Model Details

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Models of Decision Making

There are 23 different models of decision-making that we have implemented here. These models are not unique. Some of them are variants of each other. For instance "stoneEtaVarTer" is the "stoneEta" model with variability in the non decision-time.

stone, stoneEta, stoneEtaVarBase, stoneEtaVarTer, stoneEtaVarBaseVarTer, ratcliff, ratcliffVarTer models of decision-making

$$x(t+1) = x(t) + v \times dt + \sqrt(dt) \times s \times N(0,1)$$

Where x is the decision variable, v is the drift rate, dt is the step size, s is the standard deviation of the noise, $N(0,1)$ denotes the normal distribution.

A correct responseis made when $x(t+1) > a_{upper}$ and an incorrect response when $x(t+1) < a_{lower}$. The decision time is identified at the first time when the process crosses the lower or the upper bounds. The total reaction time is estimated as a sum of the decision time and a non decision time that is also estimated from the RTs. The non decision time is thought to reflect processes such as stimulus encoding and motor initation time.

stoneEta

$$x(t+1) = x(t) + v_{sample} \times dt + \sqrt(dt) \times s \times N(0,1)$$

$$v_{sample} \sim N(v, \eta)$$

Where v_{sample} is drawn from each trial from a normal distribution with mean v and a standard deviation η.

stoneEtaVarTer

Is the same formulation as the stoneEta model except it also includes variability in the non decision time.

$$x(t+1) = x(t) + v_{sample} \times dt + \sqrt(dt) \times s \times N(0,1)$$

$$v_{sample} \sim N(v, \eta)$$

The variability in the residual time for the model is defined using a uniform distribution with a range of st_0.

$$Ter = unif(Ter - st_0/2, Ter + st_0/2)$$

with $unif$ denoting the uniform distribution.

stoneEtaVarBase

This model also assumes a variability in the baseline state of accumulation. The baseline state x_0 is again drawn from a uniform distribution with limits of z_{min} and z_{max}.

$$x(t+1) = x(t) + v_{sample} \times dt + \sqrt{(}dt) \times s \times N(0,1)$$
$$x_0 = unif(z_{min}, z_{max})$$
$$v_{sample} \sim N(v, \eta)$$

stoneEtaVarBaseVarTer

This model is identical to the previous stoneEtaVarBase model except with an introduction of a variabilityin the residual time for the model which is again defined using a uniform distribution with a range of st_0.

$$x(t+1) = x(t) + v_{sample} \times dt + \sqrt{(}dt) \times s \times N(0,1)$$
$$v_{sample} \sim N(v, \eta)$$
$$x_0 = unif(z_{min}, z_{max})$$
$$Ter = unif(Ter - st_0/2, Ter + st_0/2)$$

ratcliff, ratcliffVarTer model

The ratcliff model is identical to the stoneEtaVarBase model. The ratcliffVarTer model is identical to the stoneEtaVarBaseVarTer model.

Stone UGM model

In the Urgency "gating" model that Cisek and collaborators have proposed, there is no integration of evidence instead the input evidence is low pass filtered and then multiplied by an urgency term that increases with time. Note, without a low pass filter, multiplication of instantaneous evidence by the urgency signal would lead to excessive noise especially in the later time points. So Cisek and collaborators argue that the sensory evidence is low pass filtered (time constants of either 100 or 200 ms are used). Implementation of the urgency gating model uses the exponential smoothing average approach that can be used for discrete smoothers.

x_0 is the initial evidence for each choice which is also the baseline state of the process. Again assume input drift rate is v and there is additive noise which again is appropriately scaled. Then the pair of governing equations for the UGM model are as follows. The current evidence that is used for making a decision is a weighted sum of past evidence with the present evidence.

$$\alpha = \frac{\tau}{\tau + dt}$$

$$E(t) = \alpha \times E(t-1) + (1-\alpha)(v \times dt + \sqrt{d}t \times s \times N(0,1))$$

For a quick intuition about the method. When alpha is zero, there is no filtering, however when alpha is 100 ms (and dt is 1 ms), then the previous evidence is weighted by 0.99 and the new evidence by 0.01.

