Optimal Control for Remote Patient Monitoring with Multidimensional Health Representations

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***Abstract*— Selecting the right monitoring level in Remote Patient Monitoring (RPM) systems for e-healthcare is crucial for balancing patient outcomes, various resources, and patient's quality of life. A prior work has used one-dimensional health representations, but patient health is inherently multidimensional and typically consists of many measurable physiological factors. In this paper, we introduce a multidimensional health state model within the RPM framework and use dynamic programming to study optimal monitoring strategies. Our analysis reveals that the optimal control is characterized by switching curves (for two-dimensional health states) or switching hyper-surfaces (in general): patients switch to intensive monitoring when health measurements cross a specific multidimensional surface. We further study how the optimal switching curve varies for different medical conditions and model parameters. This finding of the optimal control structure provides actionable insights for clinicians and aids in resource planning. The tunable modeling framework enhances the applicability and effectiveness of RPM services across various medical conditions.**

I. Introduction

Remote Patient Monitoring (RPM) is becoming an increasingly integral part of modern healthcare, enabling continuous observation of patients within their daily environments and enhancing both their quality of life and healthcare outcomes [1]–[3]. Advances in wearable medical devices, such as continuous glucose monitors [4] and smartwatches with vital sign monitoring capabilities [5], [6] among others have facilitated the collection and transmission of extensive health data for real-time analysis and timely medical interventions.

A significant challenge in RPM is determining the optimal intensity of patient monitoring. While intensive monitoring can lead to earlier detection of health issues and prompt responses, it may also cause patient stress or fatigue due to its invasive nature [7], reduce device battery life [6] and increase other costs. Conversely, less intensive monitoring is less intrusive to the patient but might not provide sufficient data for timely interventions. In a recent work [8], we addressed this trade-off using a single dimensional health state model and identified that threshold policies, where patients switch to intensive monitoring when their health falls below a certain threshold, are optimal. However, in reality, a patient's health state is inherently multidimensional, encompassing various physiological factors that clinicians use to make decisions [9].

Given the practical limitations of one-dimensional health states, in this paper, we extend our initial work [8] by introducing a realistic multidimensional representation of patient health states within the RPM framework. We develop a dynamic programming approach to determine optimal monitoring strategies in this more complex setting. Our analysis reveals that controls characterized by *switching curves* (in two dimensions) or *hyper-surface* (in general) are optimal when considering multidimensional health states. That is, there exist switching curves/hyper-surfaces within the multidimensional health space such that patients transition to intensive monitoring when their health indicators fall below this switching surface and return to ordinary monitoring when they improve. This finding provides clinicians with actionable guidelines for adjusting monitoring intensity based on a comprehensive multidimensional view of patient health.

The implications of the model and its analysis are significant for both patient care and resource management. By incorporating tunable parameters—such as health improvement probabilities, monitoring options, and invasiveness costs—the framework can be adapted to specific medical conditions and tailored to individual patient needs. Furthermore, although not explicitly modeled, our approach allows for estimating the resources required to effectively manage patient populations, given the known time commitments associated with different levels of care. The rest of the paper is organized as follows: In Section II, we present the multidimensional RPM service model and describe the evolution of the patient's health state under different monitoring strategies. Section III discusses critical health states and their impact on the optimal policy. Section IV provides numerical investigations of the optimal monitoring control, and Section V concludes the paper with potential extensions.

II. The Remote Patient Monitoring Model

Consider a patient whose health condition is modeled as an $n-$dimensional **health state** $\boldsymbol{h} = (h^{(1)}, \ldots, h^{(n)}) \in \mathcal{H} := \{0, 1, \ldots, H\}^n$. At each time $t \in \{0, 1, \ldots\}$, the patient's health state is given by $\boldsymbol{h}_t \in \mathcal{H}$. The remote patient monitoring (RPM) service places the patient in a monitoring state $m_t \in \mathcal{M} = \{o, i\}$ in each time period t, abbreviated

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to **monitoring state**, where o denotes ordinary monitoring and i intensive monitoring. Thus, one can view the joint monitoring and patient state $s_t \coloneqq (m_t, \boldsymbol{h}_t) \in \mathcal{M} \times \mathcal{H} =: \mathcal{S}$ as the system or service state at time t.

The n-dimensions of the patient's health state correspond to different **health measurements** monitored by the e-health service. For example, a program for Type 1 Diabetes management may have patients wear continuous glucose monitors (CGMs) and examine multiple measurements, such as time in range (the percentage of time glucose levels remained between 70-180mg/dl) and time with clinically significant hypoglycemia (the percentage of time glucose levels were below 54 mg/dl) [10], [11]. A 'higher' patient health state $\boldsymbol{h} \in \mathcal{H}$ corresponds to the patient having better health[1]. Here 'higher' is defined component-wise, i.e., for two health states $\boldsymbol{h}$ and $\boldsymbol{h}'$, with $h'^{(j)} = h^{(j)}$ for all $j \neq k$ and $h'^{(k)} > h^{(k)}$, the patient is said to have better health in $\boldsymbol{h}'$ than in $\boldsymbol{h}$.

In particular, there exist **critical** health states in the sense that, when the patient drifts into those 'worse' states, they go beyond the scope of the current e-health service; and other emergency and/or more severe medical interventions are required, which are outside the scope of this service. For example, a person with diabetes experiencing a severe hypoglycemic episode may have to go to the emergency department or hospital [12]. When the patient enters a critical health state under any monitoring state i or o, the service evolution stops, as other medical measures/interventions are initiated. We denote the set of these critical health states by $\mathcal{H}_C$. A simple example of these critical sets could just be the origin or a hypercube around origin. We discuss more such critical sets, with their medical relevance, in the next section. The origin $(0, \ldots, 0)$ is always a part of $\mathcal{H}_C$.

