Optimization of a Fed-Batch Bioreactor with Biomass Constraint

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Introduction

- **Fed-Batch Bioreactor:** A bioreactor where substrates are continuously added over time, while the culture medium is not removed. This method allows for more control over microbial growth and product formation.
- **Applications:** Fed-batch reactors are widely used in the production of pharmaceuticals, biofuels, and fermentation products.

Reactions and Conditions

Reactions:

- Substrate S is converted into Biomass X and Product P:

$$S \xrightarrow{X} P.$$

Conditions:

- Fed-batch bioreactor, isothermal.

Objective and Problem Definition

Objective: Maximize the final concentration of product P at a given time t_f.

Manipulated Variable: The feed rate of substrate u, which influences both the substrate concentration S and biomass concentration X.

Constraints:

- Input bounds on feed rate: $u_{\min} \leq u \leq u_{\max}$.
- Biomass concentration constraint: $X(t) \leq X_{\max}$ to avoid oxygen depletion.

Model Equations

The following set of differential equations describe the system dynamics:

- **Biomass Growth:**

$$\frac{dX}{dt} = \mu(S)X - \frac{u}{V}X$$

- **Substrate Consumption:**

$$\frac{dS}{dt} = -\frac{\mu(S)}{Y_x}X - \frac{\nu}{Y_p}X + \frac{u}{V}(S_{\text{in}} - S)$$

- **Product Formation:**

$$\frac{dP}{dt} = \nu X - \frac{u}{V}P$$

- **Volume Growth:**

$$\frac{dV}{dt} = u$$

Optimization Variables and Parameters

Variables:

- S: Substrate concentration
- X: Biomass concentration
- P: Product concentration
- V: Volume of the culture
- u: Feed rate of substrate

Parameters:

- μ_m, K_m, K_i, ν: Kinetic parameters
- Y_x, Y_p: Yield coefficients
- S_{in}: Inlet substrate concentration
- $X_{\max}$: Maximum biomass concentration
- $u_{\min}$, $u_{\max}$: Feed rate bounds

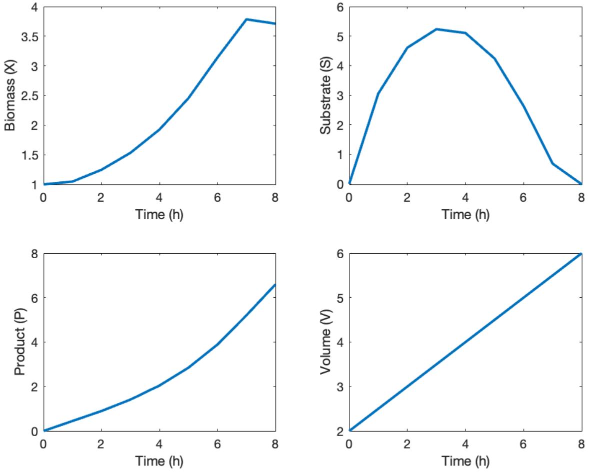
Model Parameters and Initial Conditions

The following values are used for the model parameters:

- $\mu_m = 0.53\,\mathrm{h}^{-1}$
- $K_m = 1.2\,\mathrm{g/l}$
- $K_i = 22\,\mathrm{g/l}$
- $Y_x = 0.4,\; Y_p = 1$
- $\nu = 0.5\,\mathrm{h}^{-1}$
- $S_{\mathrm{in}} = 20\,\mathrm{g/l}$
- $u_{\mathrm{min}} = 0\,\mathrm{l/h},\; u_{\mathrm{max}} = 1\,\mathrm{l/h}$
- $X_{\mathrm{max}} = 3\,\mathrm{g/l}$
- Initial concentrations: $X_0 = 1\,\mathrm{g/l},\; S_0 = 0\,\mathrm{g/l},\; P_0 = 0\,\mathrm{g/l},\; V_0 = 2\,\mathrm{l}$

ODE Solutions at Fixed Feed Rate

- at $u = 0.5l/h$




```
>> ModellingFermentor
The final product concentration P is: 6.5964 at u = 0.5000
```