The current decision variable at time t is now given as

$$u(t) = (intercept + \beta t)$$

$$x(t) = E(t) \times u(t)$$

In classical papers the intercept is set to be zero and beta to be 1.

We went a little bit crazy and designed many other variants of it.

1. stoneUGM
2. stoneEtaUGM
3. stoneEtaUGMVarTer
4. stoneEtaUGMallVar
5. stoneEtaUGMintercept
6. stoneEtaUGMinterceptVarTer
7. stoneEtaUGMVarinterceptVarTer

stoneEtaUGM

Same as the stoneUGM model except now that there is variability in the drift rate from trial-to-trial

$$\alpha = \frac{\tau}{\tau + dt}$$

$$E(t) = \alpha \times E(t-1) + (1-\alpha)(v_{sample} \times dt + \sqrt{d}t \times s \times N(0,1))$$

Where v_{sample} is drawn from a normal distribution with mean v and variance eta

$$v_{sample} \sim N(v, \eta)$$

stoneEtaUGMVarTer

Assumes the same construction as stoneEtaUGM. However the residual time is assumed to be drawn from a uniform distribution with a mean of Ter.

$$Ter = unif(Ter - st_0/2, Ter + st_0/2)$$

stoneEtaUGMintercept

The original form of the urgency gating model assumed no intercept for the urgency signal and assumed it to be zero before the stimulus arrived. This led to quantile probability plots where error trials were always slower than correct trials (see "demo.r" for plotting the output of the simple "stoneUGM").

One way to fix it is to ensure that the urgency signal also includes an intercept term, in which case one can fit this term while keeping the time constant τ and the slope term β intact. This provides slightly better fits to the behavior of the monkeys than a simple UGM

stoneEtaUGMVarintercept

There is no reason to assume that the intercept is identical on a trial-by-trial basis. This means that we can add some additional variability to the intercept of this urgency term.

stoneEtaUGMVarinterceptVarTer

This starts to get very crazy very quickly. If you assume variable intercept for the urgency signal and variable residual movement time, you can get a new family of curves.

Stone Urgency Model

The philosophy of the stone urgency model contains elements of the urgency gating model and the classical stone model except that there is no low pass filtering of the input evidence. Instead, the input evidence is multiplied by the urgency signal and then accumulated over time.

$$E(t) = (v \times dt + \sqrt{d}t \times s \times N(0,1))x(t) = x(t-1) + E(t) \times u(t)$$

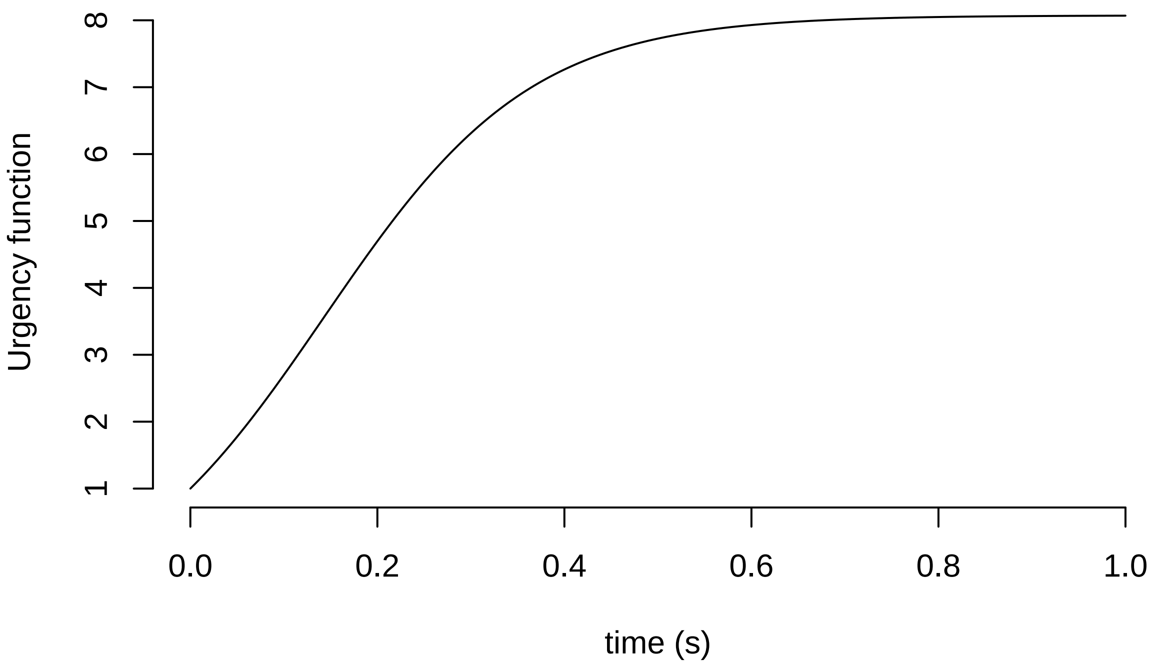
Both the input stimulus and the noise are multiplied by the urgency signal which can take on many forms. An elegant 3 parameter form with two scaling factors (s_x, s_y) and a delay (d) was proposed by Jochen Ditterich (2006).