An advantage of our model is that the costs can be interpreted from multiple perspectives. But in this section, we first take the patient's quality of life point of view when defining the various costs. Under ordinary monitoring, the patient incurs a constant cost $C_o \geq 0$ at any state (o, h) with $\boldsymbol{h} \in \mathcal{H}$. Correspondingly, under intensive monitoring, the patient incurs a constant cost $C_i \geq 0$ at any state (i, h) with $\boldsymbol{h} \in \mathcal{H}$. These costs reflect the invasiveness of the monitoring process to the patient's everyday lifestyle and quality of life. Of special interest are the critical health states, where this model ceases to apply. In such states, a cost of C_c is incurred.

A. Markov Evolution

We model the system as a controlled Markov chain. Such models are often used for medical decision making [13], [14]. Given several inherent complexities, we try to stay as simple as possible, yet still capture the essence of the problem and get insights into its solution. At the beginning of every time period t, the service takes the decision/action (control) to either keep the monitoring state the same (as in the previous time period) or switch it to the alternate monitoring state. Formally, the decision/action space is $\mathcal{A} = \{o, i\}$ and each (state, action) pair is associated with a cost given by the function $c : \mathcal{S} \times \mathcal{A} \mapsto \mathbb{R}^+$. The transition probabilities are given by $p(s'|s, a)$ where $s', s \in \mathcal{S}$ and $a \in \mathcal{A}$.

We explain the evolution in the two-dimensional plane (i.e., $n = 2$), helping us simplify the notation and better visualize the optimal control as discussed in Section IV. The definitions can easily be extended to higher dimensions. The health state is then denoted as $\boldsymbol{h} = (h^{(x)}, h^{(y)})$ and the system state is given by $(m, h^{(x)}, h^{(y)})$ where $m \in \mathcal{M}$. The cost function and transition probabilities are given below. For simplicity, we focus only on the states lying inside (and not on) the boundary. The general transition probabilities can be understood using Figure 1 and have been formally written in Appendix I-A.

1. At critical health states $\boldsymbol{h} \in \mathcal{H}_C$ —

No action is taken with the the service ceasing operation. A cost of C_c is incurred.

2. When $\boldsymbol{h} \notin \mathcal{H}_C$ and $1 \leq h^{(x)}, h^{(y)} \leq H - 1$ —

(a) *Ordinary Monitoring (m=o), no Switching (a=o):* Does not induce a monitoring change. Starting at state $(o, h^{(x)}, h^{(y)})$, the next state is:
- i) $(o, h^{(x)} + 1, h^{(y)})$ with probability $\lambda_{o,x}$
- ii) $(o, h^{(x)}, h^{(y)} + 1)$ with probability $\lambda_{o,y}$
- iii) $(o, h^{(x)} - 1, h^{(y)})$ with probability $\mu_{o,x}$
- iv) $(o, h^{(x)}, h^{(y)} - 1)$ with probability $\mu_{o,y}$

and a cost C_o is incurred.

(b) *Intensive Monitoring (m=i), no Switching (a=i):* Does not induce a monitoring change. Starting at state $(i, h^{(x)}, h^{(y)})$, the next state is:
- i) $(i, h^{(x)} + 1, h^{(y)})$ with probability $\lambda_{i,x}$
- ii) $(i, h^{(x)}, h^{(y)} + 1)$ with probability $\lambda_{i,y}$
- iii) $(i, h^{(x)} - 1, h^{(y)})$ with probability $\mu_{i,x}$
- iv) $(i, h^{(x)}, h^{(y)} - 1)$ with probability $\mu_{i,y}$

and a cost C_i is incurred.

(c) *Intensive Monitoring (m=i), with Switching (a=o):* Induces a switch to ordinary monitoring. The next state, respective transition probabilities, and the cost incurred is same as part (a): ordinary monitoring ($m = o$) with no switching ($a = o$).

(d) *Ordinary Monitoring (m=o), with Switching (a=i):* Induces a switch to intensive monitoring. The next state, respective transition probabilities, and the cost incurred is same as part (b): intensive monitoring ($m = i$) with no switching ($a = i$).

Here $\lambda_{o,x} + \lambda_{o,y} + \mu_{o,x} + \mu_{o,y} = 1$. The same relation holds for the probabilities corresponding to intensive monitoring. We assume that the decision to switch (or not) is made in the beginning of the timestep. As a result, the transition probabilities and costs are decided solely by the control and the next monitoring state.

We make the following *natural* assumptions.

Assumption 1. *We assume that:*

[1]If higher health measurement does not correspond to better health for the standard measurements we are using, we use the inverse measurement instead. For example, the standard measurement is time with clinically significant hypoglycemia. But for our model we use time without clinically significant hypoglycemia.

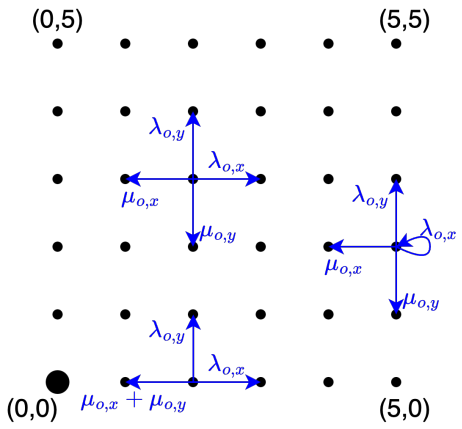


Fig. 1: The evolution for a two-dimensional model with $H = 5$. The transitions are given for states $(o, 2, 0), (o, 5, 2)$ and $(o, 2, 3)$ under the action o. Cost C_o is incurred in each case. The arrows are labeled with transition probabilities.

a) The transition probabilities satisfy: $\lambda_{i,x} \geq \lambda_{o,x}$ *and* $\lambda_{i,y} \geq \lambda_{o,y}$.

b) The costs satisfy: $0 \leq C_o \leq C_i \leq C_c$.

