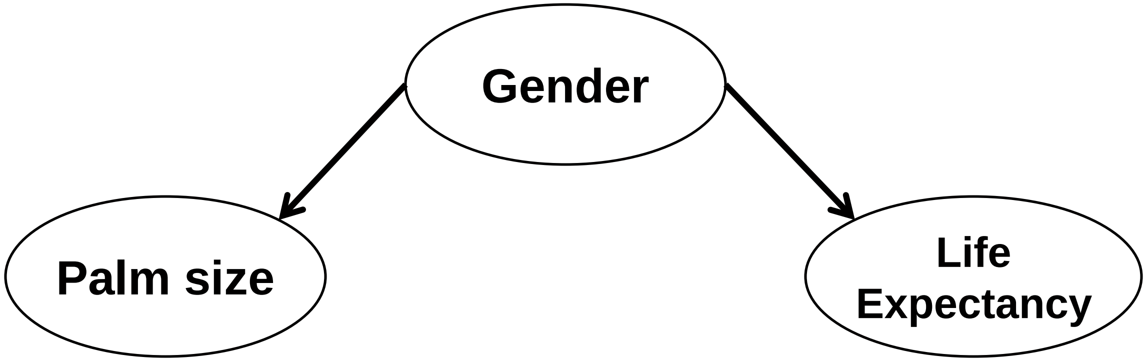
From Data To Understanding

- In machine learning, maintain critical perspective
  - Making predictions is only part of the story
  - Also try to get some <u>understanding</u> of the domain
- Example
  - True statement: palm size negatively correlates with life expectancy
    - The larger your palm size, the shorter your life (on average)
  - Why?
    - Women have smaller palms than men on average
    - Women live 5 years longer than men on average
  - Sometimes you need better model of your domain!

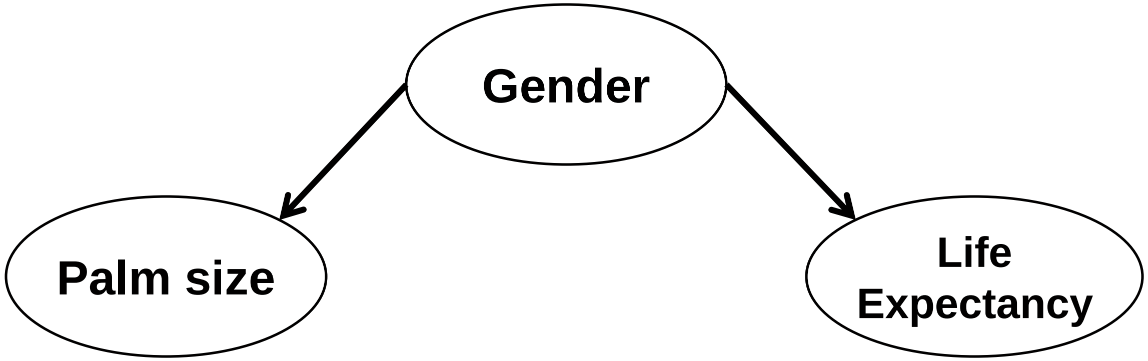
Bayesian Networks

- Bayesian Network
  - Graphical representation of joint probability distribution



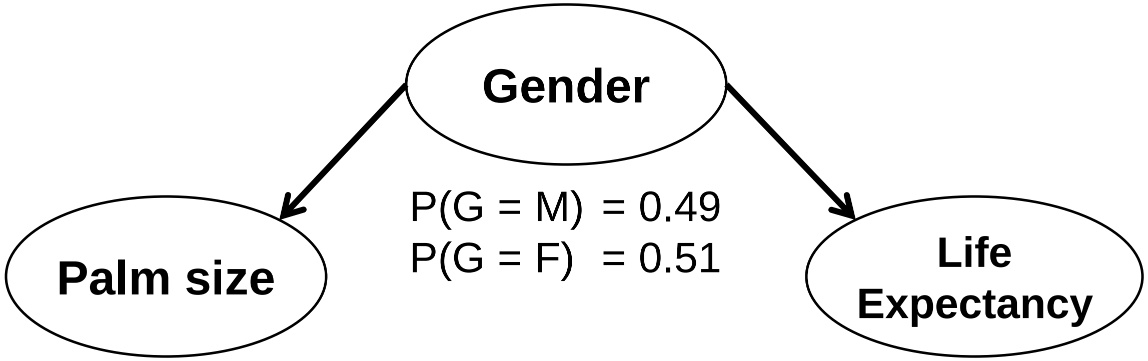
- Node: random variable
- Arc (X, Y): variable X has direct influence on variable Y
  - Call X a "parent" of Y
- Each node X has conditional probability: P(X | parents(X))
- Graph has no cycles (loops by following arcs)
  - Called "Directed Acyclic Graph" (DAG)

Network Shows Conditional Independence



- Conditional independence encoded in network
  - Each node (variable) is conditionally independent of its non-descendants, given its parents
  - In network above, Palm Size (PS) and Life Expectancy (LE) are conditionally independent, given Gender (G)
    - Formally: P(PS, LE | G) = P(PS | G) P(LE | G)
- Network structure provides insight about domain