The figure below shows an example of an urgency function estimated for a monkey performing a visual checkerboard discrimination task.

$$S_1(t) = exp(s_x(t-d));$$

$$S_2(t) = exp(-s_x d);$$

$$gamma(t) = \frac{s_y S_1(t)}{1 + S_1(t)} + \frac{1 + (1 - s_y) S_2(t)}{1 + S_2(t)}$$



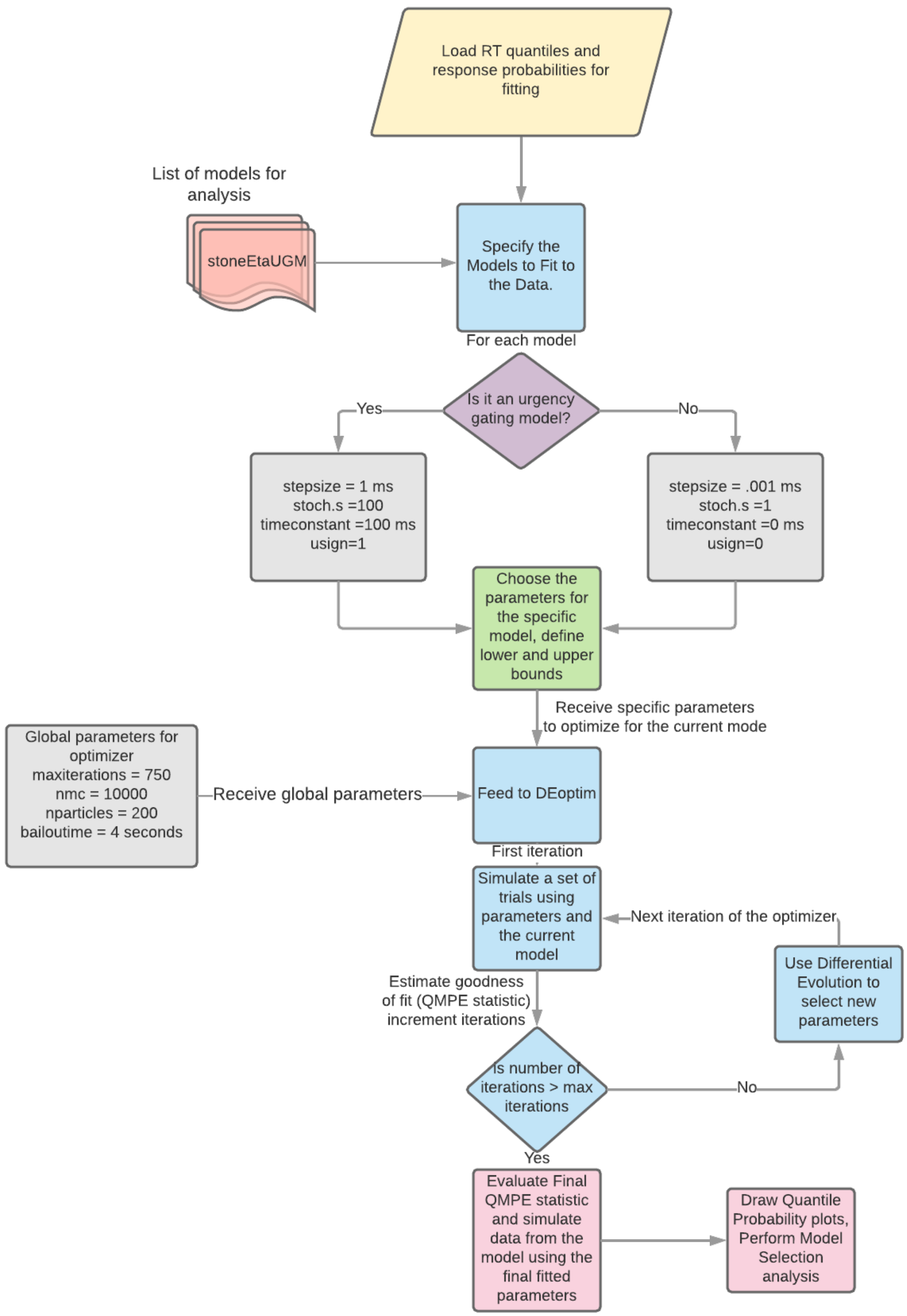
Using this toolbox.