The first assumption 1.a) intuitively states that the patient's health improves faster under intensive monitoring, rather than under ordinary. Regarding assumption 1.b), it is naturally expected that $C_o \leq C_i$, as the patient's "annoyance" is higher under intensive monitoring/intervention than under ordinary. Further, given the severity of entering the critical state $\mathcal{H}_C$, it is naturally expected that $C_i \leq C_c$, and practically C_c is expected to be *much larger* than C_i.

The above model can be generalized in multiple directions. We can easily incorporate health state dependent costs and transition probabilities. Further, there can be tiers of progressively more intensive monitoring. We could also include different number of levels for each health measurement, i.e, each $h^{(k)} \in \{0, 1, \ldots, H^{(k)}\}$. Due to limited space, we assume constant costs, probabilities, etc., and explore the solution under these simplifications.

B. Optimal Monitoring Control

We study this problem under the *discounted cost* setting of the dynamic programming methodology [15]. Costs incurred t time periods into the future are discounted by a factor of γ^t with $0 < \gamma < 1$. Starting from state $s_0 = s \in \mathcal{M} \times \mathcal{H}$, the total expected (discounted) cost to be incurred is

$$\mathbb{E}\Big[\sum_{t=0}^{T-1} \gamma^t c(s_t, a_t) + \gamma^T C_c \Big| \; s_o = s\Big].$$

At each time t, control action a_t is taken while in state $s_t = (m_t, \boldsymbol{h}_t)$, at cost $c(s_t, a_t)$. The service ceases operation when the patient enters the critical state $\boldsymbol{h} \in \mathcal{H}_C$ at time T. At time T, the critical cost C_c is incurred, discounted to $\gamma^T C_c$. Thus, discounting by γ implicitly reflects the patient's desire to stay away from the critical health states for longer.

A (stationary) monitoring control $\pi(s)$ is a maps each state $s = (m, h) \in \mathcal{M} \times \mathcal{H}$ to a control action $a \in \mathcal{A} = \{o, i\}$ taken at that state. The value function $V_\pi(s)$ for control $\pi(s)$ is the total expected (discounted) cost the system will incur when it starts from state s at time $t = 0$, i.e.,

$$V_\pi(s) = \mathbb{E}\left[\sum_{t=0}^{T-1} \gamma^t c\Big(s_t, \pi(s_t)\Big) + \gamma^T C_c \;\Big|\; s_0 = s\right],$$

and satisfies the dynamic programming equation [15]

$$V_\pi(s) = c\Big(s, \pi(s)\Big) + \gamma \sum_{s' \in \mathcal{S}} \mathbb{P}\Big(s' \mid s, \pi(s)\Big) V_\pi(s'),$$

for all $s \in \mathcal{S} = \mathcal{M} \times \mathcal{H}$.

The goal is to find an optimal control π^* which minimizes V_π over all controls π, i.e., $V_{\pi^*}(s) \leq V_\pi(s)$ for every $s \in \mathcal{S}$ over all controls π. We define $V^*(s) := V_{\pi^*}(s)$ which satisfies the following dynamic programming equation [15].

$$V^*(s) = \min_{a \in \{o,i\}} \left\{ c(s, a) + \gamma \sum_{s' \in \mathcal{S}} \mathbb{P}\Big(s' | s, a\Big) V^*(s') \right\},$$

and can be solved numerically to yield the optimal control $\pi^*(s) = \pi^*(m, \boldsymbol{h})$, i.e., what optimal decision to take when the patient is in health state $\boldsymbol{h}$ under monitoring m. For the state transition probabilities and costs defined previously for the two-dimensional case, this equation unfolds into:

(i) At critical health states $\boldsymbol{h} \in \mathcal{H}_C$:

$$V^*(i, \boldsymbol{h}) = V^*(o, \boldsymbol{h}) = C_c.$$

(ii) For health states $\boldsymbol{h} \notin \mathcal{H}_C$, recall that the transition probabilities and cost incurred for a given time step is decided by the action taken and the new monitoring state (and not the current one). This results in having $V^*(i, \boldsymbol{h}) = V^*(o, \boldsymbol{h})$. For $1 \leq h^{(x)}, h^{(y)} \leq H - 1$,

$$\begin{aligned}
&V^*(i, \boldsymbol{h}) = V^*(o, \boldsymbol{h}) \\
&= \min \Bigg\{ C_i + \gamma \Big[\lambda_{i,x} V^* \left(i, h^{(x)} + 1, h^{(y)}\right) \\
&+ \lambda_{i,y} V^* \left(i, h^{(x)}, h^{(y)} + 1\right) + \mu_{i,x} V^* \left(i, h^{(x)} - 1, h^{(y)}\right) \\
&+ \mu_{i,y} V^* \left(i, h^{(x)}, h^{(y)} - 1\right) \Big], \\
&C_o + \gamma \Big[\lambda_{o,x} V^* \left(i, h^{(x)} + 1, h^{(y)}\right) \\
&+ \lambda_{o,y} V^* \left(i, h^{(x)}, h^{(y)} + 1\right) + \mu_{o,x} V^* \left(i, h^{(x)} - 1, h^{(y)}\right) \\
&+ \mu_{o,y} V^* \left(i, h^{(x)}, h^{(y)} - 1\right) \Big] \Bigg\} \qquad (1)
\end{aligned}$$

For higher clarity, we focus only on the states lying inside (and not on) the boundary in the above dynamic programming equation. The complete equation has been given in Appendix I-A.

In the $\min\{\cdot, \cdot\}$ above, the first term indicates the control i, i.e., the patient is in intensive monitoring in the next timestep, while the second term indicates the control o.

III. Critical Health States

Critical health states are states where the patient's health has deteriorated significantly and the patient's health care is no longer within the scope of the current service model. Instead, other (emergency and more severe) medical interventions are required at that point and, hence, a large cost C_c is incurred when the patient reaches these states.

In our earlier model [8], where the health states were one-dimensional, the only critical health state was the state $h = 0$. But in this case, with our multidimensional health state model, the set of critical health states $\mathcal{H}_C$ can have diverse structures. As we discuss below, the set of critical states is dependent on the medical condition this model is applied to and is closely related to how different health measurements (dimensions of our model) interact. Choosing an appropriate set of critical health states is vital, as the structure of the optimal control is heavily influenced by this set. We discuss this further in the next section.

We consider the class of $\mathcal{H}_C$ of the form $\mathcal{H}_C = \{\boldsymbol{h} \mid g(\boldsymbol{h}) \leq c\}$, where $g : \mathcal{H} \mapsto \mathbb{R}$ and $c \in \mathbb{R}$. Intuitively, these are sets where the health goes below a certain threshold, defined by a function of the health states $g(\cdot)$. The following are some exemplary sets of critical health states, which we consider in this short paper. In Figure 2, the critical sets are marked with larger black dots. Note that these are just a few examples of the critical sets that can be represented using our general model. As we move to higher dimensions, there can be more complex sets which can be used to model medical conditions with a higher number of measurement types.

(a) **Any health measurement is very low**: In this case, the patient's health is considered critical if even a single measurement becomes 'very low' (normalized to 0 in our paper). For example, a person with diabetes experiencing a severe hypoglycemic episode would be considered to be in a critical state [12] even if they otherwise had a high time in range. Mathematically this can be represented as the set $\mathcal{H}_C = \{\boldsymbol{h} \mid \min_{1\leq i\leq n}\{h^{(i)}\} = 0\}$. In the two-dimensional setting, this set would be the states along the x and y axes (Figure 2a).

(b) **Health measurements are collectively low**: The patient's health is considered critical in this case when the measurements are together sufficiently low. This case can have multiple structures, but we consider the following two sets which correspond to the ℓ_1 and ℓ_∞ norm, respectively - $\mathcal{H}_C = \{\boldsymbol{h} \mid \sum_{1\leq i\leq n} h^{(i)} \leq c\}$ and $\mathcal{H}_C = \{\boldsymbol{h} \mid \max_{1\leq i\leq n}\{h^{(i)}\} \leq c\}$. In the two-dimensional setting, these correspond to a triangle (Figure 2b) and a square (Figure 2c) cornered at origin, respectively. Note that a critical set consisting of only the origin is a special case of this set.

(c) **Combination of the above sets**: A critical set applicable to a wide range of medical conditions is a combination of the above two sets. An example of such a set could be $\mathcal{H}_C = \{\boldsymbol{h} \mid \min_{1\leq i\leq n}\{h^{(i)}\} = 0 \text{ or } \sum_{1\leq i\leq n} h^{(i)} \leq c\}$. In the two-dimensional setting, this set would include the axes, along with a triangle cornered at origin (the larger black dots in Figure 2d). A practical example could be if we have defined health states corresponding to abnormal heart rate and blood pressure for heart failure patients [16].

IV. Optimal control: The Switching Curve

In this section, we discuss the optimal control obtained by numerically analyzing the dynamic programming equation (1) for various critical health sets and model parameters. While we discuss some analytical results to help build intuition, various theoretical results for the critical sets are not discussed, given the limited space of this short paper. We postpone these results to the journal version of the paper.

We first make an important observation about the optimal control. Based on (1), we observe that $V^*(i, \boldsymbol{h}) = V^*(o, \boldsymbol{h})$ for all health states $\boldsymbol{h}$. This implies that $\pi^*(o, \boldsymbol{h}) = \pi^*(i, \boldsymbol{h})$. Hence under the optimal control, the service chooses the same (monitoring) action irrespective of the current monitoring state. Therefore, we introduce the simpler notation $\pi^*(\boldsymbol{h}) = \pi^*(i, \boldsymbol{h}) = \pi^*(o, \boldsymbol{h})$ denoting the optimal monitoring at state $\boldsymbol{h}$. For example, suppose that $\pi^*(\boldsymbol{h}) = i$ for some health state $\boldsymbol{h}$. If the current state is $(o, \boldsymbol{h})$, then the optimal control is to switch to intensive monitoring, and if the current state is $(i, \boldsymbol{h})$, then the optimal control is to stay in intensive monitoring.

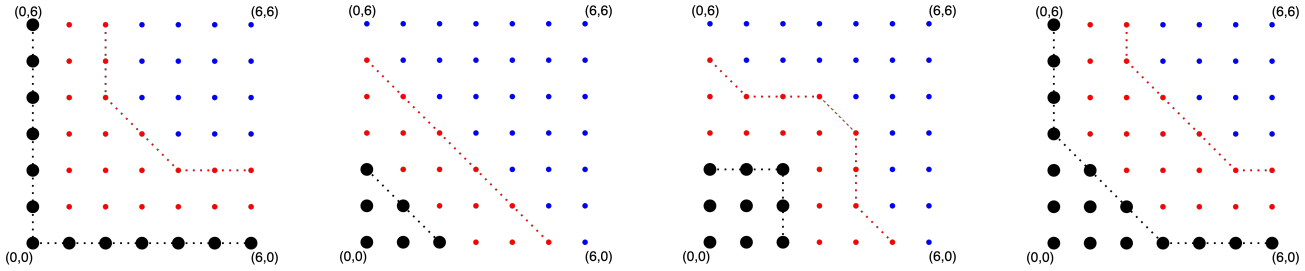
The above observation helps us better visualize the optimal control. In this section, we visualize the optimal control for a 2-dimensional health state model using a 2D grid. In Figures 2 and 3, the critical health states are marked with larger black dots. Ordinary and intensive monitoring are optimal for health sets marked with *blue* and *red*, respectively.

Before moving to optimal control for specific critical health sets and parameters, we first make another important numerical observation. For our class of $\mathcal{H}_C = \{\boldsymbol{h} \mid g(\boldsymbol{h}) \leq c\}$, we observe that the optimal control is a threshold policy $\pi_{t,f}$, characterized by some function $f : \mathcal{H} \mapsto \mathbb{R}$. Then $\pi_{t,f}(\boldsymbol{h}) = i$ if $f(\boldsymbol{h}) \leq 0$, and $\pi_{t,f}(\boldsymbol{h}) = o$ if $f(\boldsymbol{h}) > 0$. For example, consider Figure 2b. In that case, $f(\boldsymbol{h}) = h^{(x)} + h^{(y)} - 5$ and the optimal control is $\pi^*(\boldsymbol{h}) = i$ if $h^{(x)} + h^{(y)} \leq 5$ and o otherwise. A lower value for $f(\boldsymbol{h})$ implies that the patient is 'closer' to the critical set, and hence intensive monitoring may be preferred for state $\boldsymbol{h}$.

The curve $f(\boldsymbol{h}) = 0$ is called the **switching curve**. The monitoring control switches from intensive monitoring to ordinary as the patient health state crosses from the side of the curve closer to critical set to the other side. The switching curve is seen in Figures 2 and 3 as the red dotted boundary between the region of blue dots and the region of red dots. Note that in higher dimensions, the switching curve will be replaced by a switching hypersurface.

A. Impact of Critical Health States

We first present the structure of the optimal control for different critical health sets discussed in the previous section. In Figure 2, we consider 'symmetric' sets and parameters, i.e., the critical sets are symmetric in the two health measurements (dimensions) and the probabilities satisfy $\lambda_{o,x} =$



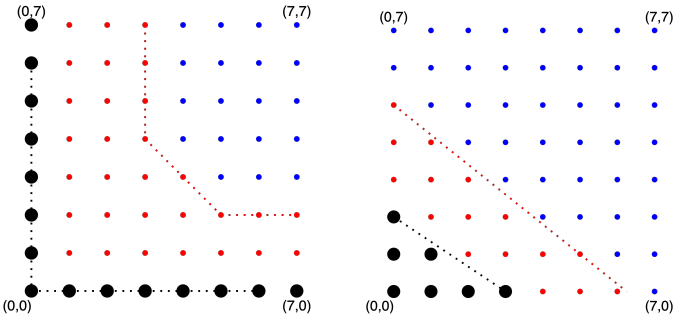
(a) The critical set comprises of the states on x- and y-axis.

(b) The critical set comprises of states in a triangle around origin.

(c) The critical set comprises of states in a square around origin.

(d) The critical set comprises of union of sets in (a) and (b).

Fig. 2: Optimal controls for a two-dimensional model with $H = 6$ for critical sets given by (a) $h^{(x)} = 0$ or $h^{(y)} = 0$, (b) $h^{(x)} + h^{(y)} \leq 2$, (c) $\max\{h^{(x)}, h^{(y)}\} \leq 2$, and (d) $h^{(x)} = 0$ or $h^{(y)} = 0$ or $h^{(x)} + h^{(y)} \leq 2$. Larger black dots represent critical health states, and dotted black line represents the boundary of the critical set. Intensive monitoring is optimal for states marked as red, and ordinary monitoring is optimal for states marked as blue. The dotted red line represents the *switching curve* $f(\boldsymbol{h}) = 0$. For each of the plots: $\gamma = 0.9, C_i = 1, C_o = 0, C_c = 35, \lambda_{o,x} = \lambda_{o,y} = 0.5 - \mu_{o,x} = 0.5 - \mu_{o,y} = 0.075$ and $\lambda_{i,x} = \lambda_{i,y} = 0.5 - \mu_{i,x} = 0.5 - \mu_{i,y} = 0.2$.



(a) The critical set comprises of the states on x- and y-axis.

(b) The critical set comprises of states in an asymmetric triangle around origin

Fig. 3: Optimal control for **asymmetric** two-dimensional models for critical sets given by (a) $h^{(x)} = 0$ or $h^{(y)} = 0$, and (b) $2h^{(x)} + 3h^{(y)} \leq 6$. For both plots: $\gamma = 0.9, C_i = 1, C_o = 0, C_c = 35$. For plot (a): $\lambda_{o,x} = \lambda_{o,y} = 0.5 - \mu_{o,x} = 0.5 - \mu_{o,y} = 0.1$ and $\lambda_{i,x} = 0.5 - \mu_{i,x} = 0.3$ and $\lambda_{i,y} = 0.5 - \mu_{i,y} = 0.25$. For plot (b): $\lambda_{o,x} = \lambda_{o,y} = 0.5 - \mu_{o,x} = 0.5 - \mu_{o,y} = 0.1$ and $\lambda_{i,x} = \lambda_{i,y} = 0.5 - \mu_{i,x} = 0.5 - \mu_{i,y} = 0.2$

$\lambda_{o,y} = 0.5 - \mu_{o,x} = 0.5 - \mu_{o,y}$ (analogous relation holds for transition probabilities for intensive monitoring).

The first case we discuss is the set where all points on the axes are considered critical states. Figure 2a shows the optimal control for this set. Closer to the origin, we observe that the optimal control assigns intensive monitoring to more health states, as the patient is at higher risk of reaching either axis (and hence a critical state). Further away from the origin, the patient's risk of reaching a critical state can be characterized by its distance to just the closer axis, and hence the switching curve is parallel to the axis.

This optimal control can also be understood using intuition gleaned from the one-dimensional system studied in [8]. In that work, we showed how the hitting time for a critical set (under the assumption of $H \uparrow \infty$) explains the optimal control. We showed that the function $\mathbb{E}[\gamma^{\tau(h)}]$ is closely related to the optimal control, where $\tau(h)$ denotes the hitting time for the critical health state starting at state h. In the present case, the time taken to hit a critical state is given by $\min\{\tau_x(\boldsymbol{h}), \tau_y(\boldsymbol{h})\}$ where $\tau_x(\boldsymbol{h})$ and $\tau_y(\boldsymbol{h})$ denote the time taken to hit x-axis and y-axis, respectively, starting at health state $\boldsymbol{h}$. We numerically observe that the level sets for $\mathbb{E}[\gamma^{\min\{\tau_x(\boldsymbol{h}), \tau_y(\boldsymbol{h})\}}]$ have a structure very similar to the optimal control we observe.

We next study the optimal control when the critical health state is a triangle cornered at origin, given by $h^{(x)} + h^{(y)} \leq c$. In this case (Figure 2b), we observe that the switching curve has the same form. We observe that the patients are in intensive monitoring when $h^{(x)} + h^{(y)} \leq k$, for some k. In this case, the hitting time for the critical set (starting at state $\boldsymbol{h}$) is a function of $(h^{(x)} + h^{(y)} - c)$ and hence the level sets are of the form $h^{(x)} + h^{(y)} = k$. We have the following result for the asymptotic case which shows that the optimal control in this case will always be of this form.

Theorem 1. *Under the assumption that $H \uparrow \infty$, for a critical set of the form $\mathcal{H}_C = \{\boldsymbol{h} \mid h^{(x)} + h^{(y)} \leq c\}$, the optimal control is of the form $\pi_{t,f}$, where $f(\boldsymbol{h}) = h^{(x)} + h^{(y)} - k$ for some $k \geq c$.*

Note that this result holds for any set of probabilities, and does not require the symmetric condition we assumed above. The proof for this theorem follows by looking at this two-dimensional grid as a one-dimensional random walk, where all health states in the set $A^{(k)} = \{\boldsymbol{h} \mid h^{(x)} + h^{(y)} = k\}$ are considered as state k of the one-dimensional random walk. A proof sketch has been presented in Appendix I-B.

We next study the critical set given by the combination of the above two sets, i.e., when all states on the axes and

states in a triangle around the origin are critical health states. Figure 2d gives the optimal control for this set. The shape of the switching curve is very similar to that in Figure 2a, but intensive monitoring is optimal for a larger region around the origin. Finally we present the optimal control for the critical health state corresponding to $\max\{h^{(x)}, h^{(y)}\} \leq c$ (Figure 2c). The switching curve in this case is structurally more complex than the previous cases. Even though the structure is complex, different pieces of the switching curve are explainable using the hitting time intuition.

B. Asymmetry Between Dimensions

In the previous subsection, we only considered 'symmetric' critical sets and parameters. Now we discuss how the optimal control is affected when the model is asymmetric.

In Figure 3a, we plot the optimal control for the two-dimensional model where health states along the axes are critical, and $\lambda_{i,x} - \lambda_{o,x} > \lambda_{i,y} - \lambda_{o,y}$. This implies that, under intensive monitoring, the probability of patient's health measurement $h^{(x)}$ improving is higher than that for $h^{(y)}$. In this case, the patient has a high incentive to pay the cost for intensive monitoring even when they are at 'mild' risk of reaching the critical state corresponding to $h^{(x)} = 0$. On the other hand, the advantage for measurement $h^{(y)}$ (in terms of probability of improvement) of intensive monitoring is lower, and hence the patient has an incentive to pay the cost for intensive monitoring only when their $h^{(y)}$ is very low.

The next plot (Figure 3b) considers an asymmetric critical set. Here $\mathcal{H}_C = \{\boldsymbol{h} \mid 2h^{(x)} + 3h^{(y)} \leq 6\}$. In this case, switching curve is $f(\boldsymbol{h}) = 4h^{(x)} + 5h^{(y)} - 25 = 0$. Note that the slope for the switching curve $f(\cdot)$ is not the same as the slope for the critical set function $g(\cdot)$. But we observed that the slopes of functions $g(\boldsymbol{h})$ and $f(\boldsymbol{h})$ are close under most cases for a large set of asymmetric critical sets.

C. Variation With Parameters

Finally, we report the impact of different model parameters on the optimal control. The observations here are very similar to that in the one-dimensional case [8]. Due to the limited space in this paper, we do not include plots for these results.

Increasing the discount factor γ, increasing the cost ratio of C_c/C_i (keeping C_o fixed at zero), or increasing the probabilities $\lambda_{i,x}$ or $\lambda_{i,y}$ (keeping $\lambda_{o,x}$ and $\lambda_{o,y}$ as fixed) have similar effects on the optimal control. Each of these *push* the switching curve away from the origin, with the patient staying under intensive monitoring for a larger set of health sets. In the first two cases, the patient incurs a higher discounted cost on reaching the critical state, and hence the patient has a higher incentive to stay in intensive monitoring. In the last case, the probability of the patient's health improving under intensive monitoring improves, incentivizing the patient to stay under intensive monitoring for longer.

V. Conclusions

We develop a model which introduces multidimensional health representations, extending our prior work [8] which considered one-dimensional health states. The monitoring service control decides whether to place the patient under ordinary or intensive monitoring, given their health state. Optimal monitoring control is then studied using a dynamic programming approach. Our observations show that the optimal control, is characterized by a *switching curve*, such that patients transition to intensive monitoring when their health is below the switching curve.

An important extension would be to analytically study the optimal control for the different cases covered in this paper. This would help develop intuition about the optimal control and characterize it better. Other possible future directions include incorporating health state dependent costs and transition probabilities or considering a multi-tiered monitoring system with tiers of progressively more intensive monitoring.

References

[1] F. A. C. d. Farias, C. M. Dagostini, Y. d. A. Bicca, V. F. Falavigna, and A. Falavigna, "Remote patient monitoring: a systematic review," *Telemedicine and e-Health*, vol. 26, no. 5, pp. 576–583, 2020.

[2] L. P. Malasinghe, N. Ramzan, and K. Dahal, "Remote patient monitoring: a comprehensive study," *Journal of Ambient Intelligence and Humanized Computing*, vol. 10, pp. 57–76, 2019.

[3] A. Zinzuwadia, J. M. Goldberg, M. A. Hanson, and J. D. Wessler, "Continuous cardiology: the intersection of telehealth and remote patient monitoring," in *Emerging Practices in Telehealth*. Elsevier, 2023, pp. 97–115.

[4] I. Lee, D. Probst, D. Klonoff, and K. Sode, "Continuous glucose monitoring systems-current status and future perspectives of the flagship technologies in biosensor research," *Biosensors and Bioelectronics*, vol. 181, p. 113054, 2021.

[5] M. Masoumian Hosseini, S. T. Masoumian Hosseini, K. Qayumi, S. Hosseinzadeh, and S. S. Sajadi Tabar, "Smartwatches in healthcare medicine: assistance and monitoring; a scoping review," *BMC Medical Informatics and Decision Making*, vol. 23, no. 1, p. 248, 2023.

[6] Y. B. David, T. Geller, I. Bistritz, I. Ben-Gal, N. Bambos, and E. Khmelnitsky, "Wireless body area network control policies for energy-efficient health monitoring," *Sensors*, vol. 21, no. 12, p. 4245, 2021.

[7] B. W. Heckman, A. R. Mathew, and M. J. Carpenter, "Treatment burden and treatment fatigue as barriers to health," *Current opinion in psychology*, vol. 5, pp. 31–36, 2015.

[8] S. Chandak, I. Thapa, N. Bambos, and D. Scheinker, "Tiered service architecture for remote patient monitoring," *arXiv preprint arXiv:2406.18000*, 2024.

[9] N. Bashi, M. Karunanithi, F. Fatehi, H. Ding, and D. Walters, "Remote monitoring of patients with heart failure: An overview of systematic reviews," *Journal of Medical Internet Research*, vol. 19, no. 1, p. e18, 2017. [Online]. Available: https://www.jmir.org/2017/1/e18

[10] D. Scheinker, P. Prahalad, R. Johari, D. M. Maahs, and R. Majzun, "A new technology-enabled care model for pediatric type 1 diabetes," *NEJM Catalyst*, vol. 3, no. 5, p. CAT.21.0438, 2022.

[11] P. Prahalad, D. Scheinker, M. Desai, V. Y. Ding, F. K. Bishop, M. Y. Lee, J. Ferstad, D. P. Zaharieva, A. Addala, R. Johari *et al.*, "Equitable implementation of a precision digital health program for glucose management in individuals with newly diagnosed type 1 diabetes," *Nature Medicine*, pp. 1–9, 2024.

[12] R. G. McCoy, K. J. Lipska, H. K. Van Houten, and N. D. Shah, "Association of cumulative multimorbidity, glycemic control, and medication use with hypoglycemia-related emergency department visits and hospitalizations among adults with diabetes," *JAMA Network Open*, vol. 3, no. 1, p. e1919099, 2020.

[13] L. N. Steimle and B. T. Denton, *Markov Decision Processes for Screening and Treatment of Chronic Diseases*. Cham: Springer International Publishing, 2017, pp. 189–222.

[14] O. Alagoz, H. Hsu, A. J. Schaefer, and M. S. Roberts, "Markov decision processes: a tool for sequential decision making under uncertainty," *Medical Decision Making*, vol. 30, no. 4, pp. 474–483, 2010.

[15] D. Bertsekas, *Dynamic programming and optimal control*. Athena scientific, 2012, vol. II.

[16] P. Ponikowski, I. Spoletini, A. J. Coats, M. F. Piepoli, and G. M. Rosano, "Heart rate and blood pressure monitoring in heart failure," *European Heart Journal Supplements*, vol. 21, no. Suppl M, pp. M13–M16, 2019.

APPENDIX I

A. Transition Probabilities and Dynamic Programming Equation

We first discuss the transition probabilities and the cost function for our model -

1. At critical health states $\boldsymbol{h} \in \mathcal{H}_C$ —

No action is taken with the the service ceasing operation. A cost of C_c is incurred.

2. When $\boldsymbol{h} \notin \mathcal{H}_C$ and $1 \leq h^{(x)}, h^{(y)} \leq H-1$ —

(a) *Ordinary Monitoring (m=o), no Switching (a=o):* Does not induce a monitoring change, Starting at state $(o, h^{(x)}, h^{(y)})$, the next state with their respective transition probabilities are:
   i) $(o, \min\{h^{(x)}+1, H\}, h^{(y)})$ w.p. $\lambda_{o,x}$
   ii) $(o, h^{(x)}, \min\{h^{(y)}+1, H\})$ w.p. $\lambda_{o,y}$
   iii) $(o, h^{(x)}-1, h^{(y)})$ w.p. $\mu_{o,x}\mathbb{1}_{\{h^{(x)}\neq 0\}} + \mu_{o,y}\mathbb{1}_{\{h^{(y)}=0\}}$
   iv) $(o, h^{(x)}, h^{(y)}-1)$ w.p. $\mu_{o,y}\mathbb{1}_{\{h^{(y)}\neq 0\}} + \mu_{o,x}\mathbb{1}_{\{h^{(x)}=0\}}$

   and a cost C_o is incurred. The $\min\{h^{(x)}+1, H\}$ above is used to account for the boundary case of $h^{(x)} = H$ since H is the highest health state. Similarly, the $\mathbb{1}_{\{h^{(x)}\neq 0\}}$ is used to account for the boundary case of $h^{(x)} = 0$ (see Figure 1).

(b) *Intensive Monitoring (m=i), no Switching (a=o):* Does not induce a monitoring change. Starting at state $(o, h^{(x)}, h^{(y)})$, the next state with their respective transition probabilities are:
   i) $(o, \min\{h^{(x)}+1, H\}, h^{(y)})$ w.p. $\lambda_{i,x}$
   ii) $(o, h^{(x)}, \min\{h^{(y)}+1, H\})$ w.p. $\lambda_{i,y}$
   iii) $(i, h^{(x)}-1, h^{(y)})$ w.p. $\mu_{i,x}\mathbb{1}_{\{h^{(x)}\neq 0\}} + \mu_{i,y}\mathbb{1}_{\{h^{(y)}=0\}}$
   iv) $(i, h^{(x)}, h^{(y)}-1)$ w.p. $\mu_{i,y}\mathbb{1}_{\{h^{(y)}\neq 0\}} + \mu_{i,x}\mathbb{1}_{\{h^{(x)}=0\}}$

   and a cost C_i is incurred.

(c) *Intensive Monitoring (m=i), with switching (a=o):* Induces a switch to ordinary monitoring. The next state, respective transition probabilities, and the cost incurred is same as part (a): ordinary monitoring ($m = o$) with no switching ($a = o$).

(d) *Ordinary Monitoring (m=o), with switching (a=i):* Induces a switch to intensive monitoring. The next state, respective transition probabilities, and the cost incurred is same as part (b): intensive monitoring ($m = i$) with no switching ($a = i$).

Next, we give the dynamic programming equations satisfied by the optimal control $V^*(\cdot,\cdot)$.

1. At critical health states $\boldsymbol{h} \in \mathcal{H}_C$ —

$$V^*(i, \boldsymbol{h}) = V^*(o, \boldsymbol{h}) = C_c.$$

2. When $\boldsymbol{h} \notin \mathcal{H}_C$ and $1 \leq h^{(x)}, h^{(y)} \leq H-1$ —

$$\begin{aligned}
&V^*(i, \boldsymbol{h}) = V^*(o, \boldsymbol{h}) \\
&= \min\Bigg\{ C_i + \gamma\Bigg[\lambda_{i,x} V^*\left(i, \min\{h^{(x)}+1, H\}, h^{(y)}\right) \\
&+ \lambda_{i,y} V^*\left(i, h^{(x)}, \min\{h^{(y)}+1, H\}\right) \\
&+ \left(\mu_{i,x}\mathbb{1}_{\{h^{(x)}\neq 0\}} + \mu_{i,y}\mathbb{1}_{\{h^{(y)}=0\}}\right) V^*\left(i, h^{(x)}-1, h^{(y)}\right) \\
&+ \left(\mu_{i,y}\mathbb{1}_{\{h^{(y)}\neq 0\}} + \mu_{i,x}\mathbb{1}_{\{h^{(x)}=0\}}\right) V^*\left(i, h^{(x)}, h^{(y)}-1\right)\Bigg], \\
&C_o + \gamma\Bigg[\lambda_{o,x} V^*\left(o, \min\{h^{(x)}+1, H\}, h^{(y)}\right) \\
&+ \lambda_{o,y} V^*\left(o, h^{(x)}, \min\{h^{(y)}+1, H\}\right) \\
&+ \left(\mu_{o,x}\mathbb{1}_{\{h^{(x)}\neq 0\}} + \mu_{o,y}\mathbb{1}_{\{h^{(y)}=0\}}\right) V^*\left(o, h^{(x)}-1, h^{(y)}\right) \\
&+ \left(\mu_{o,y}\mathbb{1}_{\{h^{(y)}\neq 0\}} + \mu_{o,x}\mathbb{1}_{\{h^{(x)}=0\}}\right) V^*\left(o, h^{(x)}, h^{(y)}-1\right)\Bigg]\Bigg\}
\end{aligned}$$

B. Proof for Theorem 1

Proof. We work under the asymptotic condition of $H \uparrow \infty$ for this proof. Recall that the one-dimensional model considered in [8] considered health state $h = 0$ as the critical set and defined the parameters γ, λ_o and λ_i. These are the discount factor, probability of health improving under ordinary monitoring, and health improving under intensive monitoring, respectively. Theorem 1 and 2 from [8] together show that the optimal control is always a threshold policy, i.e., there exists $\bar{h}$, such that $\pi^*(h) = i$ for $h \leq \bar{h}$ and $\pi^*(h) = o$ for $h > \bar{h}$. Note that the control where the optimal control at all states is ordinary monitoring is a special case of the threshold policy with $\bar{h} = 0$.

Now in our two-dimensional model, consider the sets $A^{(k)} = \{\boldsymbol{h} \mid h^{(x)} + h^{(y)} = k\}$. Then $\mathbb{P}(\boldsymbol{h}_{t+1} \in A^{(k+1)} \mid \boldsymbol{h}_t \in S^{(k)}, m_t = o) = \lambda_{o,x} + \lambda_{o,y}$. Similarly, $\mathbb{P}(\boldsymbol{h}_{t+1} \in A^{(k-1)} \mid \boldsymbol{h}_t \in S^{(k)}, m_t = o) = \mu_{o,x} + \mu_{o,y}$. Similar transitions are defined for intensive monitoring with analogous probabilities.

Now define $\lambda_i' = \lambda_{i,x} + \lambda_{i,y}$ and $\lambda_o' = \lambda_{o,x} + \lambda_{o,y}$. Suppose the set of health sets $A^{(c)} = \{\boldsymbol{h} \mid h^{(x)} + h^{(y)} = c\}$ is defined as the health set $h' = 0$. Then sets of health states $A^{(k)}$ are given by $h' = k - c$ for $k \geq c$. Then our two-dimensional model can be represented using the one-dimensional model with parameters $\gamma, \lambda_o', \lambda_i'$ and with health states given by h'. Then applying Theorems 1 and 2 from [8] gives us the result that the optimal control in the one-dimensional case can be represented using a threshold. Let that threshold in the one-dimensional case be $\bar{h}'$, then the optimal control in the two-dimensional case is $\pi_{t,f}$ where $f(\boldsymbol{h}) = h^{(x)} + h^{(y)} - (\bar{h}' + c)$. This completes the proof for Theorem 1. □