Conditional Probability Tables



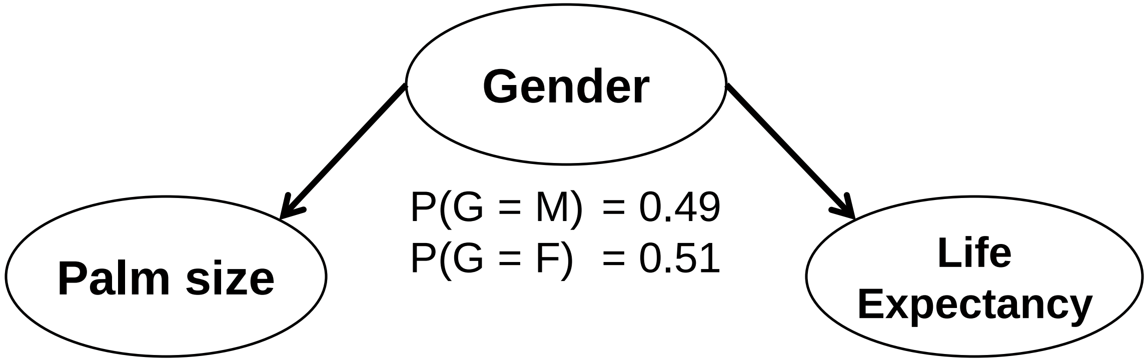
P(G = M) = 0.49
P(G = F) = 0.51

P(PS = L | G = M) = 0.45	P(LE = < 70 | G = M) = 0.20
P(PS = M | G = M) = 0.35	P(LE = 70-80 | G = M) = 0.50
P(PS = S | G = M) = 0.20	P(LE = > 80 | G = M) = 0.30
P(PS = L | G = F) = 0.10	P(LE = < 70 | G = F) = 0.10
P(PS = M | G = F) = 0.30	P(LE = 70-80 | G = F) = 0.35
P(PS = S | G = F) = 0.60	P(LE = > 80 | G = F) = 0.55

- Each node has conditional probability table (CPT)
  - For node X: P(X | Parents(X))
  - Conditional independence modularizes joint probability:

$$P(X_1, X_2, \ldots, X_m) = \prod_{i=1}^{m} P(X_i \mid \mathrm{Parents}(X_i))$$

Efficient Representation



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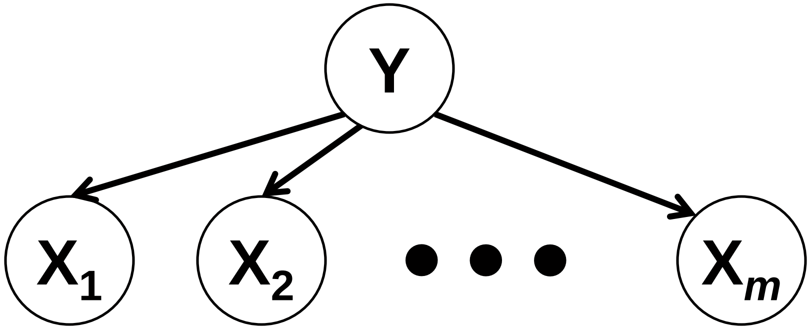
- Each node has conditional probability table (CPT)
  - Reduces number of parameters needed in model
  - Normally, need 2 x 3 x 3 – 1 = 18 – 1 = 17 parameters
  - Here, need (2 – 1) + (6 – 2) + (6 – 2) = 9 parameters

Bayesian Network for Naïve Bayes

- Welcome back, Naïve Bayes…
  - Now with new and improved "Bayesian Network" flavor!



  - Network structure encodes assumption:

$$P(\boldsymbol{X} \mid Y) = P(X_1, X_2, \ldots, X_m \mid Y) = \prod_{i=1}^{m} P(X_i \mid Y)$$

  - Full joint distribution can be computed as:

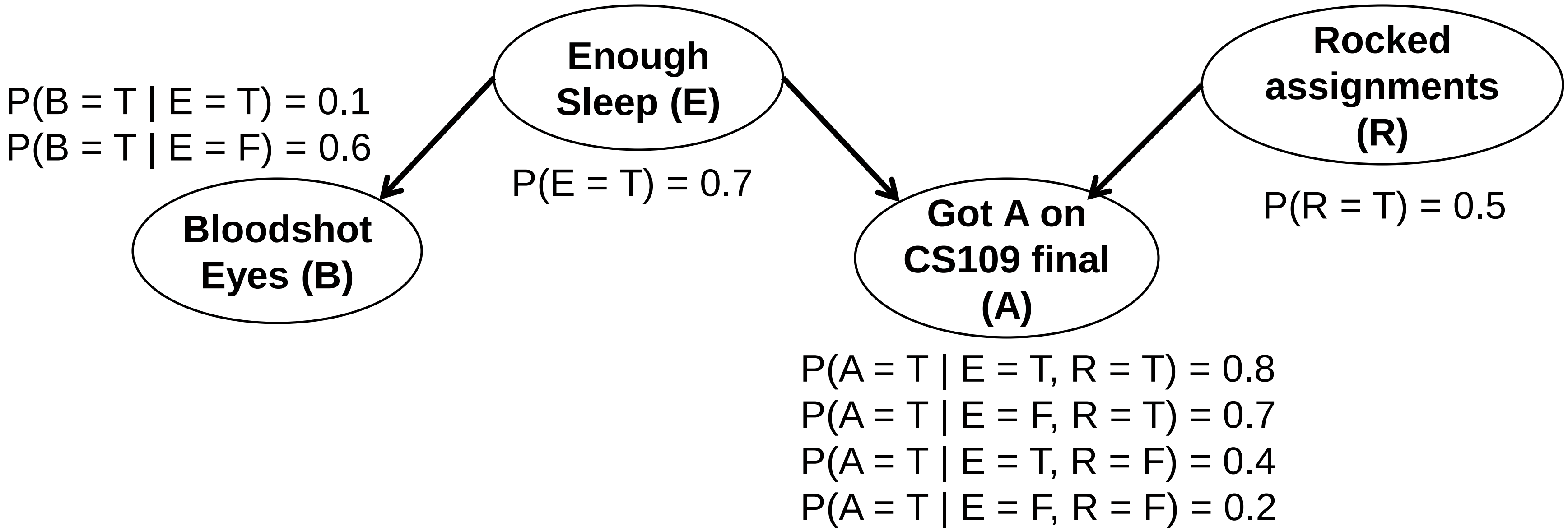
$$P(\boldsymbol{X}, Y) = P(Y) P(\boldsymbol{X} \mid Y) = P(Y) \prod_{i=1}^{m} P(X_i \mid Y)$$