This toolbox is an attempt by us (Chand and Guy) to provide the legions of researchers interested in various models of decision-making a simple and easily used toolbox for analysis of RT and discrimination accuracy behavior in decision-making tasks. The architecture of the toolbox is very simple. The choosing of which model to run and the lower and upper parameters and the

We assume that there is a reasonable working knowledge of R and C.

List of Available Models

Model	Description	Name in model list	Uses C Function Name	Involves Urgency
stone	Vanilla drift diffusion model for decision-making originally developed by stone (1960)	stone	stone	No
stoneEta	Drift diffusion model with variability in the drift rates. Drift rate variability is drawn from a normal distribution.	stoneEta	stoneEta	No
stoneEtaVarTer	Drift Diffusion Model with variability in the drift rates and variability in the residual time that is thought to reflect sensory and motor processing delays	stoneEtaVarTer	stoneEta	No
stoneEtaVarBaseVarTer	Drift Diffusion Model with variability in the drift rates, variability in the baseline state before evidence comes in and variability in the residual time that is thought to reflect sensory and motor processing delays	stoneEtaVarBaseVarTer	stoneEtaVarBase	No
ratcliff	Ratcliff model that involves variability in the baseline starting point and in the drift rate	ratcliff	ratcliff	No
ratcliffVarTer	Ratcliff model with baseline variability, drift rate and also variability in the residual time	ratcliff	ratcliff	No
stoneEtaUrgency	Drift diffusion model with variability in the drift rate and an urgency term	stoneEtaUrgency	stoneEtaUrgency	Yes
stoneEtaUrgencyVarTer	Drift diffusion model with			



Load RT quantiles and response probabilities for fitting
List of models for analysis
stoneEtaUGM
Specify the Models to Fit to the Data.
For each model
Is it an urgency gating model?
Yes
No
stepsize = 1 ms
stoch.s =100
timeconstant =100 ms
usign=1
stepsize = .001 ms
stoch.s =1
timeconstant =0 ms
usign=0
Choose the parameters for the specific model, define lower and upper bounds
Receive specific parameters to optimize for the current mode
Global parameters for optimizer
maxiterations = 750
nmc = 10000
nparticles = 200
bailoutime = 4 seconds
Receive global parameters
Feed to DEoptim
First iteration
Simulate a set of trials using parameters and the current model
Next iteration of the optimizer
Use Differential Evolution to select new parameters
Estimate goodness of fit (QMPE statistic) increment iterations
Is number of iterations > max iterations
No
Yes
Evaluate Final QMPE statistic and simulate data from the model using the final fitted parameters
Draw Quantile Probability plots, Perform Model Selection analysis

Figure 1: