CS610 (2017) Homework 2, Due 8th September

August 29, 2017

Write down the names of your collaborators. Please follow the honor code as outlined on the class webpage. Please submit a single pdf that contains typed up/scanned answers. Using Latex is strongly advised. For each problem, besides writing down the algorithm, please write down 1) a clear analysis of complexity 2) a clear claim of correctness and 3) a proof of the claim.

1. Give an efficient algorithm that takes as input a directed acyclic graph $G = (V, E)$, as well as two vertices $s, t \in V$, and outputs the number of different directed paths from s to t.

2. Suppose we are given a set of numbers $x_1, \ldots x_n$. Basic arithmetic operations on these take unit time. Define the quantity μ_∞ as follows:

$$\mu_\infty = \max_i |x_i - \mu|.$$

   (a) Give an example (of $x_1, \ldots x_n$) where μ_∞ is different from median and mean of the numbers.

   (b) Give an $O(n)$ algorithm for computing μ_∞.

3. KT Chapter 5, problem 1 (database access).

4. KT Chapter 5, problem 2 (sorting).

5. KT Chapter 5, problem 4 (calculating force).

6. KT Chapter 5, problem 5 (hidden surface removal).

7. Given two sets of integers X and Y, we define the Minkowski sum $X + Y$ as the following *set* of integers : $\{x + y, x \in X, y \in Y\}$. A naive algorithm would need $O(n^2 \log n)$ time (why?) in order to calculate the Minkowski sum of two set if both are of size n.

   Let M is the largest absolute value of any of integer in $X \cup Y$. Give an algorithm to find out the Minkowski sum in $O(M \log M)$ time.