓

$\approx 2 \times 10^6$

$10^5 \times \log_2 10^5$ 1.7×10^6

17

$N \leq 10^3$

$N^2 \Rightarrow 10^3 \times 10^3 = 10^6$ ✓

$N^3 \Rightarrow 10^9$ ×

How should we approach a problem?