“Evidence” in Bayesian Networks

- In many machine learning examples:
  - We observe all $X_1, X_2, \ldots, X_m$ input variables and predict single output variable Y
- In general case of probabilistic inference:
  - Have a set of random variables $X_1, X_2, \ldots, X_m$
  - *Subset* of the variables $X_1, X_2, \ldots, X_m$ are observed
    - Call observed variables $E_1, E_2, \ldots, E_k$ (E for “evidence”)
  - Want to determine probability of some set of *unobserved* variables given the observed evidence
    - Call unobserved variables we care about $Y_1, Y_2, \ldots, Y_c$
  - Formally, want: $P(Y_1, Y_2, \ldots, Y_c \mid E_1, E_2, \ldots, E_k)$

Evaluation of Evidence

- Consider the following Bayes Net:

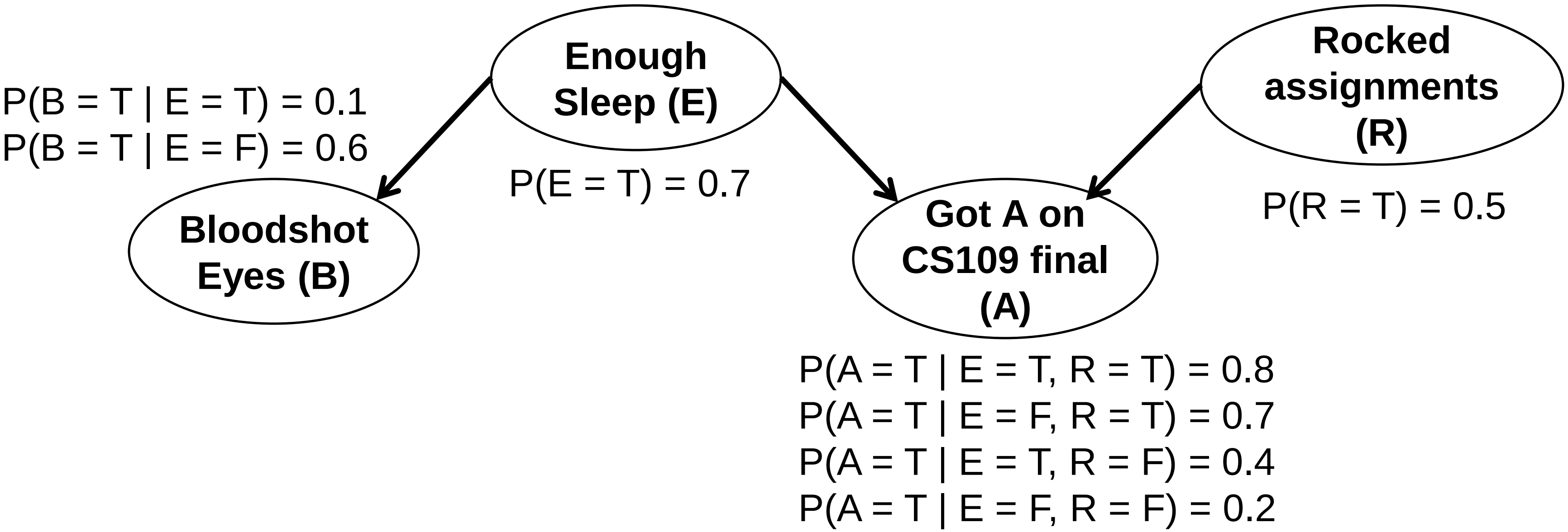


  - Determine P(A = T | B = T, R = T)
  - Sum over unseen variables:

$$P(A=T \mid B=T, R=T) = \frac{P(A=T, B=T, R=T)}{P(B=T, R=T)} = \frac{\sum_{E=T,F} P(A=T, B=T, R=T, E)}{\sum_{E=T,F} \sum_{A=T,F} P(B=T, R=T, E, A)}$$

Evaluation of Evidence

- Consider the following Bayes Net:



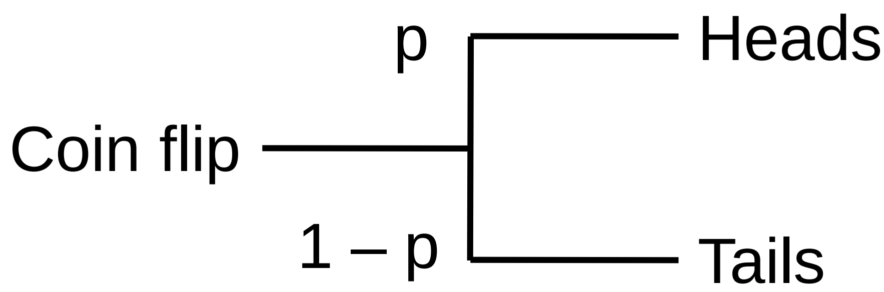
  - Determine P(A = T | B = T, R = T)
  - Note that joint probability decomposes as:

$$P(A, B, E, R) = P(E)P(B \mid E)P(R)P(A \mid E, R)$$

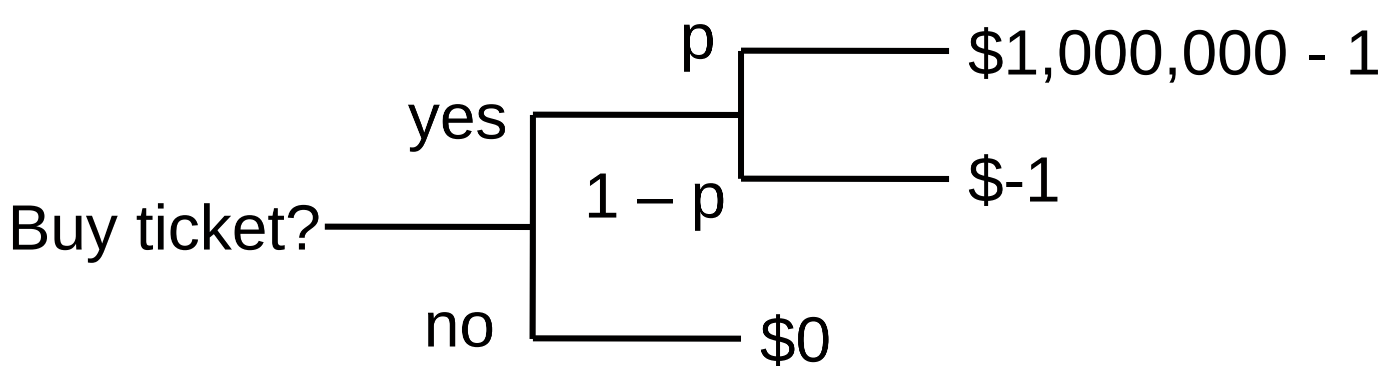
  - Plug in values from CPTs to compute joint probabilities

Probability Tree

- Model outcomes of probabilistic events with tree



- Useful for modeling decisions

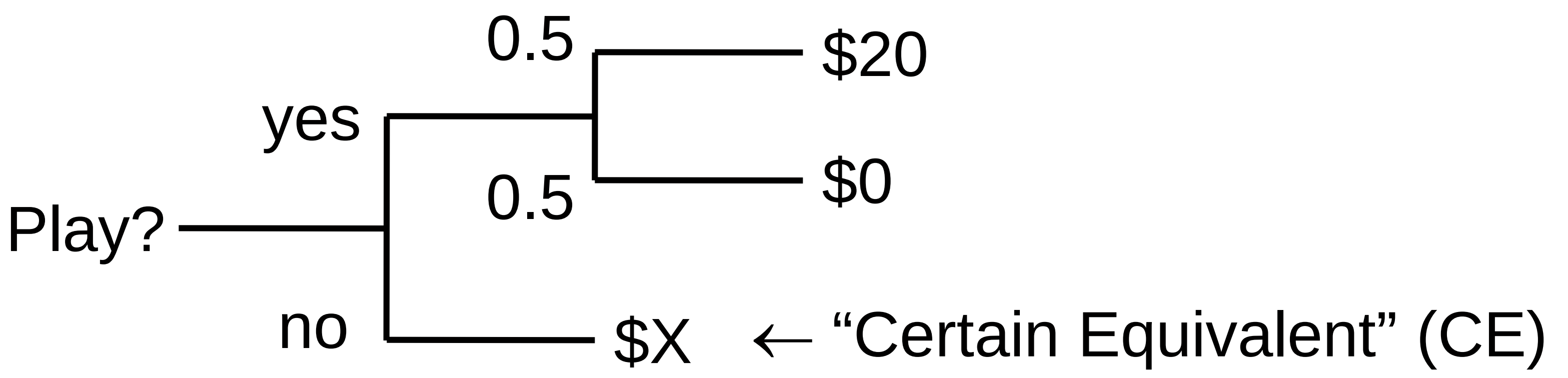


  - Expected payoff: yes = p(1000000 - 1) + (1 - p)(-1)

    no = 0

Let’s Play a Game

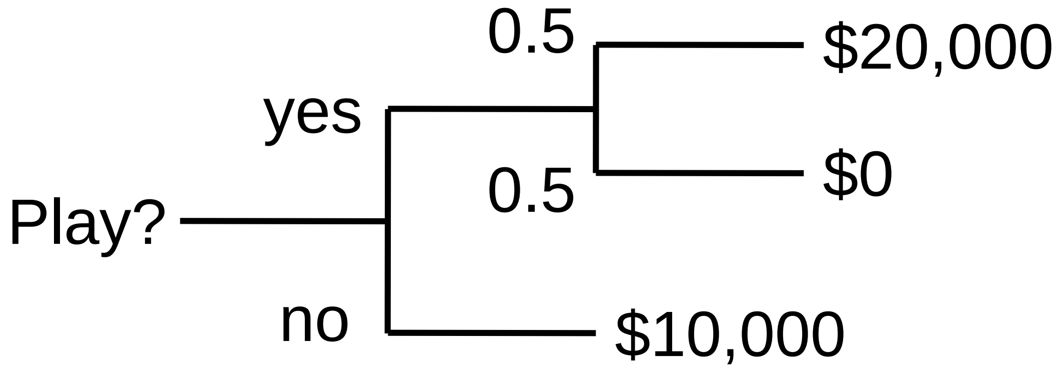
- Which choice would you make?

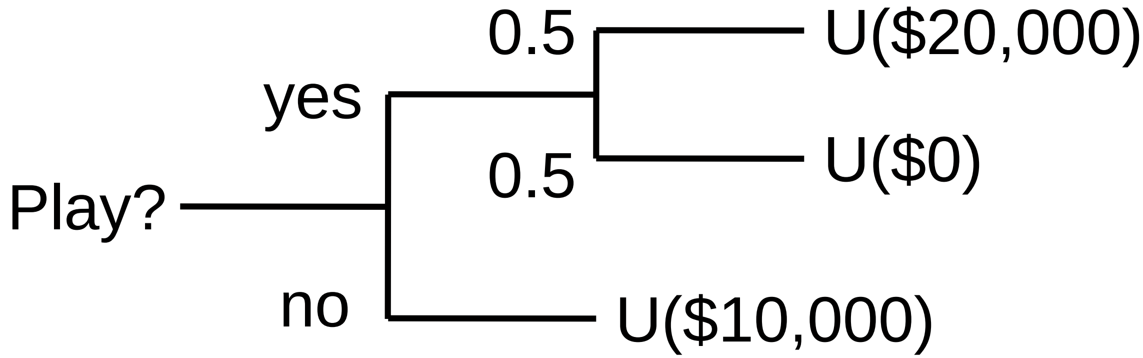


- For what value of X are you <u>indifferent</u> to playing?
  - X = 3
  - X = 7
  - X = 9
  - X = 10
- Certain equivalent is value of game to <u>you</u>

Utility

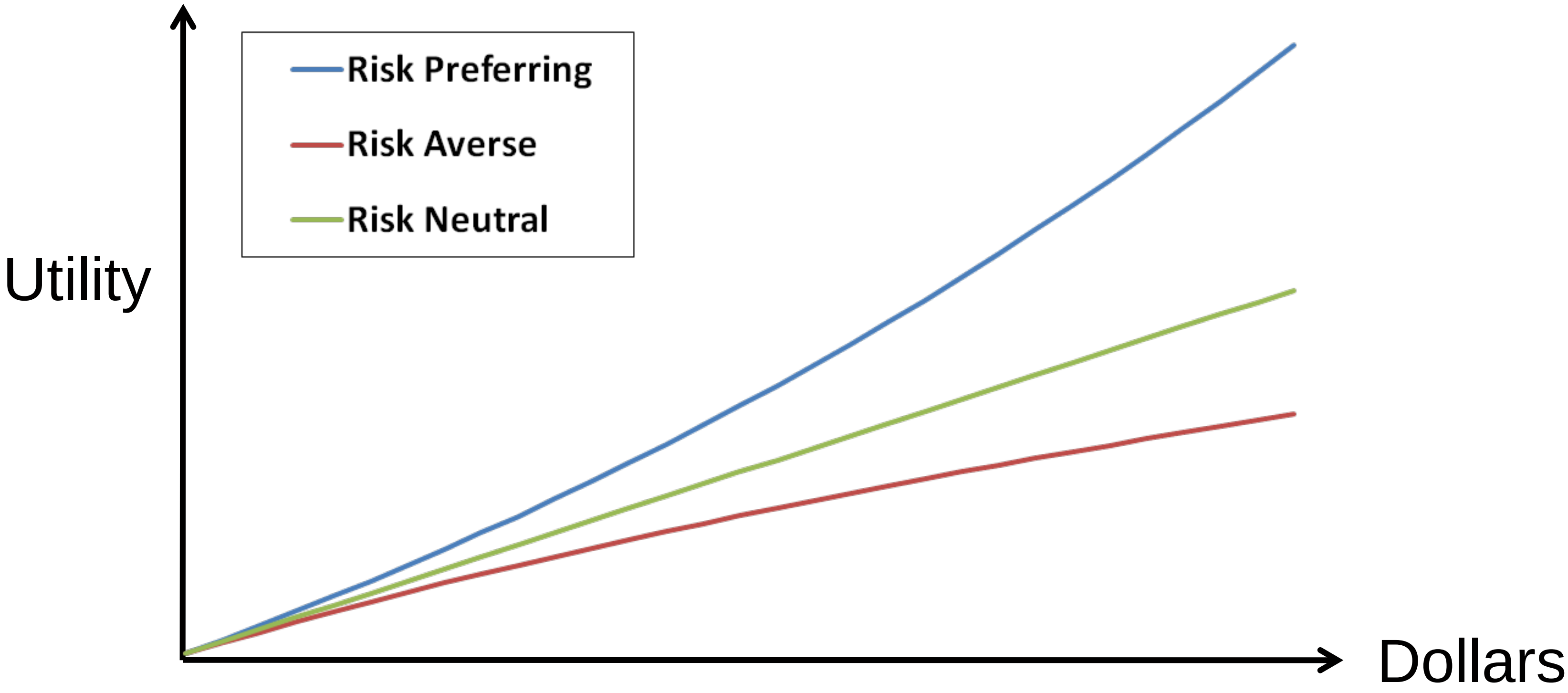
- Utility U(x) is “value” you derive from x





  - Can be monetary, but often includes intangibles
    - E.g., quality of life, life expectancy, personal beliefs, etc.

