

Bit Manipulation

TABLE OF CONTENTS

1. Number System and Conversion
2. Addition of two binary numbers
3. Bitwise Operators
4. Negative Numbers
5. Ranges of Numbers
6. Importance of Constraints



Notes



Decimal Number System *base -10*

Numbers we use everyday

$$342 \Rightarrow 300 + 40 + 2$$

$$3 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$$

$$2563 \Rightarrow 2000 + 500 + 60 + 3$$

$$\begin{matrix} 3 & 2 & 1 & 0 \\ 2 \times 10^3 & + & 5 \times 10^2 & + & 6 \times 10^1 & + & 3 \times 10^0 \end{matrix}$$

Binary Number System

base -2

0's 1's

$$\begin{matrix} 1 & 1 & 0 \\ 2 & 1 & 0 \end{matrix}$$

$$1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$= 6$$

$$\begin{matrix} 1 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{matrix}$$

$$1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= 13$$

$$\begin{matrix} 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{matrix}$$

$$1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 90$$



1. Binary to Decimal Conversion

• $(1101)_2 = (\quad)_{10}$

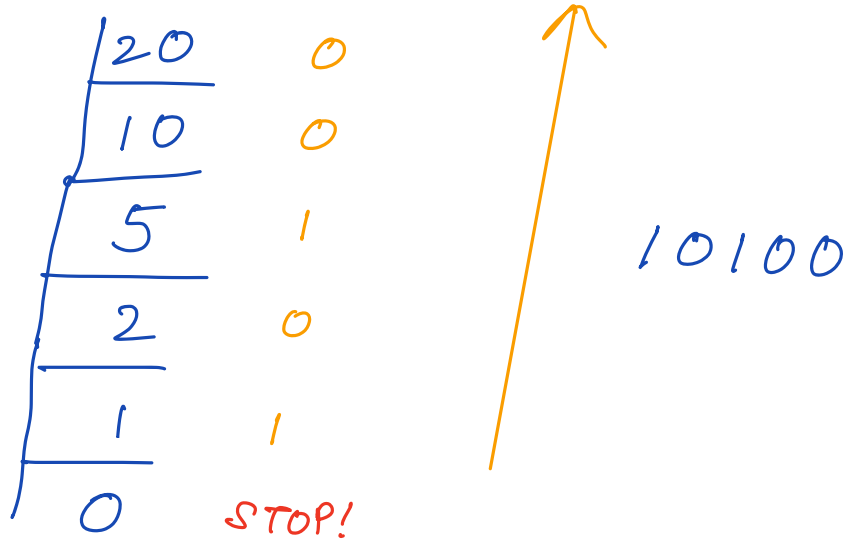
$$\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{array}$$