Optimization Problem

Objective:

$$\max_{u(t)} \quad J = P(t_f)$$

Subject to:

- Dynamic system equations (1) to (4)
- Biomass constraint: $X(t) \leq X_{\max}$
- Feed rate bounds: $u_{\min} \leq u(t) \leq u_{\max}$

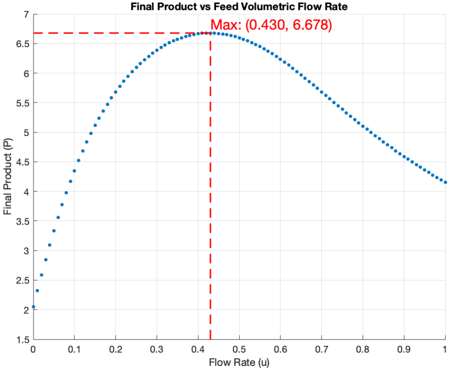
Optimization Method

Optimization method used:

- **Iterative Method:**
  - Create an array of u values from u_{min} to u_{max}.
  - Solve the ODEs for each value of u and compute the corresponding product concentration P.
  - Identify the u corresponding to the maximum P value.
  - Simple to implement but lacks precision and can be computationally expensive.

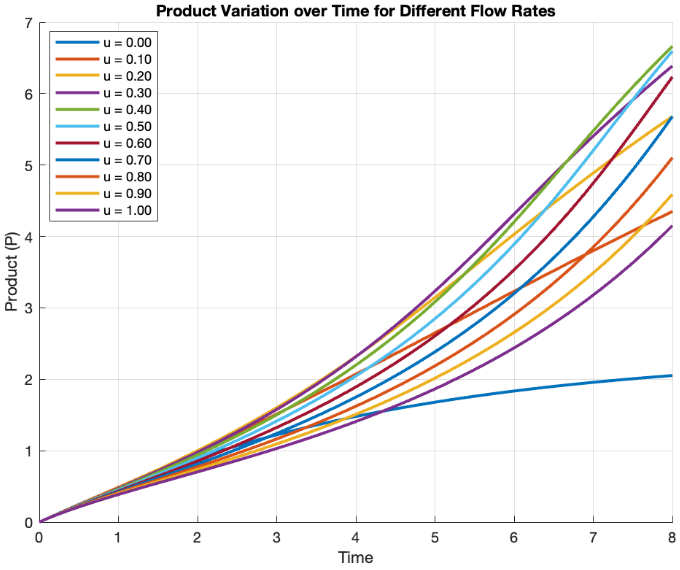
Iterative Method

- Iterative method used to solve ODEs for different feed rates u.




```
>> IterativeOptimization
We got the maximum P = 6.6778 at u = 0.4300
```

Iterative method



fmincon Optimization Method

- `fmincon` is a MATLAB function used for solving constrained nonlinear optimization problems.
- It is designed to minimize an objective function, subject to both linear and nonlinear constraints.
- In this case, the objective is to maximize the product concentration P by optimizing the feed rate u.
- The input to `fmincon` includes:
  - Initial guess for u (e.g., $u_0 = 0$).
  - Boundaries for u (e.g., $u_{\min} = 0, u_{\max} = 1$).
  - Linear and nonlinear constraints, if applicable (e.g., $X(t) \leq X_{\max}$).

How fmincon Works in Our Case

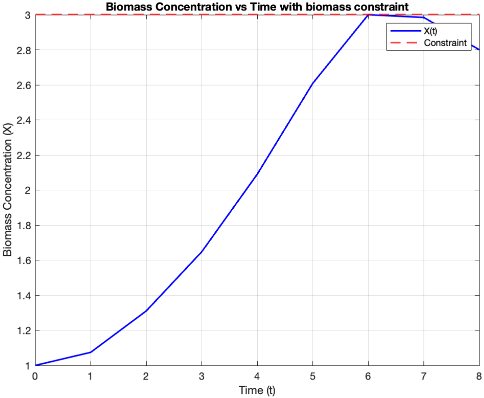
```
[uopt, fval] = fmincon(@(u) fun(u, tspan),u0, A, b, Aeq, beq, lb, ub, nonlcon);
```

- **Optimization Process**:
  - The objective function $J = -P(t_f)$ is minimized.
  - Nonlinear constraints like $X(t) \leq X_{\text{max}}$ are applied to prevent overgrowth.
  - `fmincon` iteratively adjusts u to maximize $P(t_f)$.
- **Result**: The optimal feed rate $u_{\text{opt}} = 0.3173$ is found, which maximizes P and respects the constraints.

This process ensures an optimal and sustainable bioprocess.

fmincon Optimization (With Constraints)

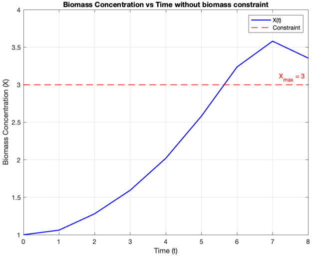
- Biomass constraint: $X(t) \leq 3$




```
fmincon optimal solution:
u = 0.3173 , Maximum value of the P = 6.4633
```

Comparison with Iterative Method (No Constraints)

- Iterative method: $u = 0.4284, P = 6.6778$.
- fmincon: $u = 0.4284, P = 6.6778$.




```
fmincon optimal solution:
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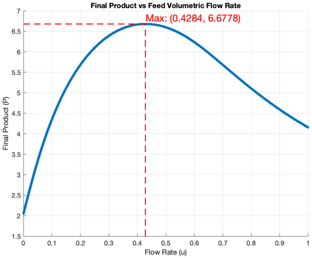
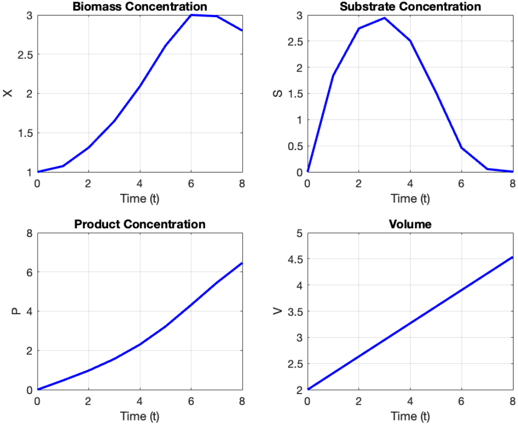



```
>> IterativeOptimization
We got the maximum P = 6.6778 at u = 0.4284
```

fmincon Method (With Constraints)

- Optimal feed rate $u = 0.3173$.
- Final product concentration $P = 6.4633$.



Future Work

- **Hourly Feed Rate Optimization:**
  - In the current work, a single optimal feed rate u is determined for the entire 8-hour period.
  - Future work aims to optimize the feed rate $u(t)$ at each hour to maximize the product concentration $P(t_f)$ at the final time $t_f = 8$ hours.
- **WWT Plant**
  - Understanding the CWM1 Model simulate it and control it.

Conclusions

- Both the iterative method and the fmincon optimization method provide similar results, with fmincon offering a more efficient and precise solution.
- Biomass concentration constraints are crucial for maintaining a stable bioprocess and preventing overgrowth and oxygen depletion.
- The optimal feed rate, determined by fmincon, ensures maximum product concentration while respecting constraints.

References

1. P.F. Stanbury, A. Whitaker, and S.J. Hall, *Principles of Fermentation Technology*. Pergamon, 2017.
2. L. Bastiaens and J. F. Van Impe, *Bioreactor dynamics and control*. Springer, 2015.
3. "fmincon" [Online]. Available: `https://www.mathworks.com/help/optim/ug/fmincon.html`
4. F. Garcia-Ochoa, E. Gomez, V. E. Santos, and J. C. Merchuk, "Oxygen uptake rate in microbial processes: an overview," *Biochemical Engineering Journal*, vol. 49, no. 3, pp. 289-307, 2010.

Thank you for your attention!

Questions and Comments?