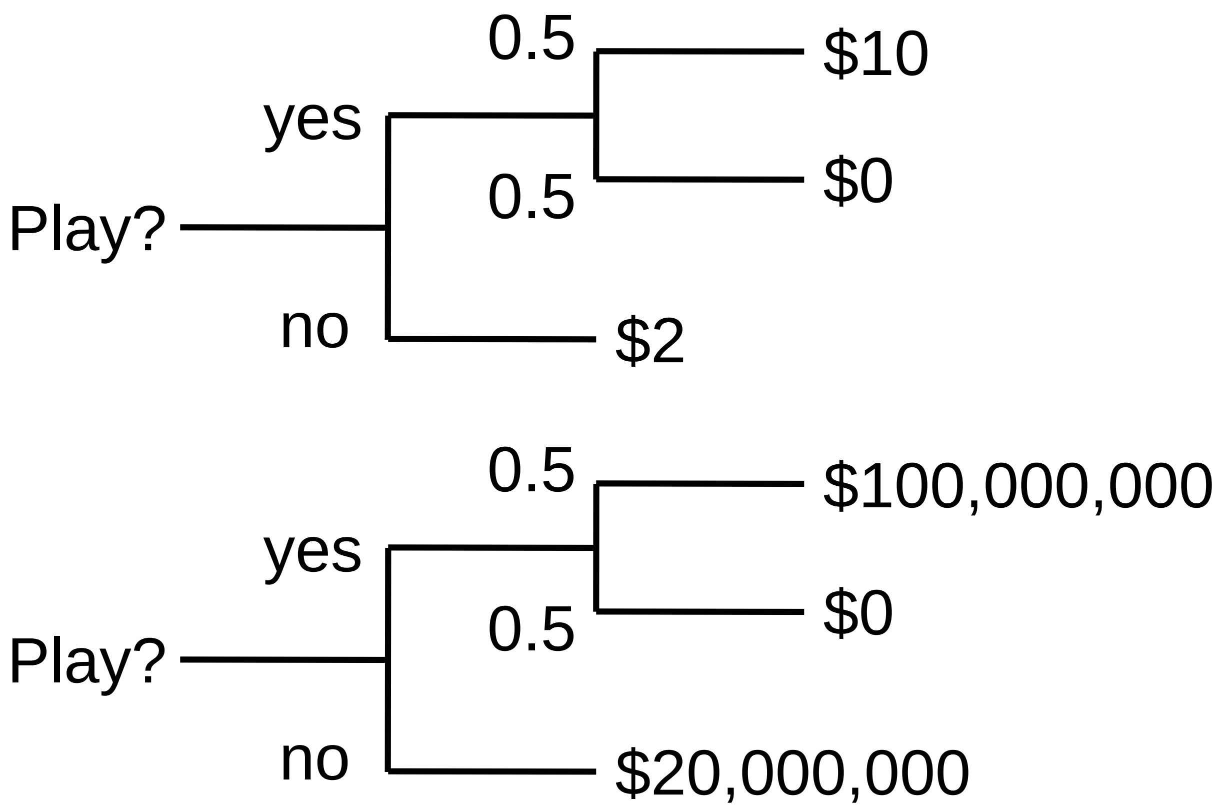
Utility Curves



- Utility curve determines your “risk preference”
  - Can be different in different parts of the curve

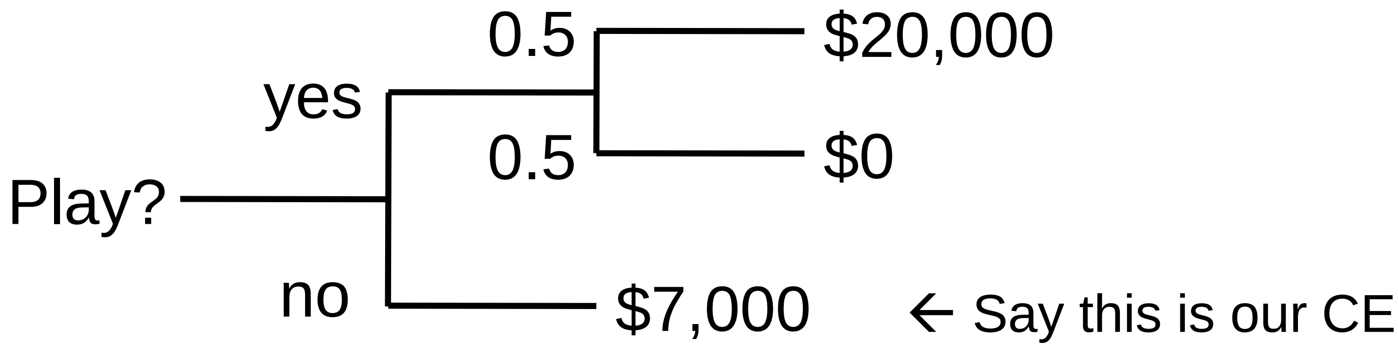
Non-Linear Utility of Money

- These two choices are different for most people

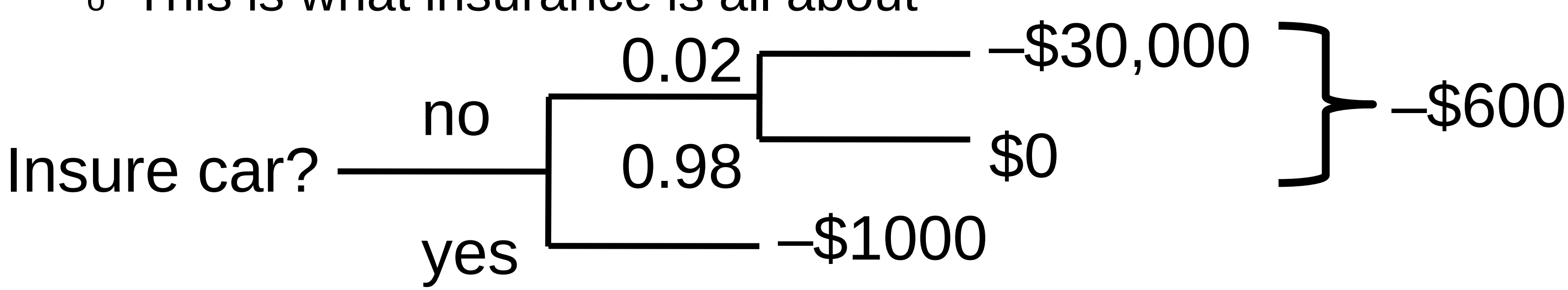


Risk Premium

- A slightly different game:



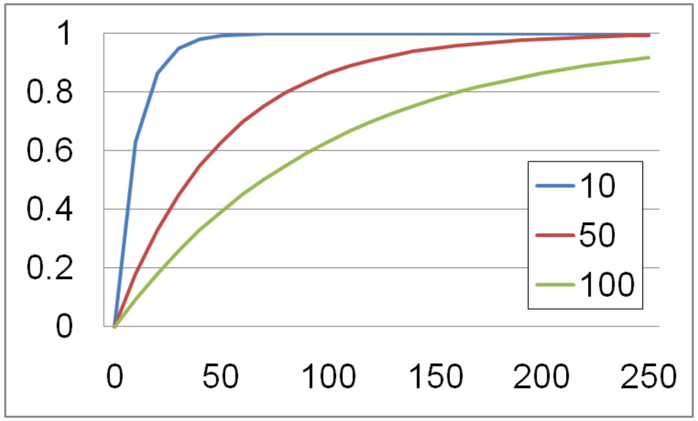
  - Expected monetary value (EMV) = expected dollar value of game (here = $10,000)
  - Risk premium = EMV – CE = $3,000
    - How much you would pay (give up) to avoid risk
    - This is what insurance is all about



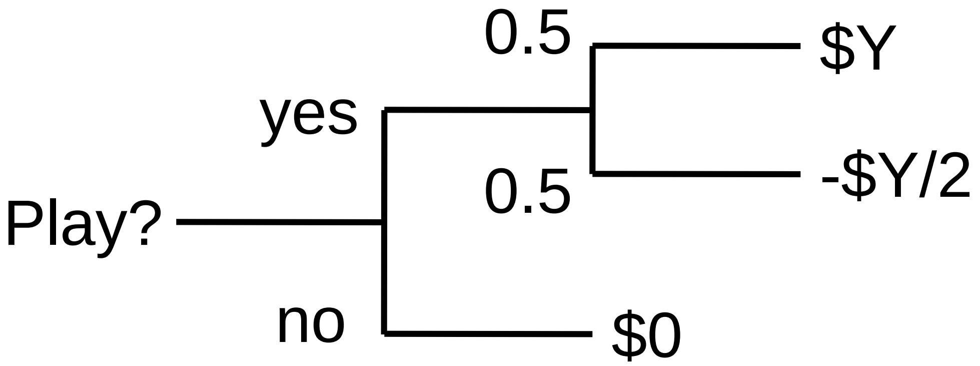
Exponential Utility Curves

- Many people have exponential utility curves

$$U(x) = 1 - e^{-x/R}$$

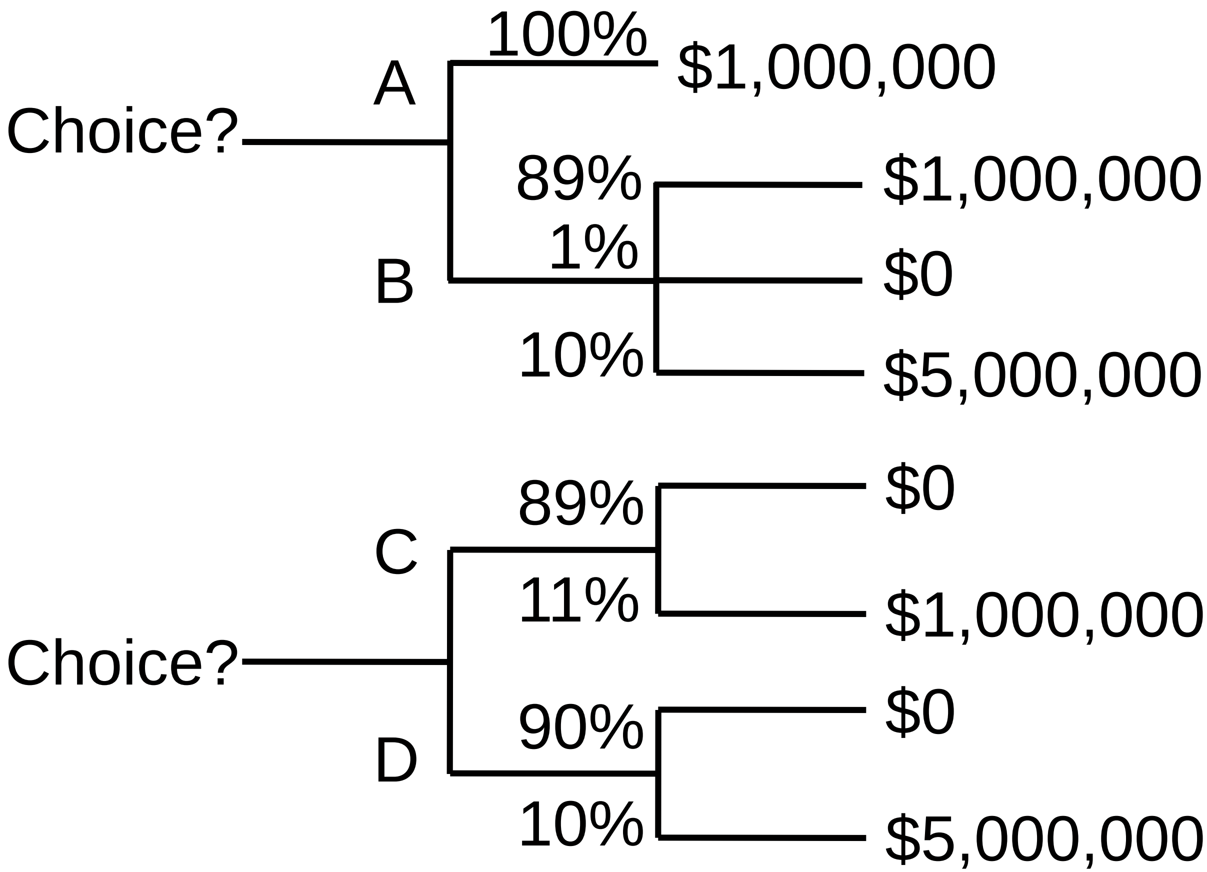


  - R is your “risk tolerance”
  - Larger R = less risk aversion
    - Makes utility function more “linear”
  - R ≈ highest value of Y for which you would play:



How Rational Are You?

- Which option would you choose?



Choice A preferred:
1.00 U(1,000,000) >
0.89 U(1,000,000) +
0.01 U(0) +
0.10 U(5,000,000)

Choice D preferred:
0.89 U(0) +
0.11 U(1,000,000) <
0.90 U(0) +
0.10 U(5,000,000)

  - How many chose A and D?

How Rational Are You?

- Which option would you choose?

Choice D preferred:
1.00 U(1,000,000) <
0.89 U(1,000,000) +
0.01 U(0) +
0.10 U(5,000,000)

X < Y

X > Y

Choice A preferred:
1.00 U(1,000,000) >
0.89 U(1,000,000) +
0.01 U(0) +
0.10 U(5,000,000)

Add 0.89 U(1,000,000)
to both sides

Choice D preferred:
0.11 U(1,000,000) <
0.01 U(0) +
0.10 U(5,000,000)

Subtract 0.89 U(0)
from both sides

Choice D preferred:
0.89 U(0) +
0.11 U(1,000,000) <
0.90 U(0) +
0.10 U(5,000,000)

  - You are inconsistent with utility theory (Allais Paradox)
    - For any choice of utility function

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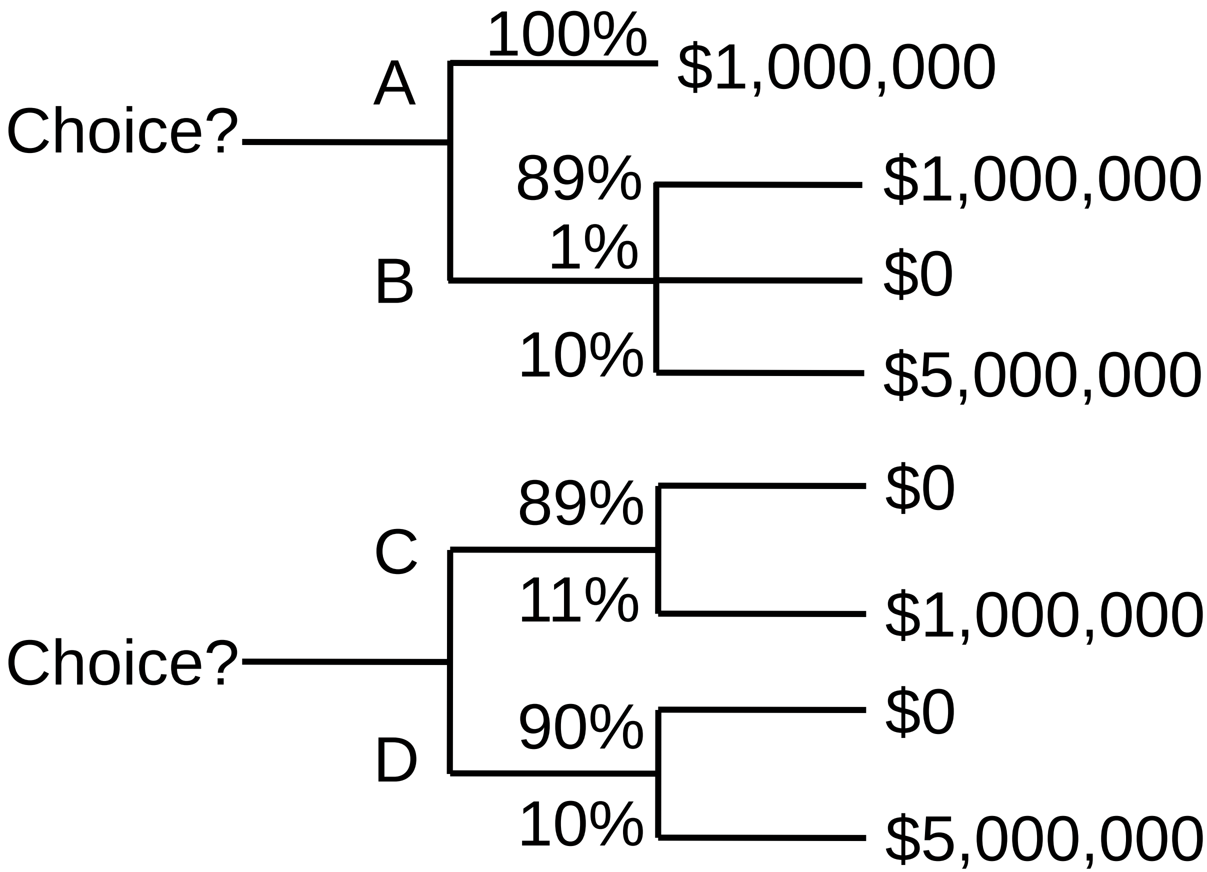

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- Human behavior is not always axiomatically consistent

Micromort

- A **micromort** is 1 in 1,000,000 chance of death
  - How much would you need to be paid to take on the risk of a micromort?
  - How much would you pay to avoid a micromort?
    - P(die in plane crash) ≈ 1 in 1,500,000
    - P(killed by lightning) ≈ 1 in 1,400,000
  - How much would you need to be paid to take on a decimort (1 in 10 chance of death)?
  - If you think this is morbid, companies actually do this
    - Car manufacturers
    - Insurance companies

Let’s Do a Real Test

- Game set-up
  - I will flip a fair coin
  - If “heads”, you win \$50. If “tails”, you win \$0
  - How much would you be willing to pay me to play?
    - \$1 ?
    - \$10 ?
    - \$20 ?
    - \$24.99 ?
    - \$25.01 ?
    - \$35 ?
  - Maximal value?
    - Come on down!
    - How did you determine that value?