• $(1011010)_2 = (\quad)_{10}$



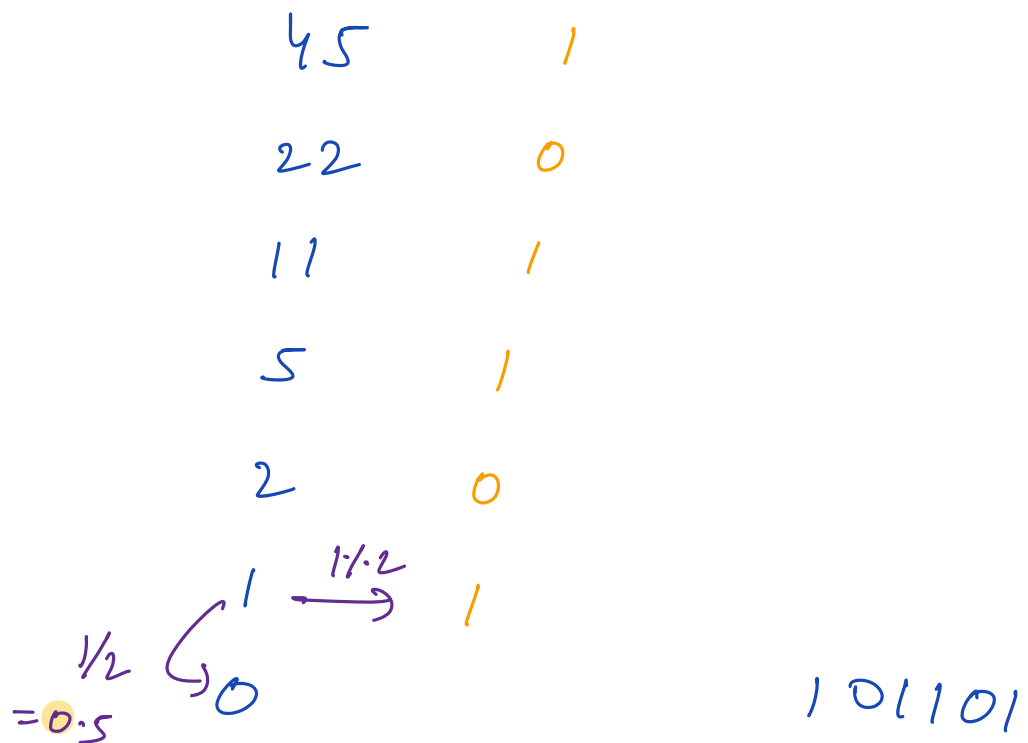
2. Decimal to Binary

• $(20)_{10} = (\quad)_2$



Ans = 10100

• $(45)_{10} = (\quad)_2$






Addition of two decimal numbers -

$$\begin{array}{r} \textcircled{1} \quad \textcircled{1} \\ 3 \quad 6 \quad 8 \\ + \quad 1 \quad 4 \quad 5 \\ \hline 5 \quad 1 \quad 3 \\ \hline \end{array}$$

Addition of two ^{binary}~~decimal~~ numbers -

$$\begin{array}{r} \textcircled{1} \quad \quad \quad \textcircled{1} \\ 1 \quad 1 \quad 0 \quad 1 \\ + \quad 1 \quad 0 \quad 0 \quad 1 \\ \hline 1 \quad 0 \quad 1 \quad 1 \quad 0 \\ \hline \end{array} \quad \begin{array}{r} 13 \\ 9 \\ \hline 22 \end{array}$$

$$\begin{array}{l} 2 \Rightarrow 10 \\ 2 \rightarrow 0 \\ 1 \rightarrow 1 \\ 0 \text{ stop} \end{array}$$




$$\begin{array}{r} \\ \\ + \\ \hline 1 1 1 1 \\ \hline \end{array}$$

$$2 \Rightarrow 10$$

$$3 \Rightarrow 11$$





Bitwise Operators

! , & , | , ^ , << , >> - Advanced BM

NOT AND

OR XOR

result is 1 if both inputs = 1

result is 1 if any of the inputs = 1

		AND	OR	XOR	NOT → reverse 3rd 1
a	b	a&b	a b	a^b	~a/!a
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

XOR = same same puppy theme

5 & 6 = ?

5 & 6 = 4

```

5 → 1 0 1
6 → 1 1 0
-----
    1 0 0
  
```

20 & 45 = ?

20 & 45 = 4

```

20 → 0 1 0 1 0 0
45 → 1 0 1 1 0 1
-----
    0 0 0 1 0 0
  
```





$$20 \mid 45 = ?$$

20 \rightarrow 0 1 0 1 0 0

45 \rightarrow 1 0 1 1 0 1

1 1 1 1 0 1

 61

$$20 \wedge 45 = ?$$

20 \rightarrow 0 1 0 1 0 0

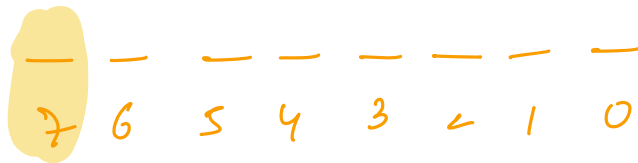
45 \rightarrow 1 0 1 1 0 1

1 1 1 0 0 1

\Rightarrow 57



Negative Numbers



8 bit number

signed
bit

\swarrow $= 0$ number is +ve
 \searrow $= 1$ number is -ve

How to get -ve number in binary

2's complement \rightarrow 1's complement + 1

\Downarrow
 !a

-5

5 \Rightarrow 0 0 0 0 0 1 0 1

!5 \Rightarrow 1 1 1 1 1 0 1 0
 +1 + 0 0 0 0 0 0 0 1

-5 \Rightarrow

 1 1 1 1 1 0 1 1



0



No. is positive

1



No. is negative

Binary representation of -3

$$\begin{array}{r} 3 \Rightarrow \quad 00000011 \\ !3 \Rightarrow \quad 11111100 \\ +1 \quad + \quad 00000001 \\ \hline 11111101 \end{array}$$

$10 \Rightarrow 00001010$
 $\sim 10 \quad 11110101$
 $+1 \quad 11110110$

$\text{int} \Rightarrow \underline{\underline{32 \text{ bits}}}$

----- } 32 bit

$\left[-2^{31}, 2^{31} - 1 \right]$ $\rightarrow \text{Integer.MAX_VAL}$
 \downarrow
 Integer.MIN_VAL

Appron $\Rightarrow 2 \times 10^9 \quad \text{INT_MAX}$

$\text{int } a = 10^5$

$\text{int } b = 10^6$

$\Rightarrow 10^{11}$

$\text{long } c = (\text{long}) a * b$

Q Find array sum

$$1 \leq N \leq 10^5$$

$$1 \leq A[i] \leq 10^6$$

sum \Rightarrow

$$\underbrace{10^6 + 10^6 + 10^6 + \dots}_{10^5 \text{ times}}$$

$$= 10^6 \times 10^5 = 10^{11}$$